Optimizing A Fuzzy Multi-Objective Closed-loop Supply Chain Model Considering Financial Resources using meta-heuristic

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Abstract
This paper presents a multi-objective mathematical model which aims to optimize and harmonize a supply chain to reduce costs, improve quality, and achieve a competitive advantage and position using meta-heuristic algorithms. The purpose of optimization in this field is to increase quality and customer satisfaction and reduce production time and related prices. The present research simultaneously optimized the supply chain in the multi-product and multi-period modes. The presented mathematical model was firstly validated. The algorithm’s parameters are then adjusted to solve the model with the multi-objective simulated annealing (MOSA) algorithm. To validate the designed algorithm’s performance, we solve some examples with General Algebraic Modeling System (GAMS). The MOSA algorithm has achieved an average error of %0.3, %1.7, and %0.7 for the first, second, and third objective functions, respectively, in average less than 1 minute. The average time to solve was 1847 seconds for the GAMS software; however, the GAMS couldn’t reach an optimal solution for the large problem in a reasonable computational time. The designed algorithm’s average error was less than 2% for each of the three objectives under study. These show the effectiveness of the MOSA algorithm in solving the problem introduced in this paper.

Keywords: Supply Chain, Metaheuristics, Logistics, Fuzzy Sets, Multi-objective.

1. Introduction
The business that competes in today’s world is based on the production of goods and services based on customer needs and, at the same time, cost-effective. In many companies, customer orientation has been adopted to reduce the amount of time spent to meet customer needs and improve products’ quality. These companies seek to gain a competitive advantage by effectively managing their purchasing processes and creating better interaction with their suppliers. Coordinating the flow of materials across multiple organizations within each organization is one
of the major management challenges in the supply chain that achieving it requires the use of technologies and tools to track materials along the route from source to destination and record information at each step. Due to its ability to recover value from returned and used products, reverse logistics has received a lot of attention and has become a key element in the supply chain. The supply chain is a chain that includes all activities related to the flow of goods and conversion of materials, from the stage of preparation of raw materials to the stage of delivery of the final goods to the consumer. There are two other streams about the flow of goods: the flow of information, and the other is the flow of financial resources and credit. The design of a reverse logistics network is critical because of the need for materials and products to flow in the opposite direction of the supply chain for a variety of reasons. Legal requirements, social responsibilities, environmental concerns, economic interests, and customer awareness have forced manufacturers to produce environmentally friendly products, reclaim and collect returned and used products. Marketing, competitive and strategic issues, and improving customer loyalty and subsequent sales are also motivations for reverse logistics. Therefore, different industrial sectors need to improve their structures and activities to meet these challenges. Hence, a decision-making tool for supply chain coordination is presented in this study based on existing contracts using heuristic algorithms. Adopting the right strategy to improve supply chain performance brings many benefits to improve productivity in companies and organizations.

Considering the supply chain optimization under different circumstances will lead to lower costs and improve quality and thus achieve a competitive advantage. Optimization problems in this area seek to increase quality and customer satisfaction and reduce production time and related costs. Several variables are considered inputs of these kinds of problems.

The goal is to find the optimal design points fitted with the mentioned objective functions. Given the pricing role in reducing the uncertainty of returned products and the impact of product returns on the number, location, and capacity of facilities needed for product revival in this paper, designing a closed-loop supply chain network (SCN) will be a model for designing a closed-loop SCN developed considering discounts, and financial resource flows. Also, the network of the mentioned model is derived from Ramezani et al. [1]. In a direct direction, the model includes the levels of suppliers, distributors, warehouses, retailers, and customers that warehouses are considered separately (allocating warehouse to a group of retailers) to make the paper's model more realistic. In the opposite direction, the network includes the collection, recycling, and disposal centers, which are produced in the direct flow of products using materials provided by suppliers, and through distribution centers to warehouses, and from there to retailers, and finally, to customers. This paper's main objective is to develop a multi-
objective contingency optimization model for closed-loop supply chain design, which involves modeling the closed-loop supply chain problem considering discounts and flow of funds under uncertainty and two secondary objectives of solving the proposed model using fuzzy perspective and obtaining optimal design points values. The rest of the article is structured as follows: the theoretical foundations, literature review, and the research gap were discussed in the second part. Then, the solution method provided in the third section, and the research data is analyzed, and the numerical results are presented in the fourth section. The results were presented in the fifth and sixth sections, and the conclusion and future suggestions were presented in the seventh section.

2. Literature Review

Logistic Network Design is a part of supply chain planning focused on long-term strategic planning [2]. The logistics network design itself is divided into three parts, Forward Logistic Network Design, Reverse Logistic Network Design, and Integrate Forward Reverse Logistic Network (closed-loop).

Forward Logistics Network: A network of suppliers, manufacturers, distribution centers, and channels between them and customers to obtain raw materials, convert them into finished products, and distribute finished products to customers efficiently (Amiri, [3]).

Reverse Logistics Network: The process of efficiently planning, implementing, and controlling the flow of incoming and storing second-hand goods and related information in the opposite direction to the traditional supply chain to recover value or disposal [4]. The previous related literature is reviewed in the following.

Peng et al. [5] designed a multi-period forward supply chain network. They presented complex linear programming to solve the problem of explaining the supply chain network. The proposed multi-period model is designed with two objective functions of optimal distribution and cost reduction. Ramezani et al. [1] presented a multi-objective and multi-product stochastic model for forward/reverse network design under uncertainty. The model objectives include maximizing profits, maximizing customer service levels, and minimizing the total number of defective raw materials purchased from suppliers, thereby determining the facilities’ locations and flows between facilities in line with capacity constraints. This model is based on the scenario. In this paper, the ε-constraint method is used to obtain a set of optimal Pareto supply chain configurations.

Hassanzadeh and Zhang [6] presented a multi-objective, multi-product problem in which communication flow is such that the products first are sent to demand markets.
Then, the products are sent from demand markets to collection centers. The product can be improved, and it is transferred to production workshops, otherwise transferred to recycling centers. This problem has been solved with two summing weights and $\varepsilon$ constraints to convert the two-objective problem into a single-objective one. Vahdani and Sharifi [7] proposed a new mathematical model for designing a closed-loop SCN that integrated the network design decisions in both forward and reversed supply chain networks. They considered that the model's parameters are uncertain and modeled this uncertainty by fuzzy parameters. They presented an inexact-fuzzy-stochastic solution methodology to deal with various uncertainties in their proposed model.

In this context, Pishvaee et al. [8] developed a feasible multi-objective programming model for designing a network of sustainable medical supply chains under uncertainty, considering the conflicting economic, environmental, and social goals. The present study provides a robust mathematical model for designing a medical needle and syringe supply chain as an essential strategic medical requirement in health systems. A product and a period have been evaluated in this research. A rapid Benders analysis algorithm using three efficient acceleration mechanisms that consider the proposed model solution's computational complexity was proposed to solve this model. Moreover, Braidio et al. [9] addressed optimizing the SCN using the Tabu search method. Considering the importance of reducing logistics costs through supply chain optimization and the complexity of realistic problems, the present study aims to implement and evaluate the Tabu search's exploratory method to optimize a supply chain network. According to their research results, the proposed exploratory optimization can be used for networks with complex supply chains and can provide acceptable results on a computer that has been sufficiently optimized.

Qin and Ji [10] designed a reverse logistics network to deal with uncertainty during the recovery process in a fuzzy environment. They formulated a single-objective, single-period, single-product model to minimize costs, applied three types of fuzzy programming optimization models based on different decision criteria, and used a hybrid smart algorithm to integrate genetic algorithm (GA) and fuzzy simulation in order to solve the proposed models. Yang et al. [11] developed a two-stage optimization method for designing a Multi-purpose SCN (MP-SCN) with uncertain transportation costs and customer requirements. They developed two objectives for the SCN problem according to the neutral and risky criteria. They also designed an improved multi-purpose biography-based optimization algorithm (MO-BBO) to solve the approximate complicated optimization problem and compare it with the Multi-Objective GA (MO-
GA). According to their results, the improved MO-BBO algorithm outperforms MO-GA in terms of solution quality.

By clicking on recent research, Avakh Darestani and Pourasadollah [12] used a multi-objective fuzzy approach to design a closed-loop SCN concerning Dynamic Pricing. The model objectives include maximizing profits, minimizing delays in delivering goods to customers, and minimizing the return on suppliers’ raw materials. Since the model is multi-objective, the fuzzy mathematical programming approach is used to convert the multi-objective model into a single objective in order to solve a large-sized version the problem. The results show the efficiency and effectiveness of the model. Sarkar et al. [13] provided optimal production delivery policies for suppliers and manufacturers in a constrained closed-loop supply chain for returnable transport packaging through a metaheuristic approach. The model objectives include profit maximization and carbon emissions minimization of the system. A weighted goal programming technique and three distinct meta-heuristic approaches are applied to obtain efficient trade-offs among model objectives. Three heuristic methods, particle swarm optimization, interior point optimization algorithm, and genetic algorithm, were used, and the best method was presented for the given data. The results provided by the interior-point optimization algorithm and GA were the best ones. The weighted goal programming results while using the single setup multi-delivery (SSMD) policy were compared with the SSMD policy. Results show an SSMD policy for supplier and manufacturer-focused decision-making in a proposed supply chain management to improve proper economic sustainability.

Rahimi Sheikh et al. [14] designed a Resilience supply chain model by identifying the factors creating instability in the supply chain. Govindan et al. [15] reviewed big data analytics and application for logistics and supply chain management. This study summarizes the big data attributes, effective methods for implementation, effective practices for implementation, and evaluation and implementation methods. Their review papers offer various opportunities to improve big data analytics and applications for logistics and supply chain management. Vanaei et al. [16] proposed a new multi-product multi-period mathematical model for integrated production-distribution three-level supply chain. They considered the uncertainty of the model's parameters using the Markowitz model and solved the presented model by GA.

Mahmoudi et al. [17] presented a new multi-product, multi-level, and multi-period mathematical model for a reverse logistic network which aimed to minimizes transportation and facilities establishing cost, and lowers purchasing from suppliers, and solved the proposed model using a genetic algorithm. Khorram-Nasab et al. [18]
presented an integrated management model for the electronic supply chain of products in gas and oil companies by investigating the effective parameters on the company's performance. Zahedi et al. [19] designed a closed-loop SCN considering multi-task sales agencies and multi-mode transportation. The proposed model has four echelons in the forward direction and five echelons in the backward direction. The model considers several constraints from previous studies and addresses new constraints to explore better real-life problems that employ different transportation modes and rely on sale agency centers. The objective function is to maximize the total profit. Besides, this study firstly considers a distinct cluster of customers based on the product life cycle. The model's structure is based on linear mixed-integer programming, and the proposed model has been investigated through a case study regarding the manufacturing industry. The findings of the proposed network illustrated that using the attributes of sale agency centers and clusters of customers increases total revenue and the number of returned products.

Srivastava and Rogers [20] researched how to manage various industries of global supply chain risks in India. They believe that in each industry sector, the global supply chain risks and their mitigation strategies differ. They used profile deviation and ideal profile methodology to identify top performers in three industry sectors (Audit, Finance and Consulting, Automotive, and IT and Software) and evaluated their best practices towards managing global supply chain risks. They then found the 'ideal' risk mitigation profiles for all three industries. These findings provide new insights to practitioners as they will serve as a helpful reference tool for Indian executives planning to internationalize.

Jaggi et al. [21] presented a multi-objective production model in the lock industry case study. In the proposed model, an attempt has been made for the production planning problem with multi-products, multi-periods, and multi-machines under a specific environment that takes into account to minimize the production cost and maximize the net profit subject to some realistic set of constraints. In a multi-objective optimization problem, objective functions usually conflict with each other, and any improvement in one of the objective functions can be achieved only by compromising with another objective function. To deal with such situations, the Goal Programming approach has been used to obtain the formulated problem's optimal solution. This optimal solution can only be obtained by achieving the highest degree of each of the membership goals.

Talwar et al. [22] reviewed big data in supply chain operations and management. Their research is a systematic review of the literature (SRL) to uncover the existing
research trends, distill key themes, and identify future research areas. For this purpose, 116 studies were identified and critically analyzed through a proper search protocol. The key outcome of this SRL is the development of a conceptual framework titled the Dimensions-Avenues-Benefits (DAB) model for adoption and potential research questions to support novel investigations in the area offering actionable implications for managers working in different verticals and sectors. Maheshwari et al. [23] reviewed the role of big data analytics in supply chain management. A review from the year 2015–2019 is presented in this study. Further, the significance of DAB in supply chain management (SCM) has been highlighted by studying 58 papers, which have been sorted after a detailed study of 260 papers collected through the Web of Science database. Their findings and observations give state-of-the-art insights to scientists and business professionals by presenting an exhaustive list of the progress made, and challenges left untackled in the field of DAB in SCM.

Recently, Atabaki et al. [24] used a priority-based firefly algorithm (FA) for the network design of a closed-loop supply chain with price-sensitive demand. A mixed-integer linear programming model is developed to make location, allocation, and price decisions maximize total profit regarding capacity and number of opened facilities constraints. The proposed FA uses an efficient solution representation based on the priority-based encoding. Moreover, the algorithm utilizes a backward heuristic procedure for decoding. For large-sized problems, the performance is compared with a differential evolution algorithm, a genetic algorithm, and an FA relying on the conventional priority-based encoding through statistical tests and a chess rating system. The results indicate the superiority of the proposed approach in both FA structure and encoding-decoding procedure. In the same year, Avakh Darestani and Hemmati [25] optimized a dual-function closed-loop SCN for corrupt commodities according to the queuing system using three multi-criteria decision-making methods, namely the weighted sum method method, the LP-Metrics. The objectives of this study are to minimize total network costs and minimize greenhouse gas emissions. The results indicate a significant difference between the mean of the first and second objective functions and the computational time. According to Zaleta & Socorrás [26], no algorithm can solve the supply chain design problem for large cases in a reasonable time period. Lee and Kwon [27] suggest that although computing power has increased, and several efficient and powerful software programs have been introduced in the market, computing time is still very long for hundreds of products and customers and dozens of plants. The research model was developed based on previous research studies and literature review and gaps identified in modeling and solution methodology.
2.1 Contribution of this work

Overall, this research offers a comprehensive yet multi-objective model for closed-loop supply chain design, and to make the model more adaptable to the real world, hence uncertainty in demand, return rates when delivering products to customers is considered that fuzzy numbers are used to describe these factors and fuzzy mathematical programming for modeling given the fuzzy capability to interact with uncertainty patterns. This paper’s contribution is to present an optimized fuzzy model based on several objective functions and consider discounts and financial flows that show the model is complicated due to the objectives mentioned above and variables mentioned in this environment and has not been presented so far. Since the closed-loop supply chain problem is one of the NP-hard problems, some extraordinary approaches to solving this problem, which is part of the paper, contribute to the research literature.

3. Problem Modelling

The structure of the studied chain was presented in Figure 1. A transportation system must be considered in this chain for each of the existing connections between the chain members. For this purpose, several predefined transportation systems are investigated, and each of them establishes material connections between different chain members. Moreover, this chain’s key parameters, including demand, return rate, and delivery time to customers, are assumed to be uncertain, aiming to get closer to the real situation.

The research assumptions can be stated as follows:

- The supply chain understudy is multi-level, multi-product and multi-period
- Discounts are considered in the supply of raw materials
- The current chain value is considered in the feasibility studies of the chain
- The problem is based on the demand uncertainty and the delivery amount and time
- Except for disposal centers, other chain components have limited capacity
- Hybrid centers can distribute and collect returned goods simultaneously
- The suppliers’ locations in the chain are fixed
- The non-deterministic parameters are provided as the triangular fuzzy numbers
• The problem objectives include maximizing the profit's present value, minimizing the total weight of the delivery time, and minimizing the defective items received from the suppliers.

A multi-echelon multi-product closed-loop supply chain is designed for this problem. The chain consists of suppliers, manufacturers, distributors, and collection and disposal centers. The 'suppliers' location is fixed, but the manufacturing 'plants' location must be determined. There is also a set of potential points that can be distribution, collection, or combination centers. Combination centers can distribute as well as collect simultaneously. The disposal center location should also be determined from among its potential points. Then, a mathematical model was presented in this research.

Moreover, the network of the current research's model is derived from Ramezani et al. [1]. Three objectives were optimized simultaneously in this model. The first objective is to maximize the value of the chain profit; the second objective is to minimize the transition times. The third objective is to minimize defective parts purchased. In this regard, due to the uncertainty of some parameters, the fuzzy theory approach was applied to the mathematical model. Professor Lotfi Asgar Zadeh first introduced fuzzy logic in new computation after setting the fuzzy theory. The fuzzy method is a very efficient method that helps managers control these uncertainties and is therefore used in our model to achieve the desired objective. Moreover, the Multi-Objective Simulated Annealing Algorithm is used to solve the model due to the complexity of the mathematical model.

3.1. Mathematical model

The proposed mathematical model is presented in the following:

Indices
- $S$: Supplier fixed location ($s = 1,2,\ldots,S$)
- $i$: Potential locations of plants ($i = 1,2,\ldots,I$)
- $j$: Potential locations for distribution centers / collection facilities / hybrid centers ($j = 1,2,\ldots,J$)
- $c$: Customers' fixed locations ($c = 1,2,\ldots,C$)
- $k$: Potential centers of goods disposal ($k = 1,2,\ldots,K$)
- $p$: Products ($p = 1,2,\ldots,P$)
- $r$: Raw materials ($r = 1,2,\ldots,R$)
- $l$: Transportation systems ($l = 1,2,\ldots,L$)
- $t$: Time periods ($t = 1,2,\ldots,T$)

Parameters
- $d_{ctp}$: Customer $c$ demand for product $p$ in period $t$,
- $PR_{ctp}$: The selling price of each unit of product $p$ to customer $c$ in period $t$, 

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Page 9
\( SC_{ir}^t \): Cost of purchasing 1 unit of raw material \( r \) from supplier \( s \) in period \( t \),
\( DS_{is}^t \): Discount on purchase of raw materials from supplier \( s \) in period \( t \),
\( MC_{ip}^t \): Production cost per unit of product \( p \) in plant \( i \) in period \( t \),
\( OC_{ip}^t \): Operating cost on product \( p \) at the collection center \( j \) in period \( t \),
\( IC_{ip}^t \): Inspection and recycling cost per unit of product \( p \) at the facility location \( j \) in period \( t \),
\( RC_{ip}^t \): Cost of recovering product \( p \) in plant \( i \) in period \( t \),
\( DC_{kp}^t \): Disposal cost per unit of product \( p \) at the disposal center \( k \) in period \( t \),
\( HC_{ip}^t \): Maintenance cost per unit of product \( p \) in the facilitation center \( j \) in period \( t \),
\( RD_{gr}^t \): The failure rate of raw material \( r \) in supplier \( s \) in period \( t \),
\( w_r \): Significance coefficient of raw material \( r \),
\( FX_{ls}^t \): Fixed cost of supplier \( s \) selection in period \( t \),
\( FX_{il}^t \): Fixed cost of setting up plant \( i \) in period \( t \),
\( FY_{lj}^t \): Fixed cost of setting up facility \( j \) in period \( t \),
\( FZ_{lj}^t \): Fixed cost of setting up a collection center \( j \) in period \( t \),
\( FU_{lj}^t \): Cost of setting up a hybrid center at point \( j \) in period \( t \),
\( FV_{lk}^t \): Fixed cost of setting up a disposal center \( k \) in period \( t \),
\( CS_{r}^t \): The capacity of supplier \( s \) for supplier \( r \) in period \( t \),
\( CX_{i}^t \): Production capacity in plant \( i \) in period \( t \),
\( CY_{j}^t \): The capacity of distribution center \( j \) in period \( t \),
\( CZ_{j}^t \): The capacity of the collection center \( j \) in period \( t \),
\( CU_{j}^t \): The capacity of the hybrid center \( j \) in period \( t \),
\( CR_{i}^t \): Plant capacity \( i \) to recover products returned in period \( t \),
\( CV_{k}^t \): The capacity of the disposal center \( k \) in period \( t \),
\( CSU_{ls}^t \): The unit cost of transporting raw material \( r \) from supplier \( s \) to plant \( i \) in period \( t \),
\( CJ_{ijp}^t \): The unit cost of transporting product \( p \) from plant \( i \) to distribution center \( j \) in period \( t \) with transportation system \( l \),
\( CJC_{jcp}^t \): The unit cost of transporting product \( p \) from the distribution center \( j \) to the customer \( c \) with the transportation system \( l \) in period \( t \),
\( CCL_{jcp}^t \): The unit cost of transporting product \( p \) from the customer \( c \) to the collection center \( j \) with the transportation system \( l \) in period \( t \),
\( CJK_{jcp}^t \): Cost of transporting product \( p \) inspected from the collection center \( j \) to the plant \( i \) for recovery in period \( t \) with the transportation system \( l \),
\( CJK_{jkp}^t \): The unit cost of transporting product \( p \) from the collection center \( j \) to the disposal center \( k \) in period \( t \),
\( TIP_{jip}^t \): Product transporting time \( p \) from plant \( i \) to distribution center \( j \) in period \( t \) with transportation system \( l \),
\( TJC_{jcp}^t \): Product transporting time \( p \) from distribution center \( j \) to customer \( c \) with transportation system \( l \) in period \( t \),
\( TCI_{jcp}^t \): Product transporting time \( p \) from customer \( c \) to collection center \( j \) with transportation system \( l \) in period \( t \),
\( TJK_{jip}^t \): Product time \( p \) inspected from collection center \( j \) to plant \( i \) for recovery in period \( t \) with transportation system \( l \),
\( n_{rp} \): Raw material consumption coefficient \( r \) in product \( p \),
\( m_p \): Rate of capacity utilization in producing product \( p \),
\( RR_p \): The return rate of product \( p \) from customers,
\( RX_p \): The reproduction rate of product \( p \),
\( RW_p \): Disposal rate of product \( p \),
\( \text{ir:} \) Interest rate,
\( \gamma: \) Discount rate,
\( \beta: \) The importance weight of the direct chain and \( 1 - \beta \) is the important factor of the reverse chain,

\( \text{BM:} \) A very large number

**Variables**

- \( \text{QSI}_{i,s,t}^r \): Amount of raw material \( r \) sent from supplier \( s \) to plant \( i \) in period \( t \),
- \( \text{QIJ}_{i,j,p,l}^t \): Quantity of product \( p \) sent from plant \( i \) to distribution center \( j \) with transportation system \( l \) in period \( t \),
- \( \text{INV}_{j,p}^t \): Inventory of product \( p \) in the distribution center \( j \) at the end of period \( t \),
- \( \text{QJC}_{j,c,p,l}^t \): Amount of product \( p \) transferred from the distribution center \( j \) to the customer \( c \) with the transportation system \( l \) in period \( t \),
- \( \text{QCI}_{j,c,p,l}^t \): Quantity of product \( p \) returned from the customer \( c \) to the collection center \( j \) with the transportation system \( l \) in period \( t \),
- \( \text{QJC}_{j,k,p}^t \): Amount of defective product \( p \) sent from the collection center \( j \) to the disposal center \( k \) in period \( t \),
- \( W_{s,t}^t \): A binary variable equal to 1 if the supplier \( s \) is selected in period \( t \),
- \( X_{i,t}^t \): A binary variable equal to 1 if plant \( i \) is started in period \( t \),
- \( Y_{j,t}^t \): A binary variable equal to 1 if the distribution center is set up at point \( j \) in period \( t \),
- \( Z_{i,t}^t \): A binary variable equal to 1 if the collection center is set up at point \( i \) in period \( t \),
- \( U_{j,t}^t \): A binary variable equal to 1 if a hybrid center is set up at point \( j \) in period \( t \),
- \( V_{k,t}^t \): A binary variable equal to 1 if the disposal center is set up at point \( k \) in period \( t \),
- \( A_{i,j,l}^t \): A binary variable equal to 1 if the transportation system \( l \) connects plant \( i \) and distribution center \( j \) in period \( t \),
- \( B_{j,c,l}^t \): A binary variable equal to 1 if the transportation system \( l \) connects the distribution center \( j \) to customer \( c \) in period \( t \),
- \( C_{c,j,l}^t \): A binary variable equal to 1 if the transportation system \( l \) connects customer \( c \) to the collection center \( j \) in period \( t \),
- \( D_{j,i,l}^t \): A binary variable equal to 1 if the transportation system \( l \) connects the collection center \( j \) to plant \( i \) in period \( t \),

### 3.2. Mathematical Model Relationships

The problem consists of three objectives that are presented in detail as follows.

- **Maximize the value of chain profit**
  
  The first objective function maximizes the chain’s net present value, derived from the difference between incomes and costs. Equation (2) is the specified income from the sale of products in each period. Equation (3) indicates the total chain costs in each period. These costs include fixed costs of setting up plants and facilities, costs of supply and purchase from suppliers, discounts from suppliers, costs of production and recovery of
defective products, operating costs in distribution centers and disposal centers, inventory costs in distribution centers, and transportation costs by different transportation systems in the supply chain.

\[
\text{Max } \text{NPV} = \sum \frac{\text{Income}_t - \text{Cost}_t}{(1 + ir)^{t-1}}
\]

\[
\text{Income}_t = \sum \sum \sum \sum \text{OJC}_j \cdot \text{PR}_{cp}
\]

\[
\text{Cost}_t = \sum \text{FX}_t \cdot (X_t - X_{t-1}) + \sum \text{FY}_t \cdot (Y_t - Y_{t-1}) + \sum \text{FZ}_t \cdot (Z_t - Z_{t-1}) + \\
\sum \text{FU}_t \cdot (U_t - U_{t-1}) + \sum \text{FV}_t \cdot (V_t - V_{t-1}) + \sum \text{FW}_t \cdot (W_t) + \sum \sum \sum \text{QSI}_{ir} \cdot SC_{ir} - \\
\sum \sum \sum \sum \text{QJC}_{jcp} \cdot OC_{ip} + \sum \sum \sum \sum \text{QIJ}_{ipl} \cdot DC_{ip} + \sum \sum \sum \sum \text{QIK}_{jkp} \cdot DC_{kp} + \\
\sum \sum \sum \sum \text{INV}_{jp} \cdot HC_{ip} + \sum \sum \sum \sum \text{QSI}_{sr} \cdot CSI_{sr} + \sum \sum \sum \sum \text{QJI}_{jpl} \cdot CJI_{ipl} + \\
\sum \sum \sum \sum \text{QII}_{jpl} \cdot CJI_{jpl} + \sum \sum \sum \sum \text{QIK}_{jkp} \cdot CJK_{jkp}
\]

- **Minimize the transition times**

  The second objective function minimizes the weighted total of the transmission times in the direct and reverse chains as follows:

\[
\text{Min } f_2 = \beta \left( \sum \sum \sum \sum \sum \text{A}_{ij} \cdot TII_{ipl} + \sum \sum \sum \sum \sum \text{B}_{ij} \cdot TCJ_{ipl} \right) + \\
(1 - \beta) \left( \sum \sum \sum \sum \sum \text{C}_{ipl} \cdot TCJ_{ipl} + \sum \sum \sum \sum \sum \text{D}_{jil} \cdot TII_{ipl} \right)
\]

- **Minimize defective parts purchased**

  The last objective function minimizes the total amount of defective raw materials in suppliers. This goal seeks to select suppliers that minimize the return of final goods as follows:

\[
\text{Min } f_3 = \sum \sum \sum \sum \sum \text{QSI}_{ir} \cdot RD_{ir} \cdot W_r
\]
The model's constraints are presented in Equations (6) to (33) as follows. Equation (6) indicates that the amount of raw material imported to each plant in each period is equal to the amount of output from that plant in the same period. Equation (7) ensures that the amount imported for each product in each period to each distribution center and the remaining inventory from the previous period is equal to the amount sent to customers and the remaining inventory at the end of the period.

\[
\sum_{j} \sum_{p} \sum_{i} n_{rp} QIJ^t_{ijpl} = \sum_{s} QSI^t_{sr} + \sum_{j} \sum_{p} \sum_{l} n_{rp} QIJ^t_{ijpl}; \quad \forall i, j, t
\]  

\[
INV^t_{jp} + \sum_{r} QIJ^t_{ijpl} = INV^t_{jp} + \sum_{c} QJC^t_{cjpl}; \quad \forall j, p, t
\]  

Equation (8) shows that for each product and each period, the amount available in each of the distribution centers or hybrid centers must meet the demand for that product. Equation (9) describes the relationship between customer demand and the amount returned to collection centers and hybrid centers. Equation (10) ensures that the total amount received from customers in collection centers and recyclable centers that can be recycled is equal to the total amount sent from these centers to plants. Equation (11) ensures that the total amount of recyclable goods received from customers at collection centers and recycling centers is equal to the total amount sent to disposal centers.

\[
\sum_{j} \sum_{c} QJC^t_{cjpl} = d^t_{cp}; \quad \forall c, p, t
\]  

\[
\sum_{j} \sum_{c} QCJ^t_{cjpl} = D^t_{cp} RR^p; \quad \forall c, p, t
\]  

\[
\sum_{i} \sum_{l} QIJ^t_{ijpl} = \sum_{c} QCI^t_{cjpl} RX^p; \quad \forall j, p, t
\]  

\[
\sum_{k} K^t_{jk} + \sum_{l} QIJ^t_{ijpl} = \sum_{c} QCI^t_{cjpl}; \quad \forall j, p, t
\]  

Equation (12) ensures that suppliers' raw material does not exceed the suppliers' capacity. Equation (13) indicates material capacity constraints in plants similar to suppliers. Equation (14) indicates that each distribution center's remaining inventory and the hybrid center should not exceed its capacity. Equation (15) ensures that the flow of goods from collection centers to plants and disposal centers does not exceed these centers' capacity. Equation (16) states that the total amount of goods returned to each plant should not exceed that plant's recovery capacity. Equation (17) states that the total amount sent to the disposal centers should not exceed these centers' capacity. Equation (18) is the maximum number of facilities that can be established.
\[
\sum_{i} QSI_{air}^t \leq CS_{sr}^t, Q_{sr}^t, \forall s, r, t \tag{12}
\]
\[
\sum_{j} \sum_{p} \sum_{l} m_{p} QIJ_{ipl}^t \leq CX_{i}^t, X_{i}^t, \forall i, t \tag{13}
\]
\[
\sum_{j} \sum_{p} \sum_{l} m_{p} QJC_{ipl}^t \leq CY_{j}^t, Y_{j}^t + CU_{j}^t U_{j}^t, \forall j, t \tag{14}
\]
\[
\sum_{j} \sum_{p} \sum_{l} m_{p} QCJ_{ipl}^t \leq CZ_{j}^t, Z_{j}^t + CU_{j}^t U_{j}^t, \forall j, t \tag{15}
\]
\[
\sum_{j} \sum_{p} \sum_{l} m_{p} QJI_{jpl}^t \leq CR_{i}^t, X_{i}^t, \forall i, t \tag{16}
\]
\[
\sum_{j} \sum_{p} \sum_{l} m_{p} QJK_{jpl}^t \leq CV_{j}^t, V_{j}^t, \forall k, t \tag{17}
\]
\[
Y_{j}^t + Z_{j}^t + U_{j}^t \leq 1; \forall j, t \tag{18}
\]

Equation (19) ensures that raw materials are received from selected suppliers. Equations (20) and (21) determine the minimum amount received from each of the selected suppliers, so that very small orders are not sent to a particular supplier.

\[
Q_{sr}^t \leq W_{s}^t; \forall s, r, t \tag{19}
\]
\[
q_{s}^t \leq \frac{1}{2} \left( \sum_{r} Q_{sr}^t \right); \forall s, t \tag{20}
\]
\[
\sum_{i} QSI_{air}^t \geq \gamma CS_{sr}^t Q_{sr}^t; \forall s, r, t \tag{21}
\]

Equation (22) to (25) requires that only one transportation system be used in each chain member.

\[
\sum_{l} A_{ijl}^t \leq 1; \forall i, j, t \tag{22}
\]
\[
\sum_{l} B_{jcl}^t \leq 1; \forall j, c, t \tag{23}
\]
\[
\sum_{l} C_{cjl}^t \leq 1; \forall c, j, t \tag{24}
\]
\[
\sum_{l} D_{jil}^t \leq 1; \forall i, j, t \tag{25}
\]

Equation (26) to (29) indicates that the transportation system is used between the chain members who send goods.

\[
A_{ijl}^t \leq \sum_{p} QIJ_{ipl}^t; \forall i, j, l, t \tag{26}
\]
\[ B'_{jcl} \leq \sum_{p} QJC'_{jcl}; \quad \forall j, c, l, t \]  
\[ C'_{cjl} \leq \sum_{p} QCJ'_{cjl}; \quad \forall c, j, l, t \]  
\[ D'_{jil} \leq \sum_{p} QJI'_{jil}; \quad \forall j, i, l, t \]  

Equation (30) to (33) indicates that the chain members with no transaction do not also send goods to each other.

\[ \sum_{p} QIJ'_{jipl} \leq BM . A'_{jil}; \quad \forall i, j, l, t \]  
\[ \sum_{p} QJC'_{jcl} \leq BM . B'_{jcl}; \quad \forall j, c, l, t \]  
\[ \sum_{p} QCJ'_{cjl} \leq BM . C'_{cjl}; \quad \forall c, j, l, t \]  
\[ \sum_{p} QJI'_{jil} \leq BM . D'_{jil}; \quad \forall j, i, l, t \]  

3.3. Fuzzification approach and model solution in fuzzy conditions

Each of the non-deterministic parameters is considered as a triangular fuzzy number displayed as \( \tilde{D} = (d_1, d_2, d_3) \). The alpha cut is used to determine the values of \( x \) with an alpha confidence level in its uncertainty. The following equation obtains these values of \( x \):

\[ x_\alpha = \{ x : x \in X, \mu_A(x) \geq \alpha, \alpha \in [0,1] \} \]  

The lower the alpha, the higher the confidence level and the smaller the confidence interval, and the higher the alpha, the lower the confidence level and the more the confidence interval. Considering the specified alpha level, the range of changes \( x \) can be reduced, and the investor can be assured that the investment risk is somewhat reduced. Determining the alpha level or the same level of confidence is the decision maker's responsibility and is added as a predefined parameter in the model.

So generally, the fuzzy demand \( \tilde{D} = (d_1, d_2, d_3) \) becomes an interval of \( D = [d^m, d^n] \) considering value for alpha. The following process is then performed to optimize the mathematical model considering the demand interval.

Step 1: Set the demand value at the lower limit of \( d^m \) and determine the optimal value of each of the objective functions and name them as \( sf_1^m, sf_2^m, sf_3^m \).
Step 2: Set the demand value at the lower limit of \( d^* \) and determine the optimal value of each of the objective functions and name them as \( f_1^n, f_2^n, f_3^n \).

Step 3: State the optimal amount of each goal using the following equation.
\[
\begin{align*}
f_1^* &= \alpha f_1^m + (1-\alpha)f_1^n \\
f_2^* &= \alpha f_2^m + (1-\alpha)f_2^n \\
f_3^* &= \alpha f_3^m + (1-\alpha)f_3^n
\end{align*}
\]

3.4. Multi-Objective Simulation Annealing Algorithm

The Multi-Objective Simulation Annulling (MOSA) is a meta-heuristic algorithm based on the Simulation Annulling (SA) algorithm’s overall structure. Due to the existence of more than one goal for optimization in this algorithm, the answers' superiority in each step is based on the concept of non-dominance. Answer \( x \) is dominant to answer \( y \) if the value of each objective function for answer \( x \) is better than its equivalent for answer \( y \). In each iteration in the MOSA algorithm, the answers' dominance relative to each other is checked after generating a neighborhood answer. If one answer is dominated by the other, we save it in the list of non-dominant answers. Otherwise, the answers are checked based on the probability of Relation 38, and one of them is deleted, and the other is used in the next step. Therefore, generally, MOSA and SA’s main difference is how to delete the answers and apply new solutions.

\[
p(\text{accept}) = \begin{cases} 
1 & \text{if } \Delta f \leq 0 \\
\frac{\Delta f}{e^c} & \text{if } \Delta f > 0
\end{cases}
\]

In the above Relation, \( P \) is the probability of accepting the next point. It \( \Delta f \) is the changes in the objective function for the established neighborhood, and \( C \) is the control parameter, which is considered equal to the current temperature. A stop criterion is required to complete this algorithm. One criterion for this purpose can be reaching the final temperature. Another criterion is the degree to which the answer does not improve in a certain number of iterations.

In this research, the initial temperature value is 1000, and the temperature reduction rate is equal to 0.01 of the previous stage temperature for the solved examples (Sharifi et al., [28]). In other words, \( T_{t+1} = 0.99 \times T_t \) the stopping criterion is no improvement in the last 100 repetitions or reaching a temperature of less than 1.

4. Computations and results

First, the proposed mathematical model was validated. In order to determine the validity of the model and the accuracy of its performance, an example of the problem
generated in GAMS software was solved with linear programming SOLVER called CPLEX on a personal computer with Intel Core i5-3230M 2.6GHz processor and 6 GB of executive RAM with Windows 8 version 1. The data for this example is provided in Table 1.

| Insert Table 1 here |

Other problem parameters are randomly assigned. Since the mathematical model is multi-objective and GAMS software solves the mathematical model in a single objective, the objects presented to this software are a total of 3 objective functions presented in the mathematical model. Problem-solving is done with GAMS software and with a BARON solver. The optimal value of each of the objective functions is shown in Table 2.

| Insert Table 2 here |

Since the most important elements of this chain are plants, distribution centers, and recycling and disposal centers, the following outputs regarding location are presented after solving the mathematical model. Then, the supplier selection is determined. The number 0 means no selection, and the number 1 means the supplier selection, which is shown in Table 3.

| Insert Table 3 here |

The plant’s location is also indicated in Table 4.

| Insert Table 4 here |

The results related to distribution centers, collection, and hybrid location are shown in Table 5.

| Insert Table 5 here |

Considering that the answers obtained for decision variables are feasible and consistent with the manual analysis, then the proposed mathematical model is efficient and valid. The efficiency of the proposed meta-heuristic algorithms for solving the desired model is analyzed in the following. First, it is necessary to optimize the value of the algorithm parameters. To do this, the technique of designing experiments will be used based on the Taguchi method.

4.1. Designing experiments for MOSA algorithm parameters
Based on the Taguchi method structure, three values are first proposed for each of the MOSA algorithm parameters. The suggested values are shown in Table 6.

The following modes of the MOSA algorithm are implemented based on the Taguchi L9 scheme, and its outputs are presented in Table 7.

After entering this information into MINITAB software and implementing the Taguchi method, the S/N diagram is presented in Figure 2.

According to the diagram above, a value with the lowest S / N value is appropriate for each parameter. Therefore, the values shown in Table 8 are optimal values relating to the MOSA algorithm, and other examples will be executed with these values.

4.2. Numerical results

It is required to measure the MOSA algorithm's performance in several examples in different dimensions to evaluate the introduced algorithm's performance. For this purpose, 11 examples in different dimensions have been generated. Information about these examples is provided in Table 9.

In Table 9, $S$ is the number of suppliers, $I$ is the potential plants, $J$ is distribution, collection, and hybrid centers, $C$ is the number of customers, $K$ is the number of potential disposal centers, $P$ is the number of products, $R$ is the number of raw materials, $L$ is the number of transportation systems, and $T$ is the number of studied periods. The examples generated in GAMS software are solved with a time limit of 3600 seconds and solved with the MOSA algorithm. It should be noted that the MOSA algorithm provides several answers in the form of the Pareto boundary. However, GAMS software only presents one answer as the optimal answer. Now, in order to better compare these two solution methods, the answer with the highest value of swarm index as a candid answer from MOSA is compared with the answer provided by GAMS. The swarm index is calculated as follows.
\[ d(k) = \sum_{i=1}^{n} \frac{f_i(k-1) - f_i(k+1)}{f_i^\text{max} - f_i^\text{min}} \]  

(39)

In Relation (39), \( d \) is the swarm index value, and \( k \) is the counter of Pareto boundary responses; \( n \) is the number of goals, and \( f \) represents the value of the goal function for each goal for the \( k \)th answer the Pareto boundary. The answer that has the highest value of the swarm index is very close to the other answers. In other words, the answer in the middle of the Pareto border is known as the answer with the highest swarm index. After identifying this answer, each of its objective functions' value is reported in Table 10 and compared with its equivalent value in GAMS. It should also be noted that the alpha cut method has been used due to the fuzzy amount of demand. In all solved examples, the alpha value is assumed to be 0.75. Table 10 summarizes the results of these examples.

According to Table 10, \( z_1 \) to \( z_3 \) are the three objective functions obtained from both methods. 'Time' is the execution time by both methods. 'GAP' provides the error rate of the MOSA algorithm. As can be seen, GAMS software has not been able to solve the last two examples. On the other hand, it has consumed the entire defined time in examples 7, 8, and 9. In other words, the optimization of these examples in GAMS software has been performed for a longer time, but it has stopped after 1 hour due to the time limit of 3600 seconds. The MOSA algorithm solves all the examples presented in less than 1 minute, while the average solution time of GAMS software was 1847 seconds. The following Figure compares the solution times of the two methods.

4.3. Checking the efficient border of the MOSA algorithm

Since this algorithm optimizes the problem in a multi-objective way and its output includes several answers (the efficient boundary of a multi-objective problem), it is
necessary to examine this algorithm’s features in terms of different solutions of the optimal center. Several indicators are provided to evaluate the performance of multi-objective meta-heuristic algorithms. These criteria include Mean Ideal Distance (MID), and Maximum spread or diversity (MD), relative distance from straight answers (SM), and outstanding achievement (RAS). The following is the method of calculating the above indicators:

The MID criterion is used to calculate Pareto’s average distance from the ideal answer or, in some cases, from the origin of the coordinates. In the following Relation, it is clear that the lower this criterion, the higher the efficiency of the algorithm. In this Relation, NOS is the number of answers, g shows the objectives, and sol shows the answers.

$$MID = \frac{1}{NOS} \sum_{sol=1}^{n} \sqrt{\sum_{g=1}^{2} f_{sol,g}^2}$$ (40)

The maximum diversity (MD), proposed by Zetzeler, measures the length of the space cube diameter used by the end values of the objectives for the set of non-dominated solutions. The Relation shows the computational procedure of this index. The larger values for the criterion are more desired.

$$MD = \sqrt{\sum_{g=1}^{2} \left(\max_{sol} f_{sol,g} - \min_{sol} f_{sol,g}\right)^2}$$ (41)

The SM index calculates how Pareto answers are distributed using the relative distance of consecutive answers.

$$SM = \frac{\sum_{m=1}^{M} d_{m}^e + \sum_{i=1}^{\left|A\right|} \left|d_{i} - \bar{d}\right|}{\sum_{m=1}^{M} d_{m}^e + \left|A\right| \left|\bar{d}\right|}$$ (42)

In this equation, M is the number of objectives, and di shows distance. $d_{m}^e$ is the distance between the optimal Pareto boundary’s side solutions and the Pareto boundary obtained in the $m^{th}$ objective function. The lower the value of this measure, the better the boundary obtained.

The RAS index, calculated based on the following equation, shows the simultaneous achievement of all objective functions’ ideal value. The lower the value of this index, the higher the efficiency of the algorithm.

$$RAS = \frac{\sum_{i=1}^{n} \left|f_{1i} (x) - f_{1i}^{best} (x) + f_{2i} (x) - f_{2i}^{best} (x)\right|}{n}$$ (43)
Then, for 11 solved examples, $MID$, $MD$, $SM$, and $RAS$ indices are calculated and presented in the Table 11 and Figure 4.

Insert Table 11 here

Insert Figure 4 here

The average MID index for the MOSA algorithm is 150878. Figure 4 shows the trend of this indicator in different examples. The value of this index will increase with increasing the problem dimensions due to this index’s nature. Accordingly, the MOSA algorithm should increase the value of this index according to the problem dimensions. As can be seen in Figure 4, the MOSA algorithm has done it well.

The average MD index for the MOSA algorithm is 5162. Figure 4 shows the value of this index for different examples. The MD index is not related to the problem dimensions. Therefore, it is expected that this index’s value has a relatively similar trend in different examples. As can be seen, there is a relatively similar trend in this index in all examples except in examples 7 and 9 (due to algorithm error).

The average of the SM index is 6164. Figure 4 shows the value of this index for different examples. As mentioned before, the lower the value of this index, the better the status. This is well seen in the first six examples, and small amounts of this index are given. The sudden increase in this index’s value from Examples 9 onwards is due to the enlargement of the problem dimensions and the complexity of finding its optimal boundary.

After running the sample examples, the average value of the RAS index is about 0.204. Figure 4 also shows the value of this index in various examples. Examining the above chart, it is clear that this index’s value, in most examples, was between 0.25 and 0.45. The index’s value does not change much due to averaging this index while increasing the problem dimensions. It should be noted that the lower the index value, the proximity of the found Pareto boundary to the optimal boundary is further approved.

4.4. Discussing the results

The numerical results obtained in this study are discussed in this section. After designing the meta-heuristic algorithm, 11 examples were run in different dimensions with this algorithm’s help, and the results are reported separately. The trend of increasing the problem dimensions has affected the objective function’s values and the studied indices, which are briefly expressed below.

1. Increasing the problem dimensions means increasing the limits of the problem indices, increasing each objective function’s values.
2. Based on the comparisons, increasing the problem dimensions leads to a sharp increase in the MID index.

3. If the problem dimensions increase, the SM and MD indices increase relatively. However, it is possible to create fluctuations in these indicators in some problems.

4. Increasing the problem dimensions does not affect the limits of the RAS index values, and this is due to the nature of averaging in this index.

Also, it is necessary to compare these results with similar research in order to prove the superiority of the obtained numerical results. Accordingly, Pishvae et al., 2014 have been found to have only evaluated one product and one period, while the present research simultaneously optimizes the supply chain in multi-product and multi-period modes. Therefore, its results will be closer to the real conditions of supply chains. Ramezani et al. [1] are another important researches in this field. In this study, the two objectives of increasing profits and increasing service levels have been evaluated. In this research, the Epsilon Constraint method has been used to solve the problem. Although the method proposed in this research is inefficient in solving large-scale problems, the method proposed in this research can solve problems in all possible scales [29].

5. Conclusion and further studies

The presented mathematical model was firstly validated. This algorithm’s parameters are first adjusted to solve the model with the MOSA\(^1\) algorithm, and then 11 different examples are designed using this algorithm. The reason for using the MOSA algorithm compared to the SA algorithm to solve the problem is the ability to optimize multiple goals simultaneously. The best way to evaluate this algorithm's performance is to compare the results' objective function values obtained from this algorithm with the exact solution value in GAMS software. For this purpose, 11 examples were produced in different dimensions to evaluate this algorithm's ability to solve different examples. Of the 11 examples solved, GAMS only managed to solve 9 of them. However, the proposed algorithm solves all 11 examples with an average error of .3% for the first objective function, 1.7% for the second objective function, and .7% for the third objective function.

On the other hand, the GAMS software time to solution on examples 7, 8, and 9 was precisely 3600 seconds, equivalent to one hour. However, the MOSA algorithm’s average solving time for all solved examples is 25 seconds, and all the examples are solved in less than 60 seconds. Therefore, it can be concluded that a trade-off is created between the quality of the solutions and time to solution to choose between the MOSA algorithm and

\(^1\) Multi-Objective Simulated Annealing
GAMS, as shown in Table (10) and Figure (3). The average time to solution by GAMS software is 1847 seconds and the average time to solution by MOSA is 25 seconds. That is, an average decrease of 730% is created, and at the same time, an average error of 0.3% for the first objective function, 1.7%, and 0.7% for the second and third objective functions should be considered in the MOSA method. The trade-off between the time and the solutions' quality shows the MOSA algorithm's outstanding performance in reducing the time to solve the problem ahead and providing near-optimal solutions.

On the other hand, since the MOSA algorithm introduces a set of solutions as the Pareto problem, it is necessary to examine the characteristics of the set of solutions from the Pareto boundary evaluation indices. Accordingly, four different indices have been introduced in this field, and the value of these indices has been calculated for all solved examples. By analyzing the trend of these indices' values on different examples, it can be well pointed out that the Pareto boundary created by the MOSA algorithm covers well an integrated boundary and all the Pareto frontal space.

5.1. Implications for researchers

As a planning process, executing and controlling operations and raw materials storage, supply chain management is critical in various industries during operations and finished products from the starting point to the endpoint of consumption. Hence, optimizing and synchronizing the supply chain is conducted in this research using heuristic algorithms to reduce costs, improve quality, and achieve a competitive advantage and position. The goal of optimization in this area is to improve the quality and 'customers' satisfaction and reduce the time of production and its related price. This research aims to design a multi-objective optimization algorithm for multi-period and multi-product reverse logistics problems. First, due to the uncertainty of some parameters and considering the discounts and financial flows, the fuzzy mathematical model is presented, then the optimal MOSA algorithm is designed to solve it. Three objectives were optimized simultaneously in this model. The first objective is to maximize the value of the chain profit; the second objective is to minimize the transition times. The third objective is to minimize defective parts purchased. This algorithm's average error for each of the three objectives understudy was less than 2%. These illustrate the efficiency of the MOSA algorithm in solving the problem presented in this study. Finally, the performance of the MOSA algorithm compared to the GAMS method shows that GAMS software cannot provide a solution for some large-scale problems, while the MOSA algorithm is well able to provide the optimal solution with minimum error for different conditions. The MOSA algorithm solves all the examples presented in less than
1 minute. However, the average time to solve was 1847 seconds for the GAMS software. This study’s results are consistent with Lee and Kwon [27] and Braido et al. [9] research. The objectives and parameters considered in this study have been increased in terms of complexity and number, but with optimized design, the algorithm has achieved an average error of 0.3% for the first objective function, 1.7% for the second objective function, and 0.7 for the third objective function. Also, despite being multi-objective, the convergence time in this study is less than 1 minute, which has also reduced the time compared to previous works (Braido et al., [9]; Lee and Kwon, [27], Yang et al., [11]), which shows the efficiency of this algorithm compared to previous research. Accordingly, if we look at previous research (Pishvaee et al., [8]; Ramezani et al., [1]), they considered only one product and in one period or used inefficient methods to solve the problem on large scales. While the present research simultaneously optimized the supply chain in the multi-product and multi-period modes, its results will be closer to the supply chains' actual conditions. Also, the method proposed in this research can solve problems in larger dimensions. Adopting the right strategy to improve supply chain performance brings many benefits, such as saving energy resources, reducing pollutants, eliminating or reducing waste, creating value for customers, and ultimately improving companies and organizations' productivity. Since the closed-loop SCN consists of facilities to achieve this goal, and since customers’ demand is uncertain, this factor is necessary to find the required number of facilities and the amount of flow transmitted between them.

5.2. Suggestions for future research

The supply chain design problem has become more complex, and more elements are needed today according to the new global regulations and considering the environmental protection rules. It is suggested to use dynamic systems and simulation models to consider different parameters. Supply chain design can also take into account the impact of uncertainties and various parameters on it. Besides, more and more parameters such as financial considerations, risks, and uncertainties can be considered in other models. Other optimization methods and fuzzy programs with different indices can also be considered. Finally, an effective and accurate heuristic solution for larger-size problems can be developed and compared with the method presented here in terms of time and accuracy.

As one of the limitations of this method, the MOSA algorithm requires many initial selections to become an optimal solution method. There should also be a trade-off between the optimization time and the convergence of the final answer so that too much
time can reduce the answer’s accuracy. The sensitivity to optimization parameters, which affects algorithm performance quality, is another limitation of this method. Therefore, to resolve each algorithm’s weaknesses, it is suggested to use a combination of different algorithms such as genetics and annealing simulation to optimally solve the multi-objective multi-period and multi-product reverse logistics problem in future research.

6. Reference


Caption of the tales

Table 1. Model validation example data.
Table 2. Value of objective functions obtained from GAMS software.
Table 3. Selected suppliers in optimal mode.
Table 4. Selected plants in an optimal mode.
Table 5. Selected distributors in an optimal mode.
Table 6. Parameters and their values levels for the MOSA algorithm.
Table 7. Value of answer variable in the Taguchi technique for MOSA.
Table 8. The optimal value of MOSA parameters.
Table 9. Information on generated problems.
Table 10. The output of solved problems.
Table 11. MOSA algorithm output for solved examples.
Caption of the figures

Fig. 1: The SCN of this work.
Fig. 2: Output for the Taguchi method in the MOSA algorithm.
Fig. 3: Comparison of computational times of GAMS and MOSA.
Fig. 4: Comparison of MOSA algorithm based on indices.
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Table 1. Model validation example data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of products</td>
<td>3</td>
</tr>
<tr>
<td>Number of suppliers</td>
<td>3</td>
</tr>
<tr>
<td>Number of factories</td>
<td>4</td>
</tr>
<tr>
<td>Number of distribution, collection, and combination centers</td>
<td>5</td>
</tr>
<tr>
<td>Number of customers</td>
<td>7</td>
</tr>
<tr>
<td>Number of disposal centers</td>
<td>3</td>
</tr>
<tr>
<td>Number of raw materials</td>
<td>2</td>
</tr>
<tr>
<td>Number of transportation systems</td>
<td>2</td>
</tr>
<tr>
<td>Number of periods</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 2. Value of objective functions obtained from GAMS software.

<table>
<thead>
<tr>
<th>Objective function</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>First goal (maximizing current value)</td>
<td>165785</td>
</tr>
<tr>
<td>The second objective function (minimizing sending times)</td>
<td>3497</td>
</tr>
<tr>
<td>Third Objective Function (minimizing Defective Items)</td>
<td>2794</td>
</tr>
</tbody>
</table>
Table 3.
Selected suppliers in optimal mode.

<table>
<thead>
<tr>
<th>Supplier</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selected/not selected</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 4.
Selected plants in an optimal mode.

<table>
<thead>
<tr>
<th>Warehouse</th>
<th>1</th>
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<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selected/not selected</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 5.  
Selected distributors in an optimal mode.

<table>
<thead>
<tr>
<th>Retailer</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selected/not selected</td>
<td>Distribution center</td>
<td>Hybrid center</td>
<td>0</td>
<td>0</td>
<td>Disposal center</td>
</tr>
</tbody>
</table>


Table 6.
Parameters and their values levels for the MOSA algorithm.

<table>
<thead>
<tr>
<th>Solving algorithm</th>
<th>Parameter</th>
<th>Values of each level</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOSA</td>
<td>Number of neighborhood production per iteration (NM)</td>
<td>2 3 5</td>
</tr>
<tr>
<td></td>
<td>Initial temperature (T)</td>
<td>500 1000 1500</td>
</tr>
<tr>
<td></td>
<td>Temperature reduction coefficient (alpha)</td>
<td>0.85 0.9 0.95</td>
</tr>
<tr>
<td></td>
<td>Max-iteration</td>
<td>100 200 300</td>
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Table 7.  
Value of answer variable in the Taguchi technique for MOSA.

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<th>Run order</th>
<th>Algorithm parameters</th>
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<td></td>
<td>NM</td>
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Table 8. The optimal value of MOSA parameters.

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<th>Solving algorithm</th>
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<th>Optimal value</th>
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<tr>
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<td>Initial temperature (T)</td>
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<tr>
<td></td>
<td>Temperature reduction coefficient (alpha)</td>
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Table 9.
Information on generated problems.

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<th>S</th>
<th>I</th>
<th>J</th>
<th>C</th>
<th>K</th>
<th>P</th>
<th>R</th>
<th>L</th>
<th>T</th>
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Table 10.
The output of solved problems.

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<th>NO</th>
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<th>GAP(%)</th>
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<td>$z_2$</td>
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<td>1294</td>
<td>671</td>
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<td>P3</td>
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<td>3478</td>
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<td>P4</td>
<td>139115</td>
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Table 11.
MOSA algorithm output for solved examples.

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