Stability Analysis of Sloping Grounds in Unsaturated Soils using the Method of Stress Characteristics

Sadeq Qojevand[1*], Ghassem Habibagahi[2] and Mehdi Veiskarami[3]

[1*] Graduate Student, School of Engineering, Shiraz University, Iran
[2] Professor, School of Engineering, Shiraz University, Iran
[3] Associate Professor, School of Engineering, Shiraz University, Iran

Email addresses: sadeqqojevand@gmail.com (Corresponding author. Tel.:)
sadeqqojevand@gmail.com (Prof. Habibagahi)
mveiskarami@shirazu.ac.ir (Dr. Veiskarami)

Abstract

An extension of the slip line theory to unsaturated soils is presented and applied to the problem of stability of slopes in unsaturated soils. The matric suction is then introduced and implemented in governing equations by the well-known Bishop effective stress concept. In this regard, the van Genuchten model is utilized to estimate the effective stress parameter required for the computation of the effective stress. The influence of the soil matric suction on the stability of slopes has been investigated for a variety of soils under steady state evaporation and infiltration, i.e. the distribution of the matric suction was assumed to remain constant with time. In addition, a measure of stability in terms of some factor of stability has been introduced.

Keywords: Unsaturated Soil, Slip Lines, Slope Stability, Suction, Van Genuchten Model

1- Introduction

The classical soil mechanics deals with a variety of typical problems including the bearing capacity of shallow and deep foundations [1-11], earth pressure problems [12-17] and stability of slopes [18-23]. In most of these problems the soil is regarded as either completely saturated or dry. However, in practice, soil above the ground water table is unsaturated and often non-zero matric suction exists. This suction gives rise to more strength in soil structure compared to those in saturated and dry states. One of the interesting subjects in the geotechnical engineering is the stability of slopes in either saturated or unsaturated state. The latter is relatively less addressed compared to other typical classical problems in geotechnical engineering [24-28]. However, this is a practical problem in many countries around the world, where the soil in those areas rarely experiences the state of fully saturation. In some areas, like Hong Kong and Malaysia there are lack of flat lands for construction purposes and cutting slopes have proved inevitable [29]. Many of such slopes are of unsaturated soils and they require special attention. Moreover, it is imperative to gain behavioral knowledge of unsaturated soils to predict the potential danger and take appropriate measures.

Slope stability problems are usually approached using the limit equilibrium method, limit analysis method and the method of stress characteristics. In limit equilibrium approach, a slip line is considered and soil above this slip line is often assumed to be a rigid material and divided into several blocks. Depending on the assumption on the distribution and magnitude of the inter-slice forces, the factor of safety is obtained in terms of force and moment equilibrium along the slip surface. The surface of the least factor of safety is regarded as the critical slip surface [24, 30-32]. Failure load in limit analysis method is reported approximately between two bounds. The lower bound theorem and the upper bound theorem specify these two limits [33-37]. The method of stress characteristics on the other hand, considers the soil as a continuous matter. Governing equations are obtained by simultaneously combining the equilibrium and yield surface equations. Sokolovskii [38] assumed the body to be at the state of limiting equilibrium and combined the equilibrium and yield equations for constant cohesion and internal friction angle, presenting a solution to some typical geotechnical problems including the bearing capacity and stability of slopes. Using this method, Harr [39] solved some problems in plasticity of soil. Later, Bolton and Lau [40] used the method to present the second and third bearing capacity factors for strip and circular foundations. Kumar and Mohan Rao [41] employed this method to find the bearing capacity under static and seismic conditions. Martin [42,43] applied this method to find a rather precise estimation of the third bearing capacity factor. Thanh and Russell [44] applied this method on unsaturated soils to investigate the effect of suction on limiting passive strength of the soil behind the retaining wall. Veiskarami and Zanj [45] investigated the stability of sheet-pile walls subjected to seepage flow by Slip Lines and Finite Elements methods. Johari *et al.* [46] evaluated the reliability analysis of seismic ultimate bearing capacity of strip footing using the Slip Line Method combined with random field theory. Each method clearly has its own restrictions and advantages. The greatest advantage of the slip line method compared to other methods is its independence of the soil constitutive model and also its compatibility with real soil behavior, e.g. in detection of failure mechanisms and estimation of limit loads.

In the current study, the soil is assumed to obey the Mohr-Coulomb yield criterion. The method of stress characteristics is then extended to the problem of stability of slopes of unsaturated soils. The shear strength of soils is related to the effective stress, using Bishop's [47] widely employed effective stress expression for unsaturated soils. The effective stress is described as the summation of two independent stress state tensors as follows:

$$\boldsymbol{\sigma}_{ij}^{'} = (\boldsymbol{\sigma}_{ij} - u_a \boldsymbol{\delta}_{ij}) + \boldsymbol{\chi}(u_a - u_w)\boldsymbol{\delta}_{ij} \tag{1}$$

In this equation σ_{ij} is a component of the stress tensor, σ_{ij}' is the effective stress, u_a is the pore air pressure and u_w is the pore water pressure. The multiplier χ is the well-known Bishop's effective stress parameter associated to the degree of saturation, but not necessarily in a linear fashion. Many researchers presented relationships for estimating Bishop's effective stress parameter. In this regard, Vanapalli *et al.* [48], Khalili and Khabbaz [49] also Khalili *et al.* [50] and Vaunat and Casini [51] have proposed the foremost correlations.

The main objective of this research is to estimate a factor of stability against failure for slopes of unsaturated soils, implementing the method of stress characteristics undergone some modifications to encompass the unsaturated condition. In connection with this point, van Genuchten [52] model is used to estimate parameter χ. The suction is assumed to be in steady state evaporation and infiltration. The factor of stability is defined in a meaningful manner to be applicable for engineering purposes. Since no flow of water is considered through the media, the matric suction varies linearly with distance from ground water table and also there will be no variation of matric suction in horizontal plane. This might seem an unrealistic assumption as the entire field could be affected by the rainfall pattern. But the variations caused by rainfall are mostly seasonal and gradual over the time. Our work in this respect could be regarded in a particular fixed instance in the time. The boundary conditions consist of a traction free surface of the slope. Governing equations are formed using the finite difference numerical solution strategy which is well described in the literature (e.g. Bolton and Lau [40]; Anvar and

Ghahramani [53]). The numerical results are then compared with conventional data wherever possible. The term conventional in this paper is referred to the state where stresses are computed without taking matric suction into account (i.e. fully saturated or dry). Five types of soils with different van Genuchten parameters including air entry value were subject of this study. The air entry value is the matric suction separating saturated state from unsaturated state. It is in fact defined as the matric suction value exceeded before air recedes into the soil pores [54]. Analyses are carried out for different slope angles and slope heights in conventional and unsaturated states. It is indicated that the increase in the matric suction tends to increase the stability of a slope in an unsaturated soil. This increase is particularly significant for fine grain soils.

Another issue which should be investigated in a separate research is the deformation problem often followed by surface settlement. This issue is out of the scope of this work. There are some relevant researches which can be found in the literature in this regard [44,55].

2- Soil Model

As it has been already stated, the soil in this study was assumed to obey a rigid-perfectly plastic behavior consisting with assumptions used in the method of stress characteristics obeying the Mohr-Coulomb yield criterion. The suction is introduced as a cohesion intercept or the so called apparent cohesion. It is important to note that both Bishop's effective stress parameter and the matric suction vary in the media and hence, the cohesion intercept changes with depth. Therefore, this makes the results different from those formerly done by assuming a constant cohesion intercept in the media using the method of stress characteristics, e.g. Jahanandish *et al.* [8]. Furthermore, the main point of the present study is using the method of stress characteristics for slope stability problems rather than other subjects such as bearing capacity and retaining walls, e.g. Thanh and Russell [44]. The soil itself shows no cohesion when it is subject to conventional analysis. Figure 1 shows the Mohr stress circle corresponding to these two states. For unsaturated state we have:

$$\frac{(\sigma_1^{'}-\sigma_3^{'})}{2}=(H+\frac{\sigma_1^{'}+\sigma_3^{'}}{2})\sin\phi' \tag{2}$$

where

σ_1' and σ_3' are principal stresses

ϕ' is the effective angle of internal friction

c' is the apparent cohesion which is defined as follows:

$$c'=-\chi u_w \tan\phi' \tag{3}$$

$$H=c'\cot\phi' \tag{4}$$

Note that the pore air pressure is assumed to be the atmospheric pressure.

The van Genuchten model has been used to estimate the Bishop's effective stress parameter χ. This model relates χ and the matric suction as follow:

$$\chi=\frac{1}{\left[1+(\alpha h)^n\right]^m} \tag{5}$$

Where h is the matric suction, α, m and n are model parameters in the Soil Water Retention Curve (SWRC) of soils. The SWRC diagrams enable us to illustrate parameters such as the degree of saturation, the coefficient of hydraulic conductivity, the normalized gravimetric water content (which in this paper is the Bishop's effective stress parameter, using van Genuchten formula) and so on, in terms of the matric suction.
The equilibrium equations in two dimensions for plane strain problem of any arbitrary material in absence of horizontal body forces (e.g. without a seismic force) are as follows:

$$\begin{cases} \dfrac{\partial \boldsymbol{\sigma}_x}{\partial x} + \dfrac{\partial \boldsymbol{\tau}}{\partial z} = 0 \\ \dfrac{\partial \boldsymbol{\tau}}{\partial x} + \dfrac{\partial \boldsymbol{\sigma}_z}{\partial z} = \boldsymbol{\gamma} \end{cases} \tag{6a}$$

$$\begin{cases} \dfrac{\partial \boldsymbol{\sigma}'_x}{\partial x} + \dfrac{\partial \boldsymbol{\tau}}{\partial z} = 0 \\ \dfrac{\partial \boldsymbol{\tau}}{\partial x} + \dfrac{\partial \boldsymbol{\sigma}'_z}{\partial z} = \boldsymbol{\gamma}_e \end{cases} \tag{6b}$$

The parameters γ and γ_e in these equations are body forces acting in vertical direction which in this case are soil density in conventional analysis and effective soil density i.e. unsaturated analysis respectively. This system of equations can be used by augmenting the yield criterion as the third equation to render it into a solvable system of three equations into three unknowns. The procedure is studied and solution can be found by the method of stress characteristics. Soil obeying the Mohr-Coulomb yield criterion possesses two directions along which slip will occur when the limiting state of equilibrium is reached. These directions are the well-known characteristics of the stress at the limiting equilibrium (i.e. at yield). At this state, θ, is the angle between the major principal stress direction and the positive x axis. Slip directions make an angle of $\mu = \frac{\pi}{4} - \frac{\phi\prime}{2}$ with θ. These directions are called *Characteristics Directions* or *Slip Lines.* These directions are depicted in Figure 2.

By introducing the stress invariant $\bar{\sigma}$, the effective mean stress, as $\bar{\sigma} = (\sigma'_x + \sigma'_z)/2$, the normal and shear stresses can be found in terms of parameters $\bar{\sigma}$ and θ as follows:

$$\boldsymbol{\sigma}'_x = (\bar{\boldsymbol{\sigma}} + H)(1 + \sin\boldsymbol{\phi}' \cos 2\boldsymbol{\theta}) - H \tag{7a}$$

$$\boldsymbol{\sigma}'_z = (\bar{\boldsymbol{\sigma}} + H)(1 - \sin\boldsymbol{\phi}' \cos 2\boldsymbol{\theta}) - H \tag{7b}$$

$$\boldsymbol{\tau} = (\bar{\boldsymbol{\sigma}} + H)\sin\boldsymbol{\phi}' \sin 2\boldsymbol{\theta} \tag{7c}$$

σ'_x and σ'_z are effective normal stresses and τ is shear stress along the soil element. Substituting Equations 7 into 6, and considering c' and ϕ' vary within the field, the set of stress characteristics equations can be found as follows:

Along the positive *characteristics* direction: $\dfrac{dz}{dx} = \tan(\boldsymbol{\theta} + \boldsymbol{\mu})$

$$d\overline{\sigma}+2(\overline{\sigma}\tan\phi^{'}+c^{'})d\theta=\gamma_e(\tan\phi^{'}dx+dz)+(\overline{\sigma}-H)(\frac{\partial\phi^{'}}{\partial z}dx-\frac{\partial\phi^{'}}{\partial x}dz)+(\frac{\partial c^{'}}{\partial z}dx-\frac{\partial c^{'}}{\partial x}dz) \quad (8)$$

Along the negative *characteristics* direction: $\frac{dz}{dx}=\tan(\theta-\mu)$

$$d\overline{\sigma}-2(\overline{\sigma}\tan\phi^{'}+c^{'})d\theta=-\gamma_e(\tan\phi^{'}dx-dz)-(\overline{\sigma}-H)(\frac{\partial\phi^{'}}{\partial z}dx-\frac{\partial\phi^{'}}{\partial x}dz)-(\frac{\partial c^{'}}{\partial z}dx-\frac{\partial c^{'}}{\partial x}dz) \quad (9)$$

In this study, the internal friction angle is taken constant as an independent variable and since the ground water level is constant, variations of apparent cohesion happen in z direction only. In addition, the negative pore water pressure is assumed to increase linearly above the groundwater level. Thus, Equations 8 and 9 yield to following equations:

$$d\overline{\sigma}+2(\overline{\sigma}\tan\phi^{'}+c^{'})d\theta=\gamma_e(\tan\phi^{'}dx+dz)+[\chi(-\gamma_w dx)+u_w\frac{\partial\chi}{\partial z}]\tan\phi^{'} \quad (10)$$

$$d\overline{\sigma}-2(\overline{\sigma}\tan\phi^{'}+c^{'})d\theta=-\gamma_e(\tan\phi^{'}dx-dz)-[\chi(-\gamma_w dx)+u_w\frac{\partial\chi}{\partial z}]\tan\phi^{'} \quad (11)$$

3- Geometry and Boundary Conditions

The geometry of the problem is illustrated in Figure 3. Ground water level in this paper has been taken constant and located at the toe of the slope. As mentioned previously, since there is no flow of water considered within the soil media, based on the theories of classical soil mechanics, the soil above the water table is subject to the negative pore pressure which varies in a fashion slightly proportional to the distance above the ground water table. This assumption is quite reasonable for most soils in normal conditions, i.e. in absence of other types or sources of pressure or highly non-homogeneous nature. Parameter h, shown in Figure 3, is the matric suction. The boundary condition involves the surface of traction free of the sloping ground. However, since the computer program is incapable of starting from zero value of traction, a negligible value of surcharge q, acting on the horizontal projection of that surface, is taken into account simply for the purpose of running the program. Having tangent and normal values of surcharge pressure along the slope and the Mohr-Coulomb yield criterion (Figures 3 and 4), the unknowns $\overline{\sigma}'$, θ, x and z can be specified along the slope surface as follows:

$$\sigma_v^{'}=q\cos\alpha \quad (12)$$

$$\tau=q\sin\alpha \quad (13)$$

To find σ_h' , we have:

$$R=\tau_{\max}=\frac{1}{2}(\sigma_h^{'}-\sigma_v^{'})=[H+\frac{1}{2}(\sigma_h^{'}+\sigma_v^{'})]\sin\phi^{'} \quad (14)$$

$$\overline{\sigma}^{'}=\frac{1}{2}(\sigma_h^{'}+\sigma_v^{'}) \quad (15)$$

τ_{max} represents the maximum shear stress which is equal to the radius of the Mohr stress circle. Then utilizing the Mohr stress circle, the magnitude of θ along the slope will be found accordingly:

$$\theta = \alpha + \frac{\beta}{2} \tag{16}$$

Where β is shown in Figure 4.

$$\beta = \tan^{-1} \frac{2\tau}{(\sigma_h^{'} - \sigma_v^{'})} \tag{17}$$

Other important issue which is one of the constrains of this paper is the spatial variability of soil layers as it could affect the outcome of the work; especially in slopes of short height where plastic zone may fall in more than one layer. However, as a central assumption in this study, the slopes are assumed to be relatively high and hence, only shallow failure mechanisms are possible. In addition, the soil layer was assumed to be homogeneous and hence, soil parameters spatial variabilities which could affect the results, was not considered. This issue, however, could be investigated in a separate study in the future by other researchers.

4- Methodology and Analyses

Conventional and unsaturated analyses have been carried out on five types of soil with different van Genuchten parameters, for which, a computer code has been developed. The algorithm and the code had been verified several times for standard bench mark problems and other conditions reported in published papers. For instance, the bearing capacity of a flat ground as a special case has been examined with very good agreement with the results obtained by the algorithm (and the code) developed for this paper [8,41,53,56,57]. Verifications can be found in next sections of this paper. In all cases, the soil cohesion was neglected, although for computational purpose, a slight value of soil cohesion was assumed in calculations (e.g. 0.1kPa). It is known that slopes of cohesionless soils are unstable where the slope inclination angle exceeds the effective angle of internal friction. Also, the van Genuchten model has been utilized in order to express Bishop’s effective stress parameter in terms of the matric suction. In this regard, the van Genuchten parameters of these five types of soil have been collected from the *Soil Vision* software (*Soil Vision* is an information bank for soils containing a series of available laboratory test data including the particle size distribution, the Atterberg limit, mechanical and engineering properties of soils and so on). The data regarding the soils subject to this study is tabulated in Table 1. Analyses have been carried out on three different slope angles, four different slope heights and also two states of conventional and unsaturated conditions. In addition, the hydraulic gradient, if any, is assumed to be very low and the effect of the water flow is neglected. Thus, a total number of 24 analyses have been carried out on each soil type.

Starting from the known state of stress along the sloping ground, the slip lines were extended towards the flat ground at the top and the bearing pressure was found as a measure of stability. Once the stress characteristics network was completed within the soil media, the distribution of the limiting stresses was obtained for two states of conventional and unsaturated conditions. Although in a few methods such as the limit equilibrium method, the pressure distribution beneath the footing is assumed to be uniform, in most other methods (like the method of stress characteristics, lower bound limit analysis, finite element methods, etc.) the pressure distribution cannot be uniform. This is the character of these methods which employ field equations (not only equilibrium equations for rigid blocks) and hence, the distributions of the stress or displacement fields make the pressure distribution, inevitably, non-uniform. Many approaches like those of Brinch Hansen [3], Bolton and Lau [40], Kumar [6], etc., by different methods have non-uniform pressure distribution. Comparison, however, is possible once the "average" of the pressure is calculated and used for this purpose. Therefore, in all these methods the bearing capacity implicitly means the "average" pressure (although not uniformly distributed) beneath the footing.

The parameter *stability factor* or some measure of the bearing ratio was defined here as the ratio of the bearing pressure obtained along a flat ground in unsaturated condition, to that calculated from the conventional condition. In other words, it indicates how stable or precarious a slope is when the soil is unsaturated. The higher the bearing ratio, the more reliable it is against the failure and vice versa. So, in order to be able to compare these two states, the widths through which analyses were done along the flat ground must be the same in both states. It was observed that suction leads to more stability, especially when the slope inclination angle approaches the angle of the effective internal friction. In this state the stability factors happened to be greater than those of smaller slope angles.

5- Results

As stated before, five types of soils of different van Genuchten parameter were selected. The diagram indicating variations of apparent cohesion and Bishop's effective stress parameter of five soil samples has also been presented in Figure 5. Since the highest slope dealt with in this study was $30m$, these diagrams were plotted for this range. By making use of the van Genuchten parameters, one can easily get to *Soil Water Retention Curve* (Figure 6). All soils under study happened to possess an effective internal friction angle ranging between $\phi' = 28°$ and $\phi' = 32°$ also taken from the *Soil Vision* databank. The slight difference was neglected and so an average value has been assigned to all soil types equal to $\phi' = 30°$. Analyses were carried out for slope angles of $\alpha = 20^{\circ}$, 25° and 30°, also heights of $H = 5m$, $10m$, $20m$ and $30m$ for both conventional and unsaturated conditions. The final results of the study are presented in Figures 7-9 and Tables 2 to 6. The parameter "AEV" noted in the figures is the Air Entry value.

6- Verifications and Comparisons

Comparison of results obtained from this study involves having them confirmed in both conventional and unsaturated analyses. Since there is not any comparable data to verify the results obtained for unsaturated cases, comparisons were made only for conventional states. Results of the conventional analyses are in a good agreement with those of Meyerhof [2], Brinch Hansen [3], Vesić [4] and Kumar and Mohan Rao [6], when the angle of slope is zero, i.e. the bearing capacity problem. These results are illustrated in Table 7, also compared with Kumar and Mohan Rao [6] with different slope angles (Tables 8 and 9 and Figure 10). Parameters P_u and x in diagram of Figure 10 indicate the bearing pressure beneath the foundation and the distance from the foundation's edge (relative to the singularity point) respectively. As observed, there is a good agreement between the results of Kumar and Mohan Rao [6] and the present study. The bearing pressure starts from values very close to zero at one edge and increases linearly along the footing base.

7- Discussion

Delving further into the results of this study, it is discernible that the bearing ratio, as a measure of slope stability, increases with increase of the slope inclination angle in all soil types (Figure 7). This sounds quite reasonable since the steeper the slope, the more precarious and unstable it is; especially when the slope angle approaches the angle of effective internal friction. However, the matric suction tends to boost the soil cohesion and contribute to a greater level of stability. Therefore, a steep slope with an inclination angle near the internal friction angle, shows higher ratio of stability when it is in an unsaturated condition. This evidence is attested in Figure 8 with slopes inclined at an angle equal to $\alpha = 30°$ exhibiting the largest bearing ratios. Since the soil Type 1 is a granular soil, the reported stability factors are significantly smaller than those of the other soil types.

It is also observed that the bearing ratio decreases with increase in the height of the slope (Figures 8 and 9). Although the matric suction increases by distance from the ground water table (which in this study lies constantly on the same level as toe of the slope), this increase cannot make up the adverse effect of the height. As it is shown in Figure 7, the bearing ratio corresponding to Type 4 soil, hardly varies with increase of the slope height (especially for slope angle of $\alpha = 30°$). This is mostly due to the higher value of the apparent cohesion with respect to the other soil types. As it is shown in Figure 5, the apparent cohesion value corresponding to soil Type 4 is approximately between 3 to 17 times of those of the other types. This leads to a significantly stronger structure of the soil Type 4 and resistance against the failure.

The effect of the air entry value as an independent parameter has also been investigated. In contrary to our expectation, it is shown that there is no specific trend between the air entry value and the bearing ratio for the soils subject to this study. Figure 9 shows the bearing pressure versus the air entry value for three different slope angles and four different slope heights. Other van Genuchten parameters appear to be of greater importance, as soil Type 4, with smaller air entry value than the Type 5 soil (almost one third), has apparent cohesion of almost threefold of the Type 5 within the 30 meter range of the slope height.

Figures 11 and 12 underscore the existence of slight rotation in the mixed zone's characteristics directions in unsaturated case which does not exist in conventional state. This rotation stands out more conspicuously in slopes with angle of $= 30°$, since the slope angle is equal to the angle of the effective internal friction in this study. Figure 12 can be held up as a proof that the failure surface of a conventional analysis is almost approaching a thin narrow strip. However, due to the presence of the apparent cohesion in unsaturated state, the failure surface is developed and it is also smoother.

8- Conclusions

The influence of the matric suction on the stability of slopes has been theoretically explored, using the method of Stress Characteristics. This research was performed to introduce a model for predicting the bearing ratio, as a measure of slope stability, the ratio of the bearing pressure along a flat ground on an unsaturated medium to that of conventional state based on the solution of plasticity problems in soil mechanics. The method is capable of modeling the soil matric suction. Variation of the matric suction with depth down to the ground water level was linear. Bishop's effective stress theory in unsaturated soils was also employed to compute the shear strength and the bearing capacity of soil. In this regard, van Genuchten model has been utilized to relate the matric suction to Bishop's effective strength parameter. The results of the conventional analyses have been validated with works of Meyerhof, Brinch Hansen and Vesić bearing capacity and also Kumar and Mohan Rao. Since there were no significant comparable results in unsaturated state, the comparison in this state was precluded. It was observed that in presence of the matric suction, the slope becomes more stable and failure surfaces turn smoother. It was also observed that in contrast to general expectations, the air entry value parameter does not influence the general behavior of the unsaturated soil samples under this study and other van Genuchten parameters are of greater significance, since they directly affect the Bishop's effective stress parameter and consequently, the apparent cohesion. The soil type and the height of the slope were also found to be important factors in the general stability of a slope. In general, soils exhibiting higher apparent cohesion will have a higher tendency towards stability for a slope with a constant height. Thus, granular soils are less stable compared to other soils. On the other hand, the slope height has a reverse influence on stability as it tends to highly contribute to destabilizing the slope regardless of the slope angle.

For the future studies some suggestions could be made. For instance, the effect of soil parameters spatial variabilities as well as the effect of infiltration as a result of rain pattern can be taken into account. Also, the variation of the matric suction in horizontal direction due to the flow of water may be examined. In addition, a more efficient way of solving the governing equations, in particular for spatially variable soil layer, can be developed which is mainly related to the numerical solution techniques.

Authors' Biography

Sadeq Qojevand has received his M.Sc. and B.Sc. degrees from Shiraz University and recognized as one top alumni of Shiraz University. He has accomplished his master's thesis on the mechanics of unsaturated soils applied to some slope stability problems under joint supervision of Prof. Ghassem Habibagahi and Dr. Mehdi Veiskarami. His main research interests cover a variety of problems in the mechanics of unsaturated soils and porous media as well technical applications of theoretical soil mechanics.

Ghassem Habibagahi is professor in the School of Engineering, Shiraz University, with main research interests in mechanics of unsaturated soils and porous media as well as soil improvement techniques and geoenvironmental engineering. Prof. Ghassem Habibagahi has been recognized as one of famous faces in the unsaturated soil mechanics with a rich background in this field and a series of valuable publications in credible worldwide journals. He has graduated a number of Ph.D. and M.Sc. students over the past 30 years of his academic career and contributed in many nationwide research plans. He is recently working on dust control in arid areas of Iran as well ground subsidence as a consequences of recent climate change.

Mehdi Veiskarami is an associate professor in the School of Engineering, Shiraz University, with main research interests in engineering mechanics and geomechanics with topics covered in continuum mechanics and plasticity. He has published some works on instability problems in classical continua with focus on instabilities caused by localization of deformation into shear bands. Dr. Mehdi Veiskarami is recently working on applications of mathematics and rational mechanics in geomechanics.

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Figure captions:

Figure 1. Mohr stress circle for (a) Saturated (Conventional analysis), (b) Unsaturated soil

Figure 2. Slip lines in Cartesian coordinates system

Figure 3. Geometry of the problem and definition of parameters

Figure 4. Application of the Mohr-Coulomb yield criterion to find parameters $\sigma'$and θ along the slope surface

Figure 5. Variation of (a) Bishop’s effective stress parameter and (b) Apparent Cohesion with distance from the groundwater table

Figure 6. SWRC curves of Soils type 1-5

Figure 7. Variation of Bearing Ratio with Slope Height for slopes of different angles

Figure 8. Variation of the bearing ratio with height of the slope

Figure 9. Variation of bearing ratio with air entry value for slopes

Figure 10. Comparison of values of $p_u/\gamma B$ vs x/B

Figure 11. Slip lines for soil type 2, slope height=5m, slope of angle α =20, corresponding to (a) Unsaturated analysis, (b) Conventional analysis

Figure 12. Slip lines for soil type 2, slope height=5m, slope of angle = 30, corresponding to (a) Unsaturated analysis, (b) Conventional analysis

Table captions:

Table 1: Air entry value and van Genuchten parameters of subjecting soils of this study

Table 2: Results corresponding to soil type 1.

Table 3: Results corresponding to soil type 2.

Table 4: Results corresponding to soil type 3.

Table 5: Results corresponding to soil type 4.

Table 6: Results corresponding to soil type 5.

Table 7. Comparison of bearing capacity factors, $N_q$and N_c

Table 8. Comparison of bearing capacity factors, N_q, on sloping ground

Table 9. Comparison of bearing capacity factors, N_c, on sloping ground

Figures:

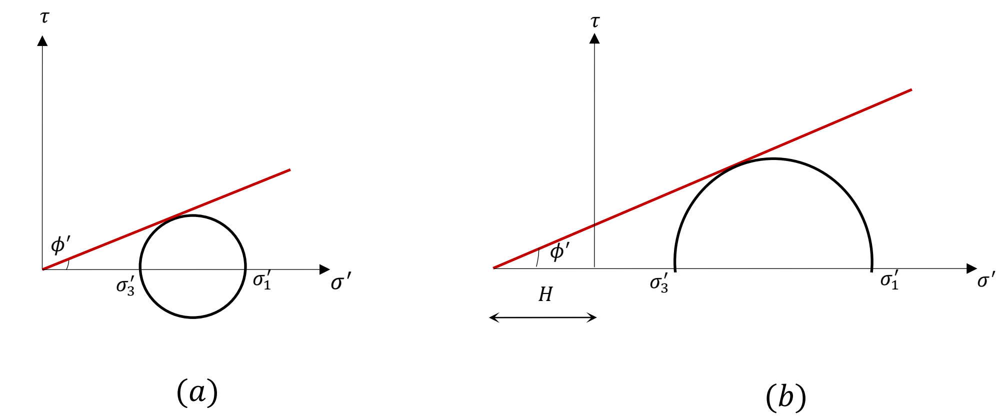


Figure 1.

Figure 2.

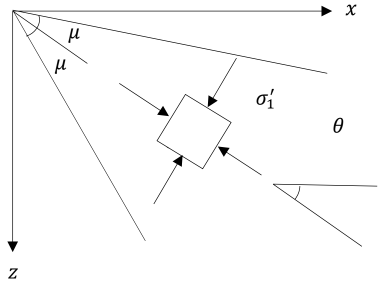


Figure 2.

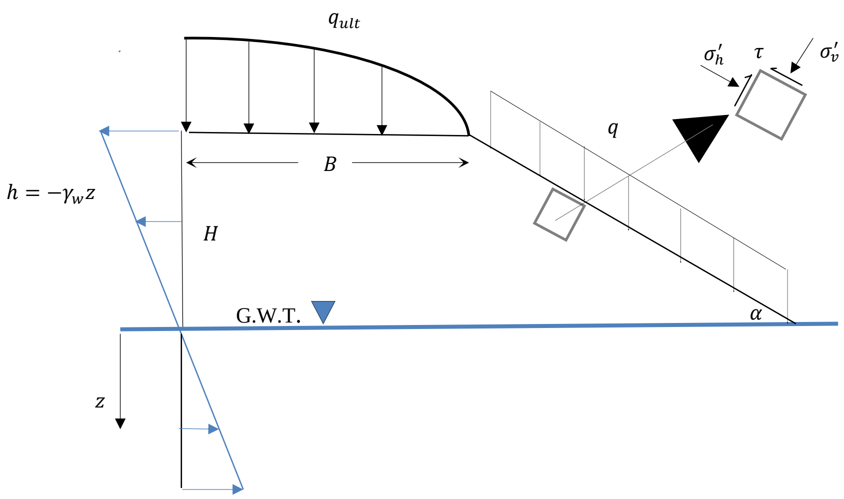


Figure 3.

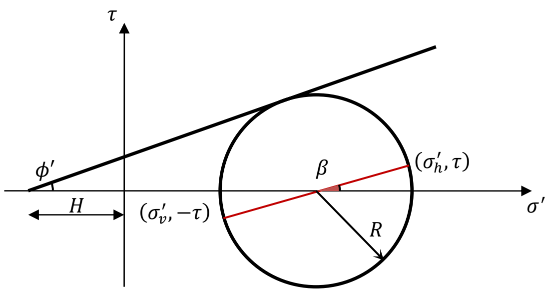


Figure 4.

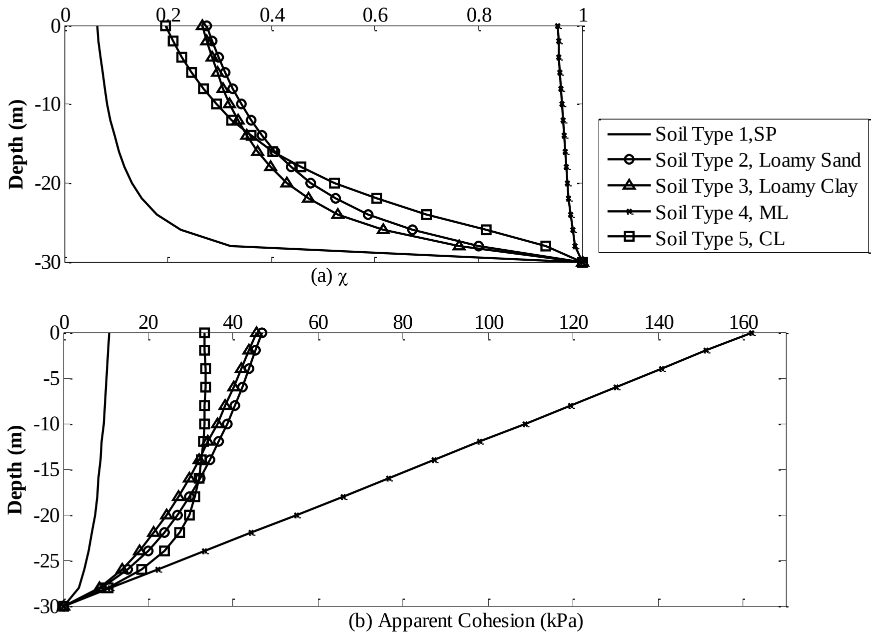

0
0.2
0.4
0.6
0.8
1
0
-10
-20
-30
Depth (m)
Soil Type 1,SP
Soil Type 2, Loamy Sand
Soil Type 3, Loamy Clay
Soil Type 4, ML
Soil Type 5, CL
(a) χ
0
20
40
60
80
100
120
140
160
0
-5
-10
-15
-20
-25
-30
Depth (m)
(b) Apparent Cohesion (kPa)

Figure 5.

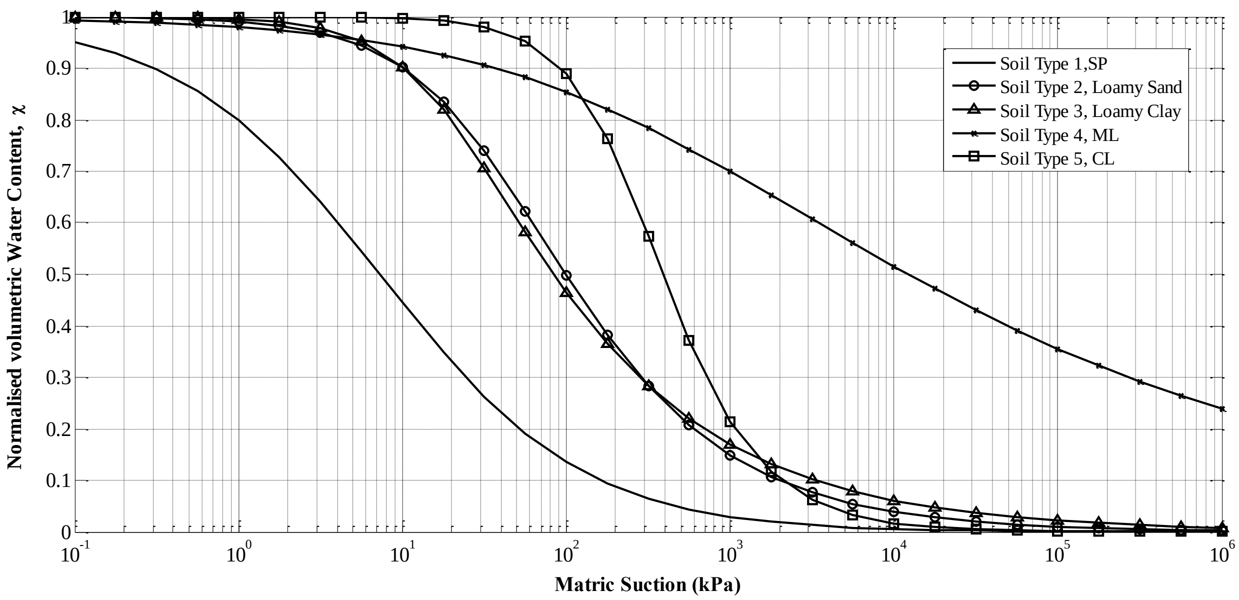
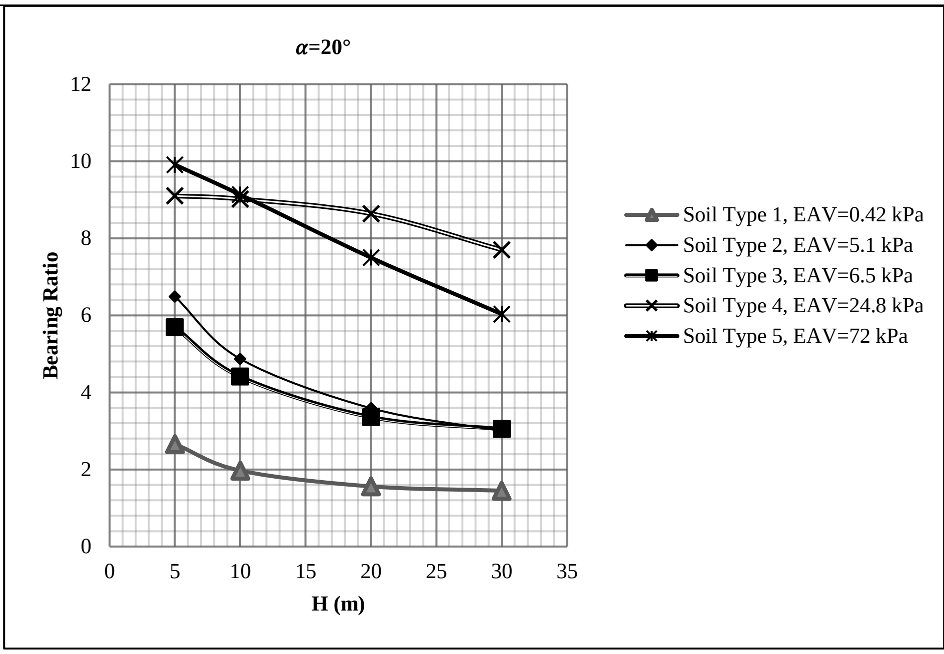
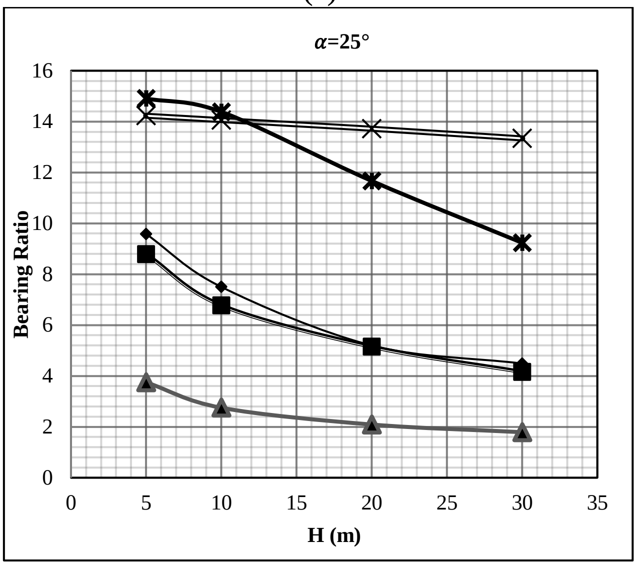

Normalised volumetric Water Content, χ
Matric Suction (kPa)
Soil Type 1,SP
Soil Type 2, Loamy Sand
Soil Type 3, Loamy Clay
Soil Type 4, ML
Soil Type 5, CL
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
10^-1
10^0
10^1
10^2
10^3
10^4
10^5
10^6

Figure 6.

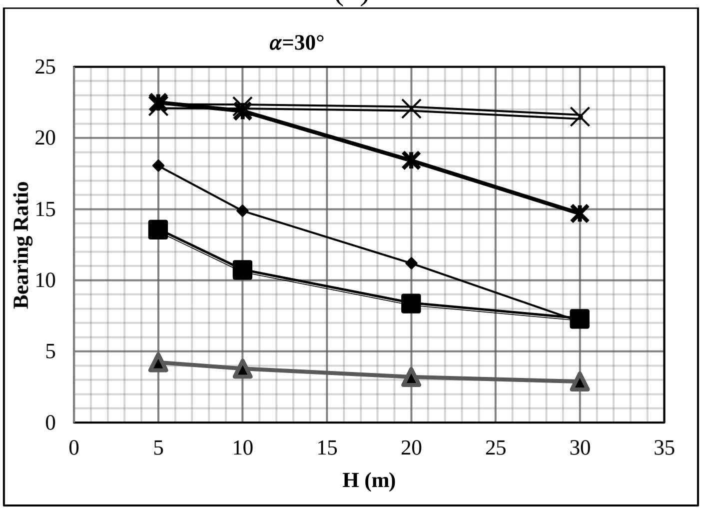
(a)- Slope of α=20°
(b)- Slope of α=25°
(c)- Slope of α=30°



(a)



(b)



(c)

Figure 7.

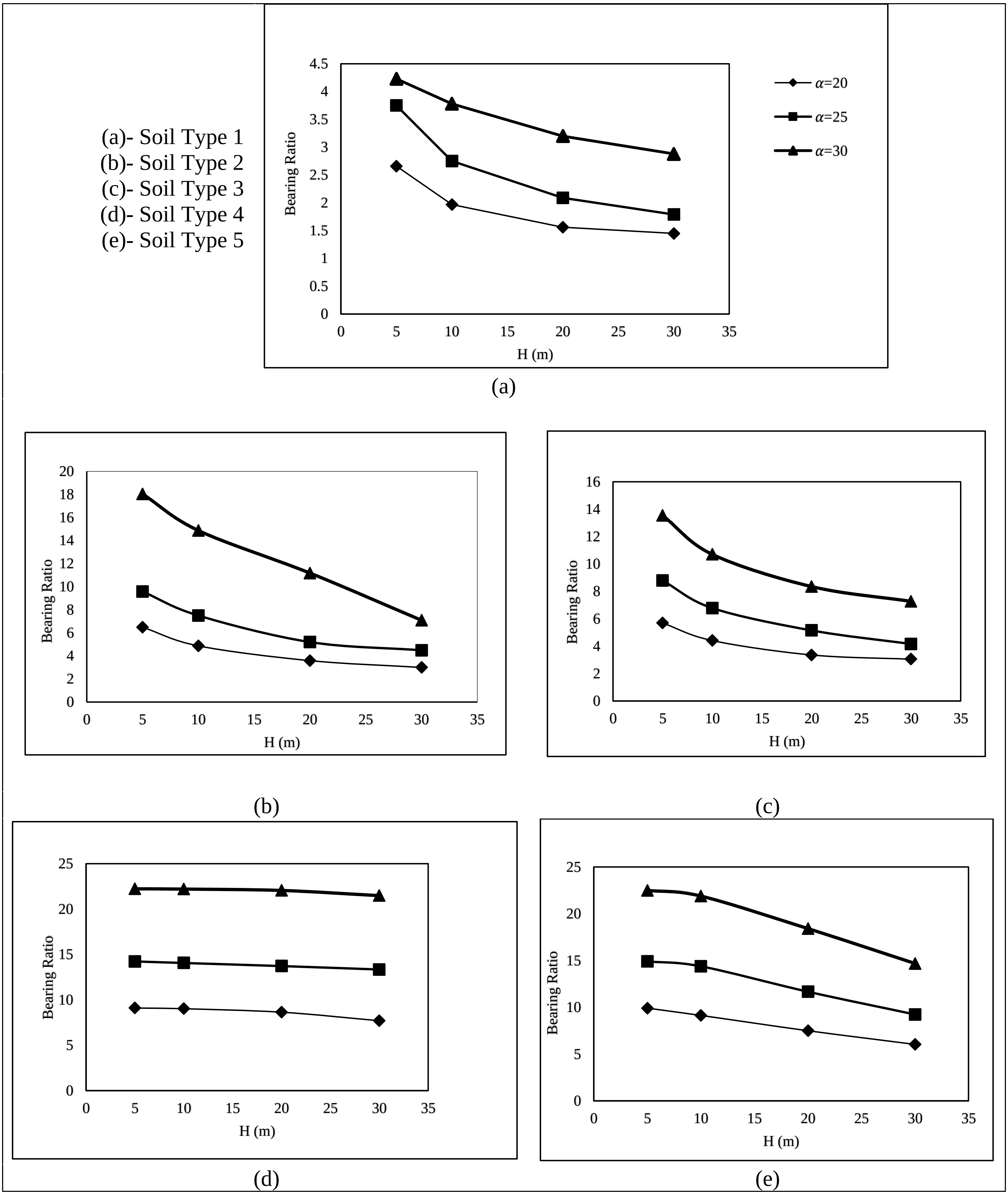
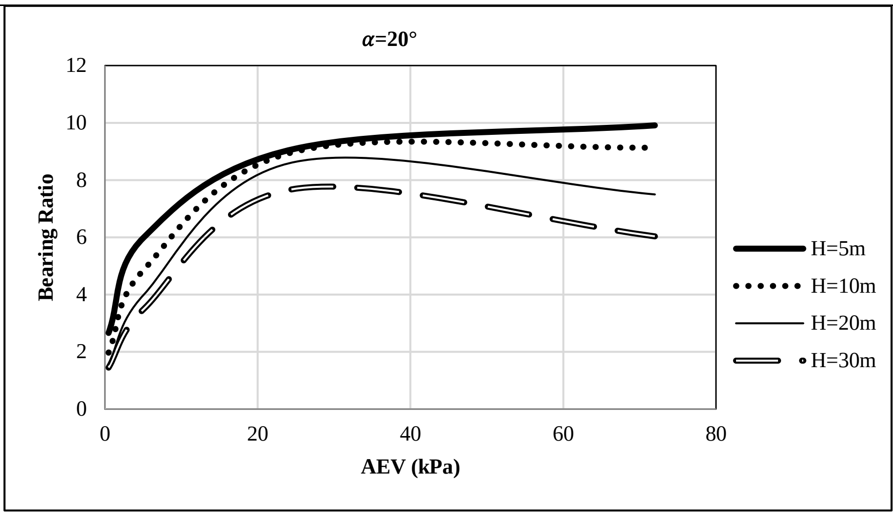
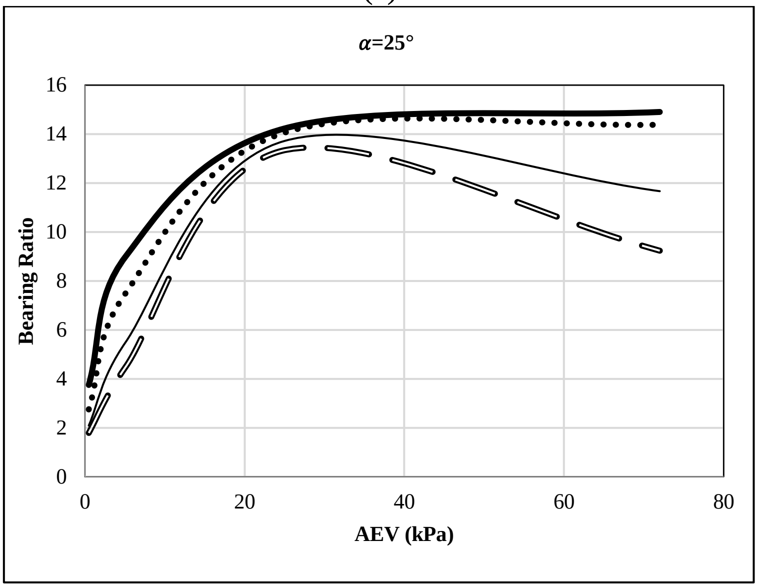

(a)- Soil Type 1
(b)- Soil Type 2
(c)- Soil Type 3
(d)- Soil Type 4
(e)- Soil Type 5
α=20
α=25
α=30
Bearing Ratio
H (m)
(a)
(b)
(c)
(d)
(e)

Figure 8.

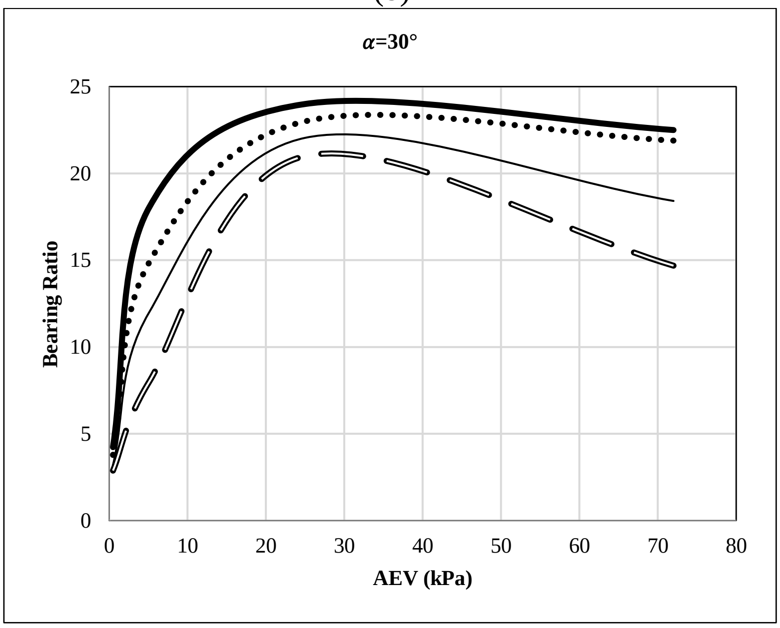
(a)- Slope of α=20°
(b)- Slope of α=25°
(c)- Slope of α=30°



(a)



(b)



(c)

Figure 9.

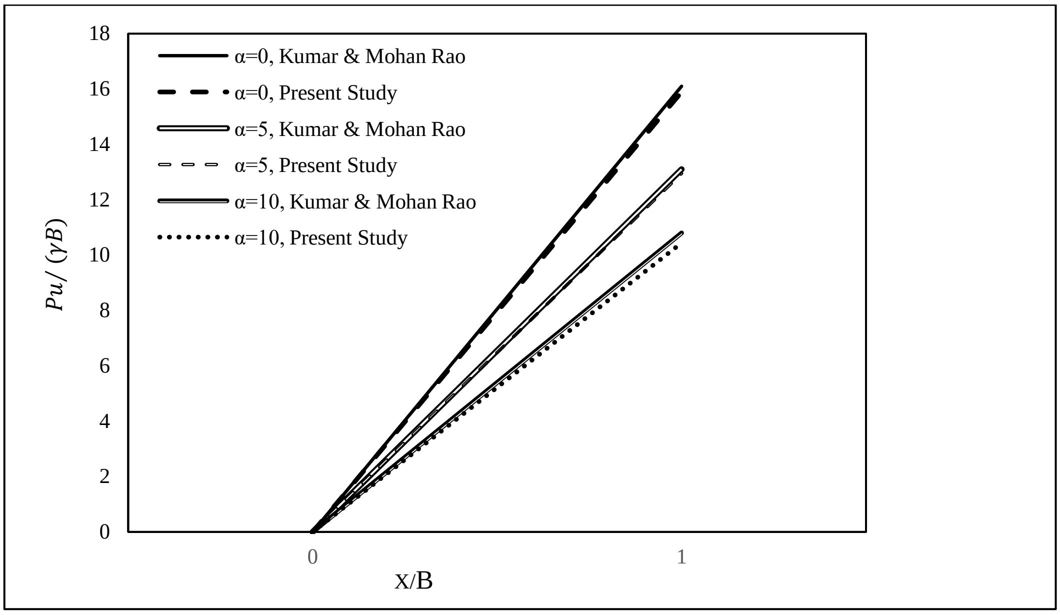
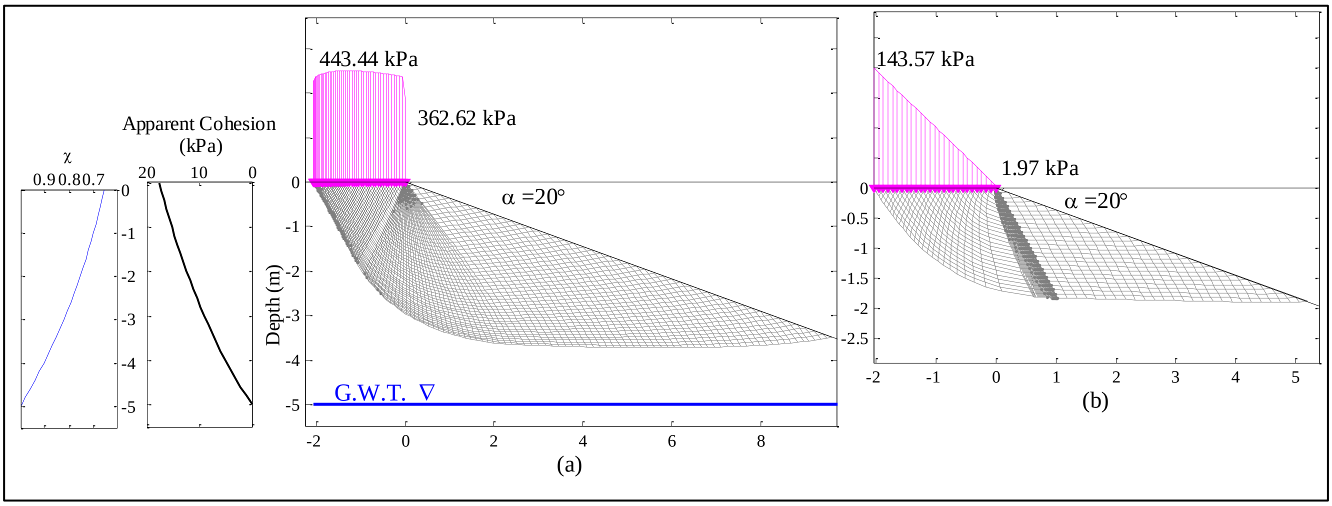

18
16
14
12
10
8
6
4
2
0
0
1
Pu/ (γB)
X/B
α=0, Kumar & Mohan Rao
α=0, Present Study
α=5, Kumar & Mohan Rao
α=5, Present Study
α=10, Kumar & Mohan Rao
α=10, Present Study

Figure 10.



χ
0.9 0.8 0.7
Apparent Cohesion (kPa)
20 10 0
443.44 kPa
362.62 kPa
α =20°
Depth (m)
G.W.T. ∇
(a)
143.57 kPa
1.97 kPa
α =20°
(b)

Figure 11.

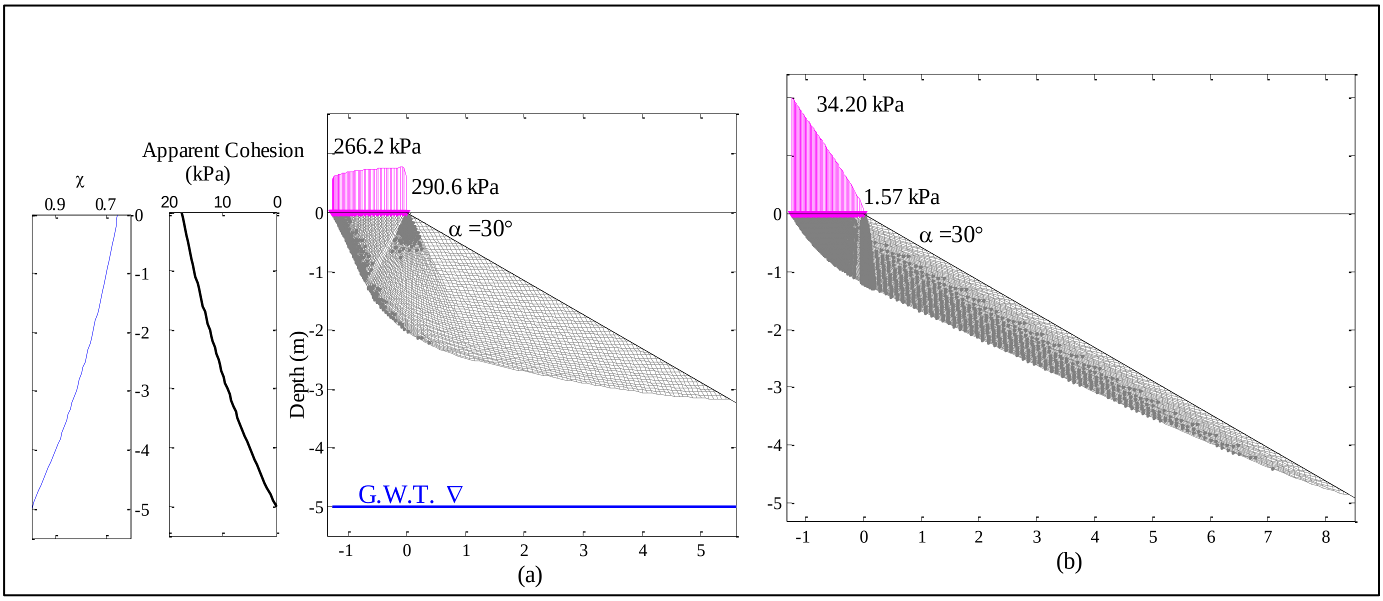

χ
0.9
0.7
Apparent Cohesion
(kPa)
20
10
0
266.2 kPa
290.6 kPa
α =30°
Depth (m)
G.W.T. ∇
(a)
34.20 kPa
1.57 kPa
α =30°
(b)

Figure 12.

Tables:

Table 1:

Type	Soil Type 1 SP	Soil Type 2 Loamy Sand	Soil Type 3 Loamy Clay	Soil Type 4 ML	Soil Type 5 CL
Air Entry Value (kPa)	0.43	5.1	6.5	25	72
α van Genuchten	0.121	0.024	0.052	0.0027	0.0036
n van Genuchten	0.685	1.094	1.447	0.486	1.643
m van Genuchten	1.066	0.543	0.309	0.372	0.696

Table 2:

Soil Type 1, AEV=0.43kPa									
Height (m)	$\alpha = 20°$ Bearing Pressure $\phi' = 30°$ & $c' = 0$		Bearing Ratio	$\alpha = 25°$ Bearing Pressure $\phi' = 30°$ & $c' = 0$		Bearing Ratio	$\alpha = 30°$ Bearing Pressure $\phi' = 30°$ & $c' = 0.5kPa$		Bearing Ratio
	Conventional (kPa)	Unsaturated (kPa)		Conventional (kPa)	Unsaturated (kPa)		Conventional (kPa)	Unsaturated (kPa)	
5	86.8	231.2	2.66	44.93	168.88	3.75	34.46	145.79	4.23
10	207.08	408.86	1.97	100.27	275.41	2.75	57.34	216.54	3.78
20	481.18	755.34	1.56	240.37	501.9	2.09	101.06	323.31	3.2
30	757.39	1097.52	1.45	364.19	650.42	1.79	142.4	410.82	2.88

Table 3:

Soil Type 2, AEV=5.1 kPa									
Height (m)	$\alpha = 20°$ Bearing Pressure $\phi' = 30°$ & $c' = 0.1kPa$		Bearing Ratio	$\alpha = 25°$ Bearing Pressure $\phi' = 30°$ & $c' = 0.1kPa$		Bearing Ratio	$\alpha = 30°$ Bearing Pressure $\phi' = 30°$ & $c' = 0.1kPa$		Bearing Ratio
	Conventional (kPa)	Unsaturated (kPa)		Conventional (kPa)	Unsaturated (kPa)		Conventional (kPa)	Unsaturated (kPa)	
5	73.45	476.57	6.49	40.73	390	9.58	18.84	340.01	18.04
10	167	812.99	4.87	87.69	657.53	7.5	37.1	551.86	14.87
20	382	1374.6	3.59	202.21	1055.69	5.2	76.3	852.82	11.18
30	620	1873.23	3	308.08	1385.67	4.49	160	1137.24	7.09

Table 4:

Soil Type 3, AEV=6.5 kPa									
Height (m)	$\alpha = 20°$ Bearing Pressure $\phi' = 30°$ & $c' = 0.1kPa$		Bearing Ratio	$\alpha = 25°$ Bearing Pressure $\phi' = 30°$ & $c' = 0.1kPa$		Bearing Ratio	$\alpha = 30°$ Bearing Pressure $\phi' = 30°$ & $c' = 0.1kPa$		Bearing Ratio
	Conventional (kPa)	Unsaturated (kPa)		Conventional (kPa)	Unsaturated (kPa)		Conventional (kPa)	Unsaturated (kPa)	
5	81.5	464.44	5.69	43.64	383.68	8.79	24.78	335.73	13.54
10	179	789.59	4.41	92.7	628.28	6.78	50.32	538.7	10.7
20	401.99	1350.03	3.36	198.43	1023.08	5.15	102.23	854.06	8.35
30	609.53	1861.22	3.05	327.59	1363.26	4.16	153.43	1115.65	7.271

Table 5:

Soil Type 4, AEV=25 kPa									
Height (m)	$\alpha = 20°$ Bearing Pressure $\phi' = 30°$ & $c' = 0.1kPa$		Bearing Ratio	$\alpha = 25°$ Bearing Pressure $\phi' = 30°$ & $c' = 0.1kPa$		Bearing Ratio	$\alpha = 30°$ Bearing Pressure $\phi' = 30°$ & $c' = 0.1kPa$		Bearing Ratio
	Conventional (kPa)	Unsaturated (kPa)		Conventional (kPa)	Unsaturated (kPa)		Conventional (kPa)	Unsaturated (kPa)	
5	63.45	578.1	9.1	35.01	498.35	14.23	20.99	445.61	22.23
10	124.14	1119.6	9.02	68.66	965.78	14.06	38.87	863.02	22.2
20	250.1	2160.1	8.64	135.21	1855.42	13.72	74.93	1652.79	22.05
30	410.17	3159.6	7.7	202.83	2707.29	13.34	111.93	2404.69	21.48

Table 6:

Soil Type 5, AEV=72 kPa									
Height (m)	$\alpha = 20°$ Bearing Pressure $\phi' = 30°$ & $c' = 0.1kPa$		Bearing Ratio	$\alpha = 25°$ Bearing Pressure $\phi' = 30°$ & $c' = 0.1kPa$		Bearing Ratio	$\alpha = 30°$ Bearing Pressure $\phi' = 30°$ & $c' = 0.1kPa$		Bearing Ratio
	Conventional (kPa)	Unsaturated (kPa)		Conventional (kPa)	Unsaturated (kPa)		Conventional (kPa)	Unsaturated (kPa)	
5	62.8	622.34	9.91	36.14	538.66	14.90	21.44	482.4	22.5
10	130.11	1188.24	9.11	71	1021.16	14.38	41.77	914.33	21.89
20	283.83	2128.21	7.49	153.93	1795.95	11.67	86.2	1587.27	18.41
30	473.9	2859.5	6.03	255.47	2359.4	9.23	140.31	2060.55	14.68

Table 7.

ϕ'	N_q			N_c		
	Mayrehof, Hansen and Vesic	Kumar and Mohan Rao (Single side mechanism)	Present study	Mayerehof, Hansen and Vesic	Kumar and Mohan Rao (Single side mechanism)	Present study
0	1	1	1	5.14	5.14	5.14
5	1.57	1.57	1.57	6.52	6.52	6.53
10	2.47	2.47	2.47	8.34	8.34	8.35
15	3.94	3.94	3.94	10.97	10.97	10.99
20	6.4	6.4	6.4	14.84	14.84	14.86
25	10.7	10.7	10.75	20.8	20.8	20.9
30	18.4	18.4	18.49	30.14	30.14	30.29
35	33.3	33.3	33.46	46.13	46.13	46.36
40	64	64	64.31	75.08	75.08	75.45

Table 8.

N_q					
α	$\phi' = 30°$		α	$\phi' = 40°$	
	Kumar and Mohan Rao	Present Study		Kumar and Mohan Rao	Present Study
0	18.4	18.49	0	65	65.15
10	12.8	13.16	15	33.5	34.68
20	7.9	8.48	30	13.1	15.18

Table 9.

N_c					
α	$\phi' = 30°$		α	$\phi' = 40°$	
	Kumar and Mohan Rao	Present Study		Kumar and Mohan Rao	Present Study
0	30.14	30.29	0	75	76.45
10	24.4	24.41	15	48	48.53
20	19.5	19.61	30	30.5	30.72