

**Experimental and numerical probe into the effects of adding one and two steps to a mono-hull planing vessel on its performance in calm water**

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## Abstract

In the current study, two different vessels with a single step and two steps are experimentally and numerically studied. The experimental tests are conducted at 7.5, 8.025, 8.5, and 9.5 m/s, i.e., at beam Froude numbers from 3.22 to 4.09. The measured parameters include bow rise-up, trim angle, and resistance. After validating the numerical set-up against the experimental measurements, simulations of fluid flow around the vessel at 10 m/s and 12 m/s speeds are conducted using STAR-CCM+ software. Two-phase flow is analyzed using the finite volume method and the volume of fluid technique with Overset meshing scheme. Based on the experimental results, the addition of the transverse step enhances the vessel's stability and reduces its trim. It is also concluded that the single stepped high-speed vessel's resistance reduces, compared to a vessel of without step, and both single-step and two-step models are stable at speeds up to 12 m/s. Finally, using Taguchi test design method, the number of numerical modeling is extracted and the interaction of the three parameters of the first and second step length and speed at two levels is investigated. Based on the findings, as the length of the second step increases, the resistance increases, as well.

**Keywords:** Experimental test; Numerical study, High-speed planing vessel; Longitudinal stability; Transverse step; STAR-CCM+.

## Nomenclature

B	Beam (m)
L	Length (m)
LCG	Longitudinal center of gravity
m	Mass (kg)
$\beta$	Deadrise angle (deg)
$\Delta$	Weight (N)
$Fr_B$	Beam Froude number
$\tau_s$	Static trim angle (deg)
Z	Rise-up
$R_T$	Total Resistance
CG	Center of gravity
LOA	Overall length of the vessel
$L_s$	Distance from the step to the transom stern
$C_\Delta$	Weight factor
$C_L$	Lift coefficient
v	Speed of models
$C_v$	Speed coefficient
Lc	Wetted Chine
Lk	Wetted keel

## 1. Introduction

Speed increase and progress in seakeeping capability have always been some of the main goals of naval architects in designing different vessels. Achieving these goals through development of displacement vessels poses serious challenges. This is due to the fact that under any circumstances, from the viewpoint of design, there will be a peak or sharp slope in the resistance-speed plot. Therefore, reaching and passing this peak in order to increase the design speed, necessitate a huge cost that will make the accomplishment of these goals completely non-economical. To overcome this difficulty, the concept of planing vessels has been proposed. A great deal of attention has been paid to this type of vessel with various approaches with are described in the following.

### **1.1 Analytical and semi-empirical studies**

In 1920 decade, first of many researches related to the planing hulls was initiated by von Karman [1]. Since analytical method can furnish suitable information and easy to achieve about the behavior of these vessels, mathematical modelling has been considered as one of main research branch in the field of planing hulls that has been adopted by many researchers. Martin [2] produced a theoretical method to predict the responses of planing hull in the presence of waves. However, his method only considers the linearized response characteristics of constant deadrise hulls. Zarnick [3] also engaged in mathematical modeling of planing vessel's motions. This work focused on vessel's motion in regular waves. Later, Sebastiani et al. [4], by adopting a 2D approach, determined the heave, pitch, and roll motions of a high-speed craft considering the momentum and wedge theories. Investigation of heave, pitch, and roll motions in regular waves was also conducted by Ghadimi *et al.* [5]. They pursued the areas that have not received much attention by other researchers. Ghadimi *et al.* [6] studied the generated spray by the motion of planing hulls and developed a computer program for predicting its dynamic behavior. Ghadimi *et al.* [7] also utilized the pressure distribution and calculated the resistance of a prismatic planing hull in calm water, analytically. Besides, Ghadimi et al. [8] proposed a mathematical model to find the roll motion of warped planing hulls. This model, which was validated by empirical data, could consider the water entry in asymmetric condition. Tavakoli et al. [9] carried out a study to find the planing hull's roll motion coefficients via 2D+t and potential theories. The method of 2D+t, as an efficient solution to predict the performance of high-speed craft, was also employed by Hasse et al. [10]. They produced a model using 2D+t and the boundary element method to study the prismatic hull in head waves. Afterwards, the behavior of the heeled planing hulls in clam water was analyzed by Ghadimi *et al.* [11] using 2D+t theory. They calculated the trim angle, rise-up at center of gravity, and resistance by integrating the pressure distribution on the wedge sections, which were fixed at heel angle and free to rise and trim. Ghadimi et al. [12] also developed a method based on the 2d+t theory to predict the hard-chine vessels in semi-planing and planing regimes, considering three resistance types of frictional, induced and spray.

### **1.2 Numerical and experimental studies**

Alongside these analytical efforts, due to the arrival of advanced computer processors, computational fluid dynamic (CFD) methods are nowadays used as a powerful application

tool for investigating the behavior of vessels under different geometries and conditions. Kazemi and Salari [13] took a great deal of effort to determine how the loading condition and weight distribution can affect a planing hull's performance, employing a finite volume method (FVM). As the meshing of the domain around the vessel with different degrees of freedom is always a challenging task for naval architects, Sukas et al. [14] conducted a study to evaluate the overset technique to be used in this specific sector. Recently, Judgea et al. [15] considered a deep V planing hull and performed some experimental and numerical investigations and gained some detailed knowledge. Whereas the finite volume method in CFD has highly attracted the designers in planing-hull sectors, it's not the only computational method. Ghassemi and Kohansal [16] showed that BEM method could also be effectively exploited to find the vessels performance and even their effect on the free surface. However, the high dependence of hydrodynamics to empirical measurement encouraged the researchers to conduct empirical tests alongside numerical studies. Ghadimi et al. [17] investigated the wedge effect on the calm water performance of planing vessels with two different wedge heights. On the other hand, Sajedi et al. [18] considered a planing hull via numerical methods and found the effect of a mounted transom wedge. Their numerical results displayed good agreement with their empirical measurements, which were conducted in calm water.

### **1.3 Stepped hulls**

On the other hand, planing hulls may display undesirable behavior in some particular conditions from the standpoint of resistance. For example, due to wetted chine at lower speeds, the planing hulls may exhibit larger resistance and lower lift to drag ratio than the displacement vessels at these speeds [19]. Among the adopted solutions for overcoming this challenge, creating a step at the bottom of the vessel is one of the most practical methods. Creating such cuts would lead to the division of the body into two segments of aft body and fore body. Through separation of water from the fore body and its reattachment with the aft body [20], the wetted surface of the vessel reduces which causes a decrease in the viscous friction. On the other hand, the higher pressure due the reattachment of water with the aft body yields an increase in the lift to drag ratio. Meanwhile, according to Savitsky and Morabito [21], aft body produces only 10% of the necessary lift and the remaining lift is generated by the fore body. However, since this 10% lift must be generated in a relatively small region, downstream of the reattachment point, the shape, height, and step location are critically important [22]. It is noteworthy that these geometrical characteristics, through a change in the way spray root is attached to the fore body, can change the performance of the stepped vessels [23]. It should be noted that longitudinal lift distribution in stepped planing hulls increases its longitudinal stability, since this lift is divided between two or three bodies (depending on the usage of one or two step(s)) and is not concentrated at a specific region. Another noticeable point about these vessels' performance is the fact that due to water separation from the body in the step location, a particular condition prevails from the viewpoint of air injection that if recognized and used in appropriate way, this capability of the step(s) can enhance the performance of the non-stepped planing [24-25]. Since there are several influential parameters involved in prediction of the behavior of the stepped planing hulls, various research has been conducted to ascertain their effects. Clement and Koelbel [26] presented a summary of up-to-date research conducted on stepped planing hulls until

1993. Indeed, some findings which were essentially carried out for the non-stepped vessels have also been employed to study the behavior of stepped planing hulls. Using the method proposed by Savitsky [27], different mathematical models have been presented to predict the behavior of the single-stepped vessels in different conditions [28-29].

#### **1.4 Experimental studies on stepped hulls**

Due to the importance of empirical data in hydrodynamics, several experimental studies have been carried out on stepped planing hulls. Najafi et al. [30] evaluated single-step planing hulls' performance in terms of drag, trim angle, and rise-up and measured the wetted surface areas behind the vessel. Sajedi et al. [31] compared a single-step model with a non-step model at Froude numbers up to 3.87. They found that a step can remove the instability at high speeds. Recently, a stepped planing hull was also compared with wedge mounted hulls by Sajedi and Ghadimi [32] in calm water and the presence of irregular waves. Their measurements showed that both step and wedge could have benefits in some conditions. More recently, Sajedi et al. [33] also considered the simultaneous transverse step and transom wedge effects on planing hulls. They reported that combined configuration could lead to the least trim angles. The wetted surface area of the stepped hulls as one of the primary factors affecting the total resistance of planing hulls was studied by Najafi et al. [34]. They found the wetted surface of the forebody and aftbody of stepped hulls and compared those with the total wetted surface of the same hull without transverse step. Najafi et al. [35] also extended their study to two-stepped hulls, and determined the effects of the fore and the aft step geometric features on the reattachment lengths and consequently, the wetted surface area of the hull.

#### **1.5 Numerical studies on stepped planing hulls**

Employing CFD techniques to study planing hulls, De Marco et al. [36] demonstrated the flow pattern beyond the transverse step for the first time. In addition, Najafi and Nowruzi [37], by comparing five step shapes, studied the hydrodynamic performance of Fridsma planing hull in terms of resistance, trim angle, rise-up, and lift to drag ratio. Due to the presence of degrees of freedom in the planing hull simulation, the moving hull presents a challenge in gridding the computational domain. In this regard, Doustdar and Kazemi [38] compared the fixed and moving mesh reliability in a stepped planing hull simulation. Sajedi and Ghadimi [39] found the optimum location of the transverse step by carrying out some tests in a towing tank and then simulated various step positions using CFD methods. The step hull angle variation was also considered by Afriantoni et al. [40], numerically. They calculated the most proper angle using stability analysis. As a result, numerical methods, with capability of considering the nonlinear motions of the vessels and complex nature of the environment, can perhaps challenge the development of analytical methods, which usually comprise considerable linearization of the flow. However, the validation criterion for any CFD analysis, is still the possibility of its comparison with available corresponding experimental data.

#### **1.6 Specific studies on two-stepped hulls**

Despite the possible advantages of two-stepped vessels like proper longitudinal stability due to pressure distribution of the bottom on three hulls, sporadic research has been conducted on these vessels as follows. Taunton *et al.* [41-42] conducted extensive experiments on various models of planing hulls including a single-stepped model and a two-stepped model. In these experiments, performance of the stated models was examined at different speeds in calm water from the viewpoints of resistance, dynamic sinkage, dynamic trim, and wetted surface area. Vitiello *et al.* [43] also examined the performance of two-stepped planing hulls from standpoints of effective ship power, total resistance, and trim angle. On the other hand, Lee *et al.* [44], through conducting experiment on a two-stepped vessel in stationary water under 7 different arrangements of front and back steps as well as three displacements at different speeds, compared their results with non-stepped high-speed vessels. They concluded that resistance related to the stepped vessels is less than that of non-stepped hulls. Nourghassemi *et al.* [45] employed RANS solver and simulated a two-stepped planing hull. They reported various findings such as resistance, trim, rise-up, and pressure distribution over the hull. Double stepped planing hulls were also undertaken by Ghadimi and Panahi [46] who defined the hydrodynamic forces and moments at various yaw angles and beam Froude numbers in calm water. On the contrary, Esfandiari *et al.* [47] compared double-stepped hull behavior with the non-stepped hull in rough water, using numerical methods. In order to gain the advantages of two-stepped hulls and also enhance their behaviors, Zou *et al.* [48] investigated the roll of stern flap on the performance of two-stepped planing hulls. More recently, Sajedi and Ghadimi [49] compared the stability and resistance of two-stepped and non-stepped hull in a towing tank and reported more longitudinal stability and less trim angle for the two-stepped hull. On the other hand, regarding the multi-hull vessels, Ma *et al.* [50] attempted to investigate planing trimaran in four conditions of no step, single step, two steps, and three steps. Kazemi *et al.* [51] have studied the cougar model in three configurations of without step, single-stepped, and two-stepped to realize how the vessel's resistance depends on the weight ratio. They also managed to investigate various parameters extensively, using artificial neural networks.

It is particularly evident that there is no sufficient experimental data about some new fields of interest related to planing crafts like two-stepped hulls. For example, Danielson and Stromquist [22] stated in their study of two-stepped vessels that, due to the lack of experimental data, the seakeeping analysis of the considered vessel was not feasible and they could only study the influence of variation of geometrical characteristics of step(s) like longitudinal position, height, and required power. On the other hand, application of planing hulls at high speeds has economical and design advantages. This is while with all the existence of extensive research in this field, behavior of these vessels at high speeds are not still easily predictable.

Close scrutiny of the presented literature indicates that the conducted research on the high-speed planing hulls has not yet yielded a sufficient understanding of their dynamic behavior and that advantages and restrictions of application of two-stepped vessels, compared to single-stepped and non-stepped vessels in each study, depends on the type of the vessel and other experimental conditions. This goes to show that not all the influential parameters on the behavior of these vessels are completely recognized or understood. Therefore, under current circumstances, it is imperative that researchers initially conduct different experimental tests

on the considered vessel to establish the necessary confidence, before adopting any of the two methods of analytical or numerical for investigating the behavior of high-speed planing hulls. This way, after comparison of the obtained results with the reported data of other studies, the possible influential parameters are identified and a good foundation is established for extensive future studies in the field of stepped vessels like those conducted on non-stepped planing hulls. One of the important features of this article is the model used, which has a chine and can be used at speeds above 10 m/s. Accordingly, in the current study, a particular model of planing hull is experimentally and numerically studied in two different conditions of single step, and two steps. Different physical parameters such as bow rise-up, transom and center of gravity (CG), trim, and resistance of the considered model are measured. The measured parameters are compared and analyzed at the speeds of 7.5, 8.025, 8.5 and 9.5 m/s to examine the influence of each of the considered geometries and physical parameters. Numerical simulations are also performed in STAR-CCM+ software, and the two-phase flow is solved using FVM and VOF techniques at 10 m/s and 12 m/s speeds to further analyze the position of steps and hydrodynamic performance of the examined hulls. To find suitable test cases in the numerical modeling, Taguchi test design method is utilized and the interaction of the three parameters of first and second step length and speed at two levels is investigated. The measured and calculated data would show how a hard-chine planing hull could take advantage of having one and two steps up to Froude number of 4.09. Having the experimental database for the performance of single and two stepped hard-chine vessels in calm water and a validated numerical method to predict the vessel's behavior with different step shapes could provide a basis for future studies on this type of vessel.

## 2. Physical description of the model

### 2.1. Problem statement

In the current paper, the vessel is set to be a planing monohull. A hard-chine vessel that has a V-shaped body. The vessel has a deadrise angle of  $\beta$  and beam of  $B$  (illustrated in Fig. 1). It has mass of  $M$  in kg. The mass is turned into non-dimensional form as in

$$C_M = \frac{M}{\rho B^3} \quad (1)$$

where  $\rho$  is the fluid density.

**Fig. 1.** A planing hull moving forward in steady condition.

The boat is supposed to reach planing speed that can be characterized by Froude Number.

$$Fr = \frac{u}{\sqrt{gL}} \quad (2)$$

Where  $u$  is the advanced moving of the boat. Froude Number smaller than 0.4 is considered as the displacement regime, while Froude Numbers between 0.4 to 1.0 is considered to be semi-planing condition, and Froude Numbers beyond 1.0 are recognized as the planing regime. The considered vessel is a 1:5 scale of the main vessel called cougar. This vessel has a constant deadrise of 24 degrees and it is made of fiberglass. Its length to width ratio is 4.7, and its weight is 86 kg. This hull is selected because it has a constant deadrise angle and experiences longitudinal instability. In order to avoid the incurring instability, a model test is constructed and examined with and without the step(s). The main vessel characteristics are presented in Table 1, while the bodyline of the tested model without step is displayed in Fig.2. Model A is without a step, Model B is a single-stepped, and Model C is a two-stepped hull.

**Table 1.** Principal characteristics of the investigated vessel [31].

**Fig.2.** Bodyline of a vessel without a step.

### 3. Experimental set up

Experimental tests are conducted by towing the vessel in the towing tank according to the recommended guidelines of ITTC related to the targeted tests on the resistance of high-speed vessels. These tests are conducted in National Iranian Marine Laboratory (NIMALA) which has 400 m length, 6 m width, and 4 m depth. National Iranian Marine Laboratory was established in 2012 with the approach to perform all designing-engineering tests for surface ship and submarines [31]. Maximum speed achieved in this towing tank is in the range of 19 m/s. The tested model is towed at the intersection of the CG with the direction of propulsion system which has 6 degrees angle from the base line. This model is free in two degrees of freedom, i.e. heave and pitch, and other components of motion are considered fixed. The measured parameters include drag and trim. A photograph of the model test in tank is illustrated in Fig. 3

**Fig. 3.** A photograph of model test at National Persian Gulf Towing Tank.

### 4. Tests conditions

Three different models are considered in the conducted tests. Two types of the model possess step; one with a single step and the other with two steps. Step location is selected differently based on the intended model test and full-scale vessels. Figure 4 shows the ratio of the length of the forward and after steps to the beam of several scaled and full-scale vessels. The length to beam ratio of the selected vessels is about 5.

**Fig. 4.** Ratio of the length of the forward and after step to the beam of several models.

As shown in Figure 4, the average ratio of the forward step length to the beam is between 1 and 1.5, and the ratio of the after-step length to the beam is between 0.5 and 1. Accordingly, the distance of the step of a single-stepped vessel from the transom is 770 mm, whereby the step length to beam ratio is 1.5 (30% of the vessel's length). On the other hand, for the two-



stepped planing hull, the distances of the forward and after steps from the transom are 15% and 30% of the vessel's length, respectively. The ratios of length of the after step and forward steps to the beam are 0.75 and 1.5, respectively. The step height for both stepped hulls is 4% of the beam, which is 0.55 m.

#### **4.1. Tests results and discussion**

The registered parameters for the single stepped hull are presented in Table 2. Based on the presented results, the single-step model is stable at these speeds. These tests are conducted at speeds of 7.5, 8.025, 8.5 and 9.5 m/s. The model trim decreases, but its rise-up increases. Figure 5 shows the movement of the single-step model at a speed of 8.025 m/s.

**Table 2.** result of model with single step.

**Fig.5.** Single stepped vessel's motion at 8.025 m/s.

Results of the last series of tests conducted on the double stepped hull are presented in Table 3. As shown in Table 3, results indicate that with addition of the second step to the vessel, longitudinal stability enhances and porpoising is avoided. These tests are conducted at speeds of 7.5, 8.025, 8.5 and 9.5 m/s.

**Table 3.** Result of double stepped vessel.

#### **4.2. Comparison of the Results of different models**

Comparison of the results obtained from the series of the conducted tests are presented in this section. Results present the influence of location and number of steps. Location of steps is at 30% and 15% of the transom distance from the vessel's length. Comparison of trim angles and rise-up for different models are presented in Figs.6 and 7, respectively.

**Fig.6.** Comparison of the trim for different models.

**Fig.7.** Comparison of the sinkage for different models.

As observed in Fig.6, with an addition of a step to the vessel, its trim reduces. The obtained results indicate that trim reduction for the double stepped hull is more than that of single-stepped vessel. Because of this reason, lift of the transom increases and as a result, its trim reduces. The computed rise-up at center of bow for the tested models are presented in Fig.7. From these plots, one may conclude that with addition of a step, the vessel rise-up reduces. This is a reasonable conclusion, since step addition causes a trim reduction and as a result of this, rise-up reduces. The computed resistance of different models is displayed in Fig.8.

**Fig.8.** Comparison of the resistance for different models.

Single-stepped vessel experiences the least resistance in the Planing regime, while double step planing hull exhibits the most resistance. This is mainly due to trim reduction which

leads to an increase in wetted surface and resistance. The most important characteristics of single-stepped vessel is its resistance reduction in planing regime and its increasing stability. Ventilation also occurs in double stepped hull which implies the existence of pressure pick point in the second body which accompanies the longitudinal stability for the vessel. One of the most important factors in choosing a step is getting the water to the chin line, which can be seen well in the first step. To better understand this, the reader is referred to Fig.9.

**Fig.9.** Flow (water) separation in the single stepped vessel: a) at 9.5 m/s and b) at 8.025 m/s.

At 9.5 m/s, flow separation becomes more apparent, but vessel's trim reduces, as shown in Fig9. Hence, vessel's bow penetrates the water and wetter surface increases. Flow separation also occurs at 8.025 m/s (Fig.9b), but the length of separation is less, and pressure pick point occurs earlier. Thus, trim of the vessel increases more than vessel at 9 m/flow separation in double stepped planing hull may be perceived as illustrated in Fig.10.

**Fig.10.** Flow (water) separation in the double-stepped vessel: (a) at low speed and (b) at a higher speed.

With an increase in velocity, trim of the vessel reduces up until a time at which appropriate ventilation occurs for the first step. This will cause a reduction in pressure resistance, but since the trim reduces, the frictional resistance increases. When the velocity increases over a particular limit, separation from the chine does not occur and water separates from the step. This will be accompanied by an increase in wetted surface and the third body becomes wet more at this speed than previous speed. Hence, with an increase in velocity, proper ventilation does not occur, and the resistance increases. According to Table 4, a comparison is made between 8 experimental works. Comparisons are made at a volume Froude number of 4.75.

**Table 4.** Experimental works.

The least amount of drag is related to Garland's work followed by the work done in this article. This comparison is illustrated in Fig. 11.

**Fig.11.** Comparison between experimental works.

### 4.3- Uncertainty

According to ITTC recommendation, uncertainties should be minimum. Drag uncertainty is calculated based on the total drag coefficient. The level of uncertainty for trim and resistance is computed based on the relations corresponding to the hydrodynamic aspect of the vessel. To compute the uncertainty, it is imperative to act on the governing relations recommended by ITTC. Calculation of uncertainty regarding the trim results, has been performed for the speed of 7.5, 8.5 and 9.5 m/s for one step model and two step model (Table 5). The results of the uncertainty calculations are based on a percentage of the trim. The Relations and value of uncertainty (as shown in Tables 6) depends on four parameters consisting of trim, sinkage, speed, and number of tests conducted.

**Table5.** Determination of trim uncertainty.

**Table 6.** The calculated drag and trim uncertainties (stepped model).

It is observed in Table 6 that the most uncertainty is related to the trim measurement.

## 5. Numerical studies

### 5.1. Governing Equations

The unsteady Navier-Stokes equations for a three-dimensional unsteady flow, using the Reynolds Averaged Navier-Stokes (RANS) method, include continuity and momentum equations, which are introduced in equations 3 and 4, respectively.

$$\frac{\partial U_i}{\partial x_i} = 0 \quad (3)$$

$$\frac{\partial U_i}{\partial t} + \frac{\partial (U_i U_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \nu \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right] - \frac{\partial \overline{u'_i u'_j}}{\partial x_j} \quad (4)$$

where  $U_i$  is the averaged velocity,  $x_i$  is the spatial coordinate,  $t$  is time,  $u'_i$  is the oscillating velocity,  $\rho$  is the fluid density,  $p$  is the averaged pressure, and  $\nu$  is the kinematic viscosity. On the other hand, the Reynolds stress tensor  $\overline{u'_i u'_j}$  could be defined according to Boussinesq approximation, as follows:

$$\overline{u'_i u'_j} = \nu_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) + \frac{2}{3} \delta_{ij} k \quad (5)$$

Where,  $\nu_t$  is the vortical viscosity. For fluid flow simulations, the k- $\omega$  Shear Stress Transport (SST) turbulence model is utilized, which is widely used in solving external flow problems. In this model, the features of two models, k-e and k-w, are used. This model predicts the free shear flow rate and can be a suitable model for free shear currents. The SST model is a model based on the equations of turbulent kinetic energy transfer  $k$  and turbulence frequency  $\omega$ . The kinetic energy of turbulent,  $k$  and frequency,  $\omega$  are obtained through the following transfer equations:

$$\frac{\partial}{\partial t} (\rho k) + \frac{\partial}{\partial x_i} (\rho k u_i) = \frac{\partial}{\partial x_j} \left( \Gamma_k \frac{\partial k}{\partial x_j} \right) + \tilde{G}_k - Y_k + S_k \quad (6)$$

$$\frac{\partial}{\partial t} (\rho \omega) + \frac{\partial}{\partial x_i} (\rho \omega u_i) = \frac{\partial}{\partial x_j} \left( \Gamma_\omega \frac{\partial \omega}{\partial x_j} \right) + G_\omega - Y_\omega + D_\omega + S_\omega \quad (7)$$

### 5.2. Free Surface Capturing

Free surface capturing, as one of the most challenging issues multiphase problems, needs some specific assumptions and calculations. The Volume of Fluid (VOF) model assumes that both phases are homogenous, only solving the governing equations once and presenting the amounts of pressure and velocity. However, using a volume fraction for the phases could indicate each phase and subsequently provides an accurate scheme of the free surface form.

### 5.3. Methods in CFD Simulation

To extend the results after the experimental modeling, numerical simulations are conducted. Numerical results are extracted at two speeds of 10 and 12 m/s. To properly estimate all unknown hydrodynamic parameters at each time step, RANS equations are solved in an implicit, unsteady, and iterative manner. The pressure-velocity coupling is based on the SIMPLE method, and the selected turbulence model is SST-K  $\omega$ . In the simulations, two degrees of freedom of heave and pitch are considered. Dynamic Fluid-Body Interaction (DFBI) model is used to consider these degrees of freedom, and the Overset dynamic mesh technique is selected to discretize the domain. The two-phase current is solved using the VOF model, which is based on tracking the free surface boundary. To reduce the computational cost, it is helpful to simulate only half of the model body, as shown in Fig.11. The reference coordinate axis is located on the center of gravity of the model. The positive direction of the x-axis is in the opposite direction of the water inlet velocity. The cubic computational domain dimensions, assuming that L is the length of the model, are selected as  $-7L < X < 1L$ ,  $-2.5L \leq Y \leq 0$ , and  $-2L \leq Z \leq 1L$ . Figure 12 shows the computational domain with mesh around the model.

**Fig. 12:** a) Boundary conditions, the stationary and moving computational domain, b) and c) the top and side views of the grid, including the boundary layer around the model.

One other point to note is the choice of time step ( $\Delta t$ ) according to Equation 8, which is expressed as a function of length ( $l$ ) and velocity ( $V$ ) based on the ITTC recommendation of 2011 [52]. Here, this length is assumed to be equal to the wetted length of the keel ( $L_k$ ) of the model. It should also be noted that the Courant number (CFL) remains below 1 as a function of the time step, speed, and minimum element length in the direction of fluid flow. Equation 9 defines the Courant number.

$$\Delta t = 0.01 \sim 0.005 \frac{l}{V} \quad (8)$$

$$c = \frac{V \times \Delta t}{\Delta x} \leq c_{max} \quad (9)$$

$\Delta x$  is the distance of the first cell, the smallest cell, from the body surface. Of course, the Courant number changes as the Froude number changes.

An important criterion to be considered in the mesh production within the boundary layer is  $Y^+$ , which refers to the dimensionless distance of the first node from the surface. According to ITTC recommendations, the value of  $Y^+$  can be up to 300. The average value of  $Y^+$  is about 40 at a speed of 9 m/s. Figure 13 shows the value of  $Y^+$  on the floor of the two-step and single-step model at 9 m/s.

**Fig.13.** The amount of  $Y^+$  in single-step and two-step models at 9 m/s speed.

### 5.4. Mesh study and uncertainty of grid and validation

The computed total resistance of the single-step models versus the number of cells at the speed of 9.5 m/s are presented in Table 7. As observed in this table, by increasing the number of cells to more than

1,400,000, the resistance change is small. Hence, 1,400,000 cells are appropriate to conduct the intended simulations.

**Table 7.** The accuracy of the drag, using different mesh.

Among different uncertainties of CFD, grid uncertainty is the most influential. The Grid Convergence Index (GCI) is calculated for three different mesh and presented in Table 7. In the present paper,  $R_K = \varepsilon_{21K}/\varepsilon_{32K}$  is in the range of  $0 < R_K < 1$ . In the present paper, the mesh modifying factors are  $r_{21} = \sqrt{2}$ , and  $r_{32} = \sqrt{2}$ . The value of GCI for single-step floating resistance at a speed of 9 m/s and different meshes in Table 6 is equal to 6.1%. Based on Table 8, validation is conducted against the experimental model. Based on the comparison between the experimental measurements and numerical results for single-step models displayed in Table 8, good agreement is achieved.

**Table 8.** Comparison of the experimental measurements and numerical data for single-step models.

### 5.5 Design of experimental (Taguchi technique)

To extend the results, number of required simulations is extracted using the Taguchi test design method. Three factors of speed, locations of the first, and the second steps in two levels are selected in Table 9 to design the test. The locations of the first and second steps are selected based on the percentage of the distance of the steps from the transom to the model's length. The results of trim, rise-up, and resistance are shown in Table 10.

**Table 9.** Variables and their levels of change.

**Table 10.** Simulation results.

The effect of each parameter is illustrated in Figure 14. This figure shows that by increasing the distance of the second step from the transom and increasing the speed, the resistance increases. However, the trim decreases with increasing speed. As the longitudinal distance of the second step from the transom increases, the trim decreases, and the wet surface is raised.

**Fig.14.** The effect of each parameter on the resistance.

The contour in Figure 15a shows that the length of the first step has little effect on the resistance and that the lines are almost vertical. However, Figure 15b shows that the resistance increases as the length of the second step increases.

**Fig.15.** The effect of length of the first step and second step on the resistance.

Figures 16 and 17 show the wetted surface area of the two single-step and the two-step vessels at a speed of 12 m/s.

**Fig.16.** The wetted surface area of the single-step model at 12 m/s speed.

**Fig.17.** The trim and the wetted surface area of the two-step model at 12 m/s speed.

It is observed that trim of two-step model trim is less than the single-step model. However, the two-step model experiences more resistance. The wetted surface of two single-step and two-step models are summarized in Table 11. The wetted surface of the second body

decreases with increasing speed. With the help of geometric relations, the amount of wetted surface can be calculated. The highest wetting level is related to the double step model.

**Table 11.** Wetted surface of models.

According to Figure 18, based on the underwater camera, the current separation starts at speeds higher than 4 m/s.

**Figure.18.** Flow separation at a) 4m/s, b) 6m/s speed.

Flow separation for four numerical models is shown in Figure 19. As evident in this figure, the separation keeps the middle body dry in models 1 and 2. The separation length in models 1 and 2 is about 20% of the model length and in models 3 and 4 is about 10% of the model length.

**Fig.19.** Flow separation in four numerical models

Figure 20 shows the velocity and pressure counters for the two models of single and double-step.

**Fig.20.** a) Single-step model pressure contour, b) Two-step model pressure contour, c) Single-step model velocity contour, d) Two-step model velocity contour.

## 6. Conclusions

In the current study, two different planing vessels of single-stepped, and two-stepped hull are experimentally and numerically investigated. The main goal of studying these models which have similar hull is to examine their longitudinal stability and performances. The tested models are only different in the number of steps considered. The targeted simulations are performed in STAR-CCM+ software, and the two-phase flow is analyzed using finite volume technique and volume of fluid scheme. The moving mesh with the Overest technique is used for discretization purposes. Using the Taguchi method, the appropriate number of tests is selected. The measured parameters at 7.5, 8.025, 8.5 and 9.5 m/s speeds in the conducted tests include the bow rise-up, the vessel's trim, and resistance. It is observed that by adding step(s) to the planing vessel, lift at the transom increases, due to the increasing pressure in the second hull. This subsequently yields a reduction in trim angle and an increase in the vessel's stability. Overall, based on the presented results, one may conclude that:

1. Among the stepped planing vessels, the model with two steps displays lower trim angle.
2. When the planing vessel gets equipped by a single step and double steps, the porpoising phenomenon is avoided and it becomes stable.
3. Based on the numerical studies at 10 m/s and 12 m/s speeds, as the second step moves away from the transom, the resistance increases, while trim decreases. It is observed that both single-step and two-step models are stable at speeds up to 12 m/s.
4. The middle bodies of models 1 and 2 remain dry due to the current separation, which makes them less resistant but can cause instability of the models at higher speeds.

5. One of the main reasons for the increase in pressure in the middle and end bodies is the separation of the current. Current separation occurs from 4 m/s speed.

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**Table 1.**

<i>Parameter</i>	<i>Value (m)</i>
L	2.64
LCG	0.79
VCG	0.19
B	0.563

**Table 2.**

$U$ (m/s)	$Fr_B$	$Z_l$ (m)	$\tau$ (deg)	$R_T/\Delta$
7.5	3.22	0.058	4.9	0.162
8.025	3.45	0.0625	4.1	0.167
8.5	3.65	0.068	3.7	0.177
9.5	4.09	0.078	2.8	0.2

**Table 3.**

$U$ (m/s)	$Fr_B$	$Z_l$ (m)	$\tau$ (deg)	$R_T/\Delta$
7.5	3.22	0.05	2.84	0.179
8.025	3.45	0.056	2.93	0.18
8.5	3.65	0.059	2.84	0.186
9.5	4.09	0.066	2.63	0.21

**Table 4.**

name	$\Delta$ (kg)	Length (m)	Number of step
Taunton et al [41]	24	2	1
De Marco et al [36]	30	0.9	1
Khazaei et al [13]	-	8	1
Kazemi et al [52]	84	2.64	1
Chooran et al [26]	1	7	1

Najafi et al [30]	2.5	48	1
Garland et al [40]	1.5	25.8	1

**Table 5.**

V m/s	One step		Two step	
	Drag %	Trim %	Drag %	Trim %
7.5	0.629	2.4	0.88	3.3
8.5	1.4	3.7	1.8	4.3
9.5	0.406	2.12	0.03	2.1

**Table 6.**

Definition	Governing Equations
Total Trim uncertainty	$(U_\tau)^2 = (B_\tau)^2 + (P_\tau)^2$
Total bias limit for trim % of $(U_\tau)^2$	$(B_\tau)^2 = (\theta_V)^2 + (\theta_{\Delta FP} B_{\Delta FP})^2 + (\theta_{\Delta AP} B_{\Delta AP})^2$
Total bias of $\Delta FP$ % of $\Delta FP$	$B_{\Delta FP}^2 = B_{\Delta FP1}^2 + B_{\Delta FP2}^2$
Calibration bias of $\Delta FP$ % of $(B_{\Delta FP})^2$	$B_{\Delta FP1} = \sqrt{Z_{inc}^2}$
Potentiometer misalignment bias of $\Delta FP$ % of $(B_{\Delta FP})^2$	$B_{\Delta FP2} = \Delta FP - \cos(\theta_M) \times \Delta FP$
Total bias of $\Delta AP$ % of $\Delta AP$	$B_{\Delta AP}^2 = B_{\Delta AP1}^2 + B_{\Delta AP2}^2$
Sensitivity coefficient for speed, V, for trim	$\theta_V = -4 \times g \times \frac{\Delta AP - \Delta FP}{V^3}$

**Table 7.**

Grid accumulation	Num. mesh $\times 10^6$	Numerical drag (R/ $\Delta$ )
coarse	1	0.180
medium	14	0.195
fine	20	0.197
Difference percentage	Coarse to medium	8.3%
	Medium to fine	1%

**Table 8.**

Model	Velocity (m/s)	Drag (R/ $\Delta$ )			Trim (degree)		
		EXP	NUM	ERROR%	EXP	NUM	ERROR%
Single step	8.5	0.17	0.175	2.9	3.7	4	8.1
Single step	9.5	0.2	0.195	2.5	2.8	3.2	14
Two step	8.5	0.186	0.18	3.2	2.84	3.3	16
Two step	9.5	0.21	0.201	4.5	2.63	2.9	10

**Table 9.**

speed	L1 step	L2 step
10	20	30
12	30	40

**Table 10.**

case	speed	L1-step	L2-step	Drag (N)	Trim(degree)	Rise-up(m)
1	10	20	30	142	4.5	80
2	10	30	40	151	4.1	77
3	12	20	40	185	1.1	70
4	12	30	30	177	2	68

**Table 11.**

Number of models	Fore body (m <sup>2</sup> )	Aft body1(m <sup>2</sup> )	Aft body 2 (m <sup>2</sup> )
1	0.59	0.0	0.15
2	0.6	0.0	0.1
3	0.62	0.1	0.1
4	0.66	0.19	-

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