Separation of the Fetal Heart Signal in a Synchronous Network Consisting of the maternal and fetus's heart

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Abstract:
This paper studies the heart's oscillation model to separate fetal and maternal ECGs from abdominal recordings. To this aim, two phases are designed. A Modified version of the Duffing-Van der Pol oscillator is considered a computational heart model in the modeling phase. To evaluate the interaction effects of the maternal and fetal heart and the differences and features of the fetal heart structure, the fetal heart model is Modified based on the maternal heart model. A non-identical network is employed as an interactive network of the mother and the fetus's heart. Then the degree of the network synchronization is measured with the help of a pattern synchronization index of the non-identical network. An attempt is made to separate the fetal signal from the mother's in abdominal signals in the separation phase. Two problem-solving approaches are explained; the step-by-step mode that calculates the signal at any given moment and the construction of general equations. These approaches end up calculating the variables, which stand for maternal and fetal signal. That makes it possible to achieve the separation of maternal and fetal ECGs.

Keywords: Fetal monitoring; non-invasive; feto-maternal monitoring; networks; synchronization; chaos; Signal Separation.

1. Introduction and background

Cardiovascular problem is one of the most common congenital diseases that can remain undiagnosed even years after birth [1]. Therefore, recording fetal cardiac activity leads to monitor fetal heart performance. The results show that fetal electrocardiogram (fEGC) can identify most mother and fetus's heart disorders [2]. Also, the prominence of using fEGC over Cardiotocography (CTG) in the fetal heart disease diagnosis has been reported in [3, 4]. Since most fetal diseases are possible to detect by studying fEGC, a complete analysis of fetal electrocardiogram signal can prevent such drawbacks [5]. However, recording the fEGC signals contains some critical challenges [6, 7].

The fEGC signal can be obtained via direct or indirect procedures [6]. The direct method is invasively using wire electrodes on the fetus's scalp or abdomen [8]. Due to the dangers of the direct method, a non-invasive procedure is utilized [9, 10]. In such methods, ECG signals are recorded over the mother's abdomen surface. The recorded signal is the abdominal ECG (aECG), which consists of fetal and maternal electrocardiograms. As the maternal Electrocardiogram
(mECG) amplitude is significantly larger than the fECG, recovery of fECG from aECG is such a challenging work [11]. Consequently, many studies have investigated different separation methods to fulfill the aim [5, 12].

Several methods are used in order to extract the fECG signal from the aECG, such as adaptive filtering method [13-15], genetic algorithm [16], Singular Value Decomposition (SVD) [11, 17], independent component analysis (ICA) [18-21], and wavelet transform analysis [22, 23]. Adaptive filters are divided into two groups, the stochastic gradient approach and the recursive least square algorithms [24]. In these algorithms, the crucial issue is to set correct parameters [25]. The SVD technique reduces noise and separates components [11], within the constraint on the number of recording electrodes that must be more than sources [26]. ICA is a Blind Source Separation (BSS) based method, used when sources are independent [27]. The method presented in [28] is based on the FastICA algorithm. Wavelet transform is another technique used for time-scale domain analysis [29]. In [22], Datian et al. detected the mECG signal's edges using spline wavelet transform and then extracted the mECG signal by finding the local maximum of aECG. In addition to the mentioned approaches, methods such as artificial intelligence [30], maternal ECG pattern [31], Gaussian moments [32], time structure information, and high-order statistical components related to fetal ECG signal [33] are utilized to extract fECG from aECG signal. Here the computational model is introduced as an approach to fECG extraction.

Computational modeling provides the best mathematical representation of a system [34]. The computational biological models provide a virtual laboratory that characterizes the complex system's features [35]. An appropriate theoretical model mainly highlights essential features and underplay insignificant details of the natural system [36], so such a model can be the right approach to illustrate heart activities. The heart is a complex, adaptable system that operates effectively due to the interactions of its dynamic parts [37]. In former studies, it has been implied that the ECG signal is periodic [36]. However, the heart as a biological system shows evolutionary properties [38]. On this matter, [39] states that the Hierarchical model is a viable way to simulate such a complex system's properties since agents are the main components in this model.

Chaos has been investigated in various biological systems as a long-term non-periodic behavior in a deterministic system, sensitive to the initial conditions [40, 41]. The unpredictability of the complex behavior of chaotic systems is one of the crucial factors in modeling biological systems with complex features [42]. The cardiovascular system as a biological system can be considered a chaotic system [43-45]. In [46], the positive Lyapunov exponents of the RR-interval signal extracted from the pulse signal of finger capillaries, which exposed that the heart has a chaotic behavior. Chaos in cardiac myocytes has also been shown in many experimental studies and modeling. For instance, Chialvo et al. indicated that the cardiac Purkinje fibers could bifurcate due to their chaotic dynamics [47].

Many studies have been investigated the heart at microscopic, mesoscopic, and macroscopic levels [48]. In this study, macroscopic models have been proposed for the heart dynamical function, based on Van der Pol oscillators [49]. Some Modified Van der Pol models have also been introduced by incorporating physiological features of the heart dynamics [50, 51]. In another study, the coefficients of a Modified Van der Pol oscillator as a computational model of the heart are
optimized with the help of neural network algorithms [52]. In [53], two Van der Pol oscillators have been coupled as a cardiac model in which one of the oscillators is assumed to represent the dynamics of the heart pacemaker. Savi et al. used a three-coupled Van der Pol oscillator (SA node, AV node, and HP complex) to discuss the heart rhythm's implementation and model it in a macroscopic state [54, 55]. To investigate the dynamics of the mother and fetus's heart interaction should consider a network.

From a systemic perspective, a network is consists of nodes and their connections that can sometimes create the most crucial form of dynamical collective behavior, synchronization [56, 57]. Each node demonstrates a dynamical system in such networks, and their connections represent the interaction between them. Networks of identical oscillators with the same parameter values, called identical lattices, can produce complete synchronization [58]. However, non-identical networks can never be fully in synch [59, 60]. Real-world systems, such as biological and engineering networks, are non-identical [61]. In 2020, Panahi and Jafari proposed an approximate synchronization index [62], inspired by the Poincaré section. Unlike conventional methods, this method provides a quantitative index for measuring the degree of behavioral synchronization, focusing on the pattern of fluctuations in the network.

This paper is organized as follows: In section 2, a heart model is presented; also, maternal and fetal heart interaction has been modeled. In section 3, the synchronization of this network has been investigated. Besides, the systems equations have been generated by changing the time in this section. Section 4 introduces the approach of separating fetal signals from the abdominal signal. Finally, sections 5 and 6 present the discussion, and conclusions, respectively.

2. Model

2.1. The heart model

Nonlinear oscillators are used as computational models to show the normal or abnormal heart rhythm. In this study, a nonlinear Modified Duffing-Van der Pol oscillator is proposed as a computational model of the heart rhythm. Since it has been claimed that heart rhythms could be considered chaotic signals [43, 44], many researchers have attempted to reconstruct the heart signal model based on the chaotic oscillator. The Modified Van der Pol oscillator grabs wide attention among different oscillator models as it can achieve different aspects of the heart signal [63]. As an effective parameter, time delays have been applied in the Van der Pol as well, so it can illustrate the heart oscillation perfectly [51].

Van der Pol equation is a proper choice for modeling heart dynamics systems such as cardiac cycles and heart rate variations. It also can show the heart's relaxation oscillations [50, 52]. The qualities of Van der Pol signals are very similar to the heart action potential characteristics. Both slow and fast types of action potentials can be easily simulated using Duffing-Van der Pol. The driving term in each oscillator is responsible for the external input, which is states in Eq.1. Figure 1 shows the time series and phase space of the Duffing-Van der Pol model in the determined parameters.

\[ \ddot{x} - a(1 - x^2)\dot{x} + x^3 = b\cos(\omega t) \]  

(1)
The parameters of the Modified Duffing-Van der Pol model are adjusted based on the actual heart signal. It can be formulated as Eq.2.

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= -a(x - w_1)(x + w_2)y - x^3 + b\cos(ct)
\end{align*}
\] (2)

Figure 2 demonstrates the time series and phase space of the heart model using Eq.2. It indicates the heart's chaotic function when the parameters are set to \( a = 2.5, b = 1.8, c = 3.8, w_1 = 1.8, \) and \( w_2 = 1.2 \) with choosing \((1,-0.7)\) as the initial conditions.

The bifurcation diagram is another way of analyzing the chaotic behavior of this model. Hence, the system dynamics are analyzed via the bifurcation diagram concerning the variation of parameter \( a \). In Fig.3, the system bifurcation diagrams are plotted for both \( x \) and \( y \) variables with two different approaches. In the first approach, the bifurcation diagram is plotted using peaks of the time series (see Fig.3-a and Fig.3-b). In the second approach, interspike intervals of the time series (ISI) are used for plotting the bifurcation diagram in Fig.3-c, and Fig.3-d. Plotting bifurcation diagrams using max values of the variables shows the variations in their amplitude. However, plotting ISI bifurcation diagrams reveals the variation in the timing of interspike. Therefore, these bifurcations present two different features of the system dynamics. For example, by changing the parameter \( a \), in the range of \([1.79,2.23]\), the spikes amplitude changes, although the ISI bifurcation shows that the intervals of these spikes are constant. Also, a positive Lyapunov exponent is employed to show chaotic dynamics in the system [64]. In Fig.3, below the bifurcation diagrams, the corresponding Lyapunov exponents are plotted via parameter \( a \). As shown in Fig.3-e and Fig.3-f, one of the Lyapunov exponents turns positive in the chaotic region.

Complex dynamics can also be seen in the model's bifurcation diagram according to changing the other parameters of the model. Figure 4 shows the different bifurcation patterns by changing each parameter \( b, c, w_1, \) and \( w_2 \).

2.2. Fetal heart model

The amplitude of the fetal ECG is 20% smaller than the maternal ECG; also, the amplitude value of the cardiac signals in adults is about a few millivolts. Consequently, the fetal signal is much weaker compared to the maternal. One of the problems in mother-fetus signal separation is the maternal ECG signal's constant presence, which is about 5 to 20 times larger than the fetal ECG signal [11]. So, the fetal heart is modeled using a Modified Duffing-Van der Pol, which fluctuates with the intensity of one-twentieth adult heart. To this end, the Modified Duffing-Van der Pol concerning the fetus heart model is formulated with scaled variables (\( \dot{x} = A\dot{X} \) and \( \dot{y} = A\dot{Y} \)) as follows:
\[
\begin{align*}
\dot{x}_2 &= \left(\frac{1}{A}\right)(By_2) \\
\dot{y}_2 &= \left(\frac{1}{B}\right)(-a(Ax_2 - w_1)(Ax_2 + w_2)y - (Ax_2)^3 + b\cos(ct))
\end{align*}
\]

(3)

To demonstrate the differentiation of the amplitude of mECG and fECG in the worst cases, two coefficients \( A \) and \( B \), are set with a value of 20. The result is plotted in Fig.5.

As Fig.5 demonstrates, coefficients \( A \) and \( B \) only affected the amplitude's intensity; thus, similar results have been obtained for the performance and pattern of the two models.

### 2.3. The interacting network of maternal and fetal heart

A dynamical network is considered to investigate the dynamics of the mother and fetus's heart interaction. The interaction between the fetus and mother’s heart is modeled using a linear coupling of \( x \) variables, which can be formulated as Eq.4:

\[
\begin{align*}
\dot{x}_1 &= y_1 + \varepsilon(x_2 - x_1) \\
\dot{y}_1 &= -a(x_1 - w_1)(x_1 + w_2)y_1 - x_1^3 + b\cos(ct) \\
\dot{x}_2 &= \left(\frac{1}{A}\right)(By_2) + \varepsilon(x_1 - x_2) \\
\dot{y}_2 &= \left(\frac{1}{B}\right)(-a(Ax_2 - w_1)(Ax_2 + w_2)y_2 - (Ax_2)^3 + b\cos(ct))
\end{align*}
\]

(4)

In Eq.4, \( \varepsilon \) shows the interaction coefficient (coupling strength), the variable \( x \) acts as a cardiac pacemaker (CP) in each of the maternal and fetal heart models, and the variable \( y \) shows the heart rate variable (HRV). The model outputs are shown in Fig.6, after setting the interaction coefficient to \( \varepsilon = 0.01 \). Sometimes chaotic behaviors appear to be irregular and random in time series but have a strong underlying order in phase space [65]. A phase space projection shows the dependency of each state to the next one; hence phase space is used to study chaotic dynamics [66] and topological characteristics of the system [67]. Numerical processes, like estimating the correlation dimension and the Lyapunov exponents or modeling and forecasting the time series, were done base on phase space [68]. In Fig.6-c and Fig.6-d, trajectories pass close together in the delayed phase space.

### 3. Analysis of synchronization in a non-identical network

From a systemic perspective, a network is consists of nodes and their connections that can sometimes create the most crucial form of dynamical collective behavior, synchronization [56, 57]. Each node demonstrates a dynamical system in such networks, and their connections represent the interaction between them. Synchronization is one of the most exciting consequences of interaction
in dynamic networks [69]. Networks of identical oscillators with the same parameter values, called identical lattices, can produce complete synchronization [58]. However, non-identical networks can never be fully in synch [59, 60]. Real-world systems, such as biological and engineering networks, are non-identical [61].

Non-identical networks can be divided into two general groups, 1) The same dynamical system with different parameters and 2) different dynamical systems placed in each node [70]. Most of the real-world researches has been done on non-identical networks of the first group [71]. In 2008, Hill et al. presented conditions for global synchronization for the first group of the non-identical network [72-74]. Then, the approximate synchronization method was considered for both groups of non-identical networks [75]. Approximate synchronization uses measuring similarity in different behavior aspects [76]. A new method (pattern synchronous) based on the network's oscillators' behavioral pattern was proposed to measure the synchronization in non-identical complex biological systems [62]. Inspired by the Poincaré section, this method takes the inconsistency in the network's oscillations as a principle and ignores parameters such as amplitude and phase similarity.

In this method, each time series peak has been considered as a sign of a complete oscillation. The algorithm of this method starts with finding the peaks of each oscillator of the network. Then, the peaks of different time series are compared, and the close peaks are counted, considering a threshold value. Finally, the synchronization degree is then defined, proportional to the number of corresponding peaks in the time series of the network oscillators.

In this paper, the synchronous study was performed using this method since non-identical oscillators interacted. The results are shown in Fig.7.

As illustrated in Fig.7, by enlarging the coupling coefficient, the degree of the synchronization increased, which was made by calculating and averaging it with 20 random initial conditions to reduce dependence on the initial condition. It can be seen that for coupling strengths larger than 0.01, the two oscillators share the same pattern. Thus, the coefficient of 0.01 has been chosen as the coupling coefficient from now on.

4. Separation

In general, there are two methods for extracting the fECG signal: direct method (invasive) and indirect method (non-invasive). The indirect fECG signal extraction shows significant advantages compared to other methods. However, this method also has some limitations.

As shown in Fig.8, a combination of the mother and the fetus's heart signal can be received with an abdominal recording. Therefore, a combined signal of the mother and fetus's heart, i.e., $y_1 + y_2$, is recorded.

In the following, we are looking for a way to separate the fetal signal from the collective signal $y_1 + y_2$. A fragment of this signal is demonstrated in Fig. 9.
Suppose that the heart's systemic model and its parameters are known, while the degree of the interaction (the coupling parameter) is unknown. The ultimate goal is to be able to calculate \( y_2 \) at any time by having the abdominal recording. It has been tried to get some additional information that could bring us closer to the answer. For instance, the sum of \( x \) variables can also be obtained based on the model. Equation 5 shows both \( x \) variables of the network.

\[
\begin{align*}
\dot{x}_1 &= y_1 + \varepsilon (x_2 - x_1) \\
\dot{x}_2 &= \left( \frac{1}{A} \right) (B y_2) + \varepsilon (x_1 - x_2) = y_2 + \varepsilon (x_1 - x_2)
\end{align*}
\] (5)

By summing the two sides of Eq. 5, it can be concluded that \( y_1 + y_2 = \dot{x}_1 + \dot{x}_2 \). Then by integrating both sides, we have:

\[
\int \text{sum}(y_1 + y_2) = \int \dot{x}_1 + \dot{x}_2 = \text{sum}(x_1 + x_2)
\] (6)

Therefore, recording the collective signal \( y_1 + y_2 \) with an abdominal recording can end with the summation of \( x \) variables. Figure 10 shows the summation of \( x \) variables, which is calculated from the collective signal \( y_1 + y_2 \).

Considering Eulerian[77], the continuous form of Eq.4 can be discretized as:

\[
\begin{align*}
\dot{x}_1(i+1) &= \dot{x}_1(i) + \Delta t \left( y_1(i) + \varepsilon (x_2(i) - x_1(i)) \right) \\
\dot{y}_1(i+1) &= \dot{y}_1(i) + \Delta t \left( -a(x_1(i) - w_1)(x_1(i) + w_2)y_1(i) - x_1(i)^3 + b \cos(\sigma t) \right) \\
\dot{x}_2(i+1) &= \dot{x}_2(i) + \Delta t \left( \frac{1}{A} (B y_2(i)) + \varepsilon (x_1(i) - x_2(i)) \right) \\
\dot{y}_2(i+1) &= \dot{y}_2(i) + \Delta t \left( \frac{1}{B} \left( -a(A x_2(i) - w_1)(A x_2(i) + w_2)y_2(i) - (A x_2(i))^3 + b \cos(\sigma t) \right) \right)
\end{align*}
\] (7)

There are two ways to solve Eq.7; step-by-step and high-order techniques. The step-by-step technique uses the \( x \) and \( y \) summation signals at each moment, it gives us Eq.8:

\[
\begin{align*}
x_1(i) + x_2(i) &= \text{sum}(x_1 + x_2)(i) \\
y_1(i) + y_2(i) &= \text{sum}(y_1 + y_2)(i)
\end{align*}
\] (8)

Based on the summation signal, the relation can be written with a further step:

\[
y_1(i+2) + y_2(i+2) = \text{sum}(y_1 + y_2)(i+2)
\] (9)

Since in Eq.9 new unknowns are added, we use the Euler relation to write present step with the previous step's variables:
By the way, five equations are obtained; two equations from the x variables summation, two equations from the y variables summation (Eq.8), and Eq.10.

\[
y_1(i + 2) + y_2(i + 2) = y_1(i + 1) + y_2(i + 1) + \Delta t \left( -a(x_1(i + 1) - w_1)(x_1(i + 1) + w_2)y_1(i + 1) - x_1^3(i + 1) + b\cos(ct) \right) \\
+ \Delta t \left( \frac{1}{B} \left( -a(Ax_2(i + 1) - w_1)(Ax_2(i + 1) + w_2)y_2(i + 1) - (Ax_2(i + 1))^3 + b\cos(ct) \right) \right)
= \text{sum}(y_1 + y_2)(i + 2)
\]  

(10)

In addition, four equations have been achieved from the Euler discretization in Eq.7. Finally, the problem ends up with a system of nine equations and nine variables. Figure 11 shows the results for a part of the signal in the time domain.

As shown in Fig.11, this system of equations has three roots for coupling parameter, \( \varepsilon \). Roots with imaginary values are omitted because they were not acceptable. One of the roots was equal to the value of 0.01 in all cases, and a continuous line can easily see this in the value of 0.01. In the histogram plot in Fig.11-c, the maximum intensity is the exact value of 0.01. Similarly, for other unknown values, \( y_1(t) \) and \( y_2(t) \), three roots can be calculated, while roots with imaginary values must be eliminated. Assume that \( M_1, M_2, \) and \( M_3 \) are three roots of \( y_1 \), and \( F_1, F_2, \) and \( F_3 \) are roots of \( y_2 \), that only one answer is correct at a time. Figure 12 shows these roots.

Figure 13 illustrates the estimated maternal and fetal heart rate variation, based on the calculated roots of \( y_1 \) (mother heart) and \( y_2 \) (fetal heart) at any given time when the coupling coefficient is set to \( \varepsilon = 0.01 \).

The second approach can be considered as an overview of the problem. By taking an overall look at the whole system of equations from beginning to end, it can be seen that our problem is broken into solving nine equations with nine unknown variables. However, there are solution methods with high-order equations too. Meanwhile, in the previous method, the next step's variables are calculated once in this step and once again in the next. So if all the system equations (Eq.4) are put together for the signal length \( N \), the problem ends with solving the \( 4(N-1) \) equations and \( 4N \) variables. However, with the summation of \( x \) and \( y \) signals in Eq.8, we have \( 2N \) equations.
\[
\begin{align*}
\begin{cases}
x_1(1) + x_2(1) &= \text{sum}(x_1 + x_2)(1) \\
y_1(1) + y_2(1) &= \text{sum}(y_1 + y_2)(1) \\
&\vdots \\
x_1(N) + x_2(N) &= \text{sum}(x_1 + x_2)(N) \\
y_1(N) + y_2(N) &= \text{sum}(y_1 + y_2)(N)
\end{cases}
\end{align*}
\]

Also in Eq.9 if \( i = N - 1 \) then:

\[
y_i(N+1) + y_2(N+1) = \text{sum}(y_i + y_2)(N+1) = \\
y_i(N) + \Delta t \left( -a(x_i(N) - w_i)(x_i(N) + w_2)y_i(N) - x_i^3(N) + b\cos(ct) \right) \\
y_2(N) + \Delta t \left( (1/B) \left( -a(Ax_2(N) - w_1)(Ax_2(N) + w_2)y_2(N) - (Ax_2(N))^3 + b\cos(ct) \right) \right)
\]

So a new set of the equation is calculated, while no new variables are added to the equations. In conclusion, by assuming the signal length \( N \), there are \( 4(N-1) \) Eulerian system equations and \( 2N+1 \) summation equations (Eq.8 and Eq.11) with \( 4N+1 \) variables (\( x, y \), and the coupling coefficient). Therefore, the number of equations is \( 6N - 3 \), and the number of unknowns is \( 4N + 1 \).

Generally speaking, it can be concluded that for any basic \( m \)-dimensional model assuming for the heart model with nonlinear interaction function and a signal with length \( N \), there are \( N + 2m(N-1) \) equations with \( 2mN+1 \) variables. The \( 2m(N-1) \) equations are captured from Eulerian system equations, and \( N \) comes from the summation relations. The \( 2mN \) variables are the state variables in each step, with one coupling coefficient.

Also, the parameters of the model can be considered as unknown variables. In such a problem, the number of equations is \( N + 2m(N-1) \), and the number of variables turns to \( 2mN + A \) (\( A \) is the number of the model's parameters).

### 5. Discussion

This paper consists of two modeling and separation phases. In the first phase, a heart model was presented. The selected model was based on Modification in the Duffing-Van der Pol system, which was entirely optional. To develop this method, any other model can be replaced with Van der Pol, as long as the model is compatible with the model chaotic dynamics of the heart system. In order to analyze the chaotic dynamics of the system, the bifurcation and Lyapunov exponents diagrams have been plotted. One of the considerable challenges on fECG separation was the distinction between maternal and fetal ECG amplitude caused by structural differences in the fetal heart. Therefore, by creating coefficients in the heart model, the fetal heart model has been obtained. These coefficients can also be changed as desired in section 2.2. Based on the worst situation, fECG amplitude has been chosen 20 times weaker than mECG's. Choosing another value between the boundary of \([5, 20]\) to show the strength of mECG, can reproduce the whole computation as well. In other words, the selected value depends on the expected ratio of maternal
to fetal amplitude. Besides, a non-identical coupling of the maternal and fetal heart shows maternal and fetus heart interaction. The coupling is assumed linear in the $x$ variables equations. That means that the effect of cardiac pacemakers on each other was studied. Finally, the pattern synchronization of this network is checked.

In the separation phase, the values of variables and the law of interaction (coupling coefficient) have assumed unknowns. The system of equations according to the interactive network equation and the abdominal signal has been constructed. Two methods for performing calculations are discussed; a step-by-step method and high-order equations. In the step-by-step method, while calculating the equation for the $i$th iteration, some extra variables were computed (the next step's variables are calculated one time in the present step and once again in the next). We have proposed a high-order equation to decrease the computational complexity by reducing the extra unknowns in the other approach. The calculations are done only for the first method, and the analytical explanation of the second method has been proposed. In the study of high-order equation, more parameters (in addition to the interaction coefficient) can be assumed to be unknown. In the end, by selecting the correct roots, the fetal heart signal can be separated. The Block diagram in Fig.14 summarizes the procedure.

6. Conclusion

This work presented the Modified Van der Pol model to separate the fECG from the abdominal record. To this end, two phases are employed, the modeling phase and the separation phase. In the modeling phase, a heart model based on a Modified Duffing-Van der Pol oscillator was proposed. Because of the fetal heart's structural changes, some parameters are added to the model, so the model can illustrate the fetal cardiac state faultlessly. During pregnancy, the mother's heart and the fetus interact with each other; hence the two heart models are coupled linearly. Based on the pattern synchronization, the coupling parameter was calculated. In the separation phase, the obtained combination of maternal and fetal heart signals was considered as the aECG. Then the system of equations with the problem assumptions was constructed by solving the system of equations and calculating the variables, which are the maternal and fetal signals. Finally, the state variables were calculated in each step, making it possible to separate the maternal and fetal signals.

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Biography

Seyyed Mohammad Reza Hashemi Golpayegani is a Professor at the Biomedical Engineering Department of Amirkabir University of Technology, Tehran, Iran, and was Minister of Culture and Higher Education in Iran from 1983 to 1987. He has authored and co-authored numerous papers in Electrical and Biomedical Engineering and has also been active in the field of Philosophy of Science and Engineering Education. He is credited with the publication of a number of articles in international scientific journals in the fields mentioned.

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Fig. 1 a) The time series, and b) phase space of Duffing-Van der Pol model, with respect of parameters $a = 0.2$, $b = 5.8$, and $c = 3$ corresponding to the initial condition $(12, 6.6)$.

Fig. 2 a) The time series, and b) phase space of Modified Duffing-Van der Pol model, in parameters $a = 2.5$, $b = 1.8$, $c = 3.8$, $w_1 = 1.8$, and $w_2 = 1.2$ with initial condition $(1, -0.7)$. 

Fig. 3. Bifurcation diagrams of the Modified Duffing-Van der Pol model, using constant initial conditions [1,-0.7]. Peak values with changing $a$ as the bifurcation parameter in a) $x$ variable and b) $y$ variable. ISIs versus $a$ for c) $x$ variable, and d) $y$ variable. e) The corresponding Lyapunov exponent of the Modified Duffing-Van der Pol model, and f) zoom of the region within [-0.2,0.2]. Other parameters are $b=1.8$, $c=3.8$, $w_1=1.8$, and $w_2=1.2$.

Fig. 4. Bifurcation diagrams of the Modified Duffing-Van der Pol model for constant initial conditions [1,-0.7]. a) Peak values of $y$ variable and b) ISIs of $y$ versus $b$. c) Peak values and d) ISIs of $y$ with changing $c$. e) Maximum values of $y$, and f) ISIs of $y$ variable concerning the variation of parameter $w_1$. g) Peak values of $y$ variable, and h) ISIs of $y$ versus $w_2$.

Fig. 5. a) The time series, and b) phase space of Eq.3, for parameters $A=20$, $B=20$, $a=2.5$, $b=1.8$, $c=3.8$, $w_1=1.8$, and $w_2=1.2$ with initial condition (0.2,0.6).

Fig. 6. a) The time series of $x$ variable in fetal in red and maternal in blue, and b) the time series of $y$ variable in fetal in red and maternal in blue. c) Delayed phase space of maternal $x$ variable, and d) delayed phase space of maternal $y$ variable, in Eq.4 for setting parameters as parameters $A=20$, $B=20$, $a=2.5$, $b=1.8$, $c=3.8$, $w_1=1.8$, $w_2=1.2$, and $\tau=20$ with initial condition (1,-0.7,0.02,1).

Fig. 7. Measuring pattern synchronization index in coupled maternal and fetal heart model in the different coupling coefficients ($\varepsilon$).

Fig. 8. Combination of the mother and the fetus's heart signal in the abdominal recording. Higher intensity of maternal heart signal than in fetuses has been shown with a thicker arrow.

Fig. 9. Assuming abdominal recording, time series of combination signal of the mother and the fetus's heart, $y_1 + y_2$ from Eq.4.

Fig. 10. Integral of the summation of $y$ variables in blue, and the generation of the $x$ variables summation in green.

Fig. 11. a) Computed unknown coupling parameter, $\varepsilon$, in the time domain. b) A zoomed plot of part a in the specified region. c) Histogram of different answers for all the times.

Fig. 12. Separation of mother and fetus heart by computed unknowns. a) $M_1$, $M_2$, and $M_3$ as three roots in computing $y_1$, and b) a zoomed plot of it in the specified region. c) $F_1$, $F_2$, and $F_3$ as three roots in computing $y_2$, and d) a zoomed plot of it in the specified region.

Fig. 13. a) Estimated maternal and fetal HRV. b) Actual maternal and fetal HRV. c) Phase space of estimated maternal ECG in green and natural maternal ECG in blue.

Fig. 14. Block diagram of the proposed algorithm that summarizes the method in two phases. In the modeling phase, a non-identical network of maternal and fetal hearts synchronized. The separation phase summarizes how abdominal signal breakdown into maternal and fetal heart signal.
Fig. 3
Fig. 5

Fig. 6
Fig. 7

Fig. 8
Fig. 9

Fig. 10
Fig. 11
Fig. 12
Fig. 13

Separation

- Obtained abdominal signal as combination of maternal and fetal heart signals.
- Assuming the values of variables and the law of interaction (coupling coefficient) unknown, construct the system of equations.
- Solving unknown parameters in the model and state variables step by step
- Choosing the right root

Modeling

- Choosing heart model
- Model correction based on fetal model characteristics
- Coupling mother and fetus’s hearts model
- Synchronous analysis on coupled model
- Solving coupled system equation and deriving output signals

Fig. 14