

Sharif University of Technology Scientia Iranica Transactions B: Mechanical Engineering http://scientiairanica.sharif.edu



# Sensitivity analysis for Walters-B nanoliquid flow over a radiative Riga surface by RSM

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Received 11 May 2021; received in revised form 21 September 2021; accepted 6 December 2021

#### **KEYWORDS**

Sensitivity analysis; Walters-B nanofluid; Newtonian heating; Moving Riga surface; Stagnation point flow; Response surface methodology.

Abstract. In this study, a sensitivity analysis is implemented using response surface strategies to control the Walters-B nanofluid stagnant point flow caused by a Riga surface. An electromagnetic actuator is known as Riga surface. The Buongiorno model is used to construct the mathematical model that includes a Newtonian heating condition as well as radiation effects. Based on the fundamental laws of mass, momentum, and energy, transformation is incorporated to obtain nonlinear ordinary differential equations. To solve the governing system, the numerical shooting approach along with Runge-Kutta scheme is employed to solve the governing system. By considering the response of Local Nusselt Number (LNN) to the variation of input variables, an experimental structure is incorporated by sensitivity analysis. As underlined, the LNN is quite sensitive to radiation number rather than other parameters of interest. Meanwhile, it is indicated that the sensitivity of LNN to Brownian number is reduced as thermophoresis is enhanced, but sensitivity value varies from positive to negative for all the values of Brownian number. It is examined that maximum LNN occurs at a higher level for thermophoresis and for Brownian motion parameters. The results are assumed to provide a tentative guidance for possible lab-based experiments.

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# 1. Introduction

There are many compliant methods including nanoparticle addition to base liquid, rendering surfaces flexible, applying a magnetic field, corrugating surfaces, attaching fin(s), incorporating artificial surface roughness, and adding obstacle(s) for improving heat transfer from thermal devices [1–6]. The study of nanofluids has recently attracted the attention of current

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doi: 10.24200/sci.2021.58293.5662

researchers owing to the extensive use of numerous processes/industries like engine cooling, machining and electronics, heat exchanger systems, improving the performance of diesel generators and heat transport of chillers, hybrid-powered air conditioners/refrigerators, solar water heating, fuel cells, and other high-consumption energy equipment. Nanofluids, in general, have more thermal characteristics than traditional liquids, which is the reason why they are often referred to as advanced/next-generation heat transport liquids. Thermal engineers have concentrated on nanoliquids due to their essential and insightful characteristics like higher conductivity and larger relative area of nanomaterials. The nanomaterials suspended in the nanoliquid significantly improve

the suspension stability and heat transfer capabilities. Nanofluids enjoy numerous use cases because they can improve heat transfer efficiency as compared to pure fluids. Choi and Eastman [7] pioneered the study of heat transport in nanoliquids. Dispersion of a small number of solid nanomaterials in pure liquid considerably improves its thermal conductivity which in turn leads to the enhancement of the heat transport rate [6]. This has received much recognition and inspired many researchers. There are numerous studies in the literature that have discussed the mechanism underlying the improved heat transport characteristics [8–19]. An outstanding collection of research studies on this matter is accessible through a review of the referenced papers [20-33]. Riga sheet is an electromagnetic actuator. Riga sheets generate electric and magnetic fields that can generate Lorentz force parallel to liquid flow [34]. It is obvious that liquids with high electrical conductivity are significantly affected when a magnetic field is applied. Several researchers have been inspired by this scenario to analyze the liquid flows through Riga sheet. Ahmad et al. [35] considered the dependability of the asymptotic technique to address the impact of strong suction and mixed convective boundary layer flow of nanoliquid through a Riga surface. With the assistance of analytical solutions, Dhif et al. [36] analyzed the thermal examination of the solar collector cum storage system by utilizing a hybrid-nanofluids. Abbas et al. [37] analyzed the importance of Electro-Magneto-Hydrodynamic (EMHD) and bioconvective nanofluid flow toward a porous Riga sheet containing gyrotactic microorganisms. Nadeem et al. [38] employed the MATLAB byp4c methodology to investigate the nanofluid stagnant point flow via Riga plate through induced magnetic impacts. Ganesh et al. [39] studied two-dimensional nanoliquid flow to achieve an expandable Riga sheet with EMHD impacts by employing shooting numerical techniques. Rasool et al. [40] studied the second-grade nanoliquid flow towards a heated vertical Riga surface. Shafiq et al. [41] investigated thermosoluted Marangoni flow towards a vertical porous Riga sheet. Zhang et al. [42] analyzed the bioconvection flow of nanoliguid through a Riga surface with Darcy-Brinkman-Forchheimer porous medium. Bhatti and Michaelides [43] reported the significance of Arrhenius activation energy concerning bioconvective nanofluid flow on a Riga surface. Shafiq et al. [44] numerically studied the Marangoni flow of carbon nanotubes over a Riga area.

Response Surface Methodology (RSM) is a multivariate optimization approach that employs a variety of statistical and mathematical methods. This method is focused on fitting a polynomial term to experimental data. In general, this approach can be utilized whenever a comeback or set of responses (output variables) is influenced by a number of independent variables (input variables). RSM optimizes the responses at the same time in order to achieve the best system efficiency. Several investigators utilized sensitivity analysis for energy issues via RSM [45,46]. For instance, Bovand et al. [47] employed sensitivity study of a vortex tube refrigerator. They found the highest cold temperature difference in a vortex tube to be the most sensitive to cold orifice diameter than to the inlet pressure or the amount of intake in the nozzle. Rashidi et al. [48] utilized response surface methods to conduct a sensitivity study for nanoliquid flow over an equilateral triangle barrier. They found Local Nusselt Number (LNN) and Skin Friction Coefficient (SFC) sensitive to the solid volume fraction of the nanoliquid in all situations. They also discovered a significant agreement between the findings of computational liquid dynamics study and those of RSM. Shafiq et al. [49] analyzed the sensitivity study of bioconvection tangent hyperbolic nanoliquid over the moving stretched sheet by RSM.

A thorough review of the literature reveals that the sensitivity analysis of the Walters-B nanofluid stagnant point flow induced by a Riga plate has not been studied previously. Therefore, the purpose of the present research work is to study non-Newtonian Walters-B nanofluid flow over stretching Riga plate with Newtonian heating. Buongiorno's model is utilized to investigate the nanofluid features. A mathematical model is suggested for use in the context of partial differential equations. To convert them into similarity equations, appropriate transformation is employed. Numerical solution is sought using Runge-Kutta-Fehlberg (RKF) method via shooting. Moreover, an experimental framework (RSM) is inextricably linked to sensitivity analysis to investigate the dependence of interested output terms over input terms. Interestingly, the authors performed sensitivity scrutiny using the LNN. This research is related to possible rules in future gadget production. To date, such research is novel for the top systematic review discovered.

# 2. Mathematical formulation

Significance of the stagnant point flow of Walters-B nanoliquid along a stretching Riga plate is investigated. Thermal radiation and Newtonian heating are employed to investigate heat transfer features. Figure 1 shows a schematic depiction of the physical model under consideration. Buongiorno's method is utilized to investigate the nanofluid properties. The governing equation is written in the following under the given assumptions by using Boussinesq approximations:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0,\tag{1}$$



Figure 1. Studied problem of Riga plate.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - U_e \frac{dU_e}{dx} = v\frac{\partial^2 u}{\partial y^2}$$
$$- \frac{k_0}{\rho} \left[ v\frac{\partial^3 u}{\partial y^3} + u\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} \right] + \frac{\pi j_0 M_0}{8\rho} \exp\left(-\frac{\pi}{a}y\right), \qquad (2)$$

$$v\frac{\partial T}{\partial y} + u\frac{\partial T}{\partial x} = \left(\frac{16\sigma^* T_{\infty}^3}{3k_1} + K\right)\frac{\partial^2 T}{\partial y^2} + \tilde{\tau}\left[\tilde{D}_b\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{\tilde{D}_t}{T_{\infty}}\left(\frac{\partial T}{\partial y}\right)^2\right],\tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \tilde{D}_b \frac{\partial^2 C}{\partial y^2} + \frac{\tilde{D}_t}{T_\infty} \frac{\partial^2 T}{\partial y^2},\tag{4}$$

$$u(x, 0) = U_w(x) = cx, v(x, 0) = 0,$$
  

$$\frac{\partial T(x, y)}{\partial y}\Big|_{y=0} = -h_s T(x, 0),$$
  

$$C(x, 0) = C_w, u \to U_e(x) = bx,$$
  

$$T \to T_\infty \quad C \to C_\infty \quad \text{as} \quad y \to \infty.$$
(5)

Proper variables are incorporated as follows:

$$v(x, y) = -\sqrt{cv}f(\eta),$$
  

$$u(x, y) = cxf'(\eta),$$
  

$$\theta = \frac{T - T_{\infty}}{T_{\infty}}, \quad \theta = \frac{T - T_{\infty}}{T_{\infty}},$$
  

$$\phi = \frac{C - C_{\infty}}{C_w - C_{\infty}}, \quad \eta = \sqrt{\frac{c}{v}}y.$$
(6)

The continuity equation is resolved instantly, whereas the remaining governing equations are reduced to the following forms:

$$f''' + A^{2} - (f')^{2} + ff'' - W_{e} \left[ 2f'f''' - (f'')^{2} - ff^{(iv)} \right]$$
  
+  $Q_{1} \exp(-\beta\eta) = 0,$   
 $f(0) = 0, f'(0) = 1, \quad f'(\infty) = A,$  (7)  
 $\left(1 + \frac{4}{3}B_{e}\right) q'' + B_{e} fq' + B_{e} N q'' + B_{e} N q^{2} = 0$ 

$$\left(1 + \frac{4}{3}R_d\right)\theta'' + Prf\theta' + PrN_2\theta'\phi' + PrN_1\theta^{2'} = 0,$$

$$\theta'(0) = -\delta \left(1 + \theta(0)\right), \qquad \theta(\infty) = 0, \tag{8}$$

$$\phi'' + S_c f \phi' + \frac{N_1}{N_2} \theta'' = 0, \quad \phi(0) = 1, \quad \phi(\infty) = 1.$$
 (9)

Here, the prime designates differentiation with respect to  $\eta$ . The parameters are as follows:

$$A = \frac{b}{c}, \quad We = \frac{k_0 c}{\mu_0}, \quad \beta = \sqrt{\frac{\pi^2 \nu}{a^2 c}},$$
$$Q_1 = \frac{\pi j_0 M_0}{8\rho U_w^2}, \quad Pr = \frac{\mu_0 c_p}{K}, \quad R_d = \frac{4\sigma^* T_\infty^3}{k_1 K},$$
$$Sc = \frac{\nu}{\tilde{D}_b}, \quad \delta = h_s \sqrt{\frac{v}{a}}, \quad N_2 = \frac{\tilde{\tau} \tilde{D}_b \left(C_w - C_\infty\right)}{v},$$
$$N_1 = \frac{\tilde{\tau} \tilde{D}_t}{v}.$$
(10)

The details are given in nomenclature. SFC  $C_f,\ {\rm LNN}$   $Nu_x,$  and LSHN are as follows:

$$C_f = \frac{\tau_w}{\rho U_w^2}, \quad N u_{\tilde{x}} = \frac{x q_w}{K (T_w - T_\infty)},$$
$$Sh_{\tilde{x}} = \frac{\tilde{D}_b j_w}{\tilde{D}_b (C_w - T_\infty)},$$

where the wall heat flux  $q_w$  and the wall skin friction  $\tau_w$  are as follows:

$$\tau_{w} = \left[ \mu_{0} \frac{\partial u}{\partial y} - k_{0} \left( u \frac{\partial^{2} u}{\partial x \partial y} + v \frac{\partial^{2} u}{\partial y^{2}} + 2 \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} \right)^{3} \right]_{y=0},$$

$$q_{w} = -K \left( \frac{\partial T}{\partial y} \right)_{y=0}, \qquad j_{w} = -\tilde{D}_{b} \left( \frac{\partial C}{\partial y} \right)_{y=0}. \quad (11)$$

Using Eq. (6), we have:

$$(Re_x)^{1/2}C_f = (1 - 3We) f''(0), \qquad (12)$$

$$\left(Re_{\bar{x}}\right)^{-1/2} N u_{\bar{x}} \delta\left(1 + \frac{3}{4}R_d\right) \left(1 + \frac{1}{\theta(0)}\right), \qquad (13)$$

$$(Re_{\bar{x}})^{-1/2}Sh_{\bar{x}} = -\phi'(0), \qquad (14)$$

where  $Re_x = cx^2/v$  is the local Re.

#### 2.1. Nonlinear polynomial model

A statistical technique based on a standard non-linear polynomial method may be utilized to evaluate the relationships between the target response variable and the independent input variables.

$$R = r_0 + r_1 A_1 + r_2 B_1 + r_3 C_1 + r_{11} A_1^2 + r_{22} B_1^2$$
$$+ r_{33} C_1^2 + r_{12} A_1 B_1 + r_{13} A_1 C_1 + r_{23} B_1 C_1.$$
(15)

The mathematical model contains one intercept term, three bilinear two-factor terms, three linear terms, and three quadratic terms. Thus, R indicates the LNN response. It is focused on three independent input parameters that are coded by symbols  $A_1$ ,  $B_1$ , and  $C_1$  (radiation, thermophoresis, and Brownian motion numbers, respectively). According to the RSM, 20 runs with 19 DOF (Degrees Of Freedom) are optimal for the selected three stages of variables. These are the low, medium, and high stages including (-1, 0, -1)1), respectively. Table 1 lists the input variables along with their symbols and levels. Furthermore, in *R*-programming, Central Composite Design (CCD) is typically utilized for the execution of a numerical or computational experiment. A set of 20 runs of experiments is arranged in terms of  $2^F + 2F + P$ , in which the number of factors is F = 3 and the number of center points is P = 6. Table 2 describes the experimental program sequence. The Analysis of Variance (ANOVA) is a statistical technique for determining the importance of the variability performance of defined variables on the RSM model. ANOVA is used to investigate the optimization criterion in the RSM model in terms of model precision, with numerical estimators such as DOF, Sum of Squares (SS), and Minimum Mean Square (MMS) as well as p-value and F-value. The LNN for the ANOVA analysis is shown in Table 3.

Table 4 shows the deterioration coefficients for responses (local Nu) based on the *p*-value set for the nonlinear polynomial model in Eq. (14). The high *p*-value is known as statistically insignificant, which implies that no relative change to the output can be noted due to the changes made to the input. On the other hand, it is possible to ignore the term with a low *p*-value ( $\leq 0.05$ ), which is statistically significant elsewhere. As a consequence,  $A, B, C, A^2, B^2, C^2, AB$ , and

Table 1. Experimental variables and their level.

				Level	
Parar	notor	symbol	Low	Medium	High
	i arameter symbol			(0)	(1)
	$R_1$	A	0.1	0.2	0.3
LNN	$N_1$	В	0.2	0.4	0.6
	$N_2$	C	0.2	0.4	0.6

Table 2.	Experiment	design	$\operatorname{and}$	$\operatorname{response}$	performance	of
$-{\rm Re}^{-1/2}$	$V u_{-}$					

Dung	Coded value			Rea	al va	lues	Responses
nulls	$A_1$	$B_1$	$C_1$	$R_d$	$N_1$	$N_2$	$-\mathrm{Re}_x^{-1/2}Nu_x$
1	-1	-1	-1	0.1	0.2	0.2	0.281664
2	1	-1	-1	0.3	0.2	0.2	0.347108
3	-1	1	-1	0.1	0.6	0.2	0.289786
4	1	1	-1	0.3	0.6	0.2	0.35714
5	-1	-1	1	0.1	0.2	0.6	0.296965
6	1	-1	1	0.3	0.2	0.6	0.370294
7	-1	1	1	0.1	0.6	0.6	0.264525
8	1	1	1	0.3	0.6	0.6	0.331376
9	-1	0	0	0.1	0.4	0.4	0.295673
10	1	0	0	0.3	0.4	0.4	0.365663
11	0	-1	0	0.2	0.2	0.4	0.330977
12	0	1	0	0.2	0.6	0.4	0.324526
13	0	0	-1	0.2	0.4	0.2	0.319590
14	0	0	1	0.2	0.4	0.6	0.317144
15	0	0	0	0.2	0.4	0.4	0.330705
16	0	0	0	0.2	0.4	0.4	0.330705
17	0	0	0	0.2	0.4	0.4	0.330705
18	0	0	0	0.2	0.4	0.4	0.330705
19	0	0	0	0.2	0.4	0.4	0.330705
20	0	0	0	0.2	0.4	0.4	0.330705

BC are significant factors for local Nu. Consequently, the mathematical Eq. (15) may be rewritten as follows:

 $Nu_x = 0.3304169 + 0.0342968A - 0.0059655B$ - 0.0014984C + 0.0006833A<sup>2</sup> - 0.0022332B<sup>2</sup> - 0.0116177C<sup>2</sup> - 0.0005710AB + 0.0009227AC - 0.0111890BC. (16)

In addition, the values of R-squared,  $R^2$  and adjusted R-squared, and  $R^2 - adj$  are shown in Table 4, giving detailed information from the goodness-of-fit of the RSM model. It is noted that Nu is reported with higher  $R^2$  and  $R^2 - adj$  values 98.78%, 99.58%, respectively, illustrating a precisely expected correlation between output response variables and independent input variables.

#### 2.2. Sensitivity analysis

As previously mentioned, sensitivity refers to the partial derivation of the response function from the model parameters. As a result, the sensitivity functions of LNN are articulated in relation to governing variables, radiation variable (A), thermophoresis variable (B),

Source							
$-\mathrm{Re}_{\pi}^{-1/2}Nu_{\pi}$	DOF	Sum of	Contribution	Adj. mean	<i>F</i> -value	<i>n</i> -value	
1002 1.02	201	square	contribution	square	1 varao	r	
Model	9	0.013953	99.78%	0.004651	505.3	$4.32 \times 10^{-12}$	
Linear	3	0.012141	86.82%	0.004047	1318.92	0	
Square	3	0.000801	5.73%	0.000267	87.02	0	
Interaction	3	0.001011	7.23%	0.000337	109.83	0	
Residual error	10	0.000031	0.22%	0.000000	—	_	
Lack of fit	5	0.000031	0.22%	0.000000	—	_	
Pure error	5	0.000000	_	0.000000	—	_	
Total	19	0.013984	-	-	_	_	

Table 3. ANOVA scrutiny for the LNN.

**Table 4.** Estimated regression coefficients via local  $N_u$ .

Term							
$-\mathrm{Re}_x^{-1/2}Nu_x$	Estimate	Std. error	<i>t</i> -value	p-value			
Constant	0.3304169	0.0006022	548.694	$2 \times 10^{-16}$			
A	0.0342968	0.0005539	61.915	$2.94 \times 10^{-14}$			
В	-0.0059655	0.0005539	-10.769	$8.03 \times 10^{-7}$			
C	-0.0014984	0.0005539	-2.705	0.0221			
$A^2$	0.0006833	0.0010563	0.647	0.5323			
$B^2$	-0.0022332	0.0010563	-2.114	0.0606			
$C^2$	-0.0116177	0.0010563	-10.998	$6.60 \times 10^{-7}$			
AB	-0.0005710	0.0006193	-0.922	0.3782			
AC	0.0009227	0.0006193	1.490	0.1671			
BC	-0.0111890	0.0006193	-18.067	$5.78 \times 10^9$			
	$R^2 = 99.7$	8%		$R^2 - adj = 99.58\%$			

and Brownian motion variable (C) through Eq. (16):

$$\frac{\partial N u_x}{\partial A} = 0.0342968 + 0.0013666A - 0.0005710B + 0.0009227C,$$

$$\frac{\partial N u_x}{\partial B} = -0.0059655 - 0.0044664B - 0.0005710A$$
$$-0.0005710C,$$
$$\frac{\partial N u_x}{\partial N u_x} = 0.0014084 - 0.00220254C + 0.000020254$$

$$\frac{\partial V}{\partial C} = -0.0014984 - 0.0232354C + 0.0009227A -0.0111890B.$$

By utilizing Eqs. (17) and (18), the sensitivity findings of the responses  $(Nu_x)$  as well as the input governing variables (A, B, and C) are established (see Table 5). A positive degree of sensitivity illustrating the increase in input governing variable causes an increment in the response function, and vice versa, for the negative sensitivity value. The sensitivity inspection results have been plotted into vertical bar charts (Figures 2 and 3) for a more noteworthy insight.

#### 3. Results and discussion

The present statistical analysis attempts to shed light on some significant insights into the stagnant point flow of Walters-B nanoliquid on a radiative Riga surface. To illustrate the significance of the considered problem, the sensitivity analysis of the LNN is investigated based on the relevant parameters. To this end, the parameters of interest including radiation, thermophoresis, and Brownian motion parameter are assumed. In addition, a Bar chart is drawn according to the numerical values of the SFC and LNN.

Table 6 shows the process of a comprehensive analysis in a limited scope when  $Q = \beta = 0 = We$ . This table points to the high level of acceptance. Table 7 is arranged to analyze the effect of related parameters like We, A, and Q on SFC. It is worth remembering that SFC is reduced for larger We, A, and Q. The influence of  $We, A, Q, R_d, \delta$ , and Pr on

				Sensitivity	7
A	B	C	$\frac{\partial N u_{x}}{\partial A}$	$\frac{\partial N u_{x}}{\partial B}$	$\frac{\partial N u_{x}}{\partial C}$
		-1	0.0325785	-0.0003571	0.0320033
	-1	0	0.0335012	-0.0009281	0.0087679
		1	0.0344239	-0.0014991	-0.0144675
-					
İ		-1	0.0320075	-0.0048235	0.0208143
A	0	0	0.0329302	-0.0053945	-0.0024211
		1	0.0338529	-0.0059655	-0.0256565
		-1	0.0314365	-0.0092899	0.0096253
	1	0	0.0323592	-0.0098609	-0.0136101
		1	0.0332819	-0.0104319	-0.0368455
		-1	0.0339451	-0.0009281	0.032926
	-1	0	0.0348678	-0.0014991	0.0096906
		1	0.0357905	-0.0020701	-0.0135448
0					
A =		-1	0.0333741	-0.0053945	0.021737
	0	0	0.0342968	-0.0059655	-0.0014984
		1	0.0352195	-0.0065365	-0.0247338
		-1	0.0328031	-0.0098609	0.010548
	1	0	0.0337258	-0.0104319	-0.0126874
		1	0.0346485	-0.0110029	-0.0359228
		-1	0.0353117	-0.0014991	0.0338487
	-1	0	0.0362344	-0.0020701	0.0106133
		1	0.0371571	-0.0026411	-0.0126221
-					
= V		-1	0.0347407	-0.0059655	0.0226597
	0	0	0.0356634	-0.0065365	-0.0005757
		1	0.0365861	-0.0071075	-0.0238111
		-1	0.0341697	-0.0104319	0.0114707
	1	0	0.0350924	-0.0110029	-0.0117647
		1	0.0360151	-0.0115739	-0.0350001

**Table 5.** Sensitivity for LNN when A = -1.

**Table 6.** Comparison of f''(0) through Hayat et al. [50] and Mahapatra and Gupta [51] for  $We = \beta = Q = 0$ .

A	Current	Hayat	Mahapatra and
	work	et al. [50]	Gupta [51]
0.2	-0.91790	-0.91692	-0.9181
0.5	-0.66740	-0.66722	-0.6673
2.0	2.01750	2.01750	2.01750
3.0	4.72920	4.72910	4.72930

LNN is illustrated through Table 8. This table demonstrates that LNN intensifies following the increment in modified Hartman number Q, ratio parameter A, conjugate parameter  $\delta$ , thermal radiation parameter

**Table 7.** Numerical values of SFC  $\operatorname{Re}_x^{1/2} C_f$  for values of different physical parameters.

$Re_x^{1/2}C_f$	$\boldsymbol{A}$	We	Q
0.7380	0.2	0.0	0.2
0.5563	_	0.1	—
0.3464	—	0.2	—
0.0957	—	0.3	—
0.5948	0.0	-	—
0.5838	0.1	-	—
0.5563	0.2	-	—
0.5149	0.3	-	—
0.6856	—	-	0.0
0.6196	—	-	0.1
0.5563	-	-	0.2
0.4947	-	-	0.3

**Table 8.** Values of LNN  $\operatorname{Re}_{x}^{-1/2} N u_{x}$  for different physical parameters values.

We	$\boldsymbol{A}$	$\overline{Q}$	$R_d$	δ	$\mathbf{Pr}$	$-\mathrm{Re}_x^{-1/2}Nu_x$
0.0	0.2	0.2	0.1	0.5	1.0	0.7074
0.1	-	-	-	-	-	0.7009
0.2	-	-	-	-	-	0.6937
0.3	_	-	-	-	-	0.6858
0.1	0.1	0.2	0.1	0.5	1.0	0.6861
_	0.2	-	-	-	-	0.7009
_	0.3	-	-	-	-	0.7154
_	0.4	-	-	-	-	0.7224
0.1	0.2	0.0	0.1	0.5	1.0	0.6657
_	_	0.1	-	-	-	0.6829
-	-	0.2	-	-	-	0.7009
-	-	0.3	-	-	-	0.7176
0.1	0.2	0.2	0.1	0.5	1.0	0.6614
-	-	-	0.2	-	-	0.6998
_	_	-	0.3	-	_	0.7389
_	_	-	0.4	-	_	0.7806
0.1	0.2	0.2	0.1	0.1	1.0	0.7590
_	_	-	_	0.2	_	0.7669
_	_	-	_	0.3	_	0.7773
_	—	—	—	0.4	—	0.7910
0.1	0.2	0.2	0.1	0.5	1.0	0.6992
—	—	—	—	—	1.2	0.7739
—	—	—	—	—	1.3	0.8105
_	_	_	_	_	1.4	0.8471





Βd

0

 $R_d$ , and Pr although it decreases upon increasing We. Based on the data of skin friction (see Table 7), the bar charts are plotted separately and in combination. Also, a bar chart is drawn for the Nusselt number data, as given in Table 8.

We

A

The box plot portrays the distribution dispersion or spread. It makes use of the highest and lowest values for the data, the quartiles  $(Q_1 \text{ and } Q_3)$ , and the median  $Q_2$ . Figure 4 plots a box plot for Nusselt number responses in the case of Nusselt number and SFC. The SFC is evenly distributed to a greater degree with a smaller range. There is a significant increase in Nusselt number and responses of Nusselt number that are not evenly distributed. The distribution of Nusselt number is positively skewed. Sometimes, high or low values appear in a set of data. The solid points are the extreme values of the SFCs and responses of the Nu. In Figure 5, the residual curves in each response are displayed individually to provide further analysis of the model accuracy. A normal probability plot is very helpful in testing normality assumptions. There are two versions of normal probability plots: P - P and Q - Q plots. It is important to keep in mind that



 $\mathbf{Pr}$ 

δ

Figure 4. Box plot for Nusselt number, responses of Nusselt number, and Skin friction coefficient.

the normality of the findings is verified by drawing the usual probability plots for residual distribution. With a few exceptions, the usual probability plots for the response are nearly straight lines. This suggests that the regression model fits with the observed values accurately and that the model is valuable and relevant



Figure 5. Normal P-P and Q-Q plot of the residuals for the Nusselt number.

enough. Figure 6 shows the density and histogram for residual distribution. It is observed that the residual histogram has a lower degree of skewness and is more similar to a symmetrical distribution.

The findings of the sensitivity analyses are demon-

strated with vertical bar charts for a better comprehension (see Figures 7(a)-(c) and 9(a)-(c)). According to this notion, the above and right bar displays a direct sensitivity value, while the inverted bar displays an inverse sensitivity value. As noted earlier, when Figure 7(a)-(c) reveal a general pattern in which the direct sensitivity of LNN  $(Nu_x)$  to A (Radiation parameter) rises upon increment in C (Brownian motion number) from -1 to 1. Meanwhile, the sensitivity of LNN to C decreases upon increase in B; however, the sensitivity value also varies from positive to negative for all values of C. As shown in Figure 8(a)–(c), in the case of A = 0, we observe that the posit2ive sensitivity of LNN to A (radiation parameter) rises with an increment in C(Brownian motion number) from -1 to 1 and also, for B (thermophoresis number) from -1 to 1. In addition, the sensitivity of LNN to B and C is reduced with an increment in B and also, in the case of C(-1, 0, 1). The sensitivity of LNN to A rises due to an increase in C and also for enhancement B is noted in Figure 9(a)-(c) when A = 1.



Figure 6. Density and histogram plots of residuals for Nu number.



**Figure 7.** Sensitivity results for LNN when A = -1.



Figure 8. Sensitivity results for LNN when A = 0.



Figure 10. Predicted responses for LNN: (a) B and C, (b) A and C, and (c) A and B.

Figure 10 plots the predicted LNN as a function of radiation (A), thermophoresis (B), and Brownian motion (C) parameters when A = -1 = C = B. The impacts of thermophoresis and Brownian number on LNN for A = -1 ( $R_1 = 0.1$ ) are given in Figure 10(a). It is to be investigated that the maximum LNN occurs at a higher level of thermophoresis (B) and for Brownian motion (C) parameters. Moreover, the opposite behavior is given in Figure 10(c). In addition, the maximum average LNN occurs in an area closer to the low levels for radiation (A) and Brownian (B) numbers (see Figure 10(b)). The predicted LNN as a function of radiation (A), thermophoresis (B), and Brownian (C) parameters is given in the case of A = 0 = B = C and A =1 = C = B in Figures 11 and 12. The impacts of thermophoresis and Brownian motion parameters over LNN for A = 0 are given in Figure 11(a). We investigate if the LNN is maximum in a region close to the high level of thermophoresis (B) and Brownian motion (C) variables. For the case of B = 0, the maximum LNN is observed when A lies from -1 to -0.5 and C varies from -1 to 1 (see Figure 11(b)). Moreover, in the case of C = 0, the maximum



Figure 11. Predicted responses for LNN: (a) B and C, (b) A and C, and (c) A and B.



Figure 12. Predicted responses for LNN: (a) B and C (b) A and C and (c) A and B.

average LNN occurs in a region near the low levels for thermophoresis (B) and for all levels of A. The same behavior is observed in Figure 12(a) in comparison to that in Figure 10(a). In the case of B = 1, LNN attains the maximum level at higher values of C and lower level of A. Moreover, in the case of high radiation and lower thermophoresis, LNN attains its maximum level for C = 1 (see Figure 12 (c)).

#### 4. Conclusions

A numerical analysis of heat transport enhancement corresponding to a sensitivity analysis was performed using response surface strategies to control the Walters-B nanofluid stagnant point flow caused by a Riga surface employing Response Surface Methodology (RSM). The main findings are given below:

- The distribution of Nusselt number was positively skewed;
- Box plot for Nusselt number responses in the case of Nusselt number and skin friction coefficient showed that the skin friction coefficient was more evenly distributed in a smaller range, whereas there was a significant increment in Nusselt number and re-

sponses of Nusselt number that were not evenly distributed;

- The normal probability plots P P and Q Q residual plot showed the excellent fitted regression model for Local Nusselt Number (LNN);
- The values of the factors A, B, C, B<sup>2</sup>, C<sup>2</sup>, and BC were significant for the LNN;
- Maximum LNN occurred at the higher values of thermophoresis and Brownian motion parameters;
- Maximum average LNN occurred at lower values of radiation (A) and Brownian (B) numbers;
- Sensitivity of LNN to C decreased with increase in B; however, the sensitivity value also varied from positive to negative for all values of C.

#### Nomenclature

- T Nanofluid temperature, K
- $M_0$  Magnetization of the permanent magnets mounted on the area of Riga plate
- *a* Width of the magnets between the electrodes

- $\tilde{D}_t s$ Thermophoretic diffusivity coefficient,  $\mathrm{m}^2\mathrm{s}^{-1}$ Applied current density in electrodes  $j_0$  $\tilde{H}$ Concentration of nanofluid, mol  $m^{-3}$  $C_w$ Wall's nanoparticle volume fraction,  $mol m^{-3}$ Ambient concentration, mol  $m^{-3}$  $C_{\infty}$ Brownian diffusivity coefficient,  $m^2 s^{-1}$  $D_b$ Heat at uniform pressure,  $Kg^{-1} J K^{-1}$  $c_p$  $R_d$ Radiation parameter Thermal conductivity, Kg m s<sup>-3</sup> K<sup>-1</sup> K $N_1$ Thermophoresis parameter  $\mathbf{Pr}$ Prandtl number  $T_w$ Wall's temperature, K (x, y)Coordinate system Radiative flux  $q_r$  $k_1$ Mean absorption coefficient (u, v)Velocity components, m  $s^{-1}$ Radiative flux, W  $m^{-2}$  $q_r$ Schmidt number  $S_c$ Ambient fluid's temperature, K  $T_{\infty}$  $U_e$ Free stream velocity δ Conjugate term  $Q_1$ Modified Hartman number  $N_2$ Brownian motion parameter  $\beta$ **Dimensionless** parameter
- $W_e$  Weissenberg number A Ratio parameter

#### Greek symbols

- $\tilde{\tau}$  Fraction of effective heat capacity
- $\phi$  Non-dimensional concentration
- $\sigma^*$  Stefan-Boltzmann constant
- $\alpha$  Fluid thermal diffusivity, m s<sup>-1</sup>
- $\sigma$  Electrical conductivity,  $\Omega^{-1} \text{ m}^{-1}$
- $\rho$  Density of liquid, kg m<sup>-3</sup>
- $\eta$  Similarity variable
- $\theta$  Nondimensional temperature
- $\tilde{\sigma}^*$  Coefficient of mean adsorption
- $\nu$  Dynamic viscosity, m<sup>2</sup> s<sup>-1</sup>

# ${\it Abbreviation}$

- LNN Local Nusselt Number
- RSM Response Surface Methodology
- SFC Skin Friction Coefficient

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