



## Research Note

# Comparative modeling of abrasive waterjet machining process based on OA-Taguchi and *D*-optimal approach and optimization using simulated annealing algorithm

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## KEYWORDS

Abrasive Water Jet Machining (AWJM) process;  
 Design Of Experiments (DOE);  
 Taguchi Orthogonal Array (OA) method;  
 Regression modeling;  
 Simulated Annealing (SA) algorithm.

**Abstract.** Nowadays, machining of hard-to-machine alloys has become a challenge in terms of coping with different approaches that have been introduced so far, among which Abrasive Water Jet Machining (AWJM) has become one of the most extensively used ones owing to its advantages. The current study provided the required data for modeling, statistical analysis, and optimization of AWJM process based on Taguchi Orthogonal Array (OA) and *D*-optimal approaches. Regression modeling was also considered to relate the process input variables (water pressure, abrasive flow rate, machining speed, and machining gap) to the output characteristic namely Surface Roughness (SR). In this regard, three sets of models were proposed using three experimental matrices namely OA-Taguchi, *D*-optimal, and their combination, and their adequacy was checked using Analysis of Variance (ANOVA). According to the findings, the most significant variable affecting SR was the machining speed with the contribution of 66%. Finally, to optimize the objective functions of the proposed models and obtain the optimized (the least) characteristic (SR), the models were embedded in Simulated Annealing (SA) algorithm. According to the computational results, the mixture matrix (with less than 4% error) was superior to OA-Taguchi and *D*-optimal, hence quite being efficient in modeling and optimizing the process.

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## 1. Introduction

AISI M24 is one of the most widely used hard-to-machine alloys in machining for which different processes have been introduced among which Abrasive Wa-

ter Jet Machining (AWJM) process is considered the most widely used one (Figure 1) owing to its advantages including small forces of cutting and imposed stresses as well as lack of thermal distortion [1,2]. Figure 2 illustrates the flow chart of AWJM process [2].

Proper selection of the process input variables is a crucial factor that affects the quality of products [3,4]. There are several tuning variables in AWJM process among which the abrasive flow rate, water pressure, machining gap, and machining speed are the most important ones which were to be modeled and opti-

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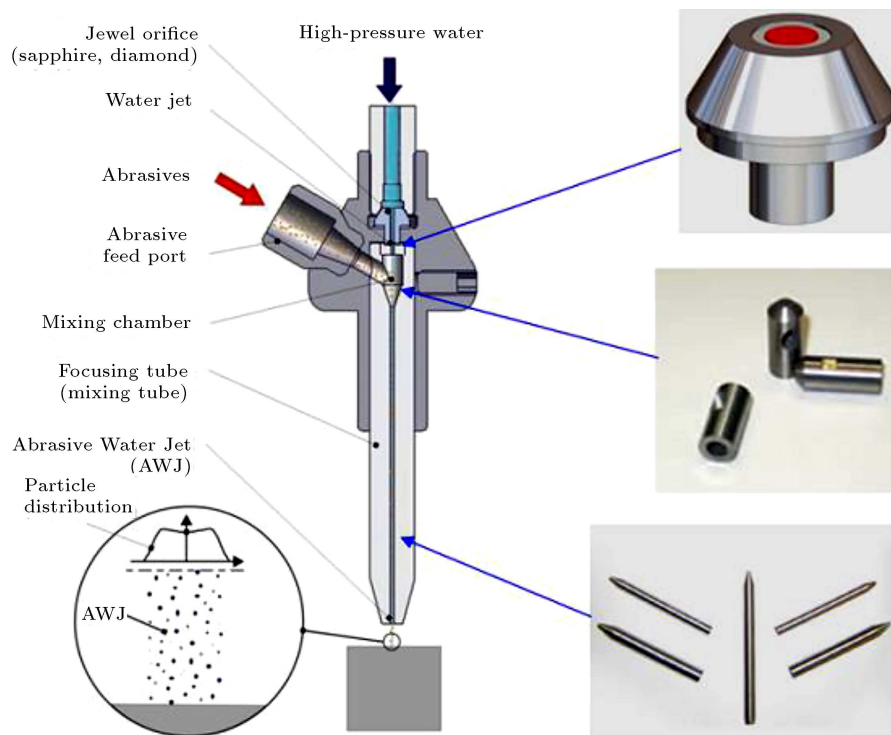


Figure 1. AWJM process schematic illustration.

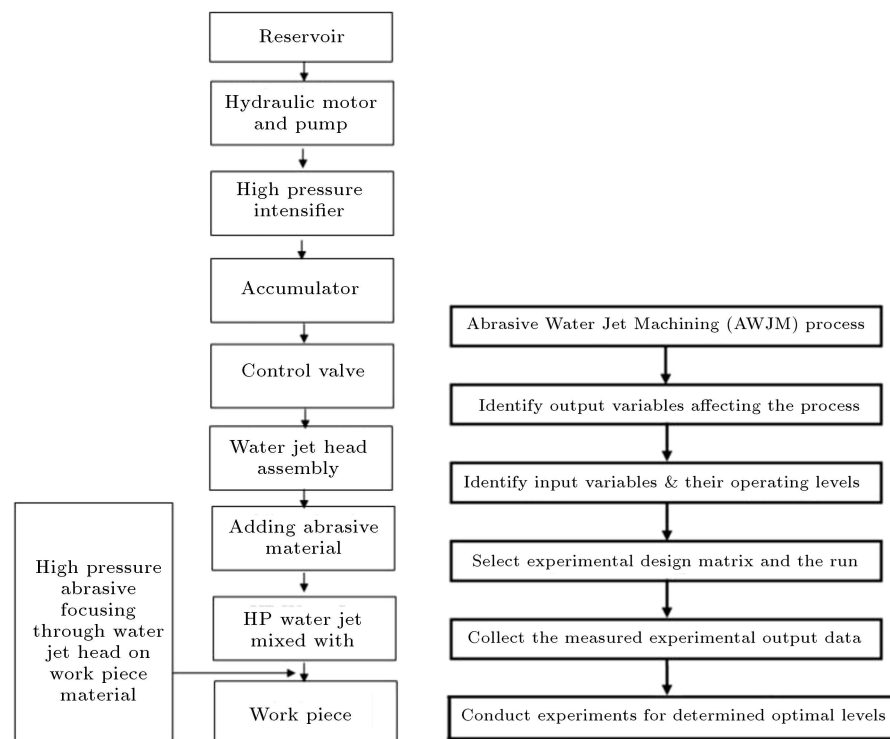


Figure 2. Flow chart of AWJM process [2].

mized in this study [5,6]. By the same token, Surface Roughness (SR) was considered as the process output performance measure to evaluate the process.

In recent years, a majority of scholars have shifted their academic focus on modeling and optimizing dif-

ferent processes to relate a set of process input-output parameters to each other and optimize them to obtain the required output characteristics [5–11].

In this regard, Kolahan and Khajavi [7] employed the regression modeling based on the OA-Taguchi

method to establish a relationship between AWJM process input-output parameters. The adequacy of the proposed model was checked using Analysis of Variance (ANOVA). Next, they embedded the proposed model into a heuristic algorithm (simulated annealing) to be optimized to obtain the desired process output.

Xie and Rittel [8] proposed a method to model the AWJM process. According to their obtained results, the proposed model could authentically model the process.

Srinivasan et al. [9] modeled and optimized the SR of the products resulting from the AWJM process using different fuzzy logic regression equations based on Response Surface Methodology (RSM). ANOVA results revealed that the proposed procedure was in good agreement with the experimental results (less than 10% error).

In the current research, the RSM approach was employed to design the experimental matrix necessary for data gathering, modeling, and optimizing purposes in the AWJM process. The relation of the process input-output parameters was determined based on the regression modeling method. The impact of the process variables, i.e., traverse speed, water pressure, and standoff distance, on the surface quality of the products was also evaluated using ANOVA. Consideration of the water pressure and standoff distance at low levels and traverse speed at high levels resulted in notable improvement in the surface quality [10].

PonSelvan et al. [11] utilized the Taguchi technique to design the matrix and conduct the required experimental tests. They optimized the process using Taguchi method and selected the process input variables such as water pressure, traverse speed, abrasive mass flow rate, and standoff distance to increase the depth of cut and decrease the SR. They found that the proposed method was quite efficient in modeling the process.

Miron et al. [12] proposed a method to model and predict the SR values in the AWJM process. They reported 10% error between the predicted values and experimental tests, thus confirming the adequacy of the proposed approach in modeling and predicting the process.

Different aspects of the AWJM process have been investigated, as can be observed in the literature. However, very few attempts have been made to model, statistically analyze, and optimize the AWJM process based on the Design Of Experiment (DOE) approach, regression modeling, ANOVA, and heuristic (simulated annealing) algorithm.

In order to model, statistically analyze, and optimize the AWJM process, this study introduced a new approach. In this regard, the OA-Taguchi and *D*-optimal methods were employed to design the matrix required for the experimental tests, data gathering,

regression modeling and statistical analysis, determination of the percent contribution, and optimization purposes. Based on the number of process input variables and their predetermined intervals and levels, different experimental matrices were proposed using the DOE approach. Of note, among the proposed matrices, the OA-Taguchi and *D*-optimal are the most extensively used ones, hence included in this study. The main objectives of this study are as follows:

1. Comparing the performance of the OA-Taguchi and *D*-optimal design matrices in terms of data gathering and process modeling;
2. Establishing a relationship among three sets of process input-output parameters using regression modeling;
3. Performing ANOVA in order to determine the adequacy of the proposed model and percent contribution of the process variables on SR;
4. Considering the appropriate models as the authentic representatives of the process (objective function) and optimizing them based on Simulated Annealing (SA) algorithm to identify the proper set of process variables that yield the desired/optimized value for SR.

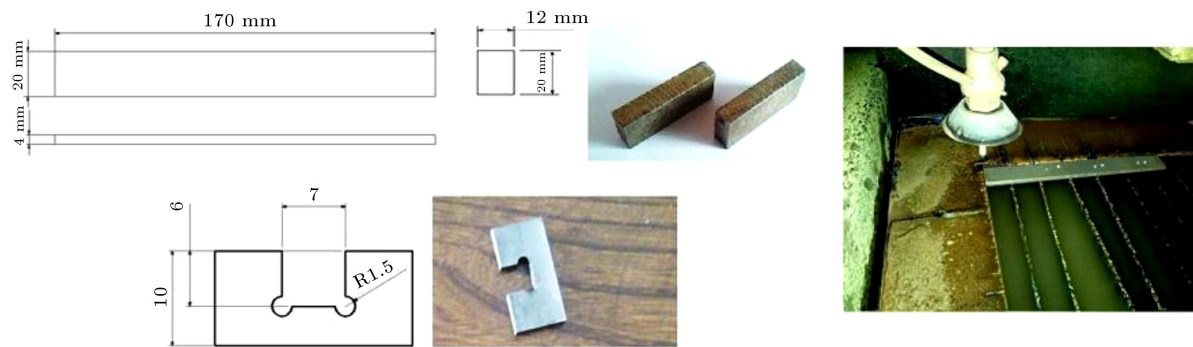
## 2. Equipment used model development and ANOVA

AISI M24 alloy belongs to the category of High-Speed Steels (HSS) that enjoys several advantages including its ability to be shaped in both soft and hard states which, in turn, makes this alloy suitable for several applications such as drilling, reaming, tapping, forming, broaching, and milling. Table 1 shows the chemical composition of AISI M24 alloy [11]. In this study, AISI M24 alloy was introduced as a material on which the experimental tests were about to be conducted using a waterjet machine (American Flow 60000 model) equipped with a numerical control (Figure 3). Table 2 lists the most important specifications of the waterjet machine.

Determination of the significant input variables of the process and their appropriate intervals and levels is the key action prior to the experimental tests. Conventionally, the intervals of the process input variables and their applicable levels have been determined based on experimental experiences. However, in this study, apart from reviewing the relevant literature survey, some preliminary experiments based

**Table 1.** AISI M24 steel chemical composition.

C	Si	Mn	Cr	Mo	V	W	Co
1.10	0.50	0.20	3.90	9.20	1.00	1.40	7.80

**Figure 3.** Waterjet machine and used workpiece.**Table 2.** Waterjet machine specifications.

Particle rate (gr/min)	Desk speed (mm/min)	Water pressure (psi)	X, Y and Z axis movement (mm)
70–140	0–4000	0–60000	400, 1500, and 2000

**Table 3.** AWJM process input variables and their appropriate intervals.

Machining variable	Symbol	Unit	Interval	Pitch
Speed	$F$	mm/min	60–180	1
Water pressure	$P$	psi	20000–40000	500
Particle debi	$N$	gr/min	70–140	0.01
Machining gap	$H$	mm	1–3	0.5

on screening method were conducted in order to determine the process input variables and levels (Table 3). Then, the most appropriate experimental matrices, i.e., OA-Taguchi,  $D$ -optimal, and their combination, were determined. In the next step, the experiments were conducted according to the proposed design matrices, and the required data were evaluated. Afterwards, the process characteristic equations were obtained by developing the mathematical models and carrying out test of significance using ANOVA ( $F$ -test and  $P$ -test). Finally, the most proper models among the linear, curvilinear, and logarithmic ones were selected as the representative of the process response based on the ANOVA findings. In order to optimize the process input variables in such a way as to achieve the desired output characteristic (the least amount of SR), the SA algorithm was employed [13].

In this study, the water pressure, abrasive flow rate, machining speed, and machining gap were considered as the AWJM process input variables (Table 4) and SR as the process response characteristic. The experimental results required for regression-based modeling and simulated annealing-based optimization purposes were obtained using OA-Taguchi and  $D$ -optimal experimental designs [14–16]. Of note, a combination

**Table 4.** AWJM process input variables and their appropriate levels.

Level	$P$ (psi)	$H$ (mm)	$N$ (gr/min)	$H$ (mm)
Level 1	20000	1	70	1
Level 2	30000	2	140	2
Level 3	40000	3	—	3

of the proposed design matrices was also taken into consideration. Tables 5 and 6 show the experimental settings and their corresponding measured SR values obtained from OA-Taguchi and  $D$ -optimal methods, respectively. As shown earlier, these tables comprise 18 and 26 experiments, respectively, based on which the statistical analysis and modeling were conducted. Moreover, both matrices were incorporated into a single matrix to study their combination and collect authentic required data with regard to regression modeling purposes.

To determine the relations between the process input and output characteristics (SR), regression modeling was employed [17–20]. Eq. (1) illustrates the general form of a mathematical model:

$$Y_1 = a_0 + a_1H + a_2F + a_3P + a_4N + a_{11}HH + a_{22}FF + a_{33}PP + a_{44}NN + a_{12}HF + a_{13}HP + a_{14}HN + a_{23}FP + a_{24}FN + a_{34}PN, \quad (1)$$

where  $H$ ,  $F$ ,  $P$ , and  $N$ , are the process input variables and  $Y_1$  is the output response (SR). In addition,  $a_0$ ,  $a_1$ ,  $a_2$ ,  $a_3$ ,  $a_4$ ,  $a_{11}$ ,  $a_{22}$ ,  $a_{33}$ ,  $a_{44}$ ,  $a_{12}$ ,  $a_{13}$ ,  $a_{14}$ ,  $a_{23}$ ,  $a_{24}$ , and  $a_{34}$  are the regression constants which are to be predicted [17]. In this study, the experiments necessary to modeling purposes were selected based

**Table 5.** OA-Taguchi design of the experiments and their corresponding measured output characteristics.

No.	Input variables				Output characteristic (SR)	No.	Input variables				Output characteristic (SR)
	<i>F</i>	<i>N</i>	<i>P</i>	<i>H</i>	<i>Ra</i>		<i>F</i>	<i>N</i>	<i>P</i>	<i>H</i>	<i>Ra</i>
1	60	70	20000	1	5.40	10	60	140	20000	3	6.80
2	120	70	20000	2	8.30	11	120	140	20000	1	8.40
3	180	70	20000	3	11.50	12	180	140	20000	2	7.10
4	60	70	30000	1	3.80	13	60	140	30000	2	5.90
5	120	70	30000	2	7.40	14	120	140	30000	3	6.90
6	180	70	30000	3	11.10	15	180	140	30000	1	12.40
7	60	70	40000	2	4.20	16	60	140	40000	3	6.70
8	120	70	40000	3	7.70	17	120	140	40000	1	7.00
9	180	70	40000	1	10.40	18	180	140	40000	2	6.80

**Table 6.** *D*-optimal design of the experiments and their corresponding measured output characteristics.

No.	Input variables				Output characteristic (SR)	No.	Input variables				Output characteristic (SR)
	<i>F</i>	<i>N</i>	<i>P</i>	<i>H</i>	<i>Ra</i>		<i>F</i>	<i>N</i>	<i>P</i>	<i>H</i>	<i>Ra</i>
1	120	70	20000	3	9.80	14	120	20000	70	3	9.80
2	180	70	40000	3	9.00	15	180	40000	70	3	9.00
3	60	140	30000	3	6.90	16	60	30000	140	3	6.90
4	120	70	30000	1	7.70	17	120	30000	70	1	7.70
5	180	70	30000	2	9.10	18	180	30000	70	2	9.10
6	180	140	20000	3	7.60	19	180	20000	140	3	7.60
7	180	140	20000	2	7.10	20	180	20000	140	2	7.10
8	180	140	20000	1	11.70	21	180	20000	140	1	11.70
9	60	70	20000	1	7.40	22	60	20000	70	1	7.40
10	60	70	20000	2	7.10	23	60	20000	70	2	7.10
11	180	70	20000	3	11.50	24	180	20000	70	3	11.50
12	180	70	40000	1	10.40	25	180	40000	70	1	10.40
13	60	140	30000	1	7.80	26	60	30000	140	1	7.80

**Table 7.** Results of ANOVA for the OA-Taguchi method.

Source	DF	Seq SS	Adj SS	Adj MS	<i>F</i>	<i>P</i>
Regression	6	98.0224	98.0224	16.3371	183.840*	0.0000000
F	1	56.4166	45.7625	45.7625	514.963*	0.0000000
PP	1	3.6261	10.9558	10.9558	123.285*	0.0000039
HH	1	0.0923	32.9741	32.9741	371.056*	0.0000001
NH	1	10.5844	24.0490	24.0490	270.622*	0.0000002
FH	1	21.1256	23.7658	23.7658	267.436*	0.0000002
NP	1	6.1774	6.1774	6.1774	69.515*	0.0000324
Error	8	0.7109	0.7109	0.0889	—	—
Total	14	98.7333	—	—	—	—

Note: DF: Degree of Freedom;  $R\text{-}Sq(adj) = 90.74\%$ ;  $R\text{-}Sq(pred) = 84.08\%$

\*: Significant variable ( $F > F_{\alpha, v1, v2}$ ), ( $F_{0.05, 6, 18} = 2.66$ )

on OA-Taguchi and *D*-optimal approaches. For each approach as well as their combination, the linear, nonlinear, and logarithmic models were developed and based on the ANOVA results, the most fitted and appropriate ones (Eqs. (2)–(4)) were regarded as the

authentic representatives of the AWJM process which were selected for optimization purposes considering the SA algorithm. Based on the ANOVA results (Tables 7–9 and Figures 4–6), the modified nonlinear model (nonlinear model devoid of trivial variables) derived

**Table 8.** Results of ANOVA for the  $D$ -optimal method.

Source	DF	Seq SS	Adj SS	Adj MS	$F$	$P$
Regression	7	118.224	118.224	16.8892	37.5938*	0.0000000
N	1	7.765	7.815	7.8145	17.3945*	0.0006410
P	1	7.058	11.820	11.8204	26.3112*	0.0000837
F	1	79.905	33.399	33.3993	74.3438*	0.0000001
HH	1	0.125	14.639	14.6392	32.5856*	0.0000256
NF	1	6.054	6.728	6.7279	14.9758*	0.0012296
NH	1	7.923	8.945	8.9448	19.9103*	0.0003424
FH	1	9.394	9.394	9.3945	20.9112*	0.0002702
Error	17	7.637	7.637	0.4493		
Total	24	125.862				

Note: DF: Degree of Freedom;  $R\text{-}Sq(adj) = 91.43\%$ ;  $R\text{-}Sq(pred) = 84.07\%$

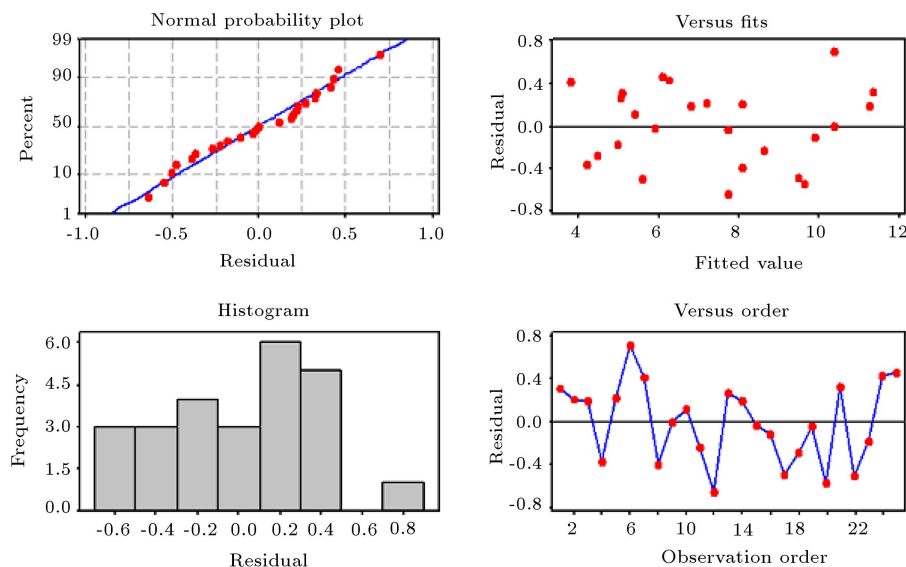
\*: Significant variable ( $F > F_{\alpha, v1, v2}$ ,  $F_{0.05, 7, 26} = 2.39$ )

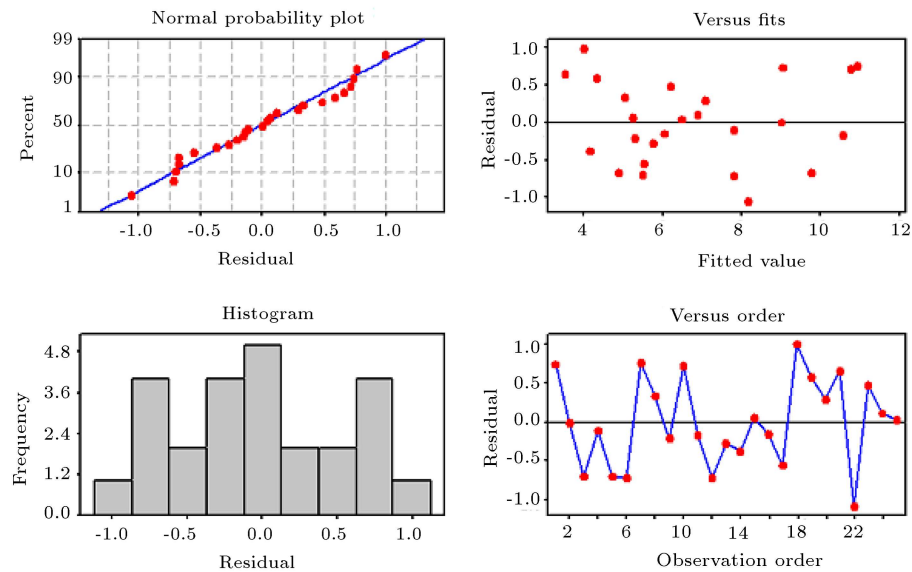
**Table 9.** Results of ANOVA for the combined OA-Taguchi and  $D$ -optimal methods.

Source	DF	Seq SS	Adj SS	Adj MS	$F$	$P$
Regression	7	129.127	129.127	18.4467	97.615*	0.0000000
N	1	8.190	6.165	6.1652	32.624*	0.0000254
P	1	4.503	12.836	12.8356	67.923*	0.0000002
F	1	88.472	27.310	27.3098	144.516*	0.0000000
HH	1	0.034	17.709	17.7095	93.714*	0.0000000
NF	1	6.625	1.869	1.8686	9.888*	0.0059118
NH	1	11.498	15.911	15.9115	84.199*	0.0000001
FH	1	9.804	9.804	9.8042	51.881*	0.0000015
Error	17	3.213	3.213	0.1890		
Total	24	132.340				

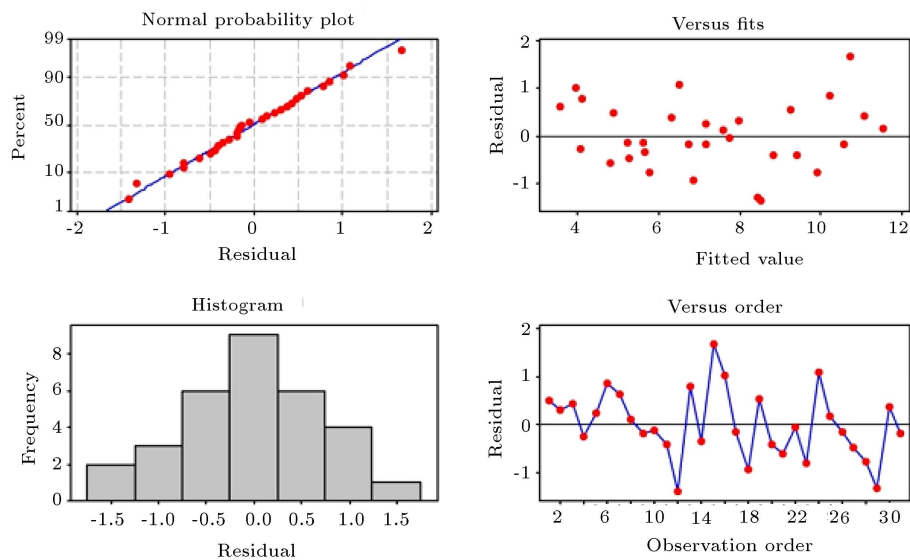
Note: DF: Degree of Freedom;  $R\text{-}Sq(adj) = 96.57\%$ ;  $R\text{-}Sq(pred) = 94.83\%$

\*: Significant variable ( $F > F_{\alpha, v1, v2}$ ), ( $F_{0.05, 7, 44} = 2.25$ )

**Figure 4.** Residual plot for surface roughness based on the OA-Taguchi method.



**Figure 5.** Residual plot for surface roughness based on the  $D$ -optimal method.



**Figure 6.** Residual plot for surface roughness based on the combined OA-Taguchi and  $D$ -optimal methods.

from the third matrix (combination of OA-Taguchi and  $D$ -optimal) was found to be superior to other ones.

$$\begin{aligned}
 Ra &= 0.501819 + 51.5809 \times N - 9.10038e - 005 \\
 &\times P + 0.0914637 \times F - 0.19745 \times N \times F \\
 &- 28.2003 \times N \times H - 0.0181511 \times F \times H \\
 &+ 1.1959 \times H \times H,
 \end{aligned} \quad (2)$$

$$\begin{aligned}
 Ra &= -0.391647 + 59.4385 \times N - 8.88813e - 005 \\
 &\times P + 0.0994871 \times F - 0.327245 \times N \times F
 \end{aligned}$$

$$\begin{aligned}
 &- 20.6863 \times N \times H - 0.015936 \times F \times H \\
 &+ 0.884686 \times H \times H,
 \end{aligned} \quad (3)$$

$$\begin{aligned}
 Ra &= 1.29146 + 0.103476 \times N + 0.4560321 \\
 &\times P - 0.0348425 \times F - 3.86679e - 009 \times H \\
 &\times H + 2.0473 \times N \times F + 0.0012907 \times N \\
 &\times H - 38.1608 \times F \times H.
 \end{aligned} \quad (4)$$

Figures 4–6 depict the residual plots for SR in Taguchi,  $D$ -optimal, and mixture matrices, respectively. Acceptable conformability of the developed model to

the real process and a normal distribution of the residuals were shown on the normal probability and histogram plots, respectively [19]. Of note, the residuals based on the residual-fitted value plot followed no pattern. In addition, the order of observation versus residuals exhibited accidental changes in the residuals [20].

As shown in Tables 7–9, the capability and adequacy of the proposed models (i.e., how well a model can represent the authentic process under study and fit the experimental data) were determined based on ANOVA results (within 95% of the confidence limit) [20–22]. According to the required confidence limit ( $Pr$ ), correlation factor ( $R^2$ ), and adjusted correlation factor ( $R^2\text{-adj}$ ), the modified second-order model (with elimination of the trivial and unimportant variables) was superior to other proposed models. As a result, the superior model was considered the best authentic representative of the AWJM process to be optimized using SA algorithm.

$F$ -test ( $F$ -value) is a factor that shows the significance of the process input variables. To be specific, a large  $F$ -value for a process variable is indicative of its significant effect on the process performance ( $R_a$ ) [22]. To evaluate the significance of the variables in this study, a confidence level of 95% was taken into account. Then, the  $F$ -values of the AWJM process variables were compared with the appropriate values from the confidence table ( $F_{\alpha, v1, v2}$ ) where  $v1$ ,  $v2$ , and  $\alpha$  are the degrees of freedom associated with the numerator, denominator, and risk given in Tables 7, 8, and 9, respectively. In addition, the  $P$ -test ( $P$ -value) was employed to determine the significance of the process input variables. As mentioned earlier, the confidence level of 95% was considered in this study; therefore, it can be concluded that each variable with the  $P\text{-value} < 0.05$  is significant.

ANOVA results were employed to determine the significance of the AWJM process input variables (percent contributions) using Eq. (5). In this equation,  $SS_i$  and  $DOF_i$  are the sums of the square and degree of freedom of the  $i$ th factor, respectively, and  $MS_{error}$  is the sum of mean squared errors [22].

$$P_i(\%) = \frac{SS_i - (DOF_i \times MS_{error})}{\text{Total sum of square}}. \quad (5)$$

Figure 7 depicts the percent contributions of the AWJM process variables according to which the machining speed is the major variable affecting SR at 66% contribution. It can be concluded that due to the presence of the uncontrollable parameters and error based on the nature of the process as well as the used equipment, 5% of error is acceptable.

Figure 8 shows the effect of two AWJM process variables, i.e., machining gap and speed, on the SR via

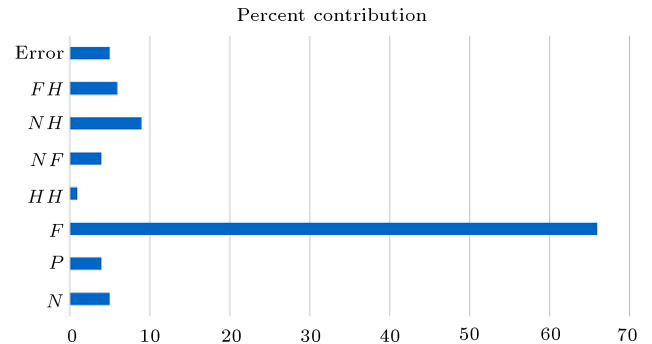


Figure 7. Percent contribution of the AWJM process variables on the surface roughness.

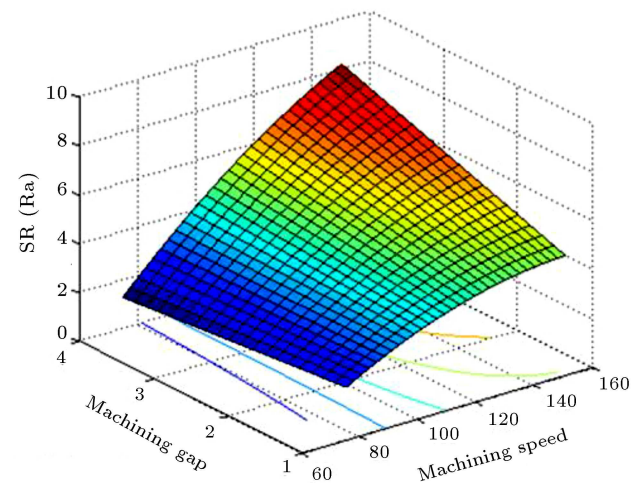


Figure 8. 3D plot of the surface roughness versus machining gap and speed.

3D response surfaces, thus demonstrating the interaction effect of the machining speed (the most important variable) and gap (the trivial factor) on the measured SR.

### 3. Optimization procedure

The most fitted mathematical model suggests a proper relationship between the AWJM process variables and their corresponding achieved SR. This model is applicable for two reasons:

1. It can predict the AWJM process characteristics (SR) for any given set of process input variables;
2. It can determine values for a set of process input variables for a desired/optimized value of SR.

While optimizing the manufacturing processes, the process input variables should be set to obtain the maximum, minimum, and desired output characteristics (in this case, the least amount of SR). Given that finding an optimal set of process input variables to achieve the optimized/desired process characteristic is challenging, different methods were proposed among

which the evolutionary algorithms were utilized as an optimizing procedure [22]. Among the most significant advantages of SA algorithm are easy programming and few parameters to be set that make it a widely used evolutionary algorithm for different optimization purposes. For this reason, the current study used SA algorithm for model optimization [22].

Generally, heuristic algorithms are the reminiscent of natural or physical phenomena. In addition, the methodology of SA algorithm is reminiscent of heat treatment (annealing) process used for cooling down the molten metal, which was proposed by Kirkpatrick et al. [21]. Molten metal temperature acts as a key factor in determining the movement of atoms. Since the temperature at the first steps of the annealing process is high, the movement of molten metal atoms with respect to each other is freer and more intense. Consequently, upon proceeding the annealing process, temperature was reduced slowly at a certain reduction rate and movement became slower and more restricted. As a result, the atoms were rearranged, thus forming a crystal structure with a minimum level of energy. Such a reduction in the temperature acts as a key factor in determining the polycrystalline state and energy level by increasing through which a higher energy level can be achieved. More details about the procedure and algorithm are documented in [21,22].

### 3.1. The mechanism of SA algorithm

Finding an initial solution to the objective function ( $C_0$ ) (the proposed model in this study), proper answer space, and random solution generation in the proper answer space ( $C_1$ ) is the first and crucial step in the SA algorithm. Next, the new and current values of the objective function are compared with each other ( $\Delta C$ ). The movement towards the new solution was made feasible under two conditions: either the new solution is improved (in comparison with the current one) or the probability function (Eq. (6)) is higher than a randomly generated number, which is between 0 and 1 [22]:

$$Pr = \exp\left(-\frac{\Delta C}{T_k}\right). \quad (6)$$

As the algorithm proceeds, the temperature is reduced using Eq. (7). In this equation,  $T_k$  and  $T_{k+1}$  are the current and new temperatures, respectively, which play a similar role to the temperatures in the physical annealing process. The cooling rate is represented by parameter  $\alpha$  [22].

$$T_{K+1} = \alpha \times T_K \quad k=0, 1, \dots \quad \text{and} \quad 0.9 \leq \alpha \leq 1. \quad (7)$$

In the first step of the SA algorithm, a temperature takes a value that decreases at each iteration based on

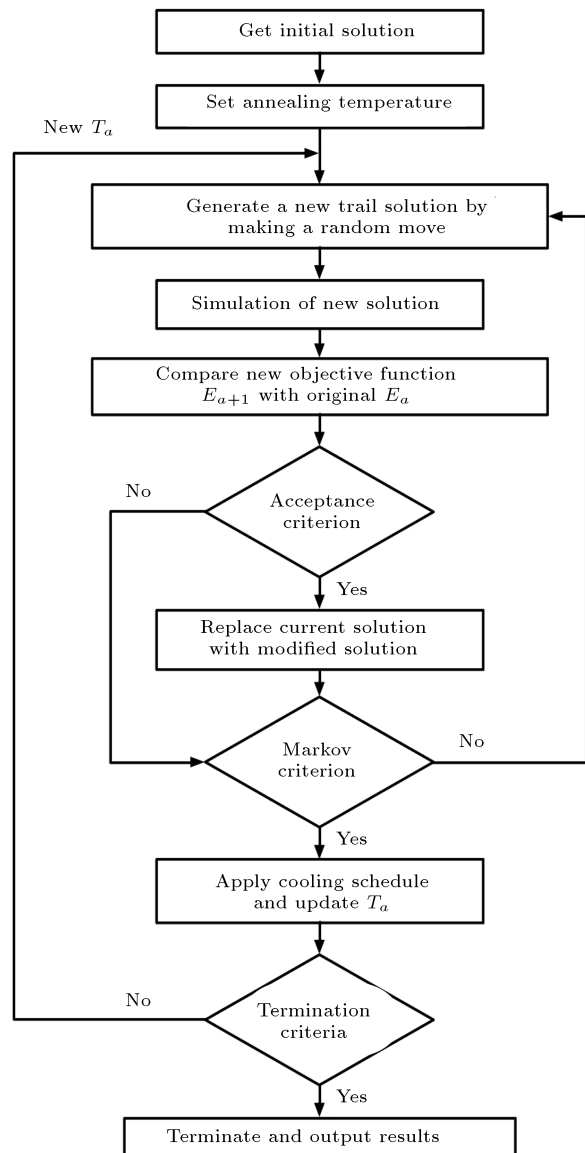
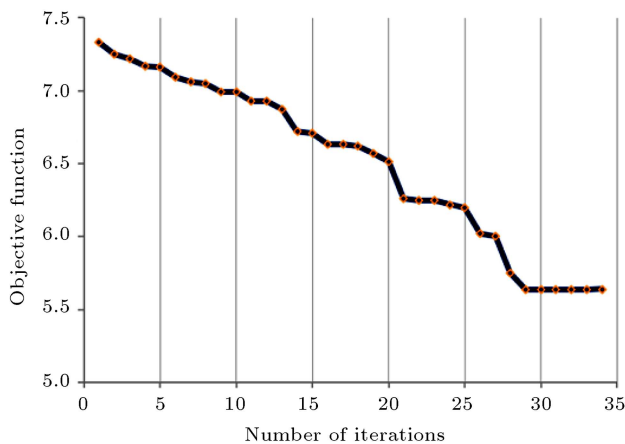


Figure 9. Illustration of the SA algorithm procedure schematic [22].

the pre-determined temperature reduction procedure. Based on the mechanism of the SA algorithm, the temperatures at the first iterations are high and a majority of the movement occurrences including improving and worsening ones can be accepted. Accepting the worsening movement lessens the chance of getting trapped in local minima. As the algorithm proceeds, followed by decreasing the temperature, only improving solutions are likely to be accepted to move towards. There are different methods that can act as algorithm terminations among the most important and extensively used ones are the pre-determined number of iterations or run time as well as the number of iterations where no development or improvement is detected [21]. Figure 9 depicts the flowchart of the SA algorithm used for optimizing the AWJM process [22].

**Table 10.** Results of SA algorithm optimization.

Experimental matrix used	Process input variables				Process output $Ra$ ( $\mu\text{m}$ )		
	$P$ (psi)	$F$ (mm/min)	$N$ (gr/min)	$H$ (mm)	Predicted	Experimental	Error (%)
Combined OA-Taguchi and $D$ -optimal	39000	178	140	2.4	5.6	5.8	3.5
OA-Taguchi	38000	174	130	3.0	5.7	6.4	12.5
$D$ -optimal	39000	170	135	2.9	5.6	6.3	12.5

**Figure 10.** SA algorithm convergence.

At the first step of any process optimization, defining the objective/fitness function (in this study, the proper SR model) is a key feature whose optimization comprises setting the process variables at proper levels to obtain the desired/optimum value for SR. Table 10 shows the results of the SA algorithm optimization based on which the least amount of optimization error (less than 4%) is attributed to the mixture matrix (OA-Taguchi and  $D$ -optimal). Application of OA-Taguchi and  $D$ -optimal method yielded the same results. Figure 10 illustrates the SA algorithm convergence.

#### 4. Conclusion

The current research study evaluated, modeled, and optimized the effects of the settings of the Abrasive Water Jet Machining (AWJM) process input variables on the Surface Roughness (SR) of AISI 24M alloy. To this end, a set of experimental data based on the OA-Taguchi method,  $D$ -optimal approach, and their corresponding combination was used for data collection, model development, statistical analysis, and optimization purposes. Followed by conducting several experiments based on the proposed matrixes, regression modeling was taken into account to develop mathematical models that related these process variables namely the water pressure, abrasive flow rate, machining speed, and machining gap to the SR. In this

study, different models such as linear, curvilinear, modified curvilinear (with elimination of trivial variables), and logarithmic ones were studied among which the modified curvilinear model was selected as the most fitted authentic representative of the AWJM process based on the Analysis of Variance (ANOVA) findings. The ANOVA results revealed that the most important process variable affecting SR was the machining speed (at 66% contribution). In addition, machining gap was regarded as a trivial variable which was eliminated from the model. Next, the most proper model was identified as an authentic representative of the process optimization based on Simulated Annealing (SA) algorithm to predict the values for the best process input variables and obtain the desired (the least) value for SR. The computational results confirmed that the proposed modeling and optimization method (regression-SA) could efficiently and accurately determine the cutting variables to obtain the optimum SR. The optimization results of both OA-Taguchi and  $D$ -optimal methods yielded the same results. However, the best result was reported while using the mixture matrix (with less than 4% error).

#### Nomenclature

AWJM	Abrasive Water Jet Machining
ANOVA	Analysis of Variance
$R^2$ -adj	Adjusted correlation factor
$R^2$	Correlation factor
$Pr$	Confidence limit
$T_{k+1}$	Current temperatures
$C_0$	Current objective function
DOF	Degree Of Freedom
$v_2$	Denominator degrees of freedom
DOE	Design Of Experiments
$F$ -value	$F$ test's value
$T_k$	Former temperatures
HSS	High-Speed Steels
$MS_{\text{error}}$	Mean sum of square of error
$v_1$	Numerator degrees of freedom
OA-Taguchi	Orthogonal array Taguchi method

$C_1$	Objective function
$P$ -value	$P$ -test's value
SA	Simulated Annealing algorithm
SR	Surface Roughness
SS	Sum of Square
$T_k$	Temperature parameter

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