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A data-driven design of the optimal investment portfolio for the industry in a two-level game using the Markowitz model by meta-heuristic algorithms: Economic analysis of condition monitoring system

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Abstract. This paper studies the investment portfolios of two players in the banking system in a two-level game and then, determines optimal portfolios of investors using the Markowitz model. This two-level game includes Bank C as the leader and customers of this bank as the game followers. The investment portfolios of the leader include investment in competitor banks (A and B), foreign exchange market, real estate market, and stocks. The data related to the mentioned assets covered the years 2010–2020, in which the optimal investment portfolios of the players were first determined using GAMS and genetic meta-heuristic algorithm. Next, the problem was solved again using the meta-heuristic algorithms of Particle Swarm Optimization (PSO) and Invasive Weed Optimization (IWO). Eventually, the optimal algorithm was chosen using TOPSIS multi-criteria decision-making. The results of the 3 algorithms indicated that the optimal portfolio for the leader player consisted of investment in properties, securities, and competitor banks.

1. Introduction

In today's turbulent world, analysis of investment options, techniques of comparison and decision-making, and selection out of solutions are based on economic, political, social, and technological conditions governing the society. The progressive industrial development in societies over the past years suggests that economic decision-making has become more difficult and sensitive than ever, and it is no longer possible to make investment decisions in the available investment portfolios by solely relying on traditional methods [1–3].

Meanwhile, in today's challenging world, economic enterprises are also heavily competing with each other and cannot make proper decisions only using traditional decision-making methods under certainty, risk, and uncertainty conditions in order to cope with both internal and foreign competitors. Thus, novel techniques should be identified under conflict

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conditions to compete with competitors. Use of such techniques facilitates economic evaluation of investment in investment portfolios and risk management and helps investors with better decision-making. Since investment in developing countries comes with numerous risks because of the large and unknown variables and the many assets rather than merely one asset. The portfolio is a set of assets in which every asset has its own specific efficiency and risk, and given the type as well as the number of assets present in it, it has a specific rate of efficiency and risk. The portfolio of every investor differs from others because the motivations as well as behavioral characteristics of every investor plus the risk-taking level of individuals are different from each other [2].

Accordingly, considering the current inflation haunting Iranian economy, which has always been a two-digit rate, people from different walks of life begin to invest in different markets out of the fear of currency depreciation and the declining value of their assets, as well as the motivation to maintain the value of future assets. These markets mostly include banking investment, foreign-exchange market, gold and valuable coin market, housing and property, car, and stock market, all of which are subject to certain risk and efficiency. Also, the crises governing the Iranian economy over the recent years have challenged economic enterprises in a new form, among which competition is absolutely evident. In order to compete with their competitors, economic enterprises (including banks) must make proper and timely decisions so that they would hopefully survive in the future. In this regard, banks as the leaders of economic games involved in the economic cycle through resource allocation and mobilization absorb customers as the followers of economic games. In addition to creating a positive profit margin, which is obtained by subtracting the interest rate assigned to the deposits from the interest rate assigned to the facilities offered, banks should spend their capital in proper markets while handling the reactions of their competitors. The trend governing capital markets (securities, housing, foreign-exchange, etc.) in Iran suggests that understanding the behaviors of competitors can help investors gain maximum utility from their investments.

With this explanation, the application of the mean-variance model attributed to "Markowitz" is one of the best ways to determine the optimal investment portfolio for a bank and its customers among the existing investment portfolios, including bank deposits, currency, coins and gold, and stock exchange, from which real estate and cars are usually considered.

In Markowitz model, investors invest their assets in a portfolio through which they would hopefully gain maximum efficiency. In addition, they prefer experiencing minimum deviation concerning the efficiency of their portfolio. In order to measure the risk of securities, the variance of expected efficiencies is used. Initially, Markowitz indicates selection of the stock portfolio using variance to measure the risk. The method presented by him is known as a mean-variance method (E-V). Markowitz model simultaneously considers the maximum expected value (E) and the minimum variance value (V). The main assumptions of Markowitz constitute the basis of his model; the investors consider efficiency as desired while they are risk-averse. In addition, they act rationally when making decisions so that their desired efficiency can be maximized. Therefore, authors in [3] found that the utility of investors was a function of the expected efficiency and risk, which were the major parameters of decision-making on any form of an investment.

In [4], the mean-variance method and the Monte Carlo simulation were employed to create an optimal investment portfolio holding 7 Shanghai stocks over a 5-year time period in definite and probabilistic terms. The results of both definite and probabilistic methods were the same. In [5], a differential approach was applied to select viable options for investing in tourism projects. Their study is a dynamic decision-making model and it helps select optimal investment portfolios for tourism destinations in the event of uncertainty. Huang [6] proposed a method of optimizing investment in the portfolio of securities using the stochastic differential equation. In this paper, the Shanghai stock was considered the research target and a new differential equation was obtained using the dynamic programming method and the optimal feedback control. Then, the optimal system index was obtained.

Sharma and Habib [7] examined the issue of creating a network between the shares of a market on the Indian Stock Exchange in 2014. Using the correlation matrix, they demonstrated that the relationships between stock returns were nonlinear. Finally, they used the Markowitz model to show that selected environmental stocks using reciprocal information performed significantly better than the stocks selected using correlation. In [8], based on the Markowitz mean-variance model, researchers discussed the problem of portfolio selection in an uncertain environment. They provided a quadratic planning model to solve the portfolio selection. The results showed that their proposed method was better and more practical than the E-V usual method.

To examine the possibility of financial crises in the future similar to September 2008 in the United States, the referenced study [9] investigated 37 major indicators associated with the US economy and used the mean-variance method to depict the variations. The behavior of an average investor can be a warning sign of the impending risk of a financial crisis.

In [10], authors used shortfall models using replica analysis and belief propagation algorithm for the portfolio optimization problem. The research results demonstrated that the answers of the mean-variance model were consistent with the answers from replica analysis and the belief propagation algorithm. Another study [3] developed a mathematical model for the production case involving an integrated distribution problem with a three-level supply chain including manufacturing factories, distribution centers, and customers for several types of products and in the course of several time periods. To consider and deal with uncertainties associated with real problems, some parameters including costs were converted into an uncertain format using the Markowitz model in the examined problem. Finally, the model was solved with probable parameters using Genetic Algorithm (GA). Another referenced piece of research [11] titled “optimizing the investment portfolio using extreme value theory in the securities exchange market of Tehran” concluded that creating an optimal stock portfolio using extreme value theory would not make any significant difference from the Markowitz E-V model. In [12], an LMP-UPM model was developed at different risk and potentiality levels using the indicators of all industries for the portfolio optimization.

The optimized portfolio was compared against E-V model and then again, their performances were comparatively studied using Sharp ratio. The results indicated that LMP-Upm had a better performance. In [13], a portfolio optimization model in Tehran securities exchange market was presented based on the sustainable Sharp ratio. It was found that the real efficiency of the Sharp model did not significantly differ from the real efficiency of Markowitz model. In [14], particle swarm optimization algorithm and Markowitz model were employed for portfolio selection, and these two methods were compared with each other. The results indicated that the particle swarm algorithm encountered less error than the Markowitz model in selecting the optimal investment portfolio.

In [15], a study titled “selecting multi-objective portfolio by combining Markowitz model and cross data envelopment analysis” was conducted. The referenced study found that the proposed model significantly enhanced the efficiency as compared to the Markowitz model, while the portfolio efficiency diminished slightly. Bayat and Abcher [16] first extracted the coefficients of fundamental and technical variables for creating an optimal portfolio through simulation. Then, using real information, they developed the rules of trade for portfolio management. In another study [17] titled “estimated the risk of investment in an asset portfolio in Iran,” the value-at-risk method was employed to calculate the risk of investment in a household asset portfolio including bank deposits, corporate bonds, stocks, foreign-exchange, valuable coins, housing, and lands. Efficiency, efficiency standard deviation, and correlation coefficients between the efficiency rates of assets as well as the value at risk of each asset were measured by applying the mean variance model of the optimal combination of assets. The results indicated that within the 14-year time horizon, the maximum risk of portfolio occurred for those with high-risk taking characteristics, while the individuals with low risk-taking levels would not experience any risk at any confidence level within this period. Further, within the one-year time horizon, the maximum risk of portfolio belongs to those with high-risk taking levels, and the minimum risk was found for those with low risk-taking degrees.

Rahimi et al. [18] investigated the relationship between decision-making models and expectations of investors about risk and investment efficiency of financial tools based on Markowitz model. They concluded that there was a positive relationship between the expected efficiency and the tendency to risk among investors. In [19], researchers developed Markowitz model and introduced the mean-semi variance-skewness three-criterion model. In [20], a novel model was presented based on ant colony algorithm and entropy optimization to select an optimal portfolio. They found the four most important criteria more suited for their research objectives.

In [21], a new method was presented using cross data envelopment analysis based on the mean-variance framework for the problem of portfolio selection. In [22], authors attempted to optimize the portfolio through hunting, searching for metaheuristic algorithms, and Markowitz model under complex optimization model conditions with a wide range of input data for the model, despite cardinal constraints. In [23], a multi-objective model was presented with fuzzy random efficiency that incorporated the criteria of risk, efficiency, and liquidity and an arbitrary programming approach was used based on GA to achieve the final solution. In [24], a fuzzy mean-variance- skewness model was suggested with cardinality constraints and focus on liquidity. The above study proposed an algorithm comprising GA, fuzzy simulation, and cardinality constraints for solving their model. An attempt was made in [25] to optimize the stocks investment portfolio using the ant colony metaheuristic algorithm in the Markowitz model in spite of cardinality constraints in Iran. In [26], a GA was employed for solving the mean-semi variance model of portfolio by benefiting from fuzzy logic and in the presence of cardinality constraints. They applied fuzzy trapezoidal numbers in order to capture uncertainty in the data algorithm. In [27], researchers investigated the selection of a household asset portfolio regarding the housing market for the first time in Iran. For that purpose, the data related to assets including stocks, foreign-exchange, valuable coins, banking deposits, securities, and housing were examined within the period of 1991 to 2006. After calculating the efficiency, risk, and correlation coefficients of the assets within the intended period and by applying the mean-variance model, the results indicated that housing was an important asset in the asset portfolio within the housing boom period, which would cause the efficiency boundary transfer.

In [28], the variance criterion was replaced with semi-variance in the Markowitz model, and harmony search algorithm was used to develop an enhanced model for creating an optimal stock portfolio. The referenced study [29] dealt with the problem of multi-objective portfolio selection including risk, efficiency, and number of shares and added the limitations of value and class to their model. They employed Non-dominated Sorting Genetic Algorithm-II (NSGA-II), Pareto Envelope-based Selection Algorithm (PESA), and Strength Pareto Evolutionary Algorithm 2 (SPEA2) algorithms to solve their model and compared their performance with each other. In [30], a research study titled "determining the efficient boundary of the portfolio using particle swarm optimization algorithm" was conducted to calculate an efficient portfolio by applying metaheuristic algorithms in Markowitz model while considering cardinal constraints in Iran. In [31], efficiency and risk criteria in the Markowitz model were defined using fuzzy trapezoidal numbers and a multi-objective model was presented. By defining a probabilistic model, they defuzzified the problem before solving it and then, solved it using a fuzzy programming method. In [32], the Markowitz model was developed considering different criteria for risk measurement (semi-variance, absolute standard deviation, and variance), and an algorithm was used based on GA.

Ehrgott et al. [33] attempted to use the value at risk as a risk measurement criterion for the creation of optimal portfolio in the Tehran stock market. They observed that the addition of the limitation of value at risk to the Markowitz model might limit the efficient boundary, change it into a point, or even eliminate it. In [1], the criterion of risk value was used as an alternative index of variance, the basis of the stock portfolio selection model in Markowitz model. Also, the study in [1] developed Markowitz model and presented a three-dimensional mean-variance-liquidity model. Another study [34] solved a developed Markowitz model including cardinality and bound constraints by applying a neural network. In [35], data envelopment analysis was utilized for analyzing financial statements of companies and determining the desired stocks. Then, they utilized the mean-variance model for selecting the portfolio among these candidate desired stocks. They used a two-stage algorithm composed of random sampling and local search for solving their model. The study in [36] developed the Markowitz model with five goals including 12-month efficiency, three-year efficiency, annual dividend per share, S&P starred ranking, and standard deviation of risk measurement. They used several meta-heuristic methods for solving their model. In [37], a local search algorithm, which combined simulated annealing and evolutionary strategy principles with each other, was presented to solve the Markowitz model with cardinality constraints. Aihong and Yuping [38] considered the Markowitz model with cardinality constraints and a constraint for the value of each stock individually. They proposed local search methods, especially Tabu search, for solving the portfolio selection problem and presented a new algorithm that combined different neighborhood relations. In [39], cardinality constraints were added to the mean-variance model, which limited the number of portfolio stocks. They presented three heuristic algorithms based on GA for solving their model.

In [40], portfolio optimization was ensured based on an improved knapsack problem with the cardinality, floor and ceiling, budget, class, class limit, and pre-assignment constraints for asset allocation by GA.

Another study [41] investigated the novel solution approaches to solve a new developed portfolio optimization model. Another study considered NSGA-II and Imperialist Competitive Algorithm (ICA) fuzzy simultaneously and used a dataset of assets from the Iran's stock market for three-year historical data and PRE method.

Based on a review of the relevant literature, most studies conducted on Markowitz model both in Iran and worldwide are related to determining the optimal investment portfolio in the stock portfolio, and limited research has been associated with determining the optimal investment portfolios in markets other than the stock market. Further, none of the researches detailed above have analyzed the Markowitz model in a two-level game between leader and follower.

With this explanation, in this study, in the framework of the mean variance algorithm attributed to Markowitz (the mean as a criterion for efficiency and variance as a criterion of risk), the optimal combination of assets in Iran (including banking deposits, foreign-exchange, gold and valuable coins, properties, car, and stocks) is extracted based on the data belonging to the years 2010–2020 using meta-heuristic algorithms in MATLAB software as well as GAMS software for two players: leader (Bank C) and follower (customers of Bank C) in a two-level game.

2. Materials and methods

The present research is a library research study that employs theoretical principles. Backed by the real

Table 1. The interest rate on banking deposits across the Iranian banking systems in 2010–2020.

Bank name	The interest rate of timed deposits within 2010–2020 (%)							
	2020	2019	2018	2017	2016	2014	2012	2010
A	14.14	15.3	16.64	16.05	13.65	12.8	2	2.1
B	13.55	15.7	15.63	15.2	10.97	11.68	9.94	10.67
C	19	16.38	17.66	16.47	14.87	11.61	8.05	7.53

Table 2. The loss and profit rates of parallel markets with banking deposits, including the housing market, foreign-exchange market, valuable coins and gold market, car market, and stocks or securities market in 2010–2020.

Year	Loss/profit rate in parallel markets (%)				
	Foreign exchange market	Housing market	Valuable coins and gold market	Car market	Capital market
2010	4	–13	24	2.5	68
2012	10	–7	51	2.44	87
2014	64	12	45	3.57	0
2015	111	28	44	95.4	60
2016	–5	31	16	8.82	108
2017	–1	10	–9	10	–21
2018	1	–8	–3	4.18	28
2019	–5	3	25	6.13	–4
2020	24	12	37	4.89	29

data, this study conducts an economic evaluation of investment choices in available portfolios between the leader player (Bank C) and follower player (Bank C customers). Evidently, each of the players seeks to maximize their utility in the available markets including banking deposits, foreign-exchange market, valuable coins and gold market, real estate market, car market, and securities or stock market. This section investigates the profit and loss rates of deposits in Bank C and its competitors (Banks A and B as the strongest competitors of Bank C on gaining profit based on their financial statements on the Codal website) as well as the loss and profit rates of parallel markets with deposits in the banking system including the real estate market, foreign-exchange market, valuable coins and gold market, car market, and stock market for investment of customers in 2010–2020. Specifically, the strategies of Bank C customers alongside their loss or profit statuses are measured. Customers' investment strategies in the investment portfolios are given in Figure 1.

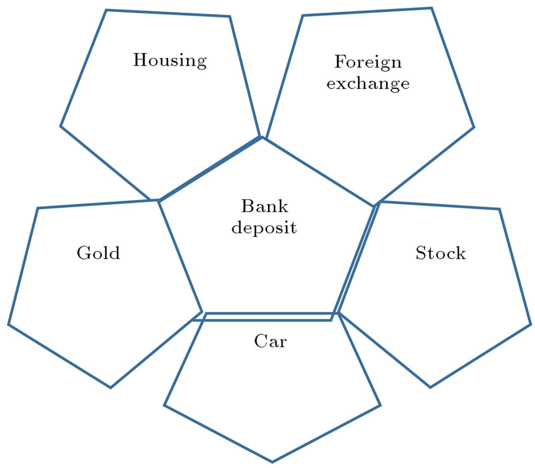

Figure 1. Investment strategies of customers in the investment portfolios (Iran).

Table 1 reports the interest rate on banking deposits in the three considered banks based on their financial statements in 2010–2020.

Table 2 lists the loss and profit rates of the parallel markets with banking deposits in 2010–2020.

2.1. Introducing Markowitz model

Markowitz model is a nonlinear programming model based on the mean and variance of the efficiency of assets and it is based on the presumption of normal distribution of asset efficiency. Based on this model, risk is associated with efficiency fluctuations and the fluctuations are measured based on the efficiency variance. The efficiency rate of a portfolio consisting of different assets is obtained based on the weighted mean of individual assets constituting that portfolio according to Eq. (1):

$$E(R_p) = \sum W_i E(R_i). \tag{1}$$

In Eq. (2), $E(R_P)$ represents the portfolio efficiency rate, R_i the asset efficiency rate, and W_i the weight of assets in the portfolio. The intended portfolio risk is obtained by Eq. (2):

$$\min \sigma_p^2=\sum_{i=1}^{n}\sum_{j=1}^{n} W_iW_j\sigma_{ij}=\sum_{i=1}^{n}\sum_{j=1}^{n} W_iW_jSR_iSR_jr_{ij}. \tag{2}$$

In Eq. (2), σ_p^2 represents the portfolio efficiency variance; SR_i and SR_j denote the standard deviations of efficiencies of assets i and j, respectively; σ_{ij} indicates the covariance between the efficiencies of assets; W_i and W_j show the weights of assets i and j in the portfolio, respectively; and n indicates the number of assets present in the portfolio. Based on this model, individuals maximize the expected efficiency of the portfolio by considering a fixed risk; alternatively, they minimize the risk of portfolio considering fixed expected efficiency. Thus, the nonlinear programming model is used as follows (Eq. (3)):

$$E(R_p)=\sum W_iE(R_i) \quad \sum_{i=1}^{n} W_i=1 \quad Wi\geqslant 0, \tag{3}$$

where $E(R_i)$ indicates the expected efficiency rate of each asset, $E(R_p)$ shows the expected efficiency rate of the portfolio, σ_{ij} denotes the covariance between the efficiency of the ith and jth assets, and W_i represents the share of each asset in the portfolio.

Eq. (2) indicates the expected efficiency of the portfolio, while Eq. (3) shows that the entire budget of a person is invested. Eq. (3) represents the positive weights on each asset in the portfolio, suggesting no short sell. By solving this model, W_i and W_j (weight of assets), which are the decision variables, are identified [42].

Accordingly, the main assumptions of the Markowitz model are:

- Investors are risk-averse and have an incremental expected utility and the final utility curve of their wealth is diminishing;
- Investors choose their investment portfolio based on the expected mean and variance of efficiency. Hence, their indifference curves are a function of the expected variance and efficiency rate;
- Every investment option is divisible ad infinitum;
- The time horizon for all investors is the same and one period;
- Investors prefer a higher efficiency level at a certain risk level; however, at a certain level of efficiency, they prefer minimum risk.

2.2. Introducing two-level programming problems

Non-centralized programming for the long term has been identified as the most important decision-making problem. Many solutions and approaches, which are based on wide-range systemic analyses, lack the ability to model samples of independent subsystems that exist in practice [43].

There is a group of mathematical programming problems that deal with the optimization of different objectives within a hierarchical structure. In such problems, there are several decision-making levels, each "controlling part of the decision variables present in the decision-making space". In such problems, every level has its own objective function, and every objective function at each hierarchical level has its own constraints. Nevertheless, there may be some common constraints for the entire problem, as well. Eq. (4) is one of the two-level problems. Of note, in two-level problems, the high-level decision-maker, who may be an organization or person, is called "leader", while the low-level problem decision-maker is called "follower" [40]. A sample of the two-level problem is between leader and follower [7]:

$$\min_{X\subseteq X,Y} F(x,y),$$

$$G(x,y)\leq 0,$$

s.t.:

$$\min_{y} f(x,y) \qquad g(x,y)\leq 0. \tag{4}$$

The problem variables are categorized into two groups of high-level $x\in R^{n_1}$ and low-level $y\in R^{n_2}$ variables, which can be discrete or continuous. Similarly, the functions $F : R^{n_1}\times R^{n_2}\rightarrow R$ and $f : R^{n_1}\times R^{n_2}\rightarrow R$ represent high-level and low-level objective functions, respectively, which can be linear, nonlinear, fractions, etc. Likewise, vector functions $G : R^{n_1}\times R^{n_2}\rightarrow R^m$ and $g : R^{n_1}\times R^{n_2}\rightarrow R^m$ indicate high-level and low-level constraints, respectively, which again can be linear, nonlinear, fractions, etc. Multilevel optimization problems are mathematical programs, where a subset of their variables should be the optimal solution for the other parametrized programs by their variable residuals. When the program at the second level is itself a mathematical programming problem, the problem will be a two-level programming problem. Multilevel programming models segregate the control over decision variables along regular levels across a ranked programming structure. Multilevel programming is often a suitable instrument for modeling special samples of decision processes, which do not interact with each other; instead, they are in conflict with each other. For example, a player optimizes a subset of decision variables, while it considers the independent reaction of every other player in relation to its own action. Although there are not many known techniques about this type of programming and it is used sparsely, it can offer various potentials [40].

Historically, multilevel optimization has a close relationship with Stackelberg H. Economic problem in

1952 in "game theory". Accordingly, the economic programming process, which involves conflicting organizations, is investigated at two separate levels: leader and follower [41].

Within the special framework of Stackelberg games, it is assumed that the leader predicts the reactions of the follower elements. Subsequently, this allows selecting the best strategy. More specifically, the leader chooses strategy X in the set $X \subseteq R^n$, and each of the follower elements i will have the strategy set $Y_i(x) \subseteq R^{m_1}$ corresponding to every $x \in X$. Sets $Y_i(x)$ are assumed close and concave [41]. Accordingly, in order to solve multilevel programming problems, two approaches can be used [40]:

(a) **Hierarchical approach:** In this approach, all stages are considered hierarchical and consecutive; the decisions transfer from one stage to another and at every stage, a simple programming is done, leading to the optimization of that subset (every subset separately);

(b) **Synchronization approach:** All stages and steps are considered concurrently in this method; although it complicates the programming, it can create a generally optimal program.

Accordingly, Table 3 reports the characteristics of the symbols of the leader-follower two-level game modeling based on the Markowitz model between Bank C and its customers.

Overall, such multilevel problems are considered to be NP-hard, for which precise methods cannot be employed. To solve such problems, authors and researchers apply heuristic and meta-heuristic methods based on optimization of hybrid problems. As such, this research used GA to solve the research model. Thus, in this research, genetic, particle swarm optimization, and invasive weed optimization metaheuristic algorithms were used to solve the research model. Eventually, the obtained solutions are compared with each other and then, the optimal portfolios of investment for the leader and follower players are chosen.

2.3. Genetic Algorithm (GA)

In order to clarify the stages of applying GA in this research, only some details about important characteristics of the algorithm are described. Briefly, the steps covering GA are as follows:

The first step in implementing a GA represents the chromosome, which plays an important role in the success and proper performance of the algorithm. Also, a chromosome of the algorithm can indicate a solution or part of a solution; with progression of generations, in addition to optimality regarding fitness, it goes through evolution. Evidently, in every metaheuristic algorithm, depending on the social or natural phenomenon inspiring data algorithm, a name is assigned to every generated response. For example, in the GA, each chromosome is the answer; in Particle Swarm Optimization (PSO) algorithm, each particle is the answer; in the Harmony search algorithm, each note is the answer; and in Imperialist Competitive

Table 3. The characteristics of the symbols for modeling the leader-follower two-level game based on the Markowitz model between Bank C and its customers.

Index (symbol)	Definition of symbols
n	The number of portfolios of the leader player
m	The number of portfolios of the follower player
j, i	The index of portfolios of the leader player
t, k	The index of portfolios of the follower player
ER_i	Investment efficiency in the investment portfolios by the leader player
SR_i	Risk (SD) of investment in the investment portfolios by the leader player
ER_k	Investment efficiency in the investment portfolios by the follower player
SR_k	Risk (SD) of investment in the investment portfolios by the follower player
R_{ij}	Correlation coefficient between the efficiency of ith and jth assets of the leader player
t_{kt}	Correlation coefficient between the efficiencies of the kth and tth assets of the follower player
w_i	Weight of assets of the leader's portfolio
v_i	Weight of assets of the follower's portfolio
$E(RP_L)$	Expected efficiency rate of the leader's portfolio
$E(RP_F)$	Expected efficiency rate of the follower's portfolio
δ_L^2	Efficiency variance of the leader's portfolio
δ_F^2	Efficiency variance of the follower's portfolio
$\delta_T^2 = \delta_L^2 + \delta_F^2$	Total variance of the portfolio efficiency

Algorithm, each country is the answer. Following the chromosome generation, the extent of the fitting of every chromosome of the objective function is calculated.

In this research, for the selection policy, the roulette wheel mechanism, first proposed by Holland, was used. This method is one of the most suitable random selections and its ideal is selection probability. In the roulette wheel selection method, for choosing every chromosome, first, a random number is generated between zero and one and then, depending on the range of the mentioned numbers, its corresponding chromosome is chosen. Nevertheless, regarding the method of implementing the roulette wheel, first, a circle is considered and then, is divided into several segments based on the number of chromosomes, with each segment corresponding to the extent of fitting of the relevant chromosome. Now, the wheel rotates and wherever it stops, the wheel index is noted and the chromosome related to that segment is chosen.

For the crossover operator, the parents are chosen first and the offspring is then generated using a **uniform crossover operator**. The crossover operations are done with all matrices present in the parent chromosomes, whereby the offspring chromosomes are formed. In this operator, per every gene in the selected parent chromosome, a binary number (zero and one) is generated randomly; if it is 1, the relevant gene into the apparent chromosomes is swapped with each other, while if it is zero, there is no swapping.

The mutation operations are performed on every element of the matrix present in the chromosome. In this operator, after choosing the intended parents, per every gene in the parent chromosome, a random number is generated between zero and one, and with a specific mutation rate, the values of parent chromosome genes are mutated. Now, in case the generated random number is smaller than the desired mutation rate, the relevant gene in the parent chromosome will be randomly mutated; however, if the generated random number is larger than the mutation rate, the related gene in the parent chromosome does not mutate.

2.4. PSO algorithm

PSO is an ensemble random optimization algorithm inspired by the social behavior of bird groups. Since it is group-based, works collaboratively, and has **Fitness** function, it is similar to evolutionary algorithms, except that in PSO, every person benefits from their past information. However, such a behavior does not exist in other evolutionary algorithms and every member of the society changes their position based on personal experience as well as the general experiences of the society. Social sharing of information between the members of a community offers a series of evolutionary advantages, with this assumption being the basis of PSO and its development. PSO can be easily implemented and has been used in solving many discrete plus continuous nonlinear optimization problems. Swarm or mass movement is a coordinated group movement usually done based on the sparse communication among the members as well as limited information of the members about the general status of the system. Particle swarm algorithm is a population-based dynamic computational method. The devised algorithm is inspired by the simulation of the social behavior of a group of birds in finding food. A group of birds search for food in a random space. There is only one piece of food in the space discussed. None of the birds know the place of food, but they know their distance up to the place of food at any stage. Accordingly, the best approach to finding food is to follow the closest bird to the food. One of the best strategies is following the bird with the minimum distance up to the food. This behavior is simulated by PSO in optimization problems [14].

This strategy is indeed the gist of the algorithm. Every bird is a possible solution in the search space of the problem, which is called particle. PSO in the algorithm is equivalent to a bird in the mass or swarm pattern of birds. Every particle has a fitness value, which is calculated by a fitness function. The closer the particle in the search space to the food target in the birds' place of motion, the greater its fitness. Also, a particle has a speed and is responsible for guiding the particles movement. By following the optimal particles in the current state, every particle continues its movement in the problem space. In this way, a group of particles in PSO is randomly generated at the beginning and tries to find the optimal solution by updating the generations. First, the mentioned algorithm is initially assigned by a group of birds randomly to a problem space called a particle and then, the search for achieving the best solution begins. At any stage of the algorithm iteration, the particles shift towards a better position. The next position for each particle is obtained based on two values:

At every step, every particle is updated using the best value. The first is the best position the particle has managed to achieve so far (p_{best}), with this position being recognized and maintained. The second value is the best position obtained by the particle population so far. This position is represented by g_{best}. This process is iterated until achieving the desired outcome (i.e., the speed of birds tends to zero or until achieving the maximum number of iterations considered for the PSO algorithm).

After finding the best values, changes in the speed and position of each particle are updated using the following equations [14]:

$$V_{i,t} = W_{ij} V_{ij} + C_1 r_{1,t} (P_{i,t} - X_{i,t} + C_2 r_{2,t} (P_{g,t} - X_{i,t}), \tag{5}$$

$$X_{i,t+1} = W_{i,t} + V_{i,t}, \tag{6}$$

where C_1 and C_2 is the learning (level of impact) for p_{best} and g_{best}; $r_{1,t}$ and $r_{2,t}$ the random numbers within [0,1]; $X_{i,t}$ the current position of particle; V_{ij} the speed of particles' movement at each stage; W_{ij} the particle movement controller.

2.5. Invasive Weed Optimization (IWO) algorithm

The nature-inspired invasive weed optimization method was introduced and used in [43]. By definition, a weed is a plant that is produced and grows in unwanted places, and it is considered a serious pest for farming plants and can hamper their growth. This algorithm, despite being simple, is very effective and quick in finding optimal points. It acts based on primary and natural features of weeds such as seed production, growth, and struggle for survival in a colony. The stages of implementing this algorithm are introduced as follows [44].

2.5.1. Determining the initial population

First, a preliminary limited population is generated and distributed randomly in the solution space.

2.5.2. Reproduction

In this optimization method, any member of the population reproduces seeds based on its abilities. The number of seeds any plant can generate changes linearly from the minimum to maximum possible number of seeds, where weeds with greater fitness produce more seeds. The relation for producing the number of seeds is as follows:

$$Seed(n) = \frac{f - f_{\min}}{f_{\max} - f_{\min}}(S_{\max} - S_{\min}) + S_{\min}, \tag{7}$$

where $seed(n)$ represents the number of generated seeds, f is fitness of the current weed, $f_{\max}$ and $f_{\min}$ indicate the maximum and minimum fitness levels of the current population, and $S_{\max}$ and $S_{\min}$ indicate the maximum and minimum possible values of seed generation, respectively.

2.5.3. Spatial distribution

In this stage, the generated seeds are randomly scattered in the multidimensional space of the problem. The random distribution function is a normal function, meaning that its mean value is zero and its standard deviation is variable at different stages. It is ensured that the seeds divided randomly are very close to their parent plant. The value of standard deviation (σ) of the normal distribution function decreases at any stage from the initial defined value $\sigma_{initial}$ to the final value σ_{final}. The relationship between the above parameters and standard deviation can be formulated as follows:

$$\sigma_{iter} = \frac{(iter_{\max} - iter)^n}{(iter_{\max})^n}(\sigma_{initial} - \sigma_{final}) + \sigma_{final}, \tag{8}$$

where $iter_{\max}$ represents the maximum number of iterations, σ_{iter} indicates the value of standard deviation in the operating stage, and n denotes the nonlinearity index of modulation or the nonlinear fluctuation index.

2.5.4. Competitive elimination

In the invasive weed algorithm, after some stages of iteration, the invader brings the number of colony seeds in response to reproduction colony to the maximum ($P_{\max}$) and then, a mechanism is applied to removal of the weak seeds. When the maximum allowed number of seeds is produced, any seed can generate new seeds according to the method stated in previous stages, which can be scattered in the space discussed. When all seeds are scattered across the place, a score is given to each seed; in the last stage, the seeds with lower scores are removed, such that the population of seeds would remain maximum again. These stages are iterated until the seeds would gradually converge to the optimal seed [43].

With these explanations, the two main hypotheses of the research are as follows:

Hypothesis 1: The strategy of customers (follower) in banking investment or deposits is gaining profit in an optimal leader-follower game;

Hypothesis 2: The current strategy of Bank C (leader) is no investment in competitor banks for the profitability of this bank in the optimal leader-follower game.

3. Theory/calculation

In the third section, a game is considered between Bank C, as the main beneficiary of the research, and the customers of this bank as investors. In this game, Bank C as the leader and customers of this bank as followers of the game invest in a leader-follower two-level model and in the form of "Markowitz model" in several markets. Eventually, the optimal portfolio investment for both the leader and follower players is determined using GA, PSO, and IWO algorithms.

3.1. Leader-follower two-level model using Markowitz model

In order to determine the leader-follower two-level model of investment between Bank C and its customers using Markowitz model, the data related to this model, covering the data of 2010–2020, were first extracted according to Tables 4 and 5. Table 4 reports the data over the leader-follower equation of the bank and customers in order to determine the optimal investment portfolio using Markowitz model in the course of 2010–2020.

Table 5 lists the correlation coefficients and covariance of the efficiency of investment portfolios of leader and follower players.

Table 4. The data of the leader-follower equation for the bank and customers in order to determine the optimal investment portfolio using Markowitz model (data: 2009–2017).

No.	Leader (bank) data	Value	Symbol	Portfolio no. $n = 1$ to 4
1	Efficiency of investment in stocks market	0.081	ER_1	1
2	Risk of investment in stocks market	0.132	SR_1	1
3	Efficiency of investment in other banks	0.48	ER_2	2
4	Risk of investment in other banks	0.356	SR_2	2
5	Efficiency of investment in foreign exchange currency (US dollar)	0.991	ER_3	3
6	Risk of investment in foreign exchange currency (US dollar)	0.664	SR_3	3
7	Efficiency of investment in real estate	0.957	ER_4	4
8	Risk of investment in real estate	0.121	SR_4	4
9	Efficiency of investment in stocks market	0.350	ER_1	1
10	Risk of investment in stocks market	0.385	SR_1	1
11	Efficiency of investment in real estate	0.199	ER_2	2
12	Risk of investment in real estate	0.232	SR_2	2
13	Efficiency of investment in valuable coins/gold market	0.258	ER_3	3
14	Risk of investment in valuable coins/gold market	0.212	SR_3	3
15	Efficiency of investment in foreign exchange currency (US dollar)	0.248	ER_4	4
16	Risk of investment in foreign exchange currency (US dollar)	0.390	SR_4	4
17	Efficiency of investment in car market	0.169	ER_5	5
18	Risk of investment in car market	0.298	SR_5	5
19	Efficiency of investment in Bank A	0.782	ER_6	6
20	Risk of investment in Bank A	1.887	SR_6	6
21	Efficiency of investment in Bank B	0.047	ER_7	7
22	Risk of investment in Bank B	0.166	SR_7	7
23	Efficiency of investment in Bank C	0.151	ER_8	8
24	Risk of investment in Bank C	0.154	SR_8	8

3.2. Measuring the normality of the data of investment portfolio efficiency

Markowitz model is a nonlinear programming model based on the mean and variance of efficiency of assets, whose main assumption is a normal distribution of efficiency of assets. Based on this model, the risk is associated with efficiency fluctuations, with the fluctuations being measured based on the efficiency variance. Thus, after extracting the relevant data, first, Kolmogorov-Smirnov test must be conducted in order to measure the normality of the efficiency of the assets of leader (Bank A) and follower (customer) players. While the Kolmogorov-Smirnov test is used for a large volume of data, Shapiro-Wilk test is more suitable for a small data volume such as 50 data sets or less [44]. Hence, in SPSS 18, Shapiro-Wilk test results were presented according to Table 6.

In the table of tests of normality, the statistics and probability value for the normality tests are presented. Concerning the Sig. value (significance level) observed in the last column of Table 6, the distribution of data is considered normal, as the significance level has been larger than the standard error level (0.05) for all the efficiency values of investment portfolios. Thus, the data normality assumption at a confidence interval of 95% is confirmed, and the Markowitz model can be used to measure the optimal portfolio of investment.

3.3. Determining the leader-follower model of investment between Bank C and customers based on Markowitz model

The leader-follower two-level model of investment between Bank C and its customers, according to the Markowitz model, was developed based on a study in [3], as can be expressed in Eq. (9):

$$\min \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j \sigma_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j SR_i SR_j r_{ij}, \qquad (9)$$

Table 5. Correlation coefficients and covariance of the efficiency of investment portfolios of leader and followers.

Player's name	Correlation coefficient	Covariance	Name/index of the second market	Name/index of the first market
Leader	$R_{11} = 1$	$COV_{11} = 0.019$	Stocks	Stocks
	$R_{12} = -0.287$	$COV_{12} = -0.015$	Banking deposition	Stocks
	$R_{13} = 0.377$	$COV_{13} = 0.009$	Foreign exchange	Stocks
	$R_{14} = -0.324$	$COV_{14} = -0.007$	Real estate	Stocks
	$R_{22} = 1$	$COV_{22} = 0.142$	Banking deposition	Banking deposition
	$R_{23} = 0.846$	$COV_{23} = 0.184$	Foreign exchange	Banking deposition
	$R_{24} = 0.377$	$COV_{24} = 0.017$	Real estate	Banking deposition
	$R_{33} = 1$	$COV_{33} = 0.588$	Foreign exchange	Foreign exchange
	$R_{34} = 0.286$	$COV_{34} = 0.034$	Real estate	Foreign exchange
	$R_{44} = 1$	$COV_{44} = 0.017$	Real estate	Real estate
Follower	$R_{11} = 1$	$COV_{11} = 0.191$	Stocks	Stocks
	$R_{12} = 0.523$	$COV_{12} = 0.057$	Property	Stocks
	$R_{13} = 0.350$	$COV_{13} = 0.035$	Gold coin	Stocks
	$R_{14} = -0.005$	$COV_{14} = -0.001$	Foreign exchange	Stocks
	$R_{15} = 0.096$	$COV_{15} = 0.013$	Car	Stocks
	$R_{16} = 0.203$	$COV_{16} = 0.148$	Bank A deposit	Stocks
	$R_{17} = -0.422$	$COV_{17} = -0.027$	Bank B deposit	Stocks
	$R_{18} = 0.634$	$COV_{18} = 0.038$	Bank C deposit	Stocks
	$R_{22} = 1$	$COV_{22} = 0.062$	Property	Property
	$R_{23} = 0.108$	$COV_{23} = 0.006$	Coins	Property
	$R_{24} = 0.454$	$COV_{24} = 0.047$	Foreign exchange	Property
	$R_{25} = 0.632$	$COV_{25} = 0.050$	Car	Property
	$R_{26} = 0.602$	$COV_{26} = 0.267$	Bank A deposit	Property
	$R_{27} = 0.028$	$COV_{27} = 0.001$	Bank B deposit	Property
	$R_{28} = 0.860$	$COV_{28} = 0.031$	Bank C deposit	Property
	$R_{33} = 1$	$COV_{33} = 0.051$	Coins	Coins
	$R_{34} = 0.582$	$COV_{34} = 0.055$	Foreign exchange	Coins
	$R_{35} = 0.287$	$COV_{35} = 0.02$	Car	Coins
	$R_{36} = 0.415$	$COV_{36} = 0.158$	Bank A deposit	Coins
	$R_{37} = -0.512$	$COV_{37} = -0.172$	Bank B deposit	Coins
	$R_{38} = 0.307$	$COV_{38} = 0.01$	Bank C deposit	Coins
	$R_{44} = 1$	$COV_{44} = 0.174$	Foreign exchange	Foreign exchange
	$R_{45} = 0.809$	$COV_{45} = 0.107$	Car	Foreign exchange
	$R_{46} = 0.820$	$COV_{46} = 0.639$	Bank A deposit	Foreign exchange
	$R_{47} = 0.058$	$COV_{47} = 0.004$	Bank B deposit	Foreign exchange
	$R_{48} = 0.620$	$COV_{48} = 0.039$	Bank C deposit	Foreign exchange
	$R_{55} = 1$	$COV_{55} = 0.101$	Car	Car
	$R_{56} = 0.999$	$COV_{56} = 0.589$	Bank A deposit	Car
	$R_{57} = 0.360$	$COV_{57} = 0.019$	Bank B deposit	Car
	$R_{58} = 0.785$	$COV_{58} = 0.038$	Bank C deposit	Car
	$R_{66} = 1$	$COV_{66} = 3$	Bank A deposit	Bank A deposit
	$R_{67} = 0.351$	$COV_{67} = 0.11$	Bank B deposit	Bank A deposit
	$R_{68} = 0.781$	$COV_{68} = 0.227$	Bank C deposit	Bank A deposit
	$R_{77} = 1$	$COV_{77} = 0.027$	Bank B deposit	Bank B deposit
	$R_{78} = 0.153$	$COV_{78} = 0.024$	Bank C deposit	Bank B deposit

Table 6. The results of Shapiro-Wilk test in SPSS in order to determine the data normality.

Tests of normality				
Player's name	**Title**	**Statistic value**	**Degree of freedom**	**Significance level**
Leader (Bank C)	Efficiency of investment in stocks	0.832	4	0.172
	Efficiency of investment in other banks	0.867	4	0.286
	Efficiency of investment in foreign exchange	0.949	4	0.71
	Efficiency of investment in real estate	0.991	4	0.962
Follower (customers)	Efficiency of investment in stocks	0.97	4	0.843
	Efficiency of investment in real estate	0.88	4	0.341
	Efficiency of investment in valuable coins/gold	0.793	4	0.09
	Efficiency of investment in foreign exchange	0.933	4	0.614
	Efficiency of investment in car market	0.681	4	0.053
	Efficiency of deposition in Bank A	0.662	4	0.051
	Efficiency of deposition in Bank B	0.873	4	0.31
	Efficiency of deposition in Bank C	0.921	4	0.544

s.t.:

$$E(RP_L)=\sum_{i=1}^{n} W_i E(R_i),$$

$$\sum_{i=1}^{n} W_i = 1 \qquad W_i \geqslant 0,$$

$$E(RP_F)=\sum_{k=1}^{m} V_k E(R_k),$$

$$\sum_{k=1}^{m} V_k = 1,$$

$$V_k \geqslant 0 \qquad W_i \geqslant 0.$$

4. Results and discussion

Upon solving the above model using GA in MATLAB software as well as GAMS, the answer of the unknowns of the problem is obtained according to Table 7.

Figure 2 indicates the convergence of GA to the leader-follower two-level problem using Markowitz model. As can be observed, in this diagram, after around 50 iterations, the value of objective function reaches its saturated or optimal state.

As shown in Table 7, based on the results of this table, the optimal portfolio for the leader (Bank C) regarding investment contains investment in real estate ($W_4 = 0.497$), investment in securities market ($W_1 = 0.473$), and investment in other banks ($W_2 = 0.0284$), respectively, while investment in the foreign exchange market is not economically justified. On the other hand, the optimal portfolio for the follower player (customers of Bank C) regarding investment includes investment in Bank B ($V_7 = 0.562$), investment in valuable coins and gold market ($V_3 = 0.335$), investment in securities market ($V_1 = 0.08$), and investment in real estate ($V_2 = 0.022$), while investments in other parallel markets (foreign-exchange and car) or investments in Banks A and C are not economically justified in this model. The final and optimal value of the objective function, 0.0126, indicates the minimum total variance of the efficiency of investment portfolio for both leader and follower players.

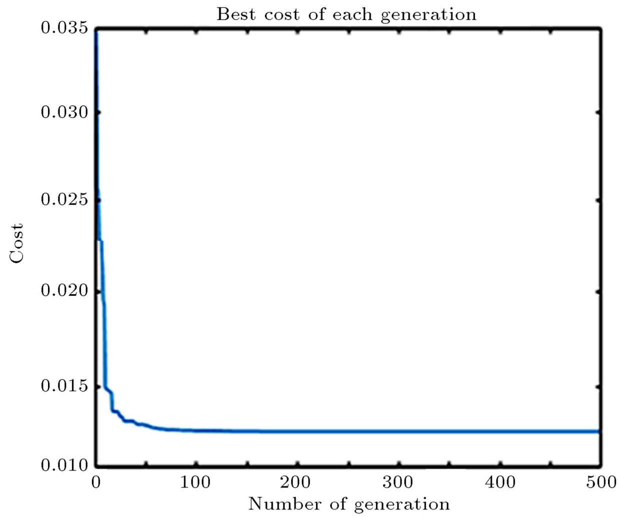

Figure 2. Convergence of genetic algorithm for the leader-follower two-level problem using Markowitz model.

Next, after solving the above model using PSO algorithm in MATLAB software, the unknowns of the problem are given according to Table 8.

Table 7. The answer of unknowns for the leader-follower two-level problem using Markowitz model.

Leader (Bank C)	Value	Symbol	Portfolio no.
The expected efficiency rate for the leader portfolio	0.528	$E(RP_L)$	
Weight of assets of the leader portfolio	0.473	W_1	Stocks (1)
	0.0284	W_2	Banking deposits (2)
	0	W_3	Foreign exchange (3)
	0.497	W_4	Property (4)
Variance of leader's portfolio efficiency	0.0053	δ^2L	
The expected efficiency rate for the follower portfolio	0.145	$E(RP_F)$	
Weight of assets of the follower portfolio	0.08	V_1	Stocks (1)
	0.022	V_2	Property (2)
	0.335	V_3	Valuable coins and gold (3)
	0	V_4	Foreign exchange (4)
	0	V_5	Car (5)
	0	V_6	Bank A deposit (6)
	0.562	V_7	Bank B deposit (7)
	0	V_8	Bank C deposit (8)
Variance of follower's portfolio efficiency	0.0073	δ_F^2	—
Total variance of the portfolio efficiency	**0.0126**		$\delta_T^2 = \delta_L^2 + \delta_F^2$

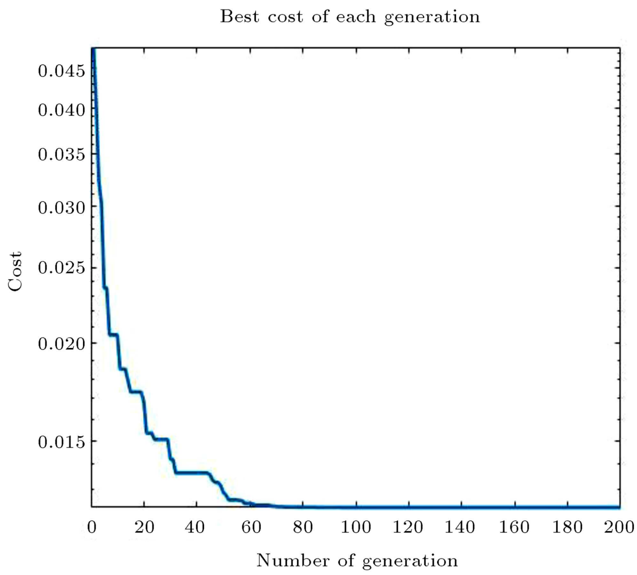

Figure 3. Convergence of PSO algorithm for leader-follower two-level problem using Markowitz model.

Figure 3 displays the convergence of PSO algorithm for solving the leader-follower two-level problem using Markowitz model. As can be seen, in this diagram, after around 60 iterations, the value of the objective function reaches its saturated and optimal state.

As shown in Table 8, based on the results of this table, the optimal portfolio for the leader (Bank C) regarding investment contains investment in real estate ($W_4 = 0.4977$), investment in securities market ($W_1 = 0.4738$), and investment in other banks ($W_2 = 0.0284$), respectively, while investment in the foreign exchange market is not economically justified. On the other hand, the optimal portfolio for the follower player (customers of Bank C) regarding investment includes investment in Bank B ($V_7 = 0.5762$), investment in valuable coins and gold market ($V_3 = 0.3260$), investment in securities market ($V_1 = 0.08$), and investment in real estate ($V_2 = 0.0178$), while investment in other parallel markets (foreign-exchange and car) or investment in Banks A and C is not economically justified in this model. The final and optimal value of the objective function, 0.0123, indicates the minimum total variance of the efficiency of investment portfolio for both leader and follower players.

In the final step, again the above model has been solved using IWO algorithm in MATLAB software, where the unknowns of the problem are given according to Table 9.

Table 8. The findings of unknowns of the leader-follower two-level problem using Markowitz model and PSO algorithm solution method.

Leader (Bank C)	Value	Symbol	Portfolio no.
The expected efficiency rate for the leader portfolio	0.5284	$E(RP_L)$	
			Stocks (1)
Weight of assets of the leader portfolio	0.4738	W_1	
	0.0284	W_2	Banking deposits (2)
	0	W_3	Foreign exchange (3)
	0.4977	W_4	Property (4)
Variance of leader's portfolio efficiency	0.0053	δ_L^2	
The expected efficiency rate for the follower portfolio	0.1427	$E(RP_F)$	
			Stocks (1)
Weight of assets of the follower portfolio	0.08	V_1	
	0.0178	V_2	Property (2)
	0.3260	V_3	Valuable coins and gold (3)
	0	V_4	Foreign exchange (4)
	0	V_5	Car (5)
	0	V_6	Bank A deposit (6)
	0.5762	V_7	Bank B deposit (7)
	0	V_8	Bank C deposit (8)
Variance of follower's portfolio efficiency	0.007	δ_F^2	
Total variance of the portfolio efficiency	**0.0123**		$\delta_T^2 = \delta_L^2 + \delta_F^2$

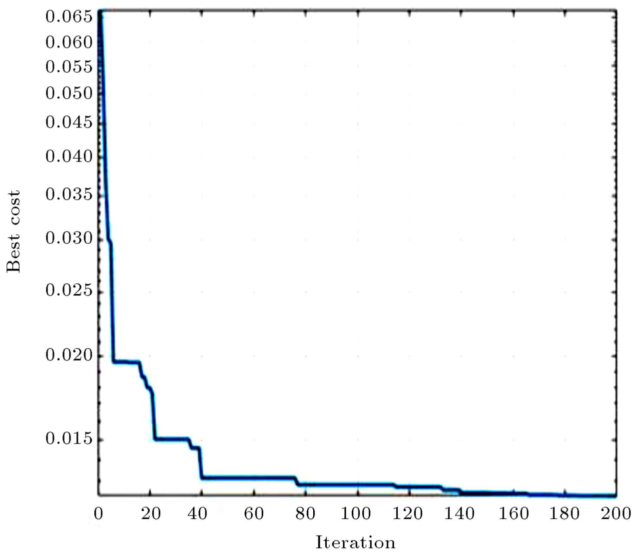

Figure 4. Convergence of the IWO algorithm for solving the leader-follower two-level problem using Markowitz model.

Figure 4 depicts the convergence of IWO algorithm for the leader-follower two-level problem using Markowitz model. As can be seen, in this diagram after around 80 iterations, the value of the objective function has reached its saturated and optimal state.

As shown in Table 9, based on the results of this table, the optimal portfolio for the leader (Bank C) regarding investment contains investment in real estate ($W_4 = 0.4986$), investment in securities market ($W_1 = 0.474$), and investment in other banks ($W_2 = 0.0275$), orderly, while investment in the foreign exchange market is not economically justified. On the other hand, the optimal portfolio for the follower player (customers of Bank C) regarding investment includes investment in Bank B ($V_7 = 0.5734$), investment in valuable coins and gold market ($V_3 = 0.3289$), investment in securities market ($V_1 = 0.0761$), and investment in real estate ($V_2 = 0.0216$), while investment in other parallel markets (foreign-exchange and car) or investment in Banks A and C is not economically justified in this model. The final and optimal value of the objective function, 0.0123, indicates the minimum total variance of the efficiency of investment portfolio for both leader and follower players.

4.1. Comparison of GA, PSO, and IWO algorithms in solving the leader-follower two-level problem for investment between Bank C and its customers according to Markowitz model

After solving the leader-follower problem for investment between Bank C and its customers according to Markowitz model, this section deals with comparing

Table 9. The results of unknowns of the leader-follower two-level problem using Markowitz model and IWO algorithm solution method.

Leader (Bank C)	Value	Symbol	Portfolio no.
The expected efficiency rate for the leader portfolio	0.5287	$E(RP_L)$	
Weight of assets of the leader portfolio	0.474	W_1	Stocks (1)
	0.0275	W_2	Banking deposits (2)
	0	W_3	Foreign exchange (3)
	0.4986	W_4	Property (4)
Variance of leader's portfolio efficiency	0.0053	δ_L^2	
The expected efficiency rate for the follower portfolio	0.1427	$E(RP_F)$	
Weight of assets of the follower portfolio	0.0761	V_1	Stocks (1)
	0.0216	V_2	Property (2)
	0.3289	V_3	Valuable coins and gold (3)
	0	V_4	Foreign exchange (4)
	0	V_5	Car (5)
	0	V_6	Bank A deposit (6)
	0.5734	V_7	Bank B deposit (7)
	0	V_8	Bank C deposit (8)
Variance of follower's portfolio efficiency	0.007	δ_F^2	—
Total variance of the portfolio efficiency	**0.0123**	$\delta_T^2 = \delta_L^2 + \delta_F^2$	

the solutions obtained from the GA, PSO, and IWO algorithms. Comparison of the results is presented in Table 10.

As seen in the above table, although the results of the leader and follower variables are almost close to each other in GA and PSO, the solution has been eventually better in PSO than in GA and GAMS software. Regarding IWO, the results of this algorithm have been better than the GA findings in terms of weight, while some others have been worse. Nevertheless, the risks of the IWO have been lower than those of GA, and eventually the objective function findings of the IWO have been better than those of GA and GAMS software. Overall, no significant difference was observed between the objective function values of the three meta-heuristic algorithms. Nevertheless, to select the optimal algorithm in terms of the time required to achieve the solution, the average portfolios of the leader and follower players, and the final value of the objective function using TOPSIS decision-making technique, these three meta-heuristic algorithms have been compared in order to choose the optimal algorithm.

4.2. Comparison of meta-heuristic algorithms using TOPSIS multi-criteria decision-making technique

In the final step, in order to compare meta-heuristic algorithms and select the optimal algorithm using TOPSIS multi-criteria decision-making technique and according to the criterion of time to achieve the final solution, the average portfolios of players, and the final value of objective function, the algorithms were compared and the meta-heuristic algorithms were prioritized using TOPSIS software in Table 11.

Upon completing the implementation stages of TOPSIS multi-criteria decision-making technique, the relative weights as well as the deviation from the positive ideal and negative ideal of the three algorithms are given in Table 12.

As observed in Table 12, the IWO with a rela-

Table 10. Comparing the results obtained from GA, PSO, and IWO algorithms in solving the leader-follower model problem for investment between Bank C and its customers according to Markowitz model.

Variable	Variable response with GA algorithm	Variable response with PSO algorithm	Variable response with IWO algorithm
Time of obtaining the final findings	43.3	12.72	9.12
Number of iterations for objective function	50	60	80
$E(RP_L)$	0.5265	0.5284	0.5287
W_1	0.4762	0.4738	0.474
W_2	0.0279	0.0284	0.0275
W_3	0	0	0
W_4	0.4958	0.4977	0.4986
δ_L^2	0.0053	0.0053	0.0053
$E(RP_F)$	0.1452	0.1427	0.1427
V_1	0.0799	0.08	0.0761
V_2	0.0221	0.0178	0.0216
V_3	0.334	0.3260	0.3289
V_4	0	0	0
V_5	0	0	0
V_6	0	0	0
V_7	0.5634	0.5762	0.5734
V_8	0	0	0
δ_F^2	0.0073	0.0070	0.0070
$\delta_T^2 = \delta_L^2 + \delta_F^2$	**0.0126**	**0.0123**	**0.0123**

Table 11. The initial table for comparing the meta-heuristic algorithms using TOPSIS decision-making technique.

Index	GA algorithm	PSO algorithm	IWO Algorithm
Time of achieving the final results	43.3	12.72	9.12
Efficiency of leader player portfolios	0.5265	0.5284	0.5287
Efficiency of follower player portfolios	0.1452	0.1427	0.1427
Final value of objective function	**0.0126**	**0.0123**	**0.0123**

Table 12. Determining the most optimal metaheuristic algorithm using TOPSIS multi-criteria decision-making technique.

Algorithm name	Deviation from positive ideal	Deviation from negative ideal	Weights (respectively)
Invasive Weed Optimization (IWO)	0	0.367	1
Particle Swarm Optimization (PSO)	0.0122	0.1	0.891
Genetic Algorithm (GA)	0.1166	0.0007	0.00596

tive weight of 1 was the most optimal meta-heuristic algorithm in the leader-follower problem of investment between Bank C and its customers according to Markowitz model, which was followed by PSO and GA.

4.3. Investigating the research hypotheses

Although the IWO algorithm was identified as the optimal algorithm, since the findings obtained by all the three algorithms were close to each other and there were no significant differences between the weights of investment portfolios of the leader and follower players, the results obtained from solving the problem of investment portfolios of the leader and follower players based on the three meta-heuristic algorithms are presented in Table 13 in order to examine the research hypotheses.

Hypothesis 1: The strategy of customers (followers) in banking deposition/investment for their profitability is optimal. Based on the results of Table 13, since investment in Bank B is considered the most profitable act by the follower players, investment

Table 13. Prioritization of the optimal portfolios in solving the leader-follower model for investment between Bank C and its customers according to Markowitz model and using GA, PSO, and IWO algorithms.

Variable	Prioritization of portfolios using GA	Prioritization of portfolios using PSO	Prioritization of portfolios using IWO
Leader player	Properties	Properties	Properties
	Securities exchange market	Securities exchange market	Securities exchange market
	Investment in other banks	Investment in other banks	Investment in other banks
Follower player	Investment in Bank B	Investment in Bank B	Investment in Bank B
	Investment in gold and valuable coins market	Investment in gold and valuable coins market	Investment in gold and valuable coins market
	Investment in securities exchange market	Investment in securities exchange market	Investment in securities exchange market
	Investment in properties	Investment in properties	Investment in properties

of customers in banks is better than investment in other markets and puts customers at a lower risk. Thus, the strategy of customers (follower) in banking deposition is optimal for their profitability; thus, the first research hypothesis is confirmed;

Hypothesis 2: The strategy of Bank C (leader) involving not investing in competitor banks is optimal for the profitability of this bank. Based on the results of Table 13, it can be concluded that since investment in the real estate market is considered the most profitable action to be taken by the leader player (Bank C), followed by investment in the securities market as well as in other banks, the strategy of Bank C as not investing in competitor banks is optimal for its profitability; thus, the second research hypothesis is also confirmed.

Regarding investment in portfolios (investment portfolios) by Bank C and its customers, the research results indicate that the optimal investments by Bank C include investment in real estate, securities market, and competitor banks (in the order of priority), while investment in the foreign exchange market within the studied years did not prove to be profitable. Regarding the investment priorities by customers in investment portfolios, the research results indicated that optimal investments by customers, in the order of priority, were "investment in Bank B", "investment in coin and gold market", "investment in stock market", and "investment in real estate". Selection of valuable coins and gold as well as securities markets as the second and third optimal options for investment in the customer's portfolios concurs with the results obtained by (x) regarding investment portfolios during the housing recession period. Also, the observational investment option in the securities market is in line with the results of the same research regarding investment portfolios within the housing boom period. Regarding the order of priority of investment in bank deposits, securities market, and real estate, the research results are in agreement with the results obtained in [20] in terms of estimating the risk of investment in an asset portfolio in Iran. In this research, three major investment portfolios for the individuals with low, medium, and high levels of risk-taking included banking deposits, land, and stocks. Based on Table 5, the maximum negative correlation was observed between the asset efficiencies of "valuable coins and gold as well as investment in Bank B". This means that the combination of these two assets in one portfolio significantly reduces the risk. Thus, for the people and investors seeking a lower level of risk, this point can be notable. Nevertheless, since the goal of investors is to achieve an optimal combination of risk and efficiency (the maximum expected utility), they are recommended to consider several markets as their investment target concurrently to achieve this aim.

5. Conclusions

The main reason of the optimality of investment in real estate is that the bank employs the grants offered by municipalities of meta-policies as the top stakeholders of Bank C. Accordingly, there is a mutual relationship between Bank C and municipalities of metropolises, based on which Bank C first offers loans and facilities to these municipalities and makes extensive investment in large-scale urban projects, thereby playing its "social responsibility and social role" and providing social welfare for citizens in metropolises. It also uses the grants offered by municipalities in an optimal way. Meanwhile, investment in securities market, which has been considered as the second optimal portfolio of investment for Bank C, is also considered an important and more valuable measure taken to support domestic production and establish economic prosperity. The lack of economic justification for investment in the foreign-exchange market can be due to the extensive international sanctions against the Iranian banking system. If these sanctions continue, investment by Bank C in this market will not be economically justified, unless this bank could change this "threat" into an "opportunity" through creating proper infrastructures, cooperating with the top brokers, and establishing extensive smart interactions with foreign banks.

Although the efficiency rate of investment in Bank C was higher than Bank B, the risk of investment in Bank C was less than in Bank B and the coefficient of variations and distribution of investment in Bank C were lower than those in Bank B. The main reason of the optimality of investment in Bank B by the customers of Bank C can be attributed to "the lower range of variations in the interest rate on Bank B deposits than on Bank C" over the studied years. Indeed, the range of changes in the interest rate on banking deposits has been around 11.5% and 5.75% in Banks C and B, respectively, indicating a twofold difference between these two banks regarding the range of changes in interest rates on bank deposits.

Given these explanations, it can be stated that the main reason of investment in parallel markets of the banking system by customers and speculators can be summarized into "devaluation of the national currency" and "fear of value reduction for the current assets". In this regard, if the value of Iran's national currency (Rial) in 1980–2020 had not diminished to one third (1/3) every eight years on average, Iran would have not faced this copious volume of liquidity wandering in the parallel markets of the banking system.

Thus, the researcher at the end of this research offers some policy recommendations and suggestions for the central bank of the Islamic Republic of Iran and the banking system in order to specify the optimal processes regarding the determination of fiscal policies including specifying the interest rate on bank deposits:

1. The banks' tendency to invest in real estate under stagflation conditions leads to higher "toxic assets" for these banks. Toxic assets refer to the financial assets whose liquidity is lost and there is no secondary market for their trade because of demand reduction. In such cases, there is typically no market for trading these assets and maintaining this type of assets is deemed a loss. Thus, the optimal measure taken by banks in this regard is to avoid investing in real estate market while attempting to sell the real estate properties;
2. In order to prevent speculation and rush of investors to parallel markets for banking deposition, the central bank should first "consolidate the value of the national currency" and "prevent the decline of the value of money and assets of investors". Evidently, consolidating the value of national currency cannot be achieved merely through economic policies; rather, various optimal foreign and domestic policies are required for establishing a communication with economic powers of the world and lifting the massive sanctions against Iran;
3. As observed, among the parallel markets of the banking deposition, any market with greater efficiency would naturally have a higher level of risk. Thus, in case of the development of parallel markets to banking deposition for investment of customers and the public, "securities market", which is considered a high-efficiency and high-risk market for investors, can be regarded suitable for investment if the shares of companies are offered with clear financial statements in Stocks Boards. This will become possible when all companies, factories, and production industries in the country are able to maximize their production capacity and services; have high efficiency, effectiveness, and productivity; and enjoy sufficient capital and necessary profitability according to statistics. Otherwise, encouraging the public to invest in the securities market would only intensify the false excitement in this market, and the possible ascending trend of the Stock total index would only indicate a "bubble"; this bubble would burst sooner or later, causing excessive losses for investors in this high-risk market;
4. Therefore, in their policies that affect the efficiency of assets, economic policymakers should consider the possible reactions of parallel markets as well as investors' behavior in line with the new efficiency of assets, thereby preventing turbulence in financial markets and other markets.

6. Suggestions for future research

Due to the level of access to information and other limitations of this research, the following topics are suggested for further studies:

1. Using the Markowitz model to determine the optimal portfolio of simultaneous investment of the entire banking network (23 banks and institutions that have financial statements in the Codal system) and their customers in parallel markets and model solving using meta-heuristic algorithms;

2. Adding a portfolio of facilities to customers for banks besides investment portfolios in parallel markets and deposits in the banking network using the Markowitz model; considering the credit risk factor as the risk of this portfolio and the amount of current facilities as the return of the portfolio; and recalculating the portfolio of Bank C and its customers;
3. Using other meta-heuristic algorithms to solve the problem of determining the optimal investment portfolios between leader and follower players and comparing it with the results of this study;
4. Using Markowitz semi-variance model to solve the problem of determining the optimal investment portfolios between leader and follower players in the banking network.

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