Vendor-managed inventory (VMI) deteriorating item model taking into account carbon emissions

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Abstract: This research investigates the primary actions that generate carbon supply chain emission generated by the transport of deteriorating products. This study applies a vendor-managed inventory (VMI) model that considers deteriorating items with a single-setup-single-delivery (SSSD) system, and also a single-setup-multiple-delivery (SSMD) system. This study's managerial insights can help both researchers and businesses make inventory management decisions that reduce the total cost of processing these deteriorating commodities, as well as the total cost of carbon emissions. A numerical example optimizing the order quantity and order frequency for each manufacturing cycle is shown. Findings from this research can assist businesses in discovering an efficient inventory management approach that is crucial to lowering carbon emissions, particularly for deteriorating goods.

Keywords: Supply-chain; Vendor-managed inventory; Carbon emissions; Deteriorating products

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1. Introduction

Inventory control and delivery are the key carbon-emitting operations in a supply chain. [1] (Jiang et al., 2015). Moreover, carbon emissions have a detrimental effect on the environment. [2] (Palmer, 2007). As a result, proper supply chain strategy is crucial for a long-term business. Product degradation must be evaluated to correctly reflect a viable

scenario. (see for example Wee, 1993 [3]). Later, Lee and Ren (2011) [4] showed the advantages of considering vendor-managed inventory (VMI) systems, but they did not consider deteriorating items and carbon emissions. Therefore, the motivation for our paper is to establish an inventory control framework with a VMI system, considering carbon emissions, as well as taking into account product deterioration.

The VMI models developed by Coelho et al. (2013) [5] were found to reduce carbon emissions costs. Setak and Daneshfar (2014) [6] researched the vendor-managed inventory methodology and devised an economic order quantity model for 2-echelon supply chains, contrasting VMI with non-VMI supply networks. Then, Jiang et al. (2015) [1] devised an ecofriendly, single-manufacturer, single-distributor VMI model that considers carbon emissions as an additional objective. Mateen and Chatterjee (2015) [7] developed a VMI supply chain that minimized the total costs and carbon emissions. However, while all three studies provided valuable insights into the benefits of VMI, they did not consider deteriorating items in making a comprehensive assessment of an optimal transport policy. Our paper serves as a stepping stone by presenting a detailed evaluation of the VMI system for deteriorating products from the standpoints of cost reduction and environmental preservation. We investigate the tradeoff between reducing carbon emissions and minimizing transport costs and investigate the most efficient solution under varying scenarios.

Cárdenas-Barrón et al. (2012) [8] created an optimization approach to solve a VMI management policy for a multi-product as well as multi-constraint EOQ model. Sarkar et al. (2016) [9] created a three-stage supply chain model with a single-setup-single-delivery (SSSD) policy for varying emissions and transportation costs.

This study advances the mathematical formula by Daryanto et al. (2019) [10] to encompass vendor-managed inventory systems. The two modes of transportation are

contrasted. Our study incorporates an SSMD policy driven by the idea of just-in-time (JIT), and we balance our approach to maximize the combined overall cost for both participants by an optimal quantity of consecutive deliveries, despite their competing aims.

2.   Literature Review

Vendor-managed inventory is an order fulfillment strategy that incorporates data management and sharing technology. VMI entails coordination between suppliers and buyers, with the suppliers managing the inventory for the buyer's end. VMI also enables shared business objectives for both suppliers and buyers, resulting in outcomes such as greater inventory turnover, decreased inventory, improved efficiency, and higher revenue. Additionally, VMI focuses on developing a scalable restocking and shipping coordination strategy, and also the impact of data exchange effectiveness (Wong et al., 2009) [11]. Wong et al. (2009) also characterized supply chain collaboration as the consolidation of dispersed choices into a centralized decision-making mechanism. This decreases the stock kept by purchasers, hence increasing the combined profit margin for both the supplier and distributor. The supplier oversees the retailer's inventory and determines the duration and amount necessary in VMI. As a result, the VMI process allows suppliers to streamline stock levels choices for the retailers. It provides information regarding the sales and stock requirements of the buyers. Furthermore, VMI shifts inventory management responsibilities from buyers to suppliers, thereby providing buyers with a competitive edge in terms of increased product availability (Rana et al., 2015) [12]. However, this shift also creates new challenges for the supply chain manager; these include the need to build a trusting relationship and manage the supplier-buyer relationship.

Yu et al. (2012) [13] investigated a vendor-managed inventory supply chain model, in which the manufacturer sets the policy on how and when to handle inventories of quickly

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deteriorating raw materials and gradually decaying products. They deduced that the traditional reorder cycle of a deteriorating product remains proportionate to the number of increasing retailers. Taleizadeh et al. (2015) [14] investigated the difference in deterioration rates between the materials and finished products. They provided a methodology for determining the best retail pricing, regularity of raw material restocking, product reorder cycle, and joint manufacturing volume while maximizing overall profit. Initially, Xiao and Xu (2013) [15] created a Stackelberg model approach of a distribution chain in a VMI system with a sole producer and retailer incorporating degrading items and service quality judgments. Furthermore, Yu et al. (2012) [13] demonstrated that when the number of retailers drops, the importance of VMI grows.

Sainathan and Groenevelt (2018) [16] provided a mathematical model to analyze contracts capacity with VMI. Wee et al. (2011) [17] created a VMI model for ecofriendly consumer electronics that takes into account both the forward and reverse supply chain operations. Bai et al. (2019) [18] assessed the effects of reducing emissions on the VMI system's deteriorating product supply chain.

Darom et al. (2018) [19] provided a framework for a supply chain with one manufacturer and retailer that considered emissions from vehicles as well as disruption rebuilding efforts. Their approach may be used to compute timetables, the ideal quantity of safety stock, and transportation operations to decrease emissions. Yu et al. (2020) [20] studied the effects of preservation equipment investment on ordering quantities under carbon tax and carbon cap-and-trade policies. Sepehri et al. (2021) [21] also examined the impact of preservation and emissions reduction initiatives on a single producer and retailer model with imperfect manufacturing. They examined the efficiencies of cumulative investment in these technologies on total profit. Tiwari et al. (2018) [22] also created a model based on time-varying non-decreasing deterioration. This model was expanded by Liu et al. (2021) [23] by

incorporating Stackelberg game rules under periodic reviews. Taleizadeh et al. (2021) [14] used the Stackelberg scenario in a dual system with one store and numerous consumers to compute cooperative optimality. Prerna et al. (2019) [24] developed two models to discuss the integrated problem-solving approaches; one of them uses the Stackelberg policy. Their overall profit function was optimized by controlling the delivery frequency, order volume, and backorder volume between both parties. Table 1 shows the research gap.

[Please insert Table 1 here]

As shown in the research gap chart above, we propose a unique inventory management policy model based on carbon emission and deterioration costs in the supply chain. The mathematical formulation is founded on these goals in mind: to reduce overall inventory costs, maximize revenue for both parties, and assure appropriate replenishment of the manufacturer's and retailer's inventories. This is accomplished by optimizing the quantity order while taking into consideration the costs of carbon emissions and product deterioration. Some implications of implementing this policy include a decrease in inventory levels, increased dependence on logistical coordination, and greater required interaction between suppliers and buyers.

Thus, this study can provide insights to supply chain managers in making informed decisions regarding transportation policies, their associated costs, and carbon emission taxes. Currently, research gaps exist in the extant literature regarding the deteriorating items and VMI models, especially the effects of carbon emission and deterioration costs on the profit of the retailer and the manufacturer. Specifically, the presence of a VMI system is not usually considered alongside these factors.

This paper's novelty addresses these research gaps by presenting a new inventory management policy model that considers carbon emission and deterioration costs in the

supply chain. In this research, we created a vendor-led model for this analysis to solve the issue of minimizing total inventory costs and optimizing profit for both parties while ensuring proper replenishment of the manufacturer's and retailer's inventories. Our model measures the impact of carbon emissions and deterioration expenses on retailer and producer income, as well as the overall cost for both sides.

The rest of this study is summarized: Section 3 covers the study's context and premises, whereas Section 4 explains the mathematical model. The numerical analysis and model findings are detailed in Sections 5 and 6. Finally, outcomes and implications of the study are presented.

3. Problem Definition

This research advances the research of Daryanto et al (2019) [10] model to consider vendor-managed inventory systems, with two cases investigated. With the first scenario, the supplier manufactures and transports the products all at once. With the latter, the supplier manufactures the items in a single setup but provides them to the consumer in batches. To save setup time and expense, the vendor manufactures nQ units of item for each manufacturing cycle. As a result, the supplier delivers the goods in equal batches and at periodic intervals (Cao et al., 2007) [25].

These following assumptions were taken into consideration in the research:

1.   The rates of manufacturing, demand, and deterioration are constant and known.

2.   Resupply is immediate.

3.   The supplier inspects and ensures the quality of all products.

4.   The cost of inspection for each cycle is fixed.

5.   The amount of carbon emissions is determined by the amount of fuel and power utilized during transportation and inventory holding.

6.   Shortages are not permitted.

7.   Both the supplier and the buyer collaborate to jointly reduce expenses.

8.   Both the supplier and the buyer are subject to significant carbon emissions fees.

The list of notations used to develop the mathematical models is provided in Table 2.

[Please insert Table 2 here]

4. Materials and Methods

4.1.Model Development with Single-Setup-Single-Delivery (SSSD) Policy

The model under a single setup is described in this part. The supplier manufactures an item that deteriorates and provides the final items to the buyer. Figure 1 depicts the supplier's inventory level (I_p) under the SSSD policy.

[Please insert Figure 1 here]

4.1.1 Buyer Cost and Emission Function

Referencing the work of Yang and Wee (2000) [26], the inventory function is:

$$I_d(t) = \frac{D}{\theta}\left(e^{\theta(T-t)} - 1\right), \quad 0\,t \le T \tag{1}$$

and

$$Q = I_d(0) = \frac{D}{\theta}\left(e^{\theta T} - 1\right) \tag{2}$$

During every cycle, the buyer's total holding cost is:

$$\frac{h_d}{T}\int_0^T I_d(t)\,dt = \frac{h_d\left(D\left(\frac{e^{\theta T} - 1}{\theta} - T\right)\right)}{T\theta} \tag{3}$$

The buyer's cost of deterioration per year is:

$$\frac{d_d}{T}(Q - DT) = \frac{d_d\left(\frac{D(e^T - 1)}{\theta} - DT\right)}{T} \tag{4}$$

By incorporating the carbon emissions cost of holding the inventory, the buyer's total carbon emissions and carbon emissions cost per year may therefore be expressed as:

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$$ETE_d = e_c E_e \left(\frac{D}{T\theta} \left(\frac{1}{\theta} \left(e^{\theta T} - 1 \right) - T \right) \right) \tag{5}$$

$$\frac{w_e \left(D \left(\frac{e^{\theta T} - 1}{\theta} - T \right) \right)}{T\theta} \tag{6}$$

From Equations (3) to (6), the buyer's expected total cost will be:

$$ETC_d = \frac{h_d \left(D \left(\frac{e^{\theta T} - 1}{\theta} - T \right) \right)}{T\theta} + \frac{d_d \left(D \left(\frac{e^{\theta T} - 1}{\theta} - DT \right) \right)}{T} + \frac{w_e \left(D \left(\frac{e^{\theta T} - 1}{\theta} - T \right) \right)}{T\theta} \tag{7}$$

For more detailed derivation, please refer to Yang and Wee (2000) [26] and Daryanto et al. (2019) [10].

4.1.2 Supplier Cost and Emission Function

Because u% of the manufactured products is defective, the production rate for acceptable products is $(1 - u) P$. The supplier's setup cost each year is $\frac{s}{T}$. The cost of inspection incurred on the supplier includes a fixed cost for each cycle (i_c) and a unit cost (u_c). Each cycle produces PT_1 units; thus, the supplier's annual inspection cost is:

$$\frac{i_c}{T} + \frac{u_c PT_1}{T} \tag{8}$$

The supplier's transport cost is:

$$\frac{\left(t_f + 2dc_1 t_v + d \frac{D}{\theta} \left(e^{\theta T} - 1 \right) wc_2 t_v \right)}{T} \tag{9}$$

where t_f is the set cost of transportation per delivery.

The next component computes the expense incurred by moving an empty truck. Since the truck travels from the provider to the buyer and then back, the distance is doubled. The third component is the truckload transportation cost, which is determined by the delivery range and

amount, item weight, extra fuel consumption per ton per kilometer, and energy prices. Consequently, the quantity of carbon emissions emitted by the supplier each year as a result of shipping may be calculated as follows:

$$T\left(2dc_1 + d\frac{D}{\theta}\left(e^{\theta T} - 1\right)wc_2\right)F_e \tag{10}$$

The inventory differential equations are obtained from Figure 2:

$$dI_{p1}(t_1) = \left((1-u)P - D\right)dt_1 - \theta I_{p1}(t_1)dt_1, \quad 0 \le t_1 \le T_1$$

$$dI_{p2}(t_2) = -Ddt_2 - \theta I_{p2}(t_2)dt_2, \quad 0 \le t_2 \le T_2$$

For the boundary condition for $t_1 = 0$, $I_1(0) = 0$ and for $t_2 = 0$, $I_2(0) = I_o$ and for $t_2 = T_2$, $I_2(T_2) = 0$; thus, the supplier's inventory for the good products are:

$$I_{p1}(t_1) = \frac{(1-u)P - D}{\theta}\left(1 - e^{-\theta t_1}\right), \quad 0 \le t_1 \le T_1 \tag{11}$$

$$I_{p2}(t_2) = \frac{D}{\theta}\left(e^{\theta(T_2 - t_2)} - 1\right), \quad 0\, t_2 \le T_2 \tag{12}$$

Using the boundary $I_{p1}(T_1) = I_{p2}(0)$, the next formulation is obtained below:

$$\frac{\left((1-u)P - D\right)}{\theta}\left(1 - e^{-\theta T_1}\right) = \frac{D}{\theta}\left(e^{\theta T_2} - 1\right) \tag{13}$$

Figure 2 depicts the supplier's inventory's production and non-production periods.

[Please insert Figure 2 here]

From the Taylor series expansion and the supposition that $\theta T \ll 1$, with Misra's (1975) [27] estimation, one has:

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$$\left((1-u)P-D\right)T_1\left(1-\frac{1}{2}\theta T_1\right)=DT_2\left(1+\frac{1}{2}\theta T_2\right)$$

$$T_1 \approx \frac{D}{(1-u)P-D}T_2\left(1+\frac{1}{2}\theta T_2\right) \tag{14}$$

$$T \approx \frac{T_2}{(1-u)P-D}\left((1-u)P+\frac{1}{2}D\theta T_2\right) \tag{15}$$

As a result, the supplier's inventory of acceptable items is as follows:

$$\int_0^{T_1}\frac{(1-u)P-D}{\theta}\left(1-e^{-\theta t_1}\right)dt_1+\int_0^{T_2}\frac{D}{\theta}\left(e^{\theta(T_2-t_2)}-1\right)dt_2-\left[\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\theta T}-1\right)-T\right)\right] \tag{16}$$

In addition, there is a stockpile of faulty items. From Figure 2, one obtains the inventory differentiation for faulty products:

$$dI_{pd}(t_1)=uPdt_1-\theta I_{pd}(t_1)dt_1, \quad 0\le t_1\le T_1$$

Under the boundary for $t_1 = 0$, $I_1(0) = 0$, the supplier's inventory level for defective products becomes:

$$I_{pd}(t_1)=\frac{uP}{\theta}\left(1-e^{-\theta t_1}\right), \quad 0\le t_1\le T_1$$

Thus, the supplier's faulty product inventory becomes:

$$\int_0^{T_1}\frac{uP}{\theta}\left(1-e^{-\theta t_1}\right)dt_1 \tag{17}$$

Therefore, the supplier's yearly cost of holding is:

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$$\frac{h_p}{T}\left(\begin{array}{l}\dfrac{(1-u)P-D}{\theta}T_1+\dfrac{(1-u)P-D}{\theta^2}\left(e^{-\theta T_1}-1\right)-\dfrac{DT_2}{\theta}-\dfrac{D}{\theta^2}\left(1-e^{\theta T_2}\right)\\[2mm]-\left(\dfrac{D}{\theta}\left(\dfrac{1}{\theta}\left(e^{\theta T}-1\right)-T\right)\right)+\dfrac{uPT_1}{\theta}+\dfrac{uP}{\theta^2}\left(e^{-\theta T_1}-1\right)\end{array}\right) \tag{18}$$

The supplier's emissions expense and net estimated annual emissions can be determined using Equations (9) and (18):

$$T\left(2de_1+d\frac{D}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)e_2\right)$$
$$+\frac{w_e}{T}\left(\begin{array}{l}\dfrac{(1-u)P-D}{\theta}T_1+\dfrac{(1-u)P-D}{\theta^2}\left(e^{-\theta T_1}-1\right)-\dfrac{DT_2}{\theta}-\dfrac{D}{\theta^2}\left(1-e^{\theta T_2}\right)\\[2mm]-\left(\dfrac{D}{\theta}\left(\dfrac{1}{\theta}\left(e^{\theta T}-1\right)-T\right)\right)+\dfrac{uPT_1}{\theta}+\dfrac{uP}{\theta^2}\left(e^{-\theta T_1}-1\right)\end{array}\right) \tag{19}$$

$$ETE_p=T\left(2dc_1+d\frac{D}{\theta}\left(e^{\theta T}-1\right)wc_2\right)F_e$$
$$+\frac{e_cE_e}{T}\left(\begin{array}{l}\dfrac{(1-u)P-D}{\theta}T_1+\dfrac{(1-u)P-D}{\theta^2}\left(e^{-\theta T_1}-1\right)\\[2mm]-\dfrac{DT_2}{\theta}-\dfrac{D}{\theta^2}\left(1-e^{\theta T_2}\right)-\left(\dfrac{D}{\theta}\left(\dfrac{1}{\theta}\left(e^{\theta T}-1\right)-T\right)\right)+\dfrac{uPT_1}{\theta}+\dfrac{uP}{\theta^2}\left(e^{-\theta T_1}-1\right)\end{array}\right)$$
$$+e_cE_e\left(T\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\theta T}-1\right)-T\right)\right) \tag{20}$$

Next, the manufacturing cost is calculated as follows:

$$\frac{(Pm+PeTx)T_1P}{T} \tag{21}$$

The quantity items that have deteriorated owned by the supplier is equivalent to the combined output for period T_1, subtracted by the total goods given to the customer and the stock of faulty products; hence, the supplier's annual cost of deterioration is:

$$\frac{d_p}{T}\left((1-u)PT_1-\left(\frac{D}{\theta}\left(e^{\theta T}-1\right)\right)+\left(uPT_1-\frac{uP}{\theta}\left(1-e^{-\theta T_1}\right)\right)\right) \tag{22}$$

The estimated total cost of the supplier equals a total of all setup, inspection, transportation, holding, manufacturing, carbon emissions, and deterioration costs is:

$$ETC_p = \frac{s}{T} + \frac{i_c}{T} + \frac{u_c P T_1}{T} + \frac{\left(t_f + 2dc_1 t_v + d\frac{D}{\theta}\left(e^{\theta T} - 1\right)wc_2 t_v\right)}{T}$$

$$+ T\left(2de_1 + d\frac{D}{\theta}\left(e^{\theta T} - 1\right)e_2\right)$$

$$+ \frac{(h_p + w_e)}{T}\left(\begin{array}{l} \frac{(1-E[u])P-D}{\theta}T_1 + \frac{(1-E[u])P-D}{\theta^2}\left(e^{-\theta T_1} - 1\right) - \frac{DT_2}{\theta} - \frac{D}{\theta^2}\left(1-e^{\theta T_2}\right) \\ -\left(\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\theta T} - 1\right) - T\right)\right) + \frac{E[u]PT_1}{\theta} + \frac{E[u]P}{\theta^2}\left(e^{-\theta T_1} - 1\right) \end{array}\right)$$

$$+ \frac{d_p}{T}\left(\left(1-E[u]\right)PT_1 - \left(\frac{D}{\theta}\left(e^{\theta T} - 1\right)\right) + \left(E[u]PT_1 - \frac{E[u]P}{\theta}\left(1-e^{-\theta T_1}\right)\right)\right)$$

$$+ \frac{\left(Pm + PeTx\right)T_1 P}{T} \tag{23}$$

4.2. Model Development with Single-Setup-Multi-Delivery (SSMD) Policy

This section provides model development with one setup and many deliveries per cycle. The buyer orders *n* deliveries of identical order quantity (Q). To save setup time and expenses, the supplier manufactures *n*Q units of items per production cycle. As per Sarkar et al. (2016) [9], when the supplier uses an SSMD strategy, delivery frequencies rise. As an added bonus, the buyer can save money on holding costs. Consequently, a compromise exists between shipping and holding costs. Thus, carbon emissions caused by transportation and inventory-keeping activities are sensitive to decisions made in this area. Figure 3 depicts how the inventory of the supplier (I_p) and buyer (I_d) includes the supplier's defective products

(I_{pd}). This figure also depicts how I_d decreases proportionately with demand rates and deterioration rates during the period T/n.

4.2.1 Buyer Cost Function

Per replenishment cycle, the total of the buyer's inventory holding cost and deterioration cost becomes:

$$\left(h_d \frac{n}{T} + \theta d_d \frac{n}{T} \right) \int_0^{T/n} I_d(t) dt \tag{24}$$

Buyer emissions are created as a result of inventory holding; hence, the emissions cost is:

$$w_e \int_0^{T/n} I_d(t) dt \tag{25}$$

The overall cost incurred on the buyer can be obtained from the total of the inventory, deterioration, and emission costs. The projected total annual cost (ETC_d) is as follows:

$$ETC_d = (h_d + w_e) \frac{n}{T} \left(\frac{D}{\theta} \left(\frac{1}{\theta} \left(e^{\frac{\theta T}{n}} - 1 \right) - \frac{T}{n} \right) \right) + d_d \frac{n}{T} \left(\frac{D}{\theta} \left(e^{\frac{\theta T}{n}} - 1 \right) - \frac{DT}{n} \right) \tag{26}$$

4.2.2 Supplier Cost Function

The supplier incurs the same setup and inspection costs for the SSMD policy as for the SSSD policy. However, for transportation-related carbon emissions, the carbon emission equation becomes:

$$\frac{n}{T} \left(t_f + 2dc_1 t_v + d \frac{D}{\theta} \left(e^{\frac{\theta T}{n}} - 1 \right) wc_2 t_v \right) \tag{27}$$

Figure 3 depicts the inventory levels of the supplier and buyer under SSMD, respectively.

[Please insert Figure 3 here]

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The supplier inventory held during the production and during the non-production periods is as follows:

$$\int_0^{T_1} \frac{(1-u)P-D}{\theta}\left(1-e^{-\theta t_1}\right)dt_1 + \int_0^{T_2} \frac{D}{\theta}\left(e^{\theta(T_2-t_2)}-1\right)dt_2 - n\left[\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\frac{T}{n}\right)\right] \tag{28}$$

The supplier's annual holding cost thus becomes:

$$\frac{h_p}{T}\left(\begin{array}{c}\dfrac{(1-u)P-D}{\theta}T_1 + \dfrac{(1-u)P-D}{\theta^2}\left(e^{-\theta T_1}-1\right)-\dfrac{DT_2}{\theta}-\dfrac{D}{\theta^2}\left(1-e^{\theta T_2}\right)\\[4mm] -n\left(\dfrac{D}{\theta}\left(\dfrac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\dfrac{T}{n}\right)\right)+\dfrac{uPT_1}{\theta}+\dfrac{uP}{\theta^2}\left(e^{-\theta T_1}-1\right)\end{array}\right) \tag{29}$$

The supplier's carbon emission cost and total estimated annual carbon emissions may be determined below:

$$\frac{n}{T}\left(2de_1+d\frac{D}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)e_2\right)+\frac{w_e}{T}\left(\begin{array}{c}\dfrac{(1-u)P-D}{\theta}T_1+\dfrac{(1-u)P-D}{\theta^2}\left(e^{-\theta T_1}-1\right)\\[4mm]-\dfrac{DT_2}{\theta}-\dfrac{D}{\theta^2}\left(1-e^{\theta T_2}\right)-n\left(\dfrac{D}{\theta}\left(\dfrac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\dfrac{T}{n}\right)\right)\\[4mm]+\dfrac{uPT_1}{\theta}+\dfrac{uP}{\theta^2}\left(e^{-\theta T_1}-1\right)\end{array}\right)$$

$$+(h_d+w_e)\frac{n}{T}\left(\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\frac{T}{n}\right)\right) \tag{30}$$

$$ETE_p=\frac{n}{T}\left(2dc_1+d\frac{D}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)wc_2\right)F_e \, \square$$

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$$+\frac{w_e}{T}\left(\begin{array}{l}\dfrac{(1-u)P-D}{\theta}T_1+\dfrac{(1-u)P-D}{\theta^2}\left(e^{-\theta T_1}-1\right)\\[2mm]-\dfrac{DT_2}{\theta}-\dfrac{D}{\theta^2}\left(1-e^{\theta T_2}\right)-n\left(\dfrac{D}{\theta}\left(\dfrac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\dfrac{T}{n}\right)\right)\\[2mm]+\dfrac{uPT_1}{\theta}+\dfrac{uP}{\theta^2}\left(e^{-\theta T_1}-1\right)\end{array}\right)$$

$$+e_c E_e\left(\frac{n}{T}\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\frac{T}{n}\right)\right)$$

(31)

The number of deteriorated goods in the supplier's inventory equals the sum of the output during the period T_1, subtracted by the total products delivered to the customer and by the quantity of defective products in the inventory; hence, the supplier's annual deteriorating cost becomes:

$$\frac{d_p}{T}\left(\left((1-u)PT_1-n\left(\frac{D}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)\right)\right)+\left(uPT_1-\frac{uP}{\theta}\left(1-e^{-\theta T_1}\right)\right)\right)$$

(32)

Taking into account the increased manufacturing costs based on Equation (18), the supplier's projected total annual cost would be:

$$ETC_p=\frac{s}{T}+\frac{i_c}{T}+\frac{u_c PT_1}{T}+\frac{n}{T}\left(t_f+2dc_1t_v+d\frac{D}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)wc_2t_v\right)$$

$$+\frac{n}{T}\left(2de_1+d\frac{D}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)e_2\right)$$

$$+\frac{(h_p + w_e)}{T}\left(\begin{array}{c}\dfrac{(1-E[u])P-D}{\theta}T_1 + \dfrac{(1-E[u])P-D}{\theta^2}\left(e^{-\theta T_1}-1\right) \\[2mm] -\dfrac{DT_2}{\theta} - \dfrac{D}{\theta^2}\left(1-e^{\theta T_2}\right) - n\left(\dfrac{D}{\theta}\left(\dfrac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\dfrac{T}{n}\right)\right) \\[2mm] +\dfrac{E[u]PT_1}{\theta} + \dfrac{E[u]P}{\theta^2}\left(e^{-\theta T_1}-1\right)\end{array}\right)$$

$$+\left(h_d + w_e\right)\frac{n}{T}\left(\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\frac{T}{n}\right)\right)$$

$$+\frac{d_p}{T}\left(\begin{array}{c}\left(1-E[u]\right)PT_1 - n\left(\dfrac{D}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)\right) \\[2mm] +\left(E[u]PT_1 - \dfrac{E[u]P}{\theta}\left(1-e^{-\theta T_1}\right)\right)\end{array}\right) \tag{33}$$

5. Numerical Examples

Parameter values from the data of Yang and Wee (2000) [26], Hariga et al. (2017) [28], and Tiwari et al. (2018) [29] were adopted: P = 2,000,000 units/year, D = 500,000 units/year, x = 1,725,000 unit/year, i_c = \$500/delivery, u_c = \$0.5/unit, c = \$2,000/order, s = \$100,000/setup, h_d = \$40/unit/year, h_p = \$40/unit/year, d_d = \$600/unit, d_p = \$400/unit, θ = 0.1, d = 100 km, t_f = \$1000/delivery, t_v = \$0.75/L, w = 0.01 ton/unit, c_1 = 27 L/100 km, c_2 = 0.57 L/100 km/ton truckload, e_c = 1.44 kWh/unit/year, T_x = \$75/tonCO$_2$, Pm=\$10/unit, Pe= 10 kgCO$_2$/unit, F_e = 2.6×10^{-3} tonCO$_2$/L, and E_e = 0.5×10^{-3} tonCO$_2$/kWh, and $E[u]$ = 0.02 where u is uniformly distributed with $\alpha = 0$ and $\beta = 0.04$. In the SSSD policy, the expected total cost $\left(ETC\right)$ is \$391,471,196/year at T_2 = 0.04458, T_1 = 0.01530, and T = 0.05988 with a Q of 30,032.340 units. The ETE produced was 15.4 tons of CO$_2$/year. The ETE generated was compared to the total cost and carbon emissions predicted under the SSMD policy. The numerical example for the SSMD policy began at n = 2 and progressed until convexity was seen. Table 3 shows the outcomes of the numerical example under SSMD.

[Please insert Table 3 here]

Where $n^* = 5$ is the minimum joint expected total cost, which is obtainable at $T_1 =$ 0.01667, $T_2 = 0.04856$, and $T = 0.06523$, and amounts to \$391,256,071/year. The minimum joint expected total carbon emission is 24.61 tonCO$_2$/year. The optimal Q is 6527.475 units. Initially, the result of the expected total cost exhibited convexity; however, total carbon emissions followed a linear trend. Fig. 5 illustrates *ETC* convexity for n = 5. By comparing the results of both policies, the SSMD policy provides the optimum results.

6. Results and Sensitivity Analysis

A sensitivity analysis was performed with the purpose of seeing how altered parameters impacted predicted outcomes. This was performed by raising parameters to 15% and 30% of the real amounts, as well as reducing them by 15% and 30% of previous numbers. There were a total of 17 parameters utilized. Moreover, by utilizing the following formula, we estimated the percentage variations between the baselines and changed predicted overall cost:

$$\frac{ETC - ETC^*}{ETC^*} x100 \qquad (31)$$

Here, *ETC** and *ETC* signify the optimal anticipated total model cost, in which the initial and tested sensitivity analysis values are used as inputs, respectively. Figure 4 depicts the graph of *ETC*, and Table 4 shows the sensitivity analysis findings.

[Please insert Figure 4 here]

[Please insert Table 4 here]

The sensitivity analysis is also depicted below in Figure 5.

[Please insert Figure 5 here]

Accordingly, these conclusions were drawn from the sensitivity analysis results:

1. The parameter n^* responds to fluctuations in P, D, s, h_d, h_p, d_d, d_p, θ, t_f, t_v, d, c_1, c_2, and Pe, and is less sensitive to the other parameters.

2. According to the percentage ETC, the estimated total cost responds strongly to changes in parameters D and Pe, and less so to i_c, t_v, d, c_1, and c_2.

3. As parameters h_d and d_d increase, the values of n^* and ETC also increase. However, when increasing the value of parameters h_p and d_p, n^* decreases.

4. When the parameters P, D, ic, hp, dp, tf, tv, c_1, c_2, and Pe increase, Q also increases. However, when s, uc, hd, dd, d, u, and Pm increase, Q decreases.

7. Conclusions and future research

This analysis gives both academicians and companies with useful managerial insights for inventory management decisions. Suppliers, for example, can use the SSMD policy to cut setup and inventory holding costs in their manufacturing and distribution operations. Furthermore, both suppliers and purchasers may lower their carbon footprint by optimizing their transportation schedules and inventory levels. The findings help businesses to identify the most effective inventory management policies that lower carbon emissions, particularly for deteriorating items. The VMI system shows how the ideal delivery amount and number of deliveries in each manufacturing cycle are determined in order to optimize total cost, while also reducing carbon emissions. For deteriorating goods, two policies, the SSSD policy and the SSMD policy, are devised and assessed. From the solutions and numerical analysis, the ETC of the SSSD policy is \$391,437,636.4/year, with net emissions of 17.47 tons CO_2/year. The ETC of the SSMD policy is \$391,256,071/year with total carbon emissions of 24.61 tons CO_2/year. The overall cost of the SSMD policy is

lower than the total cost of the SSSD strategy, but the SSMD's total carbon emissions are greater.

The ideal deliveries per order (n), delivery order quantity (Q), production amount (R), production length (T_1), and non-production length (T_2) in each cycle (T) may be calculated using the numerical example. According to the sensitivity analysis, the estimated total cost per year is highly sensitive to n. The VMI system employed in this study helps to the inventory management system's optimization. The novel aspect of this study is the invention of a new technique for optimizing the VMI system utilizing the SSMD policy. In addition to using the VMI system, we modify the supply chain model to account for decaying products. The study fills a research vacuum by providing systematic inventory management in order to establish the best method for reducing carbon emissions.

The limitations of this study include a constant deterioration rate, known demand and constant carbon emission factor. By relaxing these assumptions, more complex scenarios can be investigated. Furthermore, it would be intriguing to extend this study to incorporate multiple suppliers and buyers, as well as integrating stochastic models. Other studies may consider dynamic pricing and the processing time.

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Table captions

Table 1: Research Gaps

Source	Year	Demand	Buyer(s)	Deterioration	Emissions	VMI	Other Considerations
Wee, Hui-Ming [3]	1993	Constant	1	Constant	-	-	Partial backordering
Wong, et al. [11]	2009	Price-dependent (own price vs all prices)	Multiple	-	-	Yes	Sales rebate contract
Wee, et al. [17]	2011	Constant	1	Constant	-	Yes	Reverse supply chain
Yu, et al. [20]	2012	Deterministic	1	Constant; materials and product	-	Yes	Materials and products deteriorate
Xiao, et al. [15]	2013	Price, service level dependent	1	Constant	-	Yes	Stackelberg game
Jiang, et al. [1]	2015	Constant	1	-	Yes	-	Carbon trading
Taleizadeh, et al. [14]	2015	Constant and price-dependent	Multiple	Constant	-	-	Stackelberg game
Darom, et al. [19]	2018	Constant	1	Caused by transportation	Yes	-	Shortages, lost sales, recovery
Tiwari, et al. [22]	2018	Price-dependent	1	Time-varying non-decreasing	-	-	Trade credit
Bai, et al. [18]	2019	Price, investment-dependent	2	Constant	Carbon cap-and-trade		Decentralized and centralized model
Sainathan, et al. [16]	2019	Stochastic	1	-	-	Yes	5 VMI contracts
Yu, et al. [20]	2020	Price, stock-dependent	(EOQ)	Constant, with/without preservation	Carbon tax, cap-and-trade	-	Preservation technology and carbon policies
Sepehri, et al. [21]	2021	Price-dependent	1	Constant	Yes; carbon tax	-	Defective production, preservation/emissions reduction investment
Liu, et al. [23]	2021	Price-dependent	1	Time-varying non-decreasing	-	-	Continuous review, Stackelberg game
Taleizadeh, et al. [14]	2021	Price, green rate-dependent	2	-	Carbon cap-and-trade	-	Dual-channel, Stackelberg game
Prerna et al. [24]	2019	known, constant, uniform	1	Constant; during delivery	variable	-	Stackelberg game
This paper		Constant	1	Constant	Yes; carbon tax	Yes	Single delivery/multiple delivery, transport costs

Table 2: List of Notations

Symbol	Definition

Q	delivery lot size (unit)
D	demand rate (unit/year);
P	production rate (unit/year);
R	production volume; $R = PT_1$;
θ	deterioration rate; $(0 \le \theta < 1)$;
u	probability of defective products per delivery lot;
x	quality inspection rate (units/year);
i_c	fixed quality inspection cost ($/cycle);
u_c	unit inspection cost ($/unit);
c	buyer's ordering cost ($/order);
h_d	buyer's holding cost ($/unit/year);
d_d	buyer's deteriorating cost ($/unit);
s	supplier's setup cost ($/order);
h_p	supplier's holding cost ($/unit/year);
d_p	supplier's deteriorating cost ($/unit);
p_m	supplier's production cost ($/unit);
P_e	supplier's carbon emissions from production activities (tonCO$_2$/unit)
t_f	supplier's fixed transportation cost per delivery ($/delivery);
t_v	fuel price for supplier's variable transportation cost ($/liter);
d	distance traveled from vendor to buyer (km);
w	product weight (ton/unit);
$c1$	average vehicle fuel consumption when empty (liter/km);
$c2$	average additional fuel consumption per ton of load (liter/km/ton);
Tx	carbon emission tax ($/tonCO2);
Fe	average emissions from fuel combustion (tonCO2/liter);
Ee	average emissions from electricity generation (tonCO2/kWh);
$e1$	transportation emission cost ($/km); e1 = c1FeTx;
$e2$	average additional transportation emission cost per unit product ($/unit/km); e2 = c2wFeTx;
ec	average warehouse energy consumption per unit product (kWh/unit/year);
w_e	warehouse emissions cost per unit product ($/unit/year); w_e = ecEeTx;
T	cycle length;
$T1$	production period for the supplier in each cycle;
$T2$	nonproduction period for the supplier in each cycle;
$Ip(t)$	supplier's inventory level at time t;
$Ipd(t)$	supplier's inventory for defective products at time t;
$Id(t)$	buyer's inventory level at time t;
$ETCd$	buyer's expected total cost per year ($/year);
$ETCp$	supplier's expected total cost per year ($/year);
ETC	joint expected total cost per year ($/year);

Table 3. Expected total cost and carbon emissions for SSMD

n	T (10^{-5})	T_1 (10^{-5})	T_2 (10^{-5})	ETC	ETE
2	6239	1594	4645	391,302,436.6	20.05
3	6373	1629	4744	391,266,479.0	22.37
4	6458	1650	4808	391,256,109.5	24.61
5*	6523	1667	4856	391,256,071.0	26.80

6	6578	1681	4897	391,261,201.3	28.96
7	6626	1693	4933	391,269,268.0	31.09
8	6670	1705	4966	391,279,148.7	33.19
9	6712	1715	4997	391,290,218.2	35.27
10	6752	1726	5026	391,302,100.0	37.33

Table 4. Sensitivity Analysis

Parameter	Value Change		n^*	T	Q	ETC	%CETC
$P=2000000$	30%	2600000	6	0.060764602	5066.28	391459551.5	0.0520
	15%	2300000	5	0.062975022	6301.47	391372139.8	0.0297
	0	2000000	5	0.06523219	6527.475	391256071	0.0000
	-15%	1700000	5	0.068551615	6859.865	391099236.4	-0.0401
	-30%	1400000	5	0.073912497	7396.715	390875642.6	-0.0972
$D=500000$	30%	650000	5	0.061657	4318.654	507810860.9	29.7899
	15%	575000	5	0.063438	5395.651	449549830.4	14.8991
	0	500000	5	0.065232	6527.455	391256071	0.0000
	-15%	425000	4	0.067304	9683.092	332922889.4	-14.9092
	-30%	350000	4	0.071363	11606.84	274543801.4	-29.8301
$s=100000$	30%	130000	5	0.073912	7396.665	391687279.3	0.1102
	15%	115000	5	0.069707	6975.56	391478393.2	0.0568
	0	100000	5	0.065232	6527.455	391256071	0.0000
	-15%	85000	4	0.059775	7477.46	391014869.7	-0.0616
	-30%	70000	4	0.054548	6823.15	390752454.8	-0.1287
$i_{c=}500$	30%	650	5	0.065278	6532.065	391258369.6	0.0006
	15%	575	5	0.065255	6529.76	391257220.9	0.0003
	0	500	5	0.065232	6527.455	391256071	0.0000
	-15%	425	5	0.065209	6525.155	391254921	-0.0003
	-30%	350	5	0.065186	6522.85	391253770.8	-0.0006
$u_{c=}0.5$	30%	0.65	5	0.065229	6527.155	391332740.3	0.0196
	15%	0.575	5	0.065231	6527.355	391294405.9	0.0098
	0	0.5	5	0.065232	6527.455	391256071	0.0000
	-15%	0.425	5	0.065234	6527.655	391217736.7	-0.0098

	-30%	0.35	5	0.065235	6527.755	391179402.4	-0.0196
hd=40	30%	52	6	0.06512	5429.615	391293937.2	0.0097
	15%	46	5	0.064842	6488.405	391275590	0.0050
	0	40	5	0.065232	6527.455	391256071	0.0000
	-15%	34	4	0.065071	8140.495	391231786.8	-0.0062
	-30%	28	3	0.065017	10847.92	391202067	-0.0138
hp=40	30%	52	3	0.062218	10380.43	391345307.9	0.0228
	15%	46	4	0.063635	7960.705	391304261.7	0.0123
	0	40	5	0.065232	6527.455	391256071	0.0000
	-15%	34	5	0.066346	6639.005	391201716.9	-0.0139
	-30%	28	6	0.068249	5690.655	391143793	-0.0287
dd=600	30%	780	6	0.064799	5402.835	391310182.9	0.0138
	15%	690	5	0.064649	6469.08	391285306.3	0.0075
	0	600	5	0.065232	6527.455	391256071	0.0000
	-15%	510	3	0.064688	10792.97	391218291.5	-0.0097
	-30%	420	2	0.065244	16337.64	391158768	-0.0249
dp=400	30%	520	3	0.062902	10494.67	391309378.7	0.0136
	15%	460	4	0.063995	8005.775	391285933.5	0.0076
	0	400	5	0.065232	6527.455	391256071	0.0000
	-15%	340	5	0.06595	6599.35	391220719.7	-0.0090
	-30%	280	6	0.067403	5620.075	391182769.9	-0.0187
θ=0.1	30%	0.13	5	0.059329	5935.364	391579150.9	0.0826
	15%	0.115	5	0.062071	6210.376	391421454.4	0.0423
	0	0.1	5	0.065232	6527.455	391256071	0.0000
	-15%	0.085	4	0.068264	8541.378	391081012.1	-0.0447
	-30%	0.07	4	0.072659	9093.108	390895259.7	-0.0922
tf=1000	30%	1300	4	0.064951	8125.47	391274637.2	0.0047
	15%	1150	4	0.064767	8102.435	391265386.3	0.0024
	0	1000	5	0.065232	6527.455	391256071	0.0000
	-15%	850	5	0.065	6504.225	391244553.5	-0.0029

	-30%	700	5	0.064768	6480.995	391232994.4	-0.0059
tv=0.75	30%	0.975	4	0.064597	8081.15	391257484	0.0004
	15%	0.8625	4	0.06459	8080.27	391256796.8	0.0002
	0	0.75	5	0.065232	6527.455	391256071	0.0000
	-15%	0.6375	5	0.064574	6461.57	391255292.5	-0.0002
	-30%	0.525	5	0.065213	6525.555	391254514.4	-0.0004
d=100	30%	130	6	0.064601237	5386.335	391257845.9	0.0005
	15%	115	6	0.064591661	5385.535	391256977.5	0.0002
	0	100	5	0.06523219	6527.475	391256071	0.0000
	-15%	85	5	0.065220212	6526.275	391255088.6	-0.0003
	-30%	70	5	0.065208233	6525.075	391254105.4	-0.0005
E[u]=0.02	30%	0.026	5	0.065213	6525.555	393647218.7	0.6111
	15%	0.023	5	0.065223	6526.555	392447973.7	0.3046
	0	0.02	5	0.065232	6527.455	391256071	0.0000
	-15%	0.017	5	0.065242	6528.46	390071443.3	-0.3028
	-30%	0.014	5	0.065251	6529.36	388894023.5	-0.6037
c_1=0.275 c_2=0.0054	30%	c1= 0.3575; c2= 0.0070	4	0.064597	8081.15	391257484	0.0004
	15%	c1=0.4125; c2=0.0081	4	0.06459	8080.27	391256796.8	0.0002
	0	c1=0.275; c2= 0.0054	5	0.065232	6527.455	391256071	0.0000
	-15%	c1=0.23375; c2= 0.0046	5	0.065223	6526.555	391255292.5	-0.0002
	-30%	c1=0.1925; c2=0.0038	5	0.065213	6525.555	391254514.4	-0.0004
Pm=10	30%	13	5	0.065177	6521.95	392789448.8	0.3919
	15%	11.5	5	0.065204	6524.655	392022760.6	0.1960
	0	10	5	0.065232	6527.455	391256071	0.0000
	-15%	8.5	5	0.06526	6530.26	390489381.1	-0.1960
	-30%	7	5	0.065288	6533.065	389722691.2	-0.3919
Pe=10	30%	13	4	0.060845	7611.41	506251346.9	29.3913

|						
15%	11.5	4	0.06263	7834.88	448755210.6	14.6960
0	10	5	0.065232	6527.455	391256071	0.0000
-15%	8.5	5	0.067423	6746.85	333752651.6	-14.6971
-30%	7	5	0.069851	6989.98	276245569.2	-29.3952

Figure captions

Fig. 1: Supplier inventory under SSSD
Fig. 2: Supplier's inventory production and non-production periods
Fig. 3: Supplier and buyer inventory levels under SSMD
Fig. 4: Graphical representation of ETC
Fig. 5: Graph of the sensitivity analysis
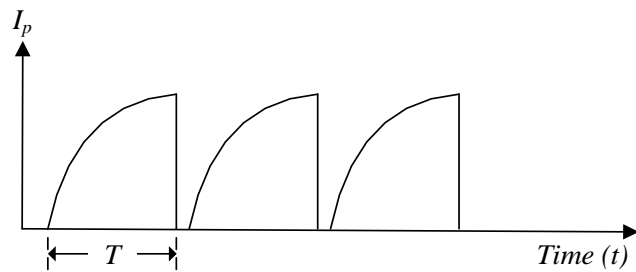
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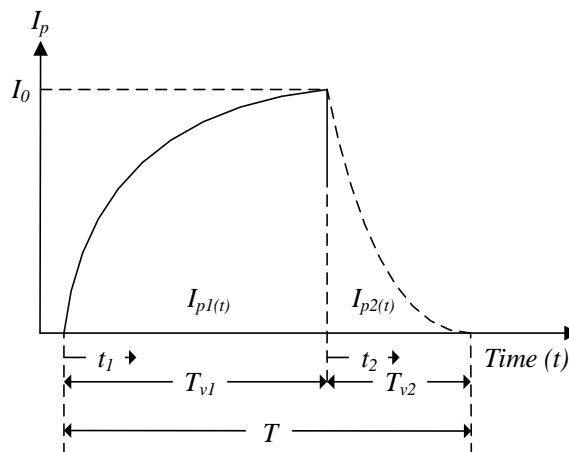
Figure 2. Supplier's inventory production and non-production periods
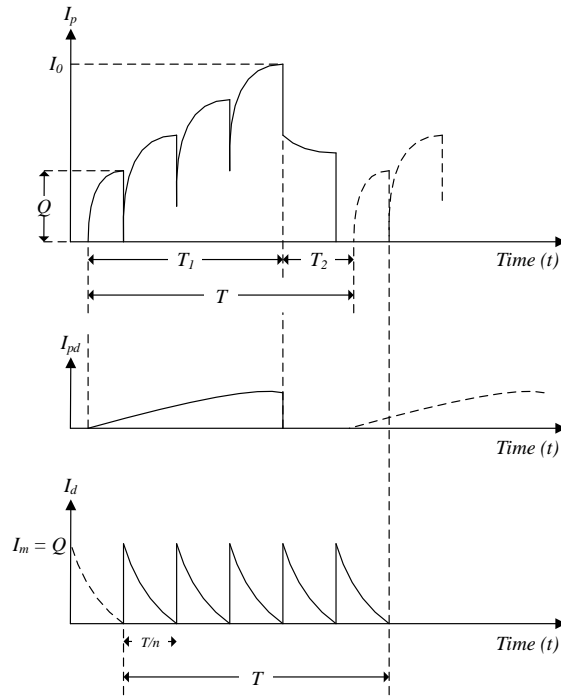
Figure 3. Supplier and buyer inventory levels under SSMD
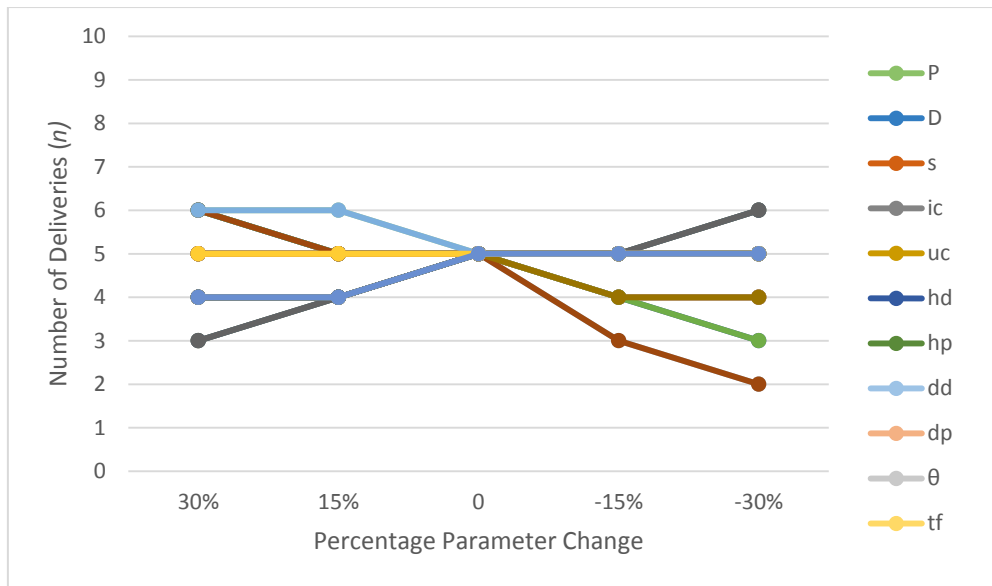

Figure 4. Graphical representation of *ETC*

Figure 5. Graph of the sensitivity analysis

Biographies

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H. M. Wee is a retired Professor from the Department of Industrial and Systems Engineering at Chung-Yuan Christian University. He has authored more than 500 papers in international peer-reviewed journals, conferences, books and book chapters. His papers have been cited in Google 11,199 times with an H-index of 57. In 2020, he is listed as top 2% scientists in Operations Research by the research team led by John Ioannidis, a distinguished professor at Stanford University. He has trained 36 PhDs and 150 Master students globally, and is the Editor-in-Chief for the *Journal of Ubiquitous Computing and Communication Technologies*, Guest Editor for *Journal of Cloud Computing,* on 'Cloud Information Technologies in Education' and *International Journal of Lean Six Sigma*, *on* "How Lean Six Sigma Improve Organizational Resilience post COVID-19.