A Modified Russell Measure for Estimating Efficiency Changes in the Presence of the Undesirable Outputs and Stochastic Data

Seyedeh Sara Hosseini[1], Reza Farzipoor Saen* and Reza Kazemi Matin[2]

(1) Department of Mathematics, Qazvin Branch, Islamic Azad University, Qazvin, Iran. (s.hosseini794@gmail.com)
(2) Department of Mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran. (rkmatin@kiau.ac.ir)
*Department of Operations Management & Business Statistics, College of Economics and Political Science, Sultan Qaboos University, Muscat, Oman, Tel. number: +9682414 2937, Mobile Number: +96895055696, Fax: +96824413415. (farzipour@yahoo.com)

Abstract. Although data envelopment analysis (DEA) assumes deterministic data, a great volume of data might be stochastic. The global Malmquist productivity index (GMPI) is a highly effective instrument for productivity analysis in DEA. This paper extends GMPI in the presence of stochastic data. Our new stochastic DEA model is a chance-constrained programming model, which is converted to a deterministic programming problem with a linear objective function and quadratic constraints. For efficiency evaluation purposes, in this paper, the weak disposability principle is used to model Russell's measure in the presence of undesirable outputs. The main contribution of this paper is to develop a global Russell model with stochastic data. A case study is presented to illustrate the applicability of the proposed models.

KEYWORDS
Data envelopment analysis (DEA);
Stochastic data;
Undesirable outputs;
Modified Russell measure;
Global Malmquist productivity index.

1. Introduction

Data envelopment analysis (DEA), developed by Charnes et al. [1], is a rigorous mathematical programming tool for evaluating the relative performance of decision making units (DMUs). In this technique, multiple inputs and outputs are fed into some mathematical programming models for analysis. DEA has been widely applied in many different fields (e.g., Seiford and Thrall [2]; Korhonen and Luptacik [3]; Kazemi Matin et al. [4]; Emrouznejad et al. [5]; Hadi-Vencheh and

Matin [6]; Wang et al. [7] and Emrouznejad et al. [8]; Tavassoli et al. [9]; Nemati et al. [10]; Tavana et al. [11]; Hajaji et al. [12]; Yousefi et al. [13]).

The standard DEA models stress technical efficiency measurement by utilizing radial measures, which are gauged in proportion to input/output isoquant by searching for the maximal reduction/expansion in all input/outputs of DMU that are likely feasible for a given output/output vector. Basic DEA models draw on several mild assumptions in terms of production possibility sets and production functions.

In production economics, it is important to estimate the efficiency changes of a production unit in different periods and determine the regress and progress of production systems. Caves et al. [14], first, proposed a Malmquist productivity index (MPI) as a highly effective instrument for efficiency analysis in DEA. Färe et al. [15] combined the technical efficiency measurement with the efficiency measurement idea of Caves et al. [14] and decomposed it into efficiency change (EC) and technical change (TC) components. MPI has many successful practical applications (e.g., Lissitsa and Odening [16]; Guzman and Reverte [17] and Emrouznejad et al. [18,19]).

Chen and Golley [20] modified MPI to measure environmental efficiency growth. Luenberger [21] and [22] presented Malmquist-Luenberger (ML) productivity index to incorporate the notions of the MPI and directional distance function (DDF). However, the ML index might be infeasible (Pastor and Lovell [23]). Some solutions have been proposed to overcome this problem. Shestalova [24] proposed a sequential Malmquist–Luenberger (SML) index to combine the concepts of successive sequential reference production sets and DDF. SML index has been utilized in some studies like Oh and Heshmati [25], Oh and Lee [26], and Wang and Lan [27]. Molinos et al. [28] examined the Luenberger productivity indicator in the water industry. Oh and Lee [26] introduced the metafrontier Malmquist productivity growth index to deal with the group heterogeneity, which was used by O'Donnell et al. [29] for calculating technical gaps and the efficiency of the agriculture sector. Portela and Thanassoulis [30] described a meta-Malmquist index to measure efficiency. Pastor and Lovell [23] introduced global MPI (GMPI) that does not suffer from the infeasibility issue. Likewise, Färe and Grosskopf [31] and Oh [32] employed the global ML productivity index in their studies. Maniadakis and Thanassoulis [33] proposed the cost Malmquist index that is decomposed to technical efficiency change (TEC), allocative efficiency change (AEC), technical change (TC), and price effect (PE) components. A global profit MPI was developed by Tohidi et al. [34,35], which can be utilized once input and output prices are available and can be decomposed into several components. Furthermore, Tohidi and Razavyan [36] introduced the convex combination of the inputs' costs for various periods. Emrouznejad et al. [19] proposed an ML index based on a range adjusted measure (RAM) model to avoid the infeasibility issue.

In the real world, there might be undesirable outputs such as pollutions and CO_2 emissions (Färe et al. [37]; Arabi et al. [38]). Intuitively, if inputs are freely available, reducing bad outputs depends on desirable outputs. In economics, this feature is referred to as weak disposable technology (Kuosmanen [39]). Shephard [40] was the first to propose the weak disposability principle between good and bad outputs. This issue was extended by Hailu and Veeman [41], Scheel [42], and Kuosmanen and Podinovski [43].

Kuosmanen [39] discussed that the desirable and undesirable outputs are dependent on each other. Kuosmanen and Podinovski [43] showed that the technology proposed by Kuosmanen [39] is free from the convexity contravention problem and it provides a correct technology for modeling undesirable outputs under weak disposability assumption. They demonstrated that this correct technology is the smallest technology, which confirms the principles of convexity, free disposability inputs and desirable outputs, and weak disposability of undesirable outputs.

Azadi and Farzipoor Saen [44] proposed a deterministic version of the stochastic model in the presence of undesirable outputs. Momeni and Farzipoor Saen [45] developed a Russell chance-constrained DEA model to help decision-makers to identify suitable third-party reverse logistics providers. Skevas et al. [46] determined the technical inefficiency and pesticides' environmental inefficiency of farms using the Russell model in the presence of undesirable inputs and outputs. Tavana et al. [47] developed DEA adaptations of a robust cross-efficiency approach to rank DMUs in the existence of uncertain data and undesirable outputs. Chen and Golley [20] proposed an enhanced Russell-based directional distance function model for dealing with undesirable outputs in the presence of zero data. Zare Haghighi and Rostamy-Malkhalifeh [48] suggested an enhanced Russell measure (ERM) to consider the undesirable outputs. Wu et al. [49] developed a fuzzy ERM to analyze the undesirable outputs. Jamshidi et al. [50] studied the Russell model given the chance-constrained technique.

Cooper et al. [51] developed the chance-constrained programming (CCP) DEA model. Then, Cooper et al. [52,53] extended the CCP model and applied it in different applications. Azadi and Farzipoor Saen [54] introduced a chance-constrained DEA model to select suppliers with non-discretionary factors and stochastic data. They developed a chance-constrained DEA model. Azadi and Farzipoor Saen [55] introduced a chance-constrained integer DEA model for evaluating production units. There are other related papers that readers can study (e.g., Khodabakhshi [56]; Hosseinzadeh Lotfi et al. [57]; Eslami et al. [58]; Ross et al. [59]; Izadikhah et al. [60]; Majumder et al. [61]; Mahmoodirad et al. [62] and Hajiagha [63]).

To the best of our knowledge, there is no study on assessing efficiency with stochastic data using GMPI. The objective of this paper is to develop a global Russell model under weak disposable technology in the presence of stochastic data and undesirable outputs. In summary, the contributions of this paper are as below:

- For the first time, a global Russell model with stochastic data is developed.
- The proposed model can deal with stochastic data.
- The proposed model inspects the criteria to improve GMPI and recommends practical solutions.
- A case study is presented.

This study is organized as follows: Section 2 briefly reviews the weak disposable technologies, Russell measure (RM) model with undesirable outputs, and the global Malmquist index. Section 3, assuming weak disposable technology, presents a new GMPI based upon the modified Russell measure (MRM) model in the presence of stochastic data and undesirable outputs. A case study is given in Section 4. Section 5 concludes the paper.

2. Preliminaries

2.1 Mathematical notations

DMU_o	The DMU under evaluation
x_{nK}	The *n*th input of *k*th DMU
v_{mK}	The *m*th good output of *k*th DMU
w_{jK}	The *j*th bad output of *k*th DMU
$\tilde{x}_{nk}^{T}$	The *n*th stochastic input of *k*th DMU in period $T \in \{t, t+1\}$
$\tilde{v}_{mk}^{T}$	The *m*th good stochastic output of *k*th DMU in period $T \in \{t, t+1\}$
$\tilde{w}_{jk}^{T}$	The *j*th bad stochastic output of *k*th DMU in period $T \in \{t, t+1\}$
Φ	The normal cumulative distribution function
Φ^{-1}	The inverse of a normal cumulative distribution function
$(\omega)^{s}$	The variance and covariance of inputs in period $s \in \{t, t+1\}$
σ_n^I	The standard deviation of *n*th input
σ_m^o	The standard deviation of *m*th good output
θ_n	The distinct abatement factors of *n*th input
φ_m	The distinct abatement factors of *m*th good output
ψ_j	The distinct abatement factors of *j*th bad output
λ_k	A portion of the intensity weight of DMU_k
μ_k	A portion of the intensity weight of DMU_k

2.2 Weak disposable technology

The common definition of production technology is the set $T = \{(\mathbf{x},\mathbf{v},\mathbf{w}) \mid \mathbf{x}\ can\ produce\ (\mathbf{v},\mathbf{w})\}$. The output is defined as $P(x) = \{(\mathbf{v},\mathbf{w}) \mid (\mathbf{x},\mathbf{v},\mathbf{w}) \in T\}$. If proportional reductions of feasible outputs are feasible, then the outputs are weakly disposable. If $(\mathbf{v},\mathbf{w}) \in P(x)\ and\ 0 \le \theta \le 1\ then\ (\theta\mathbf{v},\theta\mathbf{w}) \in P(x)$. For example, a power plant should reduce electricity production to reduce CO_2 emissions.

Assume that there are K DMUs in which $DMU_k, k \in \{1,...,K\}$ is determined by a vector $(\mathbf{x}_k,\mathbf{v}_k,\mathbf{w}_k)$, where $\mathbf{x}_k = (x_{1k}, x_{2k},...,x_{Nk}) \in R^N$, $\mathbf{x}_k \ge \mathbf{0}$, $\mathbf{x}_k \ne \mathbf{0}$ shows the vector of inputs, $\mathbf{v}_k = (v_{1k}, \mathrm{v}_{2k},...,v_{Mk}) \in R^M$, $\mathbf{v}_k \ge \mathbf{0}$, $\mathbf{v}_k \ne \mathbf{0}$ is the vector of desirable outputs and $\mathbf{w}_k = (w_{1k}, w_{2k},...,w_{Jk}) \in R^J$, $\mathbf{w}_k \ge \mathbf{0}$, $\mathbf{w}_k \ne \mathbf{0}$ is the vector of undesirable outputs.

Axioms of observations inclusion, convexity, and variants of disposability (free or weak) can be introduced to form the production technology (Sueyoshi [64]). In modeling the undesirable outputs, Shephard [40] assumes weak disposability, which implies that any proportional reduction in undesirable outputs needs the same reduction in desirable outputs. Shephard [40] applied a single abatement factor to model weak disposability. (T_s) is defined as follows:

$$T_s = \{(\mathbf{x},\mathbf{v},\mathbf{w}) \mid \sum_{k=1}^{K} z_k \mathbf{x}_k \le \mathbf{x},\ \sum_{k=1}^{K} \theta z_k \mathbf{v}_k \ge \mathbf{v},\ \sum_{k=1}^{K} \theta z_k \mathbf{w}_k = \mathbf{w},\ \sum_{k=1}^{K} z_k = 1,\ z_k \ge 0,\ 0 \le \theta \le 1,\ (k = 1,...,K)\}. \quad (1)$$

However, Kuosmanen [39] presented minimum extrapolation technology for modeling Shephard's definition and used distinct abatement factors, θ_k, $(k \in \{1,...,K\})$, for any DMU. Assuming the variable returns to scale (VRS), the following technology (T_k) is presented:

$$T_k = \{(\mathbf{x},\mathbf{v},\mathbf{w}) \mid \sum_{k=1}^{K} z_k \mathbf{x}_k \le \mathbf{x},\ \sum_{k=1}^{K} \theta_k z_k \mathbf{v}_k \ge \mathbf{v},\ \sum_{k=1}^{K} \theta_k z_k \mathbf{w}_k = \mathbf{w},\ \sum_{k=1}^{K} z_k = 1,\ z_k \ge 0,\ 0 \le \theta_k \le 1,\ (k = 1,...,K)\}. \quad (2)$$

where z_k is the intensity variable. Since T_s and T_k are nonlinear, Kuosmanen and Podinovski's [43] approach is applicable for the choice of abatement factors. They showed that the Shephard technology is not convex and cannot be linearized (Kuosmanen [39]). Regarding the weak disposability, Expression (2) introduces an abatement factor θ_k that scales down both good and bad outputs. It is shown that the technology is convex and can be linearized using the following variable substitution:

$$z_k = \underbrace{\theta_k z_k}_{\lambda_k} + \underbrace{(1-\theta_k)(z_k)}_{\mu_k}$$

The intensity weight of DMU_k has two parts, including λ_k and μ_k. The λ_k is a portion of the output of DMU_k, which is active (Kuosmanen [39]). The μ_k is a portion of the output of DMU_k that is reduced by scaling down the activity level. Using the two components, the technology is defined as follows:

$$T_k = \{(\mathbf{x},\mathbf{v},\mathbf{w}) \mid \sum_{k=1}^{K}(\lambda_k + \mu_k)\mathbf{x}_k \le \mathbf{x}\ ,\ \sum_{k=1}^{K}\lambda_k\ \mathbf{v}_k \ge \mathbf{v}\ ,\ \sum_{k=1}^{K}\lambda_k\ \mathbf{w}_k = \mathbf{w}\ ,$$
$$\sum_{k=1}^{K}(\lambda_k + \mu_k) = 1,\ \ \lambda_k \ge 0,\ \ \mu_k \ge 0,\ \ (k = 1,\ldots,K)\}. \quad (3)$$

2.3 Russell efficiency measure with undesirable outputs

RM model is a non-radial measure to assess the performance of DMUs. Cooper et al. [54] introduced the ERM model. ERM can be best described as the ratio of the average efficiency of inputs to the outputs' efficiency. A brief review of ERM in different fields is given by Hsiao et al. [65], Lozano et al. [66] , and Levkoff et al. [67].

Wang and Li [68] proposed fuzzy ERM and ranked DMUs. Esmaeili [69] developed an ERM model with interval data. Azadi and Farzipoor Saen [44] identified the most efficient suppliers with undesirable items and stochastic data in ERM. Skevas et al. [46] used a bootstrap method to discuss the performance of farmers by providing the effect of stochastic data on products. Zare Haghighi and Rostamy-Malkhalifeh [48] presented an RM to consider both the desirable and undesirable outputs. Wu et al. [49] developed a fuzzy ERM model to estimate the environmental efficiency of thermal power plants in the presence of undesirable outputs. Izadikhah et al. [60] developed a network DEA model to cope with stochastic data.

In this paper, an MRM is presented to calculate the performance of DMUs by taking the desirable and undesirable outputs into account. Suppose that there are K DMUs $(\mathrm{k} = 1,\ldots,\mathrm{K})$ with N inputs $(\mathrm{n} = 1,\ldots,\mathrm{N})$, M desirable outputs $(\mathrm{m} = 1,\ldots,\mathrm{M})$, and J $(\mathrm{j} = 1,\ldots,\mathrm{J})$ undesirable outputs. Chen et al. [70] presented a version of the weighted Russell directional distance model (WRDDM) and showed that the directional distance models with slacks are special cases of slack-based WRDDM. Chen et al. [71] extended Russell's measure to include the undesirable outputs, which is based on WRDDM. Their proposed model measures the inefficiency of inputs, desirable and undesirable outputs, and overall inefficiencies. Employing technology (3), the following model is presented:

$$
\begin{aligned}
& D(\mathbf{x},\mathbf{v},\mathbf{w}) = \max \sum_{n=1}^{N}\theta_n + \sum_{m=1}^{M}\varphi_m + \sum_{j=1}^{J}\psi_j \\
& s.t. \quad \sum_{k=1}^{K}(\lambda_k + \mu_k)x_{nk} \le (1-\theta_n)x_{no} \qquad (\mathrm{n}=1,\dots,\mathrm{N}), \\
& \qquad \sum_{k=1}^{K}\lambda_k v_{mk} \ge (1+\varphi_m)v_{mo} \qquad (\mathrm{m}=1,\dots,\mathrm{M}), \\
& \qquad \sum_{k=1}^{K}\lambda_k w_{jk} = (1-\psi_j)w_{jo} \qquad (\mathrm{j}=1,\dots,\mathrm{J}), \\
& \qquad \sum_{k=1}^{K}(\lambda_k + \mu_k) = 1, \ \lambda_k \ge 0, \ \mu_k \ge 0 \quad (\mathrm{k}=1,\dots,\mathrm{K}), \\
& \qquad 0 \le \theta_n \quad (\mathrm{n}=1,\dots,\mathrm{N}), \\
& \qquad 0 \le \varphi_m \quad (\mathrm{m}=1,\dots,\mathrm{M}), \\
& \qquad 0 \le \psi_j \quad (\mathrm{j}=1,\dots,\mathrm{J}).
\end{aligned} \tag{4}
$$

In Model (4), DMU$_o$ is the DMU under evaluation. The non-negative constraints $0 \le \theta_n, \varphi_m, \psi_j$ imply a potential improvement in the criteria of DMU$_o$ in T_K and ERM lies between zero and unity (Zare Haghighi and Rostamy-Malkhalifeh [48]).

2.4 Global Malmquist productivity index (GMPI)

The MPI is a useful and widely used tool to evaluate the efficiency in different periods and identifies the regress and progress of the DMUs during periods. Färe et al. [15] analyzed the MPI based on technology and efficiency variations. Yao et al. [72] and Aparicio et al. [73] introduced the cost-based MPI. Chen and Golley [20] modified the MPI and applied it to the directional distance function (DDF) model. The MPI has been used in different settings (e.g., Nakano and Managi [74]; Sueyoshi and Goto [75]; Zhang and Choi [76]). However, MPI suffers from the infeasibility issue. Pastor and Lovell [23] proposed the global Malmquist index and Oh [32] used the ML index to overcome the infeasibility problem. Tohidi et al. [34] proposed a global profit Malmquist index, which is based on the cost MPI. Maniadakis and Thanassoulis [33] introduced MPI to determine the efficiency change when cost and price are accessible. Giménez et al. [77] used the global ML index for dynamic analysis of the results in the presence of undesirable products.

Chung et al. [78] presented an ML productivity index to deal with the undesirable outputs. The following ML productivity index is defined as follows:

$$
ML^{t}(\mathbf{x}^t,\mathbf{v}^t,\mathbf{w}^t) = \frac{1+D^t(\mathbf{x}^t,\mathbf{v}^t,\mathbf{w}^t)}{1+D^t(\mathbf{x}^{t+1},\mathbf{v}^{t+1},\mathbf{w}^{t+1})}, \qquad ML^{t+1}(\mathbf{x}^{t+1},\mathbf{v}^{t+1},\mathbf{w}^{t+1}) = \frac{1+D^{t+1}(\mathbf{x}^t,\mathbf{v}^t,\mathbf{w}^t)}{1+D^{t+1}(\mathbf{x}^{t+1},\mathbf{v}^{t+1},\mathbf{w}^{t+1})} \tag{5}
$$

where D is the distance function. $\mathbf{x}^t$ and $\mathbf{x}^{t+1}$ are the inputs of periods t and t+1. Also, $\mathbf{v}^t$, $\mathbf{v}^{t+1}$, $\mathbf{w}^t$, and $\mathbf{w}^{t+1}$ represent desirable output in period t, desirable output in period t+1, undesirable output in period t, and undesirable output in period t+1, respectively. The geometric means ML^t and ML^{t+1} are productivity changes between periods t and t+1, where $ML = \sqrt{ML^t \times ML^{t+1}}$.

To present our new GMPI, assuming weak disposability of outputs, the production set is as $P(\mathbf{x}) = \{(\mathbf{v},\mathbf{w}) \mid \mathbf{x} \mathit{can\ produce}\ (\mathbf{v},\mathbf{w})\}$. By selecting the improvement direction $\mathbf{g} = (\mathbf{g}_x, \mathbf{g}_v, \mathbf{g}_w)$, $P_D(x)$ can be used for performance evaluation (Luenberger [21]):

$$D(\mathbf{x},\mathbf{v},\mathbf{w}) = \max\{\rho \mid (\mathbf{x} - \rho g_X, \mathbf{v} + \rho g_V, \mathbf{w} - \rho g_W) \in P_D(\mathbf{x})\} \tag{6}$$

Now, for $D(\mathbf{x},\mathbf{v},\mathbf{w})$, the GMPI for periods t and t+1 is defined as follows:

$$MLP^G(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t, \mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1}) = \frac{1 + D^G(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t)}{1 + D^G(\mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1})} \tag{7}$$

Note that the global technology set, $P_D^G(\mathbf{x})$, is formed by the same production principles. By considering all the observations of periods t and t + 1, $D^G(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t)$ denotes the DDF of the DMU in period t and $D^G(\mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1})$ is associated with period t+1.

In Expression (7), if MLP^G is bigger than 1, it implies improvement in efficiency. If MLP^G is less than 1, it indicates a decline in efficiency (Giménez et al. [78]). For a given DDF measure, one can decompose Expression (7) into the components of efficiency growth, which is as follows:

$$\begin{aligned} &MLP^G(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t, \mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1}) = \frac{1 + D^G(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t)}{1 + D^G(\mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1})} = \\ &\frac{1 + D^t(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t)}{1 + D^{t+1}(\mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1})} \times \left[\frac{(1 + D^G(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t)) / (1 + D^t(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t))}{(1 + D^G(\mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1})) / (1 + D^{t+1}(\mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1}))} \right] = \\ &\frac{TE^{t+1}(\mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1})}{TE^t(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t)} \times \left[\frac{BPG^{t+1}(\mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1})}{BPG^t(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t)} \right] = EC^{t,t+1} \times BPC^{t,t+1} \end{aligned} \tag{8}$$

where $EC^{t,t+1}$ represents the changes in technical efficiency or catching-up between periods t and t+1. If $EC^{t,t+1} > 1$, then the technical efficiency is improved. If $EC^{t,t+1} < 1$, then the technical efficiency is declined. The best practice gap ($BPG^{t,t+1}$) between a coexistent technology and a global technology frontier indicates the technological change in the corresponding periods.

In this paper, we formulate Model (4) based on the GMPI using weak disposable technology (T_K). Given the observations of periods t and t+1, $s \in \{t, t+1\}$, our new model is as follows:

$$D^G(\mathbf{x}^s,\mathbf{v}^s,\mathbf{w}^s)=\max\sum_{n=1}^{N}\theta_n+\sum_{m=1}^{M}\varphi_m+\sum_{j=1}^{J}\psi_j$$

$$\begin{aligned}
s.t.\quad & \sum_{T=t}^{t+1}\sum_{k=1}^{K}(\lambda_k+\mu_k)x_{nk}^{T}\le(1-\theta_n)x_{no}^{s} && (\mathrm{n}=1,\dots,\mathrm{N}),\\
& \sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k v_{mk}^{T}\ge(1+\varphi_m)v_{mo}^{s} && (\mathrm{m}=1,\dots,\mathrm{M}),\\
& \sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k w_{jk}^{T}=(1-\psi_j)w_{jo}^{s} && (\mathrm{j}=1,\dots,\mathrm{J}),\\
& \sum_{T=t}^{t+1}\sum_{k=1}^{K}(\lambda_k+\mu_k)=1,\ \ \lambda_k\ge0,\ \ \mu_k\ge0 && (\mathrm{k}=1,\dots,\mathrm{K}),\\
& 0\le\theta_n \quad (\mathrm{n}=1,\dots,\mathrm{N}),\\
& 0\le\varphi_m \quad (\mathrm{m}=1,\dots,\mathrm{M}),\\
& 0\le\psi_j \quad (\mathrm{j}=1,\dots,\mathrm{J}).
\end{aligned}\tag{9}$$

By solving Model (9), the GMPI under VRS assumption is as follows:

$$\begin{aligned}
& MLP^G(\mathbf{x}^t,\mathbf{v}^t,\mathbf{w}^t,\mathbf{x}^{t+1},\mathbf{v}^{t+1},\mathbf{w}^{t+1})=\frac{1+D^G(\mathbf{x}^t,\mathbf{v}^t,\mathbf{w}^t)}{1+D^G(\mathbf{x}^{t+1},\mathbf{v}^{t+1},\mathbf{w}^{t+1})}=\\
& \frac{1+D^t(\mathbf{x}^t,\mathbf{v}^t,\mathbf{w}^t)}{1+D^{t+1}(\mathbf{x}^{t+1},\mathbf{v}^{t+1},\mathbf{w}^{t+1})}\times\left[\frac{(1+D^G(\mathbf{x}^t,\mathbf{v}^t,\mathbf{w}^t))/(1+D^t(\mathbf{x}^t,\mathbf{v}^t,\mathbf{w}^t))}{(1+D^G(\mathbf{x}^{t+1},\mathbf{v}^{t+1},\mathbf{w}^{t+1}))/(1+D^{t+1}(\mathbf{x}^{t+1},\mathbf{v}^{t+1},\mathbf{w}^{t+1}))}\right]=\\
& \frac{TE^{t+1}(\mathbf{x}^{t+1},\mathbf{v}^{t+1},\mathbf{w}^{t+1})}{TE^t(\mathbf{x}^t,\mathbf{v}^t,\mathbf{w}^t)}\times\left[\frac{BPG^{t+1}(\mathbf{x}^{t+1},\mathbf{v}^{t+1},\mathbf{w}^{t+1})}{BPG^t(\mathbf{x}^t,\mathbf{v}^t,\mathbf{w}^t)}\right]=EC^{t,t+1}\times BPC^{t,t+1}
\end{aligned}\tag{10}$$

3. Global Malmquist productivity index (GMPI) with stochastic data

Expression (10) assumes deterministic inputs and outputs. However, in many real-world applications, the data often involves uncertainty. Stochastic programming is one of the main approaches to deal with uncertainty. To deal with the stochastic data, Model (9) is extended.

Assuming known mean and variance and normal distribution of inputs and outputs, Cooper et al. [54] developed a chance-constrained DEA model. This technique can be converted to a deterministic equivalent by a quadratic optimization approach. Suppose that the inputs and outputs are random vectors in periods t and t+1.

$$\tilde{\boldsymbol{x}}_k^t=(\tilde{x}_{1k}^t,\dots,\tilde{x}_{Nk}^t),\tilde{\boldsymbol{v}}_k^t=(\tilde{v}_{1k}^t,\dots,\tilde{v}_{Mk}^t),\tilde{\boldsymbol{w}}_k^t=\left(\tilde{w}_{1k}^t,\dots,\tilde{w}_{Jk}^t\right)$$
$$\tilde{\boldsymbol{x}}_k^{t+1}=(\tilde{x}_{1k}^{t+1},\dots,\tilde{x}_{Nk}^{t+1}),\tilde{\boldsymbol{v}}_k^{t+1}=(\tilde{v}_{1k}^{t+1},\dots,\tilde{v}_{Mk}^{t+1}),\tilde{\boldsymbol{w}}_k^{t+1}=\left(\tilde{w}_{1k}^{t+1},\dots,\tilde{w}_{Jk}^{t+1}\right)$$

To calculate the GMPI, Model (9) is presented as the following chance-constrained optimization model:

$$
\begin{aligned}
&SD^{G}(\tilde{\mathbf{x}}^{s},\tilde{\mathbf{v}}^{s},\tilde{\mathbf{w}}^{s}) = \max \sum_{n=1}^{N}\theta_n + \sum_{m=1}^{M}\varphi_m + \sum_{j=1}^{J}\psi_j \\
&s.t. \quad \Pr\left\{\sum_{T=t}^{t+1}\sum_{k=1}^{K}(\lambda_k+\mu_k)\tilde{x}_{nk}^{T} + s_n = (1-\theta_n)\tilde{x}_{no}^{s}\right\} \ge 1-\alpha \qquad (n=1,\dots,N), \\
&\qquad \Pr\left\{\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k\tilde{v}_{mk}^{T} - s_m = (1+\varphi_m)\tilde{v}_{mo}^{s}\right\} \ge 1-\alpha \qquad (m=1,\dots,M), \\
&\qquad \sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k\tilde{w}_{jk}^{T} = (1-\psi_j)\tilde{w}_{jo}^{s} \qquad (j=1,\dots,J), \\
&\qquad \sum_{T=t}^{t+1}\sum_{k=1}^{K}(\lambda_k+\mu_k) = 1, \quad \lambda_k \ge 0, \quad \mu_k \ge 0 \qquad (k=1,\dots,K), \\
&\qquad 0 \le \theta_n, \varphi_m, \psi_j, s_n, s_m \qquad (n=1,\dots,N), (m=1,\dots,M), (j=1,\dots,J).
\end{aligned} \tag{11}
$$

where *Pr* indicates "probability" and "~" represents a random variable with a normal distribution. The $\alpha \in (0,1]$ indicates the allowed chance of failure to satisfy the constraints. Also, $SD^{G}(\tilde{\mathbf{x}}^{s},\tilde{\mathbf{v}}^{s},\tilde{\mathbf{w}}^{s})$ shows the stochastic modified Russell model based on the global Malmquist index.

Definition 1. DMU_o in Model (11) is stochastically efficient if and only if, in optimality, $SD^{G} = 0$.

For stochastic data, it is assumed that all inputs and outputs are independent random variables. By applying the CCP approach in which the cumulative distribution function is denoted by *Φ,* it is possible to convert the chance-constrained Model (11) into a deterministic equivalent model. To this end, we proceed as follows:

Considering the constraint of good outputs in Model (11) as $\Pr\left\{\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k\tilde{v}_{mk}^{T} \ge (1+\varphi_m)\tilde{v}_{mo}^{s}\right\} \ge 1-\alpha \quad (m=1,\dots,M)$, where $s \in \{t, t+1\}$. Using slack variable $s_m \ge 0$, the constraint can be stated as $\Pr\left\{\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k\tilde{v}_{mk}^{T} - (1+\varphi_m)\tilde{v}_{mo}^{s} \ge s_m\right\} = 1-\alpha \quad (m=1,\dots,M)$. Now, by applying the definition of the expected value and variance of the elements, we write it as follows:

$$
Pr\left\{\frac{\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k\tilde{v}_{mk}^{T} - (1+\varphi_m)\tilde{v}_{mo}^{s} - E\left(\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k\tilde{v}_{mk}^{T} - (1+\varphi_m)\tilde{v}_{mo}^{s}\right)}{\sqrt{Var\left(\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k\tilde{v}_{mk}^{T} - (1+\varphi_m)\tilde{v}_{mo}^{s}\right)}} \le \frac{s_m - E\left(\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k\tilde{v}_{mk}^{T} - (1+\varphi_m)\tilde{v}_{mo}^{s}\right)}{\sqrt{Var\left(\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k\tilde{v}_{mk}^{T} - (1+\varphi_m)\tilde{v}_{mo}^{s}\right)}}\right\} = \alpha,
$$

For simplicity, let $\sqrt{Var\left(\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k\tilde{v}_{mk}^{T} - (1+\varphi_m)\tilde{v}_{mo}^{s}\right)}$ is denoted by $\sigma_m(\varphi_o, \lambda)$. Therefore,

$$Pr\left\{\frac{\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k\tilde{v}_{mk}^{T}-(1+\varphi_m)\tilde{v}_{mo}^{s}-\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k v_{mk}^{T}+(1+\varphi_m)v_{mo}^{s}}{\sigma_m\left(\varphi_o,\lambda\right)}\le\frac{s_m-\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k v_{mk}^{T}+(1+\varphi_m)v_{mo}^{s}}{\sigma_m\left(\varphi_o,\lambda\right)}\right\}=\alpha,$$

where $v_{mk}^{T}=E(\tilde{v}_{mk}^{T})$ and $v_{mo}^{s}=E(\tilde{v}_{mo}^{s})$. By denoting the left-hand side of the above inequality by z, we note that it has a normal standard distribution with zero mean and unit variance. Then, we can write $Pr\left\{Z\le\frac{s_m-\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k v_{mk}^{T}+(1+\varphi_m)v_{mo}^{s}}{\sigma_m\left(\varphi_o,\lambda\right)}\right\}=\alpha$. This leads to $\Phi\left\{\frac{s_m-\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k v_{mk}^{T}+(1+\varphi_m)v_{mo}^{s}}{\sigma_m\left(\varphi_o,\lambda\right)}\right\}=\alpha$ where Φ represents the normal cumulative distribution function. By considering the inverse, we have $(1+\varphi_m)v_{mo}^{s}-\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k v_{mk}^{T}+s_m-\Phi^{-1}(\alpha)\sigma_m\left(\varphi_o,\lambda\right)=0 \quad (\mathrm{m}=1,\ldots,\mathrm{M})$, or equivalently $\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k v_{mk}^{T}+\Phi^{-1}(\alpha)\sigma_m\left(\varphi_o,\lambda\right)\ge(1+\varphi_m)v_{mo}^{s} \quad (\mathrm{m}=1,\ldots,\mathrm{M})$. For calculating $\sigma_m\left(\varphi_o,\lambda\right)$, note that

$$(\sigma_m(\varphi_o,\lambda))^2=Var\left(\sum_{T=t}^{t+1}\sum_{k=1}^{K}\lambda_k\tilde{v}_{mk}^{T}-(1+\varphi_m)\tilde{v}_{mo}^{s}\right)=Var\left(\sum_{T=t}^{t+1}\sum_{\substack{k=1\\k\neq o}}^{K}(\lambda_k v_{mk}^{T})+((\lambda_o)-(1+\varphi_m)(v_{mo}^{s}))\right)$$

$$=Var\left(\sum_{T=t}^{t+1}\sum_{\substack{k=1\\k\neq o}}^{K}(\lambda_k v_{mk}^{T})\right)+Var\left((\lambda_o)-(1+\varphi_m)(v_{mo}^{s})\right)+2\operatorname{cov}\left(\sum_{T=t}^{t+1}\sum_{\substack{k=1\\k\neq o}}^{K}((\lambda_k v_{mk}^{T})),((\lambda_o)-(1+\varphi_m)(v_{mo}^{s}))\right)$$

The $\sigma_m(\varphi_o,\lambda)$ is represented by non-negative variables $(\omega_m)^s$, where $s\in\{t,t+1\}$. Using the property of normal distribution for inputs, considering non-negative variables $(\omega_n)^s$, Model (3.1) is finally transformed to a quadratic programming problem as follows:

$$
\begin{aligned}
& SD^G(\mathbf{x}^s, \mathbf{v}^s, \mathbf{w}^s) = \max \sum_{n=1}^{N} \theta_n + \sum_{m=1}^{M} \varphi_m + \sum_{j=1}^{J} \psi_j \\
& s.t. \quad \sum_{T=t}^{t+1} \sum_{k=1}^{K} (\lambda_k + \mu_k) x_{nk}^T - \Phi^{-1}(\alpha)(\omega_n)^s \le (1-\theta_n) x_{no}^s \quad (\mathrm{n}=1,\dots,\mathrm{N}), \\
& \qquad \sum_{T=t}^{t+1} \sum_{k=1}^{K} \lambda_k v_{mk}^T + \Phi^{-1}(\alpha)(\omega_m)^s \ge (1+\varphi_m) v_{mo}^s \quad (\mathrm{m}=1,\dots,\mathrm{M}), \\
& \qquad \sum_{T=t}^{t+1} \sum_{k=1}^{K} \lambda_k w_{jk}^T = (1-\psi_j) w_{jo}^s \quad (\mathrm{j}=1,\dots,\mathrm{J}), \\
& \qquad \sum_{T=t}^{t+1} \sum_{k=1}^{K} (\lambda_k + \mu_k) = 1, \quad \lambda_k \ge 0, \ \mu_k \ge 0 \quad (\mathrm{k}=1,\dots,\mathrm{K}), \\
& \qquad 0 \le \theta_n \quad (\mathrm{n}=1,\dots,\mathrm{N}), \\
& \qquad 0 \le \varphi_m \quad (\mathrm{m}=1,\dots,\mathrm{M}), \\
& \qquad 0 \le \psi_j \quad (\mathrm{j}=1,\dots,\mathrm{J}).
\end{aligned}
\tag{12}
$$

where $(\omega_n)^s$ and $(\omega_m)^s$ refer to the variance and covariance of inputs and good outputs, which are as follows:

$$
\forall n: \ (\omega_n^s)^2 = \begin{cases} \sum_{\substack{k=1 \\ k \ne o}}^{K} ((\lambda_k + \mu_k)^2 Var(\tilde{x}_{nk}^T)) + ((\lambda_o + \mu_o)) - (1-\theta_o))^2 Var(\tilde{x}_{no}^s), & T = s \\ \sum_{k=1}^{K} ((\lambda_k + \mu_k)^2 Var(\tilde{x}_{nk}^T)) + (1-\theta_o)^2 Var(\tilde{x}_{no}^s), & T \ne s \end{cases}
$$

$$
\forall m: (\omega_m^s)^2 = \begin{cases} \sum_{\substack{k=1 \\ k \ne o}}^{K} ((\lambda_k)^2 Var(\tilde{v}_{mk}^T)) + (\lambda_o - (1+\varphi_o))^2 Var(\tilde{v}_{mo}^s), & T = s \\ \sum_{k=1}^{K} ((\lambda_k)^2 Var(\tilde{v}_{mk}^T)) + (1+\varphi_o)^2 Var(\tilde{v}_{mo}^s), & T \ne s \end{cases}
$$

As mentioned before, all the variables have a normal distribution with a known mean and variance. The variances for the inputs and the desirable outputs can be estimated as follows:

$$
\bar{x}_n = \frac{1}{K} \sum_{k=1}^{K} x_{nk} \quad \textit{and} \quad \bar{v}_m = \frac{1}{K} \sum_{k=1}^{K} v_{mk}
$$

where

$$
Var(\tilde{x}_n) = \frac{1}{K-1} \sum_{k=1}^{K} (x_{nk} - \bar{x}_n)^2 \quad \textit{and} \quad Var(\tilde{v}_m) = \frac{1}{K-1} \sum_{k=1}^{K} (v_{mk} - \bar{v}_m)^2
$$

Note that V represents the variance-covariance matrix for inputs

$$
V = \begin{bmatrix} Var(\tilde{x}_1) & Cov(\tilde{x}_1, \tilde{x}_2) & \cdots & Cov(\tilde{x}_1, \tilde{x}_N) \\ & \vdots & \ddots & \vdots \\ Cov(\tilde{x}_N, \tilde{x}_1) & Cov(\tilde{x}_N, \tilde{x}_2) & \cdots & Var(\tilde{x}_N) \end{bmatrix}
$$

It is assumed that the inputs and good outputs are independent. This implies that $Cov(\tilde{x}_{nk}, \tilde{x}_{no}) = 0$ and $Cov(\tilde{v}_{mo}, \tilde{v}_{mk}) = 0$. Model (12) is a non-linear optimization model due to its quadratic constraints. It indicates the stochastic efficiency of DMU_o in periods t and t+1 concerning two different technologies associated with periods t and t+1. For simplicity of calculations, all inputs and outputs are assumed to be independent. Thus, the corresponding covariance in the above expressions is zero. Given Model (12), one can calculate the GMPI for an RM in the presence of stochastic data. Note that, for α=0.5, we have $\Phi^{-1}(\alpha) = 0$, which is equivalent to the deterministic Model (9).

Remark. In Model (12), for $\alpha > 0.5$ when $s \in \{t, t+1\}$, it is likely to obtain an unbounded objective value. Then, in two periods, the stochastic global Malmquist productivity index (SGMPI) of DMU_o is calculated under the condition $\alpha \in (0, 0.5)$.

To evaluate the GMPI using Model (12) and variances, Expression (10) can be written as follows:

$$SMLP^G(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t, \mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1}) = \frac{1+SD^G(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t)}{1+SD^G(\mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1})} = \frac{1+SD^t(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t)}{1+SD^{t+1}(\mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1})} \times \left[\frac{(1+SD^G(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t))/(1+SD^t(\mathbf{x}^t, \mathbf{v}^t, \mathbf{w}^t))}{(1+SD^G(\mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1}))/(1+SD^{t+1}(\mathbf{x}^{t+1}, \mathbf{v}^{t+1}, \mathbf{w}^{t+1}))} \right] \quad (13)$$

where SD^G is the stochastic Global Malmquist Index based on the modified Russell model in the presence of stochastic data. The $SMLP^G < 1$ implies the stochastic deterioration in productivity from period t to t+1. $SMLP^G > 1$ indicates the stochastic progress in productivity and $SMLP^G = 1$ implies no change in productivity.

Theorem 1. For $0 < \alpha \leq 0.5$, the optimal objective value of Model (12) is bounded.

Proof: Note that for $0 < \alpha \leq 0.5$ we have $\Phi^{-1}(\alpha) \leq 0$**.** Therefore, the left-hand side of the first set of constraints in Model (12) is non-negative. Thus, for all n, we have $0 \leq (1-\theta_n)x_{no}^s$, which leads to $0 \leq \theta_n \leq 1$, for all n. For the second set of constraints, based on $\Phi^{-1}(\alpha) \leq 0$, note that $(1+\varphi_m)v_{mo}^s \leq \sum_T \sum_k \lambda_k v_{mk}^T$. Since $\sum_k \lambda_k \leq 1$, based on the fourth constraint, we have $(1+\varphi_m)v_{mo}^s \leq \sum_T Max_k\{v_{mk}^T\}$. Therefore, $0 \leq \varphi_m \leq \frac{\sum_T Max_k\{v_{mk}^T\}}{v_{mo}^s}$ which shows that for all m, $\varphi_m$is bounded. Note that in the third set of constraints we also have $0 \leq \psi_j \leq 1$, for all j. These show that the objective function is bounded, even in the optimality. ■

Theorem 2. For $0 < \alpha \leq 0.5$, the optimal solution of Model (12) is unique.

Proof: In Model (12), the $\sum_T \sum_k (\lambda_k + \mu_k) x_{nk}^T - \Phi^{-1}(\alpha)(\omega_n)^s + \theta_n x_{no}^s - x_{no}^s$ and $-\sum_T \sum_k \lambda_k v_{mk}^T - \Phi^{-1}(\alpha)(\omega_m)^s + \varphi_m v_{mo}^s + v_{mo}^s$ are convex functions. Also, $(\omega_n)^s$ and $(\omega_m)^s$ are the convex functions with $\Phi^{-1}(\alpha) \leq 0$ for $0 < \alpha \leq 0.5$. The third and the fourth equality constraints, $\sum_T \sum_k \lambda_k w_{jk}^T + \psi_j w_{jo}^s - w_{jo}^s$ and $\sum_T \sum_k (\lambda_k + \mu_k) x_{nk}^T - 1$ are the convex functions. Therefore, Model (3.2) is a convex programming problem. Thus, the theorem is proved. ■

The above theorems show that the new GMPI is well-defined and can be used for estimating the efficiency changes and productivity analysis in the presence of undesirable output and stochastic data.

4. Case study

Bahman Diesel Co. (BDC) was founded in 2003. BDC produces and assembles various kinds of motor vehicles, different types of trucks, buses, mini trucks, and long vehicles. BDC has 60% of Iran's mini truck market and is one of the biggest business partners of Isuzu Japan in the Middle East. The main customers of BDC are food companies, consumer products distributors, and fire departments.

The kitting system is a Japanese management philosophy, which has been used in many Japanese production institutes since 1970. The kitting system feeds production lines and sends parts in small groups to the production lines without any breaks, which was introduced in the Toyota Company. The kitting system consists of a series of techniques and principles of production. The kitting system can increase the competitive advantage of companies by decreasing the dissipation of resources and improving product quality and production efficiency (Jonsson et al. [79]; Hanson and Brolin [80]). Hanson and Medbo [81] designed an efficient kit preparation system and recognized important features of the kitting system.

BDC implements a kitting delivery system (KDS) as one of the line feeding systems. The KDS involves the gathering of all the parts needed for a particular assembly from the stockroom and issuing the kit to the manufacturing line at the right time and in the right quantity. The KDS reduces waste in production lines and increases production flexibility (Hanson and Brolin [80]).

Our proposed model is used to evaluate the efficiency of the line feeding systems. Here, the efficiency of 10 KDSs (DMUs) of BDC is assessed. The criteria for efficiency assessment are as follows:

Input 1 (x_1): The number of personnel

Input 2 (x_2): The number of logistics' staff

Input 3 (x_3): The number of pallets

Good output (v_1): The number of productions

Bad output (w_1): The number of industrial wastes

In Figure. 1, the inputs and outputs of KDSs are shown.

<<Figure. 1 goes here>>

Tables 1 and 2 report the dataset of 2014 and 2016, respectively.

<<Table 1 and 2 go here>>

Using Model (9) and Equation (10), the GMPI for the deterministic data is calculated, which is reported in the last column of Table 3. The progress and regress of KDSs are calculated from 2014 to 2016. The covariance is assumed to be zero. Table 3 reports the GMPI for deterministic and stochastic data.

The comparison is depected in Table 3.

<<Table 3 goes here>>

To analyze the sensitivity of the results, different values of α are considered, which are the acceptable percentage of unsatisfied constraints of Model (12). For instance, the computed efficiency scores for Trim line N75 series and $\alpha=0.05$ are as follows: $MRMS^{t}(\mathbf{x}^{t},\mathbf{v}^{t},\mathbf{w}^{t})=0.8533301$, $MRMS^{t+1}(\mathbf{x}^{t+1},\mathbf{v}^{t+1},\mathbf{w}^{t+1})=0.902752$, $MRMS^{G}(\mathbf{x}^{t},\mathbf{v}^{t},\mathbf{w}^{t})=0.6068276$, and $MRMS^{G}(\mathbf{x}^{t+1},\mathbf{v}^{t+1},\mathbf{w}^{t+1})=0.1010444$. Therefore, by solving Equation (13), $MRMS^{t,t+1}(\mathbf{x}^{t},\mathbf{v}^{t},\mathbf{w}^{t},\mathbf{x}^{t+1},\mathbf{v}^{t+1},\mathbf{w}^{t+1})=1.45936677.$

The deterministic GMPIs are reported in the last column of Table 3. For example, the deterministic and stochastic GMPIs for all DMUs, except for Trim line N55 and Chassis line N55 series, indicate progress given the acceptable risk level of $\alpha \in [0.001, 0.04]$. Based on the stochastic GMPIs, for many levels, the Chassis line 77E series has experienced progress as its stochastic GMPIs are more than 1. However, the GMPI for deterministic data is less than 1.

SHILLER5 line, for $\alpha \in [0.001, 0.1]$, has progressed. Trim line N75, Chassis line N75 series, and MB SERIES line have progressed during 2014 and 2016. However, the MB SERIES line, for $\alpha = 0.1$ is unbounded. Compared with the deterministic GMPIs, their stochastic GMPIs are strictly more than 1. Trim line N55 and Chassis line N55 have regressed in terms of deterministic and stochastic GMPIs.

Chassis line 77E series remains almost unchanged for the risk levels $\alpha = 0.03 \; and \; 0.4$. Given Tables 1 and 2, the desirable and undesirable outputs of the Chassis line 77E series from 2014 to 2016 have remained almost fixed. Given the deterministic data, the SHILLER5 line has progressed. Since there is no significant difference between the deterministic and stochastic MPIs during 2014 and 2016, Trim line N55 and Chassis line N55 have regressed. For instance, the deterministic GMPIs of Trim line N75, Chassis line N75 series, and MB SERIES line are less than one, which have regressed.

Comparing the results of deterministic and stochastic GMPIs for α=0.05 is shown in Figure. 2. The stochastic GMPIs of DMU1, DMU2, DMU3, DMU4, DMU5, DMU8, and DMU9 are bigger than 1, which have progressed.

<<Figure. 2 goes here>>

4.1. Managerial implications

Competitiveness in the global economy has been played a significant role in the market. Productivity improvement is a vital issue for firms. The use of DEA has become very crucial for industries to evaluate productivity in the presence of undesirable outputs and stochastic data. In this paper, we explained how to analyze the progress and regress of DMUs in the presence of stochastic data. To get a better idea, the sensitivity of the results given different values of α was discussed. Usually, managers can use the proposed models in the real world as they face stochastic data whenever they wish to assess the productivity of their systems.

5. Conclusions

The efficiency assessment and determining the progress or regress of DMUs are important for decision-makers. There might be stochastic data for efficiency evaluation. In this paper, for the first time, GMPI was presented to evaluate the efficiency of DMUs with stochastic data. To this end, a new MRM model was developed. The novelty of the current paper lies in the analysis and study of progress and regress in efficiency analysis of the MRM model in the presence of stochastic data. Given the stochastic inputs and outputs, a new ERM model was developed under a weak disposability assumption. The new model assumes a normal distribution of inputs and outputs.

Also, a new stochastic version of MPI was introduced for an MRM model under weak disposability assumptions. The proposed approach was then applied in BDC for analyzing ten KDSs during 2014 and 2016. The results showed that the developed models can be implemented in the real world.

The prospective scholars can apply the developed models in other settings such as suppliers' assessment, customers' assessment, hospitals' assessment, etc. A similar method can be repeated in the presence of both stochastic data and fuzzy data. Possible extensions of the provided stochastic GMPI in the existence of skewed and truncated normally distributed data are another interesting research topics.

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Conflict of interest: None.

List of Figureures:

List of Tables:

Table 2. The dataset (2016)

Table 3. Comparison of stochastic GMPIs with different risk levels and deterministic GMPI

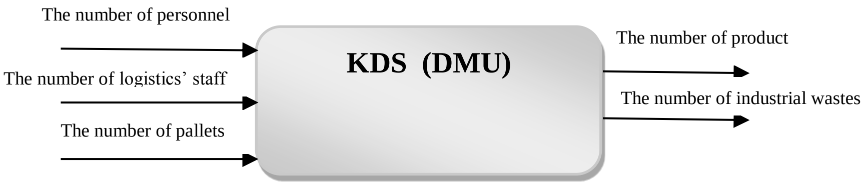

Figure 1. The inputs and outputs

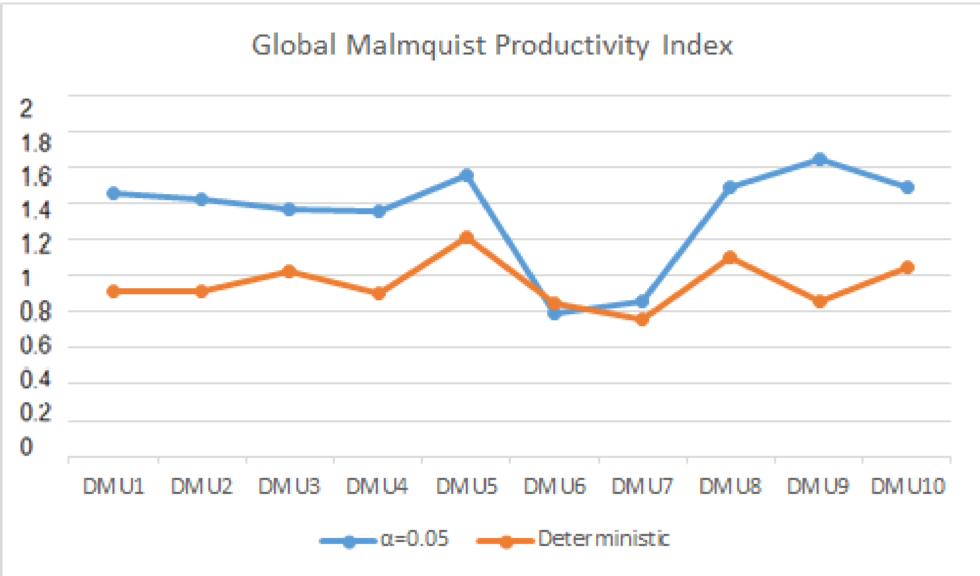

Figure 2. The deterministic and stochastic GMPI (α=0.05)

DMUs (KDSs)	Inputs			Good output	Bad output
	X_1	X_2	X_3	V_1	W_1
Trim line N75 series	20	8	42	14	8
Chassis line N75 series	18	4	38	24	10
F SERIES line	14	4	28	6	3
MB SERIES line	18	3	48	6	1
SHILLER6 line	14	4	28	7	2
Trim line N55 series	18	7	39	24	10
Chassis line N55 series	25	7	40	12	11
Trim line 77E series	18	7	40	25	6
Chassis line 77E series	20	7	42	24	7
SHILLER5 line	14	5	27	5	2

Table 1. The dataset (2014)

DMUs (KDSs)	Inputs			Good output	Bad output
	X_1	X_2	X_3	V_1	W_1
Trim line N75 series	15	2	25	21	5
Chassis line N75 series	16	2	16	28	6
F SERIES line	12	3	16	8	2
MB SERIES line	16	3	30	7	0
SHILLER6 line	14	3	27	8	1
Trim line N55 series	18	5	30	26	5
Chassis line N55 series	16	6	32	17	8

Trim line 77E series	18	6	30	26	4
Chassis line 77E series	16	6	31	26	6
SHILLER5 line	14	3	24	6	1

Table 2. The dataset (2016)

DMUs (KDSs)	Stochastic GMPI (α=0.001)	Stochastic GMPI (α=0.01)	Stochastic GMPI (α=0.05)	Stochastic GMPI (α=0.1)	Stochastic GMPI (α=0.3)	Stochastic GMPI (α=0.4)	Deterministic GMPI
Trim line N75 series	1.302	1.389	1.46	1.494	1.546	1.57	0.917
Chassis line N75 series	1.207	1.315	1.425	1.491	1.649	1.719	0.918
F SERIES line	1.083	1.224	1.373	1.466	1.744	1.811	1.021
MB SERIES line	1.745	1.756	1.357	Unbounded	1.31	1.576	0.904
SHILLER6 line	1.412	1.356	1.554	1.376	1.869	1.929	1.212
Trim line N55 series	0.785	0.792	0.798	0.801	0.806	0.808	0.853
Chassis line N55 series	0.858	0.86	0.862	0.863	0.864	0.865	0.764
Trim line 77E series	1.295	1.393	1.494	1.555	1.701	1.761	1.100
Chassis line 77E series	1.457	1.567	1.651	1.684	0.899	0.934	0.855
SHILLER5 line	1.353	1.456	1.495	1.638	0.842	0.849	1.044

Table 3. Comparison of stochastic GMPIs with different risk levels and deterministic GMPI

Technical biography of authors:

Seyedeh Sara Hosseini is an assistant professor in the Department of Mathematics, Qazvin Branch, Islamic Azad University, Qazvin, Iran. Her research interests include mathematical modelling, optimization, operations research, and data envelopment analysis.

Reza Farzipoor Saen has published over 203 refereed papers in many prestigious journals. His h indices in Google Scholar and Scopus are 40 and 36, respectively. He has 24 years of industrial and consultation experience. Reza is on the list of top 2% of scientists in the world, ranked by Stanford University (2020; 2021).

Reza Kazemi Matin is a faculty member in the the Department of Mathematics, Karaj Branch Islamic Azad University, Karaj, Iran. His research interests include supply chain management, optimization, metaheuristic algorithms, operations research, and data envelopment analysis.