Scientia Iranica A (2022) 29(3), 1154–1167

Sharif University of Technology
Scientia Iranica
Transactions A: Civil Engineering
http://scientiairanica.sharif.edu

Optimal design of a non-prismatic reinforced concrete box girder bridge with three meta-heuristic algorithms

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Received 28 October 2021; received in revised form 27 November 2021; accepted 13 December 2021

KEYWORDS
Optimal cost;
RC box girder bridge;
Non-prismatic;
Water Strider Algorithm (WSA);
Enhanced Colliding Bodies Optimization (ECBO);
Enhanced Vibrating Particles System (EVPS).

Abstract. Bridges are one of the most important structures that come with a high cost. The use of optimization methods may have a significant effect on cost reduction. In the present study, the performances of the Water Strider Algorithm (WSA), Enhanced Colliding Bodies Optimization (ECBO), and Enhanced Vibrating Particles System (EVPS) are compared so that the cost of a real-world non-prismatic reinforced concrete box girder bridge can be optimized. To this end, a computer tool linking CSiBridge to Matlab software has been used to optimize the bridge problem. For the first time, the present study uses the WSA algorithm to optimize bridges. Results show that the performance of ECBO algorithm is better than the other two algorithms and the WSA algorithm outperforms the EVPS algorithm.

1. Introduction

Structural engineers attempt to design the most economical structures that are resistant to demanding functional requirements over their service life. In traditional design methods (trial and error), structural analysis is repeated to obtain a reasonable design. The final design obtained by traditional methods is insufficient to meet economical and safety criteria simultaneously. Recently, a number of methods have been developed that can be used to determine the optimal solution of problems. One of these methods is random search algorithm, which is an effective tool for solving large-scale problems. Mainly, these algorithms are inspired by nature. These methods can find optimal or near-optimal solutions in a reasonably computational time. There are many metaheuristic algorithms in the literature, including Genetic Algorithm (GA) [1], Particle Swarm Optimization (PSO) [2], Ant Colony Optimization (ACO) [3], Firefly Algorithm (FA) [4], Cuckoo Search (CS) algorithm [5], Charged System Search (CSS) [6], and Simulated Annealing (SA) [7]. Kazemzaeh Azad [8] presented monitored convergence curve as a new framework for metaheuristic structural optimization algorithm, among others. Extensive research has been conducted on optimal design of structures (e.g., frame structures, truss buildings, plates, shells, dams, retaining walls, scissor-link foldable structures, etc.). The optimization of reinforced concrete structures is more complex than steel structures. In the optimization of steel structures, the weight of steel is considered, while in an RC structure, the costs of three items including concrete, steel, and formwork are considered and each of these parameters affects the cost. In a number of studies [9,10] on reinforced concrete frames, the objective function was concerned with the cost of

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doi: 10.24200/sci.2022.59322.6178

materials while the objective function of other studies was to reduce CO_2 emissions and ensure a trade-off between cost and CO_2 [11–20]. Bridges are one of the most important structures that incur high costs. Recently, studies have been conducted on bridge optimization. Fan et al. [21] summarized the applications of Machine Learning (ML) for designing and inspection of reinforced concrete bridges. Perea et al. [22] used four heuristic algorithms to optimize the cost of a reinforced concrete box-frame bridge. Aydin et al. [23] minimized the cost of prestressed concrete I-girder bridges using genetic algorithm. Pedro et al. [24] developed a two-stage optimization approach for designing steel-concrete composite I-girder bridges to minimize material costs. Yepes et al. [25] minimized the cost of post-tensioned concrete box-girder pedestrian decks based on the Spanish code. In another study, Yepes et al. [26] investigated the relation between optimal CO_2 emissions and cost for the precast-prestressed bridges with a double U-shape cross-section. García-Segura et al. [27] presented a method for optimizing the cost and CO_2 emissions of post-tensioned concrete box-girder pedestrian bridges. Also, García-Segura et al. [28] applied multi-objective harmony search algorithm to optimal design of the post-tensioned concrete road bridges in order to reduce CO_2 emissions and costs as well as to ensure the overall safety factor.

The main focus of this paper is to compare the performance of three metaheuristic algorithms for optimal design of a real-world 3D non-prismatic reinforced concrete box girder bridge. For the first time, the WSA algorithm has been used in bridge optimization and the performance of these algorithms is compared with those of ECBO and EVPS algorithms. To this end, a computer tool that provides a link between CSiBridge and Matlab software has been used to optimize the bridge problem.

2. Optimization algorithms

In this section, the algorithms utilized in this study are introduced. The ECBO and EVPS algorithms have been recently developed and compared with previously developed algorithms and found to be comparatively efficient. WSA is a new algorithm that was presented in 2020 [29] and in this study, the aim is to examine its performance in solving problems such as bridges.

2.1. Water strider algorithm

The Water Strider Algorithm (WSA) is a population-based algorithm that has been newly developed by Kaveh et al. [29]. This algorithm is inspired by the social behavior patterns of water striders. Water striders constitute a class of hemiptera insects that are able to live on the surface of the water using surface tension as well as their hydrophobic legs. They are creating the territories to defend their assets and use wave communications to convey their information. In order to mate, males send call signals to females and the latter respond to them. The response may be attractive or repulsive. For growing and absorbing energy, striders eat various kinds of food. After successful mating, females lay gelatinous eggs on submerged cliffs or plants. It takes about two months for the eggs to mature strider. The steps including birth, establishing territory, mating, feeding, death, and succession should be mathematically modeled. In this model, the search space is defined as a lake that includes different territories and food is a metaphor for the objective function. The steps are given below:

1. *Birth.* Striders are born from eggs that are distributed in the lake. Through random distribution, the initial population is obtained according to the following equation.

$$WS_i^0 = L_b + rand.(U_b - L_b), \qquad i = 1, 2, ..., nws, \tag{1}$$

where WS_i^0 represents the initial position of the ith water strider. U_b and L_b are the upper and lower bounds corresponding to variables' maximum and minimum allowable values, respectively. Rand is a vector with uniform random numbers between 0 and 1. The parameter nws is the number of WSs, where the objective function is calculated for them to determine their position in the lake.

2. *Establishing territory.* Striders establish territories for living. They are placed in territories according to their objective function. First, the WSs are sorted based on their fitness and divided into nws/nt groups. In this respect, nws is the total number of striders and nt is the number of territories. The members of the territories are determined, as shown in Figure 1.

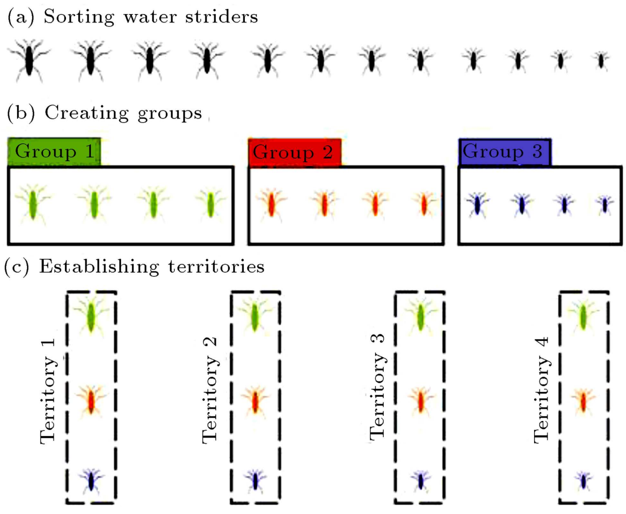

Figure 1. Establishment of a territory by striders [29].

```
Create initial population randomly as Eq. (1)
Evaluate the fitness value of WSs
While termination criteria not fulfilled
    Create the territories and allocate the WSs
    for each territory
The male keystone sends mating ripples, and the selected female decides about the response which can be an
attractive or repulsive signal.
    Update the position of keystone based on the response of female and Eq. (2)
Evaluate the new position to find food for compensating the consumed energy during the mating
    If keystone could not find food
    Forage for food resource and approach the food-rich territory by Eq. (4).
        if keystone could not find food again
          The hungry keystone will be died
           A larva will replace the killed keystone as the successor defined by Eq. (5)

        end
      end
   end
Return WS_optimal
```

Figure 2. Pseudocode of the Water Strider Algorithm (WSA) [29].

3. *Mating.* For mating, keystone sends precopulatory courtship signals to each territory and the female responds by either absorbing or repelling signals. While the probability of sending attraction responses is considered equal to p, $1-p$ is the probability of repulsive response. If the female rejects the request, the male may mount her aggressively, but the female throws him away; therefore, keystone may mate or be repelled. Inspired by this step, a new position for insects is updated.

$$\begin{cases} WS_i^{t+1} = WS_i^t + R.rand \\ \qquad \text{if mating happens (with probability of } P) \\ WS_i^{t+1} = WS_i^t + R.(1+rand) \quad \text{otherwise} \end{cases} \tag{2}$$

where WS_i^t is the position of the ith WS in the ith cycle; $rand$ is a random vector between 0 and 1; R is a vector whose starting point is at the position of male (WS_i^{t-1}) and the endpoint is at the position of a female in the same territory (WS_F^{t-1}). This female can be selected through a roulette wheel mechanism. The distance between male (WS_i^{t-1}) and female WSs (WS_F^{t-1}) is the length of R (Eq. (3)):

$$R = WS_F^{t-1} - WS_i^{t-1}. \tag{3}$$

4. *Feeding.* In the new position, WSs forage for food to absorb energy. If the value of the objective function is lower than the previous state, there is no need to change the position, but if the objective function is higher than the previous state, they move to the territories with the best position. Eq. (4) shows the new positions in this step; WS_{BL}^t is the best WS of lake:

$$WS_i^{t+1} = WS_i^t + 2.rand.\ (WS_{BL}^t - WS_i^t). \tag{4}$$

5. *Death and succession.* In the process of searching for food, if the WSs cannot find food, they are destroyed. Therefore, in this step, if the objective function is greater than the previous position, a newly matured larva is replaced with the killed WS and its position is randomly obtained according to Eq. (5):

$$WS_i^{t+1} = Lb_j^t + 2.rand.\ (Ub_j^t - Lb_i^t), \tag{5}$$

where Ub_j^t and Lb_i^t are the maximum and minimum values of the WS's position inside the jth territory.

6. *Termination of algorithm.* In the last step, if the termination criterion is satisfied, the algorithm stops and the best solution for the variables is reported. However, if the condition is not satisfied, it will return to the mating step. The pseudo-code for WSA algorithm is shown in Figure 2.

2.2. ECBO algorithm

Colliding Bodies Optimization (CBO) algorithm [30] and Enhanced Colliding Bodies Optimization (ECBO) algorithm [31] are inspired by the collision theory between two bodies. In this respect, the momentum before the collision is equal to the sum of the momentum after the collision. In order to enhance the performance of CBO algorithm, two techniques are used: The first technique is Collision Memory (CM), which stores some of the best solutions at every iteration found in previous population and substitutes them to the current worst CBs vector. In the second technique, one component of the ith CB is randomly regenerated in each generation. The probability of choosing this component is expressed by the Pro parameter. This parameter is distributed uniformly in the range of (0,1).

```
Initialize the parameters of algorithm
 Initial positions are selected randomly
The objective function and masses of the objects are calculated
while terminating criterion is not fulfilled
        Colliding memory and population are updated
        The pairs of moving and stationary groups are created
        for each CBs
                The velocity of pairs before the collision are calculated according Eq. (8)
                 The velocity of pairs after the collision are calculated according Eq. (9) or Eq. (10)
                  The position of CBs is updated according Eq. (12) or Eq. (13)
                 If rand i < pro
                       One dimension of the ith CB is selected randomly and regenerate
                 end
         end

end
```

Figure 3. Pseudocode of the Enhanced Colliding Bodies Optimization (ECBO) [32].

Introducing new objects to the population prevents the transfer of population to the local optima and increases the convergence rate without increasing computational cost.

The main procedure for implementing this algorithm is described in the following and its pseudocode is provided in Figure 3:

Step 1. First, the initial position of each colliding body is determined randomly in the research space according to the following equation:

$$x_i^0 = x_{\min} + rand(x_{\max} - x_{\min}), \quad i = 1, 2, ..., n, \tag{6}$$

where x_i^0 is the initial position of the ith CB, $x_{\min}$ and $x_{\max}$ are the minimum and maximum values of the variables, respectively, $rand$ is a random value in the range of $[0, 1]$, and n is the number of CBs.

Step 2. The mass of each CB is calculated as follows:

$$m_k = \frac{\frac{1}{fit(k)}}{\sum_{i=1}^{n} \frac{1}{fit(i)}}, \qquad k = 1, 2, ..., n, \tag{7}$$

where $fit(i)$ is the value of the objective function for CBs and n is the population size.

Step 3. In order to save a number of historically best CB vectors and their related mass and objective function values, Colliding Memory (CM) is utilized. The solution vectors that are saved in CM are added to the population and the same number of the current worst CBs is deleted. Finally, CBs are sorted in ascending order according to their objective function values. By using this technique, the performance of the algorithm can be improved.

Step 4. CBs are divided into two equal groups: (i) stationary and (ii) moving groups. In this algorithm, agents with higher fitness (moving objects) move towards the agents with lower fitness (stationary objects) and collision between these objects occurs. The collision occurs for two purposes: (1) improving the position of moving objects and (2) pushing stationary objects towards a better position.

Step 5. Before collision, the velocity of moving objects is calculated as follows:

$$v_i = x_{i-\frac{n}{2}} - x_i, \qquad i = \frac{n}{2} + 1, ..., n. \tag{8}$$

Step 6. The velocity of the colliding bodies after collision in each group is obtained as follows:

Stationary objects:

$$v_i' = \frac{(m_{i+\frac{n}{2}} + \varepsilon m_{i+\frac{n}{2}})v_{i+\frac{n}{2}}}{m_i + m_{i+\frac{n}{2}}}, \quad i = 1, 2, ..., \frac{n}{2}. \tag{9}$$

Moving objects:

$$v_i' = \frac{(m_i - \varepsilon m_{i-\frac{n}{2}})v_i}{m_i + m_{i-\frac{n}{2}}}, \quad i = \frac{n}{2} + 1, ..., n. \tag{10}$$

The coefficient of restitution ε is defined as follows:

$$\varepsilon = 1 - \frac{iter}{iter_{\max}}. \tag{11}$$

Step 7. The new positions of the objects using the generated velocities after the collision and their old positions are updated as follows:

(a) The new position of moving objects:

$$x_i^{new} = x_{i-\frac{n}{2}} + rand^{o} v_i',$$
$$i = \frac{n}{2} + 1, \quad \frac{n}{2} + 2, ..., n, \tag{12}$$

where x_i^{new} is the new position of the ith CBs, $x_{i-\frac{n}{2}}$ is the old position of the ith stationary CB, and $rand$ is a random vector uniformly distributed in the range of $[-1,\ 1]$. v_i' is the velocity of the ith moving CB after collision. The sign "^{o}" denotes an element-by-element multiplication.

(b) The new position of the stationary object:

$$x_i^{new} = x_{i-\frac{n}{2}} + rand^o v_i'$$

$$i = \frac{n}{2}+1, \quad \frac{n}{2}+2, ..., n, \tag{13}$$

where x_i^{new} is the new position of the ith CBs, $x_{i-\frac{n}{2}}$ is the old position of the ith stationary CB, and v_i' is the velocity after the collision of the ith stationary CB.

Step 8. The *Pro* parameter is compared with the random number rni ($i = 1, 2...n$). If $Pro > rn$, a CB is selected from both moving and stationary groups, and a component is randomly regenerated.

Step 9. Return to Step 2 until terminating criterion is satisfied.

2.3. Enhanced vibrating particles system

Vibrating Particles System (VPS) [33] and Enhanced Vibrating Particles System (EVPS) algorithms [34] are population-based algorithms. These algorithms are inspired by free vibration of single-degree-of-freedom systems with viscous damping. VPS consists of a number of particles that are problem variables. The VPS procedure can be outlined as follows:

Step 1. In this algorithm, the initial locations of all particles in the research space are determined randomly.

$$x_i^j = x_{\min} + rand(x_{\max} - x_{\min}), \tag{14}$$

where x_i^j is the jth variable of the ith particle. $x_{\min}$ and $x_{\max}$ denote the upper and lower bounds of variable and $rand$ is a random number in the range of [0, 1].

Step 2. The value of the objective function is calculated for each particle.

Step 3. For each candidate solution, three equilibrium positions with different weights are defined as follows:

- HB is the historically best location of the entire population;
- GP is a good particle;
- BP is a bad particle.

To obtain good and bad particles, the population must be sorted based on their objective function values in ascending order. Then, good and bad particles are selected randomly from the first and second halves, respectively. The position of the particle is updated using the following equation:

$$\begin{aligned} x_i^j =& w_1.[D.A.rand1 + HB^j] \\ &+ w_2.[D.A.rand2 + GP^j] \\ &+ w_3.[D.A.rand3 + BP^j], \end{aligned} \tag{15}$$

$$\begin{aligned} A =& [w_1.(HB^j - x_i^j)] + [w_2.(GP^j - x_i^j)] \\ &+ [w_3.(BP^j - x_i^j)] \quad w_1+w_2+w_3=1. \end{aligned} \tag{16}$$

The parameters w_1, w_2, and w_3 are three important parameters for measuring the relative importance of *HB*, *GP*, and *BP*, respectively. *Rand 1*, *Rand 2*, and *Rand 3* are random numbers between [0, 1].

In order to do modeling, the effect of damping parameter D is defined as follows:

$$D = (\frac{iter}{iter_{\max}})^{-\alpha}, \tag{17}$$

where $iter$ is the number of current iterations, $iter_{\max}$ is the total number of iterations, and α is a constant value.

For each particle, a parameter like P (0 to 1) is defined to accelerate the convergence of the VPS algorithm. Parameter P is compared with $rand$ if $P < rand$; then, $w_3 = 0$ and $w_2 = 1 - w_1$.

Step 4. Particles in the research space move to find a better result and may violate the boundary constraints. If a component violates a boundary, it is changed by the harmony search-based side constraint handling approach. In this technique, HMCR (Harmony Memory Considering Rate) parameter determines whether the violating component should be changed with the corresponding value into HB or should be selected from the permissible space. In addition, if the component of the historically best position is selected, a parameter like PAR (Pitch Adjusting Rate) determines whether this value can be changed with the neighboring value or not [33].

Step 5. If termination criterion is not fulfilled, return to Step 2.

In order to increase the convergence speed and avoid local optima, EVPS algorithm has been developed. In this method, two parameters "*Memory*" and "*OHB*" are used. The parameter of Memory acts as *HB* except that it saves memory size (number) of the best historically positions in the entire population, and *OHB* (one of the best historically positions in the entire population) is one row of memory that is selected randomly. *HB* is replaced with Memory in the EVPS algorithm. Another change in the VPS algorithm is that Eqs. (15) and (16) should be replaced with Eqs. (18) and (19). In Eqs. (18) and (19), either of Eqs. (a), (b) and (c) are used with the probability of $\omega 1$, $\omega 2$, and $\omega 3$, respectively. Other equations are the same as VPS algorithm [34]. The pseudocode of EVPS is provided in Figure 4.

$$x_i^j = \begin{cases} [D.A.rand1 + OHB^j] & (a) \\ [D.A.rand2 + GP^j] & (b) \\ [D.A.rand3 + BP^j] & (c) \end{cases} \tag{18}$$

```
 Initialize the parameters of algorithm
 Selected initial location randomly by Eq. (14)
 Calculate the value of object function and update the Memory
While termination ceriteria is not fulfilled
    for each particle
       chose the GP and BP and OHB
       if P<rand
          w3=0 and w2=1-w1
       end if
        for each component
          update the new positions by Eq. (18)
        end for
        Regenerated the violated components by harmony search-based handling approach
    end for
end while
        evaluated the values of objective function and updated HB
 end
```

Figure 4. Pseudocode of Enhanced Vibrating Particles System (EVPS) algorithm.

$$A = \begin{cases} (\pm 1)(OHB^j - x_i^j) & (a) \\ (\pm 1)(GP^j - x_i^j) & (b) \\ (\pm 1)(BP^j - x_i^j) & (c) \end{cases} \tag{19}$$

3. Formulation of optimum design of RC bridge

3.1. Objective function

In this study, the objective function is to minimize the cost of RC bridges. Herein, the problem can be stated as follows:

$$\text{Find} \quad \{X\} = [x_1, x_2, ..., x_n]$$

$$\text{To minimize} \quad f(\{X\}) = V_c.C_c + C_s.\gamma_s.A_s.L_s + C_f.A_f$$

$$\text{Subjected to} \quad g_j(x) \leq 0, \ j = 1 \text{ to } m$$

$$\text{where} \quad x_{\min} \leq x \leq x_{\max}, \tag{20}$$

where $f(\{X\})$ presents the cost of the bridge superstructure containing the volume of the concrete, the weight of reinforcements, and the area of the formwork. Parameters C_c , C_s, and C_f are the unit rate of concrete, reinforcement, and formwork, respectively. Their values for the objective function are given in Table 1. V_c is the volume of concrete that has been extracted from the CSiBridge software. γ_s is the unit weight of bars that is 7850 kg/m^3. A_s and L_s are the area and length of longitudinal bars of beams and slabs, respectively. In the objective function, the cost of shear rebars is not considered. $\{X\}$ is the vector containing the design variables. n is the number of variables. $x_{\min}$ and $x_{\max}$ are the lower and upper bounds of the design variable. $g_j(x)$ denotes design constraints and m is the number of the constraints.

Design variables cannot have any arbitrary value. They must be satisfying the limitations and specifications provided by the utilized codes. One method is the use of the penalty function. According to this method, the constrained problem is transformed into the unconstrained problem and the design variables with penalty are removed from the algorithm in the following iterations:

Table 1. Unit rates for cost [28].

Item	Unit	Description	Cost (€)
C_s	Kg	Steel B–500	1.16
C_s	m^3	Concrete (25 MPa)	95.05
	m^3	Concrete (30 MPa)	99.81
	m^3	Concrete (35 MPa)	104.57
	m^3	Concrete (40 MPa)	109.33
	m^3	Concrete (45 MPa)	114.10
	m^3	Concrete (50 MPa)	118.87
C_f	m^2	Form work	33.81

$$f_p(x) = f \times (1 + \sum_{i=1}^{m} \max(0, g_j(x)))^k, \tag{21}$$

where f_p represents the penalized objective function; f the value of the objective function; and k a penalty exponent, for which $k = 2$ is considered in this study.

3.2. Design variables

In the optimization process, the design variables are concrete strength, geometry of the cross-section, tapered length, reinforcement of box girders, and slabs. The constant parameters and variable are tabulated in Table 2. Geometrical cross-section of the bridge with some of the variables is shown in Figure 5.

Table 2. Design variables and parameters.

No.	Variable	Symbol	Step	Constraints
1	Concrete strength (ton/m^2)	f'_c	500	$2500 \leq f'_c \leq 5000$
2	Girder depth (m)	h1, h3	0.25	$1 \leq h \leq 2.5$
3	Girder depth in the mid supports (m)	h2	0.25	$1.5 \leq h \leq 3$
4	Top slab thickness (cm)	T_t	1	$18 \leq T_t \leq 35$
5	Bottom slab thickness (cm)	T_b	1	$17 \leq T_b \leq 30$
6	End thickness of cantilever (cm)	T_c	1	$18 \leq T_c \leq 30$
7	Initial thickness of cantilever (cm)	T_s	2	$20 \leq T_s \leq 50$
8	Length of cantilever (m)	L_c	0.25	$1 \leq L_c \leq 2$
9	Web thickness in intermediate cell (cm)	T_{W3}	2	$25 \leq T_{W1} \leq 50$
10	Web thickness in outside cell (cm)	T_{W1}	2	$30 \leq T_{W1} \leq 70$
11	Diameter of bars perpendicular to traffic in slabs	d_1	1	$\#3 \leq d_1 \leq \#11$
12	Number of bars perpendicular to traffic in slabs	n_1	1	$2 \leq n_1 \leq 15$
13	Diameter of bars perpendicular to traffic in cantilever	d_2	1	$\#3 \leq d_2 \leq \#11$
14	Number of bars perpendicular to traffic in cantilever	n_2	1	$2 \leq n_2 \leq 15$
15	Number of longitudinal bars in moment capacity for sections	nlt, nlb	1	$2 \leq n_{lt}, n_{lb} \leq 15$
16	Diameter of longitudinal bars in moment capacity for sections	dlt	Constant	#7
17	Diameter of shear bars (mm)		Constant	12
18	Tapered length (TLR) (m)	TLR	1	$3 \leq TLR \leq 7$
19	$t_1 = t_2 = t_3 = t_4 = t_5 = t_6 = t_7 = t_8$ (mm)		Constant	150
20	Number of cells		Constant	3

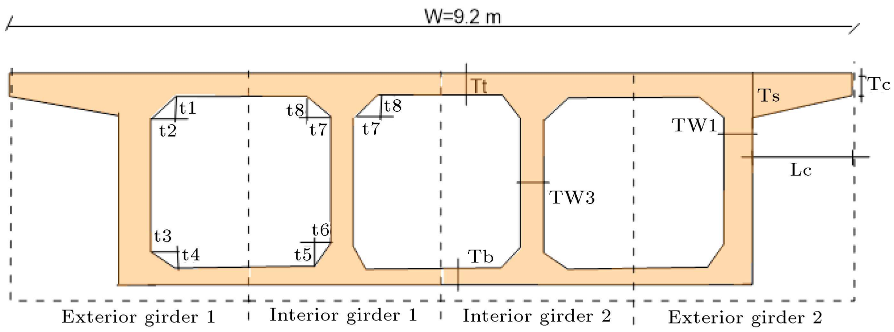

Figure 5. Geometry of superstructures.

3.3. *Loading*

The combination of dead and live loads (Eq. (22)) according to AASHTO Table 3.22.1A [35], is used to design the deck. Dead loads include the weight of girders and slabs as well as the weight of asphalt. The weight per unit volume of concrete is 2.5 ton/m^3 and the weight per unit volume of asphalt is 2.2 ton/m^3. The thickness of the asphalt is 5 cm. According to the Articles 3.7 from AASHTO 2002 [35], H20-44 and HS20-44 are considered as live loads. These loads are placed in 3.6-meter traffic lanes. The width of the deck is 9.2 meters and 2 traffic lines are considered.

$$\text{Combination load} = 1.3DL + 2.17LL, \tag{22}$$

where DL and LL are dead and live loads. In the live load, the dynamic effects are calculated as follows:

$$MI = 1 + \frac{50}{3.28L + 125} \leq 1.3, \tag{23}$$

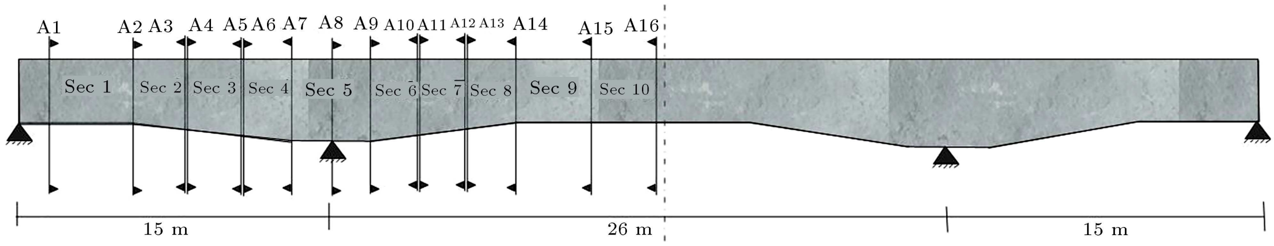

Figure 6. Bridge division for design.

where L is the length of span in meter.

3.4. Methodology implementation

In order to obtain the variables in the optimal design of 3 D non-prismatic reinforced concrete box girder bridge in continuous spans, the link between CSiBridge v22.1 and MATLAB 2016a has been used. MATLAB interacts with CSiBridge via its Application Programming Interface (API). MATLAB is used to handle the optimization algorithm and control the AASHTO 2002 standard specification while CSiBridge is used for performing finite element analysis.

3.5. Design constraints

The design of slabs and girders in the box girder bridge is based on the specifications of AASHTO 2002 [35]. In all sections, flexural capacity, shear Strength, geometry constraints, and superstructure deflection are controlled. Also, the main reinforcement, distribution reinforcement of slabs, and longitudinal skin reinforcement are obtained based on AASHTO.

4. Design example

The deck of a 3D box girder reinforced concrete bridge with three spans 15, 26, and 15 meters is optimally designed to compare the performance of algorithms in optimizing this type of bridge. The objective function is economic cost. Figure 6 shows the division of the bridge for design. The girders are divided into 16 parts (section cut) and 10 sections to satisfy the design and construction constraints. Based on this division, section cuts and related variables are shown in Table 3. In this table, *htlr* is obtained for non-prismatic sections by interpolation. The variables expressed in Table 2 are the same in all different sections, except for the items listed in Table 3.

Tables 4 and 5 list the optimal results from ECBO

Table 3. Sections and related variables.

Section cut	Depth of girders (h)	Number of longitudinal bars (top)	Number of longitudinal bars (bottom)	Space of shear bar (S)
A1	h1	nlt1	nlb1	S1
A2	h1	nlt2	nlb2	S2
A3	htlr1	nlt2	nlb2	S2
A4	htlr1	nlt3	nlb3	S3
A5	htlr2	nlt3	nlb3	S3
A6	htlr2	nlt4	nlb4	S4
A7	h2	nlt4	nlb4	S4
A8	h2	nlt5	nlb5	S5
A9	h2	nlt6	nlb6	S6
A10	htlr3	nlt6	nlb6	S6
A11	htlr3	nlt7	nlb7	S7
A12	htlr4	nlt7	nlb7	S7
A13	htrl4	nlt8	nlb8	S8
A14	h3	nlt8	nlb8	S8
A15	h3	nlt9	nlb9	S9
A16	h3	nlt10	nlb10	S10

Table 4. Optimum longitudinal bars, depth of girders, and space of shear bars for ECBO algorithm.

Section	Girders						Depth (m)	
	Exterior girders			Interior girders				
	nlt (top)	nlb (bottom)	S (m)	nlt (top)	nlb (bottom)	S (m)	h node i	h node j
Sec 1	6	6	0.4	5	6	0.5	1.25	1.25
Sec 2	8	7	0.4	7	6	0.5	1.250	1.583
Sec 3	8	9	0.4	9	7	0.5	1.5833	1.916
Sec 4	14	10	0.4	11	8	0.5	1.916	2.25
Sec 5	13	14	0.4	11	9	0.5	2.25	2.25
Sec 6	10	10	0.3	9	10	0.4	2.25	2.05
Sec 7	10	11	0.2	8	8	0.3	2.05	1.65
Sec 8	7	8	0.2	8	7	0.3	1.65	1.25
Sec 9	7	10	0.2	6	10	0.3	1.25	1.25
Sec 10	6	11	0.5	5	11	0.5	1.25	1.25

Table 5. Optimum result for ECBO algorithm.

Optimum variable	f_c' (ton/m^2)	2500
	T_t (cm)	20
	T_b (cm)	17
	T_c (cm)	19
	T_s (cm)	30
	L_c (m)	2
	T_{W3} (cm)	25
	T_{W1} (cm)	30
	Top slab reinforcement/m; (n_1, d_1)	10#3
	Cantilever slab reinforcement/m; (n_2, d_2)	8#4
	TLR1 (span 1, 3) (m)	6
	TLR2 (span 2) (m)	5
Average	87089.75 €	
Std deviation	2720.664	
Best cost	81613.86 €	

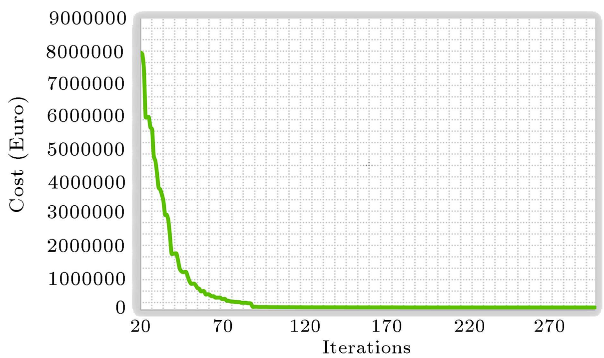

Figure 7. Convergence curve for ECBO algorithm.

algorithm. The best cost is 81613.86 Euro. The volume of concrete in this solution is 272.24 m^3 and the total weight of bars in slabs and girders is 27395.8 kg. Figure 7 shows the convergence curve of the algorithm corresponding to the lowest cost. In this algorithm, the number of population (np) and stopping criterion are considered 30 and 300 iterations, respectively. Based on the examinations, a suitable value for the parameter Pro of algorithm is 0.35 and CM is $np/2$.

Optimal results of WSA algorithm are shown in Tables 6 and 7. Herein, the best cost is 86418.11 Euro. The volume of concrete and the total weight of bars in slabs and girders are 291.47 m^3 and 28238.3 kg, respectively. Figure 8 shows the convergence curve of the algorithm for the lowest cost. In this algorithm, the number of population and the territories of WSs are assumed to be 50 and 25, respectively. The number of iterations is 200.

The results of EVPS algorithm are shown in Tables 8 and 9, in which the best cost is 86983.6 Euro. The volume of concrete in this solution is 267.4289 m^3 and the total weight of bars in slabs and girders is

Table 6. Optimum longitudinal bars, depth of girders, and space of shear bars for WSA algorithm.

Section	Girders						Depth (m)	
	Exterior girders			Interior girders				
	nlt (top)	nlb (bottom)	S (m)	nlt (top)	nlb (bottom)	S (m)	h node i	h node j
Sec 1	6	6	0.4	5	6	0.5	1.25	1.25
Sec 2	7	9	0.4	8	12	0.5	1.25	1.583
Sec 3	8	11	0.4	9	14	0.5	1.583	1.916
Sec 4	12	13	0.4	12	13	0.5	1.916	2.25
Sec 5	12	15	0.4	12	13	0.6	2.25	2.25
Sec 6	12	10	0.3	9	9	0.4	2.25	2
Sec 7	10	14	0.3	10	9	0.4	2	1.75
Sec 8	10	8	0.3	10	8	0.5	1.75	1.5
Sec 9	10	10	0.3	11	10	0.6	1.5	1.5
Sec 10	9	11	0.6	7	14	0.6	1.5	1.5

Table 7. Optimum result for WSA algorithm.

Optimum variable	f'_c (ton/m^2)	3000
	T_t (cm)	22
	T_b (cm)	17
	T_c (cm)	26
	T_s (cm)	30
	L_c (m)	2
	T_{W3} (cm)	25
	T_{W1} (cm)	30
	Top slab reinforcement/m; (n_1, d_1)	5#4
	Cantilever slab reinforcement/m; (n_2, d_2)	3#6
	TLR1 (span 1, 3) (m)	6
	TLR2 (span 2) (m)	6
Average	88197 €	
Std deviation	1751.646	
Best cost	86418.11 €	

32246 kg. Figure 9 shows the convergence curve of the EVPS algorithm corresponding to the lowest cost. In this solution, the number of population and stopping criterion are 30 and 300 iterations, respectively. A suitable value for the parameter $w1$ is 0.2 and $w2$ is 0.5. Memory size of the algorithm is assumed to be 4.

Comparison of the convergence curves of the algorithms is given in Figure 10. For each algorithm, 10 independent runs are performed. Although initial populations in our algorithms are randomly generated, the initial population can be used for efficiency seeding, as presented in [36]. In this study, although the convergence speed of the optimization process is appropriate, some particular approaches may be added to increase the convergence speed, e.g., one can utilize

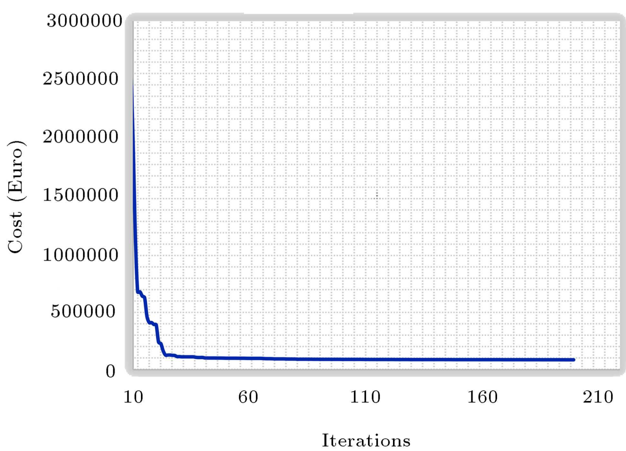

Figure 8. Convergence curve for WSA algorithm.

Table 8. Optimum longitudinal bars, depth of girders, and space of shear bars for EVPS algorithm.

Section	Girders						Depth (m)	
	Exterior girders			Interior girders				
	nlt (top)	nlb (bottom)	S (m)	nlt (top)	nlb (bottom)	S (m)	h node i	h node j
Sec 1	8	8	0.4	9	7	0.5	1.25	1.25
Sec 2	8	12	0.4	13	9	0.5	1.25	1.583
Sec 3	13	11	0.4	14	9	0.5	1.583	1.916
Sec 4	10	14	0.4	13	15	0.5	1.916	2.25
Sec 5	12	10	0.4	11	11	0.4	2.25	2.25
Sec 6	10	14	0.3	14	13	0.4	2.25	2.05
Sec 7	10	15	0.2	12	11	0.3	2.05	1.65
Sec 8	8	7	0.2	10	8	0.3	1.65	1.25
Sec 9	9	14	0.2	9	11	0.3	1.25	1.25
Sec 10	7	14	0.5	5	13	0.5	1.25	1.25

Table 9. Optimum result for EVPS algorithm.

Optimum variable	f_c' (ton/m^2)	2500
	T_t (cm)	19
	T_b (cm)	18
	T_c (cm)	19
	T_s (cm)	26
	L_c (m)	2
	T_{W3} (cm)	25
	T_{W1} (cm)	30
	Top slab reinforcement/m; (n_1, d_1)	7#4
	Cantilever slab reinforcement/m; (n_2, d_2)	6#5
	TLR1 (span 1, 3) (m)	6
	TLR2 (span 2) (m)	5
Average	8872.5 €	
Std deviation	1752.475	
Best cost	86983.6 €	

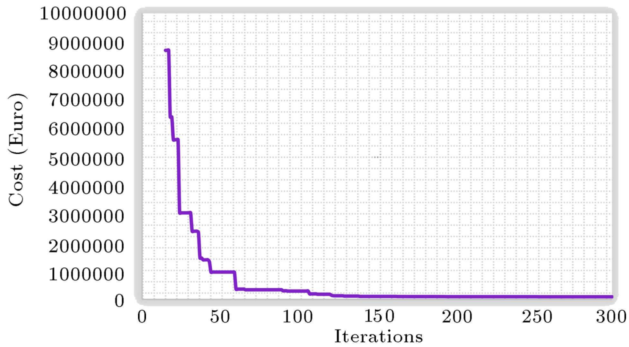

Figure 9. Convergence curve for EVPS algorithm.

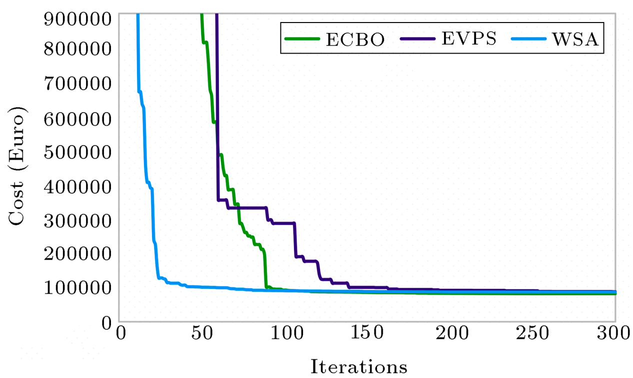

Figure 10. Convergence curves of the algorithms.

a competitive metaheuristic such as the upper bound strategy of Kazamzadeh Azad et al. [37,38].

The best costs of WSA, ECBO and EVPS algorithms are 86418.11, 81613.86, and 86983.6, respectively. The best cost of ECBO is better than other used algorithms and the best costs of WSA and EVPS algorithms are almost equal.

5. Concluding remarks

Optimal design of the multi-span real-world 3D reinforced concrete bridge with a large number of variables and several constraints based on standard codes is a challenging problem that has recently been studied by a limited number of research and optimization algorithms. Therefore, in this study, a method is implemented to optimally design the deck of non-prismatic reinforced concrete box girder bridges. The performances of the water strider algorithm, enhanced colliding bodies optimization, and enhanced vibrating particles system were compared in order to optimize the cost of bridge. The water strider algorithm was developed newly and for the first time, it we employed in this paper to optimize the bridges. A computer tool that creates a link between the CSiBridge and Matlab software has been used for the optimization process. The comparison of the results indicates that the best cost for the ECBO algorithm is lower than that for the other two algorithms and the best cost for the WSA is better than EVPS algorithm.

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