A New Hybrid Analytical Model Based on Winding Function Theory for Analysis of Unbalanced Two-Phase Induction Motors

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Abstract

The purpose of this paper is to present a new hybrid analytical model (HAM) based on winding function theory (WFT) for electromagnetic analysis of performance of one typical unbalanced two-phase induction motor (UTPIM). Different indexes of electromagnetic modeling such as winding distribution, slotting effect, and magnetic saturation can be accurately considered by using the proposed HAM. For obtaining this new hybrid technique, WFT is reformulated for considering the magnetic saturation in addition to the influence of slotting and winding distribution. The conformal mappings (CMs) are used to accurately calculate the slotted air-gap length. The magnetic equivalent circuit (MEC) model is used to consider the magnetomotive force (MMF) drop in stator and rotor cores due to the excitation of one phase-winding. The obtained results of CMs and MEC are then utilized in reformulated WFT to calculate the inductances of respective phase-winding. Transient analysis is then done to calculate the indexes of performance such as air-gap magnetic field, phase currents, electromagnetic torque, and rotor speed by using the lookup table of inductances while considering different capacitors in auxiliary phase. In each step, the accuracy of analytical results is confirmed by comparing with corresponding results obtained from finite element method (FEM).

Index Terms: air-gap; conformal mapping (CM); hybrid analytical model (HAM); inductance; magnetic field; magnetic saturation.

1. Introduction

Unbalanced two-phase induction motors (UTPIMs) are widely used in domestic appliances and low power applications. However, due to their unbalanced structure, the performance of UTPIM is affected by the backward rotating magnetic field. For this reason, the electromagnetic
modeling and analysis is necessary for careful and thorough examination of UTPIMs with different run capacitors. Various techniques can be used for modeling and analysis of UTPIMs such as finite element method (FEM) [1-2], winding function theory (WFT) [3-4], magnetic equivalent circuit (MEC) [5-6], sub-domain (S-D) model [7-8], conformal mapping (CM) method [9-11], and field reconstruction method (FRM) [12-13]. FEM is an accurate technique. However, FEM is a time-consuming method and it is conventionally used to verify the analytical results in final stage. WFT is an analytic technique to calculate the inductance matrix. However, the conventional WFT cannot consider the slotting effect and magnetic saturation. MEC model is an accurate technique for considering the magnetic saturation in iron parts. However, the air-gap region is not accurately considered by MEC model. In S-D model, the motor geometry is divided into several domains, for solving the laplace or poisson equations in each domain separately. Therefore, the computational burden of S-D model is high. CM can be used to calculate the components of flux density in slotted air-gap. However, the conventional CM cannot consider the magnetic saturation in ferromagnetic core. FRM acts based on applying the law of superposition on the basis functions obtained through FEM.

According to above explanations, the analytical techniques have some drawbacks for analysis of electric machines. For this reason, in recent years, hybrid analytical models (HAMs) have been presented which can simultaneously consider the capabilities of different techniques for accurate modeling of electric machines [14-19]. HAM based on CM and MEC models has been used to model and analysis of permanent-magnet synchronous motors in [14-17], so that the air-gap region and iron parts have been modeled by using CM and MEC, respectively. In this paper, the HAM based on WFT and MEC [18-19] is extended for electromagnetic modeling and analysis of one typical UTPIM. In real, this HAM acts based WFT which is assisted by MEC model. The main property of WFT is its easy implementation. However, the accuracy of WFT depends on the modeling accuracy of air-gap length and magnetic saturation. To this end, a virtual air-gap length function was presented in [20-22] to consider the magnetic saturation by WFT. The air-gap length function with stepped form was used in WFT while considering the slot depth of stator and rotor [23]. It is obvious that the technique presented in [23] cannot consider the real paths of flux tubes in slotted air-gap. To solve this problem in [24-25], some FEM analysis was used to calculate the air-gap permeance while considering the effect of slots and salient poles. The effect of magnetic saturation in [23] is also considered through increasing the
air-gap length under each tooth in proportion to the magneto-motive force (MMF) drop in relevant tooth. WFT has been also used to model and analyze the bearing fault through considering a suitable air-gap length function [26-27].

This paper proposes a new HAM based on WFT for transient and magnetic field analysis of one typical UTPIM, which helps from the MEC model and CMs for considering the MMF drops in iron parts and slotting effect, respectively. The organization of this paper is as follows: The proposed HAM based on WFT is introduced in section 2. The results of inductance matrix obtained through proposed HAM are presented in section 3. Transient analysis to calculate the phase currents is done in section 4. The conclusions of this work are presented in section 5.

Figure 1

2. HAM based on WFT

WFT was firstly proposed in [28] for calculating of inductance matrix through following famous formula

\[
L_{B,A} = \mu_0 r L \int_0^{2\pi} \frac{n_B(\alpha) N_A(\alpha)}{g(\alpha)} d\alpha
\]

where \( r \) is the radius in the middle of air-gap, \( L \) is the core length in axial direction, \( \alpha \) is circumferential position in air-gap, \( n_A \) is the turn function of phase A, \( N_B \) is the winding function of phase B, \( g(\alpha) \) is the distribution of air-gap length, and \( L_{A,B} \) is the element of inductance matrix between phases A and B.

Figure 1 shows a closed path in the zoomed view of motor geometry. The ampere circuit law is written on this closed path while considering the MMF drops in iron parts as [18]

\[
\oint H_A \cdot dl = n_A(\alpha) I_A - n_{sat,A}(\alpha)
\]

\[
\oint H_A \cdot dl = H_A(\alpha) g(\alpha) - H_A(0) g(0)
\]

\[
H_A(\alpha) = \frac{n_A(\alpha) I_A - n_{sat,A}(\alpha) + H_A(0) g(0)}{g(\alpha)}
\]

\( n_{sat,A}(\alpha) \) represents the distribution of MMF drops enclosed in the closed path abcd due to the excitation of only phase A with \( I_A \). \( n_{sat,A}(\alpha) \) is the virtual turn function of one virtual winding with one ampere current.
The Gauss’s law is written on the lateral surface of one cylinder in the middle of air-gap

$$\oint_S \mu_0 H_A(\alpha) \, d\mathbf{S} = 0 \rightarrow \int_0^{2\pi} H_A(\alpha) \, d\alpha = 0 $$  \hspace{1cm} (5)

By replacing (4) in (5)

$$H_A(0) g(0) = \frac{-2\pi \int_0^{2\pi} [n_A(\alpha) I_A - n_{sat,A}(\alpha)] \, d\alpha}{\int_0^{2\pi} \frac{I}{g(\alpha)} \, d\alpha} $$  \hspace{1cm} (6)

where $g(\alpha)$ is the distribution of air-gap length.

Then, (4) is written as

$$H_A(\alpha) = \frac{I_A}{g(\alpha)} n_A(\alpha) - \frac{2\pi}{\int_0^{2\pi} \frac{I}{g(\alpha)} \, d\alpha} n_{sat,A}(\alpha) + \frac{2\pi}{\int_0^{2\pi} \frac{I}{g(\alpha)} \, d\alpha} n_{sat,A}(\alpha) $$  \hspace{1cm} (7)

“Winding function” and “virtual winding function” are defined as follows:

$$N_A(\alpha) = n_A(\alpha) - \frac{2\pi}{\int_0^{2\pi} \frac{I}{g(\alpha)} \, d\alpha} $$  \hspace{1cm} (8)

$$N_{sat,A}(\alpha) = n_{sat,A}(\alpha) - \frac{2\pi}{\int_0^{2\pi} \frac{I}{g(\alpha)} \, d\alpha} $$  \hspace{1cm} (9)

The air-gap flux density and the flux-linkage with phase winding $B$ due to the excitation of only phase $A$ is calculated as follows:
\[ B_A(\alpha) = \frac{\mu_0}{g(\alpha)} \left[ N_A(\alpha)I_A - N_{sat,A}(\alpha) \right] \] (10)

\[ \lambda_{B,A} = \mu_0 r L \int_0^{2\pi} \frac{n_B(\alpha)}{g(\alpha)} \left[ N_A(\alpha)I_A - N_{sat,A}(\alpha) \right] d\alpha \] (11)

The mutual inductance between phase windings A and B is then calculated as

\[ L_{B,A} = \frac{\lambda_{B,A}}{I_A} \rightarrow L_{B,A} = \mu_0 r L \int_0^{2\pi} \frac{n_B(\alpha)}{g(\alpha)} \left[ N_A(\alpha) - \frac{N_{sat,A}(\alpha)}{I_A} \right] d\alpha \] (12)

\( N_{sat,A}(\alpha) \) depends on the non-linear characteristic of ferromagnetic core and the excitation of phase A \((I_A)\). Consequently, \( L_{B,A} \) is a non-linear function of \( I_A \), and a linear function of \( n_B(\alpha) \), \( N_A(\alpha) \), and \( g(\alpha) \).

2.1. Calculation of air-gap length function

The flux tubes in slotted air-gap have both components of radial and tangential components. To calculate the magnetizing component of inductances, \( g(\alpha) \) only considers the air-gap length for radial component of flux tubes. To this end, the radial component of specific complex permeance of air-gap due to stator and rotor slots is separately calculated by using CMs. The main parameters of analyzed UTPIM are shown in Table 1.

Table 1

The zoomed view of air-gap geometries including only the rotor slots are shown in Figure 2. Following CMs are respectively used to transform the geometry in S-domain into Z-domain, W-domain, and Ψ-domain.

\[ z = \log(s) \] (13)

\[ z = f(w) = A \prod_{w_0}^{w_l} \left( w - w_k \right)^{\gamma_k - 1} dw + C \] (14)

\[ w = \frac{j \left\{ \log(y) \frac{\Delta x}{2\pi} + \frac{\Delta y}{2} - j \frac{\Delta x}{2} \right\} }{2} \] (15)

CM (14) is called the Schwarz-Christoffel (S-C) mapping, which is used to map the slotted geometry in Z-domain into one canonical rectangle in W-domain by using the S-C Toolbox [29].
Since the air-gap length of UTPIMs is small, it is sufficient that the air-gap complex permeance is calculated in the middle of air-gap [19]. For this reason, one contour is considered in the middle of air-gap in S-domain as follows:

\[ S_g = R_g e^{j\alpha} \quad (16) \]

where \( R_g \) is the radius of contour in air-gap, and \( \alpha \) is the angular position of relevant point on the contour.

The corresponding contours in Z-domain, W-domain, and \( \Psi \)-domain are defined as [29]

\[ Z_g = \log(R_g) + j\alpha \quad (17) \]
\[ W_g = \text{evalinv}(f, Z_g) \quad (18) \]
\[ \Psi_g = e^{-j\left(\frac{2\pi}{\Delta x}\left(W_g - \frac{\Delta x}{2} - j \frac{\Delta y}{2}\right)\right)} \quad (19) \]

The complex permeance on this contour in the middle of air-gap (considering only the rotor slots) is calculated as

\[ \Lambda_{rotor}(\alpha, \theta_r) = \left(\frac{\partial z}{\partial S} \times \frac{\partial w}{\partial z} \times \frac{\partial \psi}{\partial w}\right)_{S_g, W_g, \Psi_g} \quad (20) \]
\[ \frac{\partial z}{\partial S} = \frac{1}{S_g} \quad (21) \]
\[ \frac{\partial w}{\partial z} = \text{evaldiff}(f, W_g) \quad (22) \]
\[ \frac{\partial \psi}{\partial w} = -j\left(\frac{2\pi}{\Delta x}\right)e^{-j\left(\frac{2\pi}{\Delta x}\left(W_g - \frac{\Delta x}{2} - j \frac{\Delta y}{2}\right)\right)} \quad (23) \]

The specific complex permeance of air-gap due to only the rotor slots is then calculated as

\[ \lambda_{rotor}(\alpha, \theta_r) = \frac{\Lambda_{rotor}}{\Lambda_{slotless}} \bigg|_{S_g, W_g, \Psi_g} \quad (24) \]

where \( \Lambda_{slotless} \) is the complex permeance of slotless air-gap.

\( \Lambda_{slotless} \) is calculated as
\[
\Lambda_{\text{slotless}} = \frac{2}{R_r + R_s}
\]  

(25)

where \( R_s \) and \( R_r \) are respectively the inner radius of stator and outer radius of rotor.

In similar way, the specific complex permeance of air-gap due to only the stator slots \( (\lambda_{\text{stator}}(\alpha)) \) can be calculated. Finally, the distribution of slotted air-gap length function is calculated as follows:

\[
g(\alpha, \theta_r) = \frac{g_{\text{slotless}}}{\text{real}\left(\lambda_{\text{stator}}(\alpha)\right) \times \text{real}\left(\lambda_{\text{rotor}}(\alpha, \theta_r)\right)}
\]  

(26)

where \( g_{\text{slotless}} = R_s - R_r \).

The distribution of slotted air-gap length for two typical rotor positions is shown in Figure 3.

Figure 3

2.2. Calculation of \( N_{\text{sat}, A}(\alpha) \)

To calculate the virtual winding function, a non-linear MEC model of analyzed UTPIM (Figure 4) should be prepared for calculating the MMF drops in different parts of stator and rotor cores while considering the relevant operating point. A non-linear algebraic equation system is obtained using the node equations for every rotor position:

\[
\begin{align*}
A(x)X &= B \\
\mu_r &= f(H)
\end{align*}
\]  

(27)

where ‘X’ is the matrix of scalar magnetic potential of nodes, matrix of \( A(x) \) depends on non-linear permeances, and matrix ‘B’ is including the MMF sources of stator and rotor. The relative permeability of the core \( (\mu_r) \) is a non-linear function of magnetic field intensity \((H)\). To solve this non-linear equation system, the Newton-Raphson method has been used to calculate the matrix of ‘X’ for every rotor position. The steps of Newton-Raphson method is presented briefly as follows:

a) (27) is reformed as follows:

\[
g(X) = A(x)X - B = 0
\]  

(28)

b) Non-linear equation system \( g(X) = 0 \) is transformed into a linear equation system by using
Jacobian matrix in iteration form (k\textsuperscript{th} iteration):

\[ J^k \Delta X^k = -g(X^k) \]  \tag{29} 

where \( \Delta X^k = X^{k+1} - X^k \)

c) Linear equation system \( J^k \Delta X^k = -g(X^k) \) is solved iteratively. The iteration is terminated when the magnitude of the incremental vector \( (\Delta X^k) \) becomes zero within a specified tolerance, or a prespecified maximum number of iterations are reached. Here, the convergence criteria is considered as follows:

\[ |\Delta X^k| \leq 0.001 \]

After obtaining the distribution of scalar magnetic potential in all nodes for relevant operating point, the magnetic flux density is also obtained in different branches. Then, the distribution of equivalent virtual currents in slot-openings of stator and rotor is obtained as follows [18]:

\[ I_{vs,i} = R_{ts,i} \times \phi_{ts,i} + R_{ys,i} \times \phi_{ys,i} + R_{ts,i+1} \times \phi_{ts,i+1} \]  \tag{30} 

\[ I_{vr,j} = R_{tr,j} \times \phi_{tr,j} + R_{yr,j} \times \phi_{yr,j} + R_{tr,j+1} \times \phi_{tr,j+1} \]  \tag{31} 

where \( R_{ts,i} \) is the reluctance of \( i\text{th} \) tooth of stator, \( R_{ys,i} \) is the reluctance of \( i\text{th} \) segment of stator yoke, \( R_{tr,j} \) is the reluctance of \( j\text{th} \) tooth of rotor, and \( R_{yr,j} \) is the reluctance of \( j\text{th} \) segment of rotor yoke.

As shown in Figure 5, the main phase of stator winding has two parallel branches \( A_1 \) and \( A_2 \). Figure 6 shows the variation of equivalent virtual current in one typical slot (7\textsuperslot{th} slot) of stator and rotor in terms of rotor positions when the branches \( A_1 \) and \( A_2 \) of main phase are separately excited with 0.5 ampere. The virtual turn function \( \left( n_{sat,A}(\alpha', \theta_r) \right) \) in terms of rotor positions and \( \alpha = \alpha' \) can be expressed as [18]

\[ n_{sat,A}(\alpha', \theta_r) = \int_{0}^{\alpha'} \left[ I_{vs}(\alpha, \theta_r) + I_{vr}(\alpha, \theta_r) \right] d\alpha \]  \tag{32} 

The virtual winding function \( N_{sat,A}(\alpha', \theta_r) \) is then calculated using (9) in terms of \( \theta_r \) and \( \alpha = \alpha' \).
phase are separately excited with 0.5 ampere. As shown in Figure 7, MMF drop in iron parts is about 14.2 percent due to only the excitation of main phase with 1 (A).

**Table 2**

### 3. Inductance calculation

The proposed HAM acts based on inductance calculation for electromagnetic modeling of electric machines. To this end, the elements of inductance matrix for this analyzed UTPIM are calculated using this HAM while considering the slotless, slotted, and saturated conditions. Table 2 and Figure 8 show some elements of inductance matrix of studied UTPIM under slotless condition. As seen, there is a good agreement between the corresponding results of FEM and WFT. Figure 8 also shows the elements of mutual inductance matrix between stator phases and rotor loops are periodic waveforms in terms of rotor position. To calculate the inductances of phase A, each branches of main phase is considered as a separate phase. Some elements of inductance matrix of rotor loops are shown in Figure 9 while considering the effect of stator and rotor slots.

**Figure 8**

**Figure 9**

The self-inductance of stator phases and the mutual inductance between stator phases and one rotor loop for different rotor positions and various excitations of relevant phase are shown in Figures 10-13 while considering the effect of slots and magnetic saturation. As shown, the DC component of self-inductances and the amplitude of fundamental component of mutual inductances are reduced due to the effect of slots and magnetic saturation. The effect of slots on the inductances is also clearly seen.

To extract the effect of stator and rotor slots and the magnetic saturation, the harmonic component of $L_{A,A}$ is shown in Figure 14 while considering different conditions. As shown, the DC component of $L_{A,A}$ is reduced if the magnitude of $I_A$ is increased. The $28^{th}$ harmonic and its multiple are due to the rotor slot number (28 slots). The $168^{th}$ harmonic is also due to the interaction effect of stator and rotor slots.

Figures 15-16 show the mutual inductances $L_{A,B}$ and $L_{B,A}$ between main and auxiliary phases of stator. As shown, $L_{A,B}$ and $L_{B,A}$ are not equal to zero when considering the slotting and magnetic saturation effects. Moreover, $L_{A,B}$ is not exactly equal to $L_{B,A}$ because the MMF
distribution and consequently the saturation effect of main and auxiliary phases of stator are not the same while considering similar excitations of \( I_A (L_{B-A}) \) and \( I_B (L_{A-B}) \).

To confirm the accuracy of proposed HAM for modeling the saturation and slotting effects, the corresponding results of \( L_{A-R1} \) obtained through HAM and FEM are compared in Figure 17 under the excitation \( I_A=30(A) \), and being open-circuit of other phases. As shown, there is a good agreement between results obtained through HAM and FEM.

4. Transient modeling

For transient modeling the analyzed UTPIM, the electric circuit of stator should be considered as shown in Figure 5. The electric circuit of cage-rotor is similar to the three-phase induction motors [30]. Figure 18 shows the flow-chart of simulation by proposed HAM.

As explained in [30], the set of electrical and mechanical equations are defined as shown in (33). The finite difference method (FDM) [31] is then used to solve the set of equations after creating a lookup table for each element of \( L(\theta_r) \) and its derivative. For example, Figure 19 shows the lookup of \( L_{A-A} \) as a 3-D plot in terms of the excitation \( I_A \) and the rotor position \( \theta_r \).

The no-load transient results of analyzed UTPIM obtained through proposed HAM are shown in Figures 20-22 when considering different run capacitors. As shown, the current amplitude of main phase (phase A) and auxiliary phase (phase B) are respectively reduced and increased when the capacitance of run capacitor is elevated. The total current of stator (\( I_S \)) is the sum of \( I_A \) and \( I_B \). The Root Mean Square (RMS) value of \( I_S \), the input power factor of analyzed UTPIM, and the main harmonic of \( T_e \) are compared in Table 3 when considers different capacitances of run
capacitor. Figure 22 shows the analyzed UTPIM is started in a shorter time when considers C=45 (μF).

\[ V = R \times I + \frac{d\lambda}{dt} \]

\[ \lambda = L \times I \]

\[ T_e = T_l + J \frac{d\omega_r}{dt} + D\omega_r \]

\[ W_f = \frac{1}{2} I' \times L \times I \rightarrow T_e = \frac{\partial W_f}{\partial \theta_r} \]

\[ \omega_r = \frac{d\omega_r}{dt} \]

As shown in Table 3, the better performance of UTPIM is obtained by using C=45 (μF). However, C=35 (μF) has been selected for this analyzed UTPIM for non-technical reasons [32].

To validate the accuracy of proposed HAM for transient modeling of analyzed UTPIM, the transient results obtained through HAM are compared with equivalent results obtained from FEM under no-load and loading conditions. As shown in Figures 23-25, there is a good agreement between the corresponding results of HAM and FEM. It should be noted at the moment of t=400 ms, the load torque (T_L) is abruptly increased from 0 to 5 N.m.

5. Calculation of air-gap magnetic field
To calculate the air-gap flux density considering all non-ideal effects, it is necessary to construct a non-linear MEC for relevant operating point, for calculating the MMF drops in teeth and yoke segments. Eqs. (30-31) are then used to calculate the equivalent virtual currents in stator and rotor slots. Figure 26 shows the distribution of equivalent virtual currents in stator and rotor slots under the steady state no-load condition.

Figure 26

The methods of Hague and CM [31] are then used to calculate the radial component of slotless air-gap magnetic field under the steady-state no-load condition, as shown in Figure 27.

Figure 27

The radial component of saturated slotted air-gap magnetic field is then calculated as follows:

\[ B_r(\alpha, \theta_r) = (B_{r-sl} + B_{r-stl-v}) \times \text{real}(\Lambda_s(\alpha)) \times \text{real}(\Lambda_r(\alpha, \theta_r)) \]

where \( B_{r-sl} \) and \( B_{r-stl-v} \) are respectively due to real and virtual currents in respective operating point. The result obtained through (34) and FEM are shown in Figure 28. As shown, there is a good agreement between \( B_r \) obtained through HAM and FEM. It should be noted that magnetic field calculation in this section has been done with \( C=35 \) (μF).

Figure 28

The influence of different run capacitors on the magnitude and rotation speed of fundamental component of air-gap rotating magnetic field is also studied. As shown in Figure 29 and Table 4, the best results are obtained by using \( C=35 \) (μF) and \( C=45 \) (μF) as the run capacitors.

Table 4

6. Conclusion

In this paper, a new HAM based on WFT, MEC, and CMs was introduced which can accurately consider non-ideal effects including the effects of stator and rotor slots, magnetic saturation, and winding distribution. The specific complex permeance of air-gap due to the stator and rotor slots were separately calculated through CMs for considering the effect of real paths of flux tubes in slotted air-gap length. The distribution of equivalent virtual currents in stator and rotor slots is used to consider the effect of ferromagnetic core. As was shown, different elements of inductance matrix were predicted very well by using the proposed HAM. The inductance
results also show the effect of magnetic saturation on inductance reduction can be more pronounced than the slotting effect. To perform the transient analysis in short time considering different run capacitors, a 3-D lookup table for each element of inductance matrix and its derivative was prepared in advance. It can be concluded that there are no restrictions for electromagnetic modeling of different electric machines by the proposed HAM.

References


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Figure 1
Figure 2

Figure 3
Figure 7

Figure 8

Figure 9
Figure 16

Figure 17
Start
Voltage Input
Lookup table
Solution of (30) by FDM

\[ t \leq t_{\text{end}} \]

Yes
End

No

Figure 18

Figure 19

Figure 20
Figure 21

Figure 22

Figure 23
Figure 24

Figure 25

Figure 26
Figure 27

The figure shows a graph of $B_r (T)$ vs $\alpha$ (Mech. deg.) with two curves:
- Solid line: due to stator and rotor currents
- Dotted line: due to virtual current

Figure 28

The figure compares FEM and HAM simulations for $B_r (T)$ vs $\alpha$ (Mech. deg.).

Figure 29

(a) Shows $|B_1 (T)|$ vs Time (ms) for different capacitances ($C=25(\mu F)$, $C=35(\mu F)$, $C=45(\mu F)$).
(b) Shows $n_s$ (rpm) vs Time (ms) for the same capacitances.
Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<td>Rated frequency, 60 Hz</td>
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<td>Stator slot depth, 17.1 mm</td>
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<td>Run Capacitor, 35 μF</td>
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</tr>
<tr>
<td>Axial length, 89 mm</td>
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<tr>
<td>Stator tooth-body width, 5.435 mm</td>
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</tr>
<tr>
<td>Rotor tooth-body width, 3.33 mm</td>
<td></td>
</tr>
<tr>
<td>Rotor slot depth, 19.1 mm</td>
<td></td>
</tr>
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<td></td>
</tr>
<tr>
<td>Rotor slot depth, 19.1 mm</td>
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</tr>
<tr>
<td>Number of rotor slots, 28</td>
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<tr>
<td>Inner diameter of rotor, 22 mm</td>
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<tr>
<td>Rotor tooth-body width, 3.33 mm</td>
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</tr>
<tr>
<td>Air-gap length, 0.52 mm</td>
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<tr>
<td>Stator resistance (auxiliary), 3.06 Ω</td>
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<tr>
<td>Number of rotor slots, 28</td>
<td></td>
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<tr>
<td>Stator resistance (auxiliary), 3.06 Ω</td>
<td></td>
</tr>
<tr>
<td>Stator tooth-body width, 4.73 mm</td>
<td></td>
</tr>
<tr>
<td>Rotor tooth-body width, 3.33 mm</td>
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</tr>
<tr>
<td>Stator slot depth, 17.1 mm</td>
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Table 2

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<thead>
<tr>
<th>Parameter</th>
<th>WFT</th>
<th>FEM</th>
</tr>
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<tbody>
<tr>
<td>$L_{A-A}$</td>
<td>194.4 (mH)</td>
<td>192.7 (mH)</td>
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<tr>
<td>$L_{A-B}$</td>
<td>0 (mH)</td>
<td>0 (mH)</td>
</tr>
<tr>
<td>$L_{B-B}$</td>
<td>363 (mH)</td>
<td>362.2 (mH)</td>
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<tr>
<td>$L_{R1-R1}$</td>
<td>1.6723 (μH)</td>
<td>1.669 (μH)</td>
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<tr>
<td>$L_{R1-R2}$</td>
<td>-0.1159 (μH)</td>
<td>-0.1152 (μH)</td>
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</tbody>
</table>

Table 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>RMS value</th>
<th>Power factor</th>
<th>2th harmonic $T_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C=25 (μF)</td>
<td>9.66 (A)</td>
<td>0.43 Lag</td>
<td>4.56 (N.m)</td>
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<tr>
<td>C=35 (μF)</td>
<td>8.06 (A)</td>
<td>0.52 Lag</td>
<td>3.75 (N.m)</td>
</tr>
<tr>
<td>C=45 (μF)</td>
<td>6.8 (A)</td>
<td>0.68 Lag</td>
<td>3.95 (N.m)</td>
</tr>
</tbody>
</table>

Table 4

| Parameter | Ripple of $|B_1|$ | Mean Value of $|B_1|$ | Ripple of $n_s$ | Mean Value of $n_s$ |
|-----------|-----------|--------|-----------|----------------|------------------|
| C=25(μF)  | 0.0352 (T)| 0.9317 (T)| 241.16 (rpm)| 3588.7 (rpm)|
| C=35(μF)  | 0.0283 (T)| 0.949 (T) | 172.7 (rpm) | 3591.5 (rpm)|
| C=45(μF)  | 0.0282 (T)| 0.967 (T) | 185.7 (rpm) | 3599.9 (rpm)|