Joint optimization of pricing, inventory control and preservation technology investment under both quality and quantity deteriorating

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Abstract

This study presents a model for inventory control of deteriorating product in which both quantity and quality deterioration of the products are considered overtime. In this regard, for maximizing the profit, a new model is developed through two concepts: (1) establishing a decreasing pricing policy that depends on quality in order to prevent the decline in demand and product quality deterioration, and (2) investigating the level of investment in preservation technology as a significant solution in affecting the product deterioration rate. Consequently, it maintains the quality of products and increases their expiration date. In addition, the time value of money and inflation are well noticed in making the calculation of financial flows more accurate. To demonstrate the characteristics of the model, two comparative studies are conducted. The first one emphasizes the increase in the total profit of the inventory system caused by dynamic pricing policy, and another establishes the major impact of paying attention to the time value of money and inflation on making decisions.

Keywords: Inventory control, Dynamic pricing, Preservation technology investment, Quality deterioration, Quantity deterioration, Inflation, Time value of money

1. Introduction

In today's competitive markets, the deteriorating inventories have attracted much more attention to corporates and researchers. In this regard, the development of deteriorating inventory models has begun from 1960^{th}. Deterioration may refer to the process of becoming worse as damage, spoilage, miss the properties of a material by chemical interaction. In many deteriorating products, the quality of the product deteriorates in addition to its quantity. In other words, the process of deteriorating occurs for both quantitative and qualitative aspects simultaneously.

A reasonable way to reduce the deterioration rate of products such as refrigerated food, medicine, fruits, vegetables, and several other things is an optimal investment in the preservation technology. The level of investment has a direct relation with the preservation technology level. It means that the higher investment, the higher technology level, and the lower deterioration rate. Preservation technology has different types, for example, drying, using of so2, refrigerating.

It is vital to consider some aspects that affect the demand pattern for maximizing the inventory profit. One of the elements that has a great impact on demand is the customer buying behavior. The customer profiles are characterized by several attributes, the most important of which is the price of the product. Consequently, the inventory control and pricing are two inseparable aspects which affect each other. Also, the integration of inventory control and pricing for deteriorating products has been attracted much attention during several years ago. In addition, Quality of product includes all product features that have the ability to meet the needs of the customer or buyer. In other words, the customer is sensitive to the quality of the product at the time of purchase. In this regard, there are some researches in which the effect of quality on price changes has been examined.

Since nowadays, countries face many economic problems, considering some of them in the calculations can help decision makers to be in real situations. When interest and inflation rates are high, it is necessary to consider them in calculating the financial flows more accurately.

With respect to the provided explanations, three concepts of inventory control, dynamic pricing, and preservation technology investment are integrated in a unified model in which both the product quality and quantity are deteriorated under two separate increasing functions. To maintain the demand level, the selling price is determined by a dynamic pricing strategy along the planning horizon. In other words, as products approach their expiration date, the product price decreases with respect to the decline in the product quality. To make the mathematical model more authentic, time value of money and interest rate as two important concepts for analyzing the financial flows are considered.

The remaining Sections of the research are structured as follows: In the next Section, literature review is presented. In Section 3, the problem definition is described. Then, the notations and assumptions used in the mathematical model are introduced in Sub-sections 3.1 and 3.2, respectively. In Section 4, the model description, including calculation of the objective function and constraints are explained. Section 5 expresses the solution approach for optimizing the suggested model. Then, to indicate the applicability of the proposed model, a numerical example based on a case study is investigated in Sub-section 6.1. Sub-section 6.2 consists of two comparative studies. In Section 7 conclusions as well as suggestions for further studies are given based on the obtained results in the numerical example.

2. Literature review

In this Section, the literature of mentioned issues for deteriorating products is reviewed. Hung [1] introduced a model for deteriorating products that work with any kind of deterioration rate function. Afshar-Nadjafi [2] presented a single-period inventory system for perishable goods in which two models were developed to obtain the optimal time for sale announcement and the optimal order quantity of the product. The issue of deteriorating products has not been limited to the ordering inventory systems. In this regard, Mokhtari et al. [3] provided a model for the economical production of the product in which the demand rate is stochastic and stock-dependent and shortage is permitted. Also, Mashud et al. [4] proposed product deterioration in newsboy problem according to multiple just-in-time deliveries. Marandi et al. [5] considered both distribution scheduling and production of perishable products due to fast reduction. Shi et al. [6] proposed a model for deteriorating item to minimize the average total cost by determining the optimal cycle time of replenishment while delay in payments is permissible. Yang et al.

[7] submitted a perishable inventory model which detects deterioration rate as one of the decision variables.

Among the researchers carried out about preservation technology investment, Hsu et al. [8] developed a model for the deteriorating products to assess preservation technology investment policy as well as the retailer's replenishment with the constant demand and deterioration rate. Dye and Hsieh [9] presented an inventory control model for deteriorating items under the general deterioration rate function in which the investment is intended to reduce the corruption rate. Mohammadi et al. [10] examined the high importance of the preservation technology investment in the fresh-product supply chain. In this research, developing a new approach based on the preservation-technology investment led to increase supply chain profit as well as reduce waste. Chang et al. [11] developed a multi-stage deteriorating inventory model with considering preservation technology investment for maximizing the total profit. Yu et al. [12] developed an inventory model for perishable products under carbon policy with considering preservation technology. In order to control the emission of carbon as well as deteriorations of product, Mishra et al. [13] and Mashud et al. [14] applied preservation technology.

Since demand has a key role in maximizing the profit, some scientists like Jaegler et al. [15] believe that producers need to customize their products. Regarding the role of price in demand, Nazari et al. [16] determined the optimum prices in a closed loop supply chain including a number of manufacturers. Moreover, pricing strategy of deteriorating products has always been of interest to scientists. As it is obvious, most pricing models try to maximize the profits through selling products. Maihami and Abadi [17] presented a model for integration of the pricing along with controlling the perishable goods in which delay in payment and lost sales are allowed. Tiwari et al. [18] provided a model for maximizing profits by setting the optimal selling price and optimizing replenishment cycle time for deteriorating items. Mashud et al. [19] proposed an inventory model with shortages in which demand depends on price. Hasan et al. [20] developed a model for inventory control of agricultural products in order to optimize the cycles of replenishment and the selling price. Mashud et al. [21] optimized the selling price for deteriorating products when a discount facility is provided. In terms of dynamic pricing policy, You and Hsieh [22] integrated continuing inventory management and dynamic price-setting/changing strategy for a company that has seasonal sales over a finite time of planning. Recently, several papers have been presented on the dynamic pricing of deteriorating products such as Dye and Yang [23], Rabbani et al. [24] and Chen et al. [25].

As mentioned above, Quality has a significant impact on demand .Wang and Li [26] proposed a dynamic pricing model to evaluate the product quality for the perishable food supply chain. Qin et al. [27] introduced an inventory model for the fresh productions in which: (1) the product quantity and quality deteriorate at the same time; (2) The quantity deterioration rate as well as the quality deterioration rate increase over time; and (3) Demand is defined as a function of several factors including selling price, quantity and quality of the products. Jalali et al. [28] investigated the effect of finite production capacity in determining the optimal product quality and price in a make-to-stock manufacturing system. Wei et al. [29] investigated the quality and quantity loss of perishable product to optimize the inventory and dynamic pricing strategy. Mashud et al. [30] investigated the quality of products using a proper preservation technologies for imperfect products.

Regarding two concepts of inflation and time value of money, Hou and Lin [31], Alikar et al. [32] considering both inflation rate and the time value of money in their papers. However, Pal et al. [33], and Pramanik and Maiti [34] developed EOQ models by considering only the inflation rate. Recently, because of Covid-19 pandemic, many businesses have been disrupted. In this regard, inflation as an important issue needs to be considered in the supply chain inventory systems by scientists like Mashud et al. [35]. At the end of this section, summarized literature review is given in Table 1.

Please insert Table 1 here

3. Problem definition

This paper presents a mathematical model for deteriorating inventory, which contains the quantity and quality of product decline during time. It seems logical to consider an increasing function of time based on deterioration rate in the product's quantity and quality. So according to some similar studies such as Salmasnia et al. [36], Moeini et al. [37] and Moeini et al. [38], it is assumed that the product quantity and quality deterioration follow the Weibull distribution.

The objective of the problem is maximizing the profit, which could be resulted from subtracting the costs from the sales revenue. In this regard, the product demand should be raised by increasing the inventory level and quality of the product as well as decreasing the product price. Any changes in the quality of products affects the customers' feedbacks, which has a direct influence on the demand. So, it is necessary to develop a dynamic pricing policy to maintain the product demand in an acceptable level. Accordingly, when the product quality decreases over time, its price diminishes as well.

As mentioned earlier, the product quality is known as one of effective factors on the product demand and the sales revenue. In the model, investing in the preservation technologies is employed as an influential way to reduce the deterioration rate of products. The levels of investment and technology are directly related to each other. It means that higher technology level for maintaining the product's quality requires higher amount of investment. If inflation and interest rate are not considered in the calculation of the expected sales revenue and costs imposed on the manufacturer, it will cause misleading in the results. Generally, planning horizons can be divided to finite and rolling according to the features of the inventory control models. In this study, since inventory control does not update regularly, planning horizon is finite and static.

3.1. Notations

In this Subsection, the notations used in the mathematical model are given in Table 2. They are divided into three parts: indices, decision variables and parameters.

Please insert Table 2 here

3.2. Assumptions and Definitions

The assumptions in the presented model are as follows:

1. The planning horizon L is divided into n periods with different prices, each $T = \frac{L}{n}$.
2. The order occurs at the beginning of the planning horizon.
3. Considering static parameters, a finite planning horizon is assumed.
4. The deterioration rate increases over time, and the quality and quantity deterioration follow Weibull distribution $f(t) = \alpha\beta t^{\beta-1}e^{-\alpha t^{\beta}}$ so that $t, \alpha > 0, \beta \geq 1$. So, the deterioration rate is $\mu(t) = \alpha\beta t^{\beta-1}$.
5. Shortage is not allowed.

6. Demand is a function of the quality, stock on display and the price of the product. Then, the demand rate at time t of period j is $D_j = \theta I_j(t) + g\left(N\left((j-1)\times T + t\right)\right)M\left(p_j\right)$, where $N(t)$ is a descending function of time that means the quality of the product decreases over time. $g\left(N(t)\right)$ is a non-negative function of $N(t)$ that $0 < g\left(N(t)\right) < 1$ and $g\left(N(0)\right) = 1$.is a $M\left(p_j\right)$ non-negative decreasing function of p in period j . $m(\varepsilon)$ is an ascending function of the amount of investment ε. It implicates that a higher technology needs a higher investment.

4. Model Description

This Section explains the proposed mathematical model according to the problem definition and the assumptions. For this purpose, the next Subsection calculates the inventory level, sales revenue, inventory carrying cost, purchasing cost, ordering cost, and the preservation technology investment.

4.1. Inventory Level

The inventory level of the deteriorating products reduces in two ways. Either the products get deteriorated or consumed over finite planning horizon. Therefore, the decreasing rate of the inventory level in the period *j* at time *t* is calculated as below:

$$\frac{dI_j(t)}{dt} = -\left(\theta I_j(t) + g\left(N\left(\left(j-1\right)\times T + t\right)\right)M\left(p_j\right)\right) - \mu(t) I_j(t) \qquad 0 \le t \le T \tag{1}$$

Where *0* and *T* are the beginning and the end of each period, respectively. Equation (1) consists of two terms; the first one denotes the demand relation. According to the assumption 4, the demand relation of period *j* consists of two parts. The first part indicates a decrease in the inventory level in period *j* at time *t*; while the second part comprises the quality level along the planning horizon and the price function, both of which are ascending functions, and the price increases as the quality level increases. The second term of Equation 1 also shows the decrease in the level of inventory through deteriorating.

Also, the reduction rate of in the product quality in period *j* at time *t* is:

$$\frac{dN(t)}{dt} = -N(t)\chi(t) \qquad 0 \le t \le T \tag{2}$$

According to the inventory level and quality reduction, the quantitative and qualitative deterioration rates are as Equations (3) and (4), respectively.

$$\mu(t) = \alpha_1 \beta_1 t^{\beta_1 - 1} \xrightarrow{f(t) = \alpha_1 \beta_1 t^{\beta_1 - 1} e^{-\alpha_1 t^{\beta_1}} \to \alpha_1 = \frac{E_3}{E_4 m(\varepsilon)}} \frac{E_3}{E_4 m(\varepsilon)} \beta_1 t^{\beta_1 - 1} \qquad 0 \le t \le T \tag{3}$$

$$\chi(t) = \alpha_2 \beta_2 t^{\beta_2 - 1} \xrightarrow{f(t) = \alpha_2 \beta_2 t^{\beta 2 - 1} e^{-\alpha_2 t^{\beta_2}} \to \alpha_2 = \frac{E_1}{E_2 m(\varepsilon)}} \frac{E_1}{E_2 m(\varepsilon)} \beta_2 t^{\beta_2 - 1} \qquad 0 \le t \le T \tag{4}$$

Since both the quantitative and qualitative deterioration rates must be ascending over time, according to Equations (3) and (4), the Weibull hazard rate functions with $\alpha_1, \beta_1, \alpha_2, \beta_2$ parameters are utilized for calculating the quantitative and qualitative deterioration rates, respectively. As explained in assumption 4, $m(\varepsilon)$ is an ascending function in which the higher the investment, the higher the level of technology will be. Respecting the developing technology, the deterioration rate is expected to be decreased, as it can be seen in Equations (3) and (4).

Hence by putting Equations (3) and (4) into Equations (1) and (2), the following Equations are obtained:

$$\frac{dI_j(t)}{dt} = -\Big(\theta I_j(t) + g\big(N\big((j-1)\times T + t\big)\big)M\big(p_j\big)\Big) - \left(\frac{E_3}{E_4 m(\varepsilon)}\beta_1 t^{\beta_1 - 1}\right)\times I_j(t) \qquad (5)$$

$$0 \le t \le T$$

$$\frac{dN_j(t)}{dt} = -N_j(t)\times\left(\frac{E_1}{E_2 m(\varepsilon)}\beta_2 t^{\beta_2 - 1}\right) \qquad 0 \le t \le T \qquad (6)$$

Since the inventory level at the beginning of period j is equal to q_j, the order occurs at the beginning of the finite planning horizon:

$$I_j(0) = q_j = Q \qquad (7)$$

Equation (7) denotes that the inventory level of the first period at time zero is equal to the order value.

$$I_1(\mathrm{T}) = q_2 = \left(q_1 \times e^{\left(-(T\times\theta)-\left(\frac{E_3\times T^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)} \right) - \left(e^{\left(-(T\times\theta)-\left(\frac{E_3\times T^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)} \times \int_0^T g\left(N_1(x)\right)\times e^{\left((x\times\theta)+\left(\frac{E_3\times x^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)} dx \times M\left(p_1\right) \right) \tag{8}$$

Equation (8) indicates the level of inventory at the end of the first period. In other words, the level of inventory of the first period at time T is equal to the value of q_2 which is the level of inventory at the beginning of the second period.

Consequently, since 0 and T are the beginning and end of each period respectively, the level of inventory at the end of period j equals q in period $j+1$:

$$I_j(T) = q_{j+1} = \left(q_j \times e^{\left(-(T\times\theta)-\left(\frac{E_3\times t^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)} \right) - \left(e^{\left(-(T\times\theta)-\left(\frac{E_3\times t^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)} \times \int_0^T g\left(N_j(x)\right)\times e^{\left((x\times\theta)+\left(\frac{E_3\times x^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)} dx \times M\left(p_j\right) \right) \tag{9}$$

According to Equation (9), the level of inventory at the beginning of period j can be obtained as below:

$$q_j = e^{(-T\times\theta)} \times e^{\left(-\left(\frac{E_3\times T^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)} \times \left(q_{j-1} - \left(\int_0^T g\left(N_j(x)\right)\times e^{\left((x\times\theta)-\left(\frac{E_3\times x^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)} dx \times M\left(p_{j-1}\right) \right) \right) \tag{10}$$

By replacing Q with q_{j-1}, Equation (10) could be rewritten as:

$$q_j = -\sum_{i=1}^{j}\left(e^{\left(-\frac{E_3\times T^{\beta_1}}{E_4\times m(\varepsilon)}-T\times\theta\right)^{j-i}}\times M\left(p_i\right)\times\int_0^T g\left(N_i\left(x\right)\right)\times e^{\left(-x\times\theta\times M\left(p_i\right)-\left(\frac{E_3\times x^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)}dx\right) + \left(e^{\left(-\frac{E_3\times T^{\beta_1}}{E_4\times m(\varepsilon)}-T\times\theta\right)^{j-1}}\times Q\right) \tag{11}$$

According to Equation (11), the inventory level can be obtained at the beginning of each period, by substituting it in Equation (9). Finally, the inventory level of the period *j* at time *t* is formulated as below:

$$I_j\left(t\right) = \left[\left[-\sum_{i=1}^{j}\left(e^{\left(-\frac{E_3\times T^{\beta_1}}{E_4\times m(\varepsilon)}-T\times\theta\right)^{j-i}}\times M\left(p_i\right)\times\int_0^T g\left(N_i\left(x\right)\right)\times e^{\left(-x\times\theta\times M\left(p_i\right)-\left(\frac{E_3\times x^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)}dx\right) + \left(e^{\left(-\frac{E_3\times T^{\beta_1}}{E_4\times m(\varepsilon)}-T\times\theta\right)^{j-1}}\times Q\right)\right]\times e^{\left(-t\times\theta-\left(\frac{E_3\times x^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)}\right] - \left[e^{(-t\times\theta)}\times e^{\left(-\frac{E_3\times T^{\beta_1}}{E_4\times m(\varepsilon)}\right)}\times M\left(p_j\right)\times\int_0^t g\left(N_j\left(x\right)\right)\times e^{\left(x\times\theta+\left(\frac{E_3\times x^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)}dx\right] \quad 0\le t\le T \tag{12}$$

Obviously, Equation (12) is a descending function of time which indicates that the inventory level decreases until it reaches zero at the end of the planning horizon, since the order only occurs once at the beginning of the planning horizon.

$$I_n\left(T\right) = 0 \tag{13}$$

According to Equations (12) and (13), the amount of order at the beginning of the planning horizon could be found as:

$$Q=\frac{\left(M\left(p_n\right)\times\int_0^T g\left(N_n\left(x\right)\right)\times e^{\left(-x\times\theta-\left(\frac{E_3\times x^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)}dx\right)+\left(\sum_{i=1}^{n-1}\left(e^{\left(-\frac{E_3\times T^{\beta_1}}{E_4\times m(\varepsilon)}-T\times\theta\right)^{n-i}}\times M\left(p_i\right)\times\int_0^T g\left(N_i\left(x\right)\right)\times e^{\left(x\times\theta+\left(\frac{E_3\times x^{\beta_1}}{E_4\times m(\varepsilon)}\right)\right)}dx\right)\right)}{e^{\left(-\frac{E_3\times T^{\beta_1}}{E_4\times m(\varepsilon)}-T\times\theta\right)^{n-1}}} \tag{14}$$

4.2. Sales Revenue

Let Δq_j be the sales volume over period *j*:

$$\Delta q_j=q_j-q_{j+1} \tag{15}$$

Then, $R\left(n\right)$ denotes the sales revenue when the planning horizon is divided into *n* periods. Regarding the time value of money and the inflation factors $R\left(n\right)$ is as below:

$$R\left(n\right)=\sum_{j=1}^{n}\Delta q_j p_j\times\int_{t=0}^{T}e^{(i-r)t}dt \tag{16}$$

4.3. Inventory Carrying Cost

Let H_j be the carrying cost in period *j* by considering interest and inflation rates when the planning horizon is divided into *n* periods.

$$H_j=h\times\int_{t=0}^{T}I_j(t)\times e^{(i-r)t}dt \tag{17}$$

Afterward, the total carrying cost could be calculated as follows:

$$H(n) = \sum_{j=1}^{n} H_j \tag{18}$$

4.4. Mathematical model

In this Section, according to the mentioned equations in the previous Sub-sections, the mathematical model including the objective function and constraints is defined as below:

$$MaxZ : F = R(n) - H(n) - \left(Q(n) \times c\right) - \varepsilon - c_0 \tag{19}$$

subject to :

$$M\left(p_j\right) > 0 \tag{19.1}$$

$$g_{\min} \le \varepsilon \le g_{\max} \tag{19.2}$$

$$m\left(\varepsilon\right) > 0 \tag{19.3}$$

$$\varepsilon, \mathrm{p} \ge 0; n \in N^{+} \tag{19.4}$$

The expected total profit per time unit (*F*) includes the sales revenue, carrying cost, order cost, and preservation technology cost. According to Equation (19.1), price functions must be non-negative. According to the range of available technologies in the world, the technology cost must be limited among two specific lower and upper bounds as determined in Equation (19.2). The third constraint ensures that the used technology level is a positive and ascending function of the technology cost. Equation (19.4) ensures that the number of periods should be a positive integer value and the values of ε and p_j must be the positive real numbers. If pricing policy were not dynamic, the problem would be similar to the model presented in Maihami and Abadi. [17] and could be solved by the exact methods. In this study, a dynamic pricing policy in which the planning horizon is divided to *n* periods with different product prices is developed. Since *n*, unlike the other decision variables, is discrete and in the limits of an integral in the objective function, the exact methods for solving the model are inefficient and very time-consuming. Due to the high mentioned complexity of the mathematical model, meta-heuristic algorithms which can be used to solve problems with high complexity is needed. Meta-heuristic algorithms can find suitable solutions and have shorter running times. In this regard, in order to solve the mathematical programming (19), Particle Swarm Optimization algorithm (PSO) as a unique searching method and suitable efficiency for solving the non-convex and non-linear models is employed.

5. Particle Swarm Optimization

The PSO is a population-based algorithm of possible solutions. This algorithm through members of population, which are called particles, detects proper values of decision variables. Each particle includes fitness value and velocity vector. While the first one indicates valuation of the solution suitability, the other one determines the direction fly of the particle. These particles move to track the optimum particles in the feasible solution space.

In order to gain high search efficiency, local and global searches are combined by the PSO. It starts through a particle swarm which has random velocities and positions. Subsequently, the algorithm updates particles based on the inertia force and the “best” values which are called global best (gbest) and personal best (pbest). The first one is the best solution ever seen and another is the best value witnessed through the particle. It means that the velocity and position of particle are updated in every iteration based on three factors: (1) the global best; (2) the personal best; and (3) the current velocity. To specify the PSO, Table 3 describes the algorithm parameters.

Please insert Table 3 here

$(c_1 + c_2)$ usually is set on 4 based on [39]. Also, w as a variable parameter between 0 and 1 is decreased to $w \cdot w_{damp}$ in each iteration, where w_{damp} is a fixed factor less than 1. Figure 1 and 2 indicates the computational procedure of the PSO algorithm in detail.

Please insert Figure 1 here

Please insert Figure 2 here

5.1 Initial solutions generation

The initial values of the continuous decision variables ε, p are obtained using a random uniform distribution between their corresponding upper and lower bounds. In order to generate the initial value for the discrete random variable n, a random value from a uniform distribution in the interval $[0,1]$ is chosen. The corresponding discrete value, i.e. the number of periods, is obtained by Equation (22).

$$n = \min\left(n_{\min} + \mathrm{floor}\left[\left(n_{\max} - n_{\min} + 1\right) \times \mathrm{R}\right], \mathrm{n}_{\max}\right) \tag{12}$$

where, $n_{\min}$ and $n_{\max}$ are the lower and upper bounds of *n*. In addition, *r* is a uniform random number in the interval $[0,1]$.

6. Experimental Results

As mentioned earlier, the aim of the suggested model is to maximize profit in the planning horizon, given the constraints. In Section 5.1, using the examples generated from a Taguchi L_8 experimental design, the effect of parameters on the decision variables and the objective function is investigated. Section 5.2 provides two comparative studies to illustrate the effectiveness of the proposed model. In the first comparative study to demonstrate the importance of dynamic pricing for deteriorating products, the model is compared to a similar model, called here model 1, with this difference that in which the product price remains fixed in the planning horizon. In the second comparative study, the model is compared to a similar model without considering interest and inflation rates called here model 2. It clarifies the significance of accurate calculation of the financial flows with consideration of time value of money under inflationary conditions.

6.1. Numerical Example

As mentioned earlier, a numerical example is solved to demonstrate the applicability of the suggested model. In this example, a deteriorating product vendor has an inventory system that orders once at the beginning of the planning horizon and the shortage is not allowed. He / she plans to bring the inventory level to zero by the end of the planning horizon. Since the rate of product deterioration over time follows an incremental function, in this example, it is assumed that time of deterioration follows a Weibull distribution. In order to preserve the deteriorating products, it should be decided how much will be spent for the preservation system. Since the product cannot have its initial quality and quantity after a while, the product's price could be changed. Moreover, it is essential to discover how many times the initial price is changed. Also, p_j and $M\left(p_j\right)$ functions are considered as follows:

$$M\left(p_j\right)=A-B\times p_j$$

$$p_j=p_1\times\left(\frac{j}{(j-1)\times T}\right)$$

In addition, the quality level and technology level functions are respectively considered as $g\left(N\left(\mathrm{t}\right)\right)=\mathrm{e}^{-\lambda\alpha t^{\beta}}$ and $m\left(\varepsilon\right)=\varepsilon-KH$. The parameters values for the numerical example are given in Table 4.

Please insert Table 4 here

Finally, the mathematical model is solved by the PSO algorithm using MATLAB software. The number of iterations and population size of PSO are obtained through the trial-and-error process equal to 40 and 20, respectively. The results of the numerical example are attained according to Table 5. The results indicate that until the end of the planning horizon, the product price should be lowered two times to get the maximum possible profit.

Please insert Table 5 here

In order to evaluate the economic effectiveness of the proposed model, eight examples are generated based on three parameters of the length of the planning horizon *L*, the quality parameter λ in $N(t)$ and the intercept parameter A in function $M(p_j)$ by using a L_8 Taguchi design as can be seen in Table 6.

Please insert Table 6 here

The optimal values of decision variables and the profit for each example are obtained using the PSO algorithm. The results are given in Table 7.

Please insert Table 7 here

The results of Table 7 indicate that under constant λ conditions, the number of product price changes n increases with an increment in the planning horizon L. As expected, to maximize the profit, there should be less changes in the prices and in order to do that the possibility to have less changes in the quality parameter is needed. In the mentioned problem since the amount of changes in λ is rare so the amount of changes in the price is fewer. It is obvious that the increase of λ results in reducing the product demand and profit because of the diminishing level of quality. Finally, the results show that the product price increases with enhancing A and under this situation the preservation technology investment is also enhanced.

Table 8 indicates the average profit difference when each parameter is at its upper and lower levels. From these results, it could be understood that the absolute value of parameter λ is the most, and the absolute value of parameter A is the least. Therefore, it could be concluded that among the considered parameters, the most effective one on the average profit is λ, and the least effective is A parameter.

Please insert Table 8 here

6.2. Comparative Study

In this Section, the proposed model is compared with two similar models. In the first model, the product price is assumed to be fixed over the planning horizon. The second model is designed for a situation where the time value of money and inflation are not taken into account.

6.2.1. Comparison between the proposed model and the model without changing the product price

In order to evaluate the effect of the dynamic pricing policy in the proposed model, it is compared to a model with a fixed pricing policy on2 the numerical examples given in Table 4. The results can be seen in Table 9.

Please insert Table 9 here

According to Tables 7 and 9, the obtained product price from model 1 in all the examples is lower than the initial price attained from the proposed model. In fact, the dynamic pricing strategy, at the beginning of the planning horizon, the product will be sold at a higher price due to its high quality but these changes will not happen in fixed price strategy. As the product quality decreases over time, the product price is reduced to keep the demand level along the planning horizon. On the contrary, when a fixed pricing strategy is employed, the manufacturer is forced to sell the product in lower price from the beginning to maintain the product demand in an acceptable level until the end of the planning horizon. Considering the low product price in this model, and since there is a direct relation between the products quality and its price, it is not vital to provide a product with high quality and invest in high level of preservation technology. Furthermore, the results confirm that the lower price convinces the seller to increase the ordering volume with the aim of preventing a significant reduction in the profit which of course leads to a higher purchasing cost. It also increases the inventory carrying cost due to a growth in the inventory level. As illustrated in Figure 3, the profit of the model with fixed pricing strategy is less than the model with dynamic pricing strategy in all examples despite more selling.

Please insert Figure 3 here

6.2.2. Comparison between the proposed model and the model without considering time value of money and inflation

In order to evaluate the effect of considering time value of money and inflation in the proposed model, it is compared to a model without these features, which is called model 2. To do this, the same problems used in the previous comparative study are applied. The results are given in Table 10.

Please insert Table 10 here

The results in the above table's last column indicate when the time value of money and inflation are not considered, the purchase cost increases due to an increase in the order quantity. Also, the level of product inventory in stock increases in this situation and leads to higher inventory carrying cost. Since inflation means raising the general level of prices, products can be sold more expensive than the inflation-free state. As a result, in the second column, it is noticeable that the prices are lower than the prices in the base model. In this regard, the low inflation rate in the based model is one of the factors to an increase in the prices slightly. As products are sold at a higher price in the inflation state, and there is also a direct relationship between price and quality, in the proposed model a higher level of preservation technology is used to reach the products to a higher quality level. Finally, since employing higher technology level results in a lower deterioration rate, the lower price changes is required when considering time value of money and inflation.

Please insert Figure 4 here

In order to compare the profits of the two models, Figure 4 illustrates that the profit is higher in the proposed model due to existing both the higher costs and lower income in the other model.

7. Conclusion

This study integrated three concepts of inventory control, dynamic pricing and investment of preservation technology for deteriorating products. It considered both quantity and quality deteriorations for the products over time. A dynamic pricing strategy was developed to prevent a drop in the product demand along the planning horizon due to the decrease of the product quality over time. To illustrate the effectiveness of the developed dynamic pricing strategy, a comparative study between the proposed model and a similar model with a fixed pricing strategy was conducted. The results showed that the obtained price from the model with the fixed pricing strategy is lower than the initial price achieved by the proposed model. Since there is a direct relation between the price and quality, to maintain the quality level of product, more investment in the preservation technologies is needed. Therefore, through making the quality level steadier, no price reduction or demand decrease will occur which means a higher profit. Finally, the results confirmed that employing the dynamic pricing strategy leads to a significant enhancement of the profit. Furthermore, in order to enhance the accuracy of the model in calculation of the financial flows, two concepts of the time value of money and inflation were considered. To evaluate the effect of the two mentioned concepts, the proposed model was compared to a model without these features. The results showed that the order quantity decreases when considering time value of money which involves lower purchasing cost, inventory level and carrying cost.

Due to considering the static and finite planning horizon in the suggested model, it is applicable in only special cases. Regarding this restriction, if rolling planning horizon is applied, order quantity will vary since the shortage and surplus amount of inventory are taken into account. As a result, it will be more authentic. Moreover, as the majority of businesses provide a variety of production with various deterioration rate, it is suggested that the present model develop to a model with multiple products along with different deterioration rate.

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1. Initialize

- For each particle $i = 1, 2, \ldots, N$ do:
  - Generate the initial value of each particle position with uniform distribution: $x_i^t \sim U(b_{lo}, b_{up})$
  - Initialize the $pbest$ of each particle to its initial position: $pbest_i \to x_i$
  - If $F(p_i) \geq F(gbest)$, update the $gbest$ as: $gbest \to pbest_i$
  - Generate the initial value of each particle velocity: $v_i \sim U(-|b_{up} - b_{lo}|, |b_{up} - b_{lo}|)$

2. Update

- Until a stopping criterion is met, repeat:
  - For each particle $i = 1, 2, \ldots, N$ do:
    - For each dimension $d = 1, 2, \ldots, n$ do:
    - Generate random numbers: $rand_p, rand_g \sim U(0,1)$
    - Update the particle velocity:

$$v_i^t = w\, v_i^{t-1} + c_1 rand_p (pbest_i^{t-1} - x_i^{t-1}) + c_2 rand_g (gbest^{t-1} - x_i^{t-1})$$

    - Update the particle situation: $x_i^t = x_i^{t-1} + v_i^t$
    - If $(F(x_i) \geq F(pbest_i))$: Update the $pbest$ of each particle
    - If $(F(pbest_i) \geq F(gbest))$: Update the $gbest$
  - Now $gbest$ holds the best found solution.

Figure 1. Computational procedure of the PSO algorithm

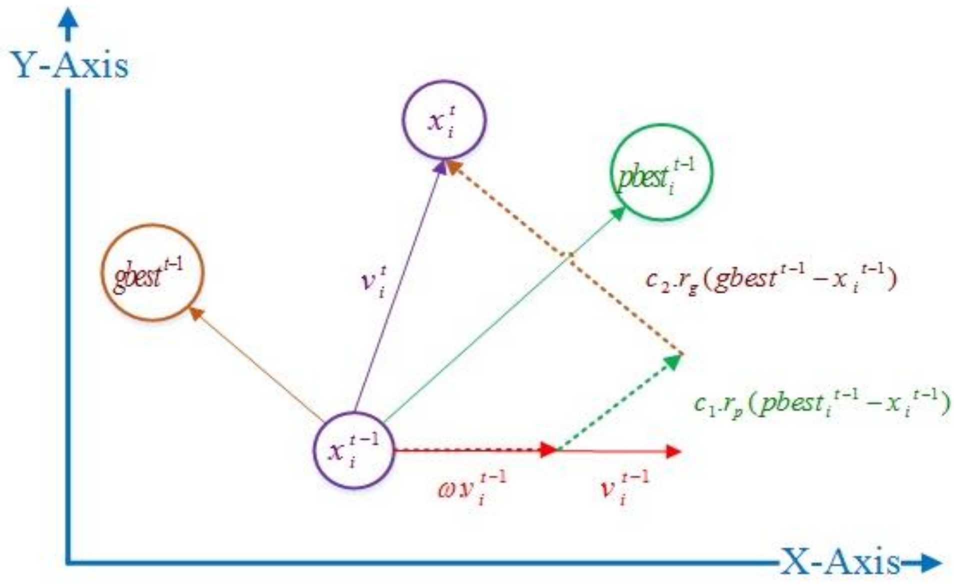

Figure 2. PSO position and velocity update [40]

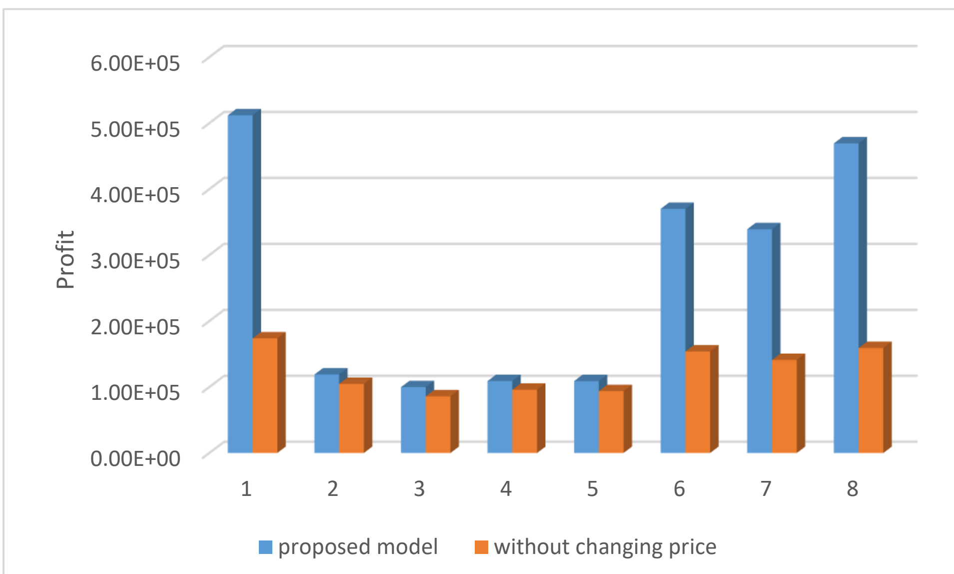

Figure 3. Comparison of the proposed model and model 1 in terms of the profit

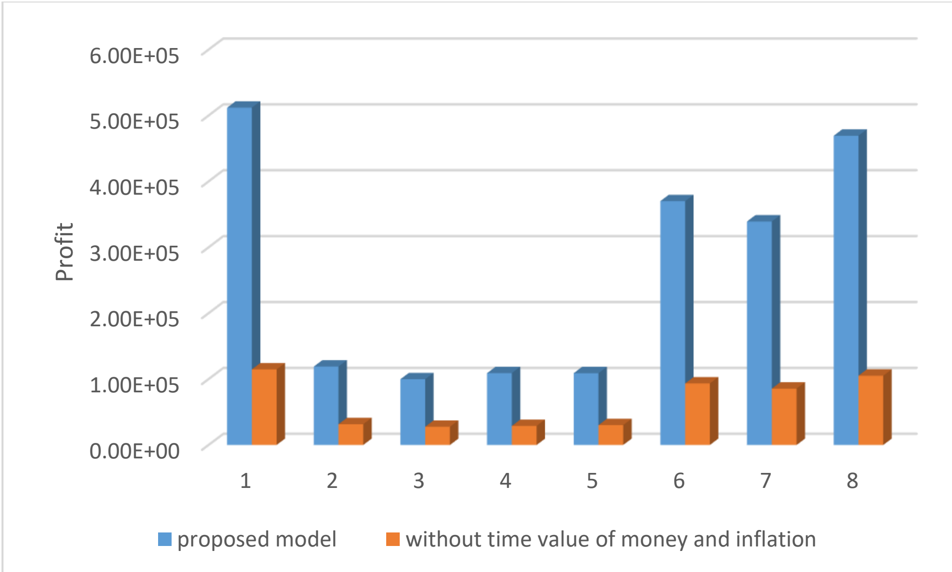

Figure 4. Comparison of the proposed model and model 2 in terms of the profit

Table 1. Summarized literature review

Papers	Pricing		Preservation technology	Deteriorate		Rate	
	Static	Dynamic		Quality	Quantity	interest	inflation
Chen et al(2018)		✓			✓		
Hsu et al(2010)	✓		✓		✓		
Dye et al(2012)			✓		✓		
Rabbani et al.(2016)		✓		✓	✓		
Hou and Lin (2006)					✓	✓	✓
Alikar et al. (2017)						✓	✓
Dye and Yang (2016)		✓	✓		✓		
Pramanik and Maiti (2019)					✓		✓
Mohammadi et al. (2019)			✓		✓		
Tiwari et al. (2018)	✓						
Qin et al(2014)	✓			✓	✓		
Wang and Li (2012)	✓			✓			
Maihami and Abadi (2012)	✓				✓		
Pal et al. (2015)					✓		✓
You and Hsieh (2007)		✓					
This research		✓	✓	✓	✓	✓	✓

Table 2. The notations

Notation	Description
Indices	
j	Index of price change period
l	index of quality and quantity
d	Index of E parameter
Decision variables	
p_j	Selling price during period j
ε	Preservation technology (PT) cost
n	The total number of periods (n - 1 also represents the number of price changes)
Parameters	
c	Unit purchasing cost
c_0	Unit ordering cost
i	Inflation rate
r	Interest rate
E_d	Non-negative constant used in calculating the scale parameter of Weibull distribution
θ	Non-negative coefficient of inventory consumption
q_j	The inventory level at the start of period j
$R(n)$	Sales revenue when the planning horizon is divided into n periods
$H(n)$	Total carrying cost when the planning horizon is divided into n periods
$M(p_j)$	Non-negative function of selling price in period j
$N(t)$	The product quality function of time
$\mu(t)$	Instantaneous deterioration rate of the quantity of inventory
$\chi(t)$	Instantaneous deterioration rate of quality of inventory
α_l, β_l	Scale and shape parameters of Weibull distribution used for the product quality and quantity deterioration
h	Carrying cost per unit per time unit
Q	Order quantity

Table 3. PSO parameters

Parameters	Description
N	Number of particles in the swarm
x_i^t, v_i^t	Position and velocity of the i^{th} particle in iteration t, respectively
$pbest_i^{t-1}$	Personal best of the i^{th} particle in the iteration t
$gbest^t$	Best solution founded until iteration t
b_{lo}, b_{up}	Lower and upper bounds of decision variables (search space), respectively
c_1, c_2, w	Recognition and social learning factors and inertia weight, respectively

Table 4. The parameters values

Parameter	c	i	r	L	E_3	E_4	θ	h
Value	2	0.1	0.2	15	1	10^4	10^{-5}	0.05
Parameter	α_1	A	B	KH	β_1	β	α	λ
Value	0.9	450	10	27	1	1	0.9	0.5

Table 5. The results of the numerical example

Objective function	***Decision variable***		
F^*	ε^*	p^*	n^*
9.9836×10^4	51.9665	44.9657	3

Table 6. The Taguchi L_8 experimental design

Problem	A	λ	L
1	470	0.3	20
2	470	0.5	20
3	450	0.5	15
4	450	0.5	20
5	470	0.5	15
6	470	0.3	15
7	450	0.3	15
8	450	0.3	20

Table 7. The optimal decision variables and profit for the numerical examples

Decision variable			***Objective function***	***Items***		
n^*	p^*	ε^*	F^*	R	H	$Q \times C$
3	46.9937	55.0545	5.1196×10^5	5.1269×10^5	176.6147	397.874
4	46.9946	50.3871	1.1904×10^5	1.1936×10^5	34.5597	134.3375
3	44.9657	51.9665	9.9836×10^4	1.0016×10^5	40.5410	127.1516

4	44.9901	55.6294	1.0909×10^5	1.0941×10^5	33.0913	128.6291
3	46.9967	44.5574	1.0896×10^5	1.0926×10^5	42.3221	132.7312
3	46.9861	57.2260	3.7008×10^5	3.7092×10^5	154.9211	522.6648
3	44.9985	47.5884	3.3926×10^5	3.4006×10^5	148.3018	500.3062
3	44.9969	55.0396	4.6934×10^5	4.7000×10^5	128.0494	380.9285

Table 8. Influence of the parameters on the profit

Level	λ	A	L
Down	422465	254296.5	229394
Top	109219	277387.5	302290
Average	-313246	23091	72896

Table 9. Results of the model with a fixed pricing policy

Decision variable			Objective function	Items		
n^*	p^*	ε^*	F^*	R	H	$Q \times C$
1	236047	30.0210	1.7374×10^5	1.7608×10^5	489.9099	1.7254×10^3
1	23.5960	43.1815	1.0465×10^5	1.0610×10^5	272.0883	1.0401×10^3
1	22.6121	39.5997	8.5925×10^4	8.7297×10^4	238.2295	993.8920
1	22.6750	40.0693	9.5869×10^4	9.7261×10^4	259.5428	992.1386
1	22.6268	47.8319	9.3794×10^4	9.5228×10^4	248.7160	1.0376×10^3
1	23.6912	34.6581	1.5386×10^5	1.5613×10^5	4456892	1.6966×10^3
1	22.6821	33.4370	1.4094×10^5	1.4313×10^5	426.7414	1.6245×10^3
1	22.6608	39.7435	1.5914×10^5	1.6139×10^5	467.8128	1.6474×10^3

Table 10. Results of the model without considering time value of money and inflation

Decision variable			***Objective function***	***Items***		
n^*	p^*	ε^*	F^*	R	H	$Q \times C$
4	46.9791	53.5092	1.1470×10^5	1.1565×10^5	234.4034	555.6457
5	46.9714	37.4368	3.1579×10^4	3.1951×10^4	54.4651	180.1521
4	43.4423	39.2275	2.7636×10^4	2.8017×10^4	53.7901	187.6715
5	44.9811	51.6314	2.8930×10^4	2.9289×10^4	35.2463	172.4500
4	45.6491	43.3817	3.0167×10^4	3.0561×10^4	55.8550	194.8008
3	46.9787	30.5738	9.3417×10^4	9.4284×10^4	213.6778	5228470
3	44.9846	38.5106	8.5576×10^4	8.6419×10^4	204.5566	500.4675
4	44.9875	40.6636	1.0513×10^5	1.0602×10^5	2244076	531.9573