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Single- and multi-stage manufacturing systems under imperfect quality items with random defective rate, rework and scrap

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Abstract. The classical manufacturing systems assume that all produced items are of perfect quality. They also need to consider the rework process in manufacturing operations. Moreover, most of the previous literature considers single-stage production-inventory systems and does not consider multi-stage options. However, in real-world production-inventory systems, defective items are inevitable, and a fraction of the produced items may be defective. In addition, to avoid extra costs and consider environmental issues, organizations tend to rework activities. We propose single and multi-stage production-inventory systems in manufacturing operations where the process is defective, rework is possible, and a percentage of items are scrapped. A main assumption in the current paper is that the defective rate is assumed to be an uncertain parameter. The grey systems theory, as a mathematical tool to address uncertain information in real-world situations, is utilized to model the random defective rate via a grey nonlinear programming problem. The proposed issues are investigated via numerical examples to assess the impact of grey parameters on optimal solutions.

1. Introduction

One of the primary assumptions of the classical production-inventory problems is that all items are of perfect quality every time. However, in real-world situations, the production processes are not necessarily of perfect quality, and producing defective items is inevitable. Usually, a certain fraction of the items are defective due to poor quality of the production process or raw materials. In a multi-stage production system, products are transferred from one stage to the next stage, and every stage may produce a fraction of defective products. A production stage is affected by a number of inevitable undesirable factors, which make it rarely possible for a production machine/production system to produce perfect quality items every time. In fact, in some defective production systems, rework is usually used for imperfect items at every stage. The perfect items go to the next stage and finally become finished products at the final stage and then go under consumption. Moreover, a fraction of imperfect items are of unacceptable quality and should be disposed of as scrap at each stage. The decision on the quality of

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a produced item is made by a quality control process in manufacturing companies.

In the real world, there are many uncertain parameters affecting the decision-making process, outstandingly. Grey systems theory is a mathematical tool to address uncertain information in real-world situations. In grey systems theory, the aim is to enable prediction and decision-making in uncertain environments. In a real system, when the information is fully known, the system is called a white system, and when the information is unknown, it is called a black system. Moreover, when the system encounters partially known information, it is called a grey system. One of the common approaches in addressing uncertain decision-making is stochastic programming, where analysis of scenarios is carried out to study an optimization problem under uncertainty. In such an approach, the uncertainty of parameters is addressed by a number of subproblems; each one is associated with a scenario on the possible values of uncertain parameters. By analyzing the subproblems and studying their solutions, one hopes to discover the solution to the original problem by using probabilistic means like expected value. However, solving the subproblem may have some difficulties, and great computational efforts are needed to solve subproblems. Grey Programming (GP) (grey optimization) is one of the potential approaches for avoiding the above problems in stochastic programming [1]. The major advantages of GP against existing approaches are:

(i) GP will generate feasible ranges of decision variables and objective functions using the interpretation of the grey solutions and grey input parameters;

(ii) GP will have lower computational efforts compared with the existing methods and then is applicable to practical problems;

(iii) GP will not require distribution information (like probability-based approaches) or membership function (like fuzzy-based approaches) for input parameters, since interval numbers are acceptable for the input parameters [1].

In this paper, as our contribution, we propose a production-inventory problem where a manufacturer produces products via a finite production rate. It is assumed that the production process is defective and produces a percentage of imperfect items. The imperfect products are also under a rework process to become perfect and return to the consumption cycle. The aim is to determine optimal/economic production quantities in such a way that the total cost of the system is minimized. The traditional literature mostly considers single-stage production systems and the multi-stage production processes are rarely considered. Moreover, many of them addressed the constant defective rate in the manufacturing process, which is not a real condition. In addition, rare studies considered the rework of defective items, and others do not consider this complementary process.

The rest of this paper is structured as follows. Section 2 introduces the grey systems theory. Then, Section 3 applies the grey systems theory principles to single and multi-stage manufacturing systems. Section 4 considers two numerical examples and studies the impact of grey parameters on optimal solutions. Finally, Section 5 concludes the paper.

2. Literature review

The manufacturing process with rework has been studied and discussed extensively in the literature (i.e., see [2,3]). Moreover, the impact of the rework process on imperfect products was evaluated as well. As one of the first contributions, Salameh and Jaber [4] considered the raw material in the economic order lot size problem where there is an inspection process within a main production process. They assumed that the defective items were sold as a single batch at a discounted price. Also, they considered a probability density function for a percentage of defective items. Besides, an inventory problem for a multi-stage production process with imperfect items was presented by Ben-Daya and Rahim [5]. In their paper, a fixed percentage of defective items has been considered. In another work proposed by Ben-Daya et al. [6], they considered different inspection policies for the inventory inspection models in a single-stage environment. Moreover, a beta distribution has been assumed for the fraction of nonconforming items. Ojha et al. [7] studied a reworkable item lot size quantity model under an imperfect production system. They considered single-stage production with producing the defective items at a constant rate. In addition, Sarker et al. [8] studied a multi-stage production environment where there is an imperfect manufacturing system. In that paper, they assumed that the proportion of defective items is constant in each cycle. In other research presented by Biswas and Sarker [9], an optimal lot size quantity was calculated in a lean production system when there are reworks and scraps in the system. A single-stage production process with a constant proportion of defective items was considered in that paper. Moreover, Wee and Widyadana [10] proposed an economic production quantity model under rework conditions for raw materials and deteriorating finished items. Their assumption was also a single-stage system with a constant rate of rework and deterioration. Krishnamoorthi and Panayappan [11] evaluated the possible sales return for poor-quality items in a production process with rework allowed.

They considered a constant proportion of defects in a single-stage manufacturing system. In another study, the various inspection options were investigated by Yoo et al. [12], who tried to manage the inventory with imperfect production. In this paper, a multi-stage system with a constant defective rate for items was considered. Colledani and Tolio [13] proposed an analytical approach to jointly assess the effect of quality and performance in a multi-stage manufacturing system with a fixed fraction of nonconforming items. Wee et al. [14] determined the production lot size for imperfect quality items when shortages are allowed for a single-stage system. In their paper, the percentage of imperfect items had a known probability density function. Additionally, Paul et al. [15] suggested a disruption system in a multi-stage manufacturing system. Moreover, Mahata [16] considered partial backlogging and fuzziness to calculate the production lot size under an imperfect production system. In other words, some costs, such as setup cost, average holding cost, backorder cost, raw material cost, labor cost, as well as a percentage of defective items, were characterized as fuzzy numbers. Jaggi et al. [17] proposed and discussed an imperfect inventory model with two warehouses with permissible payment delays. The percentage of defective items in their paper was a random variable with a probability function. Kamali et al. [18] presented a multi-objective optimization inventory model for the single-buyer, multi-vendor problem with discounts. The authors considered each vendor had a fixed rate of item deterioration in a supply chain. Nobil et al. [19] considered the scrapped and reworked items under economic production quantity, incorporating shortages and allocations in a multi-stage manufacturing system. The percentage of defective has been known as a constant parameter in this paper. Cheng et al. [20] proposed multiple disposals of defective inventories for an integrated imperfect production system. They assumed that there was a random fraction of defective items with a known probability density function. Mokhtari [21] proposed a joint production and order lot size problem considering the rework and defective manufacturing. The author considered the uncertainty for a percentage of items by a probability function. Nobil et al. [22] suggested an imperfect, shortage, rework, and scrapped single-machine system with inspection, deficiency levels, and setup times. One of their assumptions was that the expected proportion of produced defective items was known and constant. Mokhtari and Asadkhani [23] discussed an Economic Order Quantity (EOQ) model with imperfect quality inventory, including a probability density function with the inspection under batch replacement and returned items conditions. In another study, Mokhtari [24] proposed an optimal manufacturing system under both imperfect raw materials and products. In this paper, the defective rate of items has been assumed to be a constant number. Yang et al. [25] incorporated the trade credit into an inventory problem with shortage and constant defective items. Beranek and Buscher [26] suggested an integration between pricing and quality decisions under market segmentation by considering imperfect quality items. The proportion of imperfect quality items was constant and quality-dependent. Adak and Mahapatra [27] proposed a three-layer supply chain system with imperfect items and variable production costs under deterioration. In this paper, it was assumed that the defection rate of the finished item is fixed. A comparison between papers of literature review has been given in Table 1.

3. Theory of grey systems

In this section, the concept of grey systems will be defined to be used in the sequel. A grey number $G(\otimes)$ is a number whose exact value is unknown, but a bounded interval within which the value lies is known. A grey system is a system that includes information as grey numbers, and a grey decision is a decision-making system for a grey system. A grey number $G(\otimes)$ is defined as an interval with known lower limit $\underline{G}$ and known upper limit $\overline{G}$ as $G(\otimes) = [\underline{G}, \overline{G}\,]$. One of the main advantages of grey system theory is that it works even if the probability distribution and membership functions cannot be recognized [28]. A grey number $G(\otimes)$ becomes a deterministic number or white number when its upper and lower limits are equal $\overline{G} = \underline{G}$. Some useful information on this approach is summarized in the sequel.

Definition 1. A grey number $G(\otimes)$ is defined as an interval with known lower and upper limits $G(\otimes) = [\underline{G}, \overline{G}\,]$ and unknown distribution information.

Definition 2. The whitened value of grey number $G(\otimes)$, shown by $\tilde{G}(\otimes)$, is defined as a deterministic number with a value lying between upper and lower limits:

$$\underline{G} \leq \tilde{G}(\otimes) \leq \overline{G}. \tag{1}$$

The whitened value of the grey number can be formulated by defining a new variable $\gamma \in [0, 1]$ as follows:

$$\tilde{G}(\otimes) = \gamma\overline{G} + (1-\gamma)\underline{G}. \tag{2}$$

The whitened value $\tilde{G}(\otimes)$ equals to upper limit $\overline{G}$ if γ=1, and equals to lower limit $\underline{G}$, if $\gamma = 0$. Moreover, if γ gets a value between 0 and 1, the whitened value $\tilde{G}(\otimes)$ gets an intermediate value between upper and lower limits.

Grey Linear Programming (GLP) is a decision-making method under uncertainty. It is a development of the traditional linear programming method. The

Table 1. The properties of reviewed papers in the literature.

Refrences	EOQ/EPQ	Environment			Kind of uncertainty regarding the percentage of defective items
		Single -stage	Multi -stage	Supply -chain	
Salameh and Jaber (2000) [4]	EOQ	*	–	–	Probability density function
Ben-Daya and Rahim (2003) [5]	EPQ	–	*	–	Constant rate
Ben-Daya et al. (2006) [6]	EOQ	*	–	–	Beta distribution
Ojha et al. (2007) [7]	EPQ	*	–	–	Constant rate
Sarker et al. (2008) [8]	EPQ	–	*	–	Constant rate
Wee and Widyadana (2012) [10]	EPQ	*	–	–	Constant rate
Krishnamoorthi and Panayappan (2017) [11]	EPQ	*	–	–	Constant rate
Yoo et al. (2012) [12]	EPQ	–	*	–	Constant rate
Colledani and Tolio (2011) [13]	EPQ	–	*	–	Constant rate
Wee et al. (2013) [14]	EOQ	*	–	–	Probability density function
Paul et al. (2015) [15]	EPQ	–	*	–	Constant rate
Mahata (2017) [16]	EPQ	*	–	–	Fuzzy number
Jaggi et al. (2017) [17]	EPQ	–	–	*	Probability density function
Kamali et al. (2017) [18]	EPQ	–	–	*	Constant rate
Nobil et al. (2018) [19]	EPQ	–	*	–	Constant rate
Cheng et al. (2018) [20]	EOQ	–	*	–	Probability density function
Mokhtari (2018) [21]	EPQ	*	–	–	Probability density function
Nobil et al. (2019) [22]	EPQ	*	–	–	Constant rate
Mokhtari and Asadkhani (2019) [23]	EOQ	*	–	–	Probability density function
Mokhtari (2019) [24]	EPQ	–	*	–	Constant rate
Yang et al. (2019) [25]	EOQ	*	–	–	Constant rate
Beranek and Buscher (2021) [26]	EPQ	*	–	–	Constant rate
Adak and Mahapatra (2022) [27]	EPQ	–	–	*	Constant rate
This paper	EPQ	*	*	–	Grey number

GLP can be presented by following standard form:

$$\text{Max } f = c(\otimes)x, \tag{3}$$

subject to:

$$A(\otimes)\ x \leq b \qquad x \geq 0,$$

in which:

$$c(\otimes) = [c_1(\otimes),\ c_2(\otimes), \ldots, c_m(\otimes)], \tag{4}$$

$$x^T = (x_1, x_2, \ldots, x_m)\,, \tag{5}$$

$$b^T = (b_1, b_2, \ldots, b_n)\,, \tag{6}$$

$$A(\otimes) = [a_{ij}(\otimes)] \qquad \forall i = 1, 2, \ldots, n, \qquad j = 1, 2, \ldots, m, \tag{7}$$

and:

$$c_j(\otimes) = \left[\underline{c}_{ij}, \overline{c}_{ij}\right] \text{ and } a_{ij}(\otimes) = \left[\underline{a}_{ij}, \overline{a}_{ij}\right] \qquad \forall i = 1, 2, \ldots, n, \qquad j = 1, 2, \ldots, m. \tag{8}$$

The optimal solution to the problem is also a grey number since the parameters in the model Eq. (27) are grey.

$$f^*(\otimes) = \left[\underline{f}^*, \overline{f}^*\right], \tag{9}$$

$$x_{ij}(\otimes) = \left[\underline{x}_{ij}, \overline{x}_{ij}\right]. \tag{10}$$

The aim of GLP is to obtain grey objective function $f^*(\otimes)$, and grey decision variables $x_{ij}(\otimes)$ of model Eq. (27) as uncertain interval outputs. In other words, the aim is to find the lower and upper limits for optimal objective function when the input parameters are grey.

Since the proposed production-inventory problems are unconstrained nonlinear optimization problems, we focus on Grey Non-Linear Programming (GNLP). Let $X = (x_1, x_2, ..., x_n)$ be the decision vector and $(\otimes)$ a set of grey parameters. Then:

$$\max Z = f(X, \otimes), \tag{11}$$

is a grey unconstrained nonlinear programming problem, where $f(X, \otimes)$ is a grey functional. If all grey elements in $f(X, \otimes)$ are whitened, then a programming problem is called whitenized programming:

$$\max Z = \tilde{f}(X, \tilde{\otimes}). \tag{12}$$

For GNLP problems, we can first whitenize the original problem and then solve the resulting one for the solution. Let us consider that $f(X)$ is a differentiable function. Then, the solution of the gradient vector is a solution of an optimization problem:

$$\frac{\partial \tilde{f}(X, \tilde{\otimes})}{\partial X} = \left(\frac{\partial \tilde{f}(X, \tilde{\otimes})}{\partial x_1}, \frac{\partial \tilde{f}(X, \tilde{\otimes})}{\partial x_2}, \ldots, \frac{\partial f \tilde{f}(X, \tilde{\otimes})}{\partial x_n}\right) = 0. \tag{13}$$

If $f(X)$ is second-order differentiable, and its Hessian matrix;

$$H(x) = \begin{pmatrix} \frac{\partial^2 \tilde{f}(X,\tilde{\otimes})}{\partial x_1^2} & \frac{\partial^2 \tilde{f}(X,\tilde{\otimes})}{\partial x_1 x_2} & \cdots & \frac{\partial^2 \tilde{f}(X,\tilde{\otimes})}{\partial x_1 x_n} \\ \frac{\partial^2 \tilde{f}(X,\tilde{\otimes})}{\partial x_2 x_1} & \frac{\partial^2 \tilde{f}(X,\tilde{\otimes})}{\partial x_2^2} & \cdots & \frac{\partial^2 \tilde{f}(X,\tilde{\otimes})}{\partial x_2 x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial^2 \tilde{f}(X,\tilde{\otimes})}{\partial x_n x_1} & \frac{\partial^2 \tilde{f}(X,\tilde{\otimes})}{\partial x_n x_2} & \cdots & \frac{\partial^2 \tilde{f}(X,\tilde{\otimes})}{\partial x_n^2} \end{pmatrix}, \tag{14}$$

is a negative definite matrix. Moreover, in order to reach the lower and upper limits for an optimal objective function, the following mathematical models are constructed:

$$\min_{\gamma}(\underline{f}\otimes) = \max_{X} \tilde{f}(X, \tilde{\otimes}), \tag{15}$$

subject to:

$$\tilde{\otimes} = \gamma\overline{\otimes} + (1-\gamma)\underline{\otimes} \qquad 0 \le \gamma \le 1,$$

$$\max_{\gamma}(\overline{f}\otimes) = \max_{X} \tilde{f}(X, \tilde{\otimes}), \tag{16}$$

subject to:

$$\tilde{\otimes} = \gamma\overline{\otimes} + (1-\gamma)\underline{\otimes} \qquad 0 \le \gamma \le 1,$$

where $(\underline{f}\otimes)$ and $\overline{f}(\otimes)$ represents the lower and upper limits of the objective function (total profit per unit time) as a grey number. In GP, the aim is to recognize the lower and upper limits of the objective function $f(X, \otimes)$, caused by grey input parameters.

4. Grey modeling

4.1. Single-stage manufacturing system

In this section, a single-stage manufacturing system with the aim of satisfying an external demand D^S will be discussed and proposed. In this system, the production is performed through a finite rate P_1^S based on an EPQ structure. The demand rate is constant, the shortage is not permitted, and the purchase cost is assumed to be fixed. Unlike classical inventory problems, the production process is supposed to be defective, and a percentage of items $\beta^S(\otimes)$ is imperfect. That means the defective rate of the system is a grey number $G(\otimes)$, which is defined as an interval with known lower limit $\underline{\beta}^S$ and known upper limit $\overline{\beta}^S$ as $\beta^S(\otimes) = [\underline{\beta}^S, \overline{\beta}^S]$. The imperfect items are also under the rework process to become perfect and return to the consumption cycle. Every cycle of inventory, in our problem, includes periods of (i) production, (ii) reworking, and (iii) depletion. A percentage of defective items $\beta^S(\otimes)$ is produced during the production period, and then a rework process with rate P_2^S will be started. The environmental effects, in real situations, result in unsuitable variations in the quality of products. So, the percentage of defective items $\beta^S(\otimes)$ is assumed to be a random variable. During the production interval t_p^S, the products are produced at a rate P_1^S. At the end of the production period, there are imperfect items whose amount is determined by multiplying defective rate and production quantity, i.e., $\beta^S(\otimes)Q^S$. At this moment, the rework process commences for the reworkable items $\alpha\beta^S(\otimes)Q^S$. The scrapped items $(1-\alpha)\beta^S(\otimes)Q^S$ are detected and disposed of the reworkable items $\alpha\beta^S(\otimes)Q^S$ go under reworking, become perfect items, and then return to perfect items stock. During the depletion period t_D, the stored inventory is fully consumed. At every cycle, this process repeats without interruption. To ensure feasibility and prevent shortage, we consider the initial conditions as $I_{\max}^S - \alpha\beta^S(\otimes)Q^S \ge 0$. Since $I_{\max}^S = \mathrm{Q}^S\left(1 - D^S/P_1^S\right)$, this condition simplifies to $\beta^S(\otimes) \le 1 - D/P_1$. Therefore, the defective rate $\beta(\otimes)$ is assumed to be a variable on interval $[0, 1 - D^S/P_1^S]$, to avoid shortage and ensure feasibility. In order to maximize the total profit, we seek the optimal production quantity Q^S. The total profit TP^S is the subtraction of total cost TC^S from total revenue TR^S. The total revenue per cycle TR involves sales of perfect and scrapped items which is given as $TR^S = v^S\left\{Q^S - \beta^S(\otimes)\mathrm{Q}^S + \alpha\beta^S(\otimes)\mathrm{Q}^S\right\} + s^S\left\{(1-\alpha)\beta^S(\otimes)\mathrm{Q}^S\right\}$, where v^S and s^S represent the sale price of perfect and scrapped items, respectively, for a single-stage system. The total cost involves production, setup, holding, screening, and reworking costs. The production cost PC^S can be determined by $PC^S = C^S Q^S$, and the setup cost SC^S is calculated by $SC^S = A^S$. We should calculate the area below the inventory curve in order to compute the holding cost. To this end, we name areas below the production (first period) S_I^S, rework (second period) S_{II}^S, and depletion (third period) S_{III}^S. By doing so, we reach:

$$S_I^S = \frac{{Q^S}^2}{2P_1^S}\left(1 - D^S/P_1^S\right),$$

$$S_{II}^S = \frac{\alpha\beta^S(\otimes)\,\mathrm{Q}^S}{2P_2^S}\{2Q^S\left(1-D^S/P_1^S\right) - 2\beta^S(\otimes)\,\mathrm{Q}^S + \alpha\beta^S(\otimes)\,\mathrm{Q}^S\left(1-D^S/P_2^S\right)\},$$

and,

$$S_{III}^S = \frac{1}{2D^S}\{Q^S\left(1 - D^S/P_1^S\right) - \beta^S(\otimes)\,\mathrm{Q}^S + \alpha\beta^S(\otimes)\,\mathrm{Q}^S\left(1 - D^S/P_2^S\right)\}^2.$$

Then, the holding cost can be computed as follows:

$$HC^S = h^S {Q^S}^2\left\{\frac{{G^S}^2(\otimes)}{2D^S} + \frac{1}{2P_1^S}\left(1-\frac{D^S}{P_1^S}\right) + \frac{\alpha\beta^S(\otimes)}{2P_2^S}\left\{\left(1-\frac{D^S}{P_1^S}\right) - \beta^S(\otimes) + G^S(\otimes)\right\}\right\}, \tag{17}$$

where h is the unit holding cost (per product per unit time), and:

$$G^S(\otimes) = \left(1-\frac{D^S}{P_1^S}\right) - \beta^S(\otimes) + \alpha\beta^S(\otimes)\left(1-\frac{D^S}{P_2^S}\right).$$

Moreover, the cost of screening is calculated by $WC^S = d^S Q^S$, where d is the unit screening cost per item. In addition, the reworking cost is formulated as $RC^S = r^S\alpha\beta^S(\otimes)\,Q^S$ in which r^S is the cost of the reworked item.

By using the above formulation, the total cost can be written as follows:

$$TC^S = C^S Q^S + A^S + h^S {Q^S}^2 \left\{\frac{{G^S}^2(\otimes)}{2D^S} + \frac{1}{2P_1^S}\left(1 - \frac{D^S}{P_1^S}\right) + \frac{\alpha\beta^S(\otimes)}{2P_2^S} \left\{\left(1 - \frac{D^S}{P_1^S}\right) - \beta^S(\otimes) + G^S(\otimes)\right\}\right\} + d^S Q^S + r^S\alpha\beta^S(\otimes)\,\mathrm{Q}^S. \tag{18}$$

Therefore, the total profit can be calculated by $TR^S - TC^S$ as follows:

$$TP^S = v^S\left\{Q^S - \beta^S(\otimes)\,\mathrm{Q}^S + \alpha\beta^S(\otimes)\,\mathrm{Q}^S\right\} + s^S\left\{(1-\alpha)\,\beta^S(\otimes)\,\mathrm{Q}^S\right\} - C^S Q^S - A^S - h^S {Q^S}^2\left\{\frac{{G^S}^2(\otimes)}{2D^S} + \frac{1}{2P_1^S}\left(1 - \frac{D^S}{P_1^S}\right) + \frac{\alpha\beta^S(\otimes)}{2P_2^S}\left\{\left(1-\frac{D^S}{P_1^S}\right) - \beta^S(\otimes) + G^S(\otimes)\right\}\right\} - d^S Q^S - r^S\alpha\beta^S(\otimes)\,\mathrm{Q}^S. \tag{19}$$

Since the defective rate is a grey number, then the whitened value of total profit, $T\tilde{P}^S(\otimes)$, is given by substituting the whitened value of the defective rate $\tilde{\beta}(\otimes)$ into the above formula, as follows:

$$T\tilde{P}^S(\otimes) = v^S\left\{Q^S - \tilde{\beta}^S(\otimes)\,\mathrm{Q}^S + \alpha\tilde{\beta}^S(\otimes)\,\mathrm{Q}^S\right\} + s^S\left\{(1-\alpha)\,\tilde{\beta}^S(\otimes)\,\mathrm{Q}^S\right\} - C^S Q^S - A^S - h^S {Q^S}^2\left\{\frac{{\tilde{G}^S}{}^2(\otimes)}{2D^S} + \frac{1}{2P_1^S}\left(1 - \frac{D^S}{P_1^S}\right) + \frac{\alpha\tilde{\beta}^S(\otimes)}{2P_2^S}\left\{\left(1 - \frac{D^S}{P_1^S}\right) - \tilde{\beta}^S(\otimes) + \tilde{G}^S(\otimes)\right\}\right\} - d^S Q^S - r^S\alpha\tilde{\beta}^S(\otimes)\,\mathrm{Q}^S, \tag{20}$$

where,

$$\tilde{G}^S(\otimes) \text{ is } \left(1-\frac{D^S}{P_1^S}\right) - \tilde{\beta}^S(\otimes) + \alpha\tilde{\beta}^S(\otimes)\left(1-\frac{D^S}{P_2^S}\right).$$

Moreover, the whitened value of total profit per unit time $T\tilde{P}U^S(\otimes)$ can be computed by dividing $T\tilde{P}^S(\otimes)$ by the whitened inventory cycle time $\tilde{T}^S(\otimes)$. In another way, the cycle time is $t_P^S + t_R^S + t_D^S$, which simplifies to $\tilde{T}^S(\otimes) = \frac{Q}{D}\left(\tilde{\beta}^S(\otimes)(\alpha - 1) + 1\right)$. Therefore, the whitened total profit per unit time $T\tilde{P}U^S(\otimes)$ is calculated:

$$T\tilde{P}U^S(\otimes) = \frac{D^S}{\tilde{\beta}^S(\otimes)(\alpha-1) + 1}\left\{v^S\left\{\tilde{\beta}^S(\otimes)(\alpha-1) + 1\right\} + s^S\left\{(1-\alpha)\,\tilde{\beta}^S(\otimes)\right\} - C^S - h^S Q^S \left\{\frac{\tilde{G}^S(\otimes)^2}{2D^S} + \frac{1}{2P_1^S}\left(1 - \frac{D^S}{P_1}\right) + \frac{\alpha\tilde{\beta}^S(\otimes)}{2P_2^S}\right.\right.$$

$$\left\{\left(1-\frac{D^S}{P_1^S}\right)-\tilde{\beta}^S(\otimes)+\tilde{G}^S(\otimes)\right\}\right\}-d^S-\alpha r^S\tilde{\beta}^S(\otimes)\right\}$$

$$-\frac{A^S D^S}{Q^S\left(\tilde{\beta}^S(\otimes)(\alpha-1)+1\right)}. \tag{21}$$

It can be proved that $T\tilde{P}U^S(\otimes)$ is a concave function of Q^S, since its second derivative is negative. Therefore, in order to obtain the optimal quantity Q^{S^*}, we set the first derivative to zero, and arrive Eq. (22) as shown in Box I.

Note that when $\tilde{\beta}^S(\otimes)$=0, Q^{S^*} reduces to the traditional EPQ formula, $\left[2A^S D^S/\left(h^S\left(1-\frac{D^S}{P_1^S}\right)\right)\right]^{1/2}$.

Moreover, in order to reach the lower and upper limits for optimal objective function, the following mathematical models are constructed:

$$\min_{\gamma}\underline{TPU^S}(\otimes)=\max_{Q_1^S}T\tilde{P}U^S(\otimes), \tag{23}$$

subject to:

$$\tilde{\beta}^S(\otimes)=\gamma\overline{\beta}^S+(1-\gamma)\underline{\beta}^S \qquad 0\le\gamma\le 1,$$

$$\max_{\gamma}\overline{TPU^S}(\otimes)=\max_{Q_1^S}T\tilde{P}U^S(\otimes) \tag{24}$$

subject to:

$$\tilde{\beta}^S(\otimes)=\gamma\overline{\beta}^S+(1-\gamma)\underline{\beta}^S \qquad 0\le\gamma\le 1,$$

where $\underline{TPU^S}(\otimes)$ and $\overline{TPU^S}(\otimes)$ represents the lower and upper limits of the objective function (total profit per unit time) as a grey number. It is notable that the existence of the grey defective rate leads to the fact that the total profit is also a grey number with upper and lower limits. The aim of GP is to recognize the lower and upper limits of the objective function, TPU in our model, caused by grey input parameters. This will be obtained in our model by the GP model. In addition, note that the first model is a minimax problem (can be solved via conventional minimax commands in MATLAB), while the second one is a maximization problem with two variables Q_1 and γ (can be simply solved by nonlinear optimization commands in MATLAB).

4.2. Multi-stage manufacturing system

In this section, we extend the single-stage manufacturing system introduced in the previous section to a multi-stage one. Let's consider an imperfect manufacturing system with serial arrangement. As shown in Figure 1, each production stage j performs a specific operation on input items via production rate P_{1j} and then sends the perfect output items into the next manufacturing stage $j+1$. Moreover, the stage j is assumed to be defective and produces some percentages of imperfect items $\beta_j(\otimes)$. Similar to the single-stage case, the percentage of imperfect items $\beta_j(\otimes)$ is also a grey number with lower and upper limits. The defective items that are reworkable go under the rework process with the rework rate P_{2j}. At the end of the rework period, the stored inventory is sent to the next production stage $j+1$ for further processing. At each stage, there are three types of output items: (i) perfect items, (ii) imperfect but reworkable items, and (iii) imperfect and scrapped items. Once the production ends, there exist imperfect items $\beta_j(\otimes)Q_j$. Among them, the reworkable items $\alpha_j\beta_j(\otimes)Q_j$ go under the reworking process, and the scrapped items $(1-\alpha_j)\beta_j(\otimes)Q_j$ are disposed of from the system. During the rework period t_{Rj}, all the reworkable items $\alpha_j\beta_j(\otimes)Q_j$ become perfect with the rate of P_{2j} and return to the system. At the end of the rework period, the stored inventory is transferred into the next stage. The perfect and reworked items are sent to the next production stages for further processing. This is continued till the items reach to nth production stage. This stage is different from previous intermediate stages. It has one more period than previous stages, i.e., the depletion period. At this stage, the finished products are produced and depleted via demand rate D. Indeed, at the end of the rework period, the stored inventory is consumed during the depletion

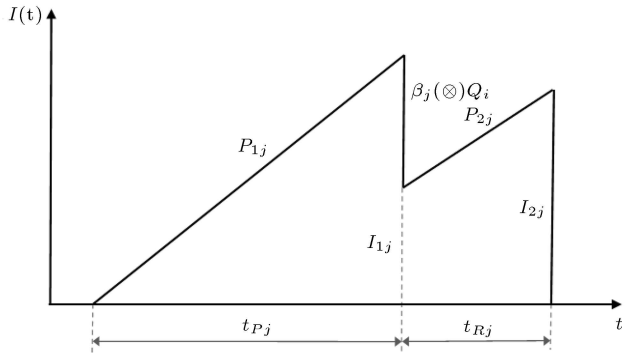

Figure 1. The inventory level for jth $j=1,2,\ldots,n-1$ stage of multi stage manufacturing system.

$$Q^{S^*}=\left[\frac{2A^S D^S}{h^S\left\{\tilde{\beta}^S(\otimes)^2+\frac{D^S}{P_1^S}\left(1-\frac{D^S}{P_1^S}\right)+\frac{\alpha\tilde{\beta}^S(\otimes)D^S}{P_2^S}\left\{\left(1-\frac{D^S}{P_1^S}\right)-\tilde{\beta}^S(\otimes)+\tilde{G}^S(\otimes)\right\}\right\}}\right]^{\frac{1}{2}}. \tag{22}$$

Box I

period t_D until it reaches zero. Figure 2 presents the inventory level of nth (final) production stage. The aim is to determine optimal/economic production quantities $Q_1, Q_2, \ldots, Q_n$, in such a way that the total profit is maximized. Moreover, Figure 3 depicts the inventory level of the entire planning horizon. To ensure feasibility and avoid shortage during stage j, the inventory level at the start of the rework period of every stage j should be greater than/equal to zero. For this purpose, we should have $P_{1j}Q_j - \beta_j(\otimes)Q_j \geq 0$, which simplifies to $\beta_j(\otimes) \leq P_{1j}$ for $j = 1, 2, \ldots, n$.

Before formulating total profit, we derive the relationship between production quantities $Q_1, Q_2, \ldots, Q_n$. The production quantity at the next stage Q_{j+1} is the sum of perfect items produced at the current stage $Q_j - \beta_j(\otimes)Q_j$ and reworked items at the current stage $\alpha_j\beta_j(\otimes)Q_j$, which is simplified to:

$$Q_{j+1} = Q_j \left\{\beta_j(\otimes)(\alpha_j - 1) + 1\right\} \qquad \forall j = 1, 2, \ldots, n-1. \tag{25}$$

For simplicity, we consider Q_1 as a decision variable in our model and formulate production quantity $Q_j\,(j \neq 1)$ in terms of Q_1, using the above recursive equation, as follows:

$$Q_1 \prod_{p=1}^{j-1} \left\{\beta_p(\otimes)(\alpha_p - 1) + 1\right\} \qquad \forall j = 2, 3, \ldots, n. \tag{26}$$

The total revenue per cycle TR involves sales of perfect

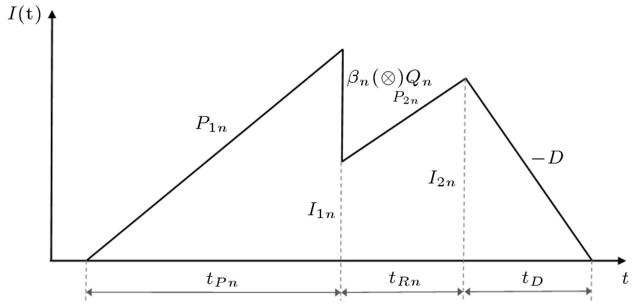

Figure 2. The inventory level for nth stage (final stage) of multi stage manufacturing system.

and scrapped items which is given as:

$$TR^M = v\left\{Q_n - \beta_n(\otimes)Q_n + \alpha_n\beta_n(\otimes)Q_n\right\} + \sum_{j=1}^{n}\left\{s_j(1-\alpha_j)\beta_j(\otimes)Q_j\right\},$$

where v denotes the unit sale price of perfect items and s_j represents the unit sale price for scrapped items at stage j. By substituting Q_n and Q_j from Eq. (26) into TR^M, it can be re-written in terms of decision variable Q_1 as follows:

$$TR^M = vQ_1\prod_{j=1}^{n}\left\{\beta_j(\otimes)(\alpha_j - 1) + 1\right\} + s_1(1-\alpha_1)\beta_1(\otimes)Q_1 + \left(\sum_{j=2}^{n}\left\{s_j(1-\alpha_j)\beta_j(\otimes)\prod_{p=1}^{j-1}\left\{\beta_p(\otimes)(\alpha_p - 1) + 1\right\}\right\}\right)Q_1. \tag{27}$$

The total cost involves production, setup, holding, screening, and reworking costs. The production cost per cycle PC^M is calculated as $PC^M = \sum_{j=1}^{n} C_jQ_j$ which is re-written as:

$$PC^M = C_1Q_1 + \left(\sum_{j=2}^{n} C_j\prod_{p=1}^{j-1}\left\{\beta_p(\otimes)(\alpha_p - 1) + 1\right\}\right)Q_1. \tag{28}$$

In addition, the setup cost SC^M is incurred per production cycle by $SC^M = \sum_{j=1}^{n} A_j$. Moreover, in order to calculate the holding cost, we first calculate the area below the inventory level in two periods, i.e., the production period S_{Ij} and the rework period S_{IIj}. The first area is calculated as $S_{Ij} = I_{\max j}.t_{pj}/2$. Since $I_{\max j} = Q_j$ and $t_{pj} = Q_j/P_{1j}$, the S_{Ij} is re-written as:

$$S_{Ij} = \frac{Q_j^2}{2P_{1j}}. \tag{29}$$

To calculate S_{IIj}, we should first formulate the inventory level at the start of the rework period I_{1j} and the inventory level at the end of the rework period I_{2j}, as

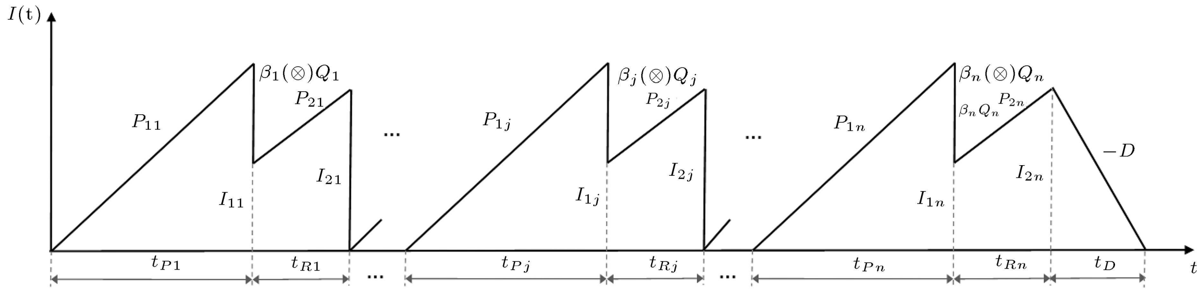

Figure 3. The inventory level for a multi stage manufacturing system.

$I_{1j} = Q_j - \beta_j(\otimes) Q_j$ and $I_{2j} = Q_j - \beta_j(\otimes) Q_j + P_{2j} t_{Rj}$. Similar to the single-stage case, we can calculate the rework period of the stage j as $t_{Rj} = \alpha_j \beta_j(\otimes) Q_j / P_{2j}$. So, the inventory level I_{2j} can be simplified as $I_{2j} = Q_j - \beta_j(\otimes) Q_j + \alpha_j \beta_j(\otimes) Q_j$. Therefore, the area below the inventory level in the rework period is calculated as $S_{IIj} = t_{Rj}(I_{1j} + I_{2j})/2$ which can be re-written as:

$$S_{IIj} = \frac{\alpha_j \beta_j(\otimes) Q_j}{2P_{2j}} \left\{2Q_j - 2\beta_j(\otimes) Q_j + \alpha_j \beta_j(\otimes) Q_j\right\}. \quad (30)$$

In addition to production and rework periods, there is one depletion period in nth (final) period. Therefore, the area below the inventory level in the depletion period is calculated as $S_{III}j = I_{2n} t_D / 2$ where the depletion period is $t_D = I_{2n}/D$; hence, we have $S_{IIIj} = I_{2n}^2 / 2D$, which is re-written as:

$$S_{IIIj} = \frac{Q_n^2}{2D}\left\{1 - \beta_n(\otimes) + \alpha_n \beta_n(\otimes)\right\}^2. \quad (31)$$

Utilizing S_{Ij}, S_{IIj}, and S_{IIIj}, the holding cost is calculated by $\sum_{j=1}^{n} h_j \{S_{Ij} + S_{IIj}\} + h_n S_{IIIj}$, as follows:

$$HC^M = \sum_{j=1}^{n} h_j \left\{\frac{Q_j^2}{2P_{1j}} + \frac{\alpha_j \beta_j(\otimes) Q_j}{2P_{2j}} \{2Q_j - 2\beta_j(\otimes) Q_j + \alpha_j \beta_j(\otimes) Q_j\}\right\} + h_n \frac{Q_n^2}{2D}\{1 - \beta_n(\otimes) + \alpha_n \beta_n(\otimes)\}^2. \quad (32)$$

By substituting Q_n and Q_j from Eq. (26) into HC, it can be re-written in terms of decision variable Q_1 as follows:

$$HC^M = h_1 Q_1^2 \left\{\frac{1}{2P_{11}} + \frac{\alpha_1 \beta_1(\otimes)}{2P_{21}}\{2 - 2\beta_1(\otimes) + \alpha_1 \beta_1(\otimes)\}\right\} + \sum_{j=2}^{n} \left\{h_j Q_1^2 \left(\prod_{p=1}^{j-1}\{\beta_p(\otimes)(\alpha_p - 1) + 1\}\right)^2 \left\{\frac{1}{2P_{1j}} + \frac{\alpha_j \beta_j(\otimes)}{2P_{2j}}\{2 - 2\beta_j(\otimes) + \alpha_j \beta_j(\otimes)\}\right\}\right\} + h_n \frac{Q_1^2}{2D}\left(\prod_{p=1}^{n-1}\{\beta_p(\otimes)(\alpha_p - 1) + 1\}\right)^2 \{1 - \beta_n(\otimes) + \alpha_n \beta_n(\otimes)\}^2. \quad (33)$$

The screening cost per cycle WC is computed as $WC^M = \sum_{j=1}^{n} d_j Q_j$, where d_j represents the screening cost per item in stage j. Moreover, the reworking cost per cycle RC is $RC = \sum_{j=1}^{n} r_j \alpha_j \beta_j(\otimes) Q_j$, where r_j denotes the rework cost per item in stage j. By using Q_j from Eq. (26), screening and rework costs are rewritten in terms of decision variable Q_1 as follows:

$$WC^M = d_1 Q_1 + Q_1 \sum_{j=2}^{n} \left(d_j \prod_{p=1}^{j-1}\{\beta_p(\otimes)(\alpha_p - 1) + 1\}\right), \quad (34)$$

$$RC^M = r_1 \alpha_1 \beta_1(\otimes) Q_1 + Q_1 \sum_{j=2}^{n} \left(r_j \alpha_j \beta_j(\otimes) \prod_{p=1}^{j-1}\{\beta_p(\otimes)(\alpha_p - 1) + 1\}\right). \quad (35)$$

Therefore, the total cost per cycle is obtained by $PC^M + SC^M + HC^M + WC^M + RC^M$ as follows:

$$TC^M = C_1 Q_1 + \left(\sum_{j=2}^{n} C_j \prod_{p=1}^{j-1}\{\beta_p(\otimes)(\alpha_p - 1) + 1\}\right) Q_1 + \sum_{j=1}^{n} A_j + h_1 Q_1^2 \left\{\frac{1}{2P_{11}} + \frac{\alpha_1 \beta_1(\otimes)}{2P_{21}}\left\{2 - 2\beta_1(\otimes) + \alpha_1 \beta_1(\otimes)\right\}\right\} + \sum_{j=2}^{n} \left\{h_j Q_1^2 \left(\prod_{p=1}^{j-1}\{\beta_p(\otimes)(\alpha_p - 1) + 1\}\right)^2 \left\{\frac{1}{2P_{1j}} + \frac{\alpha_j \beta_j(\otimes)}{2P_{2j}}\{2 - 2\beta_j(\otimes) + \alpha_j \beta_j(\otimes)\}\right\} + h_n \frac{Q_1^2}{2D}\left(\prod_{p=1}^{n-1}\{\beta_p(\otimes)(\alpha_p - 1) + 1\}\right)^2 \{1 - \beta_n(\otimes) + \alpha_n \beta_n(\otimes)\}^2 + d_1 Q_1 + Q_1 \sum_{j=2}^{n} \left(d_j \prod_{p=1}^{j-1}\{\beta_p(\otimes)(\alpha_p - 1) + 1\}\right) - r_1 \alpha_1 \beta_1(\otimes) Q_1 - Q_1 \sum_{j=2}^{n} \left(r_j \alpha_j \beta_j(\otimes) \prod_{p=1}^{j-1}\{\beta_p(\otimes)(\alpha_p - 1) + 1\}\right). \quad (36)$$

Here, the total profit per cycle TP^M can be calculated by $TR^M - TC^M$ as follows:

$$TP^M = vQ_1 \prod_{j=1}^{n} \{\beta_j(\otimes)(\alpha_j - 1) + 1\} + s_1(1-\alpha_1)\beta_1(\otimes)Q_1 + \left(\sum_{j=2}^{n}\left\{s_j(1-\alpha_j)\beta_j(\otimes)\prod_{p=1}^{j-1}\{\beta_p(\otimes)(\alpha_p-1)+1\}\right\}\right)Q_1 - C_1Q_1 - \left(\sum_{j=2}^{n} C_j \prod_{p=1}^{j-1}\{\beta_p \otimes (\alpha_p - 1) + 1\}\right)Q_1 - \sum_{j=1}^{n} A_j - h_1Q_1^2\left\{\frac{1}{2P_{11}} + \frac{\alpha_1\beta_1(\otimes)}{2P_{21}}\{2 - 2\beta_1(\otimes) + \alpha_1\beta_1(\otimes)\}\right\} - \sum_{j=2}^{n}\left\{h_jQ_1^2\left(\prod_{p=1}^{j-1}\{\beta_p(\otimes)(\alpha_p-1)+1\}\right)^2\left\{\frac{1}{2P_{1j}} + \frac{\alpha_j\beta_j(\otimes)}{2P_{2j}}\{2-2\beta_j(\otimes)+\alpha_j\beta_j(\otimes)\}\right\}\right\} - h_n\frac{Q_1^2}{2D}\left(\prod_{p=1}^{n-1}\{\beta_p(\otimes)(\alpha_p-1)+1\}\right)^2\{1-\beta_n(\otimes)+\alpha_n\beta_n(\otimes)\}^2 - d_1Q_1 - Q_1\sum_{j=2}^{n}\left(d_j\prod_{p=1}^{j-1}\{\beta_p(\otimes)(\alpha_p-1)+1\}\right) - r_1\alpha_1\beta_1(\otimes)Q_1 - Q_1\sum_{j=2}^{n}\left(r_j\alpha_j\beta_j(\otimes)\prod_{p=1}^{j-1}\{\beta_p(\otimes)(\alpha_p-1)+1\}\right). \quad (37)$$

Since the defective rate is a grey number, then the total profit is also grey, whose whitened value is given as:

$$T\tilde{P}^M(\otimes) = vQ_1\prod_{j=1}^{n}\left\{\tilde{\beta}_j(\otimes)(\alpha_j-1)+1\right\} + s_1(1-\alpha_1)\tilde{\beta}_1(\otimes)Q_1 + \left(\sum_{j=2}^{n}\left\{s_j(1-\alpha_j)\tilde{\beta}_j(\otimes)\prod_{p=1}^{j-1}\left\{\tilde{\beta}_p(\otimes)(\alpha_p-1)+1\right\}\right\}\right)Q_1 - C_1Q_1 - \left(\sum_{j=2}^{n} C_j\prod_{p=1}^{j-1}\left\{\tilde{\beta}_p(\otimes)(\alpha_p-1)+1\right\}\right)Q_1 - \sum_{j=1}^{n}A_j - h_1Q_1^2\left\{\frac{1}{2P_{11}} + \frac{\alpha_1\tilde{\beta}_1(\otimes)}{2P_{21}}\left\{2-2\tilde{\beta}_1(\otimes)+\alpha_1\tilde{\beta}_1(\otimes)\right\}\right\} - \sum_{j=2}^{n}\left\{h_jQ_1^2\left(\prod_{p=1}^{j-1}\left\{\tilde{\beta}_p(\otimes)(\alpha_p-1)+1\right\}\right)^2\left\{\frac{1}{2P_{1j}} + \frac{\alpha_j\tilde{\beta}_j(\otimes)}{2P_{2j}}\left\{2-2\tilde{\beta}_j(\otimes)+\alpha_j\tilde{\beta}_j(\otimes)\right\}\right\}\right\} - h_n\frac{Q_1^2}{2D}\left(\prod_{p=1}^{n-1}\left\{\tilde{\beta}_p(\otimes)(\alpha_p-1)+1\right\}\right)^2\left\{1-\tilde{\beta}_n(\otimes)+\alpha_n\tilde{\beta}_n(\otimes)\right\}^2 - d_1Q_1 - Q_1\sum_{j=2}^{n}\left(d_j\prod_{p=1}^{j-1}\left\{\tilde{\beta}_p(\otimes)(\alpha_p-1)+1\right\}\right) - r_1\alpha_1\tilde{\beta}_1(\otimes)Q_1 - Q_1\sum_{j=2}^{n}\left(r_j\alpha_j\tilde{\beta}_j(\otimes)\prod_{p=1}^{j-1}\left\{\tilde{\beta}_p(\otimes)(\alpha_p-1)+1\right\}\right), \quad (38)$$

where $\tilde{\beta}_j(\otimes)$ represents the whitened value of the defective rate $\tilde{\beta}_j(\otimes)$. In addition, the whitened value of total profit per unit time $T\tilde{P}U^M(\otimes)$ is calculated by dividing $T\tilde{P}^M(\otimes)$ by the whitened cycle time $\tilde{T}^M(\otimes)$. The cycle time is formulated as T^M which simplifies, by using $t_{Pj} = Q_j/P_{1j}$, $t_{Rj} = \alpha_j\beta_j(\otimes)Q_j/P_{2j}$, and $t_D = \frac{Q_n}{D}\{1-\beta_n(\otimes)+\alpha_n\beta_n(\otimes)\}$, to:

$$T^M(\otimes) = \sum_{j=1}^{n}\frac{Q_j}{P_{1j}} + \sum_{j=1}^{n}\frac{\alpha_j\beta_j(\otimes)Q_j}{P_{2j}} + \frac{Q_n}{D}\{1-\beta_n(\otimes)+\alpha_n\beta_n(\otimes)\}. \quad (39)$$

By using Q_j from Eq. (26), cycle time $T^M(\otimes)$ is re-written in terms of variable Q_1 as follows:

$$T^M(\otimes)=\frac{Q_1}{P_{11}}+\sum\nolimits_{j=2}^{n}\left(\frac{Q_1}{P_{1j}}\prod_{p=1}^{j-1}\{\beta_p(\otimes)(\alpha_p-1)+1\}\right)+\frac{\alpha_1\beta_1(\otimes)Q_1}{P_{21}}+\sum\nolimits_{j=2}^{n}\left(\frac{\alpha_j\beta_j(\otimes)Q_1}{P_{2j}}\prod_{p=1}^{j-1}\{\beta_p(\otimes)(\alpha_p-1)+1\}\right)+\frac{Q_1}{D}\left(\prod_{p=1}^{n-1}\{\beta_p(\otimes)(\alpha_p-1)+1\}\right)\{1-\beta_n(\otimes)+\alpha_n\beta_n(\otimes)\}. \tag{40}$$

The cycle time $T^M(\otimes)$ is also a grey number, and then the whitened cycle time is calculated as:

$$\tilde{T}^M(\otimes)=\frac{Q_1}{P_{11}}+\sum\nolimits_{j=2}^{n}\left(\frac{Q_1}{P_{1j}}\prod_{p=1}^{j-1}\left\{\tilde{\beta}_p(\otimes)(\alpha_p-1)+1\right\}\right)+\frac{\alpha_1\tilde{\beta}_1(\otimes)Q_1}{P_{21}}+\sum\nolimits_{j=2}^{n}\left(\frac{\alpha_j\tilde{\beta}_j(\otimes)Q_1}{P_{2j}}\prod_{p=1}^{j-1}\left\{\tilde{\beta}_p(\otimes)(\alpha_p-1)+1\right\}\right)+\frac{Q_1}{D}\left(\prod_{p=1}^{n}\left\{\tilde{\beta}_p(\otimes)(\alpha_p-1)+1\right\}\right). \tag{41}$$

For simplicity, we define $F_j=\tilde{\beta}_j(\otimes)\ (\alpha_j-1)+1$ for $2\le j\le n$, hereafter. Therefore, the whitened value of total profit per unit time $T\tilde{P}U^M(\otimes)=T\tilde{P}^M(\otimes)/\tilde{T}^M(\otimes)$ is obtained:

$$T\tilde{P}U^M(\otimes)=\{v\prod_{j=1}^{n}F_j+s_1(1-\alpha_1)\tilde{\beta}_1(\otimes)+\left(\sum\nolimits_{j=2}^{n}\left\{s_j(1-\alpha_j)\tilde{\beta}_j(\otimes)\prod_{p=1}^{j-1}F_p\right\}\right)-C_1-\left(\sum\nolimits_{j=2}^{n}C_j\prod_{p=1}^{j-1}F_p\right)-\left(\frac{\sum_{j=1}^{n}A_j}{Q_1}\right)-h_1Q_1\left\{\frac{1}{2P_{11}}+\frac{\alpha_1\tilde{\beta}_1(\otimes)}{2P_{21}}\left\{1-\tilde{\beta}_1(\otimes)+F_1\right\}\right\}-\sum\nolimits_{j=2}^{n}h_j\left\{Q_1\left(\prod_{p=1}^{j-1}F_p\right)^2\left\{\frac{1}{2P_{1j}}+\frac{\alpha_j\tilde{\beta}_j(\otimes)}{2P_{2j}}\left\{1-\tilde{\beta}_j(\otimes)+F_j\right\}\right\}\right\}-h_n\frac{Q_1{F_n}^2}{2D}\left(\prod_{p=1}^{n-1}F_p\right)^2-d_1-\sum\nolimits_{j=2}^{n}\left(d_j\prod_{p=1}^{j-1}F_p\right)-r_1\alpha_1\tilde{\beta}_1(\otimes)-\sum\nolimits_{j=2}^{n}\left(r_j\alpha_j\tilde{\beta}_j(\otimes)\prod_{p=1}^{j-1}F_p\right)\Bigg\}\Bigg/\left\{\frac{1}{P_{11}}+\sum\nolimits_{j=2}^{n}\left(\frac{1}{P_{1j}}\prod_{p=1}^{j-1}F_p\right)+\frac{\alpha_1\tilde{\beta}_1(\otimes)}{P_{21}}+\sum\nolimits_{j=2}^{n}\left(\frac{\alpha_j\tilde{\beta}_j(\otimes)}{P_{2j}}\prod_{p=1}^{j-1}F_p\right)+\frac{1}{D}\prod_{p=1}^{n}F_p\right\}. \tag{42}$$

It can be shown that the derived total profit per unit time $T\tilde{P}U^M(\otimes)$ is concave with respect to the decision variable Q_1, since $\frac{\partial^2 T\tilde{P}U^M(\otimes)}{\partial Q_1^2}\le 0$. Therefore, in order to reach the economic production quantity Q_1^*, we set the first derivative of $T\tilde{P}U^M(\otimes)$ to zero, which results obtained by Eq. (43) as shown in Box II. After calculating Q_1^*, the optimal production quantities of the next stages can be obtained as $Q_j^*=Q_1^*\prod_{p=1}^{j-1}F_p, \forall j=2,3,\ldots,n$. Moreover, in order to reach the upper and lower limits for optimal objective function, the following mathematical models are constructed:

$$\min_{\gamma_j}\underline{TPU^M}(\otimes)=\max_{Q_1}T\tilde{P}U^M(\otimes), \tag{44}$$

subject to:

$$\tilde{\beta}_j(\otimes)=\gamma_j\overline{\beta}_j+(1-\gamma_j)\underline{\beta}_j \qquad 0\le\gamma_j\le 1, \tag{45}$$

and:

$$\max_{\gamma_j}\overline{TPU}^M(\otimes)=\max_{Q_1}T\tilde{P}U^M(\otimes), \tag{46}$$

subject to:

$$Q_1^* = \left[\frac{2\left(\sum_{j=1}^{n} A_j\right) D}{h_1 D \left\{\frac{1}{P_{11}} + \frac{\alpha_1 \tilde{\beta}_1(\otimes)}{P_{21}} \left\{1-\tilde{\beta}_1(\otimes)+F_1\right\}\right\} + D\sum_{j=2}^{n}\left\{h_j\left(\prod_{p=1}^{j-1} F_p\right)^2 \left\{\frac{1}{P_{1j}} + \frac{\alpha_j \tilde{\beta}_j(\otimes)}{P_{2j}}\left\{1-\tilde{\beta}_j(\otimes)+F_j\right\}\right\}\right\} + h_n F_n^2 \left(\prod_{p=1}^{n-1} F_p\right)^2}\right]^{\frac{1}{2}} \quad (43)$$

Box II

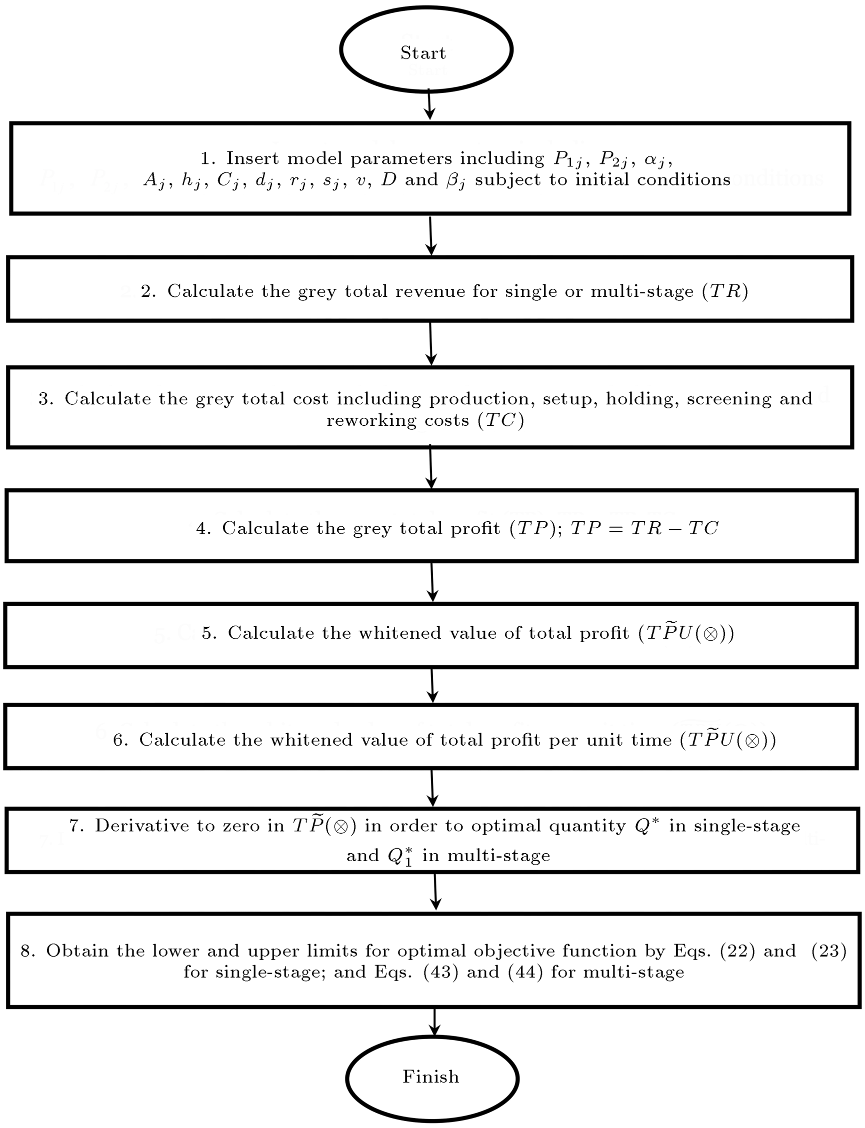

Figure 4. The flowchart of proposed approach.

$$\tilde{\beta}_j(\otimes) = \gamma_j \overline{\beta}_j + (1-\gamma_j)\underline{\beta}_j,$$

where $\underline{TPU^M}(\otimes)$ and $\overline{TPU}^M(\otimes)$ represent the lower and upper limits of the objective function (total profit per unit time) as grey numbers. As mentioned earlier, GP aims to recognize the lower and upper limits of the objective function, TPU^M in our model, caused by grey input parameters. Note that the first model is a minimax problem (which can be solved via conventional minimax commands in MATLAB). In contrast, the second one is a maximization problem with two variables Q_1 and γ_j (which can be simply solved by nonlinear optimization commands in MATLAB). A flowchart from the initial stages in order to obtain the optimum solutions has been given in Figure 4.

Table 2. The characteristics of numerical examples.

Parameters	Multi-stage $j=1$	Multi-stage $j=2$	Multi-stage $j=3$	Single stage
Production rate P_{1j}	1500	2500	2000	3000
Rework rate P_{2j}	3500	3000	4000	4500
Rate of reworkable items α_j	0.80	0.70	0.85	0.90
Setup cost A_j	100	50	200	150
Holding cost h_j	10	8	4	12
Production cost C_j	50	30	80	40
Screening cost d_j	30	35	20	25
Reworking cost r_j	10	5	8	12
Scrapped item unit price s_j	50	25	75	50
Perfect product unit price v	500	500	500	450
Demand rate D	800	800	800	1000
Defective rate β_j (grey number)	[0.10, 0.15]	[0.08, 0.12]	[0.04, 0.08]	[0.10, 0.14]

5. Examples and analysis

In order to investigate the performance of the suggested models, two numerical examples are discussed, with one and three stages. Table 2 shows the characteristics of these numerical examples. Then, we analyze sensitivity by changing the value of input parameters to assess the outputs under various inputs.

It is assumed that the whitened value of defective rates is the middle point of upper and lower limits. We first investigate the feasibility conditions for two examples. To this end, we should evaluate three conditions: (i) $P_1 > D$, (ii) $P_2 > D$, and (iii) $\beta(\otimes)$ for the single-stage example. The first two conditions are deterministic, while the third one is a grey one. Using the data of the first example, we have (i) $P_1 = 3000 > 1000 = D$ and (ii) $P_2 = 4500 > 1000 = D$. On the other hand, the right-hand equation of the third condition is $1 - 1000/3000 = 0.67$. As can be seen, both upper and lower limits of defective rate β are less than 0.67, which ensures the feasibility of the first example. We should check out three conditions: (i) $P_{1j} > D$, (ii) $P_{2j} > D$ and (iii) $\beta_j(\otimes)$ for the second example. By doing so, we can simply find that all of the conditions are satisfied for both examples. The first two conditions are satisfied since $P_{11} = 1500 > 800 = D$, $P_{21} = 3500 > 800 = D$, $P_{12} = 2500 > 1200 = D$, $P_{22} = 3000 > 1200 = D$, $P_{13} = 2000 > 500 = D$, and $P_{23} = 4000 > 500 = D$. Obviously, the third condition is also true for the second example since both the upper and lower limits of the defective rate are much less than the production rate in all three stages.

Utilizing the characteristics of the first example, the whitened values of optimal production quantity are obtained by Eq. (22) as 307.2182, and then the total profit per unit time is calculated as 381802.5. Moreover, the whitened production, rework, and depletion periods are obtained as 0.1024, 0.0074, and 0.1938, respectively. To obtain the possible upper and lower limits for optimal total profit per unit, Eqs. (23) and (24) are calculated for this example. The obtained results are $\underline{TPU}(\otimes)$ and $\overline{TPU}(\otimes)$. Moreover, the whitened values of optimal production quantity of the first stage, for the second example, are obtained by Eq. (43) as 324.0478, and then the production quantities of the second and third stages are obtained as 317.5668 and 309.9452. Moreover, the total profit per unit time is calculated as 82385.93 with lower and upper limits of 789754.58 and 865741.89.

We will evaluate the relation between the optimal solution and the input grey parameters. In the real world, changes in the parameters of the model are inevitable, and the parameters fluctuate. Moreover, changes in the parameters may have a significant impact on the values of decision variables and the objective function. In this research, we study the impact of defective rates as grey numbers in the problem on the optimal total profit per unit time. Table 3 presents the results for the first example, and Figures (5)–(7) depict the variations of the outputs. According to the results, the defective rate negatively impacts the optimal production, rework, and depletion periods. In addition, the production quantities and total profit decrease when the defective rate of the

Table 3. The impact of grey defective rate on the optimal solution (first example).

Variation on whitened β	t_P	t_R	t_D	Q^*	TPU
−50%	0.1074	0.0039	0.2090	322.1544	382509.47
−40%	0.1064	0.0046	0.2060	319.2700	382368.63
−30%	0.1054	0.0053	0.2029	316.3267	382227.54
−20%	0.1044	0.0060	0.1999	313.3321	382086.18
−10%	0.1034	0.0067	0.1968	310.2936	381944.51
0%	0.1024	0.0074	0.1938	307.2182	381802.51
+10%	0.1014	0.0080	0.1907	304.1124	381660.14
+20%	0.1003	0.0087	0.1877	300.9827	381517.39
+30%	0.0993	0.0093	0.1846	297.8350	381374.22
+40%	0.0982	0.009	0.1816	294.6749	381230.62
+50%	0.0972	0.0105	0.1786	291.5077	381086.57

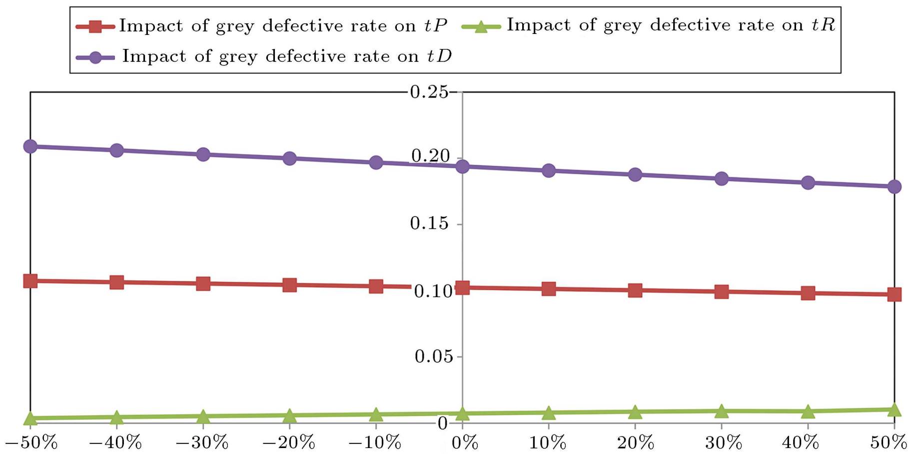

Figure 5. The impact of grey defective rate on production rework and depletion periods (first example).

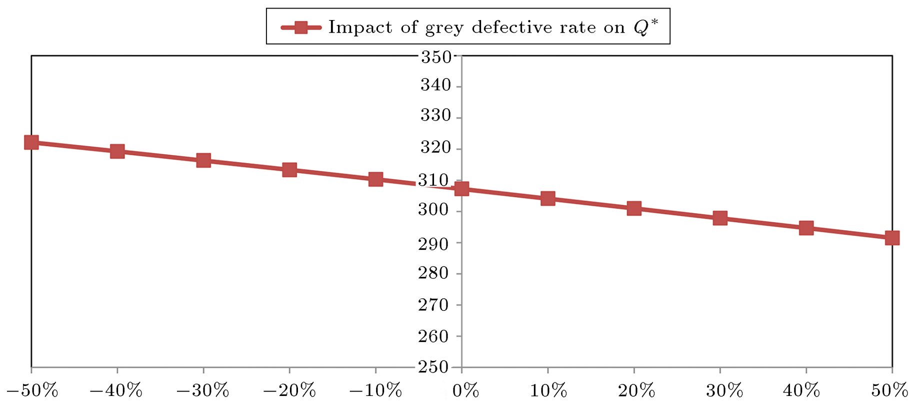

Figure 6. The impact of grey defective rate on optimal production quantity (first example).

process increases. This is a reasonable behavior in manufacturing environments.

These results are also true for the second example. Tables 4–6 present the impact of grey defective rates on the optimal solution of the second example. Moreover, Figure 8 shows the total profit per unit time under a variation of grey defective rates. As an observation, the production quantities at a special stage show a very smooth change when the defective rate(s) of the next stages change. For example, when the defective rate of the third stage increases, the production quantities at the first stage remain unchanged (very smooth change).

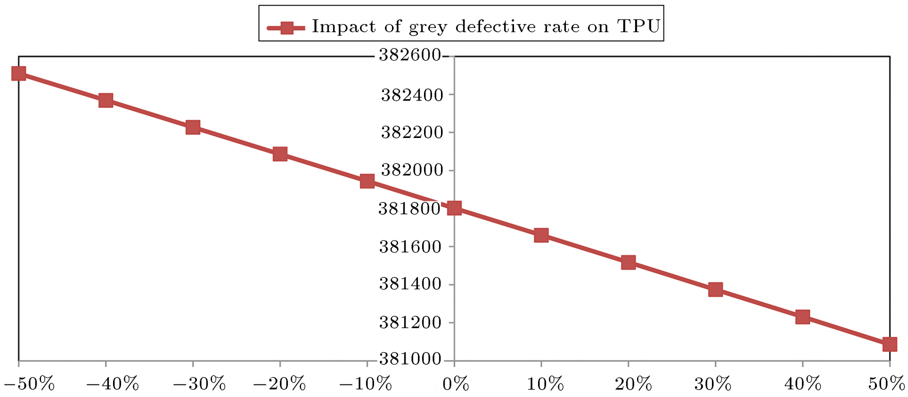

Figure 7. The impact of grey defective rate on total profit per unit time (first example).

Table 4. The impact of first-stage grey defective rate on the optimal solution (second example).

Variation on whitened β_1	Q_1^*	Q_2^*	Q_3^*	T	TPU
−50%	324.0484	319.9978	310.3979	0.8998	83415.31
−40%	324.0482	319.1875	309.6118	0.8990	83210.07
−30%	324.0480	318.3771	308.8258	0.8982	83004.52
−20%	324.0478	317.5668	308.0398	0.8974	82798.64
−10%	324.0475	316.7565	307.2538	0.8966	82592.45
0%	324.0473	315.9461	306.4678	0.8958	385.93
+10%	324.0471	315.1358	305.6818	0.8950	82179.09
+20%	324.0469	314.3255	304.8957	0.8942	81972.38
+30%	324.0467	313.5152	304.1097	0.8934	81764.43
+40%	324.0465	312.7049	303.3237	0.8926	81556.61
+50%	324.0463	311.8946	302.5377	0.8918	81348.46

Table 5. The impact of second-stage grey defective rate on the optimal solution (second example).

Variation on whitened β_2	Q_1^*	Q_2^*	Q_3^*	T	TPU
−50%	324.0473	315.9461	311.2069	0.9004	83974.49
−40%	324.0473	315.9461	310.2591	0.8995	83658.04
−30%	324.0473	315.9461	309.3113	0.8985	83340.96
−20%	324.0473	315.9461	308.3634	0.8976	83023.25
−10%	324.0473	315.9461	307.4156	0.8967	82704.91
0%	324.0473	315.9461	306.4678	0.8958	82385.93
+10%	324.0473	315.9461	305.5199	0.8949	82066.32
+20%	324.0473	315.9461	304.5721	0.8940	746.06
+30%	324.0473	315.9461	303.6242	0.8930	425.16
+40%	324.0473	315.9461	302.6764	0.8921	81103.61
+50%	324.0473	315.9461	301.7285	0.8912	80781.42

6. Implications and conclusions

From a practical point of view, it is not necessarily true that all of the produced items are of perfect quality every time, and the production processes are not necessarily of perfect quality. Consequently, the production of defective items is inevitable even in high-quality and advanced manufacturing units. Usually, a certain fraction of the items are defective due to poor quality of the production process or raw materials. From a practical point of view, the proposed model in this paper tried to address some more realistic conditions that usually happen in real-world production systems. Moreover, in a multi-stage production system, products are transferred from one stage to the next, and every stage may produce a fraction of defective products. A

Table 6. The impact of third-stage grey defective rate on the optimal solution (second example).

Variation on whitened β_3	Q_1^*	Q_2^*	Q_3^*	T	TPU
−50%	324.0473	315.9461	306.4678	0.8955	83131.53
−40%	324.0473	315.9461	306.4678	0.8956	82982.37
−30%	324.0473	315.9461	306.4678	0.8956	82833.23
−20%	324.0473	315.9461	306.4678	0.8957	82684.11
−10%	324.0473	315.9461	306.4678	0.8957	82535.01
0%	324.0473	315.9461	306.4678	0.8958	82385.93
+10%	324.0473	315.9461	306.4678	0.8959	82236.87
+20%	324.0473	315.9461	306.4678	0.8959	82087.83
+30%	324.0473	315.9461	306.4678	0.8960	81938.80
+40%	324.0473	315.9461	306.4678	0.8960	81789.79
+50%	324.0473	315.9461	306.4678	0.8960	81640.80

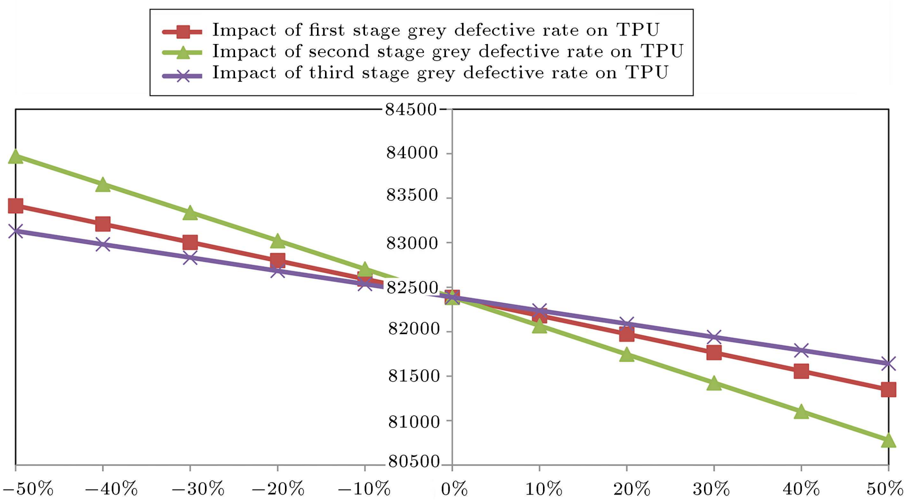

Figure 8. The impact of first, second and third stage grey defective rates on total profit per unit time.

production stage is affected by a number of inevitable undesirable factors, which make it rarely possible for a production machine/production system to produce perfect quality items every time. In fact, in some defective production systems, rework is usually used for imperfect items at every stage. In the proposed model, the perfect items go to the next stage and finally become the finished product at the final stage, and then go under consumption. To address the practical circumstances, the proposed model considers that a fraction of imperfect items are of unacceptable quality and should be disposed of as scrap at each stage. The decision on the quality of a produced item is made by a quality control process in manufacturing companies.

This paper proposed single and multi-stage production-inventory systems with defective manufacturing processes, possible rework, and scrap items. The feasibility conditions are extracted for both models to avoid shortage. It is also assumed that the defective rate is an uncertain parameter. The grey systems theory, as a mathematical tool to address uncertain information, is utilized to model the random defective rate via grey programming in both models. From a theoretical point of view, this is a new kind of mathematical programming problem with its own challenges and complications. Since the proposed production-inventory problems are unconstrained nonlinear optimization problems, we employ grey nonlinear programming principles to address the complexities that occur in the solving process. The total profit per unit time is derived from both models in terms of whitened random defective rate, and then the optimal production quantities are calculated for each model separately. Two numerical examples are presented and studied to assess the impact of grey parameters on optimal solutions. Grey programming generates feasible ranges of decision variables and objective functions using the interpretation of the grey solutions and grey input parameters; it has lower computational efforts compared with the existing methods, and then is applicable to practical problems, and does not require distribution information (like probability-based approaches) or membership function (like fuzzy-based approaches) for input parameters, since interval numbers are acceptable for the input parameters.

As an opportunity for future research, it can be considered to incorporate other uncertain parameters into proposed models like demand rate or reworkable item rate to reach a more practical problem and address it via grey programming. Moreover, it is interesting to employ other approaches like stochastic programming to handle the uncertainties in our models.

Nomenclature

J	Stages of the manufacturing system
$\beta^S(\otimes)$	A grey number for the defective rate of the single-stage system
P_1^S	The production rate in the single-stage system
P_2^S	The rework process rate in the single-stage system
α	The rate of reworkable items rate in the single-stage system
r^S	The cost of the reworked items
D^S	The demand for the single-stage system
v^S	The sale price of the perfect item in the single-stage system
s^S	The sale price of scrapped items in the single-stage system
C^S	The production unit cost in the single-stage system
SC^S	The total setup cost in the single-stage system equal to A^S
h^S	The holding unit cost in the single-stage system
d^S	The screening unit cost in the single-stage system
P_{1j}	The production rate for the stage jth in the multi-stage system
P_{2j}	The rework process rate for stage jth in the multi-stage system
$\beta_j(\otimes)$	A grey number for defective rate for stage jth in the multi-stage system
α_j	The rate of reworkable items for stage jth in the multi-stage system
t_{Pj}^S	The production interval for stage jth in the multi-stage system
α_j	The rate of reworkable items for stage jth in the multi-stage system
t_{Dj}^S	The depletion interval for stage jth in the multi-stage system
D	The demand for the multi-stage system
ν	The sale price of the perfect item in the multi-stage system
s_j	The sale price of scrapped items for stage jth in the multi-stage system
C_j	The production unit cost for stage jth in the multi-stage system
SC^S	The total setup cost in the multi-stage system
A_j	The setup cost for stage jth in the multi-stage system
h_j	The holding unit cost for stage jth in the multi-stage system
d_j	The screening unit cost for stage jth in the multi-stage system
r_j	The cost of the reworked item for stage jth in the multi-stage system
Q_j	The production quantity for stage jth in the multi-stage system
$I_{\max}^S$	The maximum inventory level for the single-stage system
Q^S	The production quantity in the single-stage system
TP^S	The total profit in the single-stage system
TC^S	The total cost of the single-stage system
TR^S	The total revenue in the single-stage system
PC^S	The total production cost for single-stage system
HC^S	The total holding cost for the single-stage system
S_I^S	The area below the first period (production) in the single-stage system
S_{II}^S	The area below the second period (rework) in the single-stage system
S_{III}^S	The area below the third period (depletion) in a single-stage system
WC^S	The total cost of screening in the single-stage system
RC^S	The total cost of rework in the single-stage system
TC^S	The total cost for the single-stage system
$T\tilde{P}^S(\otimes)$	The whitened value of total profit in the single-stage system

$T\tilde{P}U^S(\otimes)$	The whitened value of total profit per unit time in the single-stage system
$\tilde{T}^S(\otimes)$	The whitened inventory cycle time
t_P^S	The production interval in the single-stage system
t_R^S	The rework interval in the single-stage system
t_D^S	The depletion interval in the single-stage system
Q^{S^*}	The optimal quantity in the single-stage system
TP^M	The total profit in the multi-stage system
TC^M	The total cost of the multi-stage system
TR^M	The total revenue in the multi-stage system
PC^M	The total production cost for the multi-stage system
HC^M	The total holding cost for the multi-stage system
S_{Ij}	The area below the first period (production) for stage jth in the multi-stage system
S_{IIj}	The area below the second period (rework) for stage jth in the multi-stage system
S_{IIIj}	The area below the third period (depletion) for stage jth in the multi-stage system
WC^M	The total cost of screening the multi-stage system
RC^S	The total cost of rework in the single-stage system
TC^S	The total cost for the single-stage system
$T\tilde{P}^M(\otimes)$	The whitened value of total profit in the multi-stage system
$T\tilde{P}U^M(\otimes)$	The whitened value of total profit per unit time in the multi-stage system
$\tilde{T}^M(\otimes)$	The whitened inventory cycle time
t_{Pj}	The production interval for stage jth in the multi-stage system
t_{Rj}	The rework interval for stage jth in the multi-stage system
t_{Dj}	The depletion interval for stage jth in the multi-stage system
${Q_j}^*$	The optimal quantity for stage jth in the multi-stage system
I_{1j}	The inventory level at the start of the rework period for stage jth in the multi-stage system
I_{2j}	The inventory level at the end of the rework period for stage jth in the multi-stage system

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