

Launch Strategies for Newly Developed Short Life Cycle Products

Prof. Fariborz Jolai*, Ph.D.

School of Industrial Engineering, College of Engineering, University of Tehran, Iran

Address: School of Industrial Engineering, 4th Floor, Central Building, College of Engineering, University of Tehran, North Kargar st, Tehran, Iran. Postal Code: 1439955961

Email address: fjolai@ut.ac.ir (F. Jolai)

Phone: +982188335605 - Cellphone: +989122148052

Alireza Taheri-Moghadam, Ph.D

School of Industrial Engineering, College of Engineering, University of Tehran, Iran

Address: School of Industrial Engineering, 4th Floor, Central Building, College of Engineering, University of Tehran, North Kargar st, Tehran, Iran. Postal Code: 1439955961

Email address: taherimoghadam@ut.ac.ir (A. Taheri-Moghadam)

Phone: +989125017003

Prof. Jafar Razmi, Ph.D.

School of Industrial Engineering, College of Engineering, University of Tehran, Iran

Address: School of Industrial Engineering, 4th Floor, Central Building, College of Engineering, University of Tehran, North Kargar st, Tehran, Iran. Postal Code: 1439955961

Email address: jrazmi@ut.ac.ir (J. Razmi)

Phone: +982188335605

Associate Prof. Ata Allah Taleizadeh, Ph.D.

School of Industrial Engineering, College of Engineering, University of Tehran, Iran

Address: School of Industrial Engineering, 4th Floor, Central Building, College of Engineering, University of Tehran, North Kargar st, Tehran, Iran. Postal Code: 1439955961

Email address: taleizadeh@ut.ac.ir (A. Taleizadeh)

Phone: +982188335605

* Corresponding author

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Abstract

Nowadays, due to the rapid pace of technology enhancement, growing consumer expectations, and market competitiveness, life cycle of products is shortening faster than it used to be. In such situations, where new generations of products lead to the obsolescence of those currently available, the simultaneous pricing of both newly developed and available products is a challenging task. In this paper, using game theory approaches, we investigate various possible conditions in which two firms may introduce new generations of products with short life cycles. Optimum price of newly developed and planned obsolescence products are determined by the proposed method. The effectiveness of the proposed method is verified using various numerical calculations and sensitivity analyses. A real case study from textile industry illustrates the application of the proposed approach in industry.

Keywords: Planned Obsolescence; New Product Introduction; Interactive Pricing; Short Life Cycle Products; Supply Chain Management; Game Theory

1. Introduction

Introducing new products is one of the most vital activities of firms in competitive markets. The optimal selling prices of newly developed products should be determined by marketing and research and development (R&D) departments [1]. The diverse and heterogeneous distribution of customers gives rise to widely divergent preferences, which makes it difficult for managers to determine the optimal decision that provides successful new product introduction [2].

The new product development (NPD) process of regular products deals only with newly developed products that should be launched to the market, while the prior products become obsolete. Usually, at the end of the product life cycle, manufacturers lose control of the market, and there is no effort for handling the market of obsolete products [3]. The NPD process of short life cycle (SLC) products is more complicated. As the life cycle of these products (as well as the development process) is very short, there are always different products in the market that are at different phases of their life cycle. Manufacturers usually lose their control on the market after one year [3], while they introduce newly developed SLC products much faster than a year. Thus, the manufacturers should handle the market of newly developed and prior (planned obsolescence) products simultaneously. Managing newly developed products while older generations of products are not obsolete yet makes the NPD process even more complex because there are interactions between different generations of products and the uncertainty level of newly developed products is high [4]. Therefore, there is a critical need for developing proper approaches for managing the challenges related to the SLC products.

Besides, as production technologies and customer preferences are developing rapidly, the life cycles of all products are reducing over time [3], which means the products are obsoleting rapidly and new generations take the place of the previous generations in a short period of time [5]. For example, mobile devices have experienced explosive growth due to the magnificent technology improvement during recent years [6]. Thus, manufacturers should provide new generations of products to survive in competitive markets. In dynamic situations, interactions between competitors and allies make the decision making process more complicated [7], and manufacturers need more advanced tools for modeling and solving such situations. From designers and engineers' perspective, the challenges related to product development environment comprise 35% of all new product development process difficulties [8]. Decisions of a firm not only affect its own performance but also affect other firms in the market as well. Thus, to support optimal decisions, various situations should be explored [9].

The fashion industry is another example of this topic. The life cycle of fashion products is short [10]. The fashion firms are so agile; for example, for ZARA™, the life cycle from the design phase to the selling phase is about 5 to 10 days, and the selling phase lasts about 17 to 20 days. There are always several products at different life cycle phases, while their selling prices interact with each other. The quality level and launch time of fashion firms are almost constant, and determining optimal selling prices is usually of high interest for them.

In this paper, we provide various models that cover diverse situations and interactions between firms. When a firm intends to introduce its newly developed product, there are three situations: 1- there is a monopoly on the market; 2- there is a competitor/cooperator manufacturer, but does not introduce a new product at the same time; 3- there is a competitor/cooperator manufacturer that introduces its newly developed product in the same period too. The

proposed models investigate the optimal pricing and interaction strategies in all of the three main situations, while they are specially designed for SLC products.

The most important challenge for the regular NPD process is to determine the three variables: Quality level, Launch time, and Selling price of newly developed products [1]. Due to the complexity of the solution approaches, most of the studies determine only one of these variables and have rarely considered two of them. In this paper, we introduce a new variable (selling price of planned obsolescence products) that is a critical decision for the NPD process of SLC products.

This article intends to discuss the answers to the following questions:

- How does the market conditions influence launch strategies of newly developed products?
- What are SLC products' conditions and how they are handled in NPD projects?
- How do we handle planned obsolescence products to maximize profit?

2. Literature review

As is explained, the conditions of SLC products always forces the manufacturers to release a new generation of products. Hence, most of the research on SLC products assumes that the product launch time is predetermined, and accordingly, the production and design costs can be estimated as constant parameters. Several firms use such an approach for their new product launch time; for instance, Apple launches a new generation of smartphones every 12 to 16 months [11] and costumers can guess the approximate launch time of new generations of Apple smartphones. The quality level and launch time of SLC products (such as mobile phones, fashion products, high tech, and etc.) are almost constant parameters for a firm while selling prices of newly developed and planned obsolescence products play a crucial role in the competition.

Selling price is the most important variable affecting demand [12]. Klastorin and Tsai, (2004) [1] investigated a case in which two firms intended to introduce new products. They assumed that the first firm introduces new products in a monopoly until the second firm introduces the new product. In this case, they only investigated the situation that both firms determine their own prices simultaneously, without information sharing, and the determined price is fixed (finite and known) to the end of the product life cycle. Shiau and Michalek, (2009) [13] used Nash and Stackelberg methods to investigate competitor response when one of the manufacturers introduces a new product. They tested their method by three case studies from the marketing and engineering design literature. They concluded that ignoring price competition between manufacturers leads to a substantial overestimation of about 12% to 79% for profit.

In Table 1, most of the related research is introduced to clarify the research gaps in the related literature and our contributions. As this table shows, the researchers have rarely investigated both of the non-monopoly conditions. The only recent research that has considered both the simultaneous and single product introduction is [14]. It investigated three launch strategies: first entrant, fast follower, and later entry. Most of the research on SLC products has focused on demand forecasting, and there are only a few articles that determine the price of brand-new and remanufactured SLC products.

We try to address the interaction between the price of planned obsolescence and newly developed products in our demand function. It is inspired by the one proposed by [15]. They proposed a demand function to address the interaction between selling prices. They assumed that the demand for new/remanufactured products is a function of selling prices and the difference between the selling price of brand-new and remanufactured products.

In this paper, we develop three models for three different situations of introducing new SLC products. The first model considers a monopoly situation, while the other two models consider duopoly situations. The second model explores the situation where just one of the manufacturers introduces a new product, and the third one assumes both of the manufacturers introduce their own new products simultaneously. The mentioned models are studied by Nash and Stackelberg methods [16].

Increasing the number of manufacturers blows up the complexity of the models and makes them incapable of investigating other factors. Hence, rarely have researchers tried to analyze models with more than two firms. Accordingly, some of the researchers are satisfied by a duopoly situation or even have just investigated a monopoly situation.

This paper provides at least four important contributions. Firstly, to the best of our knowledge, the previous studies have rarely explored and compared the non-monopoly conditions, and none of them explored all of the three

conditions. Since these conditions are strictly related to introducing new SLC products, we explored and compared all of them.

Secondly, most of the related previous articles considered pricing decisions of only newly developed products. In other words, they assumed that when a newly developed product is being introduced to a market, its prior versions will become obsolete and there is no need to determine the selling price of prior products. However, there are always products (SLC or regular) that might be at a different phase of their life cycle [15,17], and this notion has not been considered in the available literature of the NPD process effectively. In this article, the proposed modeling approach has been designated to be capable of this, and its effect on pricing decision variables as well as the total profit evaluations is thoroughly examined and verified.

****Please insert Table 1 about here****

Thirdly, to the best of our knowledge, there is no NPD pricing research that considers interactions between selling prices of newly developed and prior (planned obsolescence) products (neither regular nor SLC products). In the best case, some researchers have investigated interactions between brand-new and remanufactured products.

Finally, we explore a real case study of the fashion industry that investigates two textile manufacturers. There is a competitive relationship between customer service and NPD in the fashion industry and it matches the model conditions for SLC products perfectly. The case study contains managerial insights from which other cases can be benefited. The proposed models determine some improvements in the manufacturers' strategies. The implementation results are briefly presented, which confirms that the proposed models can improve the total profit of the manufacturers.

3. The models designation

As mentioned before, it is assumed that the manufacturer can have a monopoly on its product. In the duopoly market, competitors can launch their new products simultaneously or singly. All of these situations are investigated by the proposed models. The first model, called MM (Monopoly Model), explores the monopoly condition. The manufacturer should determine the selling price of the new and planned obsolescence [18] products to maximize its profit.

The second model, called DM (Duopoly Model), studies a situation where the market is not exclusive but in each period just one of the manufacturers intends to launch a new product. Two different conditions are studied by this model: 1- both of the manufacturers decide simultaneously (Nash equilibrium), and 2- the first manufacturer is assumed the leader, who knows how the other manufacturer will act (Stackelberg game). These approaches are the most common approaches for solving related problems.

The third model, called DM-II (Duopoly Model II), investigates the situation where both of the manufacturers intend to launch newly developed products simultaneously. Two different conditions are studied by this model: 1- both of the manufacturers decide simultaneously (Nash equilibrium), 2- the first manufacturer knows how the other manufacturer will act (Stackelberg game). Both of the manufacturers should determine the selling prices of their new and planned obsolescence products.

****Please insert Figure 1 Table 1 about here****

Figure 1 represents the structure of the proposed models. Please note that, in all of the mentioned models, it is assumed that the demand functions are linear functions of the selling prices and the difference between the selling price of the desired product and that of the replaceable products. The linear demand function is the most common function applied by several researchers [15]. It is assumed that the manufacturers have already decided about introducing their newly developed products. Thus, the production cost of the new products (which includes development activities) is known.

3.1. Indices and Parameters

The parameters of the models are defined as follows:

i	Index of the products (n=new product, o=planned obsolescence)
j	Index of the manufacturers
M_{ij}	Maximum market size of product i made by manufacturer j
C_{ij}	Production cost of product i made by manufacturer j (which includes product development costs too)
a_{ij}	Demand sensitivity of product i made by manufacturer j to its selling price (self-price sensitivity coefficient), ($a_{ij} > b_{ij}$)
b_{ij}	Demand sensitivity of product i made by manufacturer j to the price difference between replaceable products (coefficient of the interaction between newly developed and planned obsolescence products).
β_i	Coefficient of price difference of product i made by the other manufacturer (coefficient of the interaction between manufacturers)

3.2. Variables

The variables of the proposed models are explained as follows:

p_{ij}	Selling price of product i produced by manufacturer j
D_{ij}	Demand of product i produced by manufacturer j
R_j	Revenue of manufacturer j

4. The mathematical formulations

In this section, the models are formulated and the calculations of the optimal solutions are presented by Appendix A.

4.1. Monopoly model (MM)

As mentioned before, in this model, there is one manufacturer who intends to introduce a new product while its prior product is not yet obsolete. The demand functions of the newly developed and planned obsolescence products are presented by equations (1) and (2). As these equations show, the demand for the newly developed product depends not only on its own selling price but also on the difference between its selling price and that of the replaceable product. This means that if the price of the newly developed product is equal to the price of the prior product ($p_{n1}=p_{o1}$), the interaction of the selling prices is zero. The revenue function of the manufacturer is represented by equation (3), which calculates the revenue of selling the newly developed and prior products.

$$D_{n1} = M_{n1} - a_{n1}p_{n1} + b_{n1}(p_{o1} - p_{n1}) \quad (1)$$

$$D_{o1} = M_{o1} - a_{o1}p_{o1} + b_{o1}(p_{n1} - p_{o1}) \quad (2)$$

$$R_1 = D_{n1}(p_{n1} - C_{n1}) + D_{o1}(p_{o1} - C_{o1}) \quad (3)$$

The optimum selling prices calculated in Appendix A are presented by equations (4) and (5). Since there is only one manufacturer in the monopoly situation and there is no competition, the game theory approach cannot be applied for solving this model and the optimum solution is calculated by the first-order derivatives.

$$p_{n1}^* = \frac{\left(2M_{n1}(a_{o1} + b_{o1}) + M_{o1}(b_{n1} + b_{o1}) + C_{n1}b_{n1}(b_{o1} + 2a_{o1} - b_{n1}) \right)}{\left(+C_{o1}b_{o1}(b_{n1} - a_{o1} - b_{o1}) + a_{o1}b_{n1}C_{o1} + 2C_{n1}a_{n1}(a_{o1} + b_{o1}) \right)} \quad (4)$$

$$p_{o1}^* = \frac{\left(2M_{o1}(a_{n1} + b_{n1}) + M_{n1}(b_{n1} + b_{o1}) + 2C_{o1}a_{n1}(a_{o1} + b_{o1}) \right)}{\left(+ (C_{o1}b_{o1} - C_{n1}b_{n1} - C_{n1}a_{n1})(b_{n1} - b_{o1}) + 2C_{o1}b_{n1}a_{o1} \right)} \quad (5)$$

4.2. Duopoly model (DM)

In this model, there are two manufacturers but only one of them (the first manufacturer) intends to introduce its new product to the market. Both of the manufacturers should determine the selling price of their products. As explained before, the DM is studied by two different methods: 1- The Nash equilibrium, and 2- The Stackelberg game. The demand functions for each product are determined by equations (6) to (8). As we explained before, the demand function contains three terms, where the first one is the market size, the second one determines the effect of the price on the demand, and the third one calculates the interaction between the selling prices of replaceable products. The revenue function of each manufacturer is determined by equations (9) and (10).

$$D_{n1} = M_{n1} - a_{n1}p_{n1} + b_{n1}(p_{o2} + p_{o1} - 2p_{n1}) \quad (6)$$

$$D_{o1} = M_{o1} - a_{o1}p_{o1} + b_{o1}(\beta_o p_{o2} + p_{n1} - (\beta_o + 1)p_{o1}) \quad (7)$$

$$D_{o2} = M_{o2} - a_{o2}p_{o2} + b_{o2}(\beta_o p_{o1} + p_{n1} - (\beta_o + 1)p_{o2}) \quad (8)$$

$$R_1 = D_{n1}(p_{n1} - C_{n1}) + D_{o1}(p_{o1} - C_{o1}) \quad (9)$$

$$R_2 = D_{o2}(p_{o2} - C_{o2}) \quad (10)$$

4.2.1. Nash equilibrium

Nash equilibrium assumes that the players decide simultaneously and neither player can earn any expected profit if either one decides to deviate from playing the Nash equilibrium (assuming that the other players are playing their role in the Nash equilibrium) [16]. The Nash equilibrium can be calculated by equation (11), in which K_1 , K_2 , and K_3 are defined to simplify the equations, and their formulations were provided in Appendix A.

$$\begin{cases} -2(a_{n1} + 2b_{n1})p_{n1} + (b_{n1} + b_{o1})p_{o1} + b_{n1}p_{o2} + K_1 = 0 \\ (b_{n1} + b_{o1})p_{n1} - 2(a_{o1} + b_{o1}(\beta_o + 1))p_{o1} + b_{o1}\beta_o p_{o2} + K_2 = 0 \\ b_{o2}p_{n1} + b_{o2}\beta_o p_{o1} - 2(a_{o2} + b_{o2}(\beta_o + 1))p_{o2} + K_3 = 0 \end{cases} \Rightarrow \begin{cases} p_{n1}^* \\ p_{o1}^* \\ p_{o2}^* \end{cases} \quad (11)$$

4.2.2. Stackelberg

The Stackelberg approach explores the condition in which one of the players (the first manufacturer who intends to introduce a new product to the market) knows about the best response of the other manufacturer and will decide according to the best response of the other manufacturer to maximize its profit.

Appendix A explains how the optimum decisions of the first and second manufacturers are calculated by first-order derivatives of their profit function as equations (12) to (14) show, in which K_4 to K_9 are defined to simplify the equations, and their formulations were provided by Appendix A.

$$p_{o2}^*(p_{n1}, p_{o1}) = \frac{M_{o2} + b_{o2}(\beta_o p_{o1} + p_{n1})}{2(a_{o2} + b_{o2}(\beta_o + 1))} + \frac{C_{o2}}{2} \quad (12)$$

$$p_{n1}^* = \frac{C_{n1}k_5^2 - k_5k_9 + 2k_6k_8 - k_7k_9 - 2C_{n1}k_4k_8 + C_{n1}k_5k_7 + C_{o1}k_5k_8 - C_{o1}k_7k_8}{k_5^2 + 2k_5k_7 + k_7^2 - 4k_4k_8} \quad (13)$$

$$p_{o1}^* = \frac{C_{o1}k_7^2 - k_5k_6 + 2k_4k_9 - k_6k_7 - C_{n1}k_4k_5 + C_{n1}k_4k_7 - 2C_{o1}k_4k_8 + C_{o1}k_5k_7}{k_5^2 + 2k_5k_7 + k_7^2 - 4k_4k_8} \quad (14)$$

4.3. Duopoly model II (DM-II)

The DM-II model assumes that there are two manufacturers both of which intend to introduce their newly developed products in the same period. The manufacturers should determine the selling price of their products (the new and planned obsolescence products). As mentioned before, the DM-II is surveyed by two different approaches: 1- The Nash equilibrium, and 2- The Stackelberg game. The demand and profit functions are determined by equations (15) to (20), which are similar to the previous models.

$$D_{n1} = M_{n1} - a_{n1}p_{n1} + b_{n1}(\beta_n p_{n2} + p_{o2} + p_{o1} - (\beta_n + 2)p_{n1}) \quad (15)$$

$$D_{n2} = M_{n2} - a_{n2}p_{n2} + b_{n2}(\beta_n p_{n1} + p_{o1} + p_{o2} - (\beta_n + 2)p_{n2}) \quad (16)$$

$$D_{o1} = M_{o1} - a_{o1}p_{o1} + b_{o1}(\beta_o p_{o2} + p_{n1} + p_{n2} - (\beta_o + 2)p_{o1}) \quad (17)$$

$$D_{o2} = M_{o2} - a_{o2}p_{o2} + b_{o2}(\beta_o p_{o1} + p_{n2} + p_{n1} - (\beta_o + 2)p_{o2}) \quad (18)$$

$$R_1 = D_{n1}(p_{n1} - C_{n1}) + D_{o1}(p_{o1} - C_{o1}) \quad (19)$$

$$R_2 = D_{n2}(p_{n2} - C_{n2}) + D_{o2}(p_{o2} - C_{o2}) \quad (20)$$

4.3.1. Nash equilibrium

In order to determine the Nash equilibrium (p_{n1}^* , p_{o1}^* , p_{n2}^* , p_{o2}^*), four equalities shown by equation (21) have to be solved, where K_{10} , K_{11} , K_{12} , and K_{13} are defined to simplify the equations, and their formulations were provided by Appendix A.

$$\begin{cases} -2(a_{n1} + b_{n1}(\beta_n + 2))p_{n1} + (b_{n1} + b_{o1})p_{o1} + b_{n1}\beta_n p_{n2} + b_{n1}p_{o2} + K_{10} = 0 \\ (b_{n1} + b_{o1})p_{n1} - 2(a_{o1} + b_{o1}(\beta_o + 2))p_{o1} + b_{o1}p_{n2} + b_{o1}\beta_o p_{o2} + K_{11} = 0 \\ b_{n2}\beta_n p_{n1} + b_{n2}p_{o1} - 2(a_{n2} + b_{n2}(\beta_n + 2))p_{n2} + (b_{n2} + b_{o2})p_{o2} + K_{12} = 0 \\ b_{o2}p_{n1} + b_{o2}\beta_o p_{o1} + (b_{n2} + b_{o2})p_{n2} - 2(a_{o2} + b_{o2}(\beta_o + 2))p_{o2} + K_{13} = 0 \end{cases} \quad (21)$$

4.3.2. Stackelberg

The Stackelberg approach surveys the condition that the first manufacturer decides according to the best rational response of the second manufacturer in order to maximize its profit. The rational reaction functions of the second manufacturer ($p_{n2}^*(p_{n1}, p_{o1})$, $p_{o2}^*(p_{n1}, p_{o1})$) are determined by equations (22) and (23), in which K_{14} to K_{20} are defined to simplify the equations, and their formulations were provided in Appendix A.

$$p_{n2}^*(p_{n1}, p_{o1}) = \frac{(k_{16}b_{o2} - k_{14}k_{19})p_{n1} + (k_{16}k_{18} - k_{19}b_{n2})p_{o1} + (k_{16}k_{20} - k_{17}k_{19})}{k_{15}k_{19} - k_{16}^2} \quad (22)$$

$$P_{o2}^*(P_{n1}, P_{o1}) = \frac{(k_{14}k_{16} - k_{15}b_{o2})P_{n1} + (k_{16}b_{n2} - k_{15}k_{18})P_{o1} + (k_{16}k_{17} - k_{15}k_{20})}{k_{15}k_{19} - k_{16}^2} \quad (23)$$

$$P_{n1}^* = \frac{\left(\begin{aligned} &P_{n2}^* (b_{o1} (b_{n1} (2\beta_n (\beta_o + 2) + 1)) + 2\beta_n b_{n1} a_{o1}) \\ &+ P_{o2}^* (b_{o1} (\beta_o (3b_{n1} + b_{o1}) + 4b_{n1}) + 2a_{o1} b_{n1}) \\ &+ C_{n1} \left(\begin{aligned} &2a_{o1} b_{n1} (2 + \beta_n) + b_{n1} b_{o1} (4(\beta_n + \beta_o) + 2\beta_n \beta_o + 7) \\ &+ 2a_{n1} (b_{o1} (\beta_o + 2) + a_{o1}) - b_{n1}^2 \end{aligned} \right) \\ &+ C_{o1} (b_{o1} (b_{n1} - b_{o1}) (\beta_o + 2) + a_{o1} (b_{n1} - b_{o1})) \\ &+ 2M_{n1} (b_{o1} (\beta_o + 2) + a_{o1}) + M_{o1} (b_{n1} + b_{o1}) + 4\beta_n b_{n1} b_{o1} \end{aligned} \right)}{\left(\begin{aligned} &4(a_{o1} + 2b_{o1})(a_{n1} + \beta_n b_{n1}) + 4\beta_o b_{o1} (a_{n1} + b_{n1} (\beta_o + 2)) \\ &+ 2b_{n1} (4a_{o1} + 7b_{o1}) - b_{n1}^2 - b_{o1}^2 \end{aligned} \right)} \quad (24)$$

$$P_{o1}^* = \frac{\left(\begin{aligned} &P_{o2}^* (b_{n1} (b_{o1} (2\beta_o (\beta_n + 2) + 1)) + 2\beta_o b_{o1} a_{n1}) \\ &+ P_{n2}^* (b_{n1} (\beta_n (3b_{o1} + b_{n1}) + 4b_{o1}) + 2a_{n1} b_{o1}) \\ &+ C_{o1} \left(\begin{aligned} &2a_n b_{o1} (2 + \beta_o) + b_{o1} b_{n1} (4(\beta_o + \beta_n) + 2\beta_o \beta_n + 7) \\ &+ 2a_{o1} (b_{n1} (\beta_n + 2) + a_{n1}) - b_{o1}^2 \end{aligned} \right) \\ &+ C_{n1} (b_{n1} (b_{o1} - b_{n1}) (\beta_n + 2) + a_{n1} (b_{o1} - b_{n1})) \\ &+ 2M_{o1} (b_{n1} (\beta_n + 2) + a_{n1}) + M_{n1} (b_{o1} + b_{n1}) + 4\beta_o b_{o1} b_{n1} \end{aligned} \right)}{\left(\begin{aligned} &4(a_{n1} + 2b_{n1})(a_{o1} + \beta_o b_{o1}) + 4\beta_n b_{n1} (a_{o1} + b_{o1} (\beta_n + 2)) \\ &+ 2b_{o1} (4a_{n1} + 7b_{n1}) - b_{o1}^2 - b_{n1}^2 \end{aligned} \right)} \quad (25)$$

Equations (24) and (25) show the equalities of p_{ni}^* and p_{oi}^* that determine the Stackelberg optimum solution. The numerical solutions are presented in the following sections.

5. Analyzing the optimal solutions

5.1. Sensitivity analysis

In this subsection, we analyze the behavior of the models in different conditions. Please note that, as the demand and profit functions of the manufacturers are similar, we analyze only the parameters of the first manufacturer. The analysis of the second manufacturer is similar.

The parameters of the basic model are determined in Table 2. In each step, only one category of these parameters is multiplied by 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2 and other parameters are similar to those of the basic model. We have examined the impact of all parameters on the profit of the first and second manufacturers. Figure 2 to Figure 4 show the sensitivity of optimal profit functions to the mentioned parameters.

****Please insert Table 2 about here****

As shown in Figure 2, an increase in the market size of one manufacturer (M_{ij}) increases the optimal profits of both manufacturers. By increasing the market size, the first manufacturer can increase the selling price of his products, without facing demand reduction. Accordingly, as the selling price of the first manufacturer has been increased, the second manufacturer can increase its selling price too, although this increase cannot be as much as that of the first manufacturer.

Moreover, if the production cost of the first manufacturer (C_{it}) increases, he will be forced to increase its selling price to keep the profit margin. On the other hand, increasing the selling price leads to demand reduction. Hence, the total profit of the first manufacturer will decrease. On the other side, the second manufacturer can increase its selling price in order to expand its profit margin, without facing demand reduction, because the selling price of the first manufacturer is raised and the total profit of the second manufacturer will increase. This issue shows that product development cost and launch time are critical issues. Spending so much time (and/or money) on development activities may reduce total profit in the short term horizon. Therefore, market conditions highly influence product development activities.

Besides, the analyses show that by increasing the demand sensitivity to the selling price (a_{ij}), the profit of both manufacturers decreases. This is because, by increasing a_{ij} , the first manufacturer demand becomes more sensitive to the selling price and he needs to decrease its selling price to avoid demand reduction. This makes the second manufacturer reduce its selling price, too, to maintain in the competition and the total profit of both manufacturers will be decreased.

****Please insert Figure 2Figure 1Table 1 about here****

****Please insert Figure 3Table 1 about here****

****Please insert Figure 4Table 1 about here****

As Figure 3 shows, increasing b_{n1} decreases the optimal profits of both manufacturers while increasing b_{o1} increases the optimal profits. Increasing b_{n1} forces the first manufacturer to make the selling price of its newly developed product close to the planned obsolescence product; consequently, the selling price of the newly developed product will be limited. Besides, newly developed products make a greater profit than planned obsolescence products and a great amount of profit will be lost due to the price limit. On the other side, increasing b_{o1} encourages the first manufacturer to determine the selling price of its planned obsolescence product close to the newly developed product. Rationally, when the price difference is not significant, the customers prefer to buy newly developed products instead of the planned obsolescence product and the demand for newly developed products will increase. This increases manufacturer profit. Such behavior is more significant for lower values of b_{n1} . Besides, as explained previously, the second manufacturer needs to behave similarly to the first manufacturer. In other words, if the first manufacturer decreases its selling prices, the second manufacturer should decrease its selling prices too and the optimal profits of both manufacturers will increase or decrease similar to each other.

Usually, by increasing β_i , the optimal profits will decrease. It was indicated that by increasing β_i , the interaction between the manufacturers has been increased. Consequently, the manufacturers should determine their selling prices close to each other. Thus, we expect that the manufacturer with a lower selling price has more control over its profit. For example, if we have $p_{n1}^* > p_{n2}^*$ and $p_{o1}^* > p_{o2}^*$, the first manufacturer will lose much more profit than the second one because of the interaction between the selling prices. Besides, if the gap between the selling prices is significant, not only the first manufacturer will lose profit, but also the second manufacturer can make more profit than the basic values of β_i . Figure 4 confirms that by increasing β_o , both the manufacturers lose their profits and the second manufacturer is more sensitive because in this example we have $p_{o2}^* > p_{o1}^*$. But as Figure 4 shows, by increasing β_n , the first manufacturer will lose profit while the second manufacturer can make a profit in some conditions. The reason for such behavior is the difference between selling prices. In this example we have $p_{n1}^* \gg p_{n2}^*$, i.e. there is a significant difference between p_{n1}^* and p_{n2}^* , and the second manufacturer can use the benefits of increasing the interaction. Obviously, the DM is not sensitive to β_n because in such condition only the first manufacturer introduces its newly developed product.

The analyses of various models and parameters confirm the validity and rationality of the optimal solutions provided by the proposed models, and it can be concluded that the proposed approach is able to provide reasonable and reliable solutions for various industries.

5.2. Performance simulation

As we cannot be sure that one of the proposed models always provides a larger profit (except the monopoly model), we should compare the models with each other by simulation and statistical tools. In this subsection, several numerical examples are solved to compare the proposed models with each other.

Various case studies in different industries have shown that a_{ij} and b_{ij} are usually real numbers in the range of [0,0.5] and β_j is usually in the range of [0,2]. The parameters of the test problems are therefore generated uniformly in these ranges. Besides, clearly, it is infeasible for demands and selling price values to be negative numbers. Accordingly, 250 test problems that hold the feasibility and rationality conditions are generated randomly. As these test problems cover a wide range of parameters, they are able to cover most of the real cases of different industries with various parameters. The test problems are solved and Table 3 shows the statistical results.

As Table 3 shows, the superiority of one of the models cannot be confirmed (except the MM) and the optimal solutions are highly affected by the parameters, but we can statistically compare the models with each other. As the results show, R_I^* is maximized in the MM. In other words, as it is expected, manufacturers prefer a monopoly on the market. Nevertheless, we know that a pure monopoly market does not exist and always there are other manufacturers not considered by the monopoly models. In other words, the monopoly models do not consider the reactions and interactions of other manufacturers. Although the manufacturers prefer a monopoly market, monopoly models cannot imitate a real market. As Table 3 shows, if the market is a duopoly on the market, usually both

manufacturers prefer the condition where they both introduce a newly developed product simultaneously. Besides, if one of the manufacturers leads the game, the profit of both manufacturers will increase. Although there could be special cases where one of the manufacturers prefers the DM condition (like the proposed case study), the statistical results confirm that most of the time the DM-II (Stackelberg) model provides more profit than the other models.

****Please insert Table 3 about here****

The analyses show that manufacturers should note that their optimum strategies can be different for different products. For example, for one product they may choose the singly launch strategy, while for other products simultaneous launch strategy is preferred. However, the simultaneous launch strategy statistically provides more profit. Besides, for both of the manufacturers, the Stackelberg approach always provides more profit than the Nash approach for all of the conditions. In other words, to make more profit, the manufacturers most of the times should share their best reaction functions to the others.

6. Case study (Textile industry)

The fashion industry is one of the greatest industries of SLC products. The life cycle of fashion products is about 1 to 3 months. Besides, fashion and apparel industries pose a real challenge of competition, pricing, and new product development [19]. Customer preferences for textile products alter continuously; therefore, textile manufacturers should introduce their newly developed products several times a year. This confirms that this industry quite fits the proposed models.

We consider two Iranian brands of this industry: Golriz (the first manufacturer) and Nono (the second manufacturer). These brands are deciding whether to form a coalition or not. Their products are currently available on a common market in Iran, but their selling prices are not optimal. The selling prices have interactions on each other's demand, on one hand, and on the other hand, both of them should optimize their selling prices simultaneously. Nono usually sets lower selling prices, which makes it able to absorb the demand and disturb the balance of the market share and optimality. Both of the firms are suffering from this situation and want to change the situation for their benefit. We tried to solve this problem by determining the optimal prices and exploring what happens if they both use the provided optimal solution. Besides, we explored the situation where one of them prefers not to follow the provided optimum solution.

****Please insert Table 4 about here****

To simplify the model, the products are divided into different categories, to one of which the proposed model is applied. All of the products in each category are assumed as one product. Selling data are collected for these products from 2012 to 2017. The proposed models use just one demand function for all seasons while the demand for fashion products varies in different seasons. Thus, the seasonal factor should be excluded from the selling data. In addition, the inflation rate in Iran is relatively high, and it also should be excluded from the data. This can be done using engineering economy tools [20]. After this, the demand function is able to fit the selling data.

6.1. Parameter estimation

As mentioned before, in this paper we assume that the parameters have already been determined and we do not discuss market research methods here. However, the present case study does not provide the parameters. Hence, we need to estimate the parameters. The parameters are estimated by a heuristic method (please refer to Appendix B). The dataset indicates three different conditions: 1- Only Golriz introduces new products during a month (DM_G). 2- Only Nono introduces new products during a month (DM_N). 3- Both of them introduce new products during a month (DM-II). As both of the brands were active during the time horizon of our study, the monopoly condition was not available in this case. Table 4 represents the estimated values of the parameters in different conditions. The average production costs of the manufacturers are $C_{n1}=8122$, $C_{o1}=7045$, $C_{n2}=8459$, $C_{o2}=7196$.

6.2. Solution

The optimum solutions of each condition are calculated by the Nash and Stackelberg methods. It is assumed that in conditions 1 and 2, the manufacturer who introduces a new product is the Stackelberg leader. The results are presented in Table 5.

****Please insert Table 5 about here****

The results show that the Stackelberg method always increases the profit of both manufacturers. Thus, the manufacturers should share their response functions with each other to increase their total profit. In other words, if one of the players knows about the best response of the other player, both of the players will benefit.

When only one of the manufacturers introduces a new product, its profit increases and the profit of the other manufacturer decreases, which is an obvious fact and we expected that before solving the model. Figure 5 and Figure 6 show the different strategies as a two-person nonzero-sum game [16]. In this game, there are two strategies for each player: 1- Introduce a new product (S_1). 2- Do not introduce a new product (S_2). The profits of each player are obtained from Table 5. Figure 5 represents the situation where the manufacturers do not know about each other's best response (Nash method), while Figure 6 represents the situation where one of the manufacturers knows about the best response of the other one (Stackelberg method). Please note that since in the third condition both of the players can decide to be the leader, the average profits of the Stackelberg methods are considered. However, there is no difference in the equilibrium strategy. Besides, there is no information about the situation where none of the manufacturers introduces a new product. In such a situation, we assume that there are other manufacturers who introduce new products and the profit of the first and second manufacturers is the same as the second and first conditions, respectively. Both Figure 5 and Figure 6 confirm that the best strategy for the first manufacturer (Golriz) is to introduce its newly developed product as soon as possible, while the best strategy for the second manufacturer (Nono) is to delay the introduction of its newly developed product unless the first manufacturer does not intend to introduce a new product in a period or the decision-makers determine that introducing a new product is necessary for a period.

****Please insert Figure 5 Table 1 about here****

****Please insert Figure 6 Table 1 about here****

Currently, the first manufacturer (Golriz) acts unexpectedly. That is, in 73 periods we studied (each month is considered as a period), in 12 months just Golriz introduced a new product, in 22 months just Nono introduced a new product, and in 38 months both of them introduced a new product, which is contrary to the best strategy. The historical data and our solutions confirm that the current strategy is not optimum for Golriz. Besides, in most of the periods, none of the manufacturers sets the optimum selling price for their products. Table 6 represents the average profit of each manufacturer (after eliminating the impact of the inflation rate and seasonal factor) in different conditions and the estimated optimal profit in such conditions.

As Table 6 shows, by implementing the optimum solution, Golriz is able to increase its profit between 10% and 114% (depending on different conditions). On the other hand, Nono can increase its profit in the first and second conditions between 9% and 38%, while its profit decreases in the third condition. If Golriz implements the optimum solution, the total profit will be shared between the manufacturers more reasonably. Therefore, Nono has no choice but to follow the optimum solution.

6.2.1. Implementing the solutions

Golriz agreed to implement the results for a trial period (just for one category of the products). Nono prefers not to implement the suggested plan in a trial period because the results of the third condition show that its profit is unfairly high and he should share a part of its profit with Golriz. Thus, if it follows the results, his profit decreases (only in the third condition). The trial period was a specific month matching the third condition. In other words, in the trial month, both of the manufacturers intended to introduce new products to the market.

We assumed that Nono will almost set the average values of historical data of similar conditions (the third condition) for p_{n2} and p_{o2} . Hence, we can use the Stackelberg method, in which Golriz is the leader because it knows about the decision of Nono. The results of applying the proposed model to the trial period are represented in Table 7.

Please note that the accuracy of the estimations relates to several factors. Firstly, customer preferences and demand for newly developed products are extremely uncertain [21,22], and this issue, due to several reasons, is even more serious for fashion industry [23]. Secondly, only two manufacturers were considered by the proposed model to simplify the modeling and solving processes, while there are more manufacturers in the market. Thirdly, it is assumed that Nono sets average values of historical data of similar conditions for p_{n2} and p_{o2} , while its real decision is a little different. Fourthly, the experiment lasts for just a month. Extending this duration may increase the accuracy of estimations.

****Please insert Table 6 about here****

****Please insert Table 7 about here****

As Table 7 shows, by reducing p_{n1} and p_{o1} by 11% and 14% respectively, Golriz can increase its profit by 32%. Although the selling prices are decreased, the demand increased, which led to higher profit. On the other hand, the evidence shows that the profit of Nono decreased by 27%, while if it had implemented the suggested plan, it would have to share just about 9% of its profit. In other words, the solutions suggest that while Nono preferred not to

decrease its selling prices, it should have done this to increase its profit. Doing this action would give Golriz a chance to draw a significant part of the demand to itself. The trial experiment confirms that a manufacturer should implement the proposed solution, through which it can make the other manufacturer follow him to stay in the competition. Indeed, if Nono had participated in the experimental program too, the profit of both Golriz and Nono would have increased.

The provided case study clarified the application of the proposed models to fit the conditions of SLC products and confirmed the accuracy of the proposed approach. At last, as was explained, the results of the trial experiment showed how the model can be implemented and how the manufacturers can use the results to improve their performance. Besides, the brands of the case study have two problems: 1- Determining optimum selling prices. 2- As their selling prices have interactions on each other's demand, they should decide whether to form a coalition or not. The proposed case study solved both of the problems. The optimal selling prices were provided for each condition, the best strategy for the NPD process of each firm was determined, and finally, it was revealed that if one of the firms decide not to join the coalition, its profit reduces significantly.

6.3. Managerial insights into the case study

Based on the results of the models, we propose some insights for the textile case study. There are similar suggestions for other cases.

Both of the manufacturers prefer the situation where at least one of them knows about the other one's best response because the solutions showed that the Stackelberg method always leads to greater profit than the Nash equilibrium. They can increase their profit by cooperating and sharing their best response functions.

Our analyses show that the best strategy for the first manufacturer is to introduce new products in every period and the first manufacturer should improve its NPD process. However, the second manufacturer can reduce its NPD activities (if the first manufacturer behaves according to the best strategy). The results confirm that, unless the marketing department of the second manufacturer insists that introducing a new product is necessary for a period, or the first manufacturer does not intend to introduce a new product in a period, the equilibrium strategy for the second manufacturer is to introduce a new product as late as possible.

The sensitivity analysis shows that increasing the market size of one manufacturer leads to an increase in the total profit of both manufacturers; the manufacturers should focus on promoting their product instead of obstructing the marketing of the other manufacturer.

Finally, the results confirm that if the manufacturers follow the optimum solutions determined by the proposed models, they can increase their profit significantly.

Applying the proposed models is beneficial for various industries that produce SLC products, such as cell phones, electronics, fashion, or even news agencies. Newspapers have the shortest life cycle between all of the products. The life cycle of a newspaper is one day and publishers should design and develop their new product each day while their planned obsolescence products may still contain valid content.

7. Conclusion

This paper explores how manufacturers should introduce their newly developed products, while their planned obsolescence products are still available in the market and there is a competition between the manufacturers. Such a situation is very common for short life cycle (SLC) products, which has not been considered by the research published on the new product development (NPD). It is expected that in the future almost all of the products may be categorized as SLC products because of the rapid development of technology. Our proposed models provided the optimum selling prices of the products as well as the launch strategy of the firms.

We considered different possible situations where two manufacturers may introduce new SLC products. We also addressed the interaction between the selling prices of newly developed and planned obsolescence products. The proposed models were validated by sensitivity analyses and several numerical examples. Moreover, a case study of textile industry explored the application of the proposed models. The results showed that the proposed models were able to provide optimal decisions and strategies with acceptable accuracy.

By analyzing the results, we conclude that the manufacturers who intend to introduce new products to a market can highly benefit from the proposed model. The pricing decision is one of the most vital decisions which can lead NPD projects to success or failure. Besides, as the development activities constitute most of the product life cycle costs (especially for the SLC products), determining the best development strategy can provide the firms with a

significant profit. The proposed model suggests the best strategy of each firm in both monopoly and duopoly markets.

Certainly, the proposed approach has also some limitations, which can be extended in several ways as future research: 1- Considering other decision variables such as quality/technology levels, development cost, and launch time of newly developed products, informs exciting future research direction of NPD strategies. 2- Using the determined solutions in other supply chain management problems (such as network design, supplier selection, production planning and etc.) as hierarchical or decomposed solving approaches may increase the total profit of the supply chain and/or computational time of solving integrated models. 3- The number of manufacturers can be increased to cover more complicated realistic situations. 4- Considering nonlinear demand functions may extend the application of the model to explore more various situations. However, handling the additional complexity of nonlinear demand functions will be an important challenge. 5- The bargaining approach can be considered. It can help decision-makers to determine how to join the game. 6- Investigating the application of the model in more case studies of various industries is an interesting future research topic.

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Appendix A: Calculations of optimal solutions

Derivatives, Hessian matrixes, and Calculations of the optimal solutions for all of the proposed models are provided here.

1. Monopoly model (MM)

Equation (A. 1) calculates the Hessian matrix [24] of the revenue function in order to check its concavity. As equation (A. 1) shows, the Hessian matrix is negative definite (ND) and the revenue function is concave. Please note that all of the parameters of the models are positive numbers. As the revenue function is jointly concave, the maximum value of it can be achieved by first-order derivatives, as equation (A. 2) shows. By solving the equalities of equation (A. 2), the optimum selling prices are calculated, that is presented by equations (A. 3) and (A. 4).

$$H(R_1(p_{n1}, p_{o1})) = \begin{bmatrix} -2(a_{n1} + b_{n1}) & (b_{n1} + b_{o1}) \\ (b_{n1} + b_{o1}) & -2(a_{o1} + b_{o1}) \end{bmatrix}$$

$$\Rightarrow \begin{cases} H \equiv \begin{bmatrix} - & + \\ + & - \end{bmatrix} \\ H_1 = -2(a_{n1} + b_{n1}) \stackrel{a_{ij}, b_{ij} > 0}{<} 0 \\ H_2 = 4(a_{n1} + b_{n1})(a_{o1} + b_{o1}) - (b_{n1} + b_{o1})^2 \stackrel{a_{ij}, b_{ij} > 0}{\stackrel{a_{ij} > b_{ij}}{<}} 0 \end{cases} \quad (\text{A. 1})$$

$$\begin{cases} \frac{\partial R_1}{\partial p_{n1}} = 0 \\ \frac{\partial R_1}{\partial p_{o1}} = 0 \end{cases} \Rightarrow \begin{cases} -2(a_{n1} + b_{n1})p_{n1} + (b_{n1} + b_{o1})p_{o1} + M_{n1} + C_{n1}(a_{n1} + b_{n1}) - C_{o1}b_{o1} = 0 \\ (b_{n1} + b_{o1})p_{n1} - 2(a_{o1} + b_{o1})p_{o1} + M_{o1} + C_{o1}(a_{o1} + b_{o1}) - C_{n1}b_{n1} = 0 \end{cases} \quad (\text{A. 2})$$

$$p_{n1}^* = \frac{\begin{pmatrix} 2M_{n1}(a_{o1} + b_{o1}) + M_{o1}(b_{n1} + b_{o1}) + C_{n1}b_{n1}(b_{o1} + 2a_{o1} - b_{n1}) \\ + C_{o1}b_{o1}(b_{n1} - a_{o1} - b_{o1}) + a_{o1}b_{n1}C_{o1} + 2C_{n1}a_{n1}(a_{o1} + b_{o1}) \end{pmatrix}}{2b_{n1}(b_{o1} + 2a_{o1}) + 4a_{n1}(b_{o1} + a_{o1}) - b_{n1}^2 - b_{o1}^2} \quad (\text{A. 3})$$

$$p_{o1}^* = \frac{\begin{pmatrix} 2M_{o1}(a_{n1} + b_{n1}) + M_{n1}(b_{n1} + b_{o1}) + 2C_{o1}a_{n1}(a_{o1} + b_{o1}) \\ + (C_{o1}b_{o1} - C_{n1}b_{n1} - C_{n1}a_{n1})(b_{n1} - b_{o1}) + 2C_{o1}b_{n1}a_{o1} \end{pmatrix}}{2b_{n1}(b_{o1} + 2a_{o1}) + 4a_{n1}(b_{o1} + a_{o1}) - b_{n1}^2 - b_{o1}^2} \quad (\text{A. 4})$$

2. Duopoly model (DM)

2.1. Nash equilibrium

As it is shown by equations (A. 5) and (A. 6), R_1 and R_2 are concave functions according to their variables (please note that R_2 is a single-variable function and there is no need for calculating its Hessian matrix). Hence, the Nash equilibrium can be calculated by first-order derivatives of the R_1 and R_2 . Equations (A. 7) to (A. 9) calculate the first-order derivatives of the profit functions.

$$H(R_1(p_{n1}, p_{o1})) = \begin{bmatrix} -2(a_{n1} + 2b_{n1}) & (b_{n1} + b_{o1}) \\ (b_{n1} + b_{o1}) & -2(a_{o1} + b_{o1}(\beta_o + 1)) \end{bmatrix} \stackrel{a, b, \beta > 0}{\Rightarrow} H \text{ is ND.} \quad (\text{A. 5})$$

$$\frac{\partial^2 R_2(p_{o2})}{\partial p_{o2}^2} = -2(a_{o2} + b_{o2}(\beta_o + 1)) \stackrel{a, b, \beta > 0}{\leq} 0 \quad (\text{A. 6})$$

$$\frac{\partial R_1(p_{n1}, p_{o1})}{\partial p_{n1}} = -2(a_{n1} + 2b_{n1})p_{n1} + (b_{n1} + b_{o1})p_{o1} + b_{n1}p_{o2} + \underbrace{(M_{n1} + C_{n1}(a_{n1} + 2b_{n1}) - C_{o1}b_{o1})}_{K_1} \quad (\text{A. 7})$$

$$\frac{\partial R_1(p_{n1}, p_{o1})}{\partial p_{o1}} = (b_{n1} + b_{o1})p_{n1} - 2(a_{o1} + b_{o1}(\beta_o + 1))p_{o1} + b_{o1}\beta_o p_{o2} + \underbrace{(M_{o1} - C_{n1}b_{n1} + C_{o1}(a_{o1} + b_{o1}(\beta_o + 1)))}_{K_2} \quad (\text{A. 8})$$

$$\frac{\partial R_2(p_{o2})}{\partial p_{o2}} = b_{o2}p_{n1} + b_{o2}\beta_o p_{o1} - 2(a_{o2} + b_{o2}(\beta_o + 1))p_{o2} + \underbrace{(M_{o2} + C_{o2}(a_{o2} + b_{o2}(\beta_o + 1)))}_{K_3} \quad (\text{A. 9})$$

There are three equalities and three variables (p_{n1} , p_{o1} , p_{o2}) in order to achieve the Nash equilibrium as equation (A. 10) shows. Please note that K_1 , K_2 , and K_3 are defined to simplify the equations, and their formulations are presented by equations (A. 7), (A. 8), and (A. 9) respectively.

$$\begin{cases} -2(a_{n1} + 2b_{n1})p_{n1} + (b_{n1} + b_{o1})p_{o1} + b_{n1}p_{o2} + K_1 = 0 \\ (b_{n1} + b_{o1})p_{n1} - 2(a_{o1} + b_{o1}(\beta_o + 1))p_{o1} + b_{o1}\beta_o p_{o2} + K_2 = 0 \\ b_{o2}p_{n1} + b_{o2}\beta_o p_{o1} - 2(a_{o2} + b_{o2}(\beta_o + 1))p_{o2} + K_3 = 0 \end{cases} \Rightarrow \begin{cases} p_{n1}^* \\ p_{o1}^* \\ p_{o2}^* \end{cases} \quad (\text{A. 10})$$

2.2. Stackelberg

We should determine the best response of the second manufacturer, before calculating the Stackelberg optimum solution. As it is explained before, the profit functions are concave, hence the best response of the second manufacturer ($p_{o2}^*(p_{n1}, p_{o1})$) can be determined by equation (A. 11).

$$\begin{aligned} \frac{\partial R_2(p_{o2})}{\partial p_{o2}} = 0 \Rightarrow \\ b_{o2}p_{n1} + b_{o2}\beta_o p_{o1} - 2(a_{o2} + b_{o2}(\beta_o + 1))p_{o2} + (M_{o2}C_{o2}(a_{o2} + b_{o2}(\beta_o + 1))) = 0 \\ \Rightarrow p_{o2}^*(p_{n1}, p_{o1}) = \frac{M_{o2} + b_{o2}(\beta_o p_{o1} + p_{n1})}{2(a_{o2} + b_{o2}(\beta_o + 1))} + \frac{C_{o2}}{2} \end{aligned} \quad (\text{A. 11})$$

If we replace p_{o2} in equation (9), with the best response function ($p_{o2}^*(p_{n1}, p_{o1})$) which is calculated by equation (A. 11), the profit function of the first manufacturer will be changed as equation (A. 18), in which, k_4 to k_9 are defined by equations (A. 12) to (A. 17).

$$k_4 = b_{n1} \left(\frac{b_{o2}}{2(a_{o2} + b_{o2}(\beta_o + 1))} - 2 \right) - a_{n1} \quad (\text{A. 12})$$

$$k_5 = b_{n1} \left(1 + \frac{b_{o2}\beta_o}{2(a_{o2} + b_{o2}(\beta_o + 1))} \right) \quad (\text{A. 13})$$

$$k_6 = M_{n1} + b_{n1} \left(\frac{M_{o2}}{2(a_{o2} + b_{o2}(\beta_o + 1))} + \frac{C_{o2}}{2} \right) \quad (\text{A. 14})$$

$$k_7 = b_{o1} \left(1 + \frac{\beta_o}{2(a_{o2} + b_{o2}(\beta_o + 1))} \right) \quad (\text{A. 15})$$

$$k_8 = b_{o1} \left(\frac{b_{o2}\beta_o^2}{2(a_{o2} + b_{o2}(\beta_o + 1))} - (\beta_o + 1) \right) - a_{o1} \quad (\text{A. 16})$$

$$k_9 = M_{o1} + b_{o1} \left(\frac{\beta_o M_{o2}}{2(a_{o2} + b_{o2}(\beta_o + 1))} + \frac{C_{o2}}{2} \right) \quad (\text{A. 17})$$

$$\begin{aligned} R_1(p_{n1}, p_{o1}) = & (k_4 p_{n1} + k_5 p_{o1} + k_6)(p_{n1} - C_{n1}) \\ & + (k_7 p_{n1} + k_8 p_{o1} + k_9)(p_{o1} - C_{o1}) \end{aligned} \quad (\text{A. 18})$$

Optimum decisions of the first manufacturer are calculated by first-order derivatives of his profit function as equations (A. 19) to (A. 21) show.

$$\begin{cases} \frac{\partial R_1(p_{n1}, p_{o1})}{\partial p_{n1}} = 0 \\ \frac{\partial R_1(p_{n1}, p_{o1})}{\partial p_{o1}} = 0 \end{cases} \quad (\text{A. 19})$$

$$p_{n1}^* = \frac{C_{n1}k_5^2 - k_5k_9 + 2k_6k_8 - k_7k_9 - 2C_{n1}k_4k_8 + C_{n1}k_5k_7 + C_{o1}k_5k_8 - C_{o1}k_7k_8}{k_5^2 + 2k_5k_7 + k_7^2 - 4k_4k_8} \quad (\text{A. 20})$$

$$p_{o1}^* = \frac{C_{o1}k_7^2 - k_5k_6 + 2k_4k_9 - k_6k_7 - C_{n1}k_4k_5 + C_{n1}k_4k_7 - 2C_{o1}k_4k_8 + C_{o1}k_5k_7}{k_5^2 + 2k_5k_7 + k_7^2 - 4k_4k_8} \quad (\text{A. 21})$$

3. Duopoly model II (DM-II)

3.1. Nash equilibrium

As the Hessian matrixes of the profit functions show (equations (A. 22) and (A. 23)), the profit functions (R_1 and R_2) are concave according to their own variables. Hence, the Nash equilibrium can be calculated by first-order derivatives of the profit functions. Equations (A. 24) to (A. 27) show the first-order derivatives of the profit functions.

$$H(R_1(p_{n1}, p_{o1})) = \begin{bmatrix} -2(a_{n1} + b_{n1}(\beta_n + 2)) & (b_{n1} + b_{o1}) \\ (b_{n1} + b_{o1}) & -2(a_{o1} + b_{o1}(\beta_o + 2)) \end{bmatrix} \begin{matrix} a, b, \beta > 0 \\ a > b \end{matrix} \Rightarrow H \text{ is ND} \quad (\text{A. 22})$$

$$H(R_2(p_{n2}, p_{o2})) = \begin{bmatrix} -2(a_{n2} + b_{n2}(\beta_n + 2)) & (b_{n2} + b_{o2}) \\ (b_{n2} + b_{o2}) & -2(a_{o2} + b_{o2}(\beta_o + 2)) \end{bmatrix} \begin{matrix} a, b, \beta > 0 \\ a > b \end{matrix} \Rightarrow H \text{ is ND} \quad (\text{A. 23})$$

$$\frac{\partial R_1(p_{n1}, p_{o1})}{\partial p_{n1}} = -2(a_{n1} + b_{n1}(\beta_n + 2))p_{n1} + (b_{n1} + b_{o1})p_{o1} + b_{n1}\beta_n p_{n2} + b_{n1}p_{o2} + \underbrace{(M_{n1} + C_{n1}(a_{n1} + b_{n1}(\beta_n + 2)) - C_{o1}b_{o1})}_{K_{10}} \quad (\text{A. 24})$$

$$\frac{\partial R_1(p_{n1}, p_{o1})}{\partial p_{o1}} = (b_{n1} + b_{o1})p_{n1} - 2(a_{o1} + b_{o1}(\beta_o + 2))p_{o1} + b_{o1}p_{n2} + b_{o1}\beta_o p_{o2} + \underbrace{(M_{o1} - C_{n1}b_{n1} + C_{o1}(a_{o1} + b_{o1}(\beta_o + 2)))}_{K_{11}} \quad (\text{A. 25})$$

$$\frac{\partial R_2(p_{o2})}{\partial p_{n2}} = b_{n2}\beta_n p_{n1} + b_{n2}p_{o1} - 2(a_{n2} + b_{n2}(\beta_n + 2))p_{n2} + (b_{n2} + b_{o2})p_{o2} + \underbrace{(M_{n2} + C_{n2}(a_{n2} + b_{n2}(\beta_n + 2)) - C_{o2}b_{o2})}_{K_{12}} \quad (\text{A. 26})$$

$$\frac{\partial R_2(p_{o2})}{\partial p_{o2}} = b_{o2}p_{n1} + b_{o2}\beta_o p_{o1} + (b_{n2} + b_{o2})p_{n2} - 2(a_{o2} + b_{o2}(\beta_o + 2))p_{o2} + \underbrace{(M_{o2} - C_{n2}b_{n2} + C_{o2}(a_{o2} + b_{o2}(\beta_o + 2)))}_{K_{13}} \quad (\text{A. 27})$$

In order to determine the Nash equilibrium, four equalities have to be solved, that is shown by equation (A. 28). Please note that K_{10} , K_{11} , K_{12} , and K_{13} are defined to simplify the equations, and the formulations of them are presented by equations (A. 24), (A. 25), (A. 26), and (A. 27) respectively.

$$\begin{cases} -2(a_{n1} + b_{n1}(\beta_n + 2))p_{n1} + (b_{n1} + b_{o1})p_{o1} + b_{n1}\beta_n p_{n2} + b_{n1}p_{o2} + K_{10} = 0 \\ (b_{n1} + b_{o1})p_{n1} - 2(a_{o1} + b_{o1}(\beta_o + 2))p_{o1} + b_{o1}p_{n2} + b_{o1}\beta_o p_{o2} + K_{11} = 0 \\ b_{n2}\beta_n p_{n1} + b_{n2}p_{o1} - 2(a_{n2} + b_{n2}(\beta_n + 2))p_{n2} + (b_{n2} + b_{o2})p_{o2} + K_{12} = 0 \\ b_{o2}p_{n1} + b_{o2}\beta_o p_{o1} + (b_{n2} + b_{o2})p_{n2} - 2(a_{o2} + b_{o2}(\beta_o + 2))p_{o2} + K_{13} = 0 \end{cases} \Rightarrow \begin{cases} p_{n1}^* \\ p_{o1}^* \\ p_{n2}^* \\ p_{o2}^* \end{cases} \quad (\text{A. 28})$$

3.2. Stackelberg

As it is explained before, the profit functions are concave and the rational reaction functions of the second manufacturer ($p_{n2}^*(p_{n1}, p_{o1})$, $p_{o2}^*(p_{n1}, p_{o1})$) can be calculated by first-order derivatives, as equation (A. 29) shows. By solving the equalities of equation (A. 29), $p_{n2}^*(p_{n1}, p_{o1})$, $p_{o2}^*(p_{n1}, p_{o1})$ are determined as equations (A. 30) and (A. 31), in which, k_{14} to k_{20} are defined by equation (A. 29).

$$\begin{cases} \frac{\partial R_2(p_{n2}, p_{o2})}{\partial p_{n2}} = 0 \\ \frac{\partial R_2(p_{n2}, p_{o2})}{\partial p_{o2}} = 0 \end{cases} \Rightarrow \begin{cases} b_{n2}\beta_n p_{n1} + b_{n2}p_{o1} - 2(a_{n2} + b_{n2}(\beta_n + 2))p_{n2} + (b_{n2} + b_{o2})p_{o2} + \underbrace{(M_{n2} + C_{n2}(a_{n2} + b_{n2}(\beta_n + 2)) - C_{o2}b_{o2})}_{k_{17}} = 0 \\ b_{o2}p_{n1} + b_{o2}\beta_o p_{o1} + \underbrace{(b_{n2} + b_{o2})}_{k_{18}} p_{n2} - 2(a_{o2} + b_{o2}(\beta_o + 2))p_{o2} + \underbrace{(M_{o2} - C_{n2}b_{n2} + C_{o2}(a_{o2} + b_{o2}(\beta_o + 2)))}_{k_{20}} = 0 \end{cases} \quad (\text{A. 29})$$

$$p_{n2}^*(p_{n1}, p_{o1}) = \frac{(k_{16}b_{o2} - k_{14}k_{19})p_{n1} + (k_{16}k_{18} - k_{19}b_{n2})p_{o1} + (k_{16}k_{20} - k_{17}k_{19})}{k_{15}k_{19} - k_{16}^2} \quad (\text{A. 30})$$

$$p_{o2}^*(p_{n1}, p_{o1}) = \frac{(k_{14}k_{16} - k_{15}b_{o2})p_{n1} + (k_{16}b_{n2} - k_{15}k_{18})p_{o1} + (k_{16}k_{17} - k_{15}k_{20})}{k_{15}k_{19} - k_{16}^2} \quad (\text{A. 31})$$

The best response of the second manufacturer ($p_{n2}^*(p_{n1}, p_{o1})$, $p_{o2}^*(p_{n1}, p_{o1})$) should be replaced in equation (19) in order to determine the profit of the first manufacturer. Then the optimal decision of the first manufacturer (p_{n1}^* , p_{o1}^*) according to the best rational response of the second manufacturer is determined. Equations (A. 32) and (A. 33) show equalities which determine p_{n1}^* and p_{o1}^* .

$$p_{n1}^* = \frac{\left(\begin{aligned} & p_{n2}^*(b_{o1}(b_{n1}(2\beta_n(\beta_o+2)+1)) + 2\beta_n b_{n1} a_{o1}) \\ & + p_{o2}^*(b_{o1}(\beta_o(3b_{n1}+b_{o1}) + 4b_{n1}) + 2a_{o1} b_{n1}) \\ & + C_{n1} \left(\begin{aligned} & 2a_{o1} b_{n1} (2+\beta_n) + b_{n1} b_{o1} (4(\beta_n+\beta_o) + 2\beta_o \beta_n + 7) \\ & + 2a_{n1} (b_{o1}(\beta_o+2) + a_{o1}) - b_{n1}^2 \end{aligned} \right) \\ & + C_{o1} (b_{o1}(b_{n1}-b_{o1})(\beta_o+2) + a_{o1}(b_{n1}-b_{o1})) \\ & + 2M_{n1} (b_{o1}(\beta_o+2) + a_{o1}) + M_{o1} (b_{n1}+b_{o1}) + 4\beta_n b_{n1} b_{o1} \end{aligned} \right)}{\left(\begin{aligned} & 4(a_{o1}+2b_{o1})(a_{n1}+\beta_n b_{n1}) + 4\beta_o b_{o1} (a_{n1}+b_{n1}(\beta_o+2)) \\ & + 2b_{n1} (4a_{o1}+7b_{o1}) - b_{n1}^2 - b_{o1}^2 \end{aligned} \right)} \quad (\text{A. 32})$$

$$p_{o1}^* = \frac{\left(\begin{aligned} & p_{n2}^*(b_{n1}(b_{o1}(2\beta_o(\beta_n+2)+1)) + 2\beta_o b_{o1} a_{n1}) \\ & + p_{o2}^*(b_{n1}(\beta_n(3b_{o1}+b_{n1}) + 4b_{o1}) + 2a_{n1} b_{o1}) \\ & + C_{o1} \left(\begin{aligned} & 2a_n b_{o1} (2+\beta_o) + b_{o1} b_{n1} (4(\beta_o+\beta_n) + 2\beta_o \beta_n + 7) \\ & + 2a_{o1} (b_{n1}(\beta_n+2) + a_{n1}) - b_{o1}^2 \end{aligned} \right) \\ & + C_{n1} (b_{n1}(b_{o1}-b_{n1})(\beta_n+2) + a_{n1}(b_{o1}-b_{n1})) \\ & + 2M_{o1} (b_{n1}(\beta_n+2) + a_{n1}) + M_{n1} (b_{o1}+b_{n1}) + 4\beta_o b_{o1} b_{n1} \end{aligned} \right)}{\left(\begin{aligned} & 4(a_{n1}+2b_{n1})(a_{o1}+\beta_o b_{o1}) + 4\beta_n b_{n1} (a_{o1}+b_{o1}(\beta_n+2)) \\ & + 2b_{o1} (4a_{n1}+7b_{n1}) - b_{o1}^2 - b_{n1}^2 \end{aligned} \right)} \quad (\text{A. 33})$$

Appendix B: Parameter estimation of the case study

In this section calculation of the case study for eliminating inflation rate, and seasonal factor is explained as well as the heuristic method which is used for parameter estimation.

The inflation rate affects selling prices and manufacturing costs. Besides, as it is mentioned before, the demand for the textile products alters continuously as season changes. Hence, the impacts of the inflation rate and seasons should be eliminated before parameter estimation. The inflation rate of Iran between 2012 to 2017 is presented by Table B1 [25].

The seasonal factor is implemented in order to eliminate the impact of seasons. One month is determined as an origin month, and the demands of other months are compared with the origin month, in order to show how the demands change during different months. The seasonal factor is determined as a ratio of the average demand of each month over the average demand of April (as the origin month), equation (B. 1) represents the calculation of the seasonal factor, and Table B2 presents the seasonal factors determined for each month.

$$\text{Seasonal factor}_m = \frac{\text{Average demand of month } (m)}{\text{Average demand of "April"}} \quad (\text{B. 1})$$

****Please insert Table B1 about here****

****Please insert Table B2 about here****

In order to modify the datasets by using seasonal factor (to modify demand) and inflation rate (to modify costs and prices), equations (B. 2) and (B. 3) are employed. Equation (B. 3) is a common formula that is being used for calculating future value (FV) of a present payment (PV), in which the inflation rate of period t is equal to i_t and n is the number of periods [20].

$$\text{modified } (D_{ij}^s) = \frac{D_{ij}^s}{\text{related seasonal factor}} \quad (\text{B. 2})$$

$$FV = PV \times \prod_{t=1}^n (1+i_t)^n \quad (\text{B. 3})$$

There are two basic methods for parameters estimation. The first method is market research, which requires a long time process. The second method is estimating the parameters of the demand functions by the collected dataset. Although the second method is less accurate than the first method, it requires fewer resources and time. We prefer to use the second method for parameters estimation because we do not intend to focus on the market research methods here.

After modification of the raw data, in order to eliminate the impact of the inflation rate and seasonal effect, we should estimate parameters of the proposed model. The demand functions are multivariable functions. Besides, they are not regular polynomial functions and some of the parameters are common between the demand functions and they should be estimated simultaneously. Hence, regular regression methods are not appropriate for this case and we developed a heuristic method in order to estimate the parameters. The proposed heuristic approach is explained step by step as follows:

Step 1: Determine lower bound and upper bound for each parameter (usually a_{ij} & $b_{ij} \in [0, 0.5]$, $\beta_j \in [0, 2]$, and the ranges of production cost (C_{ij}) and market size (M_{ij}) can be easily estimated by previous data).

Step 2: Divide the determined window of each parameter into 1000 equal sections.

Step 3: Calculate the demand functions for each set of predetermined parameters (middle of the mentioned sections).

Step 4: Calculate mean square errors (as equation (B. 4) shows) for each set of predetermined parameters. Determine the minimum mean square error (MMSE).

Step 5: Compare the MMSE with MSE that is evaluated by the previous iteration. If it was improved less than 0.01%, save the related parameters and **stop**. Else, narrow lower bound and upper bound of parameters and go to **step 2** in order to increase the accuracy of parameter estimation.

Please note that we can increase the accuracy of the heuristic method if it is needed, but calculation time increases too. As it is mentioned, equation (B. 4) determines the mean square error. In which, S is the number of datasets, D_{ij}^s is actual demand of product i produced by manufacturer j achieved by dataset s , and D_{ij}^{*k} is estimated demand of product i produced by manufacturer j estimated by k^{th} predetermined parameters set.

$$MSE_k = \frac{\sum_{s=1}^S \left(\left(D_{n1}^s - D_{n1}^{*k} \right)^2 + \left(D_{o1}^s - D_{o1}^{*k} \right)^2 + \left(D_{n2}^s - D_{n2}^{*k} \right)^2 + \left(D_{o2}^s - D_{o2}^{*k} \right)^2 \right)}{4 \times S} \quad (\text{B. 4})$$

Usually, these brands introduce their products simultaneously (DM-II condition). Hence, the data set of the first and second conditions are less than the DM-II condition. That is why the accuracy of the estimation of the parameters for the third condition is higher than the other conditions.

Figure B1, Figure B2, and Figure B3 indicates the accuracy of parameters' estimation for the first, second, and third conditions respectively.

The raw dataset, modified dataset, modification codes, parameter estimation code, and solving codes are presented by supplementary data.

****Please insert Figure B1 about here****

****Please insert Figure B2 about here****

****Please insert Figure B3 about here****

List of Tables

Table 1. The research gaps of the NPD literature.

Article	Year	Number of manufacturers	Conditions			Solving approach			Decision variables					
			Monopoly	Non-monopoly (separate product launch)	Non-monopoly (simultaneous product launch)	Game theory (without sharing information)	Game theory (shared information)	Other approaches	Selling price of new product	Selling price of prior product	Time	Service/ Quality/ Technology levels	Considering SLC products	
[26]	1964	>2			✓	✓					✓			
[1]	2004	2		✓		✓				✓			✓	
[13]	2009	>2		✓		✓	✓			✓				
[27]	2010	1		✓				✓		✓				
[28]	2010	1		✓			✓			✓				
[29]	2015	2		✓		✓				✓			✓	
[11]	2015	1	✓			✓					✓		✓	
[30]	2015	2			✓	✓	✓						✓	
[31]	2016	1	✓				✓						✓	
[14]	2016	1		✓	✓						✓			
[32]	2017	2			✓		✓				✓			
[33]	2018	1	✓					✓					✓	
[34]	2018	1	✓			✓	✓			✓				
[35]	2019	1	✓				✓			✓				
[36]	2019	1	✓					✓					✓	
[37]	2019	1	✓				✓			✓			✓	
[38]	2020	1	✓					✓					✓	
[39]	2020	1	✓			✓	✓			✓				
[40]	2020	1	✓					✓		✓				
[41]	2020	1	✓				✓			✓				
[42]	2020	1	✓				✓			✓				
[43]	2020	1	✓			✓				✓				
[44]	2020	1	✓				✓			✓				
[45]	2020	1	✓				✓			✓			✓	
[46]	2021	1	✓				✓			✓				
[47]	2021	2		✓			✓			✓				
[48]	2021	1	✓			✓				✓				
[49]	2021	1	✓				✓			✓				
This research		2	✓	✓	✓	✓	✓			✓	✓			✓

Table 2. Parameters of the basic model for sensitivity analysis.

Parameter Value	M_{n1}	M_{n2}	M_{o1}	M_{o2}	a_{n1}	a_{n2}	a_{o1}	a_{o2}	b_{n1}
	2212	1933	2203	1677	0.0114	0.0371	0.3074	0.0278	0.0496
Parameter Value	b_{n2}	b_{o1}	b_{o2}	C_{n1}	C_{n2}	C_{o1}	C_{o2}	β_n	β_o
	0.4982	0.0398	0.0510	8230	8048	6008	7449	1.9066	1.667

Table 3. Results of the statistical comparison.

Model	Solving method	Mean of Profits			Standard deviation of Profits	
		Total	R_1^*	R_2^*	R_1^*	R_2^*
MM	Nash	8.466E+06	8.466E+06	-	1.010E+07	-
DM	Nash	8.765E+06	5.240E+06	3.5249E+06	6.519E+06	4.4412E+06
	Stackelberg	9.302E+06	5.307E+06	3.9946E+06	6.570E+06	5.1706E+06
DM-II	Nash	1.152E+07	5.925E+06	5.5951E+06	9.524E+06	6.9833E+06
	Stackelberg	1.309E+07	6.232E+06	6.8632E+06	1.103E+07	9.3294E+06

Table 4. The estimated values of the parameters.

Parameters	Conditions		
	DM _G (Golriz)	DM _N (Nono)	DM-II
M_{n1}	2056.2126	-	1879.0941
M_{n2}	-	3036.0149	2359.1181
M_{o1}	1998.1320	1896.4957	1683.5095
M_{o2}	2725.8221	2764.1493	2120.3434
a_{n1}	0.0497	-	0.0477
a_{n2}	-	0.0898	0.0638
a_{o1}	0.1313	0.1287	0.1032
a_{o2}	0.0707	0.0707	0.0476
b_{n1}	0.0640	-	0.0899
b_{n2}	-	0.1672	0.1578
b_{o1}	0.0000	0.0576	0.0475
b_{o2}	0.2714	0.2245	0.1164
β_n	1.0375	1.0375	0.6421
β_o	1.0358	0.2414	2.7512

Table 5. The results of the case study.

Condition	Solving method	Profits ($\times 10^6$)		Selling prices ($\times 10^4$)				Demands			
		R_1^*	R_2^*	P_{n1}^*	P_{o1}^*	P_{n2}^*	P_{o2}^*	D_{n1}	D_{o1}	D_{n2}	D_{o2}
DM _G	Nash	8.534	12.810	1.423	1.262	-	1.173	1086	341	-	2825
	Stackelberg	8.569	13.849	1.460	1.306	-	1.191	1061	282	-	2938
DM _N	Nash	2.981	18.146	-	1.090	1.483	1.469	-	773	1022	1552
	Stackelberg	3.094	18.200	-	1.098	1.526	1.501	-	787	907	1539
DM-II	Nash	6.990	10.952	1.226	1.058	1.198	1.099	1013	791	1249	1727
	Stackelberg (Golriz as the leader)	7.223	13.263	1.294	1.152	1.231	1.139	959	582	1365	1909
	Stackelberg (Nono as the leader)	8.017	11.304	1.253	1.086	1.264	1.186	1077	858	1136	1406

Table 6. The comparison between the optimal and real situations.

Conditions	Solutions	Variables									
		R_1 ($\times 10^6$)	R_2 ($\times 10^6$)	P_{n1} ($\times 10^4$)	P_{o1} ($\times 10^4$)	P_{n2} ($\times 10^4$)	P_{o2} ($\times 10^4$)	D_{n1}	D_{o1}	D_{n2}	D_{o2}
DM _G	Average of real values	7.76	11.75	1.39	1.24	-	1.09	1076	365	-	3188
	Nash optimum solution	8.53	12.81	1.42	1.26	-	1.17	1086	341	-	2825
	Stackelberg optimum solution	8.57	13.85	1.46	1.31	-	1.19	1061	282	-	2938
DM _N	Average of real values	1.45	13.17	-	1.15	1.16	1.00	-	404	1693	2906
	Nash optimum solution	2.98	18.15	-	1.09	1.48	1.47	-	773	1022	1552
	Stackelberg optimum solution	3.09	18.20	-	1.10	1.53	1.50	-	787	907	1539
DM-II	Average of real values	5.32	14.55	1.38	1.23	1.24	1.08	741	302	1410	2632
	Nash optimum solution	6.99	10.95	1.23	1.06	1.20	1.10	1013	791	1249	1727
	Stackelberg optimum solution (M_1 as leader)	7.22	13.26	1.29	1.15	1.23	1.14	959	582	1365	1909
	Stackelberg optimum solution (M_2 as leader)	8.02	11.30	1.25	1.09	1.26	1.19	1077	858	1136	1406

Table 7, Results of the trial experiment.

Data/Results	Variables					
	R_1 ($\times 10^6$)	R_2 ($\times 10^6$)	D_{n1}	D_{o1}	D_{n2}	D_{o2}
Average of historical data	5.32	14.55	741	302	1410	2632
Estimated value (determined by the model)	6.98	10.78	1015	787	993	1915
Actual results of the trial experiment	7.01	10.60	1046	759	711	2074
Percentage of estimation error	0.39%	-1.68%	3%	-4%	-28%	8%
Percentage of improvement	32%	-27%	-	-	-	-

Table B1, Inflation rate of Iran.

Year-Month	2012-12	2013-12	2014-12	2015-12	2016-12	2017-12
Point to point inflation rate	25.7%	39.3%	17.2%	13.7%	8.6%	10.0%

Table B2, Seasonal factors of each month.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Seasonal factor	0.33	0.24	1.05	1.00	0.94	0.84	0.81	0.89	0.89	0.76	0.43	0.31

List of Figures

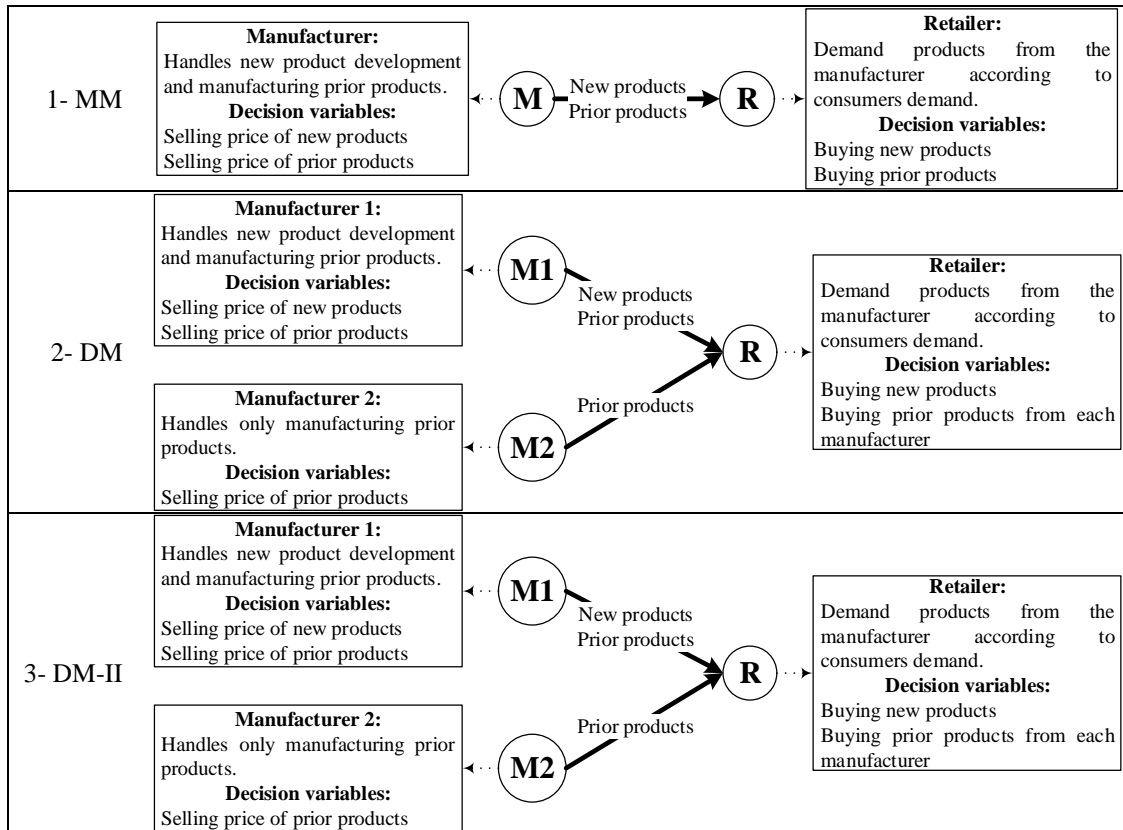


Figure 1, The proposed models.

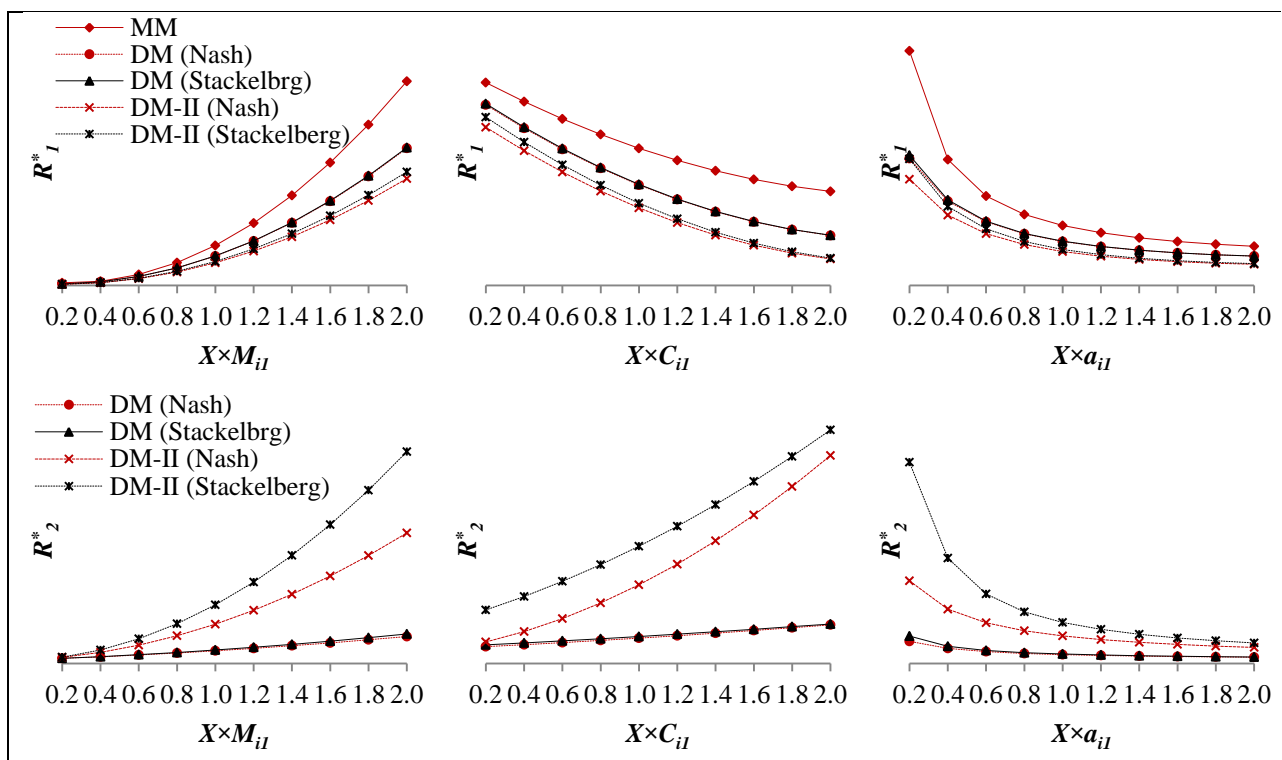


Figure 2, The sensitivity of the optimal profit functions to M_{ii} , a_{ii} , C_{ii} .

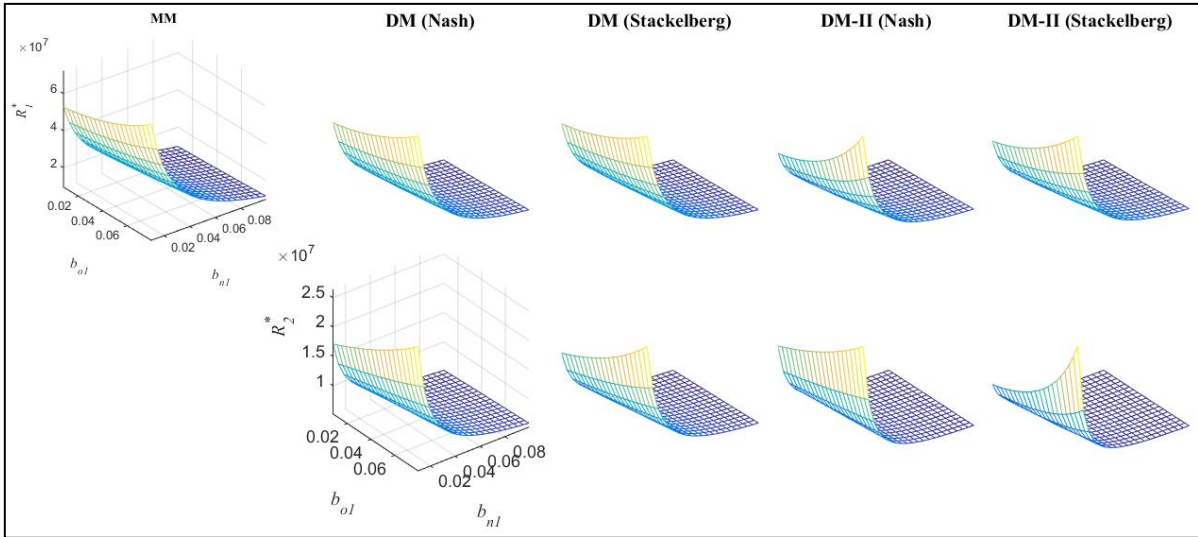


Figure 3, The sensitivity of the optimal profits to b_{il} .

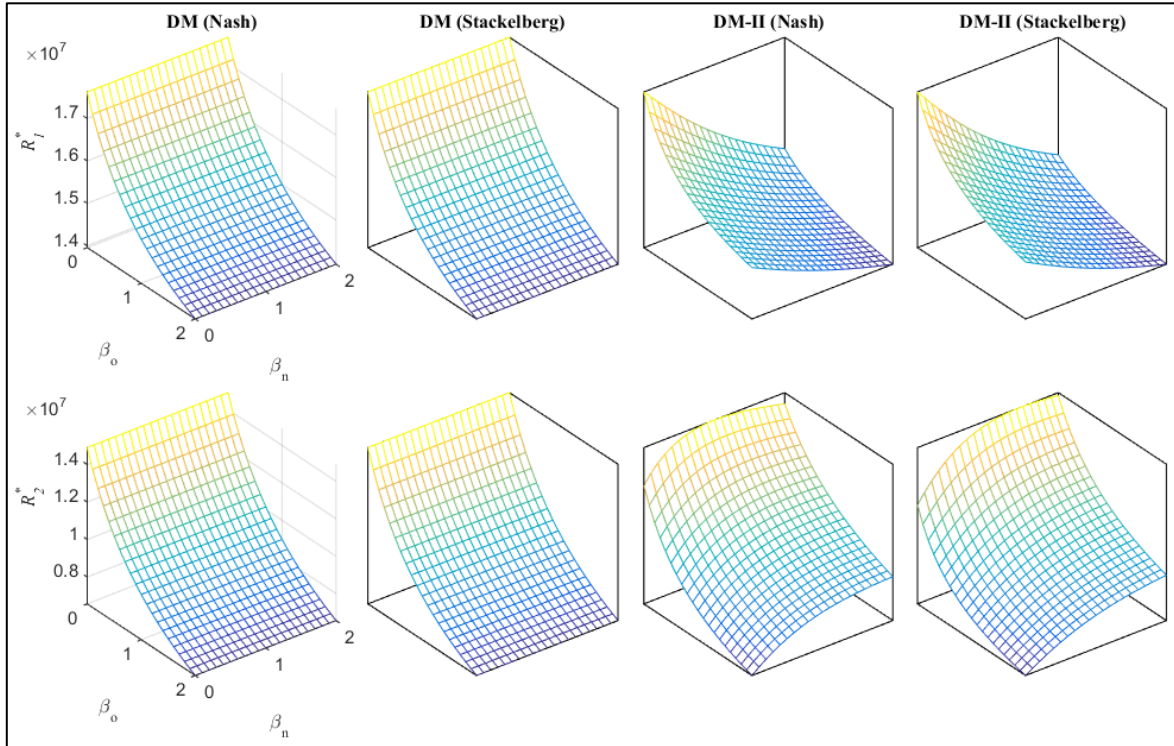


Figure 4, The sensitivity of the profits to β_i .

		Nono Strategies	
		S_1	S_2
Golroz Strategies	S_1	$(6.99, 10.952)$	$(8.534, 12.81)$
	S_2	$(2.981, 18.146)$	$(2.981, 12.81)$

$(R_1, R_2) \times 10^6$

Figure 5, The nonzero-sum matrix of strategies (Nash).

		Nono Strategies	
		S_1	S_2
Golriz Strategies	S_1	(7.62 , 12.283)	(8.569 , 13.849)
	S_2	(3.094 , 18.2)	(3.094 , 13.849)
		$(R_1, R_2) \times 10^6$	

Figure 6, The nonzero-sum matrix of strategies (Stackelberg).

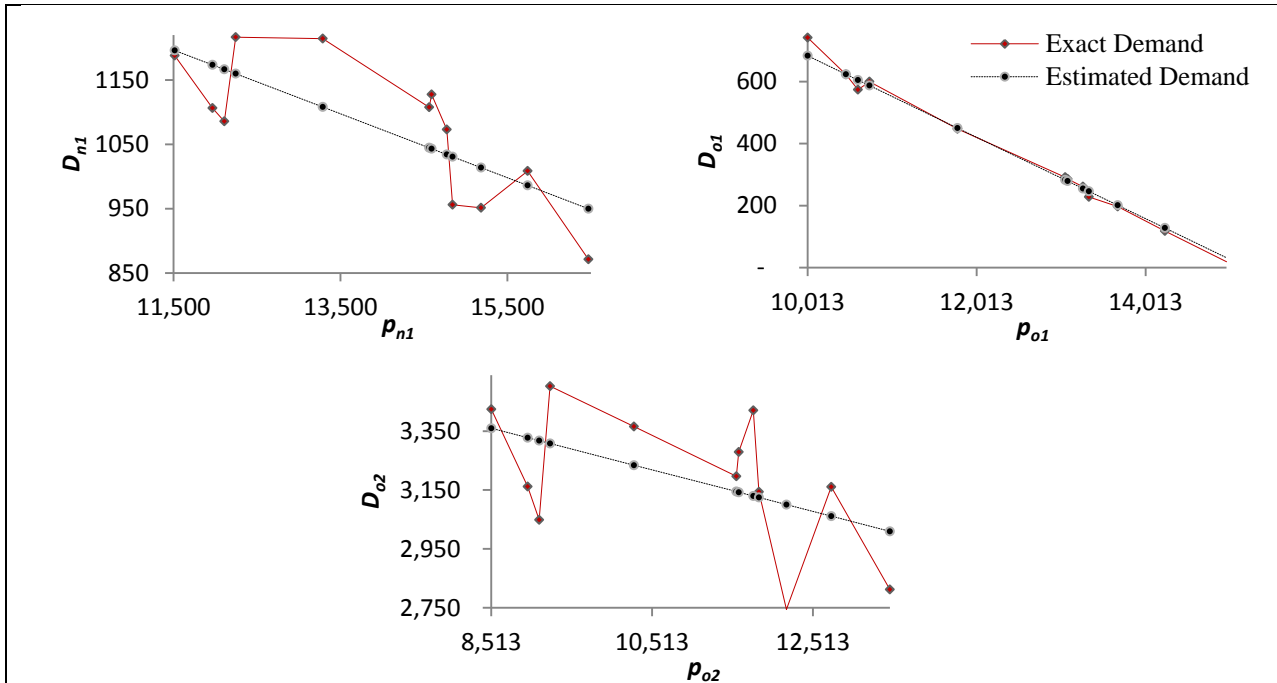


Figure B1, Evaluation of the estimation of the parameters for the first condition.

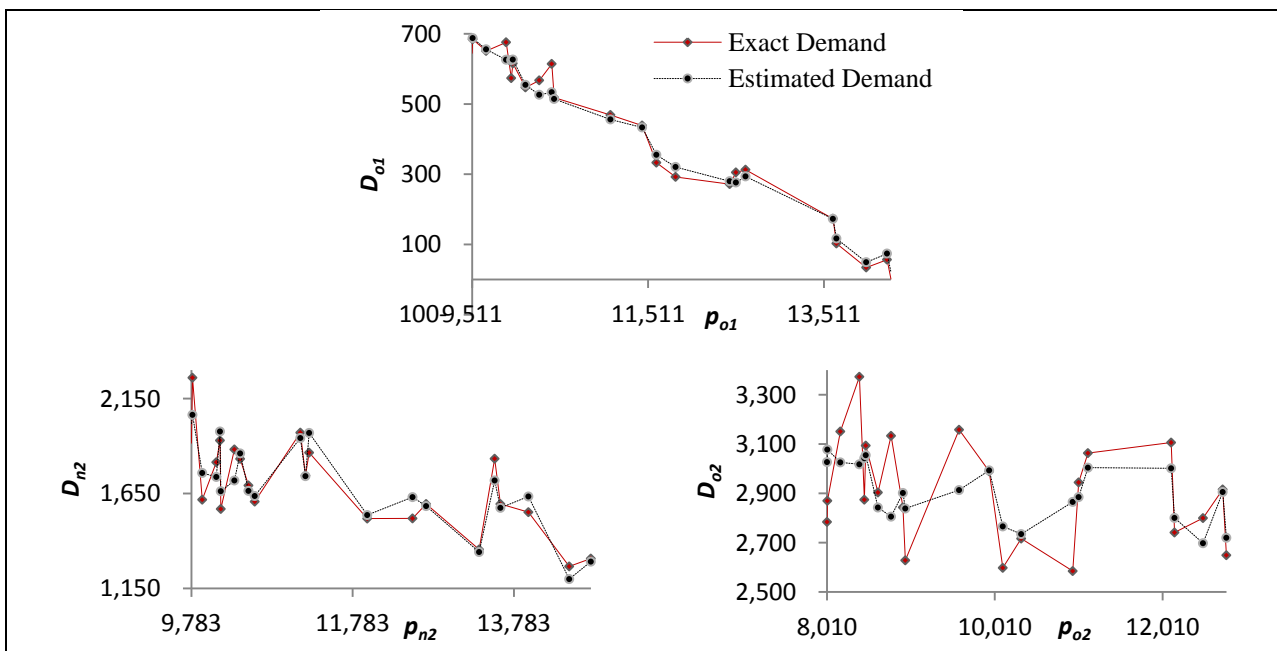


Figure B2, Evaluation of the estimation of the parameters for the second condition.

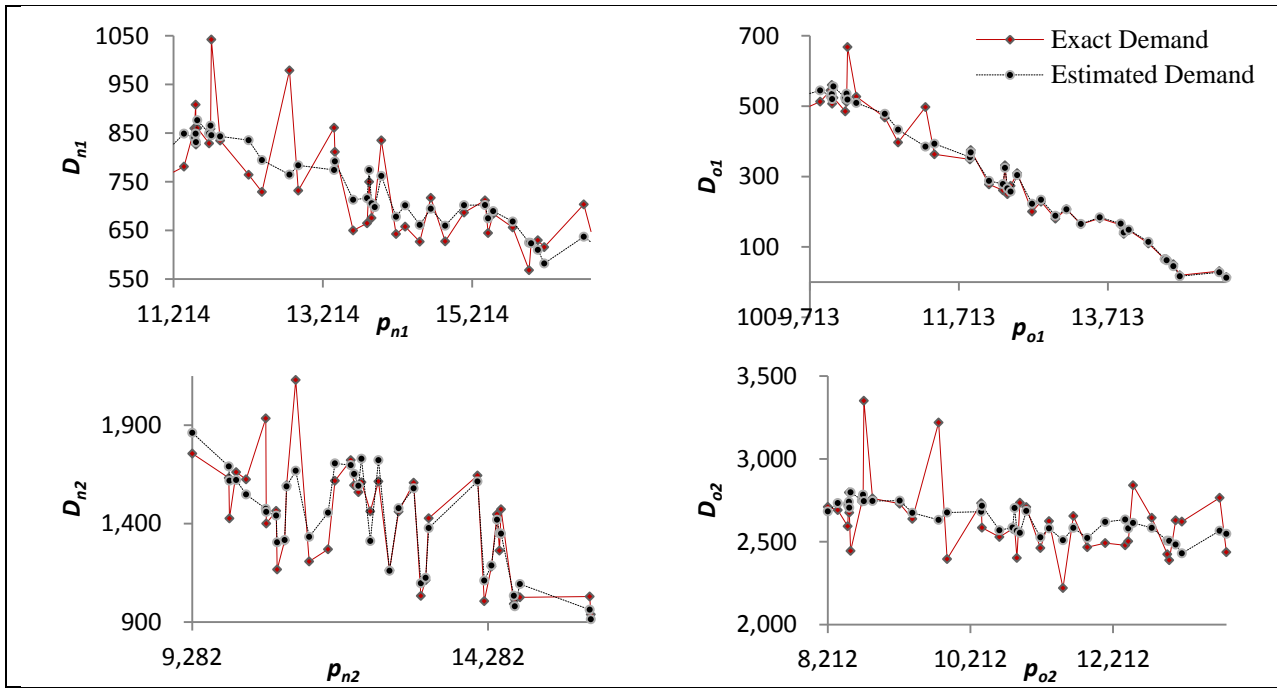


Figure B3, Evaluation of the estimation of the parameters for the third condition.

Biographical note

Dr. Alireza Taheri-Moghadam received his PhD in Industrial Engineering from University of Tehran. His research interests are mainly on sustainable supply chain management, pricing, logistics, operations research, mathematical programming, and meta-heuristic algorithms.

Prof. Fariborz Jolai is currently a professor of Industrial Engineering at College of Engineering, University of Tehran, Tehran, Iran. He obtained his Ph.D. degree in Industrial engineering from INPG, Grenoble, France in 1998. He completed his B.Sc. and M.Sc. in Industrial Engineering at Amirkabir University of Technology, Tehran, Iran. His current research interests are scheduling and production planning, supply chain modeling, optimization problems under uncertainty conditions.

Prof. Jafar Razmi is a Professor in the School of Industrial Engineering at the University of Tehran, Tehran, Iran. He teaches undergraduate and graduate courses in industrial engineering, operations management, and MS. He has published over 70 papers in peer-reviewed journals and published more than 70 papers in international conferences. He is in the editorial board of several academic journals. His research interests include supply chain management, operations management, production planning and control, lean manufacturing, and manufacturing measurement and evaluation.

Prof. Ata-Allah Taleizadeh is an Associate Professor in School of Industrial Engineering, College of Engineering, at University of Tehran in Iran. He received his PhD in Industrial Engineering from Iran University of Science and Technology. Moreover, he received his BSc and MSc degrees both in Industrial Engineering from Islamic Azad University of Qazvin and Iran University of Science and Technology, respectively. His research interest areas include inventory control and production planning, pricing and revenue optimization, and Game theory. He has published books, chapter books, and several papers in reputable journals and serves as an editor/editorial board member for a number of international journals.