A new class of robust ratio estimators for finite population variance

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Abstract

It is a general practice to use robust estimates to improve ratio estimators using functions of the parameters of an auxiliary variable. In this study, a new class of robust estimators based upon the minimum covariance determinant (MCD) and the minimum volume ellipsoid (MVE) robust covariance estimates have been suggested for estimating population variance in the presence of outlier values in the data set for the simple random sampling. The expression for the mean square error (MSE) of the proposed class of estimators is derived from the first degree of approximation. The efficiency of the proposed class of robust estimators is compared with some competing estimators discussed in the literature, and found that proposed estimators are better than other mentioned estimators here. In addition, real data set and simulation studies are performed to present the efficiencies of the estimators. We demonstrate theoretically and numerically that the proposed class of estimators performs better than all other competitor estimators under all situations.

Keywords: Finite population variance; Robust covariance estimates; Auxiliary information; Mean square error; Efficiency, Simple random sampling.

1 Introduction

The use of auxiliary variables can increase the precision of estimators. The ratio, product, and regression estimators are good examples for improving the performance of estimators. Estimating the population variance has great significance in various fields such as Industry, Agriculture, Medical, Economic, and Biological sciences. Efficient estimators for the population variance has been discussed by various authors referred to Kadilar and Cingi [1], Khan and Shabbir [2], Singh et al. [3], Yadav et al. [4], Yaqub and Shabbir [5], Singh and Pal [6], Sanaullah et al. [7], Muneer et al. [8], Housila et al. [9] and Sharma et al. [10]. However, in the presence of unusual observations in the data, since the classical estimators are sensitive to these extreme values, their efficiencies decrease [11]. Therefore, to reduce the negative effect of the unusual observation problem in the data, it is suggested to use the robust regression estimate, the minimum covariance determinant (MCD), and the minimum volume ellipsoid (MVE) estimators instead of the classical ones. Abid et al. [12] presented the ratio estimators of variance-based using robust measures in the presence of unusual values. Naz et al. [13] proposed the ratio-type estimators developing the efficiency of the ratio-type estimators of population variance using robust location measures. Zaman and Bulut [14] proposed ratio-type estimators using robust regression estimators and robust covariance matrices for stratified random sampling. Bulut and Zaman [15] presented the ratio-type estimators utilizing MCD estimates. Zaman and Bulut [16] provided the ratio estimators for population variance considering MCD and MVE robust covariance estimates, both simple and stratified random sampling. Zaman et al. [17] presented the robust regression-ratio-type estimators of the mean utilizing two auxiliary variables. Grover and Kaur [18] developed the regression-type estimators of population mean with two auxiliary variables using the robust regression technique. Zaman and Bulut [19] proposed the robust ratio double sampling estimator of finite population mean in the presence of outliers. Unlike other studies, this study proposes regression-ratio-type estimators of population variance-based MCD and MVE covariance estimates for simple random sampling.

We give notations used in Subsection 1.1 and some estimators in Subsection 1.2.

1.1 Notations

Consider a finite population $U = (U_1, U_2, ..., U_N)$ having N units. Let y_i and x_i be the values of the study variable and the auxiliary variable, respectively. The notations used in this paper can be described as follows:

$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$: the population mean of the auxiliary variable x,

$\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$: the population mean of the study variable y,

$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \overline{X}\right)^2$: the population variance of the auxiliary variable x,

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \overline{Y}\right)^2$: the population variance of the study variable y,

$\lambda_{rq} = \frac{\mu_{rq}}{\left(\mu_{20}^{r/2}\mu_{02}^{q/2}\right)}$ and $\mu_{rq} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \overline{Y}\right)^r\left(x_i - \overline{X}\right)^q$, r and q are non-negative integers.

$\lambda_{04} = \beta_{2x}$: population coefficient of kurtosis of the auxiliary variable x,

$\lambda_{40} = \beta_{2y}$: population coefficient of kurtosis of the study variable y.

$b = \frac{S_y^2\left(\lambda_{22} - 1\right)}{S_x^2\left(\lambda_{04} - 1\right)}$: the sample regression coefficient,

$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$: the sample mean of the auxiliary variable x,

$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$: the sample mean of the study variable y,

$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2$: the sample variance of the auxiliary variable x,

$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2$: the sample variance of the study variable y.

The sample means $\left(\overline{y}, \overline{x}\right)$ are unbiased estimators of the population means $\left(\overline{Y}, \overline{X}\right)$, respectively, and $\left(s_y^2, s_x^2\right)$ are unbiased estimators of population variances $\left(S_y^2, S_x^2\right)$, respectively.

1.2 Some Existing Estimators

We discuss the following estimators, and we show which estimators are more efficient under what conditions. We assume that the population variance S_x^2 of the auxiliary variable x is available in this study.

The variance of the unbiased estimator $\left(t_0 = s_y^2\right)$ as

$$V\left(t_0\right) = \frac{S_y^4}{n}\left(\lambda_{40} - 1\right) \tag{1}$$

If the population variance S_x^2 of the auxiliary variable x is known, Isaki [20] introduced the ratio estimator for s_y^2 as

$$t_R = s_y^2 \frac{S_x^2}{s_x^2} \tag{2}$$

The MSE of the estimator t_R, is given by

$$MSE\left(t_R\right) = \frac{S_y^4}{n}\left[\left(\lambda_{40} - 1\right) + \left(\lambda_{04} - 1\right)\left(1 - 2C\right)\right] \tag{3}$$

Examining (1) and (3), Isaki's [20] estimator provides a lower MSE than the unbiased estimator under the condition $C > 0.5$.

If the population variance S_x^2 of the auxiliary variable x is known and when s_y^2 in (2) is replaced with t_R, then Singh et al. [21] provided the chain ratio estimator as

$$t_{CR} = t_R \frac{S_x^2}{s_x^2} \tag{4}$$

We can rewrite (4) using (2) as

$$t_{CR} = s_y^2 \frac{S_x^4}{s_x^4} \tag{5}$$

The MSE of the estimator t_R, is given by

$$MSE\left(t_{CR}\right) = \frac{S_y^4}{n}\left[\left(\lambda_{40} - 1\right) + 4\left(\lambda_{04} - 1\right)\left(1 - C\right)\right] \tag{6}$$

$$C = \frac{\lambda_{22} - 1}{\lambda_{04} - 1}$$

Examining (1) and (6), Singh's et al. [21] estimator provides a lower MSE than the unbiased estimator under the condition $C > 1$. From (3) and (6), Singh's et al. [13] estimator provides a lower MSE than the Isaki’s [20] estimator under the condition $C > 1.5$.

If the population variance S_x^2 of the auxiliary variable x is known, Isaki [20] defined the following regression estimator for S_y^2, given by

$$t_{reg} = s_y^2 + b\left(S_x^2 - s_x^2\right) \tag{7}$$

where b is the sample regression coefficient.

The MSE of the estimator t_R, given by

$$MSE\left(t_{reg}\right) = \frac{S_y^4}{n}\left(\lambda_{40} - 1\right)\left(1 - \rho^2\right) \tag{8}$$

where $\rho = \dfrac{\left(\lambda_{22} - 1\right)}{\sqrt{\left(\lambda_{40} - 1\right)\left(\lambda_{04} - 1\right)}}$.

Examining (1) and (8), Isaki's [20] regression ratio-type estimator provides a lower MSE than the unbiased estimator under the condition $\rho^2 > 0$, because the condition is always satisfied.

When the population variance S_x^2 of the auxiliary variable x is known, Upadhyaya and Singh [22] introduced the ratio estimator for S_y^2 as

$$t_{US} = s_y^2\left(\frac{S_x^2 + \lambda_{04}}{s_x^2 + \lambda_{04}}\right) \tag{9}$$

The MSE of the estimator t_{US}, given by

$$MSE\left(t_{US}\right) \cong \frac{S_y^4}{n}\left[\left(\lambda_{40} - 1\right) + g_0^2\left(\lambda_{04} - 1\right) - 2g_0\left(\lambda_{22} - 1\right)\right] \tag{10}$$

where $g_0 = \dfrac{S_x^2}{S_x^2 + \lambda_{04}}$.

Examining (1) and (10), Upadhyaya and Singh's [22] estimator provides a lower MSE than the unbiased estimator under the condition $g_0 < \dfrac{2\left(\lambda_{22} - 1\right)}{\left(\lambda_{04} - 1\right)}$.

When the population variance S_x^2 of the auxiliary variable x is known, Kadilar and Cingi [1] provided the ratio estimator for S_y^2 as

$$t_{KC1} = s_y^2\left(\frac{S_x^2 + C_x}{s_x^2 + C_x}\right) \tag{11}$$

$$t_{KC2} = s_y^2\left(\frac{\lambda_{04}S_x^2 + C_x}{\lambda_{04}s_x^2 + C_x}\right) \tag{12}$$

$$t_{KC3} = s_y^2 \left(\frac{C_x S_x^2 + \lambda_{04}}{C_x s_x^2 + \lambda_{04}} \right) \quad (13)$$

where $C_x = \frac{S_x}{\bar{X}}$ is the population coefficient of variation.

The MSEs of the estimators $t_{KCi}\left(i=1,2,3\right)$, given by

$$MSE\left(t_{KCi}\right) \cong \frac{S_y^4}{n}\left[\left(\lambda_{40}-1\right)+g_i^2\left(\lambda_{04}-1\right)-2g_i\left(\lambda_{22}-1\right)\right] \quad (14)$$

where $g_1 = \frac{S_x^2}{S_x^2 + C_x}$, $g_2 = \frac{\lambda_{04} S_x^2}{\lambda_{04} S_x^2 + C_x}$, $g_3 = \frac{C_x S_x^2}{C_x S_x^2 + \lambda_{04}}$ [4].

Examining (1) and (14), Kadilar and Cingi's [1] estimators provide a lower MSE than the unbiased estimator under the condition $g_i < \frac{2\left(\lambda_{22}-1\right)}{\left(\lambda_{04}-1\right)}, i=1,2,3$. From (10) and (14), Kadilar and Cingi's [1] estimators provide a lower MSE than the Upadhyaya and Singh's [22] estimator under the condition $g_i < \frac{2\left(\lambda_{22}-1\right)}{\left(\lambda_{04}-1\right)} - g_0, i=1,2,3$.

In this paper, we have suggested proposed regression-ratio-type estimators and the proposed class of robust estimators for simple random sampling in Section 2. The expression for the MSEs of the proposed regression-ratio-type estimators the proposed class of robust estimators are provided in Section 3. The performance comparisons of various estimators are demonstrated in Section 4. A numerical and simulation studies are given in Sections 5 and 6, respectively. Conclusion is presented in Section 7.

2 Robust Estimators for the mean vector and the scatter matrix

MVE and MCD estimators are the most commonly used estimators in the literature for multivariate location and scatter parameters. The MVE estimator selects h observations out of n observation units that will make the volume of the ellipsoid the smallest and takes the sample mean vector of these h observations and the covariance matrix as the MVE estimator of the location and scatter parameter of the multivariate data. Similarly, the MCD estimator chooses h observations with the smallest determinant of the covariance matrix among n observations and takes the sample mean vector and covariance matrix of these h observations as the MCD estimator of the location and scatter parameter of the multivariate data [23]. To calculate MVE and MCD estimators in the R program, MASS package is used [24].

3 The Proposed Estimators

In this section, we propose regression-type estimators of population variance for simple random sampling. However, the effectiveness of these classical estimators decreases when there is an outlier in the data set. Therefore, the classical ratio estimators proposed to eliminate the negative effect of the outlier problem have been extended to robust ratio-type estimates in Subsection 3.2.

3.1 The Proposed Regression-Type Estimators

We consider the following regression-ratio-type estimators for the population variance s_y^2 as

$$t_{rZB} = \frac{s_y^2 + b\left(S_x^2 - s_x^2\right)}{\left(\zeta s_x^2 + \xi\right)}\left(\zeta S_x^2 + \xi\right) \tag{15}$$

where ζ and ξ are either constant or the functions of the parameters of auxiliary variable such as C_x, β_{2x} and ρ. To obtain the MSE of the estimator in (15), the terms with e's are defined as follows: Let $e_0 = \left(s_y^2 - S_y^2\right)/S_y^2$. and $\left(s_x^2 - S_x^2\right)/S_x^2$, such that $E\left(e_i\right) = 0, i = 0,1$. $E\left(e_0^2\right) = \frac{\left(\lambda_{40} - 1\right)}{n}$, $E\left(e_1^2\right) = \frac{\left(\lambda_{04} - 1\right)}{n}$, and $E\left(e_0 e_1\right) = \frac{\left(\lambda_{22} - 1\right)}{n}$.

Following Singh and Malik [25], the expressing (15) in terms of e's, we have

$$t_{rZB} = \left[S_y^2\left(1 + e_0\right) - bS_x^2 e_1\right]\left[1 + A_1 e_1^2\right]^{-1}$$

Up to first order of approximation, the expressions of MSE for t_{ZB} is given by

$$\left(t_{rZB} - S_y^2\right)^2 = \left(S_y^2 e_0 - bS_x^2 e_1 - A_1 S_y^2 e_1\right)^2$$

$$\left(t_{rZB} - S_y^2\right)^2 \cong \left[S_y^4 e_0^2 + \left(b^2 S_x^4 + A_1^2 S_y^4 + 2A_1 b S_y^2 S_x^2\right)e_1^2 - 2S_y^2 e_0 e_1\left(bS_x^2 + A_1 S_y^2\right)\right]$$

$$MSE\left(t_{rZB}\right) \cong \frac{1}{n}\left\{S_y^4\left(\lambda_{40} - 1\right) + \left(\lambda_{04} - 1\right)\left[B^2 S_x^4 + A_1^2 S_y^4 + 2A_1 B S_y^2 S_x^2\right] - 2S_y^2\left(\lambda_{22} - 1\right)\left[BS_x^2 + A_1 S_y^2\right]\right\} \tag{16}$$

where $A_1 = \frac{\zeta S_x^2}{\zeta S_x^2 + \xi}$.

Table 1 presents some of the estimators for the population variance s_y^2, which can be obtained by suitable choice of constants ζ and ξ.

[Table 1 Here]

3.2 The Proposed Class of Robust Estimators

We define to apply the following ratio estimators for the population variance s_y^2 using robust covariance estimates to data which have outliers.

$$t_{rZB(j)} = \frac{s_{y(j)}^2 + b_j\left(S_{x(j)}^2 - s_{x(j)}^2\right)}{\left(\zeta_{(j)} s_{x(j)}^2 + \xi_{(j)}\right)}\left(\zeta_{(j)} S_{x(j)}^2 + \xi_{(j)}\right) \tag{17}$$

where $s_{y(j)}^2$, b_j , $S_{(j)}^2$, $s_{x(j)}^2$, $\zeta_{(j)}$ and $\xi_{(j)}$ are obtained by considering MCD and MVE covariance estimates, respectively.

Using (17), the following MSE for all suggested estimators belonging to robust covariance estimates in interest are obtained as below:

$$MSE\left(t_{rZB(j)}\right) \cong \frac{1}{n}\{S_{y(j)}^{4}\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)\left[B_{j}^{2}S_{x(j)}^{4}+A_{1(j)}^{2}S_{y(j)}^{4}+2A_{1(j)}B_{j}S_{y(j)}^{2}S_{x(j)}^{2}\right]$$
$$-2S_{y(j)}^{2}\left(\lambda_{22(j)}-1\right)\left[B_{j}S_{x(j)}^{2}+A_{1(j)}S_{y(j)}^{2}\right]\}; j = MCD\, and\, MVE \tag{18}$$

We remark that the expression for the MSE of the proposed class of robust estimators is in the same form as expression for the MSE presented in (16), but it is clear that S_y^4, λ_{40}, B, S_x^4, λ_{22}, and A_1 in (16) should be replaced by $S_{y(j)}^4$, $\lambda_{40(j)}$, B_j , $S_{x(j)}^4$, $\lambda_{22(j)}$, and $A_{1(j)}$, whose values as obtained by robust covariance estimates $\left(\text{j} = \text{MCD and MVE}\right)$.

Table 2 presents the proposed class of robust estimators for the population variance s_y^2, which can be obtained by suitable choice of constants $\zeta_{(j)}$ and $\xi_{(j)}$.

[Table 2 Here]

5 Efficiency Comparisons

We compare the proposed class of robust estimators with the other competing estimators,

5.1 With the proposed class of robust estimators

(i) With the MSE of estimators given in (16) and (18)

$$MSE\left(t_{rZB(j)}\right) < MSE\left(t_{rZB}\right)$$

$$\frac{1}{n}\left\{S_{y(j)}^{4}\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)\left[B_{j}^{2}S_{x(j)}^{4}+A_{1(j)}^{2}S_{y(j)}^{4}+2A_{1(j)}B_{j}S_{y(j)}^{2}S_{x(j)}^{2}\right]-2S_{y(j)}^{2}\left(\lambda_{22(j)}-1\right)\left[B_{j}S_{x(j)}^{2}+A_{1(j)}S_{y(j)}^{2}\right]\right\}$$
$$<\frac{1}{n}\left\{S_{y}^{4}\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)\left[B^{2}S_{x}^{4}+A_{1}^{2}S_{y}^{4}+2A_{1}BS_{y}^{2}S_{x}^{2}\right]-2S_{y}^{2}\left(\lambda_{22}-1\right)\left[BS_{x}^{2}+A_{1}S_{y}^{2}\right]\right\} \tag{19}$$

Let

$L_{(j)} = \left(\lambda_{04(j)}-1\right)\left[B_{j}^{2}S_{x(j)}^{4}+A_{1(j)}^{2}S_{y(j)}^{4}+2A_{1(j)}B_{j}S_{y(j)}^{2}S_{x(j)}^{2}\right]$, and

$M_{(j)} = S_{y(j)}^{2}\left(\lambda_{22(j)}-1\right)\left[B_{j}S_{x(j)}^{2}+A_{1(j)}S_{y(j)}^{2}\right]$; $j = MCD\, and\, MVE$.

$K = S_y^4\left(\lambda_{40}-1\right)$, $L = \left(\lambda_{04}-1\right)\left[B^2S_x^4+A_1^2S_y^4+2A_1BS_y^2S_x^2\right]$, $M = S_y^2\left(\lambda_{22}-1\right)\left[BS_x^2+A_1S_y^2\right]$, $N = S_y^4\left(\lambda_{04}-1\right)$

Thus, (20) becomes

$$\left(K_{(j)}-K\right)+\left(L_{(j)}-L\right)-2\left(M_{(j)}-M\right)<0 \tag{20}$$

(ii) With the MSE of estimators given in (1) and (18)

$$MSE\left(t_{rZB(j)}\right) < V\left(t_0\right)$$

$$\frac{1}{n}\left\{S_{y(j)}^{4}\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)\left[B_{j}^{2}S_{x(j)}^{4}+A_{1(j)}^{2}S_{y(j)}^{4}+2A_{1(j)}B_{j}S_{y(j)}^{2}S_{x(j)}^{2}\right]-2S_{y(j)}^{2}\left(\lambda_{22(j)}-1\right)\left[B_{j}S_{x(j)}^{2}+A_{1(j)}S_{y(j)}^{2}\right]\right\}$$

$$<\frac{S_{y}^{4}}{n}\left(\lambda_{40}-1\right)$$

$$\left(K_{(j)}-K\right)+L_{(j)}-2M_{(j)}<0 \tag{21}$$

(iii) With the MSE of estimators given in (3) and (18),

$$MSE\left(t_{rZB(j)}\right)<MSE\left(t_{R}\right)$$

$$\frac{1}{n}\left\{S_{y(j)}^{4}\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)\left[B_{j}^{2}S_{x(j)}^{4}+A_{1(j)}^{2}S_{y(j)}^{4}+2A_{1(j)}B_{j}S_{y(j)}^{2}S_{x(j)}^{2}\right]-2S_{y(j)}^{2}\left(\lambda_{22(j)}-1\right)\left[B_{j}S_{x(j)}^{2}+A_{1(j)}S_{y(j)}^{2}\right]\right\}$$

$$<\frac{S_{y}^{4}}{n}\left[\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)\left(1-2C\right)\right]$$

$$\left(K_{(j)}-K\right)+L_{(j)}-2M_{(j)}-N\left(1-2C\right)<0 \tag{22}$$

(iv) With the MSE of estimators given in (6) and (18),

$$MSE\left(t_{rZB(j)}\right)<MSE\left(t_{CR}\right)$$

$$\frac{1}{n}\left\{S_{y(j)}^{4}\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)\left[B_{j}^{2}S_{x(j)}^{4}+A_{1(j)}^{2}S_{y(j)}^{4}+2A_{1(j)}B_{j}S_{y(j)}^{2}S_{x(j)}^{2}\right]-2S_{y(j)}^{2}\left(\lambda_{22(j)}-1\right)\left[B_{j}S_{x(j)}^{2}+A_{1(j)}S_{y(j)}^{2}\right]\right\}$$

$$<\frac{S_{y}^{4}}{n}\left[\left(\lambda_{40}-1\right)+4\left(\lambda_{04}-1\right)\left(1-C\right)\right]$$

$$\left(K_{(j)}-K\right)+L_{(j)}-2M_{(j)}-4N\left(1-C\right)<0 \tag{23}$$

(v) With the MSE of estimators given in (8) and (18),

$$MSE\left(t_{rZB(j)}\right)<MSE\left(t_{reg}\right)$$

$$\frac{1}{n}\left\{S_{y(j)}^{4}\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)\left[B_{j}^{2}S_{x(j)}^{4}+A_{1(j)}^{2}S_{y(j)}^{4}+2A_{1(j)}B_{j}S_{y(j)}^{2}S_{x(j)}^{2}\right]-2S_{y(j)}^{2}\left(\lambda_{22(j)}-1\right)\left[B_{j}S_{x(j)}^{2}+A_{1(j)}S_{y(j)}^{2}\right]\right\}$$

$$<\frac{S_{y}^{4}}{n}\left(\lambda_{40}-1\right)\left(1-\rho^{2}\right)$$

$$\left(K_{(j)}-K\right)+L_{(j)}-2M_{(j)}+K\rho^{2}<0 \tag{24}$$

(vi) With the MSE of estimators given in (10) and (18),

$$MSE\left(t_{rZB(j)}\right)<MSE\left(t_{US}\right)$$

$$\frac{1}{n}\left\{S_{y(j)}^{4}\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)\left[B_{j}^{2}S_{x(j)}^{4}+A_{1(j)}^{2}S_{y(j)}^{4}+2A_{1(j)}B_{j}S_{y(j)}^{2}S_{x(j)}^{2}\right]-2S_{y(j)}^{2}\left(\lambda_{22(j)}-1\right)\left[B_{j}S_{x(j)}^{2}+A_{1(j)}S_{y(j)}^{2}\right]\right\}$$
$$<\frac{S_{y}^{4}}{n}\left[\left(\lambda_{40}-1\right)+g_{0}^{2}\left(\lambda_{04}-1\right)-2g_{0}\left(\lambda_{22}-1\right)\right]$$
$$\left(K_{(j)}-K\right)+L_{(j)}-2M_{(j)}-g_{0}N\left(g_{0}-2C\right) \tag{25}$$

(vii) With the MSE of estimators given in (14) and (18),

$$MSE\left(t_{rZB(j)}\right)<MSE\left(t_{KCi}\right)$$

$$\frac{1}{n}\left\{S_{y(j)}^{4}\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)\left[B_{j}^{2}S_{x(j)}^{4}+A_{1(j)}^{2}S_{y(j)}^{4}+2A_{1(j)}B_{j}S_{y(j)}^{2}S_{x(j)}^{2}\right]-2S_{y(j)}^{2}\left(\lambda_{22(j)}-1\right)\left[B_{j}S_{x(j)}^{2}+A_{1(j)}S_{y(j)}^{2}\right]\right\}$$
$$<\frac{S_{y}^{4}}{n}\left[\left(\lambda_{40}-1\right)+g_{i}^{2}\left(\lambda_{04}-1\right)-2g_{i}\left(\lambda_{22}-1\right)\right],i=1,2,3$$
$$\left(K_{(j)}-K\right)+L_{(j)}-2M_{(j)}-g_{i}N\left(g_{i}-2C\right);i=1,2,3 \tag{26}$$

The proposed class of robust estimators $\left(t_{rZB(j)}\right)$ perform better than all other estimators considered here if Conditions (i)-(vii) are satisfied.

5.2 With the proposed regression-type estimators

(i) With the MSE of estimators given in (1) and (16)

$$MSE\left(t_{rZB}\right)<V\left(t_{0}\right)$$

$$\frac{1}{n}\left\{S_{y}^{4}\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)\left[B^{2}S_{x}^{4}+A_{1}^{2}S_{y}^{4}+2A_{1}BS_{y}^{2}S_{x}^{2}\right]-2S_{y}^{2}\left(\lambda_{22}-1\right)\left[BS_{x}^{2}+A_{1}S_{y}^{2}\right]\right\}<\frac{S_{y}^{4}}{n}\left(\lambda_{40}-1\right)$$
$$L-2M<0 \tag{27}$$

(ii) With the MSE of estimators given in (3) and (16)

$$MSE\left(t_{rZB}\right)<MSE\left(t_{R}\right)$$

$$\frac{1}{n}\left\{S_{y}^{4}\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)\left[B^{2}S_{x}^{4}+A_{1}^{2}S_{y}^{4}+2A_{1}BS_{y}^{2}S_{x}^{2}\right]-2S_{y}^{2}\left(\lambda_{22}-1\right)\left[BS_{x}^{2}+A_{1}S_{y}^{2}\right]\right\}$$
$$<\frac{S_{y}^{4}}{n}\left[\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)\left(1-2C\right)\right]$$

$$L-2M-N\left(1-2C\right)<0 \tag{28}$$

(iii) With the MSE of estimators given in (8) and (16)

$$MSE\left(t_{rZB}\right)<MSE\left(t_{reg}\right)$$

$$\frac{1}{n}\left\{S_y^4(\lambda_{40}-1)+(\lambda_{04}-1)\left[B^2S_x^4+A_1^2S_y^4+2A_1BS_y^2S_x^2\right]-2S_y^2(\lambda_{22}-1)\left[BS_x^2+A_1S_y^2\right]\right\}<\frac{S_y^4}{n}(\lambda_{40}-1)(1-\rho^2)$$

$$K\rho^2+L-2M<0 \tag{29}$$

(iV) With the MSE of estimators given in (10) and (16)

$$MSE(t_{rZB})<MSE(t_{US})$$

$$\frac{1}{n}\left\{S_y^4(\lambda_{40}-1)+(\lambda_{04}-1)\left[B^2S_x^4+A_1^2S_y^4+2A_1BS_y^2S_x^2\right]-2S_y^2(\lambda_{22}-1)\left[BS_x^2+A_1S_y^2\right]\right\}$$
$$<\frac{S_y^4}{n}\left[(\lambda_{40}-1)+g_0^2(\lambda_{04}-1)-2g_0(\lambda_{22}-1)\right]$$

$$L-2M-g_0N(g_0-2C)<0 \tag{30}$$

(v) With the MSE of estimators given in (14) and (16)

$$MSE(t_{rZB})<MSE(t_{KCi});\mathrm{i}=1,2,3.$$
$$\frac{1}{n}\left\{S_y^4(\lambda_{40}-1)+(\lambda_{04}-1)\left[B^2S_x^4+A_1^2S_y^4+2A_1BS_y^2S_x^2\right]-2S_y^2(\lambda_{22}-1)\left[BS_x^2+A_1S_y^2\right]\right\}$$
$$<\frac{S_y^4}{n}\left[(\lambda_{40}-1)+g_i^2(\lambda_{04}-1)-2g_i(\lambda_{22}-1)\right]$$

$$L-2M-g_iN(g_i-2C),i=1,2,3. \tag{31}$$

The proposed classical ratio estimators $\left(t_{rZB(j)}\right)$ perform better than all other classical estimators considered here if Conditions (i)-(v) are satisfied.

6 Applications

We use the data in Zaman and Bulut [26] and Zaman et al. [27] in order to compare the performances between the proposed classical ratio estimators and the proposed class of robust estimators given in (16) and (18), respectively. The statistics of the population are presented in Table 3. We have contaminated the last observation of this data set by multiplying the Y value by 50 and the value of X by 25.

[Table 3 Here]

[Table 4 Here]

We proposed two different estimators for simple random sampling in the study. The first estimators are as given in (15). The MSE of these estimators is given in (16). We obtained the MSE values of proposed classical estimators in (15), the unbiased estimator, Isaki estimator in (2), Singh et al. estimator in (4), the regression estimator in (7), Upadhyaya and Singh estimator in (9), and Kadilar and Cingi estimators in (11). These values are given in the uncontaminated data part of Table 4.

The efficiency of these estimators decreases when there are outliers in the data. Here, a second estimator using robust covariance estimates is proposed to eliminate this negative effect of the outlier. The estimators are given in (17). The MSE of these estimators is given in (18). In the presence of outliers in the data, we obtained the MSE values of proposed classical estimators in (15), the proposed class of robust estimators in (17) and estimators considered here. These values are given in the contaminated data part of Table 4. According to this part, as inferred by the theoretical comparisons, we observe that all of the proposed class of robust estimators have smaller MSE values than the proposed classical estimators and some existing estimators in data with outliers for simple random sampling. These results are expected results because the conditions (19)-(26) are satisfied for the proposed class of robust estimators. These situations are clearly seen in Tables 5, 6, 7, 8, 9, 10, and 11. The most efficient estimators are t_{rZB10} robust estimator based on MCD covariance estimate for the dataset. On the other hand, proposed classical estimators in given (15) do not provide lower MSE than unbiased, and estimators proposed by Isaki, Singh et al., the regression, Upadhyaya and Singh, and Kadilar and Cingi. This situation is expected because the conditions (27)-(31) are not satisfied for all estimators under the dataset. This situation is clearly seen in Table 12. Note recall that the proposed classical estimators should be perform better than existing estimators in here if the conditions (27)-(31) were satisfied.

[Table 5 Here]

[Table 6 Here]

[Table 7 Here]

[Table 8 Here]

[Table 9 Here]

[Table 10 Here]

[Table 11 Here]

[Table 12 Here]

7 Simulation Study

In this section, we use the following simulation study for numerical comparisons. We have used the following models:

$Y_i = 5X_i + \varepsilon_i$ which we generate ε_i and X_i independently and calculate Y_i for $i = 1, 2, \ldots, N$.

(1) X is from $U(0,1)$ and ε is from $N(0,1)$ and independent of X

(2) X is from $Exp(1)$ and ε is from $N(0,1)$ and independent of X

(3) X is from $N(5,1)$ and ε is from $N(0,1)$ and independent of X

Simulation can be summarized with the steps below. X is generated from above given distributions by taking as $N = 200$. The ratios of outliers are 10 and we have guaranteed that there is the least an outlier in sample selection.

Firstly, classical estimators given in Section 2 are obtained for each sample size, using SRSWOR (simple random sampling without replacement).

Then, for each sample taken, the proposed estimators, say s_i^2, such as t_{ZB}, given in Section 3 and the proposed class of robust estimators, $t_{rZB(j)}$, given in Section 4 in simple random sampling are obtained.

The values of MSE for all cases are obtained with the help of (32)

$$MSE = \frac{1}{10000}\sum_{i=1}^{10000}\left(s_{yi}^2 - S_y^2\right)^2 \tag{32}$$

where S_y^2 is the population variance.

Sample sizes are taken as $n = 20, 30 \text{ and } 40$ under simple random sampling. Tables 13, 14 and 15 show the values of MSE of the proposed class of robust estimators, proposed regression-type estimators and some existing estimators for the various sample sizes when it comes to uniform, exponential and normal distributions, respectively. These values are computed using (32). From Tables 13, 14 and 15 it is concluded that the proposed class of robust estimators are perform better than the proposed classical estimators and some existing estimators for all sample sizes in simple random sampling. All of these findings support the theoretical results in the contaminated data part of Table 4. It is worth to point out that the values of MSE of the proposed class of robust estimators with respect to the classical estimators in Tables 13, 14, and 15 would decrease notably, when there were more extreme observations in data.

[Table 13 Here]

[Table 14 Here]

[Table 15 Here]

From theoretical and empirical study, the proposed regression-type estimators did not provide a significant advantage over the estimators known in the literature. For example, when X comes from the uniform distribution, the proposed classical estimator t_{rZB6} has a worse result than the estimator t_R, while it has a more efficient results than the estimator t_{US}. In this context, we have developed a new class of robust estimators while there is an outlier in the data. The proposed class of robust estimators provided a significant superiority to the classical estimators considered here in empirical and simulation study. These results are clearly seen in Tables 13, 14, and 15. The contaminated data part of Table 4 shows these findings clearly.

8 Conclusion

This study has proposed a new class of robust estimators using MCD and MVE estimates in the simple random sampling The expression for MSE of the proposed class of robust estimators are obtained. Conditions are obtained under which the proposed class of robust estimators perform better than the classical estimator and the existing estimators in terms of MSE. In addition, robustness to outliers is a characteristic of the proposed class of estimators. Finally, it is recommended to use the proposed estimators over the classical and other existing estimators, especially in the presence of extreme observations in the data. In future work, we hope to extend the proposed class of robust estimators given in this article to the stratified two-stage sampling.

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List of table captions

Estimators	Values of	
	ζ	ξ
$t_{rZB1}=\dfrac{s_y^2+b\left(S_x^2-s_x^2\right)}{s_x^2}S_x^2$	1	0
$t_{rZB2}=\dfrac{s_y^2+b\left(S_x^2-s_x^2\right)}{\left(s_x^2+\beta_2\left(x\right)\right)}\left(S_x^2+\beta_2\left(x\right)\right)$	1	$\beta_2\left(x\right)$
$t_{rZB3}=\dfrac{s_y^2+b\left(S_x^2-s_x^2\right)}{\left(s_x^2+C_x\right)}\left(S_x^2+C_x\right)$	1	C_x
$t_{rZB4}=\dfrac{s_y^2+b\left(S_x^2-s_x^2\right)}{\left(s_x^2+\rho\right)}\left(S_x^2+\rho\right)$	1	ρ
$t_{rZB5}=\dfrac{s_y^2+b\left(S_x^2-s_x^2\right)}{\left(s_x^2\beta_2\left(x\right)+C_x\right)}\left(S_x^2\beta_2\left(x\right)+C_x\right)$	$\beta_2\left(x\right)$	C_x
$t_{rZB6}=\dfrac{s_y^2+b\left(S_x^2-s_x^2\right)}{\left(s_x^2C_x+\beta_2\left(x\right)\right)}\left(S_x^2C_x+\beta_2\left(x\right)\right)$	C_x	$\beta_2\left(x\right)$
$t_{rZB7}=\dfrac{s_y^2+b\left(S_x^2-s_x^2\right)}{\left(s_x^2C_x+\rho\right)}\left(S_x^2C_x+\rho\right)$	C_x	ρ
$t_{rZB8}=\dfrac{s_y^2+b\left(S_x^2-s_x^2\right)}{\left(s_x^2\rho+C_x\right)}\left(S_x^2\rho+C_x\right)$	ρ	C_x
$t_{rZB9}=\dfrac{s_y^2+b\left(S_x^2-s_x^2\right)}{\left(s_x^2\beta_2\left(x\right)+\rho\right)}\left(S_x^2\beta_2\left(x\right)+\rho\right)$	$\beta_2\left(x\right)$	ρ
$t_{rZB10}=\dfrac{s_y^2+b\left(S_x^2-s_x^2\right)}{\left(s_x^2\rho+\beta_2\left(x\right)\right)}\left(S_x^2\rho+\beta_2\left(x\right)\right)$	ρ	$\beta_2\left(x\right)$

Table 1: Suggested estimators (Equation (15))

Estimators	Values of	
	$\zeta_{(j)}$	$\xi_{(j)}$
$t_{rZB1(j)} = \dfrac{s^2_{y(j)} + b_j\left(S^2_{x(j)} - s^2_{x(j)}\right)}{s^2_{x(j)}} S^2_{x(j)}$	1	0
$t_{rZB2(j)} = \dfrac{s^2_{y(j)} + b_j\left(S^2_{x(j)} - s^2_{x(j)}\right)}{\left(s^2_{x(j} + \beta_{2j}(x)\right)}\left(S^2_{x(j)} + \beta_{2j}(x)\right)$	1	$\beta_{2j}(x)$
$t_{rZB3(j)} = \dfrac{s^2_{y(j)} + b_j\left(S^2_{x(j)} - s^2_{x(j)}\right)}{\left(s^2_{x(j)} + C_{x(j)}\right)}\left(S^2_{x(j)} + C_{x(j)}\right)$	1	C_x
$t_{rZB4(j)} = \dfrac{s^2_{y(j)} + b_j\left(S^2_{x(j)} - s^2_{x(j)}\right)}{\left(s^2_{x(j)} + \rho_{(j)}\right)}\left(S^2_{x(j)} + \rho_{(j)}\right)$	1	$\rho_{(j)}$
$t_{rZB5(j)} = \dfrac{s^2_{y(j)} + b_j\left(S^2_{x(j)} - s^2_{x(j)}\right)}{\left(s^2_{x(j)}\beta_{2j}(x) + C_{x(j)}\right)}\left(S^2_{x(j)}\beta_{2j}(x) + C_{x(j)}\right)$	$\beta_{2j}(x)$	$C_{x(j)}$
$t_{rZB6(j)} = \dfrac{s^2_{y(j)} + b_j\left(S^2_{x(j)} - s^2_{x(j)}\right)}{\left(s^2_{x(j)}C_{x(j)} + \beta_{2j}(x)\right)}\left(S^2_{x(j)}C_{x(j)} + \beta_{2j}(x)\right)$	$C_{x(j)}$	$\beta_{2j}(x)$
$t_{rZB7(j)} = \dfrac{s^2_{y(j)} + b_j\left(S^2_{x(j)} - s^2_{x(j)}\right)}{\left(s^2_{x(j)}C_{x(j)} + \rho_{(j)}\right)}\left(S^2_{x(j)}C_{x(j)} + \rho_{(j)}\right)$	$C_{x(j)}$	$\rho_{(j)}$
$t_{rZB8(j)} = \dfrac{s^2_{y(j)} + b_j\left(S^2_{x(j)} - s^2_{x(j)}\right)}{\left(s^2_{x(j)}\rho_{(j)} + C_{x(j)}\right)}\left(S^2_{x(j)}\rho_{(j)} + C_{x(j)}\right)$	$\rho_{(j)}$	$C_{x(j)}$
$t_{rZB9(j)} = \dfrac{s^2_{y(j)} + b_j\left(S^2_{x(j)} - s^2_{x(j)}\right)}{\left(s^2_{x(j)}\beta_{2j}(x) + \rho_{(j)}\right)}\left(S^2_{x(j)}\beta_{2j}(x) + \rho_{(j)}\right)$	$\beta_{2j}(x)$	$\rho_{(j)}$
$t_{rZB10(j)} = \dfrac{s^2_{y(j)} + b_j\left(S^2_{x(j)} - s^2_{x(j)}\right)}{\left(s^2_{x(j)}\rho_{(j)} + \beta_{2j}(x)\right)}\left(S^2_{x(j)}\rho_{y,x(j)} + \beta_{2j}(x)\right)$	$\rho_{(j)}$	$\beta_{2j}(x)$

Table 2: Suggested estimators using robust covariance estimates (Equation (17))

	Classical	MCD	MVE	Real Values (without outlier)
$\bar{Y}$	36.34234	18.02439	18.35366	29.27928
$\bar{X}$	448.8649	235.3171	231.8519	394.1622
S_y^2	5999.464	70.07523	78.74556	651.3122
S_x^2	476858.7	15063.36	13397.14	160288.2
C_x	1.538435	0.04707897	0.04640262	1.015724
$\lambda_{04} = \beta_{2x}$	48.54609	47.0208	46.86658	8.994593
$\lambda_{40} = \beta_{2y}$	85.99221	84.78439	84.96302	10.26559
λ_{22}	64.25012	62.63676	62.59346	8.8654
C	1.33029067	1.33932396	1.342883206	0.983839953
ρ	0.9949801	0.9926157	0.9925 B 272	0.9138736
	0.01673668	0.00623058	0.007893181	0.003997717
g_0	0.99989821	0.99688818	0.996513942	0.999943888
g_1	0.99999677	0.99999687	0.999996536	0.999993663
g_2	0.99999993	0.99999993	0.999999926	0.999999295
g_3	0.99993383	0.93781866	0.929895963	0.999944757
n			30	
N			111	

Table 3: Data statistics used for simple random sampling

Estimators	Uncontaminated Data	Contaminated Data		
		Classical	MCD	MVE
t_0	131017.7843	1.02E+08	--	--
t_R	131010.0481	7244378	--	--
t_{CR}	131018.3011	26606542	--	--
t_{reg}	21596.32214	1021215	--	--
t_{US}	35627.27383	85386937	--	--
t_{KC1}	35628.89487	7328424	--	--
t_{KC2}	35630.48609	7246107	--	--
t_{KC3}	35627.33348	9000031	--	--
t_{ZB1}	134641.8707	144531.9	1565.182	2159.938
t_{rZB2}	134629.1846	144518.2	1563.343	2157.107
t_{rZB3}	134640.438	144530.4	1565.174	2159.925
t_{rZB4}	134640.5816	144530.5	1565.033	2159.712
t_{rZB5}	134641.7114	144531.7	1565.181	2159.936
t_{rZB6}	134629.381	144518.4	1526.682	2100.859
t_{rZB7}	134640.6016	144530.5	1562.001	2155.131
t_{rZB8}	134640.3029	144530.3	1565.172	2159.922
t_{rZB9}	134633.2569	144531.7	1565.168	2159.915
t_{rZB10}	134627.9891	144517.2	1562.973	2156.455

Table 4: Theoretical results for the MSE of estimators

	MCD		MVE	
	Condition Values (C.V)	Results (R)	Condition Values (C.V)	Results (R)
t_{rZB1}	-1.7E+09	TRUE	-1.7E+09	TRUE
t_{rZB2}	-1.7E+09	TRUE	-1.7E+09	TRUE
t_{rZB3}	-1.7E+09	TRUE	-1.7E+09	TRUE
t_{rZB4}	-1.7E+09	TRUE	-1.7E+09	TRUE
t_{rZB5}	-1.7E+09	TRUE	-1.7E+09	TRUE
t_{rZB6}	-1.7E+09	TRUE	-1.7E+09	TRUE
t_{rZB7}	-1.7E+09	TRUE	-1.7E+09	TRUE
t_{rZB8}	-1.7E+09	TRUE	-1.7E+09	TRUE
t_{rZB9}	-1.7E+09	TRUE	-1.7E+09	TRUE
t_{rZB10}	-1.7E+09	TRUE	-1.7E+09	TRUE

Table 5: The results of condition in Equation (20)

	MCD		MVE	
	C.V	R	C.V	R
t_{rZB1}	-3.1E+09	TRUE	-3.1E+09	TRUE
t_{rZB2}	-3.1E+09	TRUE	-3.1E+09	TRUE
t_{rZB3}	-3.1E+09	TRUE	-3.1E+09	TRUE
t_{rZB4}	-3.1E+09	TRUE	-3.1E+09	TRUE
t_{rZB5}	-3.1E+09	TRUE	-3.1E+09	TRUE
t_{rZB6}	-3.1E+09	TRUE	-3.1E+09	TRUE
t_{rZB7}	-3.1E+09	TRUE	-3.1E+09	TRUE
t_{rZB8}	-3.1E+09	TRUE	-3.1E+09	TRUE
t_{rZB9}	-3.1E+09	TRUE	-3.1E+09	TRUE
t_{rZB10}	-3.1E+09	TRUE	-3.1E+09	TRUE

Table 6: The results of condition in Equation (21)

	MCD		MVE	
	C.V	R	C.V	R
t_{rZB1}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB2}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB3}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB4}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB5}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB6}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB7}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB8}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB9}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB10}	-3E+09	TRUE	-3E+09	TRUE

Table 7: The results of condition in Equation (22)

	MCD		MVE	
	C.V	R	C.V	R
t_{rZB1}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB2}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB3}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB4}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB5}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB6}	-3E+09	TRUE	-3E+09	TRUE

t_{rZB7}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB8}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB9}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB10}	-3E+09	TRUE	-3E+09	TRUE

Table 8: The results of condition in Equation (23)

	MCD		MVE	
	C.V	R	C.V	R
t_{rZB1}	-6.1E+09	TRUE	-6.1E+09	TRUE
t_{rZB2}	-6.1E+09	TRUE	-6.1E+09	TRUE
t_{rZB3}	-6.1E+09	TRUE	-6.1E+09	TRUE
t_{rZB4}	-6.1E+09	TRUE	-6.1E+09	TRUE
t_{rZB5}	-6.1E+09	TRUE	-6.1E+09	TRUE
t_{rZB6}	-6.1E+09	TRUE	-6.1E+09	TRUE
t_{rZB7}	-6.1E+09	TRUE	-6.1E+09	TRUE
t_{rZB8}	-6.1E+09	TRUE	-6.1E+09	TRUE
t_{rZB9}	-6.1E+09	TRUE	-6.1E+09	TRUE
t_{rZB10}	-6.1E+09	TRUE	-6.1E+09	TRUE

Table 9: The results of condition in Equation (24)

	MCD		MVE	
	C.V	R	C.V	R
t_{rZB1}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB2}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB3}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB4}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB5}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB6}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB7}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB8}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB9}	-3E+09	TRUE	-3E+09	TRUE
t_{rZB10}	-3E+09	TRUE	-3E+09	TRUE

Table 10: The results of condition in Equation (25)

	MCD (g_1)		MCD (g_2)		MCD (g_3)		MVE (g_1)		MVE (g_2)		MVE (g_3)	
	C.V	R	C.V	R	C.V	R	C.V	R	C.V	R	C.V	R
t_{rZB1}	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE
t_{rZB2}	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE
t_{rZB3}	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE
t_{rZB4}	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE
t_{rZB5}	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE
t_{rZB6}	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE
t_{rZB7}	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE
t_{rZB8}	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE
t_{rZB9}	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE
t_{rZB10}	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE	-3E+09	TRUE

Table 11: The results of condition in Equation (26)

	Eq. (27)		Eq. (28)		Eq. (29)		Eq. (30)		Eq. (31 (g_1))		Eq. (31 (g_2))		Eq. (31 (g_3))	
	C.V	R	C.V	R	C.V	R	C.V	R	C.V	R	C.V	R	C.V	R
t_{rZB1}	108722.5914	FALSE	3390480.257	FALSE	3700585.629	FALSE	3390469.468	FALSE	3390480.951	FALSE	3390480.334	FALSE	3390486.303	FALSE
t_{rZB2}	108342.0095	FALSE	3390099.675	FALSE	3700205.047	FALSE	3390088.886	FALSE	3390100.369	FALSE	3390099.752	FALSE	3390105.721	FALSE
t_{rZB3}	108679.6106	FALSE	3390437.276	FALSE	3700542.648	FALSE	3390426.487	FALSE	3390437.971	FALSE	3390437.353	FALSE	3390443.322	FALSE
t_{rZB4}	108683.9204	FALSE	3390441.586	FALSE	3700546.958	FALSE	3390430.797	FALSE	3390442.28	FALSE	3390441.663	FALSE	3390447.632	FALSE
t_{rZB5}	108717.8129	FALSE	3390475.478	FALSE	3700580.85	FALSE	3390464.689	FALSE	3390476.173	FALSE	3390475.556	FALSE	3390481.524	FALSE
t_{rZB6}	108347.9007	FALSE	3390105.566	FALSE	3700210.938	FALSE	3390094.777	FALSE	3390106.261	FALSE	3390105.643	FALSE	3390111.612	FALSE
t_{rZB7}	108684.519	FALSE	3390442.185	FALSE	3700547.557	FALSE	3390431.395	FALSE	3390442.879	FALSE	3390442.262	FALSE	3390448.23	FALSE
t_{rZB8}	108675.56	FALSE	3390433.225	FALSE	3700538.598	FALSE	3390422.436	FALSE	3390433.92	FALSE	3390433.303	FALSE	3390439.271	FALSE
t_{rZB9}	108464.1784	FALSE	3390221.844	FALSE	3700327.216	FALSE	3390211.055	FALSE	3390222.538	FALSE	3390221.921	FALSE	3390227.89	FALSE
t_{rZB10}	108306.1456	FALSE	3390063.811	FALSE	3700169.183	FALSE	3390053.022	FALSE	3390064.505	FALSE	3390063.888	FALSE	3390069.857	FALSE

Table 12: The results of condition in Equations (27)-(31)

n:	20			30			40		
Estimators	Classical	MCD	MVE	Classical	MCD	MVE	Classical	MCD	MVE
t_R	1.25E+15	--	--	1.37E+13	--	--	1.10E+15	--	--
t_{CR}	4.27E+19	--	--	2.38E+15	--	--	4.02E+19	--	--
t_{reg}	3.36E+12	--	--	2.56E+12	--	--	2.25E+12	--	--
t_{US}	3.79E+12	--	--	2.69E+12	--	--	2.27E+12	--	--
t_{KC1}	9.74E+12	--	--	4.60E+12	--	--	4.35E+12	--	--
t_{KC2}	2.38E+14	--	--	1.19E+13	--	--	1.46E+14	--	--
t_{KC3}	5.18E+12	--	--	3.20E+12	--	--	2.63E+12	--	--
t_{rZB1}	2.14E+15	182403.3	182389.6	2.44E+13	211756.7	211741.2	2.46E+15	306725	306705.4
t_{rZB2}	4.45E+12	182404.9	182392	2.88E+12	211756.3	211741.5	2.44E+12	306725.2	306719.1
t_{rZB3}	1.52E+13	182404.8	182391.9	6.77E+12	211756.3	211741.4	7.21E+12	306725.2	306718.2
t_{rZB4}	1.02E+14	182404.7	182391.8	1.48E+13	211756.3	211741.5	5.91E+13	306725.2	306717.3
t_{rZB5}	4.17E+14	182404	182390.5	2.08E+13	211756.5	211741.2	3.23E+14	306725.1	306711.8
t_{rZB6}	7.03E+12	182404.9	182392	4.03E+12	211756.3	211741.5	3.40E+12	306725.2	306719.1
t_{rZB7}	3.14E+14	182404.6	182391.8	1.95E+13	211756.3	211741.5	2.14E+14	306725.2	306717.2
t_{rZB8}	8.47E+12	182404.9	182391.9	4.84E+12	211756.3	211741.4	4.05E+12	306725.2	306718.6
t_{rZB9}	1.39E+15	182403.7	182390.2	2.34E+13	211756.5	211741.3	1.27E+15	306725	306709.6
t_{rZB10}	3.76E+12	182404.9	182392	2.62E+12	211756.3	211741.4	2.20E+12	306725.2	306719.1

Table 13: Simulation results for the MSE of estimators for various sample sizes when it comes to Uniform distribution $\left(X \sim U\left(0,1\right)\right)$

Estimators	Classical	MCD	MVE	Classical	MCD	MVE	Classical	MCD	MVE
t_R	9.06E+17	--	--	4.56E+16	--	--	5.29E+14	--	--
t_{CR}	1.70E+23	--	--	1.94E+21	--	--	4.80E+16	--	--
t_{reg}	2.95E+14	--		1.29E+14	--	--	1.38E+14	--	--
t_{US}	6.50E+14	--	--	2.55E+14	--	--	1.64E+14	--	--
t_{KC1}	1.19E+16	--	--	2.10E+15	--	--	3.52E+14	--	--
t_{KC2}	6.60E+17	--	--	3.43E+16	--	--	5.19E+14	--	--
t_{KC3}	1.93E+15	--	--	6.20E+14	--	--	2.25E+14	--	--
t_{rZB1}	9.39E+17	24190505	24188468	1.55E+17	4975362	4974004	6.74E+14	3406051	3407571
t_{rZB2}	6.83E+14	24190424	24196828	3.58E+14	4975075	4978358	1.76E+14	3406186	3409347
t_{rZB3}	1.27E+16	24190468	24195776	5.49E+15	4975154	4977771	4.23E+14	3406154	3409007
t_{rZB4}	7.71E+17	24190466	24192613	4.40E+16	4975283	4976777	6.30E+14	3406048	3408230
t_{rZB5}	6.85E+17	24190511	24189739	1.16E+17	4975341	4974567	6.58E+14	3406056	3407716
t_{rZB6}	2.07E+15	24190423	24196823	1.16E+15	4975076	4978352	2.54E+14	3406186	3409346
t_{rZB7}	8.84E+17	24190462	24192421	9.42E+16	4975286	4976593	6.56E+14	3406048	3408198
t_{rZB8}	1.01E+15	24190432	24196788	2.19E+15	4975089	4978094	2.39E+14	3406186	3409318

t_{rZB9}	9.36E+17	24190502	24188756	1.47E+17	4975357	4974209	6.72E+14	3406049	3407603
t_{rZB10}	3.06E+14	24190423	24196867	1.85E+14	4975073	4978371	1.40E+14	3406186	3409356

Table 14: Simulation results for the MSE of estimators for various sample sizes when it comes to Exponential distribution $\left(X \sim Exp(1)\right)$

n:	20			30			40		
Estimators	Clsical	MCD	MVE	Classical	MCD	MVE	Classical	MCD	MVE
t_R	9.84E+15	--	--	9.65E+15	--	--	9.91E+15	--	--
t_{CR}	6.48E+16	--	--	1.14E+16	--	--	1.80E+16	--	--
t_{reg}	9.71E+15	--	--	1.02E+16	--	--	9.87E+15	--	--
t_{US}	9.73E+15	--	--	9.65E+15	--	--	9.89E+15	--	--
t_{KC1}	9.82E+15	--	--	9.65E+15	--	--	9.91E+15	--	--
t_{KC2}	9.84E+15	--	--	9.65E+15	--	--	9.91E+15	--	--
t_{KC3}	9.79E+15	--	--	9.65E+15	--	--	9.90E+15	--	--
t_{rZB1}	1.38E+16	9.01E+08	9.01E+08	9.94E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB2}	1.33E+16	9.01E+08	9.01E+08	9.88E+15	9.91E+08	9.91E+08	1.22E+16	1.47E+09	1.47E+09
t_{rZB3}	1.37E+16	9.01E+08	9.01E+08	9.93E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB4}	1.38E+16	9.01E+08	9.01E+08	9.93E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB5}	1.38E6	9.01E+08	9.01E+08	9.94E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB6}	1.36E+16	9.01E+08	9.01E+08	9.91E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB7}	1.38E+16	9.01E+08	9.01E+08	9.94E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB8}	1.37E+16	9.01E+08	9.01E+08	9.92E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB9}	1.38E+16	9.01E+08	9.01E+08	9.94E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB10}	1.31E+16	9.01E+08	9.01E+08	9.87E+15	9.91E+08	9.91E+08	1.22E+16	1.47E+09	1.47E+09

Table 15. Simulation results for the MSE of estimators for various sample sizes when it comes to Nmal distribution $\left(X \sim N(5,1)\right)$

Author's Biography

Tolga Zaman is an Associate Professor at the Department of Statistics in Cankiri Karatekin University, Cankiri, Turkey. He received his MS and PhD degrees in Statistics from Ondokuz Mayis University Samsun, Turkey in 2013 and 2017, respectively. His research interests are sampling theory, resampling methods, robust statistics, and statistical inference. He has published more than 60 research papers in international/national journals and conferences. He has papers published in journals like Applied Mathematics and Computations, Communication in Statistics: Simulation and Computation, Communication in Statistics: Theory and Methods, Mathematical Population Studies, RevStat-Statistical Jounal, Scientia Irenica.

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