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An integrated method for sustainable performance-based optimal seismic design of RC frames with non-prismatic beams

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Abstract: In the performance-based optimal seismic design, one attempts to obtain structural design variables to meet the minimum objective function satisfying the strength-based and performance-based constraints. A limited number of studies have been conducted on the performance-based optimal seismic design of reinforced concrete frames. On the other hand, due to the importance of environmental impacts, further study is necessary for the design of RC buildings with the aim of reducing CO₂ emissions. In this study, a computational procedure is developed for performance-based optimal seismic design of RC frames with prismatic beams and frames with non-prismatic beams. The objective functions consist of minimizing the cost and CO₂ emissions. Nonlinear pushover analysis is performed for analysis of the frames. The described procedure is applied to a 4-story reinforced concrete frame and the relationship between optimal cost and optimal CO₂ emissions is studied for frames with prismatic beams and frames with non-prismatic beams.

Keywords: Performance-based seismic design; Optimal cost; Optimal CO₂ emissions; Prismatic beams; Non-prismatic beams; Metaheuristic algorithms.

1. Introduction

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Performance Based Seismic Design (PBSD) is a methodology for designing structures and has been considered by earthquake engineers. PBSD has been introduced by various guidelines for rehabilitation of existing buildings as well as designing new buildings. The purpose of PBSD is to increase the safety of the structure against natural hazards and to design the structure that has predictable performance. Depending on the importance of the structure and performance of the structure after an earthquake, the codes classify possible damages and determine the level of performance. In accordance with FEMA-273[1], Immediate Occupancy (IO), Life Safety (LS) and Collapse Prevention (CP) are considered as the performance levels. Pushover analysis is a very popular technique for nonlinear analysis of structures and seismic demand assessment of structures in PBSD.

In traditional methods (trial and error) for design of structures, analysis is repeated until a reasonable design is achieved, and the quality of the design produced by traditional methods depends on the experience and ability of the engineer. The final design obtained by this method is not sufficient to meet economic and safety criteria simultaneously. Therefore, studies have been carried out using optimization methods to solve the problems. In these methods, objective functions, constraints, type of structural analysis and optimization process may be different. In a number of studies on the performance-based optimal seismic design, the objective function was to minimize construction cost [2-7]. In some other studies, the objective has been to minimize the total life cycle costs [8-11]. The total life cycle costs come from the initial operation cost and the costs such as maintenance, damage and repair costs, which are expressed as a function of their seismic performance level in the probability of failure. Park et al. [12] have also used a process for multi-objective performance-based optimal seismic design for reinforced concrete buildings with prismatic beams. In their study the objectives functions were the cost of materials, CO₂ emission, and seismic performance.

On the other hand, nowadays reducing greenhouse gas emissions is a major global challenge. CO₂ emissions account for the largest amount of greenhouse gases, which has accounted for about 77% of greenhouse gases since 2004 [13]. The environmental impact of the construction industry on greenhouse gases is remarkable. Strategies have been employed to reduce these effects. One strategy is the use of optimization techniques during the structural design, where the objective function is reducing the amount carbon dioxide emissions. Reinforced concrete
buildings are made of two type material, concrete and steel, and they have different amount of carbon dioxide emissions. Therefore, they have high potential to reduce CO$_2$ emissions in optimization methods. Camp et al. [14] optimally designed the RC frames by using big bang-big crunch optimization algorithm. In which the objective functions were the minimization of the CO$_2$ emissions and costs. In another study, Camp et al. [15] minimized the cost and CO$_2$ emissions of reinforced concrete footings. Yepes et al. [16] proposed a methodology for optimal design of RC cantilever retaining walls, using a hybrid multistart optimization strategic method based on a variable neighborhood search threshold acceptance strategy (VNS-MTAR) algorithm to minimize the cost and CO$_2$ emissions. Kaveh et al. [17] investigated the relationship between optimal cost and optimal carbon dioxide emissions in design of RC frames with different heights. The optimal solutions for each frame were obtained by three metaheuristic algorithms. Mergos [18] has studied the effect of factors such as ductility classes, peak ground accelerations (PGAs), concrete classes on decrease of CO$_2$ emissions in design of seismic resistant RC frames. Yeo et al. [19] developed a methodology to optimally design of RC frames with the objectives of minimizing CO$_2$ emission and economic cost. In which a portal RC frame is investigated under gravitational and lateral loads. They stated that, depending on the parameters considered in the calculations, the design aiming to minimize CO$_2$ emission would reduce CO$_2$ emissions by 5-10\% compared to CO$_2$ emission obtained from optimal design with the aim of minimizing cost. Oh et al. [20] established the relationship between optimal carbon dioxide emissions, optimal cost, and structural parameters of composite columns (concrete-filled steel tube). In which the type of section (circular, rectangular), the amount of materials and the strength of each material were considered as design variables.

Furthermore, a number of studies have been conducted to optimally design of frames with non-prismatic elements. In non-prismatic beams, width, depth or both of them can be varied. An acceptable height in the floors of buildings can be obtained by using these types of beams. They are mostly used in bridges, industrial structures, amphitheatres, sport arenas, parking and etc. McKinstray et al. [21] optimally designed the single-span industrial steel frames. In which three types of sections (rolled, fabricated and non-prismatic) were used in the frames. Kaveh et al. [22] optimally designed the three dimensional steel structures with non-prismatic beams and columns. The optimal weight of frames with non-prismatic elements were compared with the optimal design of frames with prismatic elements. Yavari et al. [23] optimized CO$_2$ emissions and cost of
concrete slab frame bridges during the design phase, with the slab being considered as non-prismatic. Kaveh et al. [24] presented a methodology for the sustainable design of reinforced concrete frames with non-prismatic beams and investigated the relationship between optimal cost and optimal carbon dioxide emissions in the design of these frames.

A review of the literature shows that, a limited number of studies have been conducted on the Performance Based Optimal Seismic Design (PBOSD) of reinforced concrete buildings with the objective of minimize carbon dioxide emissions and, a limited study is conducted to optimally design the RC frames with non-prismatic elements. In a previous study, the authors of this paper investigated the relationship between the optimal cost and optimal CO$_2$ emission in RC frames with non-prismatic beams [24]. Where equivalent linear static analysis was used to obtain the demand for elements and strength-based constraints for structures are controlled, in which the nonlinear behavior of the structures and performance-based constraints were not considered.

In this paper, RC frames with prismatic beams and again RC frames with non-prismatic beams have been optimized with the aim of minimizing CO$_2$ emission and construction cost. The nonlinear static (pushover) analysis is incorporated with structural optimization strategy for optimal design of frames at the performance levels. The relationship between optimal cost and optimal CO$_2$ emissions of frames at performance levels have been investigated. Five metaheuristic algorithms consisting of the Particle Swarm Optimization (PSO), Colliding Bodies Optimization (CBO), Enhanced Colliding Bodies Optimization (ECBO), Vibrating Particles System (VPS) and Enhanced Vibrating Particles System (EVPS) are utilized for optimal design the frames with prismatic beams at the IO performance level. The most competent algorithm is selected and used to solve the remaining problems.

### 2. Formulation of Optimal Design

#### 2.1. Performance-based optimal seismic design

In the performance based optimal design, the formulation of problem is according Eq. (1). Where $f(x)$ is objective function, $g^{SB}$ is strength-based constrains, $g^{PB}$ is performance-based constraints and $x$ is design variables. Initially equivalent static analysis is performed for structures and strength-based constraints based ACI 318-08 [25] code are checked. Then the
performance-based constraints that are related to the maximum inter-story drift are checked under nonlinear static analysis in three performance levels based FEMA code [1].

\[
\min f(x) \\
g_j^{SBD}, \quad j = 1, ..., m \\
g_j^{PBD}, \quad j = 1, ..., k \\
x^L \leq x \leq x^U
\]  

According to the ACI 318-08 code the following load combinations Eq. (2) are used for equivalent static analysis. Here, \( D \) is the dead load, \( L \) is the live load and \( E \) is lateral equivalent static loads.

\[
Q_{G}^{SBD} : 1.2D + 1.6L \\
0.9D + 1.4E
\]

In pushover analysis, lateral load should be applied to the building incrementally while the gravity load is applied constantly. In this study, the load combination Eq. (3), and lateral load with the pattern based on the first mode shape are used in pushover analysis.

\[
Q_{G}^{PBD} = 1.1(D + L)
\]

In nonlinear static analysis it is assumed that the structure response is associated with an equivalent single degree of freedom system. The model of structures is pushed by lateral load until the displacement of the roof reaches to target displacement or mathematical instability occurs. According to FEMA-356 [26] the target displacement calculated as:

\[
\delta_t = C_0C_1C_2C_3S_x \frac{T^2}{4\pi^2} g
\]

Where the \( C_0 \) is the modification factor for relating the spectral displacement of a single degree of freedom to the roof of a multi degree of freedom. \( C_1 \) is the modification factor for converting the calculated displacements from the linear elastic response to the expected maximum inelastic displacements. \( C_2 \) is the modification factor to represent the effect of hysteresis shape on the
maximum displacement response. The value of this coefficient is determined according to structural performance level, the type of frame and the period of the structure. The $C_3$ is the modification factor to account for the increase in displacement due to the dynamic effects of P-Delta. $S_a$ is the Spectral response acceleration vs structural period domain, and it is calculated in three performance level as follows:

$$S_{a}^i = \frac{S_{xs}^i}{B_s} \left(0.4 + 3 \frac{T}{T_0}\right) \quad \text{for} \quad 0 < T < 0.2T_0^i$$

$$S_{a}^i = \frac{S_{xs}^i}{B_s} \quad \text{for} \quad 0.2T_0^i < T < T_0^i \quad i = IO, LS, CP$$

$$S_{a}^i = \frac{S_{x1}^i}{B_1T} \quad \text{for} \quad T > T_0^i$$

Where $T$ is the elastic fundamental period of the structure that is obtained here from modal analysis of the structure and $i$ show three performance level IO, LS and CP.

$T_0^i$ is given by Eq. (6), and $B_s$, $B_1$ in accordance FEMA 273 are assumed to be equal 1.

$$T_0^i = \left(S_{x1}^i / B_s\right) / \left(S_{xs}^i / B_1\right)$$

$$S_{x1}^i = F_x^i S_1^i$$

$$S_{xs}^i = F_a^i S_s^i$$

Where $F_x^i$ and $F_a^i$ are the site coefficients that are determined based on the site class and the values of the response acceleration parameters $S_1^i$ and $S_s^i$. Table 1 [27] lists the parameters for site class $D$.

The procedure for automatic performance-based optimal seismic design by using the selected algorithms is presented in Fig. 1.

2.2. Design variables and database for sections

In this study 11 variables for beams and 4 variables for columns are considered. Fig. 2 shows the variables for non-prismatic beams. The variables are dimensions of the cross sections of column
elements (depth and width), diameter and number of longitudinal bars of the columns, depth of the cross section of beams in prismatic zone \((h_2)\), depth of the cross section of beams in non-prismatic zone \((h_1)\), diameter and number of longitudinal bars in prismatic section of beams \((A_{s2})\) (in top and bottom of sections), diameter and number of longitudinal bars in non-prismatic section of beams \((A_{s1})\) (in top and bottom of sections) and also the length of tapered section that is defined as a percentage of the length of the beam (tapered length ratio \((TLR)\)). The width of cross section for beams is assumed to be constant. The search space parameters for beams and columns are shown in Table 2.

In order to formulate the variables in discrete form and to reduce the constraint in solving of problem, two databases are created for beams and columns. In databases, constraints that do not require structural analysis, such as percentage of permissible bars, the depth-to-width ratio, and etc., are controlled based on ACI code and removed from the database.

**Database for beam sections**

In the formulation of database for beams, the width, depth, area of cross section, moment of inertia, number and diameter of bars, bending capacity and cost or amount of CO\(_2\) emission per unit length of the beams are given. In the cross sections of beams the ratio of depth to width is varied between as 1 and 3.

The moment resisting capacities of a beam section is defined by Eq. (9) as:

\[
M_n = A_y f_y (d - \frac{a}{2})
\]  

(9)

In which \(A_y\) is the total area of tensile reinforcing bars, \(f_y\) is the yield strength of bars, \(d\) is the distance of the compressive edge of the section to the center of the tensile bars, and \(a\) is the depth of the equivalent rectangular stress block defined as:

\[
a = \frac{A_y f_y}{0.85 f'_c b}
\]  

(10)

In which \(f'_c\) is the compressive strength of the concrete and \(b\) is the width of the cross sections.

**Database for column sections**
In the formulation of database for column sections, width, depth, area of cross section, moment of inertia, number and diameter of bars, the cost or the amount of CO\textsubscript{2} emission per unit length of the column are given. Furthermore, the parameters related to the $P$-$M$ interaction diagram, as shown in Fig. 3, are calculated according to the ACI code and saved in database. In the cross section of the columns, the rebars are distributed along all four faces of sections according to the patterns shown in Fig. 4.

2.3. Objective functions

In this work, the aim of optimization is to minimize the construction cost and the amount of CO\textsubscript{2} emissions, as expressed by Eq. (11). Here, $n_b$ and $n_i$ are the number of beams and columns, respectively; $b_i$, $h_i$, and $A_{si}$ and $L_i$ are the width, depth of the sections, area of the bars, and the length of the beams and columns, respectively; $t_i$ is the thickness of the slab that is considered to be 290 mm. $C_C$, $C_s$, $C_f$ and $C_i$ are the unit rate of concrete, bars, formwork, and scaffolding, respectively, and their values are shown in Table 3. The parameter $\gamma_s$ is unit weight of steel as 7849 kg/m\textsuperscript{3}. In the objective function of CO\textsubscript{2} emission, scaffolding is not considered.

$$f_K = \sum_{i=1}^{n_b+n_i} \{C_CC_i + C_s\gamma_sA_{si}\}L_i + \sum_{i=1}^{n_b} \{C_f(b_i + 2(h_i - t_i)) + C_f(b_i)\}L_i + \sum_{i=1}^{n_i} \{2C_f(b_i + h_i)\}L_i \quad (11)$$

2.4. Design constraints

According to design codes, strength-based constraint and performance-based constraints should be checked during the design process. Equivalent static analysis performed for structures and the strength-based constraints are checked according to ACI code, then performance-based constraints that are related to the maximum inter-story drift are controlled under pushover analysis in three performance levels. The variables that do not satisfy these constraints are removed from calculations. One method to avoid repetitions of calculations for these variables is the penalty function method. In this approach, by adding a penalty value to the objective function based on the extent of violation of the constraints, the problem becomes an unconstrained
problem. In Eq. (12) the parameters, \( g_i, x \) and \( n \) are the penalty of the \( i \)th constraint, elements and number of constraints, respectively. Furthermore \( f_p \) is the penalized objective function and \( f \) is the value of the objective function. In this study \( k \) is considered as 1.5.

\[
f_p(x) = f \times (1 + \sum_{i=1}^{g} \max(0, g_i(x)))^k
\]  

(12)

2.4.1. Strength based constraints

Constraints of the beams

For evaluating the moment capacity of the reinforced concrete beams, penalty function is calculated as Eq. (13). In non-prismatic beams, this constraint is examined in sections 1 to 5 as in Fig. 5. Section 3 is where the positive bending moment has maximum value.

\[
g_1 = \frac{|M_u| - \phi M_n}{\phi M_n}
\]  

(13)

Here \( M_u \) is the ultimate applied bending moment under applied loading, \( \phi \) is the strength reduction factor which is equal to 0.9. \( M_n \) is the moment capacity of the RC beams that is defined in Eq. (9).

According of ACI code, the ratios of minimum and maximum reinforcement of the beam sections are limited. The penalties of these constraints are as follows:

\[
\rho_{\min} = \frac{\sqrt{f_c}}{4f_y} \geq \frac{1.4}{f_y}, \quad g_2 = \rho_{\min} - \rho
\]  

(14)

\[
\rho_{\max} = 0.85\beta_1 \frac{f_y}{f_c} \frac{600}{600 + f_y}, \quad g_3 = \rho - \rho_{\max}
\]  

(15)

In order to control the deflection of the beams, the following penalty is considered in this study.

\[
h_{\min} = \frac{L}{21}, \quad g_4 = \frac{h_{\min} - h}{h_{\min}}
\]  

(16)
In the section of beams, if the effective depth \( d \) is less than the compression-block depth \( a \), the penalty is defined as:

\[
g_5 = \frac{a - d}{d}
\]  

(17)

The minimum distance between the bars should be limited. The penalty of this constraint is defined as:

\[
g_6 = \frac{s_{\text{min}} - s}{s_{\text{min}}} \quad , \quad s_{\text{min}} = \max(d_b, \text{inch})
\]  

(18)

Considering the Fig. 2, secondary height \( h_2 \) must not be greater than initial height \( h_1 \), the penalty defined as:

\[
g_7 = \frac{h_2 - h_1}{h_2}
\]  

(19)

**Constraint of columns**

When the combination of \((M_u, P_u)\) under the applied loads falls inside the interaction \( P-M \) diagram, a column sections is suitable. The penalty function for the capacity of the columns can be expressed as:

\[
g_8 = \frac{L}{L_0} - 1
\]  

(20)

Based on Fig. 3, \( l \) is the distance between the origin of the interaction diagram \((O)\) and the point indicating the position of combination \((M_u, P_u)\) \((B)\), and \( l_0 \) is the radial distance between the origin of the interaction diagram \((O)\) and the point \((A)\) indicating the intersection point of the vector \( l \) with the interaction diagram.

The total area of bars \( (A_s) \) in the cross-section of reinforced concrete column is limited between 1% and 8% of the gross section \( (A_g) \). The penalties of the minimum and maximum reinforcement for the columns are expressed as:

\[
g_9 = \frac{0.01 \times A_s}{A_g} - 1 \leq 0
\]  

(21)
\[ g_{10} = \frac{A_g}{0.08 \times A_g} - 1 \leq 0 \]  \hspace{1cm} (22)

The function penalty defined for the distance between longitudinal bars is as:

\[ g_{11} = \frac{s_{\text{min}} - s}{s_{\text{min}}} , \quad s_{\text{min}} = \max(1.5d_b, 1.5\text{inch}) \]  \hspace{1cm} (23)

The dimensions of columns in each story should be smaller or equal than the dimensions of columns in bottom story, so the constraints expressed as follows:

\[ g_{12} = \frac{b_T}{b_B} - 1 \]  \hspace{1cm} (24)

\[ g_{13} = \frac{h_T}{h_B} - 1 \]  \hspace{1cm} (25)

Where \( B \) and \( T \) present the bottom column and the top column, \( b \) and \( h \) are the width and depth of the column cross section, respectively.

### 2.4.2. Performance based constraint

Lateral drift is an important indicator for measuring damage in structures and should be controlled in seismic design [28]. Performance-based constraints are expressed as lateral drift at various performance levels and are expressed as follows:

\[ g_{14} = \frac{\theta_{\text{max}}^{\text{IO}}}{\theta_{\text{allow}}^{\text{IO}}} - 1 \]  \hspace{1cm} (26)

\[ g_{15} = \frac{\theta_{\text{max}}^{\text{LS}}}{\theta_{\text{allow}}^{\text{LS}}} - 1 \]  \hspace{1cm} (27)

\[ g_{16} = \frac{\theta_{\text{max}}^{\text{CP}}}{\theta_{\text{allow}}^{\text{CP}}} - 1 \]  \hspace{1cm} (28)
Where $\theta_{\text{IO}}^\text{max}$, $\theta_{\text{LS}}^\text{max}$ and $\theta_{\text{CP}}^\text{max}$ are maximum inter-story drift of frame in performance levels of IO, LS and CP, respectively. $\theta_{\text{IO}}^\text{allow}$, $\theta_{\text{LS}}^\text{allow}$ and $\theta_{\text{CP}}^\text{allow}$ are allowable drifts that are chosen as 1%, 2% and 4% for IO, LS and CP performance levels, respectively, according FEMA-273.

2.5. Structural analysis model

The finite element software Opensees [29] is used to model the frames. Linear static and nonlinear static analysis of the structures and determining the demand of the elements performed in this software. The limitations of the ACI and FEMA codes and the optimization algorithm are handled in MATLAB [30] software. In linear static analysis the beams and columns are modeled with elastic beam column element and in nonlinear static analysis nonlinear beam column element with distributed plasticity are used to model of beams and columns. Also, to model non-prismatic beams by dividing each non-prismatic element into 12 parts, the step-by-step method has been used. Nonlinear concrete and steel material properties are provided in Table 4. As it is mentioned in Table 4, the effects of confinement and unconfined parts of the fiber section are imposed in the definition of concrete properties. Also, the P-Delta effects are included as a geometric transformation. Thus, both material and geometry nonlinearity are considered.

3. Optimization algorithms

In this section, the algorithms used in this paper are introduces. PSO is a well-known algorithm that has been widely used in many researchers. The ECBO, EVPS, VPS, CBO algorithms have been recently developed and compared with previously developed algorithms and found to be comparatively efficient.

3.1. Colliding Bodies Optimization

The colliding bodies optimization (CBO) algorithm [31] is inspired by the laws of momentum and energy of the physics. Where the bodies collide with each other and move to the lower cost. Each colliding bodies (CB) is a solution candidate that contains a number of variables. The procedure of the CBO algorithm can be expressed as:
Step 1: First, the initial position of each colliding bodies is randomly obtained in the search space as follows:

\[ x_i^0 = x_{\text{min}} + \text{rand}(x_{\text{max}} - x_{\text{min}}), \quad i = 1, 2, ..., n \]  

(29)

Where \( x_i^0 \) is the initial position of the \( i \)th CB, \( x_{\text{max}} \) and \( x_{\text{min}} \) are the minimum and the maximum allowable values of variables, respectively. The \text{rand} parameter is a random value in the range \([0, 1]\) and \( n \) is the number of CB.

Step 2: In the next step, the mass of each object is determined as follows:

\[ m_k = \frac{1}{\sum_{i=1}^{n} \text{fit}(i)} \quad \text{fit}_k, \quad k = 1, 2, ..., n \]  

(30)

Where \( \text{fit}(i) \) presents the objective function value of the \( i \)th colliding body and \( n \) is the number of populations. Objects are arranged in descending order and are divided into two equal groups of stationary and moving objects. To improve the position of moving objects and move stationary objects toward a better position, the moving object moves toward a stationary object and a collision occurs, Fig. 6.

Step 3: The velocity of stationary objects before collision is zero and the velocity of moving objects before collision is calculated as follows:

\[ v_i = 0, \quad i = 1, 2, ..., \frac{n}{2} \]  

(31)

\[ v_i = x_{\frac{i}{2}} - x_i, \quad i = \frac{n}{2} + 1, ..., n \]  

(32)

Step 4: After the collision of the moving and stationary objects, velocity of the objects is calculated as follows:

Stationary objects:
\[ v'_i = \frac{(m_i \frac{n}{2} + \varepsilon m \frac{i+n}{2}) v_i}{m_i + m \frac{i-n}{2}}, \quad i = 1, 2, ..., \frac{n}{2} \] (33)

Moving objects:

\[ v'_i = \frac{(m_i - \varepsilon m \frac{i-n}{2}) v_i}{m_i + m \frac{i-n}{2}}, \quad i = \frac{n}{2} + 1, ..., n \] (34)

Coefficient of Restitution \((\varepsilon)\) is defined as:

\[ \varepsilon = 1 - \frac{\text{iter}}{\text{iter}_{\text{max}}} \] (35)

Step 5: Using the generated velocities after the collision and their old position, the new positions of the objects for both groups are updated as follows:

The new position of moving object:

\[ x^{\text{new}}_i = x \frac{i-n}{2} + \text{rand}^o v'_i, \quad i = \frac{n}{2} + 1, \frac{n}{2} + 2, ..., n \] (36)

In which \(x^{\text{new}}_i\) is the new position of \(i\)th CBs, \(x \frac{i-n}{2}\) is old position of \(i\)th stationary CB and rand is a random vector uniformly distribution in the range (-1, 1). \(v'_i\) is the velocity of \(i\)th moving CB after collision. The sign "\(\cdot\)" denotes an element-by-element multiplication.

The new position of stationary object:

\[ x^{\text{new}}_i = x \frac{i-n}{2} + \text{rand}^o v'_i, \quad i = \frac{n}{2} + 1, \frac{n}{2} + 2, ..., n \] (37)

Where \(x^{\text{new}}_i\) is the new position of the \(i\)th CBs, \(x \frac{i-n}{2}\) is old position of \(i\)th stationary CB and \(v'_i\) is the velocity after the collision of the \(i\)th stationary CB.

Step 6: The procedure is repeated from step 2 until a terminating criterion is accepted.
3.2. Enhanced Colliding Bodies Optimization

The performance of the CBO algorithm has been modified using two techniques. These techniques are used to achieve reliable solutions and fast convergence. This algorithm is developed by Kaveh and Ilchi Ghazaan [32] and have been used in many studies, e.g. [33-34]. Using Colliding Memory (CM), the obtained solutions are modified in each step. It stores some of the best collision bodies (CBs) in the previous population in each iteration and replaces them with the worst CBs in the current population. Introducing new objects to the population can improve the performance of the algorithm without increasing the computational cost. To improve exploration capabilities and prevent premature convergence, a component of CBs is randomly regenerated in each iteration. This parameter that is expressed as $pro$ is within $(0, 1)$. The steps of this algorithm are as follows:

Step 1: The initial position of all CBs vectors with a number of variables is randomly selected.

Step 2: The mass of each CB is calculated according to Eq. (30)

Step 3: Some of the best CBs vectors are stored in collision memory (CM) and replaced with the worst CBs vectors.

Step 4: The objects are divided into two equal groups of stationary and moving objects according Fig. 6

Step 4: The velocity of moving objects before collision is obtained according to the Eq. (32).

Step 5: The velocity of objects after collision is obtained according to the Eq. (33,34).

Step 6: The new position of objects is obtained according Eq. (36,37)

Step 7: The $pro$ parameter is compared with the random number $rni$ ($i = 1, 2… n$). If $pro > rni$, a CB is randomly selected from both moving and stationary groups and one related component regenerate.

Step 8: Return to Step 2 until terminating criterion is satisfied.

3.3. Vibrating Particles System
The vibrating particles system (VPS) algorithm is proposed by Kaveh and Ilchi Ghazaan [35]. This algorithm is inspired by the free vibration of single degree of freedom systems with viscous damping. VPS is made up of particles that contain a number of variables. In this algorithm, three parameters with different weights are defined to form the new position: $HB$, the historically best position of the entire population; $GB$, a good particle; and $BP$, a bad particle. With a combination of the current population and historically best position, a balance between diversification and intensification is established, and the particles approach equilibrium positions. The procedure of this algorithm is defined as:

Step 1: In this algorithm, the initial position of all particles in the research space is randomly determined.

$$x_i^j = x_{\text{min}} + \text{rand}(x_{\text{max}} - x_{\text{min}}), \quad i = 1, 2, ..., n$$

(38)

Where $x_i^j$ is the $j$th variable of the particle $i$, $x_{\text{min}}$ and $x_{\text{max}}$ are the minimum and maximum allowable values of the research space. $\text{rand}$ is a rand number between [0,1] and $n$ is number of particles.

Step 2: The value of the target function is calculated for each particle.

Step 3: For each particle three balanced positions with different weights are defined and their positions is updated. The positions are $HB$ (the historically best position of the entire population), $GP$ (a good particle) and $BP$ (a bad particle). In order to select a good and bad particle, the population must be arranged in ascending order according to the values of their objective function. Finally, the $GP$ and $BP$ particles are randomly selected from the first and second half, respectively.

The positions are updated by:

$$x_i^j = w_1[D.A.rand1 + HB^j] + w_2[D.A.rand2 + GP^j] + w_3[D.A.rand3 + BP^j]$$

(39)

Where $w_1$, $w_2$, $w_3$ are three parameters to measure the relative importance of the $HB$, $GB$ and $BP$, respectively. In which $w_1 + w_2 + w_3 = 1$. $\text{rand}$ 1, $\text{rand}$ 2 and $\text{rand}$ 3 are random numbers that are uniformly distributed in the range [0, 1].

The parameter $D$ used to measure the effect of damping level on vibration is as follows:
\[ D = \left( \frac{\text{iter}}{\text{iter}_{\text{max}}} \right)^{-\alpha} \]  

(40)

Where \( \text{iter} \) is the current iteration number, \( \text{iter}_{\text{max}} \) is the total number of iterations for optimization process and \( \alpha \) is a constant parameter.

The parameter \( A \) is defined as:

\[
A = [w_1(H_{Bj} - x_i^j)] + [w_2(G_{Pj} - x_i^j)] + [w_3(B_{Pj} - x_i^j)]
\]

(41)

The parameter is defined like \( p \) within \((0,1)\) to accelerate the convergence of the VPS algorithm. For each particle, \( P \) is compared with \( \text{rand} \) if \( P < \text{rand} \) is then \( w_3 = 0 \) and \( w_2 = 1 - w_1 \).

Step 4: The particles move in the search space to find a better result and may violate the side boundary. In order to handling this constraint a harmony search-based approach is used in this algorithm.

Step 5: Step 2 to step 4 is repeated until the termination criterion is completed.

3.4. Enhanced Vibrating Particles System

In order to improve the results of the VPS algorithm by using techniques to increase convergence speed and prevent local optima, the EVPS algorithm has been developed [36]. In this algorithm, the memory parameter acts as \( H_B \) with the difference that it saves \( \text{Memory}_\text{size} \) number of the best historically positions in the entire population. If the best answer in each iteration is better than the worst value of Memory, it is replaced with worst value in memory. \( O_{HB} \) (one of the best historically positions in entire population) is one row of Memory that is selected randomly. Equations 39 and 41 of the VPS algorithms are replaced by equations 42 and 43 of the EVPS algorithm. where \((\pm 1)\) are used randomly

\[
[D.A.\text{rand1} + O_{HB}^j]
\]

(a)

\[
x_i^j = [D.A.\text{rand2} + G_{Pj}^j]
\]

(b)

\[
[D.A.\text{rand3} + B_{Pj}^j]
\]

(c)

\[
(\pm 1)(O_{HB}^j - x_i^j)
\]

(a)
A = (±1)(GP^i − x_i^i) \quad \text{(b)}

(±1)(BP^i − x_i^i) \quad \text{(c)}

w_1 + w_2 + w_3 = 1 \quad \text{(44)}

### 3.5. Particle swarm optimization

Particle Swarm Optimization (PSO) is one of the well-known algorithms and it is presented in many papers. This algorithm is inspired by swarm intelligence [37]. The positions and velocity of the particles are updated using the following equations to find the global optimal.

\[
V_i[t + 1] = WV_i[t] + r_1 c_1 (P_i[t] − X_i[t]) + r_2 c_2 (P_g[t] − X_i[t])
\]

\[
X_i[t + 1] = X_i[t] + V_i[t + 1]
\]

In these equations, V and X express the velocity and position of each particle, respectively. The \( P_i \) is the best position for the \( i \)th particles and the \( P_g \) is global best position among all particles. \( W \) is Inertia Weight. \( c_1 \) and \( c_2 \) are the personal learning coefficient and the global learning coefficient, respectively. \( r_1 \) and \( r_2 \) are two random numbers that are uniformly distributed within \((0, 1)\).

### 4. Numerical examples

Four-story reinforced concrete frame with prismatic beams and again frame with non-prismatic beams is considered to investigate the objectives of this paper. The performance based optimal seismic design of frames are performed at three performance levels IO, LS and CP according to FEMA. The objective functions are minimizing the construction cost and CO₂ emissions. The live and dead load considered as 24 kN/m and 36 kN/m, respectively. There are some special approaches to increase the convergence speed of algorithms [38,39]. In this study,
in order to search efficiently for obtaining the best results, the strategy described by Kazemzadeh Azad et al. [38] is used. In which to accelerate the convergence speed of the algorithms, the initial population with feasible solutions is replaced with random population. In this study, the frames are optimized multiple times and the candidate solution with the lowest penalty is placed as a candidate in the initial population and then the optimization is performed.

4.1. RC frame with prismatic beams

This example is a four-story and two-spans frame with prismatic beams. The height of each story is 3 meters and the length of each span is 10 meters. The geometry, lateral loading, and grouping of members are shown in Fig. 7. It consists of 8 column groups and 4 groups of beams. The procedure described for performance-based seismic design is used to minimize the cost and carbon dioxide emissions at three performance levels, and the results are presented below.

At the IO level of performance, five metaheuristic algorithms used for optimization. The appropriate parameters used for each of the algorithms are shown in Table 5. The convergence curves of the algorithms for the lowest cost and lowest CO\textsubscript{2} emissions are compared in Fig. 8 and Fig. 9, respectively. Table 6 presents the optimal results for all 5 algorithms. Comparison of the results shows that the ECBO algorithm performs better than the other algorithms in both objective functions, so this algorithm is used to solve the rest of the problem.

The results of the ECBO algorithm show that, in the solution with objective of minimizing cost, the best reported solution is 13770 euro with 15629 kg of carbon dioxide emissions (Table 7). In the solution with minimizing CO\textsubscript{2} emissions, the best reported solution is 15348 kg with 14082 euro of cost (Table 8). Percentage comparison of results show that in the solution based on the objective function of minimizing carbon dioxide emissions, by increase of 2.2 % in cost one can reduce the CO\textsubscript{2} emissions by 1.8%.

The distribution of the inter-story drift ratios obtained from pushover analysis for optimal designs at IO, LS and CP performance levels with the objectives of minimizing the cost and CO\textsubscript{2} emissions are shown in Figures 10 and 11. Here, the vertical dash line indicates the permissible drift. The drift of storys should not exceed the specified limits.

Figure 12 shows the relationship between optimal cost and optimal CO\textsubscript{2} emissions in the performance based optimal seismic design at three performance levels. Where the objective
functions are either CO$_2$ emissions or economic cost. These relationships indicate that in reinforced concrete frames with prismatic beams, a design based on minimizing CO$_2$ emissions can reduce the CO$_2$ emissions compared to a cost-optimization design. At the LS and CP performance levels, optimal design of frames with the objective of minimizing CO$_2$ has more effect on reducing CO$_2$ emissions.

4.2. The frame with non-prismatic beams

In this example, the frame of first example has been extended. Where the beams are non-prismatic. The lateral loading, and grouping of beams and columns are shown in Fig. 13. Performance-based optimal seismic design has been used for this frame to minimize objective functions at three performance levels.

The results of the ECBO algorithm in the IO performance level show that, in the solution with objective of minimizing cost, the best reported solution is 12752.19 euro with 15004 kg of carbon dioxide emissions (Table 9). In the solution with objective of minimizing CO$_2$ emissions, the best reported solution is 14368 kg at a cost of 12759 euro (Table 10). Percentage comparison of results show that in the solution based on the objective function of minimizing carbon dioxide emissions, by increase 0.054% in cost can reduce CO$_2$ emissions by 4.2%.

Figures 14 and 15 show the distribution of the inter-story drift ratios for the frame with non-prismatic beams at IO, LS and CP performance levels with the objectives of minimizing the cost and CO$_2$ emissions, respectively.

Figure 16 shows the relationship between optimal cost and optimal CO$_2$ emissions in the performance based optimal seismic design of frame with non-prismatic beams. At the IO and LS performance levels the increase in optimal cost is smaller. Therefore, at these levels, design with the objective of minimizing CO$_2$ emissions in addition to reduction CO$_2$ have optimal cost.

5. Concluding remarks
In this paper, nonlinear static pushover analysis is incorporated with a structural optimization strategy for performance-based optimal seismic design of RC frames with prismatic beams and frames with non-prismatic beams. The constraints are controlled in two steps: first, the equivalent static analysis is performed and strength-based constraint are controlled, then the nonlinear pushover analysis is carried out at three performance levels according to the FEMA code and the maximum inter-story drift is investigated. In order to identify the most competent algorithm for solving the problem, five metaheuristic algorithms including PSO, CBO, ECBO, VPS and EVPS are used to optimize the objectives at the IO performance level, and the rest of the problems are solved using the selected competent algorithm. Comparison of the performance of the algorithms in the first example demonstrated that ECBO performed better than the other algorithms. The relationships between optimal cost and optimal CO$_2$ emissions in PBOSD for three performance levels show that, in the design of reinforced concrete frames with the objective as minimizing CO$_2$ emissions, the CO$_2$ can be reduced compared to design with the objective of optimizing cost. In frames with prismatic beams at performance levels, with a 2.2% to 4% increase in cost, CO$_2$ is reduced by 1.8% to 12%, and for frames with non-prismatic beams with a 0.05% to 2.3% increase in cost, the reduced CO$_2$ is 1.1% to 10%.

References


Biographies

Ali Kaveh was born in 1948 in Tabriz, Iran. After graduation from the Department of Civil Engineering in the University of Tabriz in 1969, he continued his studies on
Structures at Imperial College of Science and Technology in London University and received his MSc, DIC and PhD degrees in 1970 and 1974, respectively. He then joined the Iran University of Science and Technology. Professor Kaveh is the author of 700 papers published in international journals and 150 papers presented at national and international conferences. He has authored 23 books in Persian and 10 books in English published by Wiley, Research Studies Press, American Mechanical Society, and Springer.

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**Ramezan Ali Izadifard** was born in 1966 in Frydonkenar, in the north of Iran. He studied structural engineering at Isfahan University of Technology and Shiraz University and received his bachelor, MSc and PhD degrees in 1989, 1992 and 2008, respectively. He is the author of more than 50 papers published in international journals or presented at professional conferences. He is currently teaching and researching at Imam Khomeini International University.

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- **Table 2**: Search space parameters
- **Table 3**: Unit prices and CO$_2$ emissions [14]
- **Table 4**: The properties of materials
- **Table 5**: Suitable parameters used for each of the algorithms
- **Table 6**: Comparative results of the algorithms for frame with the prismatic beams at IO level
- **Table 7**: Optimal results of the cost objective for frame with the prismatic beams at IO level
- **Table 8**: Optimal results of CO$_2$ emissions objective for frame with the prismatic beams at IO level
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Fig. 7. Geometry and grouping of the elements for frame with prismatic beams

Fig. 8. Comparison of the convergence curves of the algorithms for frame with prismatic beams to the lowest cost

Fig. 9. Comparison of the convergence curves of the algorithms for frame with prismatic beams to the lowest CO₂ emissions

Fig. 10. Inter-story drift ratio for the frame with prismatic beams at the three performance levels in the cost objective

Fig. 11. Inter-story drift ratio for the frame with prismatic beams at the three performance levels in the CO₂ objective

Fig. 12. The relationship between optimal cost and optimal CO₂ emissions for frame with prismatic beams

Fig. 13. Geometry and grouping of the elements for the frame with non-prismatic beams

Fig. 14. Inter-story drift ratio for the frame with non-prismatic beams at the three performance levels in the cost objective

Fig. 15. Inter-story drift ratio for the frame with non-prismatic beams at the three performance levels in the CO₂ objective

Fig. 16. The relationship between optimal cost and optimal CO₂ emissions for frame with non-prismatic beams

<table>
<thead>
<tr>
<th>Performance level</th>
<th>Earthquake level</th>
<th>$S_5 (g)$</th>
<th>$S_1 (g)$</th>
<th>$F_a$</th>
<th>$F_v$</th>
<th>$T_0(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IO</td>
<td>20%/50</td>
<td>1.143</td>
<td>0.403</td>
<td>1.04</td>
<td>1.60</td>
<td>0.542</td>
</tr>
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Table 2: Search space parameters

<table>
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<th>Width (mm)</th>
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<th>Number of bars</th>
<th>Bar size</th>
<th>TLR %</th>
</tr>
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<td>3</td>
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<td>Max</td>
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<td>11</td>
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<td>Increment</td>
<td>50</td>
<td>50</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Beam</td>
<td>Min</td>
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<td>350</td>
<td>2</td>
<td>3</td>
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<tr>
<td></td>
<td>Max</td>
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<td>1050</td>
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<td>11</td>
</tr>
<tr>
<td></td>
<td>Increment</td>
<td>50</td>
<td>1</td>
<td>1</td>
<td>5</td>
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</table>

Table 3: Unit prices and CO₂ emissions [14]

<table>
<thead>
<tr>
<th>Description</th>
<th>Unit</th>
<th>Cost (€)</th>
<th>CO₂ (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Beam</td>
<td>Column</td>
</tr>
<tr>
<td>Steel B-500</td>
<td>kg</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td>Concrete (40 MPa)</td>
<td>m³</td>
<td>105.93</td>
<td>105.17</td>
</tr>
<tr>
<td>Form work</td>
<td>m²</td>
<td>25.05</td>
<td>22.75</td>
</tr>
<tr>
<td>Scaffolding</td>
<td>m²</td>
<td>38.89</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4: The properties of materials

Concrete (Uniaxial Material Concrete01)

<table>
<thead>
<tr>
<th>Material type</th>
<th>$f_c'(MPa)$</th>
<th>$\varepsilon_{c0}$</th>
<th>$f_{cu}'(MPa)$</th>
<th>$\varepsilon_{cu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core concrete of beams (confined)</td>
<td>44</td>
<td>0.00296</td>
<td>15.3</td>
<td>0.0148</td>
</tr>
<tr>
<td>Core concrete of columns (confined)</td>
<td>48</td>
<td>0.0032</td>
<td>16.8</td>
<td>0.048</td>
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<tr>
<td>Cover concrete (unconfined)</td>
<td>40</td>
<td>0.0025</td>
<td>14</td>
<td>0.0055</td>
</tr>
</tbody>
</table>

Steel (Uniaxial Material Steel01)

<table>
<thead>
<tr>
<th>Material type</th>
<th>$f_y(MPa)$</th>
<th>$E_0$</th>
<th>hardening ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforcing steel</td>
<td>500</td>
<td>2e5</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 5: Suitable parameters used for each of the algorithms

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<th>Pop. Size</th>
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</thead>
<tbody>
<tr>
<td>CBO</td>
<td>Cost object</td>
</tr>
<tr>
<td></td>
<td>CO₂ object</td>
</tr>
<tr>
<td>Algorithm</td>
<td>Cost object</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
</tr>
<tr>
<td>ECBO</td>
<td>Pop. Size 16</td>
</tr>
<tr>
<td></td>
<td>CM size 0.45</td>
</tr>
<tr>
<td>VPS</td>
<td>Pop. Size 18</td>
</tr>
<tr>
<td></td>
<td>W1 0.3</td>
</tr>
<tr>
<td>EVPS</td>
<td>Pop. Size 18</td>
</tr>
<tr>
<td></td>
<td>W1 0.2</td>
</tr>
<tr>
<td>PSO</td>
<td>Pop. Size 26</td>
</tr>
<tr>
<td></td>
<td>C1 2</td>
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</table>

**Table 6**: Comparative results of the algorithms for frame with the prismatic beams at IO level

<table>
<thead>
<tr>
<th></th>
<th>EVPS</th>
<th>ECBO</th>
<th>VPS</th>
<th>CBO</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost objective</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Best cost (€)</td>
<td>15340</td>
<td>13770</td>
<td>16226</td>
<td>17692</td>
<td>16160</td>
</tr>
<tr>
<td>SD</td>
<td>1717</td>
<td>307</td>
<td>2780.66</td>
<td>551.3</td>
<td>2577</td>
</tr>
<tr>
<td>Average (€)</td>
<td>16278.6</td>
<td>14143.7</td>
<td>17821.6</td>
<td>18815</td>
<td>17627.6</td>
</tr>
<tr>
<td>CO₂ objective</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Best CO₂ (kg)</td>
<td>17745</td>
<td>15348</td>
<td>18674</td>
<td>22621</td>
<td>19483</td>
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<tr>
<td>SD</td>
<td>866.2</td>
<td>635.9</td>
<td>1750.7</td>
<td>901.3</td>
<td>1144</td>
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<tr>
<td>Average (kg)</td>
<td>18830.11</td>
<td>16212.2</td>
<td>20953.4</td>
<td>23940.88</td>
<td>20981</td>
</tr>
</tbody>
</table>

**Table 7**: Optimal results of the cost objective for frame with the prismatic beams at IO level

<table>
<thead>
<tr>
<th>Beam group no.</th>
<th>Cost objective</th>
<th>b (mm)</th>
<th>h (mm)</th>
<th>Aₜ (mm)</th>
<th>Aₘ (mm)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>350</td>
<td>750</td>
<td>3#11</td>
<td>3#8</td>
<td></td>
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<td>2</td>
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<td>3</td>
<td>350</td>
<td>900</td>
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<td>4</td>
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<table>
<thead>
<tr>
<th>Column group no.</th>
<th>Cost objective</th>
<th>b (mm)</th>
<th>h (mm)</th>
<th>Aₜ (mm)</th>
<th>Aₘ (mm)</th>
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<tbody>
<tr>
<td>1</td>
<td>350</td>
<td>850</td>
<td>8#8</td>
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<tr>
<td>2</td>
<td>350</td>
<td>800</td>
<td>6#8</td>
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<tr>
<td>3</td>
<td>350</td>
<td>800</td>
<td>10#6</td>
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<td>4</td>
<td>300</td>
<td>750</td>
<td>8#6</td>
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<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>14143.7 €</th>
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<tr>
<td></td>
<td>Std deviation</td>
<td>307</td>
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Table 8: Optimal results of CO₂ emissions objective for frame with the prismatic beams at IO level

<table>
<thead>
<tr>
<th>Beam group no.</th>
<th>b (mm)</th>
<th>h (mm)</th>
<th>A₁ top</th>
<th>A₂ bottom</th>
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<tbody>
<tr>
<td>1</td>
<td>350</td>
<td>950</td>
<td>5#5</td>
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<td>3</td>
<td>350</td>
<td>1050</td>
<td>3#10</td>
<td>4#6</td>
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<tr>
<td>4</td>
<td>350</td>
<td>1000</td>
<td>3#9</td>
<td>4#6</td>
</tr>
<tr>
<td>Column group no.</td>
<td>b (mm)</td>
<td>h (mm)</td>
<td>A₁ top</td>
<td>A₂ bottom</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>8</td>
<td>250</td>
<td>250</td>
<td>12#3</td>
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</table>

Average: 16212.2 kg
Std deviation: 635.9

Best solution: CO₂ 15347.8 kg (at a cost of 14082 €)

Table 9: Optimal results of cost objective for frame with the non-prismatic beams at IO level

<table>
<thead>
<tr>
<th>Beam group no.</th>
<th>b (mm)</th>
<th>h₁ (mm)</th>
<th>h₂ (mm)</th>
<th>A₁ top</th>
<th>A₂ bottom</th>
<th>A₂ top</th>
<th>A₂ bottom</th>
<th>TLR %</th>
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Average: 14058 €
Std deviation: 1731.8

Best solution: Cost 12752.19 € (with 15004 kg of CO₂ emissions)

Table 10: Optimal results of CO₂ objective for frame with the non-prismatic beams at IO level

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**Average** 15030 kg  
**Std deviation** 801.77  
**Best solution** CO2: 14368 kg (at a cost of 12759 €)
Fig. 1 Flowchart of the optimization by the selected algorithms
Fig. 2. Variables and shape of a non-prismatic beam

Fig. 3. Column load-moment interaction diagram
Fig. 4. Column reinforcement patterns

Fig. 5. Critical sections along the non-prismatic beams

Fig. 6. The pairs of objects for collision
Fig. 7. Geometry and grouping of the elements for frame with prismatic beams

Fig. 8. Comparison of the convergence curves of the algorithms for frame with prismatic beams to the lowest cost
Fig. 9. Comparison of the convergence curves of the algorithms for frame with prismatic beams to the lowest CO₂ emissions

Fig. 10. Inter-story drift ratio for the frame with prismatic beams at the three performance levels in the cost objective
Fig. 11. Inter-story drift ratio for the frame with prismatic beams at the three performance levels in the CO$_2$ objective

Fig. 12. The relationship between optimal cost and optimal CO$_2$ emissions for frame with prismatic beams
Fig. 13. Geometry and grouping of the elements for the frame with non-prismatic beams

Fig. 14. Inter-story drift ratio for the frame with non-prismatic beams at the three performance levels in the cost objective
**Fig. 15.** Inter-story drift ratio for the frame with non-prismatic beams at the three performance levels in the CO$_2$ objective

**Fig. 16.** The relationship between optimal cost and optimal CO$_2$ emissions for frame with non-prismatic beams