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Development of a 2D depth-averaged model for calculating scouring and deposition in alluvial streams

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KEYWORDS

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Abstract. A depth-averaged 2D model is developed to study deposition and scouring in rivers. To calculate the bed load, different empirical formulas were implemented. A two-dimensional depth integrated convection-diffusion equation was solved to determine the suspended load. This study also considered different widely used empirical relationships to determine the sediment exchange term with the bed in the suspended load equation. The model was used for calculating the scouring and deposition in two complicated cases of scouring in a bridge location and scour-hole migration in an erodible channel bed. The results of the model were then compared with experimental measurements as well as 3D numerical model results given in the literature. Based on these comparisons, the most appropriate combination of the empirical formulas for sediment exchange coefficient and sediment load to be used for computing scouring and deposition is introduced. The results of numerical simulations confirmed the applicability of the proposed model in river engineering for design purposes.

1. Introduction

Simulation of scouring and deposition in rivers, reservoirs, and other locations adjacent to hydraulic structures can be a challenging task for hydraulic engineers [1]. These phenomena are among the main reasons behind the reduction of the channel capacity and the natural changing morphology of streams.

In the last few decades, numerical models have become a widely used tool for simulation of many complex phenomena in hydraulic engineering [2–5]. Flow in nature is a 3D phenomenon. Complex boundaries of natural streams as well as scouring and deposition add to the complexity of flow simulations in rivers. Simulation of 3D models is quite time consuming due to high computational effort, especially in long rivers. The accuracy of 3D models in calculating deposition and scouring is also limited to the accuracy of sediment transport equations used in these models.

In shallow flows with no significant flow acceleration in the vertical direction, the hydrostatic pressure distribution can be assumed. With this assumption, the governing equations of the flow and sediment transport can be integrated over the depth. As a result of such integration, two-dimensional depth averaged equations are derived. These kind of two-dimensional models are particularly applicable to modeling long rivers with a large surface area and a relatively shallow depth. 2D shallow model can help to simulate the flow condition better in flood events with a very long-time hydrograph (say with 120 hours base time) within an acceptable computation time.

Kuipers and Vreugdenhill were among the pioneers who had proposed a two-dimensional mathematical model to solve the depth integrated flow equa-

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tions [6]. A number of studies have been conducted in the last two decades on the depth-integrated models and their application to simulating the flow and sediment transport in rivers. These studies have employed different constant values or empirical equations to calculate the sediment exchange rate with the bed. Among these studies, the followings can be mentioned: Spasojevic and Holly (1990) [7], Kassem and Chaudhry (2002) [8], Jia and Wang (1999) [9], Wu (2001) [10], Kolahdoozan et al. (2003) [11], Wu (2004) [12], Zarrati and Tamai (2005) [13], Huang (2007) [14], Huybrechts, et al. (2010) [15], Kolahdoozan et al. (2011) [16], Peng and Tang (2015) [17], Churuksaeva and Starchenko (2015) [18], Horvat et al. (2015) [19], Kasvi et al. (2015) [20], Nguyen et al. (2015) [21], Bohorquez and Ancey (2015) [22], Abderrezzak et al. (2015) [23], Voullieme et al. (2017) [24], and Rowan and Saied (2017) [25].

Despite previous research works, no comprehensive study has been conducted on the accuracy of depth-averaged models in modeling the scouring and deposition, especially in complicated river flows. In the present study, various suggested empirical equations for sediment exchange coefficient with bed are combined with different sediment transport equations and the accuracy of the model in two complicated cases of scouring and deposition is shown.

2. Governing equations

The depth-averaged form of continuity and momentum equations are as follows [26,27]:

Continuity equation:

$$\frac{\partial \zeta}{\partial t}+\frac{\partial UH}{\partial x}+\frac{\partial VH}{\partial y}=0, \tag{1}$$

where ζ is the level of water surface, H the total depth, and U and V the depth-integrated components of velocity in the x and y directions, respectively, which are defined as:

$$U=\frac{1}{H}\int_0^H u dz, \tag{2}$$

$$V=\frac{1}{H}\int_0^H v dz, \tag{3}$$

where u and v are the horizontal velocity components in the x and y directions, respectively. Depth-averaged form of the momentum equations in the x and y directions, neglecting the effect of Coriolis and wind induced shear stresses, as shown below:

$$\frac{\partial (UH)}{\partial t}+\frac{\partial}{\partial x}(U^2H)+\frac{\partial}{\partial y}(UVH)=-gH\frac{\partial \zeta}{\partial x} -\frac{\tau_{xz_b}}{\rho}+\nu^t H\left[\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right], \tag{4}$$

$$\frac{\partial (VH)}{\partial t}+\frac{\partial}{\partial x}(UVH)+\frac{\partial}{\partial y}(V^2H)=-gH\frac{\partial \zeta}{\partial y} -\frac{\tau_{yz_b}}{\rho}+\nu^t H\left[\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right], \tag{5}$$

where ν^t is the turbulence viscosity, and τ_{xz_b} and τ_{yz_b} are bed shear stresses in the x and y directions, respectively, which can be calculated as:

$$\tau_{xz_b}=\rho\frac{g}{C^2_{\text{chezy}}}U\sqrt{U^2+V^2}, \tag{6}$$

$$\tau_{yz_b}=\rho\frac{g}{C^2_{\text{chezy}}}V\sqrt{U^2+V^2}, \tag{7}$$

where ρ is the density of water, g is the gravitational acceleration, and C_{chezy} is the Chézy coefficient. ν^t is calculated from an equation based on the bed shear velocity [28].

3. Suspended sediment transport equation

The convection-diffusion equation is used for determining the suspended sediment concentration profiles. The depth-integrated form of this equation can be written as [15,27,29]:

$$\frac{\partial HC}{\partial t}+\frac{\partial}{\partial x}(HUC)+\frac{\partial}{\partial y}(HVC)=HD_x\frac{\partial^2 C}{\partial x^2} +HD_y\frac{\partial^2 C}{\partial y^2}-\left[w_s c+\varepsilon_{s,z}\frac{\partial c}{\partial z}\right]_b, \tag{8}$$

where C is the average concentration of sediment in depth, w_s is the fall velocity of the sediment particles, c is the sediment volume concentration in every point, and $\varepsilon_{s,z}$ is the near bed turbulent diffusivity. In addition, D_x and D_y represent the depth-averaged turbulent diffusion coefficients in the x and y directions, respectively [28]. Subscript 'b' in the last term of Eq. (8) refers to the bed location.

The last right-hand-side term of Eq. (8), $E=-[w_s c+\varepsilon_{s,z}\frac{\partial c}{\partial z}]_b$, indicates the sediment exchange between the flow and the stream bed. In equilibrium condition with no erosion or deposition, the vertical upward sediment flux resulting from the turbulence diffusion will be equal to the downward sediment flux due to the fall velocity that is [30]:

$$\left(\varepsilon_{s,z}\frac{\partial c}{\partial z}\right)_b=-w_s c_e, \tag{9}$$

where c_e is the sediment concentration near the bed in equilibrium condition. In a non-equilibrium condition, Eq. (9) can be written as:

$$\left(\varepsilon_{s,z}\frac{\partial c}{\partial z}\right)_b = -\alpha_1 w_s c_e,$$

$$c_e = \alpha_2 C_e, \tag{10}$$

where C_e is the depth-averaged capacity of the sediment transport in the equilibrium condition. Combining Eqs. (9) and (10) yields:

$$\left(\varepsilon_{s,z}\frac{\partial c}{\partial z}\right)_b = -\alpha_1\alpha_2 w_s C_e. \tag{11}$$

If c_b is assumed as $c_b = \alpha_3 C$, then one can write:

$$w_s c_b = \alpha_3 w_s C. \tag{12}$$

Rewriting the term E using Eqs. (11) and (12) and assuming that $\alpha_1\alpha_2 = \alpha_3 = \gamma$ [30], the sediment flux from the bed can be written as:

$$E = -\left[w_s c + \varepsilon_{s,z}\frac{\partial c}{\partial z}\right]_b = -[\alpha_3 w_s C - \alpha_1\alpha_2 w_s C_e]$$

$$= \gamma w_s (C_e - C). \tag{13}$$

Consequently, Eq. (8) can be written as follows:

$$\frac{\partial HC}{\partial t} + \frac{\partial}{\partial x}(HUC) + \frac{\partial}{\partial y}(HVC) = HD_x\frac{\partial^2 C}{\partial x^2}$$

$$+ HD_y\frac{\partial^2 C}{\partial y^2} + \gamma w_s(C_e - C), \tag{14}$$

where γ is a coefficient obtained through the model calibration.

The bed boundary condition, $E = \gamma w_s(C_e - C)$, is a very important part of the model, as it represents sediment exchange between the bed and flow. A number of empirical equations are presented in the literature for calculating C_e and γ.

4. Calculation of C_e

In the present study, the proposed Van Rijn [31] equation and the modified Acker and White formulae [32], and SEDTRA model [33] are used for determining the equilibrium concentration (C_e). Van Rijn [30] defined C_e as the ratio of the suspended load to the flow discharge in equilibrium condition. An empirical equation was also proposed by Van Rijn [34] to calculate the suspended sediment load based on the bed shear stress related to grains and reference concentration. Ackers and White [35] proposed an empirical equation for total load calculation. Proffit and Sutherland (1993) further modified this equation and defined C_e based on it. It should be mentioned that for fine bed material suspended sediment transport prevails. This equation is used in CCHE2D software to simulate the fluvial processes in rivers with good results [10]. it is also reported that this equation is not very accurate for very fine material [10]. SEDTRA model uses different sediment transport equations for different sediment sizes to calculate C_e. In this model, Laursen [36] equation is used for the fine sediment sizes between 0.1 mm and 0.25 mm, Yang [37] equation for the sediment sizes between 0.25 mm and 2 mm, and Meyer-Peter and Mueller [38] for the sediment sizes between 2 mm and 50 mm. Then, the results based on the percentage of each size are added. Methods used for calculating γ are described in the next section.

5. Calculation of γ

Physically, γ is defined as the ratio of the near bed concentration to the depth-averaged concentration, i.e.:

$$\gamma = \frac{c_b}{C}, \tag{15}$$

where γ coefficient is considered as a calibration parameter in most of models which can be adjusted by the user.

Guo and Jin [30] evaluated the effect of γ on the changes in the bed level. They concluded that upon increasing γ, more erosion and deposition occurs which is in agreement with the physical concept of γ. This happened mainly because an increase in both γ and E is indicative of an increase in the sediment exchange between the bed and flow. Moreover, based on the physical concept of γ and since the concentration of the sediment close to the bed is higher, γ should normally be more than 1. Many parameters are involved in determining γ among which the size of sediment particles is the most important one. In case the sediment size is fine, distribution of the sediment concentration in depth is more uniform; and consequently, the value of γ is expected to be closer to 1.

In order to determine the value of γ in non-equilibrium states of the suspended sediment transport, Armanini and Di Silvio [39] proposed the following equation:

$$\frac{1}{\gamma} = \frac{b}{H} + \left(1 - \frac{b}{H}\right)e^{\left[-1.5\left(\frac{b}{H}\right)^{-\frac{1}{6}}\frac{w_s}{u_*}\right)\right]}, \tag{16}$$

where b is the layer thickness of the bed load which can be defined as:

$$b = 33He^{-\left[1+k_{\text{von}}\left(C_{\text{chezy}}/\sqrt{g}\right)\right]}, \tag{17}$$

where k_{von} is the Von Kármán constant and u_* the bed shear velocity.

In accordance with the results from the study of Wu et al. [40], the calculated γ based on Eq. (16) is usually larger than 1. However, the values less than 1 have been occasionally found in the literature. Han [41] and Wu and Li [42] proposed $\gamma = 1$ for the case of high scouring and $\gamma = 0.25$ for the cases of low scouring and deposition. These values were obtained based on the data from different rivers and reservoirs. In both Yellow [43] and Rio Grande rivers [44], where the sediment concentration is high, and extreme erosion and deposition are frequent, much lower value of 0.001 is reported for γ.

In the present study, both Eq. (27) derived from the study of Armanini and Di Silvio [39] and constant values proposed by different researchers were used, and the results were compared.

6. Equation of bed level changes

In order to predict the bed level changes, the mass conservation equation of the sediment load was taken into consideration. Followed by integrating the conservation equation of the sediment mass and after some algebraic manipulations, one can derive the following equation as [13]:

$$\frac{\partial z_b}{\partial t} + \frac{1}{1-p}\left\{\frac{\partial}{\partial t}(HC) + \frac{\partial}{\partial x}(q_{b,x} + q_{s,x}) + \frac{\partial}{\partial y}(q_{b,y} + q_{s,y})\right\} = 0, \quad (18)$$

where p is the porosity of bed material; $q_{b,x}$, and $q_{b,y}$ are the bed loads; $q_{s,x}$ and $q_{s,y}$ represent the suspended load transport per unit width in the x and y directions, respectively; and z_b is the bed level above the datum. To calculate the bed load transport, Van Rijn equation [31] was used in all simulations in the present research.

7. The numerical model

The numerical model used in this study was originally developed and verified in previous research works [28,45–47]. This model was further developed in the present study. Finite difference method in a staggered grid system was used to solve the governing differential equations. Water surface levels and sediment concentrations were defined in cell centers, and velocities and bed levels were defined at faces of the corresponding computational cells. In order to solve the convection terms in convection-diffusion equations, ultimate quickest scheme was used as a precise method [48].

A regular space discretization approach was also used for modeling purposes. In addition, the Cartesian co-ordinate system was utilized in this study. Figure 1 shows the finite difference mesh near the boundary [28]. As can be seen in Figure 1, the curved boundaries are simulated by steps. The mesh size in each test was refined to a degree where the results were independent of the mesh size and the boundaries were accurately modeled. In order to solve the equations, first, the governing equations of the fluid flow were solved and the flow velocities and water elevation in the domain were calculated. In the next stage, the computed velocities were used in the sediment transport model to calculate the sediment load and bed level changes. The flow was considered to be steady during bed level calculations. When the changes in the bed level were considerable, the flow domain was updated.

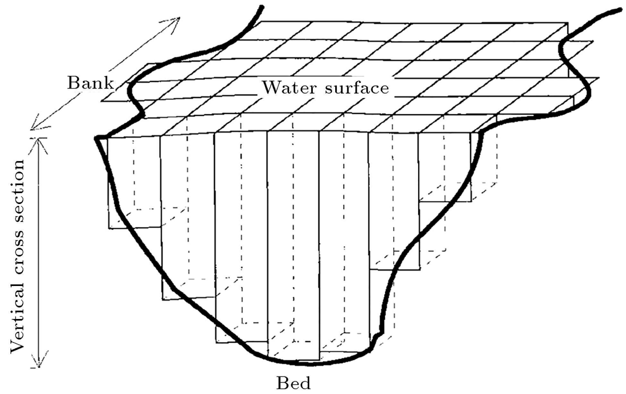

Figure 1. Mesh generation.

8. Results and discussion

8.1. Modeling the flow and changes in the bed level at a bridge constriction

The numerical model was first used for flow simulation and bed level changes at a bridge constriction with lateral guide walls. Experiments were conducted in a laboratory flume with a width of 2 m and length of 15 m with a horizontal bed [49]. The flume bed was covered with non-cohesive materials of $D_{50} = 0.95$ mm. In the upstream section of the bridge, an elliptical guide wall with the length of 1.32 m and diameter ratio of 2.2 was installed. Figure 2 shows the schematic layout of the flume and the bridge opening. Experiments were carried out at the threshold condition of the bed material to get the maximum scour depth. The bed topography was measured with a point gauge with the accuracy of 0.1 when the bed was reached to its equilibrium condition. Figure 3 shows the bed topography in this experiment. The maximum measured scour depth along the constriction was 22 cm, and the maximum local scouring near the guide wall was 33.8 cm [49]. For modeling this case, grid space was 10 cm with 80 × 21 grid point in total. The downstream depth was set as the boundary condition equal to 11.5 cm.

Comparison of calculated maximum scour depth at the constriction with experimental results are given in Table 1. From his table, it can be concluded that in

Table 1. Comparison of the calculated maximum scour depth along the constriction with experimental results.

Used formulas	γ used	Difference (%)
Bed load (Van Rijn, 1987)+suspended load, C_e (Van Rijn, 1987)	Armanini and Di Silvio (1988) γ = constant	+118 –18
Bed load (Van Rijn, 1987)+suspended load, C_e (SEDTRA)	Armanini and Di Silvio (1988) $\gamma = 1.0$	+18 –45
Bed load (Van Rijn, 1987)+suspended load, C_e (modified Ackers and White)	Armanini and Di Silvio (1988) $\gamma = 0.55$	+68 –27

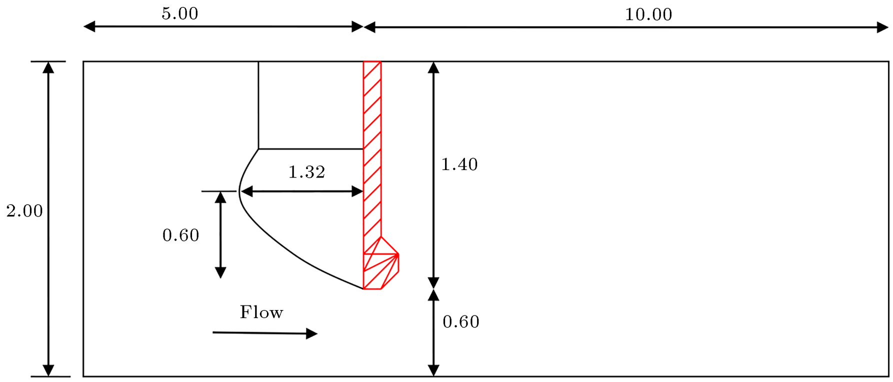

Figure 2. Schematic layout of the bridge constriction (distances are given in m) [49].

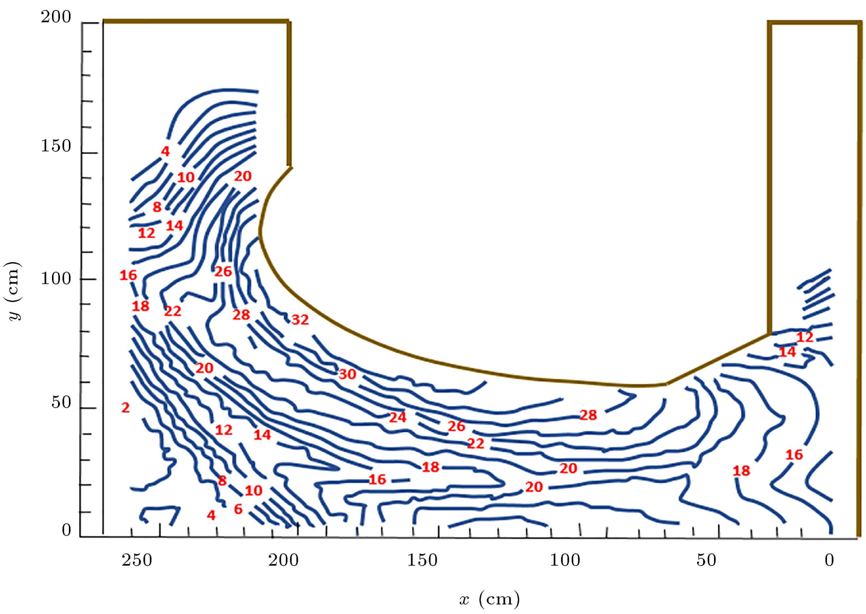

Figure 3. Experimental results of the bed topography in bridge constriction (numbers are elevation below the original bed in cm) [49].

this case, from different empirical equations, Armanini and Di Silvio [39] in combination with SEDTRA method with 26 cm scour depth (+18% of difference), yielded the best results. This case was also simulated based on SSIIM2 3D model [50], and the results showed a maximum of 15.4 cm (−30% of difference) scour along the bridge constriction [51]. Van Rijn equations are used in SSIIM2 for bed load as well as suspended load calculations.

Comparison of the present 2D model results given in Table 1 with 3D SSIIM2 results shows comparable results of the present 2D model with a 3D model for calculating the maximum scouring in a constriction. It should also be mentioned that since the 2D model is unable to simulate vertical flow circulations, local scour at the guide wall cannot be predicted correctly by the present model.

8.2. Migration of a scour-hole

Migration of a scour hole in the bed of a channel was simulated by the proposed model, and the results were compared with the experimental data reported by Van Rijn (1987) [31]. This case is too complicated and difficult to simulate even by a 3D model mainly because it involves both local deposition and scouring. However, despite this complexity, the present model was deliberately selected so that the accuracy of different suggested coefficients could be better differentiated in a case with an extreme bed level change. Experiments were conducted in a flume 30 m long, 0.5 m wide, and 0.7 m deep (Van Rijn, 1987) [31]. The flume bed was covered with sediment by 0.2 m deep with $D_{50} = 160$ μm and $D_{90} = 200$ μm. A scour-hole was created on the flume bed with the depth of 0.15 and lateral slope of 1:10. The fluid velocity at the flume entrance was 0.51 m/s, and the downstream depth was 39 m. The concentration of the sediment entering the flume was 0.04 kg/ms, 0.03 kg/ms of which was the suspended load. After 15 hours, the variation of the scour hole in the flume was measured. Figure 4 illustrates the schematic form of the flume and the scour hole. For modeling this case, grid space was 25 cm in both directions. Downstream depth equal to 39 cm was set as boundary condition.

Two methods were used for calculating γ namely:

i) Armanini and Di Silvio [39];

ii) Different constants suggested by different researchers [10].

With Van Rijn [31] equation for C_e, the value of 17.5 was used for γ based on the suggestion of Owens [52]. In SEDTRA and modified Ackers and White, γ was changed in the range 0.25 to 1 according to Wu [12] for the best results.

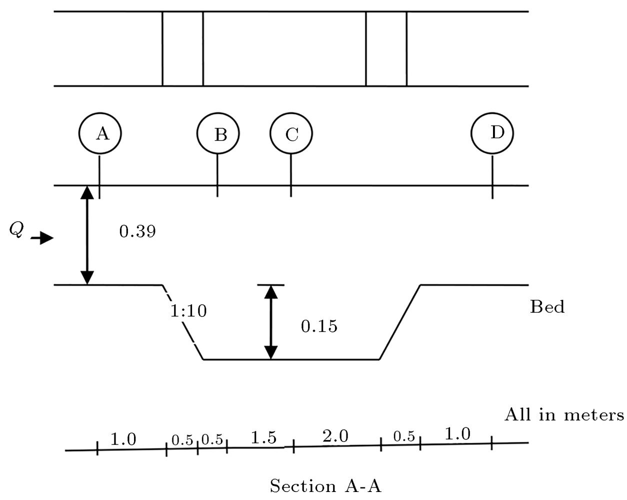

Figure 4. Schematic form of the laboratory flume used by Van Rijn (1987) [31].

According to the experimental results after 15 hours, the equilibrium condition was achieved and deposition and scouring were observed at the upstream and downstream ends of the hole, respectively. Tables 2 and 3 present the results from the numerical model with different values of γ, and Figure 5 makes a comparison between the obtained results and experimental data regarding the deposition and scouring.

Table 2 shows that after the SEDTRA method with a constant γ which is calibrated for the best results, the minimum difference in deposition depth belongs to Armanini and Di Silvio [39] combined with Van Rijn [31] as well as SEDTRA method. Results of Armanini and Di Silvio in combination with Ackers and White equation are also close to SEDTRA. Table 3 shows that Ackers and White equation conforms better with experimental results of scouring. SEDTRA model, however, underpredicts the scour depth. From the results, it can also be seen that constant γ may lead to large errors.

Figure 5 illustrates the scouring and deposition of the pre-excavated scour hole calculated by different formulas. Experimental data of Van Rijn [31] is

Table 2. Comparison between the maximum calculated deposition depth and experimental results.

Used formulas	γ used	Maximum error (percent)
Bed load (Van Rijn, 1987) + suspended load, C_e (Van Rijn, 1987)	Armanini and Di Silvio (1988)	–12
	$\gamma = 17.5$	+26
Bed load (Van Rijn, 1987) + suspended load, C_e (SEDTRA)	Armanini and Di Silvio (1988)	+23
	$\gamma = 1.0$	+1.6
Bed load (Van Rijn, 1987) + suspended load, C_e (modified Ackers and White)	Armanini and Di Silvio (1988)	+37
	$\gamma = 0.55$	+64

Table 3. Comparison between the maximum calculated scour depth and experimental results.

Used formulas	γ used	Maximum error (percent)
Bed load (Van Rijn, 1987) + suspended load, C_e (Van Rijn, 1987)	Armanini and Di Silvio (1988)	+134
	$\gamma = 17.5$	+134
Bed load (Van Rijn, 1987) + suspended load, C_e (SEDTRA)	Armanini and Di Silvio (1988)	–86
	$\gamma = 1.0$	–53
Bed load (Van Rijn, 1987) + suspended load, C_e (modified Ackers and White)	Armanini and Di Silvio (1988)	–14
	$\gamma = 0.55$	+116

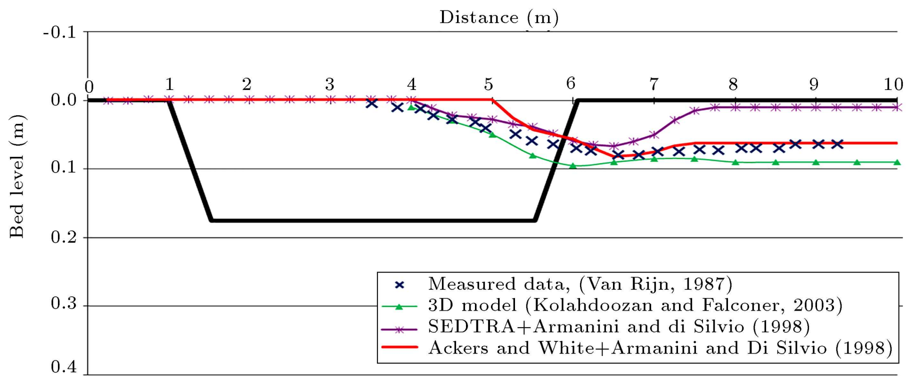

Figure 5. Migration of the scour-hole in the flow direction after 15 hours.

also shown in this figure. As can be seen from this figure, though the model underpredicts the scouring downstream of the trench, the trend of the channel bed is the same as in experiment. It can also be concluded that deposition at the upstream end of the hole is modeled better than the scouring at the downstream end.

3D model results of Kolahdoozan and Falconer [53] for this case are also shown in Figure 5. The comparison of bed topography and 3D results shows that in such a complicated case, SEDTRA and modified Ackers and White method, combined with Armanini and Di Silvio formula, yielded acceptable results in the same range as in the 3D model. It should be mentioned here that in this example, the bed material were in range of silt and sand. For coarser material, the accuracy of the present model needs to be tested.

9. Conclusion

Determining the flow characteristics in rivers considering the effect of deposition and erosion is one of the most important issues in river engineering. High computational efficiency makes depth-averaged models a useful tool for solving river engineering problems. In the present work, a 2D depth-averaged model was developed to study deposition and scouring in erodible beds. Upon integrating convection-diffusion equation in depth, an empirical term appears in the equation which is an indicator of sediment exchange between bed and the flow. This term includes two empirical parameters: sediment equilibrium transport load, C_e, and calibration coefficient, γ. Different constant values and empirical equations are suggested for calculating these parameters. In this paper, the accuracy of these empirical equations was studied in two complicated deposition and scouring cases. The numerical results were then compared with experimental data and results of 3D numerical models with good agreement.

It was concluded that in both complicated case studies, Armanini and Di Silvio [39] equation combined with the SEDTRA method [33] and the modified formula of Ackers and White [32] yielded reasonable results. Form the results, one can state that Armanini and Di Silvio [39] equation that considered the movability parameter and bed load layer thickness in calculating bed load exchange parameter γ is working well in both scouring and deposition conditions in complicated flow cases. It should also be mentioned that the results of these models were comparable to 3D simulation. Based on these results, it can be concluded that the present model is used in simulating fluvial processes in rives for design purposes. In case studies here, the bed material was in the range of silt and sand. Further studies are necessary to check the accuracy of the model for coarser material.

Nomenclature

ζ	Water surface level
H	Total depth
U	Depth integrated components of velocity in the x direction
V	Depth integrated components of velocity in the y direction
u	Velocity in the x direction
v	Velocity in the y direction
ν^t	Turbulence viscosity
τ_{xz_b}	Bed shear stresses in the x direction
τ_{yz_b}	Bed shear stresses in the y direction
ρ	Density of water
g	Gravitational acceleration
C_{chezy}	Chézy coefficient
C	Average concentration of sediment in depth
w_s	Fall velocity of sediment particles
$\varepsilon_{s,z}$	Near bed turbulent diffusivity
D_x	Depth-averaged turbulent diffusion coefficients in the x direction
D_y	Depth-averaged turbulent diffusion coefficients in the y direction
c_e	Sediment concentration near the bed in equilibrium condition
C_e	Depth averaged capacity of sediment transport in equilibrium state
γ	Coefficient
b	Bed load layer thickness
k_{von}	Von Kármán constant
u_*	Bed shear velocity
p	Porosity of bed material
$q_{b,x}$	bed load in the x direction
$q_{b,y}$	Bed load in the y direction
$q_{s,x}$	Suspended load transport per unit width in the x direction
$q_{s,y}$	Suspended load transport per unit width in y direction
z_b	Bed level above the datum
c	Sediment volume concentration in every point

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