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Decompositions of soft sets and soft matrices with applications in group decision making

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Abstract. The decompositions of soft sets and soft matrices are important tools for theoretical and practical studies. In this paper, firstly, we study the decomposition of soft sets in detail. Later, we introduce the concepts of α-upper, α-lower, α-intersection and α-union for soft matrices and present some decomposition theorems. Some of these operations are set-restricted types of existing operations of soft sets/matrices, others are α-oriented operations that provide functionality in some cases. Moreover, some relations of decompositions of soft sets and soft matrices are investigated and the newfound relations are supported with numerical examples. Finally, two new group decision making algorithms based on soft sets/matrices are constructed, and then their efficiency and practicality are demonstrated by dealing with real life problems and comparison analysis. By using these proposed approaches, solutions can be presented to soft set-based multi-criteria decision making problems, both ordinary and involving primary assessments. These allow to handle soft set-based multi-criteria decision making from different perspectives.

1. Introduction

Many problems encountered in real-life scenarios contain ambiguity or unknown data. Sometimes existing mathematical models are insufficient to deal with such situations, and therefore new models are developed. Zadeh [1] proposed fuzzy sets as a powerful and effective approach for modeling uncertainty, and in the following years, the general forms of these sets were studied (e.g., [2–6]). In 1999, Molodtsov [7] initiated the theory of soft set to parametrically classify objects (or elements of the universe) in uncertain environments. Since these sets classify alternatives according to parameters (attributes), they can be easily applied to many different areas. In 2003, Maji et al. [8] published a seminal paper on the adaptation of set operations for soft sets. In the following years, Ali et al. [9] and Pei and Miao [10] contributed to operational

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research by developing different types of intersection and union of soft sets. In [11–13] the authors focused on the operations of difference and symmetric difference on the soft sets. Aygün and Kamacı [14] introduced some generalized operations of the soft sets and discussed their related characteristic properties. In 2010, Çağman and Enginoğlu [15] revisited some of the operations of Molodtsov's soft set to make them more useful in some situations. Feng et al. [16] studied attribute analysis of information systems based on the soft sets and logical formulas over them. In 2007, Aktaş and Çağman [17] compared soft sets to fuzzy sets and rough sets, gave some basic concepts of soft set theory and defined the concept of soft group. In the following years, many papers were published on soft algebraic structures such as soft intersection semigroups [18–20], soft semirings [21], soft rings [22,23], soft near-rings [24], soft intersection Lie algebras [25], soft lattices [26–28], soft uni-Abel-Grassmann's groups [29], soft graphs [30,31], soft BCK/BCI-algebras [32–34], soft BL-algebras [35]. Moreover, many researchers were studied on the extended models of soft sets such as fuzzy soft sets [36–38], intuitionistic fuzzy soft sets [39–42], T-spherical soft sets [43], neutrosophic soft sets [44,45], three-valued soft sets [46] and N-soft sets [47–50] and new researches are currently ongoing.

In recent years, Çağman and Enginoğlu [51] published a seminal article on the matrix representations of soft sets, and thus they argued that in some cases, the use of matrix operations matching soft set operations can provide practicality to the calculations. Furthermore, they developed a matrix-based soft max-min decision making algorithm to deal with decision making problems under the soft set environment. Atagün et al. [52] constructed the soft distributive max-min decision making algorithm improving the algorithm proposed in [51]. In 2018, Kamacı et al. [53] introduced the row-products of soft matrices, and then proposed a novel decision making algorithm that can obtain an optimal choice from each of the disjoint sets of alternatives with respect to the specified parameters. Petchimuthu and Kamacı [54,55] developed some multicriteria decision making procedures based on the r-product and c-product of inverse (fuzzy) soft matrices. In [56–59], the authors derived some basic operations of soft matrices and proposed solutions to decision making problems by using these new operations. At present, the studies on the theories of soft set and soft matrix are progressing rapidly in both theoretical and practical aspects.

The set-oriented approaches based on inclusion, restriction and extension of soft sets allow the expansion of the range of operations, algebraic structures, topological structures, application aspects of soft sets. Supporting this idea, Sezer et al. [20] described the lower α-inclusion and upper α-inclusion of a soft sets over the universal set U, where $\alpha \subseteq U$. Moreover, by using the upper α-inclusion of a soft set, they introduced the upper α-semigroups, upper α-ideals and upper α bi-ideals of soft sets. The emergence of α-inclusion of soft sets leads to the idea that the fundamentals of soft sets can be revisited so that the α-oriented operations/structures of soft sets can be proposed. This paper aims to contribute to the theories of soft set and soft matrix by introducing new concepts, such as the α-intersection and α-union of soft sets, and the α-upper, α-lower, α-intersection and α-union soft matrices. Thus, initial findings and results of decompositions of soft sets and soft matrices are presented. By employing the proposed α-oriented concepts, new decision making algorithms are elaborated and then their applicability to real-world problems are illustrated.

The rest of this paper is arranged as follows. Section 2 reviews the concepts of soft sets and soft matrices. Sections 3 and 4 are devoted to theoretical findings on the decompositions of soft sets and soft matrices, respectively. Section 5 presents set-restricted soft decision making models with applications and comparative study. Section 6 gives the concluding remarks.

2. Preliminaries

In this section, we review some basic concepts for future sections.

Definition 1 [7,15]. Let $U = \{\hbar_i | i = 1, 2, ..., m\}$ be an initial universe set, $\mathfrak{X} = \{x_j | j = 1, 2, ..., n\}$ be a set of parameters (attributes), $P(U)$ be the power set of U and $A \subseteq \mathfrak{X}$. A soft set (F, A) or simply f_A on the universe U is defined as the ordered pairs:

$$(F, A) = \{(x, f_A(x)) | x \in \mathfrak{X}, f_A(x) \in P(U)\},$$

where $f_A(x) : \mathfrak{X} \to P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here f_A is called approximate function of the soft set (F, A).

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Definition 2 [15]. Let (F, A) and (F, B) be soft sets over U:

a) If $f_A(x) \subseteq f_B(x)$ for all $x \in \mathfrak{X}$, then (F, A) is a soft subset of (F, B), denoted by $(F, A)\widetilde{\subseteq}(F, B)$;

b) If $(F, A)\widetilde{\subseteq}(F, B)$ and $(F, B)\widetilde{\subseteq}(F, A)$, then the soft sets (F, A) and (F, B) are equal and denoted by $(F, A) = (F, B)$;

c) The soft union of (F, A) and (F, B) is a soft set, denoted by $(F, A)\widetilde{\cup}(F, B)$, and defined as $\{(x, f_A(x) \cup f_B(x)) | x \in \mathfrak{X}, f_A(x), f_B(x) \in P(U)\}$;

d) The soft intersection of (F,A) and (F,B) is a soft set, denoted by $(F,A)\widetilde{\cap}(F,B)$, and defined as $\{(x, f_A(x)\cap f_B(x))|x\in\mathfrak{X}, f_A(x), f_B(x)\in P(U)\}$.

Definition 3 [51]. Let f_A be a soft set over the universal set U. Then, a subset of $U\times\mathfrak{X}$ is uniquely defined as:

$$R_A=\{(\hbar_i,x_j)|x_j\in A,\hbar_i\in F(x_j)\},$$

which is termed to be a relation form of f_A. The characteristic function of R_A is written as follows:

$$\chi_{R_A}:U\times\mathfrak{X}\longrightarrow\{0,1\},\chi_{R_A}(\hbar_i,x_j)=\begin{cases}1, & (\hbar_i,x_j)\in R_A\\ 0, & (\hbar_i,x_j)\notin R_A.\end{cases}$$

If $U=\{\hbar_1,\hbar_2,...,\hbar_m\}$, $\mathfrak{X}=\{x_1,x_2,...,x_n\}$ and $A\subseteq\mathfrak{X}$, then R_A can be represented by a table as shown in Box I. If $a_{ij}=\chi_{R_A}(\hbar_i,x_j)$, the matrix:

$$[a_{ij}]_{m\times n}=\begin{bmatrix}a_{11} & a_{12} & . & . & . & a_{1n}\\ a_{21} & a_{22} & . & . & . & a_{2n}\\ . & . & . & & & .\\ . & . & & . & & .\\ . & . & & & . & .\\ a_{m1} & a_{m2} & . & . & . & a_{mn}\end{bmatrix},$$

is called an $m\times n$ soft matrix of the soft set f_A over the universal set U. The set of all $m\times n$ soft matrices over U will be denoted by $SM_{m\times n}$. From now on, $[a_{ij}]\in SM_{m\times n}$ means that $[a_{ij}]$ is an $m\times n$ soft matrix.

According to the definition of $m\times n$ soft matrix, a soft set f_A is uniquely characterized by the matrix $[a_{ij}]$. It means that a soft set f_A is formally equal to its soft matrix $[a_{ij}]$ [51].

Definition 4 [51]. Let $[a_{ij}],[b_{ij}]\in SM_{m\times n}$. Then, the soft matrix $[c_{ij}]$ is called:

a) Union of $[a_{ij}]$ and $[b_{ij}]$, denoted $[a_{ij}]\widetilde{\cup}\,[b_{ij}]$, if $c_{ij}=\max\{a_{ij},b_{ij}\}$ for all i and j;

b) Intersection of $[a_{ij}]$ and $[b_{ij}]$, denoted $[a_{ij}]\widetilde{\cap}\,[b_{ij}]$, if $c_{ij}=\min\{a_{ij},b_{ij}\}$ for all i and j.

Definition 5 [52]. Let $[a_{ij}]\in SM_{m\times n}$ and $[b_{ik}]\in SM_{m\times n'}$. Then, the soft matrix $[c_{ip}]$ is called:

a) Generalized And-product of $[a_{ij}]$ and $[b_{ik}]$, denoted $[a_{ij}]\widetilde{\wedge}\,[b_{ij}]$, if $c_{ip}=\min\{a_{ij},b_{ik}\}$ where $p=(j-1)n'+k$;

b) Generalized Or-product of $[a_{ij}]$ and $[b_{ik}]$, denoted $[a_{ij}]\widetilde{\vee}\,[b_{ij}]$, if $c_{ip}=\max\{a_{ij},b_{ik}\}$ where $p=(j-1)n'+k$.

Theorem 1 [52]. *The operation generalized And-product is associative, i.e., if $[a_{ij}]\in SM_{m\times n_1}$, $[b_{ik}]\in SM_{m\times n_2}$ and $[c_{il}]\in SM_{m\times n_3}$, then:*

$$([a_{ij}]\widetilde{\wedge}\,[b_{ik}])\widetilde{\wedge}[c_{il}]=[a_{ij}]\widetilde{\wedge}([b_{ik}]\widetilde{\wedge}\,[c_{il}]).$$

3. Decomposition of soft sets

In this section, we describe the α-intersection and α-union of soft sets and provide further theoretical results regarding these new concepts.

Definition 6 [21]. Let (F,A) be soft set over U. Then, the set:

$$supp(F,A)=\{x\in A|f_A(x)\neq\emptyset\}$$

is named the *support* of the soft set (F,A). The *null soft set* is a soft set with empty support and denoted by $\emptyset_{\mathfrak{X}}$. A soft set (F,A) is said to be a *non-null* if $supp(F,A)\neq\emptyset$.

By the following definition, we can eliminate the elements in which second component is empty set of the soft set.

Definition 7. Let (F,A) be soft set over U. Then, the soft set $(F,A)_s$ defined as:

$$(F,supp(F,A))=\{(x,f_A(x))|x\in supp(F,A)\}$$

is called the *supported soft set* of the soft set (F,A).

Now we are ready to define decomposition of soft sets.

Definition 8. Let (F,A) be soft set over U. If there exist soft sets (F,B) and (F,C) over U such that:

a) $supp(F,A)=supp(F,B)\cup supp(F,C)$;

b) $(F,A)_s=((F,B)\widetilde{\cup}(F,C))_s$;

c) $(F,B)\widetilde{\cap}(F,C)=\emptyset_{\mathfrak{X}}$.

R_A	x_1	x_2	. . .	x_n
$\hbar_1$	$\chi_{R_A}(\hbar_1,x_1)$	$\chi_{R_A}(\hbar_1,x_2)$	. . .	$\chi_{R_A}(u_1,x_n)$
$\hbar_2$	$\chi_{R_A}(\hbar_2,x_1)$	$\chi_{R_A}(\hbar_2,x_2)$	. . .	$\chi_{R_A}(\hbar_2,x_n)$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$\hbar_m$	$\chi_{R_A}(\hbar_m,x_1)$	$\chi_{R_A}(\hbar_m,x_2)$	. . .	$\chi_{R_A}(\hbar_m,x_n)$

Box I

Then the soft set (F, A) is said to be a *composition* of (F, B) and (F, C), and denoted by:

$$(F, A) = (F, B) \oplus (F, C).$$

Definition 9 [20]. Let (F, A) be soft set over U and $\alpha \subseteq U$. Then:

a) *Upper α-inclusion* of (F, A) is defined as $(F, A)^{\supseteq \alpha} = \{x \in A \mid f_A(x) \supseteq \alpha\}$;

b) *Lower α-inclusion* of (F, A) is defined as $(F, A)^{\subseteq \alpha} = \{x \in A \mid f_A(x) \subseteq \alpha\}$.

Theorem 2. *If $(F, A) = (F, B) \oplus (F, C)$ and $\alpha \subseteq U$, then:*

i) $(F, A)_s^{\subseteq \alpha} \supseteq (F, B)_s^{\subseteq \alpha} \cap (F, C)_s^{\subseteq \alpha}$;

ii) $(F, A)_s^{\subseteq \alpha} \subseteq (F, B)_s^{\subseteq \alpha} \cup (F, C)_s^{\subseteq \alpha}$;

iii) $(F, A)_s^{\supseteq \alpha} \supseteq (F, B)_s^{\supseteq \alpha} \cup (F, C)_s^{\supseteq \alpha}$.

Proof.

i) Let $(F, A) = (F, B) \oplus (F, C)$ and $x \in (F, B)_s^{\subseteq \alpha} \cap (F, C)_s^{\subseteq \alpha}$. Then $f_B(x) \subseteq \alpha$ and $f_C(x) \subseteq \alpha$. Hence $f_B(x) \cup f_C(x) = f_A(x) \subseteq \alpha$. Finally, we have $x \in (F, A)_s^{\subseteq \alpha}$;

ii) Let $x \in (F, A)_s^{\subseteq \alpha}$. Since $f_A(x) = f_B(x) \cup f_C(x)$ and $f_A(x) \subseteq \alpha$, we have $f_B(x) \subseteq \alpha$ and $f_C(x) \subseteq \alpha$. Therefore, $x \in (F, B)_s^{\subseteq \alpha} \cup (F, C)_s^{\subseteq \alpha}$;

iii) Let $x \in (F, B)_s^{\supseteq \alpha} \cup (F, C)_s^{\supseteq \alpha}$. Then $f_B(x) \supseteq \alpha$ or $f_C(x) \supseteq \alpha$ which implies $(f_B(x) \cup f_C(x)) \supseteq \alpha$. Since $f_A(x) = f_B(x) \cup f_C(x)$, then $f_A(x) \supseteq \alpha$. Hence we have $x \in (F, A)_s^{\supseteq \alpha}$, which completes the proof. □

Example 1. Let $U = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4, \hbar_5, \hbar_6\}$ be a universal discourse set and $\mathfrak{X} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ be a set of parameters. Also, $A = \{x_1, x_2, x_3, x_4, x_5\}$, $B = \{x_1, x_2, x_3, x_4\}$ and $C = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ are three subsets of $\mathfrak{X}$. Suppose that the soft sets related to the parameter subsets A, B and C are:

$$f_A = \{(x_1, \{\hbar_1, \hbar_2, \hbar_3\}), (x_2, \{\hbar_1, \hbar_4\}),$$
$$(x_3, \{\hbar_2, \hbar_3, \hbar_5\}), (x_4, \{\hbar_1, \hbar_5\}), (x_5, \emptyset)\},$$

$$f_B = \{(x_1, \{\hbar_1, \hbar_2\}), (x_2, \emptyset), (x_3, \{\hbar_2\}), (x_4, \{\hbar_1\})\}$$

and:

$$F_C = \{(x_1, \{\hbar_3\}), (x_2, \{\hbar_1, \hbar_4\}), (x_3, \{\hbar_3, \hbar_5\}),$$
$$(x_4, \{\hbar_5\}), (x_5, \emptyset), (x_6, \emptyset)\}.$$

Then, the support sets of these soft sets are:

$$supp(F, A) = \{x_1, x_2, x_3, x_4\},$$

$$supp(F, B) = \{x_1, x_3, x_4\},$$

$$supp(F, C) = \{x_1, x_2, x_3, x_4\}.$$

Obviously, we see that:

$$supp(F, A) = supp(F, B) \cup supp(F, C).$$

The soft union of (F, B) and (F, C) is the soft set:

$$(F, B)\tilde{\cup}(F, C) = \{(x_1, \{\hbar_1, \hbar_2, \hbar_3\}), (x_2, \{\hbar_1, \hbar_4\}),$$
$$(x_3, \{\hbar_2, \hbar_3, \hbar_5\}), (x_4, \{\hbar_1, \hbar_5\}), (x_5, \emptyset), (x_6, \emptyset)\}$$

and the support set of $(F, B)\tilde{\cup}(F, C)$ is $supp((F, B) \tilde{\cup}(F, C)) = \{x_1, x_2, x_3, x_4\}$.

Now we can write the soft sets $(F, A)_s$ and $((F, B)\tilde{\cup}(F, C))_s$ as follows:

$$(F, A)_s = \{(x_1, \{\hbar_1, \hbar_2, \hbar_3\}), (x_2, \{\hbar_1, \hbar_4\}),$$
$$(x_3, \{\hbar_2, \hbar_3, \hbar_5\}), (x_4, \{\hbar_1, \hbar_5\})\},$$

and:

$$((F, B)\tilde{\cup}(F, C))_s = \{(x_1, \{\hbar_1, \hbar_2, \hbar_3\}), (x_2, \{\hbar_1, \hbar_4\}),$$
$$(x_3, \{\hbar_2, \hbar_3, \hbar_5\}), (x_4, \{\hbar_1, \hbar_5\})\}.$$

Then, we have:

$$(F, A)_s = ((F, B)\tilde{\cup}(F, C))_s.$$

Since:

$$(F, B)\tilde{\cap}(F, C) = \{(x_1, \emptyset), (x_2, \emptyset), (x_3, \emptyset), (x_4, \emptyset),$$
$$(x_5, \emptyset), (x_6, \emptyset)\} = \emptyset_{\mathfrak{X}},$$

then $(F, A) = (F, B) \oplus (F, C)$. Now let $\alpha = \{\hbar_2, \hbar_3\} \subseteq U$. Then:

$$(F, A)_s^{\subseteq \alpha} = \emptyset, \quad (F, B)_s^{\subseteq \alpha} = \{x_3\} \quad \text{and}$$

$$(F, C)_s^{\subseteq \alpha} = \{x_1\}.$$

Hence $(F, A)_s^{\subseteq \alpha} \supseteq (F, B)_s^{\subseteq \alpha} \cap (F, C)_s^{\subseteq \alpha}$ and $(F, A)_s^{\subseteq \alpha} \subseteq (F, B)_s^{\subseteq \alpha} \cup (F, C)_s^{\subseteq \alpha}$ are satisfied. Now:

$$(F, A)_s^{\supseteq \alpha} = \{x_1, x_3\} \quad \text{and}$$

$$(F, B)_s^{\supseteq \alpha} = (F, C)_s^{\supseteq \alpha} = \emptyset.$$

Finally, we see that $(F, A)_s^{\supseteq \alpha} \supseteq (F, B)_s^{\supseteq \alpha} \cup (F, C)_s^{\supseteq \alpha}$ is satisfied.

Definition 10. Let (F, A) be soft set over U and $\emptyset \neq \alpha \subseteq U$. Then:

a) *α-intersection* of (F, A) is defined as $(F, A)^{\cap \alpha} = \{x \in A \mid f_A(x) \cap \alpha \neq \emptyset\}$,

b) *α-union* of (F, A) is defined as $(F, A)^{\cup \alpha} = \{x \in A \mid f_A(x) \cup \alpha = U\}$.

By this definition, it is easily seen that $(F,A)^{\cap\emptyset} = \emptyset$, $(F,A)^{\cap U} = A$, $(F,A)^{\cup U} = A$. If $f_A(x) \neq U$ for all $x \in A$, then $(F,A)^{\cup\emptyset} = \emptyset$.

Theorem 3. *If* $(F,A) = (F,B) \oplus (F,C)$ *and* $\emptyset \neq \alpha \subseteq U$*, then:*

i) $(F,B)_s^{\cap\alpha} \cup (F,C)_s^{\cap\alpha} \subseteq (F,A)_s^{\cap\alpha}$,

ii) $(F,B)_s^{\cup\alpha} \cup (F,C)_s^{\cup\alpha} \subseteq (F,A)_s^{\cup\alpha}$,

iii) $(F,B)_s^{\cap\alpha} \cap (F,C)_s^{\cap\alpha} \subseteq (F,A)_s^{\cap\alpha}$.

Proof.

i) Let $x \in (F,B)_s^{\cap\alpha} \cup (F,C)_s^{\cap\alpha}$, then $f_B(x) \cap \alpha \neq \emptyset$ or $f_C(x) \cap \alpha \neq \emptyset$. Since:

$$f_A(x) \cap \alpha = (f_B(x) \cup f_C(x)) \cap \alpha$$
$$= (f_B(x) \cap \alpha) \cup (f_C(x) \cap \alpha),$$

then $f_A(x) \cap \alpha \neq \emptyset$, i.e., $x \in (F,A)_s^{\cap\alpha}$ and:

$$(F,B)_s^{\cap\alpha} \cup (F,C)_s^{\cap\alpha} \subseteq (F,A)_s^{\cap\alpha}.$$

ii) Let $x \in (F,B)_s^{\cup\alpha} \cup (F,C)_s^{\cup\alpha}$. Then $f_B(x) \cap \alpha \neq \emptyset$ or $f_C(x) \cap \alpha \neq \emptyset$. Since:

$$f_A(x) \cup \alpha = (f_B(x) \cup f_C(x)) \cup \alpha$$
$$= (f_B(x) \cup \alpha) \cup (f_C(x) \cup \alpha) = U,$$

then $x \in (F,A)_s^{\cup\alpha}$. Therefore $(F,B)_s^{\cup\alpha} \cup (F,C)_s^{\cup\alpha} \subseteq (F,A)_s^{\cup\alpha}$ is obtained.

iii) Let $x \in (F,B)_s^{\cap\alpha} \cap (F,C)_s^{\cap\alpha}$. Then $f_B(x) \cap \alpha \neq \emptyset$ and $f_C(x) \cap \alpha \neq \emptyset$. Since:

$$f_A(x) \cap \alpha = (f_B(x) \cup f_C(x)) \cap \alpha \neq \emptyset,$$

then $x \in (F,A)_s^{\cap\alpha}$. Hence $(F,B)_s^{\cap\alpha} \cap (F,C)_s^{\cap\alpha} \subseteq (F,A)_s^{\cap\alpha}$.

Example 2. Let the soft sets $(F,A), (F,B)$ and (F,C) over U given in Example 1 and $\alpha = \{\hbar_1, \hbar_4, \hbar_6\}$. Then:

$$(F,A)_s^{\cap\alpha} = \{x_1, x_2, x_4\} \quad \text{and} \quad (F,A)_s^{\cup\alpha} = \{x_3\},$$

$$(F,B)_s^{\cap\alpha} = \{x_1, x_4\} \quad \text{and} \quad (F,B)_s^{\cup\alpha} = \emptyset,$$

$$(F,C)_s^{\cap\alpha} = \{x_2\} \quad \text{and} \quad (F,C)_s^{\cup\alpha} = \emptyset.$$

Then, we see that the following are satisfied.

$$(F,B)_s^{\cap\alpha} \cup (F,C)_s^{\cap\alpha} \subseteq (F,A)_s^{\cap\alpha},$$

$$(F,B)_s^{\cup\alpha} \cup (F,C)_s^{\cup\alpha} \subseteq (F,A)_s^{\cup\alpha},$$

$$(F,B)_s^{\cap\alpha} \cap (F,C)_s^{\cap\alpha} \subseteq (F,A)_s^{\cap\alpha}.$$

4. Decomposition of soft matrices

In this section, the support, lower α-inclusion, upper α-inclusion, α-intersection and α-union of soft matrices are defined and their remarkable properties are given.

Throughout this section, $U = \{\hbar_1, \hbar_2, \cdots, \hbar_m\}$ is the universal set, $\mathfrak{X} = \{x_1, x_2, \cdots, x_n\}$ is the parameter set and $[a_{ij}] \in SM_{m\times n}$ denotes a soft matrix over U.

Notations:

1. $|a_{ij}|_j$ denotes j, column of the soft matrix $[a_{ij}]$,
2. $|0|_j$ means that the j, column of $[a_{ij}]$ consists of zeros,
3. $|1|_j$ means that the j, column of $[a_{ij}]$ consists of 1.

So, we can express a soft matrix $[a_{ij}]$ as $\bigsqcup_{j=1}^{n} |a_{ij}|_j$ by using its columns.

By the following definition, we give the concept of support set of a soft matrix:

Definition 11. Let $[a_{ij}] \in SM_{m\times n}$. Then the *support set* of $[a_{ij}]$ is the subset of $\{1, 2, \cdots, n\}$ and defined as:

$$supp[a_{ij}] = \{j \mid a_{ij} \neq 0, \exists i = 1, 2, \cdots, m\}.$$

Now, we are ready to give the concept "composition of soft matrices".

Definition 12. Let $[a_{ij}] \in SM_{m\times n}$. If there exist soft matrices $[b_{ij}], [c_{ij}] \in SM_{m\times n}$ such that:

a) $supp[a_{ij}] = supp[b_{ij}] \cup supp[c_{ij}]$,

b) $[a_{ij}] = [b_{ij}] \tilde{\cup} [c_{ij}]$,

c) $[b_{ij}] \tilde{\cap} [c_{ij}] = [0]$,

then the soft matrix $[a_{ij}]$ is said to be a *composition* of $[b_{ij}]$ and $[c_{ij}]$, and denoted by:

$$[a_{ij}] = [b_{ij}] \oplus [c_{ij}].$$

Example 3. We consider the soft sets $(F,A), (F,B)$, and (F,C) given in Example 1 Then, the corresponding soft matrices of soft sets $(F,A), (F,B)$, and (F,C) over U are respectively:

$$[a_{ij}] = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$[b_{ij}] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$[c_{ij}] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Then:

$$supp[a_{ij}] = \{1,2,3,4\}, \quad supp[b_{ij}] = \{1,3,4\}, \quad \text{and}$$

$$supp[c_{ij}] = \{1,2,3,4\}.$$

It is seen that:

$$supp[a_{ij}] = supp[b_{ij}] \cup supp[c_{ij}], \quad [a_{ij}] = [b_{ij}]\widetilde{\cup}[c_{ij}],$$

$$\text{and} \quad [b_{ij}]\widetilde{\cap}\,[c_{ij}] = [0]$$

are satisfied. Hence, we have $[a_{ij}] = [b_{ij}] \oplus [c_{ij}]$.

We can express a soft set (F,A) over U by its soft matrix, mutually. Following theorem shows that this is valid for composition of soft sets and composition of soft matrices.

Theorem 4. *Let $(F,A), (F,B)$, and (F,C) be soft sets over U and let $[a_{ij}], [b_{ij}]$, and $[c_{ij}]$ be corresponding soft matrices, respectively. Then:*

$$(F,A) = (F,B) \oplus (F,C) \quad \text{if and only if}$$

$$[a_{ij}] = [b_{ij}] \oplus \; [c_{ij}].$$

Proof. Let $(F,A) = (F,B) \oplus (F,C)$. By the definitions of $supp(F,A)$ and $supp[a_{ij}]$, it is seen that:

$$x_j \in supp(F,A) \;\; \text{if and only if} \;\; j \in supp[a_{ij}].$$

Then $supp(F,A) = supp(F,B) \cup supp(F,C)$ iff $supp\,[a_{ij}] = supp[b_{ij}] \cup supp[c_{ij}]$.

Assume that R_A, R_B, and R_C are relation forms of the soft sets $(F,A), (F,B)$, and (F,C), respectively.

By Definitions 3 and 4:

$$(\hbar_i, x_j) \in R_A \quad \text{implies that} \quad (\hbar_i, x_j) \in R_B \quad \text{or}$$

$$(\hbar_i, x_j) \in R_C \quad \text{if and only if}$$

$$a_{ij} = \chi_{R_A}(\hbar_i, x_j) = \max\{b_{ij} = \chi_{R_B}(\hbar_i, x_j),$$

$$c_{ij} = \chi_{R_C}(\hbar_i, x_j)\},$$

and:

$$(\hbar_i, x_j) \in R_B \quad \text{and} \quad (\hbar_i, x_j) \in R_C \quad \text{if and only if}$$

$$\min\{b_{ij} = \chi_{R_B}(\hbar_i, x_j),$$

$$c_{ij} = \chi_{R_C}(\hbar_i, x_j)\} = 1.$$

Then:

$$(F,A)_s = ((F,B)\widetilde{\cup}(F,C))_s \quad \text{if and only if}$$

$$[a_{ij}] = [b_{ij}]\widetilde{\cup}\;[c_{ij}],$$

and:

$$(F,B)\widetilde{\cap}(F,C) = \emptyset_{\mathfrak{X}} \quad \text{if and only if}$$

$$[b_{ij}]\widetilde{\cap}\;[c_{ij}] = [0].$$

Therefore, by considering Definitions 3.3 and 4.2, we have:

$$(F,A) = (F,B) \oplus (F,C) \quad \text{if and only if}$$

$$[a_{ij}] = [b_{ij}] \oplus \; [c_{ij}].$$

□

Definition 13. Let $[a_{ij}] \in SM_{m\times n}$ and let $\alpha = \{\hbar_i \mid i \in I\} \subseteq U$, where $I \subseteq \{1,2,\cdots,m\}$. Then:

a) *α-upper soft matrix* of $[a_{ij}]$ denoted by $[a_{ij}]^\alpha$ is defined as:

$$[a_{ij}]^\alpha = \bigsqcup_{j=1}^{n} \begin{cases} |a_{ij}|_j, & \text{if} \quad \forall i \in I, a_{ij} = 1 \\ |0|_j, & \text{otherwise} \end{cases}$$

b) *α-lower soft matrix* of $[a_{ij}]$ denoted by $[a_{ij}]_\alpha$ is defined as:

$$[a_{ij}]_\alpha = \bigsqcup_{j=1}^{n} \begin{cases} |0|_j, & \text{if} \; \forall i \in \{1,2,\cdots,m\}\backslash I, a_{ij} = 1 \\ |a_{ij}|_j, & \text{otherwise} \end{cases}$$

It is clear that $[a_{ij}]^\alpha$ and $[a_{ij}]_\alpha$ are also $m \times n$ soft matrices.

Example 4. We consider the soft matrix $[a_{ij}]$ given in Example 3 and $\alpha = \{\hbar_5\}$. Then $I = \{5\}$ and:

$$[a_{ij}]^\alpha = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Now let $\beta = \{\hbar_1, \hbar_2, \hbar_3, \hbar_5\}$. Then $I = \{1,2,3,5\}$ and:

$$[a_{ij}]_\beta = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Theorem 5. *Let (F,A) be a soft set over $U = \{\hbar_1, \hbar_2, \cdots, \hbar_m\}$, $\emptyset \neq \alpha \subseteq U$ and let $[a_{ij}]$ be the soft matrix of (F,A). Then:*

i) $[a_{ij}]^\alpha$ is the soft matrix of the soft set $(F,(F,A)^{\supseteq\alpha})$.

ii) $[a_{ij}]_\alpha$ is the soft matrix of the soft set $(F,(F,A)^{\subseteq\alpha})$.

Proof. Let $\alpha=\{\hbar_i \mid i\in I\}$, where $\emptyset\neq I\subseteq\{1,2,...,m\}$.

i) Since:

$$(F,A)^{\supseteq\alpha}=\{x_j\in A \mid f_A(x_j)\supseteq\alpha\},$$

then $\hbar_i\in\alpha$ implies that $(\hbar_i,x_j)\in R_{A^\alpha}$, where R_{A^α} is the relation form of the soft set $(F,(F,A)^{\supseteq\alpha})$. If $\hbar_i\in\alpha$, then $\chi_{R_{A^\alpha}}(\hbar_i,x_j)=a_{ij}$, which is the ijth component of $[a_{ij}]^\alpha$.

ii) Since:

$$(F,A)^{\subseteq\alpha}=\{x_j\in A \mid f_A(x_j)\subseteq\alpha\},$$

then $(\hbar_i,x_j)\in R_{A_\alpha}$, where R_{A_α} is the relation form of the soft set $(F,(F,A)^{\subseteq\alpha})$. That's mean, if $\hbar_i\in U\setminus\alpha$, then $\chi_{R_{A_\alpha}}(\hbar_i,x_j)=0$. Hence the proof is seen by Definition 13. □

Corollary 1. Let $[a_{ij}]\in SM_{m\times n}$ and let $\alpha=\{\hbar_i \mid i\in I\}$, where $\emptyset\neq I\subseteq\{1,2,\cdots,m\}$. If $[a_{ij}]=[b_{ij}]\oplus[c_{ij}]$, then:

i) $[a_{ij}]^\alpha\supseteq[b_{ij}]^\alpha\widetilde{\cap}[c_{ij}]^\alpha$;

ii) $[a_{ij}]^\alpha\supseteq[b_{ij}]^\alpha\widetilde{\cup}[c_{ij}]^\alpha$;

iii) $[a_{ij}]_\alpha\subseteq[b_{ij}]_\alpha\widetilde{\cup}[c_{ij}]_\alpha$.

Proof. The proofs are seen by Theorems 2, 4, and 5.

Example 5. Consider the soft matrices $[a_{ij}]$, $[b_{ij}]$, and $[c_{ij}]$ given in Example 3 and assume $\alpha=\{\hbar_3,\hbar_5\}$. Then $I=\{3,5\}$ and:

$$[a_{ij}]^\alpha=\begin{bmatrix}0&0&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&0&0&0&0\end{bmatrix},$$

$[b_{ij}]^\alpha=[0]$ and

$$[c_{ij}]^\alpha=\begin{bmatrix}0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&0&0&0&0\end{bmatrix}.$$

Moreover, we obtain:

$[a_{ij}]_\alpha=[0]$, $[b_{ij}]_\alpha=[0]$, and

$$[c_{ij}]_\alpha=\begin{bmatrix}0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\1&0&1&0&0&0&0\\0&0&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&0&0&0&0\end{bmatrix}.$$

Then it is seen that $[a_{ij}]^\alpha\supseteq[b_{ij}]^\alpha\widetilde{\cap}[c_{ij}]^\alpha$, $[a_{ij}]^\alpha\supseteq[b_{ij}]^\alpha\widetilde{\cup}[c_{ij}]^\alpha$ and $[a_{ij}]_\alpha\subseteq[b_{ij}]_\alpha\widetilde{\cup}[c_{ij}]_\alpha$ are satisfied.

Definition 14. Let $[a_{ij}]\in SM_{m\times n}$ and let $\alpha=\{\hbar_i \mid i\in I\}\subseteq U$, where $I\subseteq\{1,2,\cdots,m\}$. Then:

a) *α-intersection soft matrix* of $[a_{ij}]$ denoted by $[a_{ij}]^{\cap\alpha}$ is defined as:

$$[a_{ij}]^{\cap\alpha}=\bigsqcup_{j=1}^{n}\begin{cases}|a_{ij}|_j, & \text{if } \exists i\in I, a_{ij}=1\\ |0|_j, & \text{otherwise}\end{cases}$$

b) *α-union soft matrix* of $[a_{ij}]$ denoted by $[a_{ij}]^{\cup\alpha}$ is defined as:

$$[a_{ij}]^{\cup\alpha}=\bigsqcup_{j=1}^{n}\begin{cases}|a_{ij}|_j, & \text{if } \forall i\in\{1,2,\cdots,m\}\setminus I, a_{ij}=1\\ |0|_j, & \text{otherwise}\end{cases}$$

It is clear that $[a_{ij}]^{\cap\alpha}$ and $[a_{ij}]^{\cup\alpha}$ are also $m\times n$ soft matrices.

Theorem 6. *Let (F,A) be a soft set over $U=\{\hbar_1,\hbar_2,\cdots,\hbar_m\}$, $\emptyset\neq\alpha\subseteq U$ and let $[a_{ij}]$ be the soft matrix of (F,A). Then:*

i) $[a_{ij}]^{\cap\alpha}$ is the soft matrix of the soft set $(F,(F,A)^{\cap\alpha})$;

ii) $[a_{ij}]^{\cup\alpha}$ is the soft matrix of the soft set $(F,(F,A)^{\cup\alpha})$.

Proof. Let $\alpha=\{\hbar_i \mid i\in I\}$, where $\emptyset\neq I\subseteq\{1,2,...,m\}$.

i) Since:

$$(F,A)^{\cap\alpha}=\{x_j\in A \mid f_A(x_j)\cap\alpha\neq\emptyset\},$$

then $\hbar_i\in f_A(x_j)\cap\alpha$ implies that $(\hbar_i,x_j)\in R_{A^{\cap\alpha}}$, where $R_{A^{\cap\alpha}}$ is the relation form of the soft set $(F,(F,A)^{\cap\alpha})$. That's mean, if $\hbar_i\in f_A(x_j)\cap\alpha$, then $\chi_{R_{A^{\cap\alpha}}}(\hbar_i,x_j)=a_{ij}$, which is the ijth component of $[a_{ij}]^{\cap\alpha}$;

ii) Let $R_{A^{\cup\alpha}}$ is the relation form of the soft set $(F,(F,A)^{\cup\alpha})$. Since:

$$(F,A)^{\cup\alpha}=\{x_j\in A \mid f_A(x_j)\cup\alpha=U\},$$

then $(\hbar_i,x_j)\in R_{A^{\cup\alpha}}$ implies that for $\exists i\in\{1,2,...,m\}\setminus I$, $\chi_{R_{A^{\cup\alpha}}}(\hbar_i,x_j)=a_{ij}$, which is the ijth component of $[a_{ij}]^{\cup\alpha}$. Hence the proof is seen by Definition 14. □

Corollary 2. Let $[a_{ij}] \in SM_{m\times n}$ and let $\alpha = \{\hbar_i \mid i \in I\}$, where $\emptyset \neq I \subseteq \{1, 2, \cdots, m\}$. If $[a_{ij}] = [b_{ij}] \oplus [c_{ij}]$, then:

i) $[a_{ij}]^{\cap\alpha} \supseteq [b_{ij}]^{\cap\alpha} \widetilde{\cup} [c_{ij}]^{\cap\alpha}$;

ii) $[a_{ij}]^{\cap\alpha} \supseteq [b_{ij}]^{\cap\alpha} \widetilde{\cap} [c_{ij}]^{\cap\alpha}$;

iii) $[a_{ij}]^{\cup\alpha} \supseteq [b_{ij}]^{\cup\alpha} \widetilde{\cup} [c_{ij}]^{\cup\alpha}$.

Proof. The proof is seen by Theorems 3, 4, and 6. □

Example 6. We take the soft matrices $[a_{ij}], [b_{ij}]$ and $[c_{ij}]$ given in Example 3 and $\alpha = \{\hbar_3, \hbar_4, \hbar_5, \hbar_6\}$. Then $I = \{3, 4, 5, 6\}$ and:

$$[a_{ij}]^{\cap\alpha} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = [a_{ij}],$$

$$[b_{ij}]^{\cap\alpha} = [0], \quad \text{and}$$

$$[c_{ij}]^{\cap\alpha} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = [c_{ij}].$$

Moreover, we obtain:

$$[a_{ij}]^{\cup\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$[b_{ij}]^{\cup\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad [c_{ij}]^{\cup\alpha} = [0].$$

Then, it is seen that $[a_{ij}]^{\cap\alpha} \supseteq [b_{ij}]^{\cap\alpha} \widetilde{\cup} [c_{ij}]^{\cap\alpha}$, $[a_{ij}]^{\cap\alpha} \supseteq [b_{ij}]^{\cap\alpha} \widetilde{\cap} [c_{ij}]^{\cap\alpha}$, and $[a_{ij}]^{\cup\alpha} \supseteq [b_{ij}]^{\cup\alpha} \widetilde{\cup} [c_{ij}]^{\cup\alpha}$ are satisfied.

5. Set-restricted soft decision making

In this section, we define the concepts of aggregate-row function, aggregate decision triple, first and second decision values. By using these concepts, we construct two soft decision making algorithms, the first of which is to deal with classical group decision making problems and the other is to deal with set-restricted group decision making problems.

Definition 15. Let $[a_{ij}] \in SM_{m\times n}$. Then aggregate-row function, denoted by τ_i, is defined by:

$$\tau_i : SM_{m\times n} \to \mathbb{Z}, \quad \tau_i([a_{ij}]) = \sum_{j=1}^{n} a_{ij}$$

for $i \in \{1, 2, ..., m\}$.

Definition 16. Let $[a^1_{ij}], [a^2_{ij}], ..., [a^r_{ij}] \in SM_{m\times n}$. Also let $[c_{ip}] = \widetilde{\wedge}^r_{t=1}[a^t_{ij}]$ and $[d_{ij}] = \widetilde{\cap}^r_{t=1}[a^t_{ij}]$. Then, the triple:

$$\kappa_i = (\kappa^1_i, \kappa^2_i, \kappa^3_i)$$

is called an aggregate decision triple of $h_i \in U$, where the decision components κ^1_i, κ^2_i, and κ^3_i are calculated as below:

$$\kappa^1_i = \tau_i([c_{ip}]), \quad \kappa^2_i = \sum_{t=1}^{r} \tau_i([a^t_{ij}]), \quad \text{and}$$

$$\kappa^3_i = r \times \tau_i([d_{ij}]).$$

Definition 17. Let $\kappa_i = (\kappa^1_i, \kappa^2_i, \kappa^3_i)$ be an aggregate decision triple of $\hbar_i \in U$. Then:

a) The value $\ell^1_i = \kappa^1_i + \kappa^2_i$ is called a first decision value for $\hbar_i \in U$;

b) The value $\ell^2_i = 2\kappa^3_i - \kappa^2_i$ is called a second decision value for $\hbar_i \in U$.

For $h_i, h'_i \in U$, the selection order of κ_i and κ'_i is found as follows:

- If $\ell^1_i > \ell^1_{i'}$ then we have $\kappa_i > \kappa'_i$;
- If $\ell^1_i = \ell^1_{i'}$, we consider the second decision values ℓ^2;
- If $\ell^2_i > \ell^2_{i'}$ then we have $\kappa_i > \kappa'_i$;
- If $\ell^1_i = \ell^1_{i'}$ and $\ell^2_i = \ell^2_{i'}$, we have $\kappa_i = \kappa'_i$.

Definition 18. Let $U = \{\hbar_1, \hbar_2, ..., \hbar_m\}$ be a universal set. By using the aggregate decision triple κ_i, the first decision value ℓ^1_i and the second decision value ℓ^2_i, we find the ranking order of objects as follows:

$$\hbar_{i_1} \succ \hbar_{i_1} \succ ... \succ \hbar_{i_m} \quad \text{if}$$

$$\kappa_{i_1} > \kappa_{i_2} > ... > \kappa_{i_m}.$$

Then, we obtain a subset of U as follows:

$$Opt_\kappa(U) = \{\hbar_i : \hbar_i \in U \text{ and } \kappa_i > \kappa_{i'} \text{ for each } i' \neq i\}$$

which is called an optimum set of U.

By using the emerged notions in the soft matrix theory, let us create an algorithm for group decision making.

Algorithm 1. Soft decision making algorithm

The detailed steps of the soft decision making algorithm are below:

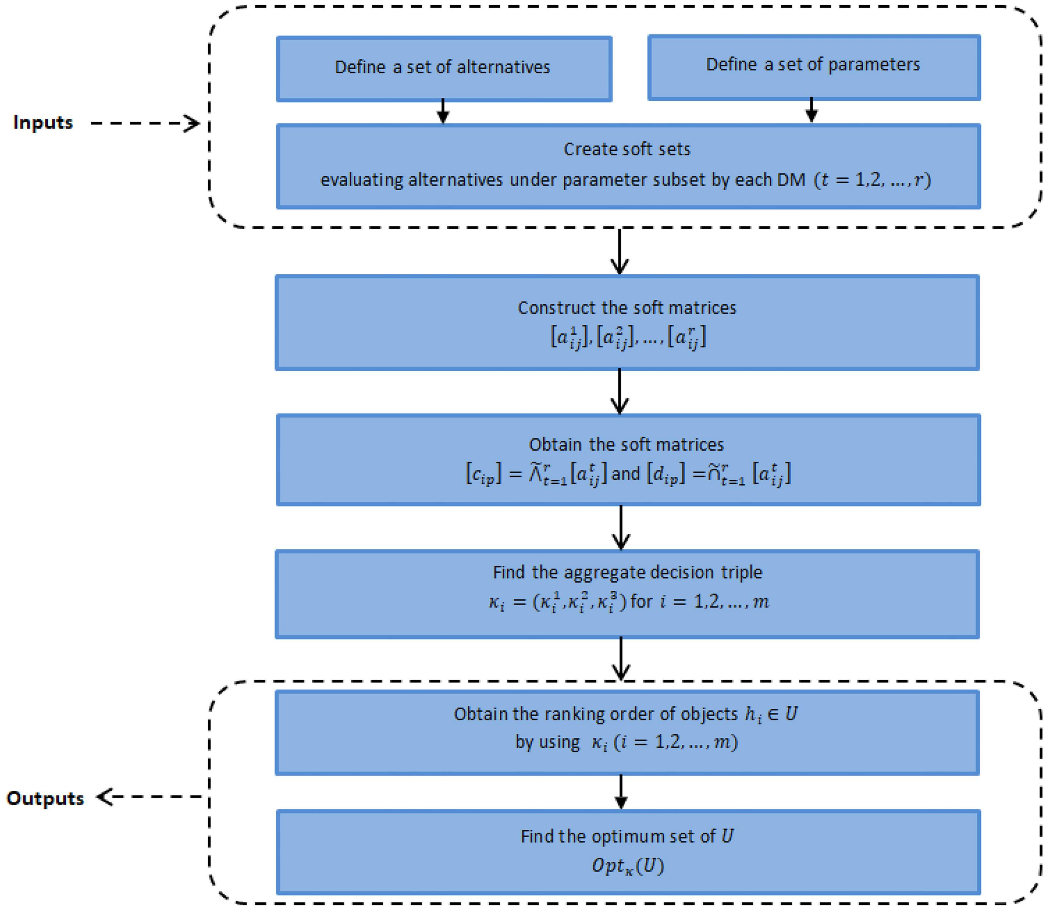

Figure 1. The framework of Algorithm 1.

Step 1. Decision Makers (DMs) choose feasible subsets of the parameter set and then create soft sets;

Step 2. Construct the soft matrices $[a^1_{ij}], [a^2_{ij}], \ldots, [a^r_{ij}]$;

Step 3. Obtain the soft matrices $[c_{ip}] = \widetilde{\wedge}^r_{t=1}[a^t_{ij}]$ and $[d_{ij}] = \widetilde{\cap}^r_{t=1}[a^t_{ij}]$;

Step 4. Find the aggregate decision triple $\kappa_i = (\kappa^1_i, \kappa^2_i, \kappa^3_i)$ for $i = 1, 2, \ldots, m$;

Step 5. Obtain the ranking order of objects $h_i \in U$ ($i = 1, 2, \ldots, m$) and find the optimum set of U.

The step by step procedure of Algorithm 1 is illustrated in Figure 1.

Now, we present an illustrative example for the Algorithm 1.

Example 7. Mr. X, Mrs. X and daughter X are moving into a big city. They are planning to buy a house in this city, so they set the criteria they want. Their common criteria are x_1 = Budget-friendly, x_2 = Modern and x_3 = Beautiful designed. The real estate agent offers them nine houses $U = \{\hbar_1, \hbar_2, \cdots, \hbar_9\}$ that carry some of these criteria.

Now they are ready to evaluate the houses under the criteria and then determine the optimal house to buy by following the steps of soft decision making algorithm:

Step 1. By determining the houses that correspond for each of their criteria, they created the following soft sets as follows, respectively:

$$f_{A_1} = \{(x_1, \{\hbar_3, \hbar_6, \hbar_7\}), (x_2, \{\hbar_1, \hbar_2, \hbar_4, \hbar_5, \hbar_7\}), (x_3, \{\hbar_1, \hbar_3, \hbar_4, \hbar_6, \hbar_7, \hbar_8\})\},$$

$$f_{A_2} = \{(x_1, \{\hbar_1, \hbar_3, \hbar_7, \hbar_9\}), (x_2, \{\hbar_2, \hbar_4, \hbar_5, \hbar_7, \hbar_8, \hbar_9\}), (x_3, \{\hbar_3, \hbar_4, \hbar_6, \hbar_9\})\},$$

$$f_{A_3} = \{(x_2, \{\hbar_2, \hbar_3, \hbar_4, \hbar_5, \hbar_6, \hbar_7, \hbar_9\}), (x_3, \{\hbar_3, \hbar_5, \hbar_6, \hbar_9\})\}.$$

Step 2. The soft matrices of soft sets f_{A_1}, f_{A_2}, and f_{A_3} are respectively:

$$[a^1_{ij}] = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad [a^2_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad \text{and}$$

$$[a^3_{ij}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

Step 3. They obtain the soft matrices are shown in Box II.

Step 4. The aggregate decision triple for each $i = 1, 2, ..., 9$ is calculated as in Table 1.

Figure 2 gives a graphical representation of aggregate decision triples in Table 1.

Step 5. Then, the ranking order of houses is obtained as:

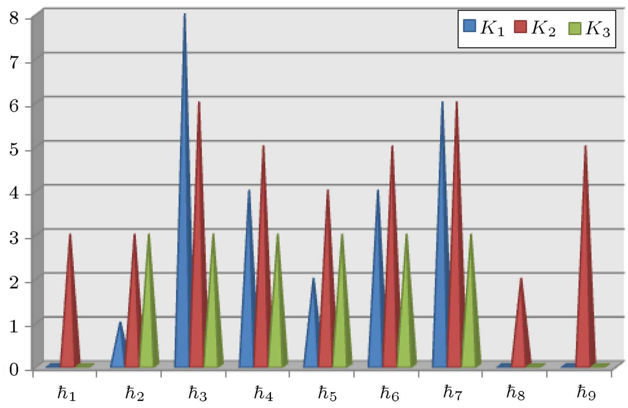

Figure 2. Graphical representation of aggregate decision triples.

$$\hbar_3 \succ \hbar_7 \succ \hbar_4 = \hbar_6 \succ \hbar_5 \succ \hbar_9 \succ \hbar_2 \succ \hbar_1 \succ \hbar_8.$$

Then, an optimum set of U is found as follows:

$$Opt_\kappa(U) = \{\hbar_3\}.$$

Consequently, $\hbar_3$ is an optimal house to buy in this city.

Comparison: Algorithm 1 is compared with the preexisting decision making algorithms based on the soft sets and soft matrices. Thus, it is seen that Algorithm 1 gives more convincing results. The results of these comparisons are in Table 2.

$$[c_{ip}] = \widetilde{\wedge}^3_{t=1}[a^t_{ij}]$$

$$= \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&1&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0&1&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&1&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1 \\ 0&1&0&0&1&0&0&0&0&0&1&0&0&1&0&0&0&0&0&1&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

and

$$[d_{ij}] = \widetilde{\cap}^3_{t=1}[a^t_{ij}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Box II

Table 1. Tabular form of the aggregate decision triples.

κ_i/i	1	2	3	4	5	6	7	8	9
$\kappa_i=(\kappa_i^1,\kappa_i^2,\kappa_i^3)$	(0,3,0)	(1,3,3)	(8,6,3)	(4,5,3)	(2,4,3)	(4,5,3)	(6,6,3)	(0,2,0)	(0,5,0)

Table 2. Comparison result of Algorithm 1 with some existing soft decision making algorithms in the literature.

Ref.	Problem in the paper	Result of their algorithm	Result of Algorithm 1 for same problem
[52]	Example 5.1 in [52]	$\{u_1,u_3\}$	$u_3\succ u_1\succ u_4\succ u_5\succ u_2$
[56]	Example 5.1 in [56]	$\{d_2,d_3\}$	$d_3\succ d_2\succ d_1\succ d_4$
[56]	Example 5.2 in [56]	$\{f_1\}$	$f_1\succ f_4\succ f_2\succ f_3$
[51]	Example 6 in [51]	$\{u_1\}$	$u_1\succ u_2\succ u_3\succ u_5\succ u_4$
[15]	Example 4 in [15]	$\{u_4,u_{13},u_{21},u_{36},u_{42}\}$	$u_{13}=u_{36}\succ u_{21}\succ u_{42}\succ u_{28}\succ u_4$
[60]	Application in [60]	$u_1>u_3>u_2$	$u_1\succ u_3\succ u_2$
[61]	Example 5.17 in [61]	$\{h_1,h_2,h_3\}$	$h_1=h_3\succ h_2\succ h_4=h_5$
[61]	Example 5.19 in [61]	$\{u_{13},u_{36}\}$	$u_{13}=u_{36}\succ u_{21}\succ u_{42}\succ u_{28}\succ u_4$
[62]	Example 3.3 in [62]	$\{h_1,h_2,h_3\}$	$h_1=h_3\succ h_2\succ h_4=h_5$
[63]	Example 31 in [63]	$\{u_3\}$	$u_3\succ u_2\succ u_4\succ u_5\succ u_1$
[59]	Example 3.10 in [59]	$\{m_3\}$	$m_3\succ m_5\succ m_2\succ m_1\succ m_4$
[59]	Example 3.12 in [59]	$\{m_3\}$	$m_3\succ m_1\succ m_2=m_5\succ m_4$

In Definition 16, we can write α-intersection soft matrices and α-union soft matrices instead of the classical soft matrices. Then, we consider the following algorithm instead of Algorithm 1.

Algorithm 2. Set-restricted soft decision making algorithm

The detailed steps of the set-restricted soft decision making algorithm are below:

Step 1. DMs choose feasible subsets of the parameter set and then create soft sets.

Step 2. Determine α_t-sets $(t=1,2,...,r)$.

Step 3. Construct the soft matrices $[a_{ij}^1],[a_{ij}^2],...,[a_{ij}^r]$.

Step 4. According to the α_t-sets, create the α_t-intersection soft matrices $[a_{ij}^1]^{\cap\alpha_1}[a_{ij}^2]^{\cap\alpha_2},...,[a_{ij}^r]^{\cap\alpha_r}$.

Step 5. Obtain the soft matrices $[c_{ip}]=\widetilde{\bigwedge}_{t=1}^r[a_{ij}^t]^{\cap\alpha_t}$ and $[d_{ij}]=\widetilde{\cap}_{t=1}^r[a_{ij}^t]^{\cap\alpha_t}$.

Step 6. Find the aggregate decision triple $\kappa_i=(\kappa_i^1,\kappa_i^2,\kappa_i^3)$ for $i=1,2,...,m$.

Step 7. Obtain the ranking order of objects $\hbar_i\in U$ $(i=1,2,...,m)$ and find the optimum set of U.

Note: In Step 4 of this algorithm, it can be taken α_t-union soft matrices instead of α_t-intersection soft matrices according to the real scenario of the problem.

The step by step procedure of Algorithm 2 is illustrated in Figure 3.

Example 8. Let us consider the decision making problem in Example 7.

Now they are ready to apply the set-restricted soft decision making algorithm:

Step 1. We consider the soft sets f_{A_1}, f_{A_2}, and f_{A_3} in Step 1 of Example 7.

Step 2. Due to the transportation problems of the big city, Mr. X specifies α_1 as "close to his workplace", Mrs. X specifies α_2 as "close to her workplace" and daughter X specifies α_3 as "close to her school". Then, the real estate agent offers $\alpha_1=\{\hbar_1,\hbar_2,\hbar_9\}$, $\alpha_2=\{\hbar_6,\hbar_8\}$ and $\alpha_3=\{\hbar_2,\hbar_4,\hbar_7,\hbar_8\}$, respectively.

Step 3. We consider the soft matrices $[a_{ij}^1]$, $[a_{ij}^2]$, and $[a_{ij}^3]$ in Step 2 of Example 7.

Step 4. Then, they create the α_t-intersection soft matrices for $(t=1,2,3)$ as follows:

$$[a_{ij}^1]^{\cap\alpha_1}=\begin{bmatrix}0&1&1\\0&1&0\\0&0&1\\0&1&1\\0&1&0\\0&0&1\\0&1&1\\0&0&1\\0&0&0\end{bmatrix},\quad [a_{ij}^2]^{\cap\alpha_2}=\begin{bmatrix}0&0&0\\0&1&0\\0&0&1\\0&1&1\\0&1&0\\0&0&1\\0&1&0\\0&1&0\\0&1&1\end{bmatrix},$$

and

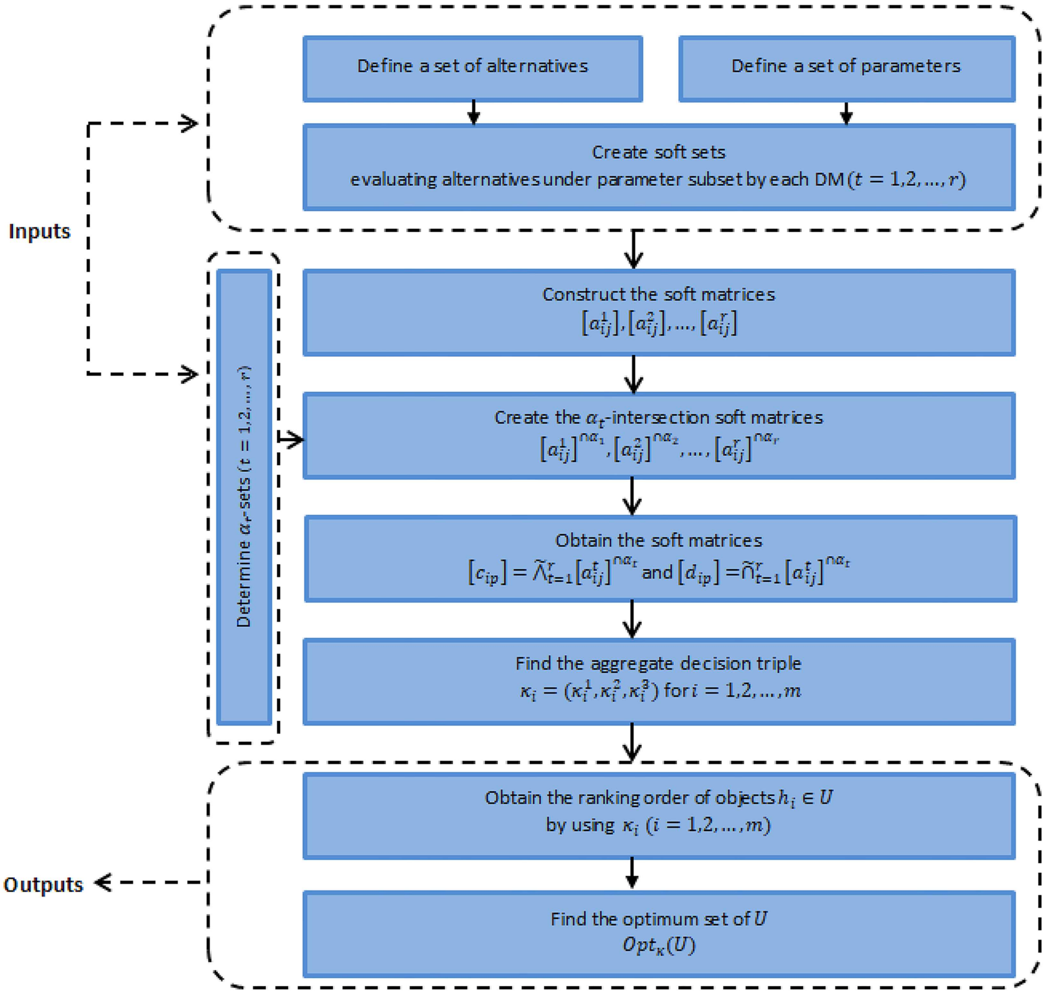

Figure 3. The framework of Algorithm 2.

Table 3. Tabular form of the aggregate decision triples.

κ_i/i	1	2	3	4	5	6	7	8	9
$\kappa_i = (\kappa_i^1,\kappa_i^2,\kappa_i^3)$	(0,2,0)	(1,3,3)	(1,3,0)	(4,5,3)	(1,3,3)	(1,3,0)	(2,4,3)	(0,2,0)	(0,3,0)

$$[a_{ij}^3]^{\cap\alpha_3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Step 5. They obtain the soft matrices are shown in Box III.

Step 6. The aggregate decision triple for each $i = 1,2,\ldots,9$ is found as in Table 3.

Figure 4 presents a graphical representation of aggregate decision triples in Table 3.

Step 7. Then, the ranking order of houses is obtained as:

$$\hbar_4 \succ \hbar_7 \succ \hbar_2 = \hbar_5 \succ \hbar_3 = \hbar_6 \succ \hbar_9 \succ \hbar_1 = \hbar_8.$$

Then, an optimum set of U is found as follows:

$$Opt_\kappa(U) = \{\hbar_4\}.$$

Consequently, $\hbar_4$ is an optimal house to buy in this city under the α_t-sets.

Advantages and limitations of proposed approaches

First, let us talk about the advantages of the proposed decision making models. The proposed soft decision making model (Algorithm 1) presents more convincing

$$[c_{ip}] = \widetilde{\wedge}_{t=1}^{3}[a_{ij}^{t}]^{\cap \alpha_t} =$$

$$\begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&1&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}$$

and

$$[d_{ij}] = \widetilde{\cap}_{t=1}^{3}[a_{ij}^{t}]^{\cap \alpha_t} = \begin{bmatrix}
0&0&0\\
0&1&0\\
0&0&0\\
0&1&0\\
0&1&0\\
0&0&0\\
0&1&0\\
0&0&0\\
0&0&0
\end{bmatrix}.$$

Box III

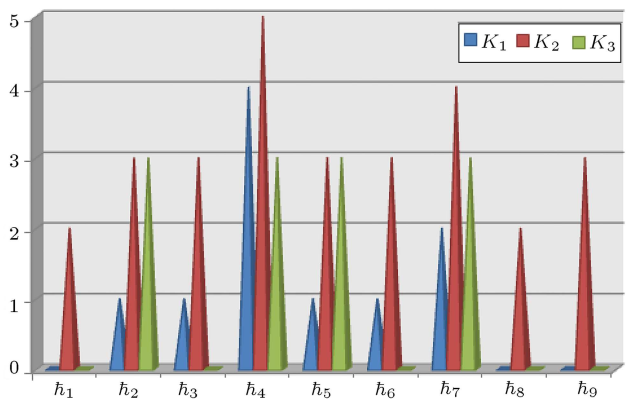

Figure 4. Graphical representation of aggregate decision triples.

outputs than some existing soft decision making models for multi-criteria decision making problems under the soft set environment (see comparison results in Table 2). The set-restricted soft decision making model (Algorithm 2) can deal with soft set-based multi-criteria decision making involving primary assessments. In conclusion, the advantages of the proposed decision making approaches is that both produce more satisfactory results than existing soft decision making approaches and present solutions for multi-criteria decision making problems involving primary considerations that existing approaches cannot cope with.

To present the limitations of the proposed approaches, we critically analyze Algorithms 1 and 2 and thus give following drawbacks: In Examples 7 and 8, possible dependencies between parameters are neglected. It is not always possible in practice to suppose in the multi-criteria decision making that each parameter is independent of other parameters. Any parameter in the multi-criteria decision making could be related to, or dependent on other parameters. In the proposed approaches, evaluating the dependencies among parameters should contribute to the objectivity of decisions. By considering dependencies between parameters in the soft set-based multi-criteria decision making, the quality of decision making process may be improved.

6. Conclusion

In this study, we introduced many new and useful concepts such as, the α-intersection set and α-union set of soft sets, and the types of α-upper, α-lower, α-intersection and α-union of soft matrices. Also, we gave some decomposition theorems for both soft sets and soft matrices. It was proved that the relationships between soft sets and soft matrices are also valid for their decompositions. To show that these emerged decompositions will be very useful in solving decision making problems involving constraint conditions, new soft decision making models were proposed.

In the future, these decompositions are expect to lead to new studies in soft sets and soft matrices. Furthermore, the proposed decision making approaches would be beneficial for applications in new research areas. In addition to these, the α-oriented concepts proposed in this paper can be adapted for the fuzzy soft sets, intuitionistic fuzzy soft sets, Pythagorean fuzzy soft sets, q-rung orthopair fuzzy soft sets, picture fuzzy soft sets, spherical fuzzy soft sets, T-spherical fuzzy soft sets, linear Diophantine fuzzy soft sets, neutrosophic soft sets and their extensions. Our future research topic will also serve these purposes.

Appendix

Scilab codes: We give the following Scilab codes for convenience of the steps in the above soft decision making algorithms. Through these codes, it is possible to solve the group decision making problems involving a large number of decision makers.

for *And*-product and *Multi-And-product* ($\widetilde{\wedge}$)

```
function c=andproduct(a1,a2)
[m,n1]=size(a1);
[m,n2]=size(a2);
c=zeros(m,n1*n2);
 for i=1:m
  for j=1:n1
   for k=1:n2
    p=(j-1)*n2+k;
    c(i,p)=min(a1(i,j),a2(i,k));
   end
  end
 end
endfunction

function C=multiandprod(varargin)
r=argn(2);
c=andproduct(varargin(1),varargin(2));
 for i=3:r
  c=andproduct(c,varargin(i));
 end
  C=c
endfunction
```

for *intersection* and *Multi-intersection* ($\widetilde{\cap}$)

```
function d=intersection(a1,a2)
[m,n]=size(a1);
[m,n]=size(a2);
d=zeros(m,n);
 for i=1:m
  for j=1:n
   d(i,j)=min(a1(i,j),a2(i,j));
  end
 end
endfunction

function D=multiint(varargin)
r=argn(2);
d=varargin(1);
 for i=2:r
  d=intersection(d,varargin(i));
 end
D=d
endfunction
```

for κ^1

```
function K1=aggregate1(C)
 K1=sum(C,'c');
endfunction
```

for κ^2

```
function K2=aggregate2\\
(varargin)
r=argn(2);
s=varargin(1);
 for i=2:r
  s=s+varargin(i);
 end
k2=sum(s,2);
K2=k2
endfunction
```

for κ^3

```
function K3=aggregate3\\
(varargin)
 r=length(varargin);
K3=r*sum(D,'c');
endfunction
```

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