Convection Heat Transfer and Flow Phenomena from a Rotating Sphere in Porous Media

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Abstract

The study of the flow and convective heat transfer from rotary sphere in fluid mechanics, astrophysics and astronaut subjects are important. Today, porous mediums use has become widespread because of the heat transfer characteristics as well as their lightweight and low volume. Many numerical studies in heat transfer and fluid mechanics have been done regarding rotary sphere. The present project studies the phenomena of flow and heat transfer due to the rotation of the sphere at a constant temperature around itself in a porous medium, assuming a laminar, steady and incompressible flow. Analytical solution of equations used are based on power series and the porosity coefficient is assumed between 0 to 1 in this problem. In the spherical coordinate system used here, changes in azimuthal angle direction are ignored and the body force and pressure gradient for the problem are considered zero. The presence of porous medium is expected to increase thermal parameters.

***Keywords**: Power series, Porous media, Rotating sphere, Flow phenomena, Heat transfer.*

Nomenclature

$A_1, B_1, a, b, c, d, e, F, G, H, M, s, s_1$	Constant
a_0	Radius of the sphere (m)

C_P	Specific heat capacity coefficient at constant pressure $(kJ/kg.K)$
$C_{P_{eff}}$	Effective specific heat capacity coefficient at constant pressure $(kJ/kg.K)$
$\bar{h}$	Average convection heat transfer coefficient $(W/m^2.K)$
K	Permeability coefficient (m^2)
k	Conductivity coefficient $(W/m.K)$
k_{eff}	Effective conductivity coefficient $(W/m.K)$
Nu	Nusselt number
$\overline{Nu}$	Average Nusselt number, $2\bar{h}r/k$
$\Pr$	Prandtl number, $\rho C_p \nu / k$
q'''	Heat generation per unit volume (W/m^3)
r	Radial coordinate (m)
Re	Reynolds number, $2ur/\nu$
T	Temperature $(^\circ C)$
t	Time (s)
T_∞	Temperature at infinity $(^\circ C)$
T_s	Temperature at sphere surface $(^\circ C)$
u	Velocity (m/s)

Z	Dimensionless variable
Greek symbols	
Ω	Angular velocity of sphere (rad/s)
δ	Dimensionless hydrodynamic boundary layer thickness
Δ_1	Dimensionless thermal boundary layer thickness
ε	Porosity coefficient
μ	Dynamic viscosity $(N.s/m^2)$
ν	kinematic viscosity (m^2/s)
θ	Acute angle subtended at the center of the sphere by any point and the nearest poleor latitude co-ordinate $(^\circ)$
ϕ	Meridian angle coordinate $(^\circ)$
ρ	Density (kg/m^3)
ρ_{eff}	Effective density of the fluid (kg/m^3)

1. Introduction

Porous mediums are used in a variety of industries, such as nuclear fuel flasks, optimal insulation of buildings, crude oil and heat recovery exchangers. Usage of the porous medium is one of the ways to improve its heat transfer performance [1]. It is also one of the examples of application of a cylinder or rotary sphere in mixers, which in some cases uses this rotation to increase the heat transfer rate.

Many studies have been conducted to measure the amount of natural and forced heat transfer rate around the sphere and other shapes by different researchers [2-5]. Kishore and Ramteke [6] numerically investigated the heat transfer phenomenon of spherical particles in Newtonian fluids with velocity slip and uniform thermal boundary condition at the fluid-solid interface using a computational fluid dynamics (CFD) based in-house solver. New results were obtained over the range of conditions as the Reynolds number, $\mathrm{Re} = 0.1 - 200$; the Prandtl number, $\mathrm{Pr} = 1 - 100$; and dimensionless slip parameter, $\lambda = 0.01 - 100$. The problem of flow and heat transfer between rotating spheres has been of interest to many researchers [7-10]. In a major study, Moghadam and Rahimi [7] numerically studied the heat transfer and flow between two concentric rotating spheres with time-dependent angular velocities. They reported that long delays in the heat transfer of a large portion of the fluid in the annulus is produced by spheres rotation. Also, they [8] investigated the same problem with constant angular velocities using similarity method and showed the temperature distribution, flow pattern and heat transfer characteristics. The unsteady free convection flow at large Grashof numbers from a differentially heated rotating sphere investigated by D'Alessio [11]. His analytical study in the form of an asymptotic expansion leaded to the heat transfer coefficient determining.

Flow and heat transfer in porous media have been numerically investigated by many researchers [12-14]. Smith et al. published a paper in which they described Other researchers, including Bijan, Ganapathy, Linan, and Kurdyumov [15], carried out studies regarding sphere in the porous medium in which the surface of the sphere was exposed to a constant heat flux. Merkin [16] studied natural heat transfer for two-dimensional and axial symmetry of objects with any desired shape in a porous medium of a saturated fluid. Cheng [17] conducted the natural and forced heat transfer studies around a horizontal cylinder and a sphere saturated with a fluid in a porous medium using similarity solution. Available work concerning such problems is considered by Sano and Okihara [18], Juncu [19], Gaffar et al. [20], Taherzadeh and Saidi [21], Pepona and Favier [22], Rao et al. [23] and Sano [24]. Chen et al. [25] numerically studied the mixed convection heat transfer from a rotating sphere within an enclosure. According to their results, the heat transfer coefficient increases by rotation. The laminar forced convection of a heated rotating sphere in air has been studied by Feng [26] using a three-dimensional immersed boundary based direct numerical simulation method. The flow structures and the mean Nusselt numbers for flow Reynolds number ranging from 0 to 1000 was obtained. They developed a new equation that correlates the mean Nusselt number of a heated rotating sphere for flows of $0 < \mathrm{Re} < 500$. Nigam [27] showed that by applying the analytic method of power series to the problem of a laminar flow due to the uniform rotation of a sphere, a solution can be found that corresponds almost to the physical condition of the problem in reality. After integrating and solving several systems of equations, he was able to calculate the velocities in spherical coordinates and the thickness of the hydrodynamic boundary layer. Singh [28] continued the work of Nigam by studying the heat transfer due to the laminar flow generated by the uniform rotation of the sphere and introduced different forms of power expansions for temperature and obtained temperature distribution and thickness of the thermal boundary layer. Later, Kreith et al. [29] demonstrated that experimental and analytical studies of the flow generated by uniform rotating of a sphere in the certain range of Reynolds, Grashof and Prandtl numbers are consistent with each other.

It is clear from the review of the previous works that the problem of analytical study of the forced convection and flow phenomena caused by a rotating sphere in a porous medium has not been investigated so far. In this paper, flow and heat transfer from a rotating sphere in porous media is analytically investigated. Firstly, the governing equations in the porous medium are simplified for the given problem with respect to the hypotheses of the problem and then solved by the analytic method of power series in a spherical geometry. It is proved that use of a porous medium will increase the thermal parameters such as Nusselt number relative to the rotating sphere in quiescent water.

2. Mathematical Formulation

The geometry discussed for the present problem is a rotating sphere with a constant radius a_0 that rotates at the angular velocity of Ω and uniformly around the axis shown in Fig. 1. The porous medium around this sphere is considered to be a sphere with a radius much larger than the sphere with radius a_0 $(r \succ\succ a_0)$.

Due to the geometry of the problem, spherical coordinates are used. Due to the lack of importance of velocity in r direction due to the small speed in this direction, in two directions, the momentum equations are solved only in θ and ϕ directions. Also, due to the symmetry of the considered geometry in ϕ direction, we consider the derivatives of this variable in all zero-governing equations. In this project, the flow is assumed laminar, steady and incompressible, and the motion of the sphere is considered to be uniform in its round. The surface temperature of the sphere and the ambient temperature are also assumed to be constant. The body force is assumed to be zero and the pressure gradient is considered zero due to minor changes in the boundary layer as well as symmetry for the considered problem. The Forchheimer equation is used for mathematical modeling of flow in porous media. Mathematic model can be developed with following assumptions: (a) the porous media are isotropic and homogeneous with no contraction or distension; (b) the local thermal equilibrium is considered between solid and liquid phases; (c) the generation of heat due to the viscous effects is negligible. Taking into account the assumption of a laminar, steady and incompressible flow, the symmetry of the sphere versus the direction ϕ, the constant of the radius of the sphere $(r = a_0)$, and the large radius, in order to remove the inequal-equivalence term with the rest of the terms in the solution of the equations, the negligibility of velocities in r due to the small thickness of the boundary layer, also due to the order of terms used in the momentum equation: $u_r \sim O(\delta)$ (due to the insignificance of the velocity in this direction), $u_\theta \sim O(1)$, $u_\phi \sim O(1)$ and $\frac{\partial}{\partial r} \sim O(\delta^{-1})$ (because of the sharp changes in this direction) assuming that the terms in the equation are all of order 1, the assumption of thermal equilibrium in a porous medium and the absence of q''' in the energy equation, the governing equations are:

$$\frac{\partial u_r}{\partial r} + \frac{1}{a_0}\frac{\partial u_\theta}{\partial \theta} + \frac{\cot\theta}{a_0}u_\theta = 0 \tag{1}$$

$$\frac{1}{\varepsilon^2}(u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{a_0}\frac{\partial u_\theta}{\partial\theta}-\frac{u_\phi{}^2}{a_0}\cot\theta)=\frac{\nu}{\varepsilon}(\frac{\partial^2 u_\theta}{\partial r^2})-\frac{\nu}{K}u_\theta \quad (2)$$

$$\frac{1}{\varepsilon^2}(u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\theta}{a_0}\frac{\partial u_\phi}{\partial\theta}-\frac{u_\theta u_\phi}{a_0}\cot\theta)=\frac{\nu}{\varepsilon}(\frac{\partial^2 u_\phi}{\partial r^2})-\frac{\nu}{K}u_\phi \quad (3)$$

$$(\rho C_P)_{eff}(u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{a_0}\frac{\partial T}{\partial\theta})=k_{eff}(\frac{\partial^2 T}{\partial r^2})+\mu[(\frac{\partial u_\theta}{\partial r})^2+(\frac{\partial u_\phi}{\partial r})^2]+\frac{\mu}{K}(u_r{}^2+u_\theta{}^2+u_\phi{}^2) \quad (4)$$

The boundary conditions are as follows:

$$r=0 \qquad T=T_s\ , \qquad u_r=0\ , \qquad u_\theta=0\ , \qquad u_\phi=a_0\Omega\sin\theta \quad (1)$$

$$r\to\infty \qquad T=T_\infty\ , \qquad u_r=0\ , \qquad u_\theta=0\ , \qquad u_\phi=0 \quad (2)$$

In which, u_r , u_θ, and u_ϕ are velocity components in directions r, θ, and ϕ, respectively, a_0 radius of the sphere , ε porosity coefficient, ν kinematic viscosity, K permeability coefficient, ρ fluid density, C_P specific heat capacity at constant pressure, T temperature, k thermal conductivity coefficient, μ dynamic viscosity and eff subscript related to effective properties.

The non-dimensional parameters for present problem are defined as follows:

$$\eta=\frac{T-T_\infty}{T_s-T_\infty}, \qquad r^*=\frac{r}{a_0}, \qquad u_r^*=\frac{u_r}{4a_0\Omega}, \qquad u_\theta^*=\frac{u_\theta}{4a_0\Omega}, \quad (7)$$

$$u_\phi^*=\frac{u_\phi}{4a_0\Omega}, \qquad \Pr=\frac{\nu}{\alpha}, \qquad \mathrm{Re}=\frac{4a_0^2\Omega}{\nu}, \qquad Da=\frac{K}{a_0^2}, \qquad Eck=\frac{(4a_0\Omega)^2}{C_{P_{eff}}(T_s-T_\infty)}$$

The non-dimensional boundary conditions are as follows:

$$r^* = 0 \qquad \eta = 1 \qquad u_r^* = 0, \qquad u_\theta^* = 0, \qquad u_\phi^* = \frac{\sin\theta}{4} \tag{8}$$

$$r^* \to \infty \qquad \eta = 0 \qquad u_r^* = 0, \qquad u_\theta^* = 0, \qquad u_\phi^* = 0 \tag{9}$$

The governing equations are as follows:

$$\frac{\partial u_r^*}{\partial r^*} + \frac{\partial u_\theta^*}{\partial \theta} + u_\theta^* \cot\theta = 0 \tag{10}$$

$$\frac{1}{\varepsilon^2}(u_r^* \frac{\partial u_\theta^*}{\partial r^*} + u_\theta^* \frac{\partial u_\theta^*}{\partial \theta} - u_\phi^{*2} \cot\theta) = \frac{\mathrm{Re}}{\varepsilon}(\frac{\partial^2 u_\theta^*}{\partial r^{*2}}) - \frac{1}{Da\,\mathrm{Re}} u_\theta^* \tag{11}$$

$$\frac{1}{\varepsilon^2}(u_r^* \frac{\partial u_\phi^*}{\partial r^*} + u_\theta^* \frac{\partial u_\phi^*}{\partial \theta} - u_\theta^* u_\phi^* \cot\theta) = \frac{\mathrm{Re}}{\varepsilon}(\frac{\partial^2 u_\phi^*}{\partial r^{*2}}) - \frac{1}{Da\,\mathrm{Re}} u_\phi^* \tag{12}$$

$$u_r^* \frac{\partial \eta}{\partial r^*} + u_\theta^* \frac{\partial \eta}{\partial \theta} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}(\frac{\partial^2 \eta}{\partial r^{*2}}) + \frac{Eck}{\mathrm{Re}}[(\frac{\partial u_\theta^*}{\partial r^*})^2 + (\frac{\partial u_\phi^*}{\partial r^*})^2] + \frac{Eck}{\mathrm{Re}\,Da}(u_r^{*2} + u_\theta^{*2} + u_\phi^{*2}) \tag{13}$$

3. Problem Solving Using Power Series Method

The power series expansions for velocities in spherical coordinates according to the Nigam method [27] are as follows:

$$u_r = \frac{1}{2}(\nu\,\Omega)^{0.5}(2 - 3\sin^2\theta)(H_1 + H_3\sin^2\theta + H_5\sin^4\theta + ...) \tag{14}$$

$$u_\theta = a_0 \Omega \cos\theta (F_1 \sin\theta + F_3 \sin^3\theta + F_5 \sin^5\theta + ...) \tag{15}$$

$$u_\phi = a_0 \Omega \sin\theta (G_1 + G_3 \sin^2\theta + G_5 \sin^4\theta + ...) \tag{16}$$

All constants in relations (14) to (16) are expressed in terms of a dimensionless variable called z. This means we have:

$$H = H(z), F = F(z), G = G(z), z = \left(\frac{\Omega}{\nu}\right)^{0.5}(r - a_0) \tag{17}$$

By substituting the velocities above in governing equations and collecting powers of sine terms and after simplification we have:

$$F^2{}_1 - G^2{}_1 + H_1 F_1' = \varepsilon F_1'' - \left(\frac{\nu\varepsilon^2}{K\Omega}\right) F_1 \tag{18}$$

$$4F_1F_3 - 2F^2{}_1 - 2G_1G_3 - 1.5H_1F_1' + H_3F_1' + H_1F_3' = \varepsilon F_3'' - \left(\frac{\nu\varepsilon^2}{K\Omega}\right) F_3 \tag{19}$$

$$-6F_1F_3 + 6F_1F_5 + 3F^2{}_3 - 2G_1G_5 - G^2{}_3 - 1.5H_3F_1' + H_5F_1' - 1.5H_1F_3' + H_3F_3' + H_1F_5' = \varepsilon F_5'' - \left(\frac{\nu\varepsilon^2}{K\Omega}\right) F_5 \tag{20}$$

$$2F_1G_1 + H_1G_1' = \varepsilon G_1'' - \left(\frac{\nu\varepsilon^2}{K\Omega}\right) G_1 \tag{21}$$

$$-2F_1G_1 + 2F_3G_1 + 4F_1G_3 - 1.5H_1G_1' + H_3G_1' + H_1G_3' = \varepsilon G_3'' - \left(\frac{\nu\varepsilon^2}{K\Omega}\right) G_3 \tag{22}$$

$$-2F_3G_1 + 2F_5G_1 - 4F_1G_3 + 4F_3G_3 + 6F_1G_5 - 1.5H_3G_1' + H_5G_1' - 1.5H_1G_3' + H_3G_3' + H_1G_5' = \varepsilon G_5'' - \left(\frac{\nu\varepsilon^2}{K\Omega}\right) G_5 \tag{23}$$

For the energy equation, in addition to the velocity expansions, we consider the following expansion for temperature in accordance with Singh's method [28]:

$$C_{P_{eff}} T = C_{P_{eff}} T_\infty + a_0^2 \Omega^2 (M_1 + M_3 \sin^2\theta + M_5 \sin^4\theta + \ldots), M = M(z) \tag{24}$$

By inserting the velocities given in relations (14) to (16) and temperature expansion in relation (24) in Eq. (4) and taking into account relations (17) and (24), and after simplifying and performing algebraic operations, and finally equating the sentences with equal power of $\sin\theta$, on the other side of the equation, we get the following relations to the energy equation (for the aggravation of relations, the constants are not written by the z variable):

$$H_1 M_1' = \frac{M_1''}{\Pr} \tag{25}$$

$$H_1 M_3' + H_3 M_1' - 1.5 H_1 M_1' + 2F_1 M_3 = \left(F_1'\right)^2 + \left(G_1'\right)^2 + \left(\frac{\nu}{K\Omega}\right)\left(F_1^2 + G_1^2\right) + \frac{M_3''}{\Pr} \tag{26}$$

$$\begin{aligned} &H_1 M_5' + H_3 M_3' + H_5 M_1' - 1.5 H_1 M_3' - 1.5 H_3 M_1' + 4F_1 M_5 - 2F_1 M_3 + 2F_3 M_3 = 2G_1' G_3' + 2F_1' F_3' \\ &-\left(F_1'\right)^2 + \left(\frac{\nu}{K\Omega}\right)\left(2F_1 F_3 - F_1^2 + 2G_1 G_3\right) + \frac{M_5''}{\Pr} \end{aligned} \tag{27}$$

The relations (25) to (27) of the three differential equations obtained by applying the power series expansion on the energy equation.

The boundary conditions governing the problem are expressed in terms of the constants in the expansions of the velocity as follows:

at $z = 0$: $\quad F_1 = F_3 = F_5 = 0 \qquad G_1 = 1, G_3 = G_5 = 0 \qquad H_1 = H_3 = H_5 = 0$ (28)

at $z \to \infty$: $\quad F_1 \to 0, F_3 \to 0, F_5 \to 0 \qquad G_1 \to 0, G_3 \to 0, G_5 \to 0$ (29)

In order to keep the hydrodynamic boundary layer flow as a constant on the surface of the sphere, the following boundary conditions must also exist:

at $z \to \infty$: $\quad F_1' \to 0, F_3' \to 0, F_5' \to 0 \qquad G_1' \to 0, G_3' \to 0, G_5' \to 0$ (30)

Also, the boundary conditions governing the problem are expressed in terms of the constants in the expansions of temperature as follows:

at $z = 0$:
$$M_1 = \frac{C_{P_{eff}}(T_s - T_\infty)}{{a_0}^2 \Omega^2}, M_3 = M_5 = 0 \tag{31}$$

at $z \to \infty$:
$$M_1 \to 0, M_3 \to 0, M_5 \to 0 \tag{32}$$

In order to keep the transfer phenomenon in the thermal boundary layer on the surface of the sphere, the following boundary conditions must also exist:

$$M_1' \to 0, M_3' \to 0, M_5' \to 0 \tag{33}$$

The differential equations obtained in relations (18) to (23) and (25) to (27), along with the boundary conditions introduced in relations (28) and (31) are the equations and boundary conditions that we use for the constants used in differential equations to solve these equations from the following relations proposed by Nigam [27]:

$$F_1 = as(1-s)^2(1+2s) - \frac{1}{2}\delta^2 s^2 (1-s)^2, F_3 = bs(1-s)^2(1+2s), F_5 = ds(1-s)^2(1+2s) \tag{34}$$

$$G_1 = 0.5(2+s)(1-s)^2, G_3 = cs(1+2s)(1-s)^2, G_5 = es(1+2s)(1-s)^2 \tag{35}$$

$$\begin{aligned} M_1 &= \frac{\left(C_{P_{eff}}(T_s - T_\infty)\right)(2+s_1)(1-s_1)^2}{2{a_0}^2\Omega^2}, M_3 = A_1 s_1(1+2s_1)(1-s_1)^2 - \frac{1}{2}(0.5835\cdot \mathrm{Pr})\Delta_1^2 s_1^2(1-s_1)^2 \\ &, M_5 = B_1 s_1(1+2s_1)(1-s_1)^2 + \frac{1}{2}(0.3136\cdot \mathrm{Pr})\Delta_1^2 s_1^2(1-s_1)^2 \end{aligned} \tag{36}$$

In these relations we have for the dimensionless parameters s and s_1:

$$z = s\delta = s_1 \Delta_1 \tag{37}$$

In this relation, δ and Δ_1 represent the dimensionless thickness of the hydrodynamic and thermal boundary layer, respectively.

In order to solve the twelve differential equations, we first need to insert $(\Omega, \nu, k, \varepsilon, \mathrm{Pr})$ values per solving time according to the problem conditions into these equations. These values are shown in Table 1.

All of these values, except Pr, calculated for the porous medium, were at the $\mathrm{Re} = 1000$ for pure liquid water, and the porosity and permeability values were also selected for admission of spherical balls of wood.

To facilitate the continuation of operations in all relationships (34) to (36), we put:

$$s = \frac{z}{\delta}, s_1 = \frac{z}{\Delta_1} \tag{38}$$

Given that the constants in the differential equations derived from the continuity can be obtained in terms of other constants, we reduce the equations to 9 of the equation. The resulting relations for H_1, H_3, H_5 are as follows:

$$\begin{aligned} H_1 &= \frac{z^3}{3} - \frac{4az^5}{5\delta^4} + \frac{3az^4}{2\delta^3} + \frac{z^5}{5\delta^2} - \frac{az^2}{\delta} - \frac{z^4}{2\delta}, H_3 = -\frac{8bz^5}{5\delta^4} + \frac{3bz^4}{\delta^3} - \frac{2bz^2}{\delta} \\ , H_5 &= -\frac{2bz^5}{5\delta^4} - \frac{12dz^5}{5\delta^4} + \frac{3bz^4}{4\delta^3} + \frac{9dz^4}{2\delta^3} - \frac{bz^2}{2\delta} - \frac{3dz^2}{\delta} \end{aligned} \tag{39}$$

Now, by substituting the relations (34) through (36) into the remaining 9 equations, we transform the differential equations into algebraic equations. Of course, at this stage, we must put the $(a_0, C_{P_{eff}}, T_s, T_\infty)$ values in relation (36), the values of which are based on the boundary conditions and the geometry of the problem, as well as the fluid used, namely, water. These values are shown in Table 2.

After integrating, the calculation results of which are shown in Table 3.

4. Results Discussions

4.1. Validation of the results

It is clear from the review of the previous works that the problem of analytical study of the forced convection and flow phenomena caused by a rotating sphere in a porous medium has not been investigated so far. Therefore, in order to validate the results, a comparison between the analytical power series solution carried out by Singh [28] and the results of the present project has been made. In Figs. 2 and 3, we can compare the validity of the temperature distribution around a quadrant of a sphere at different intervals from the surface of the sphere. These graphs indicate the accuracy and precision of the power series method on this problem.

4.2. Velocity Distribution

In Fig. 4, the velocity changes in r directions is plotted for different values of s, which represents the radial distance from the surface of the sphere. According to Fig. 4 and expansion of the power series considered in Section 3, u_r is negative when $(2-3\sin^2\theta)>0$, and positive when $(2-3\sin^2\theta)<0$, and is zero when $(2-3\sin^2\theta)=0$ meaning that $\theta=54.45^\circ$ in the upper hemisphere and $\theta=125.15^\circ$ in the lower hemisphere. In Fig. 5, both of variables are non-dimensional. Changes in the dimensionless hydrodynamic boundary layer thickness in terms of Reynolds number are shown in Fig. 5. As can be seen, with the increase of the Reynolds number due to the vortex penetration, or, in other words, the penetration of the velocity, the thickness of the boundary layer increases.

4.3. Temperature Distribution

In Fig. 6, temperature variations are plotted in θ at various s_1, which represent a radial distance from the surface of the sphere. That means, in this figure, temperature change shown in relation to θ and radius, and s_1 represents the radius changes within the thermal boundary layer. As it can be seen, the temperature approaches the ambient temperature at the boundary of the thermal boundary layer and outside of it. In Fig.

7, the temperature distribution in the boundary layer for the quadrant is given, which in fact is equivalent to placing the series of power expansions for temperature with a constant number and showing homogeneous regions.

4.4. Mean Nusselt Number

According to Kreith et al. [29] the final relation for calculating the average Nusselt number on the surface of the sphere is as follows:

$$\overline{Nu} = -\frac{2a_0}{(T_s - T_\infty)} \frac{a_0^2 \Omega^{2.5}}{\nu^{0.5} C_{P_{eff}}} [M_1'(0) + \frac{2}{3} M_3'(0) + \frac{8}{15} M_5'(0) + \cdots] \tag{40}$$

In Fig. 8, the variation of the mean Nusselt number is compared to the Reynolds number for rotating sphere in the porous media and quiescent water. As it turns out, the presence of a porous medium causes the mean Nusselt number to be higher than ones in the quiescent water in the constant Reynolds number. Also, with the increase of the Reynolds number, the mean Nusselt number increases, which is due to the increase of the gradient of the average temperature inside the boundary layer due to the reduction of the thickness of the thermal boundary layer. This phenomenon is due to the approach of the velocity profile behavior to the plug flow velocity profile. In addition, the porous media itself is a thermal bridge. Also, in a porous medium, the Nusselt number increases due to the increase in the effective heat transfer surface.

5. Conclusions

In this study, heat transfer and flow phenomena caused by a rotary sphere in porous media has been investigated. Equations governing the problem including consistency equations, momentum and energy have been obtained using available models for a laminar flow. Finally, after considering the expansions of the power series for velocities and temperature, the equations have been solved by using this analytical method.

Some of the results are as follows:

- As the Reynolds number increases, the velocity and vortex diffusion increase in the boundary layer, and as a result, the thickness of the boundary layer increases.
- With the increase of the Reynolds number as the velocity gradient increases in the boundary layer, the temperature gradients also increase and, given that the overall shape of the temperature distribution is constant, the thickness of the thermal boundary layer decreases, which results the mean Nusselt number to increase on the surface of the sphere.
- The presence of a porous medium causes the mean Nusselt number to be higher than the ones in the quiescent water in the constant Reynolds number.
- The effect of the rotation of the sphere on the velocity and temperature distribution is only up to the boundary of the boundary layer and is governed by the ambience outside the boundary layer.

References

[1] Nield, D. A. and Bejan, A. "Convection in porous media", 4th Edn., pp. 1-30, Springer, New York, (2006).

[2] Tetsu, F. and Haruo, U. "Laminar natural-convective heat transfer from the outer surface of a vertical cylinder", *International Journal of Heat and Mass Transfer,* 13(3), pp. 607-615 1970/03/01/ (1970).

[3] Nakayama, A. and Koyama, H. "Free Convective Heat Transfer Over a Nonisothermal Body of Arbitrary Shape Embedded in a Fluid-Saturated Porous Medium", *Journal of Heat Transfer,* 109(1), pp. 125-130 (1987).

[4] Yan, B., Pop, I., and Ingham, D. B. "A numerical study of unsteady free convection from a sphere in a porous medium", *International Journal of Heat and Mass Transfer,* 40(4), pp. 893-903 1997/03/01/ (1997).

[5] Feng, Z.-G. and Michaelides, E. E. "A numerical study on the transient heat transfer from a sphere at high Reynolds and Peclet numbers", *International Journal of Heat and Mass Transfer,* 43(2), pp. 219-229 2000/01/01/ (2000).

[6] Kishore, N. and Ramteke, R. R. "Forced convective heat transfer from spheres to Newtonian fluids in steady axisymmetric flow regime with velocity slip at fluid–solid interface", *International Journal of Thermal Sciences*, 105(pp. 206-217 (2016).

[7] Moghadam, J. A. and Rahimi, B. A. "A numerical study of flow and heat transfer between two concentric rotating spheres with time-dependent angular velocities", *Scientia Iranica,* 16(3), pp. 197-211 (2009).

[8] Moghadam, J. A. and Rahimi, B. A. "Similarity Solution in the study of flow and heat transfer between two rotating spheres with constant angular velocities", *Scientia Iranica,* 16(4), pp. 1-11 (2009).

[9] Mannix, P. M. and Mestel, A. J. "Bistability and hysteresis of axisymmetric thermal convection between differentially rotating spheres", *Journal of Fluid Mechanics,* 911(p. A12 (2021), Art. no. A12.

[10] Hao, X., Yang, X., Peng, C. et al. "Heat transfer between rotating sphere and spherical-surface heat sink", *Journal of Thermal Analysis and Calorimetry,* 141(1), pp. 413-420 2020/07/01 (2020).

[11] D'Alessio, S. "An analytical study of the early stages of unsteady free convective flow from a differentially heated rotating sphere at large Grashof numbers", *International Journal of Computational Methods and Experimental Measurements,* 7(1), pp. 57-67 (2018).

[12] González-Neria, I., Yáñez-Varela, J. A., Martínez-Delgadillo, S. A. et al. "Analysis of the turbulent flow patterns generated in isotropic porous media composed of aligned or centered cylinders", *International Journal of Mechanical Sciences*, 199(p. 106396 2021/06/01/ (2021).

[13] Kardgar, A. and Jafarian, A. "Numerical simulation of turbulent oscillating flow in porous media", (in en), *Scientia Iranica*, 28(2), pp. 743-756 (2021).

[14] Alsabery, A. I., Ismael, M. A., Chamkha, A. J. et al. "Unsteady flow and entropy analysis of nanofluids inside cubic porous container holding inserted body and wavy bottom wall", *International Journal of Mechanical Sciences*, 193(p. 106161 2021/03/01/ (2021).

[15] Kurdyumov, V. and Linan, A. "Free convection from a point source of heat, and heat transfer from spheres at small Grashof numbers", *International Journal of Heat and Mass Transfer*, 42(20), pp. 3849-3860 (1999).

[16] Merkin, H. J. "Free convection boundary layers on axi-symmetric and two-dimensional bodies of arbitrary shape in a saturated porous medium", *International Journal of Heat and Mass Transfer*, 22(10), pp. 1461–1462 (1979).

[17] Cheng, P. "Mixed convection about a horizontal cylinder and a sphere in a fluid-saturated porous medium", *International Journal of Heat and Mass Transfer*, 25(8), pp. 1245–1246 (1982).

[18] Sano, T. and Okihara, R. "Natural convection around a sphere immersed in a porous medium at small Rayleigh numbers", *Fluid dynamics research*, 13(1), pp. 39-44 (1994).

[19] Juncu, G. "The influence of the porous media permeability on the unsteady conjugate forced convection heat transfer from a porous sphere embedded in a porous medium", *International Journal of Heat and Mass Transfer*, 77(pp. 1124-1132 (2014).

[20] Gaffar, S. A., Prasad, V. R., Reddy, E. K. et al. "Thermal radiation and heat generation/absorption effects on viscoelastic double-diffusive convection from an isothermal sphere in porous media", *Ain Shams Engineering Journal*, 6(3), pp. 1009-1030 (2015).

[21] Taherzadeh, M. and Saidi, M. "Natural circulation in vertical porous annular enclosure with heat generation", *Scientia Iranica*, 22(1), pp. 208-219 (2015).

[22] Pepona, M. and Favier, J. "A coupled Immersed Boundary–Lattice Boltzmann method for incompressible flows through moving porous media", *Journal of Computational Physics*, 321(pp. 1170-1184 (2016).

[23] Rao, A. S., Prasad, V. R., Beg, O. A. et al. "Free convection heat and mass transfer of a nanofluid past a horizontal cylinder embedded in a non-Darcy porous medium", *Journal of Porous Media*, 21(3), pp. 279–294 (2018).

[24] Sano, T. "Unsteady heat transfer from a circular cylinder immersed in a Darcy flow", *Journal of Engineering Mathematics*, 14(3), pp. 177-190 (1980).

[25] Chen, Z., Yang, L. M., Shu, C. et al. "Mixed convection between rotating sphere and concentric cubical enclosure", *Physics of Fluids*, 33(1), p. 013605 2021/01/01 (2021).

[26] Feng, Z.-G. "Direct numerical simulation of forced convective heat transfer from a heated rotating sphere in laminar flows", *Journal of heat transfer*, 136(4), pp. 1-8 (2014).

[27] Nigam, S. D. "Note on the boundary layer on a rotating sphere", *Zeitschrift für angewandte Mathematik und Physik ZAMP*, 5(2), pp. 151-155 (1954).

[28] Singh, S. "Heat transfer by laminar flow from a rotating sphere", *Applied Scientific Research*, 9(1), pp. 197-205 (1960).

[29] Kreith, F., Roberts, L., Sullivan, J. et al. "Convection heat transfer and flow phenomena of rotating spheres", *International Journal of Heat and Mass Transfer*, 6(10), pp. 881-895 (1963).

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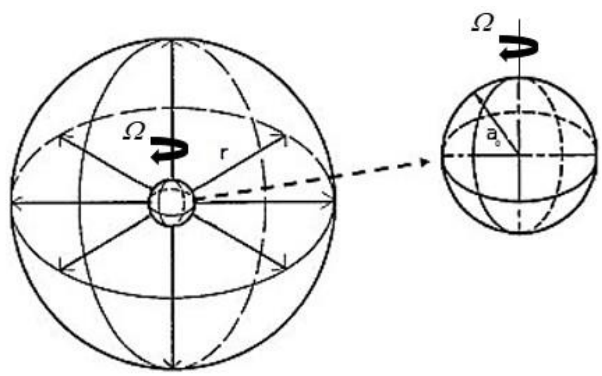


Fig. 1

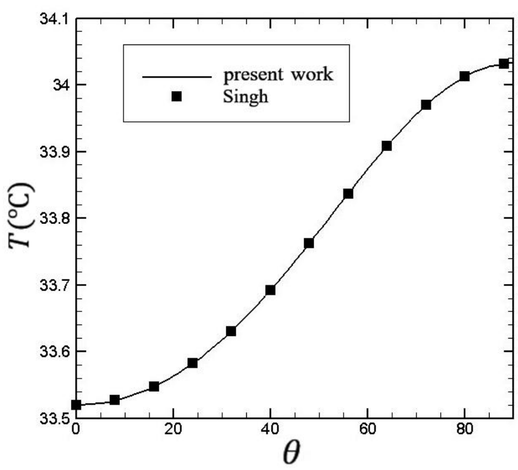
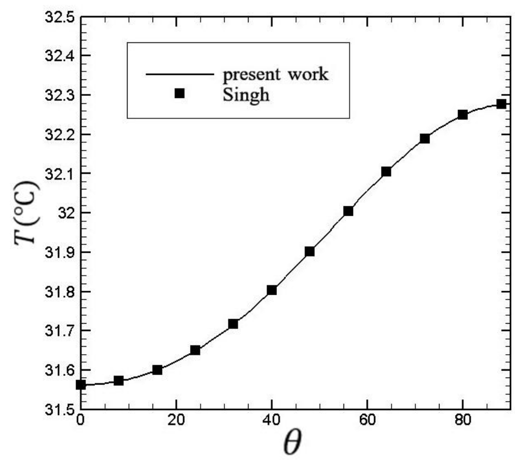


Fig. 2



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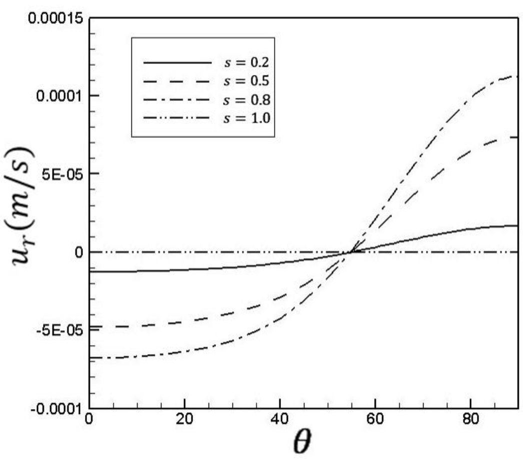

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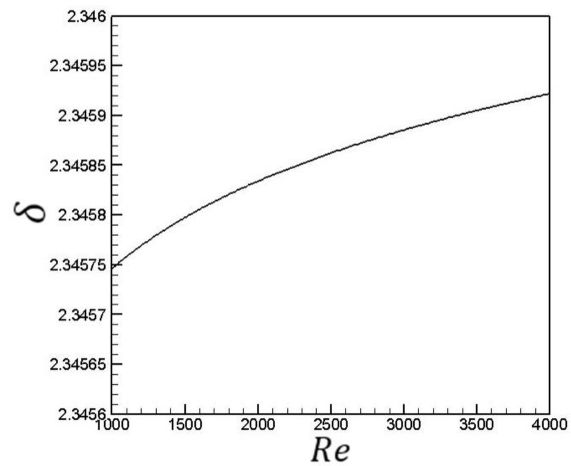

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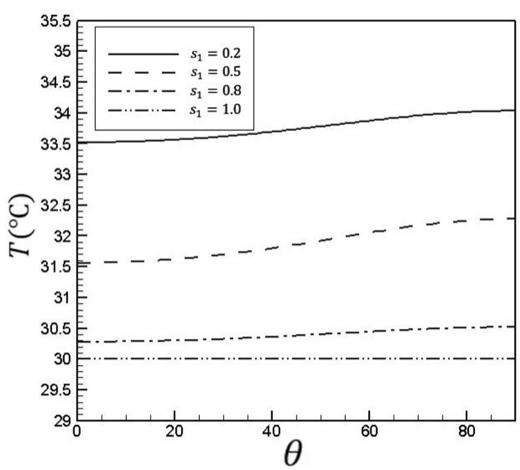

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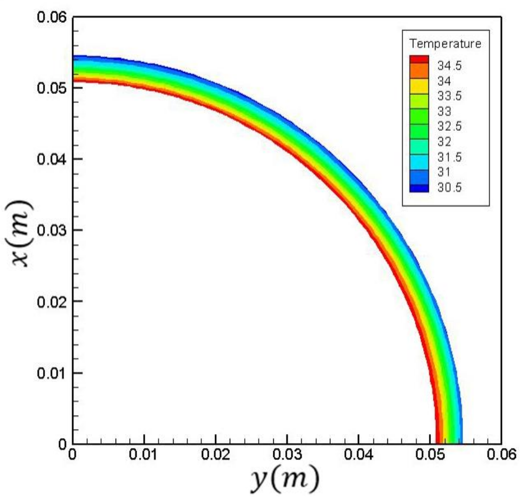

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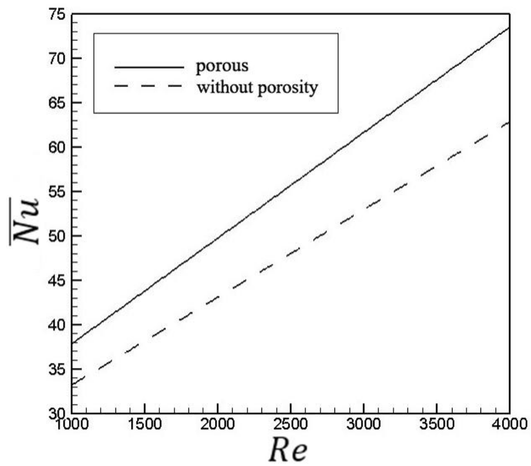

Fig. 8

Table 1

Ω	ν	K	ε	Pr
$0.097(\frac{rad}{s})$	$0.000001(\frac{m^2}{s})$	$0.025(m^2)$	0.75	6.98

Table 2

a_0	$C_{P_{eff}}$	T_s	T_∞
$0.0508(m)$	$4398(\frac{J.kg}{k})$	$35(^\circ C)$	$30(^\circ C)$

Table 3

a	δ	b	c	d	e	Δ_1	A_1	B_1
1.354	2.346	0.383	0.43	0.263	0.197	1.5	333030.54	156206.397