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Developing a two-stage multi-period stochastic model for asset and liability management: A real case study in a commercial bank of Iran

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Abstract

This paper develops a novel two-stage multi-period stochastic model to obtain a comprehensive plan. This plan aims to manage the assets and liabilities to satisfy all legal and budget constraints. Assets in the model include short- and long-term loans with reasonable interest rates, investments in the stock market, varied bonds with different expirations, investments in other banks, and the legal budget in the Central Bank. However, liabilities encompass all types of sight and investment deposits with different maturities. In the model, each type of deposit's amount is considered a decision variable, while its total amount is assumed to be stochastic. The mathematical model is constructed in an innovative way such that all previous loans and bonds with possible transactions in the planning horizon could be considered initial parameters. Real data for a commercial bank in Tehran, the Islamic Republic of Iran capital, are used to construct and check the optimization model. The total revenues obtained through the mathematical model and one achieved based on the experiences of financial experts in the commercial bank for four years are compared.

1. Introduction

Banking is an important part of today's economy, and its proper enhancement would lead to the country's economic development. In the current modern world, it is a known fact that banks, as intermediary financial institutions, should attempt to absorb more deposits from different types and provide more opportunities for investors and entrepreneurs to use these sources. In other words, having a proper banking system in each country would improve the operations of industries, which positively affects the rates of unemployment and inflation and brings more welfare for people in a society. Furthermore, the mismanagement of the assets and liability in the financial institutions could negatively influence the economy and cause the waste of resources. One of the major goals of financial banks is to find an optimal tradeoff between return and risk. In this regard, one of their significant challenges is the mismatching of the debit and credit accounts' duration. For example, a bank mainly attracts short-term deposits, whereas it usually grants long-term loans, and therefore, it might seriously be exposed to liquidity and interest rate risks. In other words, on the one hand, keeping additional cash could mostly lead to more opportunity costs and, on the other hand, the lack of sufficient cash, depending on the type of banking system, might have costs such as fines imposed by the Central Bank in a short-term and the loss of credibility in a long-term view. Therefore, the financial banks should constantly monitor their current financial structure considering possible future changes. The mathematical models for asset and liability management could be used to reach this paper's defined goal.

This study proposes a novel stochastic mathematical model for asset and liability management in financial institutions, especially financial banks. An attempt is to consider challenges that financial experts/managers usually face in real-world applications. The main goal in the proposed model is to determine the optimal amount of different deposits (i.e., liabilities) and the variety of investments (i.e., assets) by considering all respective risks and legal, budget, balance sheet, and policy constraints. The followings are the main decision variables in the proposed model, which should be optimally determined:

1. The amount of cash reserved in each period;
2. The amount of loans issued with different maturities in each period;

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3. The amount of money invested in bonds with different maturities in each period;
4. The amount of investment in the stock market in each period;
5. The amount of money invested in the interbank market in each period;
6. The amount of different deposits attracted in each period;
7. The amount of money borrowed from the Central Bank in each period.

Assuming that the total attracted deposit (i.e., the summation of all types of deposits, including sight and saving deposits) is stochastic, we use Two-stage Stochastic Programming (TSP) to construct the respective optimization model for asset and liability management problems. Furthermore, because the total benefits of investment decisions (e.g., lending, purchasing bonds) in the current period are usually obtained in the future, the proposed stochastic model is multi periods.

The rest of this paper is dedicated to the following issues: the next section is assigned to review the literature of the bank's asset-liability mathematical models. Some basic concepts on stochastic programming are presented in Section 3. Our proposed model is then presented in Section 4. The implementation of the proposed model using a real data set for a well-known Iranian commercial bank is discussed in Section 5. Section 6 is assigned to conclusions and future works.

2. Banking in Iran

There are few financial investment organizations in Iran, such as venture capital cooperation, to handle the needed investment for different firms and factories. Therefore, the banking industry in the Iranian economy has a crucial role in financing economic enterprises and directing individuals' deposits towards investment goals. The banking sector in the Iranian economy can be considered the most important bridge between the supply and demand of monetary resources.

Bank ownership in Iran is divided into three types: public, semi-public, and private. Some financial and credit institutions and funds also carry out activities similar to banking in Iran. Because of existing rules in our official religion in Iran, receiving usury from a borrower is forbidden. Therefore, receiving a deposit and giving a bank loan in Iran is based on Islamic contracts.

Banks' resources come mainly from shareholders, deposits, borrowing from the central bank, and other banks (left of the balance sheet).

Depending on the status of resources in the banking network, banks have funds that should be allocated to resources (right side of the balance sheet) by relying on absorbed resources. Allocation of Iran's banking resources includes lending to the governmental and non-governmental sectors, purchasing bonds, investing in the stock market, and purchasing fixed assets.

Therefore, it is necessary for Iranian banks, like the banks of the world, to constantly monitor their balance sheets and determine the amount of their assets according to the long-term perspective (considering future periods). So that it can achieve maximum profitability so that it can minimize risks and penalties by the central bank supervisor.

3. Models presented for asset and liability management

Asset Liability Management (ALM) models have been used in a variety of environments, ranging from pension funds [1,2] insurance companies [3-8], banks [9,10], corporate and public debt management [11-14] to personal finance [15-19] provide a comprehensive overview of the theoretical and methodological developments in the ALM field and illustrate their application with a few case studies.

All developed mathematical models on assets and liabilities management are categorized into two main groups: deterministic and stochastic. The first deterministic assets and liabilities model was proposed by Chambers and Charnes [20] using a linear programming model used to describe the current operating conditions. They addressed determining an optimal portfolio for a bank, considering its risk level as the main constraint. The analogous approach was followed by Francis [21], Cohen and Hammer [22], Fielitz and Loeffler [23]. Eatman and Sealey [24] also applied this methodology with multiple objectives handled by the goal programming technique in a Greek bank for a predefined planning horizon. They constructed a balance sheet for 2001, based on the previous year's balance sheet, and accomplished simulation using three interest rates bonds, deposit rate, and lending rate. They used an optimistic approach leading to a solution with maximum efficiency. Giokas and Vassiloglou [25] also proposed a goal programming technique for a large Greek bank. Abdollahi [26] proposed a multi-objective programming model for decreasing banks' portfolio risk and augmenting its return by using the data from financial statements of an Iranian commercial bank. Devjak and Bogataj [27] presented a mathematical model for determining the optimal amount of short-term commercial bank loans to the corporate sector in Slovenia.

Contrary to the deterministic models, stochastic models could consider uncertainties of financial parameters such as the investments' rate of return, borrowing rates, and receivable deposits. These stochastic models have been constructed based on the Monte Carlo simulation by Robinson [28]. There are also other stochastic models which are based on the Markowitz model, such as those proposed by Pyle [29], Yao et al. [30], Wei et al. [31], Li [32,33], Shen et al. [34], Cui et al. [35], Zhu et al. [36] , and Zhang et al. [37]. The stochasticity has also been handled by the Chance Constraints techniques such as models developed by Charnes and Littlechild [38], Pogue and Bussard [39], and Haneveld et al. [40]. Bradley and Crane [41] used decision theory to handle this issue. Stochastic Dynamic Programming (SDP) is another way of incorporating stochasticity into the models. Consigli and Dempster [42] and Papi and Sbaraglia [43] used SDP to cope with the issue of stochasticity. Gülpinar and Pachamanova [44], and Platanakis and Sutcliffe [45] applied robust optimization and Cohen and Thore [46], Kusy and Ziemba [47], Ziemba et al. [48], Sodhi [49], Dupačová and Polivka [50], Ferstl and Weissensteiner [51], Valladão et al. [52], and De Oliveira et al. [53] used TSP. It should be noted that the most important outcome of these models is to extract a portfolio for investing in the stock market where no withdrawal from the available accounts is permitted. Oguzsoy and Gu [54] proposed a comprehensive multi-period stochastic model. The main decisions of their model, which are used as the basis for our research study in this paper, are the optimal amount of liabilities and assets considering the withdrawals from different deposit accounts in a Turkish bank. They maximize profit subject to the structural, budget, legal, deposit flow, and balance sheet constraints considering different types of deposits, bonds, and one type of loan. The model considers the possibility of early withdrawals; however, it is assumed that there is no commitment from previous periods and no permission to pay the total or partial installments before the due date of loans.

Moreover, the portfolio investment in the stock market is embedded as a parameter in their multi-period models. However, in our proposed model, an extended stochastic multi-period model is presented in which the restrictions mentioned above are eliminated. It could more appropriately reflect the reality of asset-liability management in financial institutions, especially in commercial banks. The detail of advantages of our proposed methodology compared to current approaches in the literature could be summarized as follows:

1. We propose a novel multi-period two-stage stochastic optimization model, which is considerably different from the previous models in terms of variables and constraints. Our model is sufficiently flexible in considering many real situations, such as various loan payments and different withdrawals in financial institutions (e.g., banks and insurance companies);
2. Our proposed model can consider assets/liabilities with different maturities and types (e.g., having saving accounts for one to four years, sight deposit, bonds, and loans with one to four years' maturity dates). Contrary to other developed asset/liability models, different types of assets and liabilities with maturities out of the predefined planning horizon can be considered in the proposed model. Therefore, the optimal decision policy for one planning horizon can be used to calculate the initial values of assets and liabilities. They are needed to construct the optimization model for the next planning horizon (i.e., the model can easily be implemented in a rolling horizon approach that matches the real-world situation). In other words, the results at the end of the time horizon (e.g., the total balance of loans of different types) could be used to calculate the initial parameters (e.g., the total remaining outstanding loan) for the next round of implementing the optimization model;
3. Commonly, some customers intend to pay their loans' total or partial installments before their maturities. The customers would also like to withdraw from their saving accounts before expiration. In this regard, the proposed model is solved to provide decision-makers with a more accurate estimation of the asset/liabilities;
4. The proposed model is constructed and implemented using a real data set obtained for a financial bank in Tehran, the capital of the Islamic Republic of Iran.

The asset and liability management problem in a financial institution such as a bank could be mathematically formulated as TSP. In the first stage, the bank determines the optimal level of all types of assets (e.g., short-term loan and bond with specific maturity date) and liabilities (e.g., short-term investment/certificate and sight/saving deposit). However, in the second stage, the actual condition is considered after the stochastic events, and other corrective decisions might be made. The details of the proposed mathematical model are described as follows.

4. Presenting the proposed model

The proposed ALM model in this study is a dynamic, two-stage, and multi-period decision-making optimization model. The main outputs of the model are the optimal composition of assets and liabilities of the financial institution. Based on what was presented in Oguzsoy and Gu (1997) [54], all respective objectives and constraints can be reported as follows:

- Objective function: Maximizing the net profit from banking operations;
- Structural constraints: These equality constraints calculate the amount of all different available assets and liabilities, including bonds, loans, investment deposits, interest-free deposits, total deposits in other banks, investments in the stock market, and the total borrowings from the Central Bank and their relations between periods;
- Budget constraints: These constraints control the total available sources at the end of each period considering the total consumptions of the financial institution;
- The flow of deposit constraints: the total stochastic deposits in these constraints are connected to all the amounts of different deposits (i.e., saving, sight, and investment deposit), which are our decision variables in the proposed optimization model;
- Legal constraints: These constraints include the legal deposit and capital adequacy;
- Balance sheet constraints: These constraints ensure that the total amount of all assets should be equal to the summation of total liabilities and equity;
- Policy constraints: These constraints are imposed into the model by the management department and accounted as the financial Institution's policy priorities.

It is worth mentioning that the most critical information of our models, including different types of assets and liabilities, are originated from the financial statements, especially the balance sheets during the years under study. In our case, six types of assets, including cash, receivables loans, fixed-income investments (i.e., bonds), equity investments (e.g., investing in the stock market and other companies), and fixed assets, are considered. Our liabilities are also classified into three categories: interest-free deposits (i.e., saving and sight deposits), investment/certificate deposits (i.e., short- and long-term deposits with predefined interest rates), and borrowings from the Central Bank. Furthermore, our proposed mathematical model is quite flexible to incorporate all remaining assets from the previous years as an input to the mathematical model. A complete set of notations used in the model is provided in Appendix A.

5. Structural constraints

The total available assets and liabilities in all periods of the planning horizon are calculated using these equality constraints. They include bonds, loans, investment/certificate deposits, investing in the stock market, investments in other banks, and borrowings from the Central Bank.

5.1. Bonds

Purchasing bonds is usually a good opportunity for a safe investment in all financial institutions. Important parameters of bonds are par value, interest rate (coupon), the remaining time-to-maturity, and its issuer. In developing countries, the par value of all variety bonds on the market is the same, and the issuers of bonds are mainly public and private companies with almost zero default risk. Therefore, it could be assumed that only interest rates and the time remaining to maturity would be important parameters to distinguish available bonds.

The total remaining bonds with type i (i.e., having a defined interest rate) at the end of each period equal the total amount of bonds remaining in the previous period plus the new bonds purchased at the beginning of the same period. These constraints are written for the first and other planning periods as follows:

$$NB_1^{i,n-1} = NBI^{i,n} + B_1^{i,n}\Delta_{i,n} \qquad t=1, \quad \forall i,n \tag{1}$$

$$NB_t^{i,n-1} = NB_{t-1}^{i,n-1} + B_t^{i,n}\Delta_{i,n} \qquad t>1, \quad \forall i,n \tag{2}$$

$$0 \le B_t^{i,n} \le \Delta_{i,n} M \qquad \forall i,n,t \tag{3}$$

For example, the remaining amount of bonds of 16% (e.g., i=1) and the 2-year remaining time-to-maturity at the end of each period is equal to the sum of the total remaining amount of the bond of 16% having 3-year remaining time-to-maturity at the beginning of the period and the amount of new bonds of 16% purchased during the same period. It should be mentioned that some bonds might not be available in some periods. For example, a bond with an 18% interest rate (e.g., i=3) with a 3-year remaining time-to-maturity would not be available in the market. Here, we use a zero-one matrix, $\Delta_{i,n}$ to determine the availability of different types of bonds in each period. If the respective value is one, it means that this type of bond is available in the market; otherwise, the bond is not available. Eq. (3) checks the availability of different types of bonds in the market so that the decision-maker can only purchase the available bonds in the market. It is worth mentioning that the zero-one matrix can be prepared based on historical data.

5.2. Loans

Loans are one of the most important banking services of interest to all society segments. It is usually assumed that each customer payment includes both parts of the principal and interests (profits). In this study, two different loans are taken into consideration. The first one includes those loans issued before the planning horizon is started. The total remaining of these loans with different types is considered input parameters. It is also possible to have new loans with different types (i.e., short- and long-term loans) for customers during the planning horizon. The amount of newly

Step 1: calculate the total remaining unpaid principal (total balance) of the loan issued in period q at period t as follows:

$$A_1 = L_q^j \times (A/P, RL_q^j, j) \times (P/A, RL_q^j, j-(t-q+1)), t=1, j>t-q, \tag{4}$$

$$A_1 = L_q^j \times (A/P, RL_q^j, j) \times (P/A, RL_q^j, j-(t-q+1)) \times \prod_{g=1}^{t-q}(1-PB_{j,g}) \forall t \ne 1, j>t-q \tag{5}$$

Step 2: because of having the possibility of the earlier payment of a loan before its respective maturity, find the mean amount of this payment using:

$$RPL_{q,t}^j = A_1 \times PB_{j,t-q+1}, \tag{6}$$

and then compute the total balance of the customer through the following formula:

$$A_2 = A_1 \times (1-PB_{j,t-q+1}) \tag{7}$$

Step 3: find the new installment for the remaining future periods using:

$$A_3 = A_2 \times (A/P, RL_q^{\,j}, j-(t-q+1)), \tag{8}$$

Step 4: calculate the new principles and the respective interests using:

$$PL_{q,t}^j = A_3 \times (P/F, RL_q^{\,j}, j-(t-q)), \quad \forall t,j,q, t-q \le j. \tag{9}$$

$$IL_{q,t}^j = A_3 \times (1-(P/F, RL_q^{\,j}, j-(t-q)), \quad \forall t,j,q, t-q \le j. \tag{10}$$

Algorithm 1: Calculating the early payment, the principal, and the interest of the new loan.

Step 1: calculate the total remaining unpaid principal (total balance) of the old loan issued before the planning horizon is started:

$$A_1 = NLI^{j,m} \times (A/P, RLI^{j,m}, m) \times (P/A, RLI^{j,m}, m-t) \quad t=1, t \le m \tag{11}$$

$$A_1 = NLI^{j,m} \times (A/P, RLI^{j,m}, m) \times (P/A, RLI^{j,m}, m-t) \times \prod_{g=1}^{t-1}(1-PB_{j,j-m+g}) \quad t \ne 1, t \le m \tag{12}$$

Step 2: because of having the possibility of the earlier payment of a loan before its respective maturity, find the mean amount of this payment using:

$$RPL_t^{j,m} = A_1 \times PB_{j,j-m+t} \tag{13}$$

and then compute the total balance of the customer through the following formula:

$$A_2 = A_1 \times (1-PB_{j,j-m+t}) \tag{14}$$

Step 3: find the new installment for the remaining future periods using:

$$A_3 = A_2 \times (A/P, RLI^{j,m}, m-t), \tag{15}$$

Step 4: calculate the new principles and the respective interests of the old loan using:

$$PLI_t^{j,m} = A_3 \times (P/F, RLI^{j,m}, m-t+1)), \quad t \ne 1, \ t \le m \tag{16}$$

$$ILI_t^{j,m} = A_3 \times (1-P/F, RLI^{j,m}, m-t+1)), \quad t \ne 1, \ t \le m. \tag{17}$$

Algorithm 2: Calculating the early payment, the principal, and the interest of the old loan.

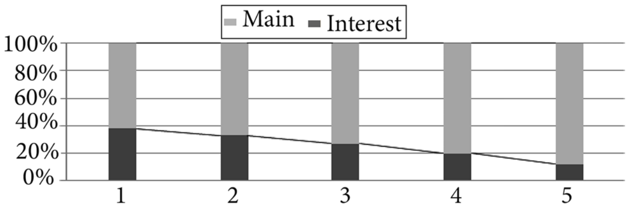

Figure 1. A view of installments including interest and the principal amount of loans.

issued loans are taken as a decision variable. The model is also constructed quite flexible so that customers could pay their future-scheduled installments earlier. Installments for each loan could be easily obtained through $L_q^j \times (A/P, RL_q^j, j)$ where $(A/P, i, n)$ is a known index in Engineering Economy used to calculate a uniform installment with the interest rate, RL_q^j, and the number of periods, n. It is a known fact that each uniform installment is composed of two parts: the principle and the respective interest of the loan. As observed in Figure 1, the amount of interest for five consecutive installments decreases; however, the principal amount of facility increases in those installments.

In Algorithm 1, we elaborate how to calculate the early payment, $RPL_{q,t}^j$, and the principle, $PL_{q,t}^j$, and the respective interest, $IL_{q,t}^j$ for each installment of a new loan issued in period q, which is within the planning horizon. Moreover, Algorithm 2 was proposed for a new loan.

As the total balance at the time of maturity is zero, there is no early payment in this period; that is $RPL_{1,3}^3 = 0$.

$PB_{j,t}$ is calculated based on all earlier payments performed by different borrowers for a loan with type j in period t. The total loan outstanding (i.e., unrepaid loan), which is a needed component in the budget constraints and the balance equation (i.e., the balance of assets and liabilities plus the shareholder's equity), at the end of period t with $m-1$ times maturity is calculated as follows:

$$NL_t^{m-1} = \overbrace{\sum_{j=m+t}^{J}(NLI^{j,m+t-1} - \sum_{r=1}^{t}(PLI_r^{j,m+t-1} + RPLI_r^{j,m+t-1}))}^{w1}$$

$$\overbrace{\sum_{h=0}^{min(t-1,J-m)}(L_{t-h}^{m+h} - \sum_{r=0}^{h}(PL_{t-h,t-r}^{m+h} + RPL_{t-h,t-r}^{m+h}))}^{w2}$$

$$\forall j,m,t \quad m>0, m+t<J \tag{18}$$

The above calculated outstanding at the end of each period consists of two different parts specified by $w1$ and $w2$. The first section of the above formulation calculates the total outstanding of different loans issued before the planning horizon. As previously explained, there is no index for the issuance time of the respective loans, and only the times to maturity are available. Assume that there are five types of loans, one to five years ($j=1... 5$), all possible times to maturity in the first period for each type of the respective loan could be obtained through Table 1.

For example, to find the total outstanding of loans with a three-year time-to-maturity at the end of the first period (i.e.,

Table 1. The possible times to maturity for the loan delivered.

J	Time-to-maturity			
5	4	3	2	1
4	3	2	1	-
3	2	1	-	-
2	1	-	-	-

$m=4$ at the beginning of the first period), only $j=5$ should be accounted for. It means that the total five-year loans delivered one year before the planning horizon should be considered in the summation parts of Eq. (18). In addition, for $m=3$ at the beginning of the first period, two types of loans, $j=4$ and $j=5$, should be accounted for in the summation. It means that the total loans with these types issued one and two years before the planning horizon should be considered in the equation. As a general form, all possible types of loans for a specific time-to-maturity m in period t could be obtained through $j=m+t... J$. It is evident that $m+t$ should be less than J, which is five in our example. Furthermore, the first part of the equation includes all principles and additional earlier payments, which correspond to those loans with type j having a time-to-maturity of $m+t-1$ at the beginning of the first period. For example, to find the total outstanding of loans with two-year time-to-maturity ($m=2$) at the beginning of the second period in the above instance, all principles and additional earlier payments for the first two periods related to $j=5$ and $j=4$ with $m=3$ at the beginning of the first period should be added to each other. It should be mentioned that if $m+t>J$, the first part of Eq. (18) ($w1$) is eliminated.

However, in the second part of Eq. (18), the entire outstanding for all new loans delivered within the planning horizon is calculated. As previously mentioned in the mathematical notations, the issuance time and the type of loan by which the time-to-maturity could be obtained need to be given in these loans. The respective summation in every period should be included in all loans with time-to-maturity higher than zero in that period. For example, to find the outstanding at the end of the third period with time-to-maturity $m=2$ at the beginning of that period in the above example ($J=5$), there could be three types of loans with $j=4$ delivered at the beginning of the first period, $j=3$ delivered at the beginning of the second period, and $j=2$ delivered at the beginning of the third period. h in the above equation is a parameter by which all possible new loans could be determined within the time intervals between the first and the current periods. The domain of this parameter is calculated through the following rules:

1. As the outstanding is determined in terms of time-to-maturity (NL_t^{m-1}), the respective type of loan is calculated using $m+h$ in which h is the time interval that should be added to the corresponding time-to-maturity. It is evident that $m+h \le J$ or $h \le J-m$;
2. All delivered loans and their principles and additional earlier payments are accomplished in periods $t-h$ for all h obtained in the first rule. It is evident that $t-h>0$ or $h \le t-1$;
3. Using the above rules, the upper bound of h is obtained through h=min$(J-m$ & $t-1)$; and the lower bound is zero, which means that all new loans

Table 2. Estimation of the mean percentage of early withdrawal for a three-year deposit.

Sample	First year	Second year	Third year	Summation
1	pw_{11}	pw_{12}	pw_{13}	1
2	pw_{21}	pw_{22}	pw_{23}	1
……………..				
N	pw_{n1}	pw_{n2}	pw_{n3}	1
The average of percentage	$\frac{pw_{11}+pw_{21}+...+pw_{n1}}{n}$	$\frac{pw_{12}+pw_{22}+...+pw_{n2}}{n}$	$\frac{pw_{13}+pw_{23}+...+pw_{n3}}{n}$	1

delivered in the current period should also be considered in Eq. (18).

5.3. Investment deposits

Investment/certificate deposit is the money someone invests in a financial bank or institution to receive an interest based on an approved mutual contract. The total balance for deposits with different times to maturity in each period is obtained using the following equation:

$$NLD_t^{z-1} = \overbrace{\sum_{k=z+t}^{K} \left(\frac{NLDI^{k,z+t-1} \times}{1-\sum_{l=1}^{k-(z+t-1)} PA_{k,l}} (1-\sum_{l=1}^{k-z+1} PA_{k,l}) \right.}^{w3} + \overbrace{\sum_{h=0}^{\min(k-z,t-1)} LD_{t-h}^{z+h} \times (1-\sum_{l=1}^{h+1} PA_{z+h,l})}^{w4}$$

$$\forall z,t \quad z>0, \quad z+t \le K \tag{19}$$

The total balance for deposit with a given time-to-maturity *z-1* at the end of period *t* in Eq. (19) is composed of two parts specified by $w3$ and $w4$. In the first part, all deposits received in some periods before the planning horizon and having a time to maturity higher than zero at the beginning of the first period are considered. The upper and lower bounds for the respective sigma's index variable are determined as analogous to what has been done for loan balance in which *j* and *m* should be replaced with *k* and *z*, respectively. Therefore, the lower and upper bounds for each deposit type (*k*) are *z+t* and *K*, respectively. It is evident that *z+t* should be less than *K*. Moreover, as the mean percentages of early withdrawals are estimated based on the total money endorsed in the mutual contract, we should estimate the total initial deposit at the beginning of the time horizon, although it could be a given input in many real case studies. To do such a task in the first sigma, the total remaining deposit given at the beginning of the first period of the planning horizon should be divided by one subtracted from all respective percentages of early withdrawals accomplished before the planning horizon. The number of these percentages is obtained through *k-(z+t-1).* Finally, the total deposit balance with time-to-maturity *z-1* at the end of period *t* is calculated by considering all percentages within the time interval from the first deposit up to the end of period *t* (the number of these percentages is obtained through *k-z+1*). It should be mentioned that if *z+t>K*, the first part of the equation is eliminated. The second part in Eq. (19) is pertinent to all deposits received within the time horizon. The lower and upper bound for *h* (sigma's index variable) are determined similar to what was accomplished for the loan balance. Eq. (19) demonstrates that the lower and upper bounds are zero and min*(t-1, K-z*), respectively. In this part, the means of all percentages from 1 to *h+1* should be considered. For example, if a two-year deposit is received in the first period, to find the total balance of deposit at the end of the first period (i.e., the total balance of deposit with one-year time-to-maturity), only the mean percentage of $PA_{2,1}$ should be considered.

The mean percentages of early withdrawal for the deposit with type *k* could be estimated using pertinent historical data. Assume that *n* customers are investing in a three-year deposit. pw_{i1}, pw_{i2}, and pw_{i3} are the percentages of early withdrawal of the *i*th customer in the first, second, and third years, respectively. As illustrated in Table 2, the mean percentages of early withdrawal for all years are simply the average of all respective samples.

5.3.1. Interest-free deposits

Some customers deposit their funds in some interest-free deposits (i.e., saving and sight deposits) in the financial banks or institutions for cooperation in charitable and pious deeds. These are ways to handle the population's needs by holding interest-free loan contracts. The total balance for these accounts in every period could be obtained through the following equations:

$$NSD_1 = NIBI(1-PC) + SD_1, \tag{20}$$

$$NSD_t = NSD_{t-1}(1-PC) + SD_t \quad \forall t>1. \tag{21}$$

As the owners of these accounts can withdraw the whole or part of their money in every period, interest-free deposits at the end of period *t-1* should be multiplied by the *1-PC*, in which *PC* is a given parameter indicating the average percentage of withdrawal from saving and sight accounts in every period. The respective percentage could be obtained by finding the proportions of all withdrawals compared to the total balances in all periods using the historical information and getting a simple average.

5.4. Investment in other banks

Banks usually prefer to have transactions with other financial institutions for (short-term) funding as the interbank market. The total balance of investments in other banks could be obtained through the following equations:

$$NIB_1 = NIBI + IB_1, \tag{22}$$

$$NIB_t = NIB_{t-1} + IB_t \quad \forall t>1. \tag{23}$$

It is noteworthy that IB_t, the net investment in other banks in period t, might be positive or negative. In the case of negative value, other banks or financial institutions invest in the bank.

5.5. Investment in the stock market

The total amount of investment in the stock market in period t could be calculated through the following equation:

$$NIS_1 = NISI + IS_1, \tag{24}$$

$$NIS_t = NIS_{t-1} + IS_t \qquad \forall t > 1. \tag{25}$$

It should be noted that IS_t, as the summation of all new investments, including the total amount of purchases and sales in the stock market in period t, could be positive or negative. The negative means that some current stocks would be sold in the stock market in period t where there is no purchase or in total the amount of purchases is less than the amount of sales in that period. However, NIS_t should always be positive, which means no short sell in the trading stock market.

5.6. Borrowings from the Central Bank

The total balance of borrowings from the Central Bank in period t is obtained through the following functions:

$$NBC_1 = NBCI + BC_1 \tag{26}$$

$$NBC_t = NBC_{t-1} + BC_t \qquad \forall t > 1 \tag{27}$$

The value of BC_t (the amount of borrowings from the Central Bank) could also be negative, which would mean that the bank reimburses a partial amount of borrowings to the Central Bank in period t.

5.6.1. Budget constraints

This constraint puts sources and uses of funds against each other. In other words, total inputs (e.g., different incomes) called cash inflows in period t plus the cash balance at the beginning of the period should be equal to the total outputs (e.g., expenditures) called cash outflows plus the cash balance at the end of the same period. This constraint is written as Eq. (28). There are different components specified by w5 to w14 in every period, which are explained in detail as follows:

Sources:

w5: The amount of received installments including interests and principals for new loans and other loans delivered before the planning horizon and had time-to-maturity higher than zero at the beginning of the current period;

w6: The total early payments of different loans, including the new loans and other loans delivered before the planning horizon and had time-to-maturity higher than zero at the beginning of the current period;

***w7*:** The sum of new deposits with different types (e.g., having different interests), new interest-free deposits (savings and sight deposits), and new borrowings from the Central Bank;

w8: The sum of the profit of investments in the stock market and received interests from other banks/financial institutions;

w9: The sum of all face values of matured bonds, the interests of all current bonds, and current cash at the beginning in period t;

$$\begin{aligned}
&\overbrace{\Sigma_{m=t}^{J-1}\Sigma_{j=m+1}^{J}(PLI_t^{j,m}+ILI_t^{j,m})+\Sigma_{j=1}^{J}\Sigma_{h=0}^{\min(j-1,t-1)}(PL_{t-h,t}^{j}+IL_{t-h,t}^{j})}^{w5}\\
&\overbrace{\sum_{m=t}^{J-1}\sum_{j=m+1}^{J}(RPLI_t^{j,m})+\sum_{j=1}^{J}\sum_{h=0}^{\min(j-1,t-1)}(RPL_{t-h,t}^{j+h})}^{w6}\\
&+\overbrace{\sum_{k=1}^{K}LD_t^k+SD_t+BC_t}^{w7}\ \overbrace{RIB_t\times NIB_t+RIS_t\times NIS_t}^{w8}\\
&\overbrace{+\sum_{i=1}^{I}NB_t^{i,0}+\sum_{n=1}^{N}(RIB_t^i\times NB_t^{i,n-1}))+CD_{t-1}}^{w9}\\
&\overbrace{=CD_t+\sum_{j=1}^{J}L_t^j+\sum_{i=1}^{I}\sum_{n=1}^{N}NB_t^{i,n}+IB_t+IS_t}^{w10}\\
&\overbrace{+\Sigma_{z=t}^{K-1}\Sigma_{k=z+1}^{K}NLDI^{k,z}\times PA_{k,k-z+t}+\Sigma_{k=1}^{K}\Sigma_{h=0}^{\min(K-1,t-1)}LD_{t-h}^{k}\times PA_{k,h+1}}^{w11}\\
&\overbrace{+\Sigma_{z=t}^{K-1}\Sigma_{k=z+1}^{K}(\frac{CLDI^k\times NLDI^{k,z}\times}{1-\Sigma_{l=1}^{k-z}PA_{k,l}}(1-\Sigma_{l=1}^{k-z+t}PA_{k,l})}^{w12}\\
&\overbrace{-\Sigma_{k=1}^{K}\Sigma_{h=0}^{\min(k-1,t-1)}CLD_{t-h}^{k}\times LD_{t-h}^{k}\times(1-\Sigma_{l=1}^{h+1}PA_{k,l}))}^{w13}\\
&\overbrace{CBC_t\times NBC_t+NSD_t\times PC+EX_t}^{w14}\\
&+\overbrace{(LR_t-LR_{t-1})+0.15\times PF_t}^{w15}\quad \forall t.
\end{aligned} \tag{28}$$

Uses:

w10: The sum of available cash at the end of period t, new delivered loans, newly purchased bonds, and the total new investments in the stock market and other banks or financial institutions in the same period;

w11: The total early withdrawals from all deposits received before and after the planning horizon and had time-to-maturity higher than zero in period t;

w12: The total interest paid to investors for those deposits received before the planning horizon with time-to-maturity higher than zero and all deposits matured in period t;

w13: The total interest paid to investors for those deposits received within the planning horizon with time-to-maturity higher than zero and all deposits matured in period t;

w14: The summation of early withdrawals for saving and sight deposit, the total interest paid to the Central Bank, and the operational and administrative costs for banking affairs;

w15: The respective change in the legal deposit at the Central Bank (i.e., all negative or positive changes that might occur because of changing for deposit at the end of the respective period) and the legal reserves commensurate with the bank's annual operating profit, which is mathematically obtained through the following equation:

$$PF_t = \sum_{m=t}^{J-1}\sum_{j=m+1}^{J}ILI_t^{j,m} + \sum_{j=1}^{J}\sum_{h=0}^{\min(j-1,t-1)}IL_{(t-h,t)}^{j}$$

$$+RIB_t \times NIB_t + RIS_t \times NIS_t + \sum_{i=1}^{I}\sum_{n=1}^{N} RIB_t^i \times NB_t^{i,n-1}$$
$$-\sum_{z=t}^{K-1}\sum_{k=z+1}^{K}\left(\frac{CLDI^k \times NLDI^{k,z} \times}{1-\sum_{l=1}^{k-z} PA_{k,l}}(1-\sum_{l=1}^{k-z+t} PA_{k,l}\right)$$
$$-\sum_{k=1}^{K}\sum_{h=0}^{\min(k-1,t-1)} CLD_{t-h}^k \times LD_{t-h}^k \times (1-\sum_{l=1}^{h+1} PA_{k,l}))$$
$$-CBC_t \times NBC_t \tag{29}$$

It should be mentioned that the following terms related to those deposits/loans received/delivered before the planning horizon are eliminated from Eq. (28):

- If $t>m$, the terms of w5 and w6 are deleted,
- If t>z, the terms w12 and w11 are deleted.

5.6.2. The constraint on deposit flow

As previously mentioned, deposits are categorized into interest-free deposits (i.e., saving and sight deposits) and certificate deposits with different time-to-maturity and interest rates. As the amount of total deposit ($RNDEP_{t,s}$) is defined as a stochastic variable, the relationship between the summation of all types of deposits as decision variables could be written as the following equation:

$$\sum_{z=1}^{K-1} NLD_t^z + NSD_t + SL_{1,t,s}^- - SL_{1,t,s}^+ = RNDEP_{t,s} \tag{30}$$

In fact, in the above equation, we seek to find a relation between the stochastic nature of all deposits as a stochastic parameter and strategic plans for receiving different types of deposits as the first-stage variables. Obviously, if they are not matched with each other, some penalty costs should be incorporated into the objective. Here, we penalize the respective shortcoming using $PEN_{1,t,s}^-$ as a result of having insufficient currency to invest in different scheduled plans and the respective surplus using $PEN_{1,t,s}^+$ because of having no scheduled plans for investment. It should be mentioned that determining the proper values for these parameters would be a challenging issue. Oguzsoy and Gu [54] (1997) suggested a formulation in which the respective parameters are calculated by subtracting the rate of deposit (F) from the interest rate of investment (R) plus the rate of a risk-free asset. It should be noted that the rate of the Treasury bond in the United States or the rate of interest in some financial banks around the world is commonly entitled as a Minimum Attractive Rate of Return (MARR). In other words, if a shortage in a scenario occurs, it can be assumed that a part of the planned revenues is not fulfilled, and it should be therefore subtracted from the objective function (i.e., adding a penalty cost to the objective). The analogous thing occurs where the total deposit is more than what has been planned for investing in different assets. However, the total revenue obtained through the optimization process would not be matched with what is obtained in reality. Suppose that after solving the two-stage stochastic optimization model, the bank manager/s plan is to issue \$200 and \$400 as short and long-term loans to customers, respectively, at the beginning of that financial year. However, after some months, the decision-makers can more accurately estimate the total attracted deposit for that year based on what happens during the last months of the year (e.g., the estimation of the attracted deposits up to the end of the year becomes \$450). Commonly, the respective short and long loans could not be paid to customers. In this situation, bank managers would decide to grant proper ratios of optimal facilities obtained through the stochastic model (e.g., 150 and 300 for short and long loans). Hence, the total revenue obtained in this situation would be remarkably different from what is calculated through the optimization problem. We should cautiously choose a proper value for the respective parameter to prevent this issue. To do so, we design an iterative routine, including a simulation started with an initial investment rate. The initial rate could be a simple or weighted average of investment rates in different assets such as short, long loans, stock market, and bonds in previous years. The parameter's value changes in each iteration of this routine; therefore, the optimization function's objective gets close to what is obtained through the simulation. In other words, to validate the value of the optimal objective function using simulation, we consider no penalty cost in the case of shortage; however, a proper opportunity cost should be considered in the case of having deposits more than what has been planned.

5.6.3. Legal constraints

Two types of legal constraints are usually considered in asset/liability management: capital adequacy and legal reserve of deposits. These are regular constraints for all financial banks and institutions imposed into the model by supervisory organizations such as the Money and Credit Council of the Central Bank of Iran.

Capital adequacy constraint

Capital is an important element of a banks' financial support by which they could repay their debts when faced with economic problems. Capital adequacy is not only crucial for the banking system; it is also vital for society and especially investors who want to deposit in the banking system. Based on the regulatory rule approved in the Central Bank, each investment opportunity, including buying bonds, delivering loans, investing in the stock market, and buying fixed assets, has a specific risk measure. The investment portfolio of the bank or financial institutions should be constructed with consideration of these risk measures. The capital adequacy constraint, which is designed to adjust the portfolio risk, is obtained as follows:

$$CP_t \geq 0.08 \times WA_t \qquad \forall t \tag{31}$$

$$WA_t = 0.2 \times NIB_t + 0.1 \times \sum_{i=1}^{I}\sum_{n=1}^{N} NB_t^{i,n-1} + \sum_{m=1}^{J-1} NL_t^m + NIS_t + FA_t, \qquad \forall t \tag{32}$$

where the ratio of capital adequacy is 8% for the risk-weighted assets determined based on the capital adequacy regulations approved in the Central Bank. it stems from guidelines of the Basel Committee that are very accredited in banking supervision. In Eq. (32), the risk measures corresponding to investing in other banks, the bonds, the stock market, and fixed assets and delivering loans are 0.2, 0.1, 1, 1, and 1, respectively. Furthermore, the above hard constraint is changed to a soft constraint by adding a new variable, SCP_t^-, to the left-hand side of Eq. (31). If this variable takes a positive value, the Central Bank penalizes the bank by $PENCP_t^- \times SCP_t^-$.

Constraints on the legal reserve of deposits

Banks must hold a part of the deposits as the legal deposit at the Central Bank. This ratio can vary according to the monetary policy. The banks' legal deposit is one of the Central Bank's monetary policy tools that control liquidity and interest rate. That is, when liquidity is high in the community, the Central Bank adopts the contractionary policy by increasing the rate of legal deposits (RT_t) and consequently attracts more amount of money from all current banks. Therefore, the respective banks would grant fewer loans, reducing liquidity and inflation in the community. The Central Bank also reduces this rate for expansionary policies mainly used to exit from the recession. The following equation is mainly used for the legal deposit constraint:

$$LR_t = RT_t \times RNDEP_{t,s} \tag{33}$$

The above hard constraint would change to a soft constraint by adding a new variable, $SL^-_{2,t,s}$, to the left-hand side of this equation. So if the constraint is not satisfied, the whole objective is penalized by $PEN^-_{2,t,s} \times SL^-_{2,t,s}$

5.6.4. Balance sheet constraint

This constraint is a basic rule for the balance sheet in all financial institutions and firms. The equality constraint corresponding to this rule is that the summation of the total amount of assets should be equal to the sum of liabilities and equity. This constraint is written in the following mathematical formulation:

$$\begin{aligned} & CD_t + \sum_{m=1}^{J-1} NL_t^m + \sum_{i=1}^{I} \sum_{n=1}^{N} NB_t^{i,n-1} \\ & + NIB_t + NIS_t + LRT_t + FA_t + OA_t \\ & = \sum_{z=1}^{K-1} NLD_t^z + NSD_t + NBC_t + OL_t + SE_t. \end{aligned} \tag{34}$$

5.6.5. Policy constraints

All banks and a few financial institutions have unlimited authority to allocate resources and funds for different investments. In other words, they would seek to identify the best ways of investment and profitability. After reducing the total amount of legal deposit for all respective financial institutions, their policymakers could invest all remaining deposits and other monetary resources in loans and other facilities such as buying bonds and investing in the stock market. However, many banks might have high intentions to deliver loans, leading to a problematic issue called unbalanced bank's risk. Also, due to the nature of banks, as a financial intermediary, they cannot ignore issuing loans and invest all deposits into the stock market or buy bonds. Therefore, the acceptable amount of delivering loans should be bounded between lower and upper bounds. These constraints are written as:

$$\sum_{m=1}^{J-1} NL_t^m \geq RateL^{lower} \times RNDEP_{t,s}, \tag{35}$$

$$\sum_{m=1}^{J-1} NL_t^m \leq RateL^{upper} \times RNDEP_{t,s}. \tag{36}$$

The above hard constraints could also be converted to soft constraints by subtracting the new variable, $SL^-_{3,t,s}$, and adding a new variable, $SL^+_{3,t,s}$ to the left-hand sides of constraints (35) and (36), respectively. If these variables take negative values, the objective is penalized with $PEN^-_{3,t,s} \times SL^-_{3,t,s}$ and $PEN^+_{3,t,s} \times SL^+_{3,t,s}$.

Furthermore, according to a popular rule approved in the Central Bank, all respective banks are required to hold a percentage of their total attracted deposit as cash to meet the minimum liquidity,

$$CD_t \geq RateC^{lower} \times RNDEP_{t,s} \tag{37}$$

Similar to Constraints (35) and (36), the above hard constraint is converted to a new soft constraint by subtracting a new variable, $SL^-_{4,t,s}$, to the left-hand side of Constraint (37). The objective is therefore penalized by $PEN^-_{4,t,s} \times SL^-_{4,t,s}$ if the respective new variable takes a negative value.

To have proper liquidity, some banks prefer to have a minimum amount of bonds because they would be so close to cash in terms of the level of liquidity. The following equation controls the minimum total amount of bonds outstanding in each period:

$$\sum_{i=1}^{I} \sum_{n=1}^{N} NB_t^{i,n-1} \geq RateB^{lower} \times RNDEP_{t,s} \tag{38}$$

Analogous to previous equations for cash and loans outstanding, the above hard constraint would be converted to a soft constraint by subtracting a new variable, $SL^-_{5,t,s}$ to the left-hand side of Eq. (38). Therefore, if the variable becomes negative, the objective is penalized with $PEN^-_{5,t,s} \times SL^-_{5,t,s}$.

5.7. Objective function

The objective is to maximize the net present value of profits minus the expected total penalty costs for violations of stochastic constraints over the planning horizon,

$$\begin{aligned} \max \ & \sum_{t=1}^{T} MARR_t \times \Big(\Big(\sum_{j=1}^{J} \sum_{h=0}^{\min(j-1,t-1)} IL_{t-h,t}^{j} \\ & + RIB_t \times NIB_t + RIS_t \times NIS_t \\ & + \sum_{i=1}^{I} \sum_{n=1}^{N} (RIB_t^i \times NB_t^{i,n-1}) \Big) \\ & - \Big(\sum_{k=1}^{K} \sum_{h=0}^{\min(K-1,t-1)} (CLD_{t-h}^{k} \times LD_{t-h}^{k} \times (1 - \sum_{l=1}^{h+1} PA_{k,l}) \Big) \\ & + CBC_t \times NBC_t - (PENCP_t^- \times SCP_t^- \\ & + \sum_{s=1}^{S} P(s) \times (PEN^-_{1,t,s} \times SL^-_{1,t,s} + PEN^+_{1,t,s} \times SL^+_{1,t,s} \\ & + PEN^-_{2,t,s} \times SL^-_{2,t,s} + PEN^-_{3,t,s} \times SL^-_{3,t,s} \\ & + PEN^+_{3,t,s} \times SL^+_{3,t,s} + PEN^-_{4,t,s} \times SL^-_{4,t,s} + PEN^-_{5,t,s} \times SL^-_{5,t,s}))). \end{aligned} \tag{39}$$

The whole proposed methodology for asset and liability management in this paper, including the preprocessing step, is summarized in a schematic view in Figure 2. This figure demonstrates how to do preprocessing step to initialize and prepare the respective input for the mathematical model and evaluate the final results schematically so that the readers could more appropriately understand the process of asset-liability management proposed in this paper.

5.7.1. Model implementation

The proposed optimization model is constructed and implemented for a real case of a commercial bank in the Islamic Republic of Iran. Most of the data used to run the model are based on the bank's financial statements, which the bank's auditor has approved, and since the bank is a member of the capital market company, its data has been publicly published. Other information, such as each asset's interest rate, is disclosed in detail because it is confidential and not published for public sectors. Moreover, all respective parameters are tuned using some available information provided by financial experts of the respective bank.

After implementing the model, the optimal decisions are compared with those made by the bank's respective financial managers for managing its asset and liability in previous years. In what follows, the assumptions are initially explained, and then the details of the information used for making and implementing the proposed model are asserted. Some analytical results are then discussed at the end of this section.

5.7.2. Model assumptions

As mentioned earlier, the model presented in the previous section is a comprehensive multi-period model considering different types of deposits, loans, bonds, and investments.

There are also some assumptions in constructing and implementing the model. They are highlighted as follows:

1. The investment deposits have four types in terms of time-to-maturity from one to four years, where their interests are paid every year, and their respective principals are reimbursed on the maturity date;
2. The loans have four different types in terms of time-to-maturity from one to four years in which each yearly installment includes both interest and a part of the principal;
3. It is assumed that the bank purchases those stocks in the stock market that are highly correlated with the market index so that the obtained profit would be sufficiently close to the profit obtained through the trading index;
4. The bonds with a coupon rate of 14%, 16%, 18%, and 19% and the remaining time-to-maturity of 1 to 4 years are assumed to be available on the stock market every year.

5.7.3. Data needed to implement the optimization model

All the information used to construct and implement the model is listed below. This information is generally derived using the balance sheet, consultation with financial experts in the bank, and the published documents available for all researchers in this area.

1. Bank's balance sheets given for five years from 2009 to 2013,
2. Historical data of index called TEPIX (Tehran Price Index) for the Tehran stock exchange market for years 1999 to 2008,
3. Some information about the interest rate of investment in other banks, the interest rate of borrowing from the Central Bank, the rate of legal

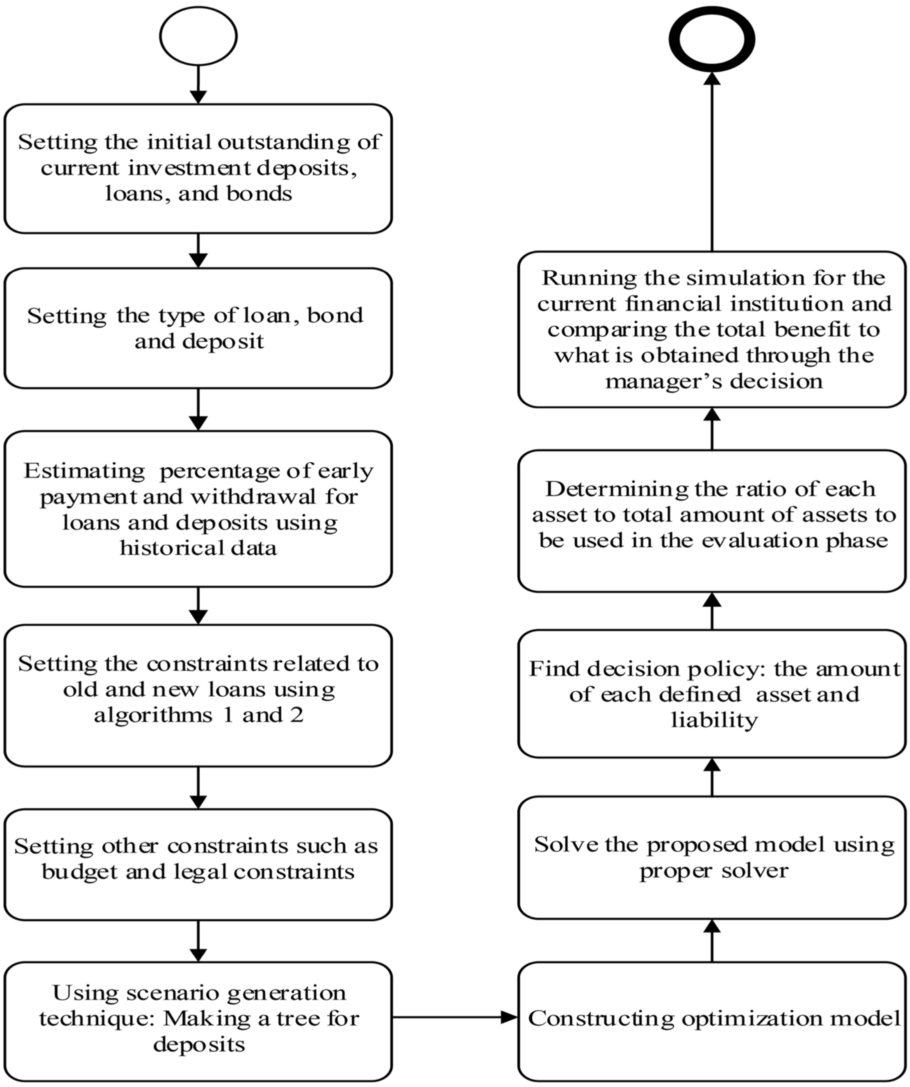

Figure 2. A schematic view of the proposed methodology for asset and liability management.

Table 3. Interest rates for different types of loans, deposits, and others.

Type period	Loans				Deposit				Investment in other banks	Borrowing from the Central Bank
	1	2	3	4	1	2	3	4		
1	21%	22%	22.5%	23.5%	18%	18.5%	19%	20%	18%	25%
2	21%	22%	23%	24%	18%	18.5%	19%	20%	23%	26%
3	22%	23%	24%	24.5%	19%	20%	20.5%	21%	25%	26%
4	23%	24%	25%	26%	20%	21%	22%	23%	22%	28%

Table 4. The percentage of early payment and withdrawal for loans and deposits.

Type Period	Early payment for a loan			Early withdrawal for deposit		
	2	3	4	2	3	4
1	0.15	0.1	0.05	0.25	0.15	0.1
2	-	0.2	0.15	-	0.20	0.2
3	-	-	0.2	-	-	0.25

Table 5. The initial outstanding of investment deposits, loans, and bonds at the beginning of the first period.

Type time-to-maturity	Loans			Deposit			Bonds			
	2	3	4	2	3	4	1	2	3	4
1	21546005	6245294	16995425	458621579	52649752	65249870	1772217	0	0	0
2	-	0	1026458	-	53691115	52015485	4113658	0	0	1051
3	-	-	36974442	-	-	56214589	547008	566536	322957	0

reserve, the interest rates of loans, and deposits for all different types, all of which were obtained through consultation with the financial experts in the destination bank. The needed interest rates for different loans, deposits, and others are summarized in Table 3.
Furthermore, the percentages of early payments of loans and early withdrawals of deposits before their maturity date were derived from historical data provided in the destination bank. These are reported in Table 4.
All the total initial outstanding for bonds, loans, and deposits from different types at the beginning of the first period of the planning horizon is listed in Table 5.

Based on the collected information from the bank's managers, the total outstanding borrowings from the Central Bank, investment in other banks, investment in the stock market, and saving and sight deposits at the beginning of the first periods are 36788928, 3789704, 236514, and 102368447, respectively. Furthermore, in the financial market of Iran, there is no difference between the interest rate of bonds with different time-to-maturity. Therefore, it is assumed that all types of bonds for all time-to-maturity are available in each of four periods. Finally, the indexes of the stock exchange market of Iran (TEPIX) for four years are obtained through ARIMA, a well-known forecasting method.

5.7.4. Scenario generation: Making a tree for deposits

For the stochastic optimization model, it is required to generate good scenarios for the total deposits received in each period. Here, we used an algorithm used in [54] to generate the scenarios tree. The process of generating scenarios is explained as follows:

1. The total balance of bank's deposits from 2000 to 2009 are collected,
2. All deposits are discounted using the corresponding inflation rate in each year to find the respective values based on the prices of 1999,
3. Three different categories are made using a clustering technique,
4. Given four periods and three clusters, 81 scenarios are generated.

It should be mentioned that the probabilities for all the three respective categories in each period are assumed to be 0.3, 0.5, and 0.2, respectively. Therefore, the probabilities of each scenario for a four-period length can be easily obtained. It is worth mentioning that the respective probabilities could be obtained through a frequency-based method.

5.7.5. Analysis of the results

We consider four years from 2008 until 2011. The model is solved using a Gams solver in a Pentium five II core computer. There is full access to the decisions made by financial managers for the respective bank in the sample years mentioned above. Therefore, the two-stage stochastic optimization model is implemented to find the optimal decisions for these years. The following experiments and comparisons are then accomplished to verify the performances of the proposed model and to compare the effect of both sets of decisions for asset/liability management in these years:

1. Comparing the profits of the TSP with those which are conventionally performed by the bank's managers in the real situation;
2. Comparing the level of liquidity (i.e., cash availability);
3. Comparing the situation of capital adequacy constraint;
4. Finding VSS (Value of Stochastic Solution) and EVPI (Expected Value of Perfect Information) for the proposed TSP;

5. Performing a sensitivity analysis for the objective function in terms of the rate of the legal reserve.

5.7.6. Performing the TSP

The optimal results of the TSP include the amount of different assets (i.e., loans, bonds, and investment in other banks and the stock market) and the amount of different liabilities (i.e., investment deposits with three types, interest-free deposits, and the amount of borrowing from the Central Bank) are summarized in Tables 6 and 7 for four consecutive years.

As previously mentioned, the investment decisions obtained through the stochastic optimization model could be a good direction for managers to use suitable strategies to attract such deposits; however, it might not be fulfilled in a financial year for some social or economic reasons. In this situation, the managers should follow the correct direction suggested by some professional experts or obtained through some structural models to find optimal strategies in different years.

Here, we would like to introduce some important ratios, which might give good direction to financial decision-makers. In other words, using these ratios, bank managers could take proper steps when encountering different attracted deposits. It means that decision-makers do not know in advance how successful their attempts to attract planned deposits would be. Therefore, they prefer to use the relevant stochastic optimization model results similar to the ratios illustrated in Table 8.

As illustrated in Table 8, the optimal percentages for all assets for each year could be easily calculated by dividing all individual values in Table 6 to the corresponding amount of total liabilities in Table 8. For example, 1.17 in Table 8, the percentage of investment in the stock market for the first year, is equal to the bold cell in Table 6 divided by the bold cell in Table 8. These ratios are essential references for decision-makers in reality as they could most accurately guess what happens in some future days/months based on what they observed through the past days/months and making proper decisions for them.

Given the whole deposit attracted for the four consecutive years and the ratios in Table 8, this is demonstrated that how to use the outputs of the optimization model in real-time decision-making (Table 9). As observed in Table 9, the respective ratios are used to find the amount of investments in the stock market and other banks and the amount of loans and bonds in different types. The table also illustrates these amounts, which are currently performed by the bank managers for the same four years using their conventional asset-liability approaches.

Table 6. The outstanding of assets for TSP.

Type	Loans			Bonds				Investment in other banks	Investment in the stock market
period	1	2	3	1	2	3	4		
1	157096535	292198573	179278573	3382488	4589986	6249551	5153612	52481629	**8583990**
2	281189775	247410936	255613433	7200148	21637028	18243020	21506685	31298669	15457488
3	573921996	224141737	244964875	27612510	35684283	20980288	28830591	26007171	23041468
4	454340684	397366822	264530877	13125445	29924506	36095801	22127781	61792619	67899709

Table 7. The outstanding of liabilities for TSP.

Type period	Investment deposit			Interest-free deposits	Borrowing from the Central Bank
	1	2	3		
1	176651817	190901163	175914829	171157950	17843255
2	277758624	216155200	178274304	258381920	20147469
3	235116025	470113340	325237346	226003231	29321915
4	443095677	336767682	488650153	174468162	31624187

Table 8. The ratios obtained from the results of TSP.

Year asset/liability		1	2	3	4
Total deposits		714625758	930570047	1256469942	1442981674
Borrowing from the Central Bank		17843255	20147469	29321915	31624187
The total liabilities		**732469013**	950717516	1285791857	1474605861
Investment in the stock market (%)		1.17	1.63	1.83	4.60
Investment in other banks (%)		7.17	3.29	2.07	4.19
Loans %	Type 1	24	27	19	18
	Type 2	40	26	17	27
	Type 3	21	30	45	31
Bonds %	Type 1	1	2	2	2
	Type 2	1	2	2	2
	Type 3	1	2	3	2
	Type 4	0	1	2	1

Table 9. The comparison of decision making using ratios obtained through the optimal values of the TSP with those achieved through the conventional way.

Model/year assets/ liabilities		Optimization model				Conventional			
		1	2	3	4	1	2	3	4
The amount of total liabilities		**350599761**	407349758	497304486	664234817	350599761	407349758	497304486	664234817
Investment in the stock market		4108768	6623002	8911727	30585360	5012549	9520737	11977964	29221031
Investment in other banks		25120580	13410403	10058769	27834427	43909230	19120767	29771173	45357148
Loans	1	**85812538**	109521565	94744830	119157684	109455200	149944088	158988545	139805295
	2	139862230	106007077	86691085	178993510	86167102	82027821	60843748	97364352
	3	75195000	120480147	221975261	204657332	43964957	54353103	101203108	141756347
Bonds	1	2466801	9214875	11150780	9967438	3891337	9375814	15990258	19638883
	2	2991377	7816506	8114526	16259319	2915235	5309346	10093670	11648157
	3	2197018	9270722	13801576	13479465	1522231	7426596	12618220	12220491
	4	1619044	3085016	10679664	5912344	384836	1413251	4880492	2625792
Borrowing from the Central Bank		8540759	8632498	11249739	14245085	12896542	12253997	15243262	38398956
Profit		43133977				35459541			

As illustrated in Table 9, all the respective deposits for both situations (i.e., using the ratios of optimization model and the decisions taken by the bank managers) are the same; however, the investment decisions are quite different. For instance, in the first financial year, the total amount of the first type loan is 85812538 (i.e., 0.24×350599761), while it is 109455200 based on the conventional decision-making. Furthermore, as shown in the last row of the table, the total profit obtained through the respective ratios is slightly better than the profit achieved by conventional decision-making. Of course, the real benefit (not expected benefit) would also be reversed for each of four consecutive years. It means that the optimal solution is supposed to be better than all other solutions, such as one is implemented by the bank managers, in terms of the total expected profit obtained using all different scenarios, not one single scenario such as what is used in Table 9.

Moreover, Tables 10 and 11 demonstrate liquidity and capital adequacy for these four years based on the decisions of bank managers and those obtained through the respective ratios. Although the results in TSP seem to be mostly better than those obtained by the bank managers, these results cannot be a reliable basis for comparison. The simulation process should be repeated for four years with new data sequences; however, this dataset is unavailable for the central bank under study.

Table10 . The level of liquidity in each period (Cash at the end of each period).

Year Decisions	2010	2011	2012	2013
TSP	4803123	5215423	5893043	8206848
Bank's managers	4695936	4452192	3665116	7373837

Table 11. The level of capital adequacy in each period.

Year Decisions	2010	2011	2012	2013
TSP	10.28	8.45	7.83	8.78
Bank's managers	8.9	8.1	8.0	8.2

Table 12. Expected Value of Perfect Information (EVPI) and Value of Stochastic Solution (VSS).

Features	Value
VSS	3865711
EVPI	2593144

5.7.7. Finding the EVPI and VSS

Finding Expected Value of Perfect Information (EVPI) and Value of Stochastic Solution (VSS) are two main indexes usually calculated for stochastic optimization. These are used as lower and upper bounds for the optimal value of the objective function in TSP (Table 12). One can refer to [55] for more detailed information.

The value of VSS verifies that stochasticity would be beneficial for decision-makers in terms of the expected value. Moreover, EVPI confirms that we might improve the expected benefit if other attempts are performed to find new knowledge about the stochastic data.

5.7.8. Analysis of the model sensitivity

Sensitivity analysis is a postal optimization method that tries to identify the effect of uncertain parameters on the optimal solution and the optimal value of the objective function. It usually assesses the effect of changing only one parameter where other parameters are constant. Here, we would like to know the effect of change in the rate of legal deposit on the objective function of TSP.

5.7.9. The rate of legal deposit

As previously discussed in the third section, the legal deposit rate is one of the tools for controlling monetary policy. Figure 3 illustrates the sensitivity of the objective function of TSP, where the rate of legal deposit changes from 10% to

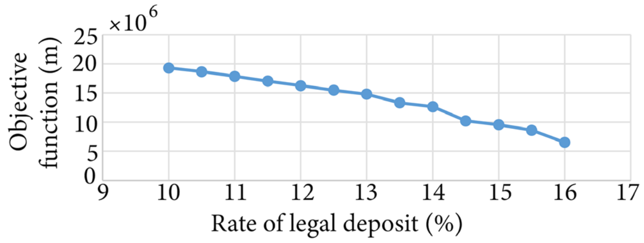

Figure 3. Changes of the objective function (m) to changes in the legal deposit rate.

16%. As expected, by increasing the rate of legal deposit, the total expected benefit decreases. It means that the bank managers should keep more deposits in the Central bank, which usually leads to trivial profitability.

6. Conclusions

Applying suitable assets and liability management techniques is vital in financial institutions like banks, insurance companies, and pension funds. The mismanagement of assets and liabilities, especially in the financial banks that usually attract short-term deposits and grant long-term loans, leads to a shortcoming or waste of resources. This situation could generally have a negative impact on their financial performance. In other words, keeping more-than-needed cash and lacking sufficient cash resources generally end up with higher opportunity costs and other costs such as fines imposed by the Central Bank.

This research study presented a novel two-stage multi-period stochastic mathematical optimization model for asset and liability management in financial institutions. Our proposed model considers many real issues that financial experts and managers usually consider. The main goal in our proposed model was to determine the amount of different deposits (i.e., liabilities) and variety of investments (i.e., assets) with consideration of some risk measurements and different legal, budget, balance sheet, and policy constraints. In our proposed model, each type of asset's amount was considered the decision variable while the total amount of attracted deposit (i.e., the summation of all types of deposits, including sight and saving deposits) is assumed stochastic. It means that bank managers could take proper strategies to reach the goals of the financial institutions using the outcome of the optimization model.

The developed optimization model could consider assets/liabilities with different maturities and types (e.g., sight and saving deposits, bonds, and loans with one to four-year maturity dates, investment in other banks and the stock exchange market, and borrowing from the Central Bank). Moreover, the outstanding feature of our proposed model compared to other works in the literature was that all types of assets and liabilities with maturities out of the predefined planning horizon could be considered. In other words, the proposed model can easily be implemented in a rolling horizon approach considering the total loans and deposits coming from previous years. A real data set for a commercial bank located in Tehran, the Islamic Republic of Iran capital, was used to validate the proposed model for four successive years. This data set was prepared using the balance sheets and the information given by the financial experts in the bank. Considering that the optimal amount of different assets achieved through the optimization model might not be fulfilled within the financial year, these amounts were converted to appropriate ratios. These ratios were used for decision-making in reality. We compared the results of decision-making obtained using these ratios and what was conventionally implemented in the commercial bank and showed the superiority of our proposed model in terms of the total profit, the capital of adequacy, and the liquidity for stakeholders. In the real case study in the commercial bank, we have shown that the total benefit, as the most important goal for stakeholders, obtained through the proposed mathematical model increases by 21%.

It should be noted that the proposed asset and liability management model could be utilized for insurance companies and pension funds in which some constraints need to be added to make it fit the individual cases. The framework of the model is the same proposed in the paper for all financial institutions having assets and liabilities.

The optimization model would also be extended for multistage situations where credibility risk is also considered. Furthermore, the risk consideration using C-VaR as a widely-used risk factor could also be another extension of the mathematical model. These are left for future studies.

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Appendix A.

Notation used in the model

Parameters of the model

RB_t^i	The rate of return on bonds by type *i,* in period *t,*
$NLI^{j,m}$	The initial remaining loans (issued before the first period) by type *j* and remaining time-to-maturity *m,*
$PLI_t^{j,m}$	The received principal of the loan by type *j* and maturity *m* at the time *t* issued before the first period,
$ILI_t^{j,m}$	The received interest of the loan by type *j* and maturity *m* at the time *t* issued before the first period,
$RPLI_t^{j,m}$	The received early payment of the loan with type *j* and maturity *m* at the time *t* issued before the first period,
$NBI^{i,n}$	The initial remaining amount of bonds with type *i*, and remaining time-to-maturity *n* for bonds purchased before the first period,
NIBI	The initial remaining receivables from other banks at the beginning of the first period,
NISI	Remaining of investment in the stock market at the beginning of the first period,

$NLDI^{k,z}$	The initial remaining investment/certificate deposits with type k and maturity z, for investment/certificate deposits received before the first period,
$NSDI$	The initial remaining interest-free deposits (i.e., saving and sight deposits) at the beginning of the first period,
$NBCI$	The initial remaining borrowings from the Central Bank at the beginning of the first period,
CDI	The initial remaining cash at the beginning of the first period,
LR_0	The legal deposit at the Central Bank at the beginning of the first period,
RL_t^j	The rate of return on loans with type j, in period t,
$RLI^{j,m}$	The rate of return on loans (issued before the first period) by type j and remaining time-to-maturity m,
RIB_t	The rate of return on investments in other financial institutions in period t,
RIS_t	The rate of return on investments in the stock market in period t,
CLD_t^k	The interest paid on investment/certificate deposit of type k in period t,
$CLDI^{k,z}$	The interest paid on investment/certificate deposits with type k and maturity z, for investment/certificate deposits received before the first period,
CBC_t	The interest paid for borrowing from the Central Bank, in period t,
FA_t	Net and intangible fixed assets, at the end of period t,
EX_t	Total operating costs of the bank in period t,
CP_t	The primary capital of the bank in period t,
CR_t	Capital adequacy ratio at period t,
RT_t	Legal reserve ratio for the total amount of different deposits in period t,
$\Delta_{i,n}$	Zero-One matrix. One, if a bond with type i and maturity n exists and zero, otherwise,
$PA_{k,l}$	The mean percentage of early withdrawal of investment/certificate deposit with type k after l periods ($l \leq k$),
$PB_{j,g}$	The mean percentage of the early payment of the total outstanding loan with type j after g periods ($g \leq j$),
PC	The average withdrawal percentage for interest-free deposits (i.e., saving and sight deposits).
$MARR_t$	The Minimum Attractive Rate of Return (*MARR*) in period t,
$RNDEP_{t,s}$	The total amount of different deposits in period t under scenario s,
$PEN_{y,t,s}^{-}$	Penalty rate of negative deviation for deposit flow constraint ($y=1$, Eq. (30), a legal reserve of deposit constraint (y=2, Eq. (33), policy constraint for a loan (y=3, Eq. (35), policy constraint for cash (y=4, Eq. (37), and policy constraint for bonds (y=5, Eq. (38) under scenario s in period t,
$PEN_{y,t,s}^{+}$	Penalty rate of positive deviation for deposit flow constraint (y=1, Eq. (30), and policy constraint for a loan (y=3, Eq. (36) under scenario s in period t,
$PENCP_{1t}^{-}$	Penalty rate of negative deviation for Capital Adequacy constraint in period t,
$P(s)$	Probability of scenario s,
PF_t	The legal reserves at the Central Bank,
$RateL^{lower}$	The lower limit percentage of total deposit used for finding the minimum loan outstanding,
$RateL^{upper}$	The upper limit percentage of total deposit used for finding the maximum loan outstanding,
$RateC^{lower}$	The lower limit percentage of total deposit used for finding the minimum cash at the end of each period,
$RateB^{lower}$	The lower limit percentage of total deposit used for finding the minimum bond outstanding in every period,

Decision variables

CD_t	The remaining cash assets at the end of period ***t***,
$B_t^{i,n}$	The bonds sold (negative value) and purchased (positive value) in period ***t***, with type ***i***, and maturity ***n***,
$NB_t^{i,n}$	The total amount of bonds with type ***i*** and remaining time-to-maturity ***n*** in period ***t***,
L_t^j	The new loans issued with type ***j*** in period ***t***,
NL_t^m	The loans outstanding with remaining time-to-maturity m in period ***t***,
$PL_{q,t}^j$	The principal of loan with type ***j*** issued at time ***q*** received in period ***t***,
$IL_{q,t}^j$	The interest of loan with type ***j*** issued at time ***q*** received in period ***t***,
$RPL_{q,t}^j$	The early payment of the loan with type ***j*** issued at time ***q*** received in period ***t***,
LR_t	The legal deposit at the Central Bank in period ***t***,
IS_t	The amount of new purchasing (positive value) and selling (negative value) or investment in the stock market in period ***t***,
NIS_t	The total amount of investment in the stock market at the end of period ***t***,

IB_t	The new investment amount in other banks in period ***t***,
NIB_t	The total investment amount in other banks at the end of period ***t***,
SD_t	The new amount of interest-free deposits (i.e., saving and sight deposits) in period ***t***,
NSD_t	The total amount of interest-free deposits (i.e., saving and sight deposits) at the end of period ***t***,
LD_t^k	The new investment/certificate deposits with type ***k*** in period ***t***,
NLD_t^z	The total investment/certificate deposits with the remaining time-to-maturity ***z*** at the end of period ***t***,
BC_t	The new borrowing from the Central Bank in period ***t***,
NBC_t	The total borrowing from the Central Bank at the end of period ***t***,
$SL^-_{y,t,s}$	The amount of negative deviation in deposit flow constraint (***y***=*1*, Eq. (30), a legal reserve of deposit constraint (***y***=2, Eq. (33), policy constraint for loans (***y***=3, Eq. (35), policy constraint for cash (***y***=4, Eq. (37), and policy constraint for bonds (***y***=5, Eq. (38) under scenario ***s*** in period ***t***,
$SL^+_{y,t,s}$	The amount of positive deviation in deposit flow constraint (***y***=1, Eq. (30), and policy constraint for loans (***y***=3, Eq. (36) under scenario ***s*** in period ***t***,
SCP_t^-	The amount of negative deviation in Capital Adequacy constraint in period ***t***,
WA_t	The total weighted assets divided by risk at the end of period ***t***,
PF_t	The bank profits at the end of period ***t***.

Biographies

Behzad Mousavi received a BSc degree in Industrial Engineering from the Urmia University of Technology University in 2012 and an MSc degree in Industrial Engineering from the Amirkabir University of Technology in 2015. He is a PhD candidate in the industrial engineering and Management Systems Department at the Amirkabir University of Technology, Tehran, Iran. His research interests are stochastic programming, simulation-based optimization, and machine learning methods in health systems.

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