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Robust bi-objective operating rooms scheduling problem regarding the shared resources

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Abstract. In recent years, many efforts have been made to propose different strategies for enhancing the scheduling and planning of Operating Rooms (ORs). Efficient planning and scheduling of ORs is a complex task since it must account for the availability of human resources, medical equipment, and medication required for each surgery and they are often shared between different ORs. This paper proposes a mathematical approach to enhance the management of OR resources. It presents a bi-objective robust optimization approach for scheduling surgeries in the ORs and recovery room, regarding the uncertainty of the surgery time, uncertainty of hospitalization time in the recovery room, and shared resources. The first objective function aims to minimize the maximum completion time of the surgeries, while the second one minimizes the sum of the earliness-tardiness of the surgical operations. The suggested approach utilizes the multi-choice goal programming approach with utility function to solve the proposed model. The proposed approach is applied to a real case in the Shahid Beheshti hospital, Babol, Iran. The obtained results show that the suggested bi-objective robust optimization approach can enhance OR scheduling and should be designed into a decision support system for OR management.

1. Introduction

Operating Room (OR) is one of the most vital but expensive parts of a hospital. Due to the importance of surgical services in the hospital, the OR is known as the heart of the hospital. In fact, it allows for surgical services for patients who require an offensive operation (surgery). Proper scheduling of surgical operations and other related limited resources is quite important for the stakeholders (managers, staff, and patients) to improve the effectiveness of staff and resources and to increase satisfaction in serving patients and satisfaction of the staff, maintain capital, and reduce costs [1]. The efficient scheduling of the sections before and after the ORs can reduce the patients' waiting and hospitalization time [2]. This reduction further contributes to both reducing costs and increasing the satisfaction of patients and employees [3]. OR scheduling follows patient prioritization; accordingly, a patient in an emergency condition is more important and should be surgically activated sooner [4]. Meanwhile, the availability of limited resources and the type of surgery are important factors that must be scheduled in OR surgery planning [5].

A priori undetermined factors of surgeries including patients' conditions, duration of surgery, and hospitalization time in the recovery room are not

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deterministic and they should be considered in the planning and scheduling of the ORs [6]. If insufficient time is allocated for planning, subsequent surgeries may not commence as scheduled, leading to delays, dissatisfaction among patients and staff, and increased overtime costs for hospital management in ORs. Conversely, if more time than necessary is allotted for a surgery, it may result in ORs being unused, causing the next patient and staff to be unprepared for the subsequent surgery. Besides, availability of some equipment pieces required for specific surgeries needs further consideration by the management. Due to the high costs of equipment, only a limited number of pieces are available and must be shared between ORs and different surgeries. Hereafter, they are referred to as shared resources. If a surgery necessitates a shared resource, other surgeries that require the same resources may need to wait until the current surgery is completed. Consequently, neglecting the use of shared resources in the planning and scheduling of ORs leads to inefficiency in the daily ORs plan, idleness of the ORs, and overtime costs. The efficient management of ORs consequently requires a highly adaptable solution to account for uncertainties and resource limitations.

This study considers the described OR-scheduling problem and accounts for different challenges in their management, such as uncertain surgery time in the ORs, uncertain time of hospitalization in the recovery room, and the use of shared resources. In the considered case study, two types of surgeries, including orthopedic surgeries and some general surgeries, share radiology equipment during the procedure. In the case, the surgery time and hospitalized time of each patient cannot be considered fixed or pre-determined. Rather, these are dependent on factors such as the general health condition of the patient, skill of surgeons and OR staff, and the availability of needed equipment. Consequently, we consider surgery time and hospitalized time as uncertain parameters. In different service centers, it is quite important to consider the costumers' and managers' preferences, simultaneously. For this purpose, two objective functions are considered. The first objective seeks to minimize the maximum completion time of surgical operation and minimize the OR-related costs. Put simply, the operating costs of a hospital increase as the length of time an OR is utilized increases. This includes expenses such as staff salaries, operating equipment costs, and maintenance and repair costs. To minimize these expenses, it is necessary to reduce the maximum completion time for surgical operations. The second objective function, which aims to minimize the sum of tardiness-earliness time, takes into account patient satisfaction. To achieve this, a bi-objective robust Mixed-Integer Linear Programming (MILP) model is proposed. A Multi-Choice Goal Programming (MCGP) approach with a utility function is used to handle the bi-objective model.

The rest of the study is organized as follows. Section 2 discusses existing findings on the scheduling problem for ORs. Section 3 provides the problem description, assumptions, and the mathematical model developed for the research problem. The robust counterpart of the mathematical model and MCGP approach with utility function are introduced in Section 3. Section 4 represents the computational results to evaluate the performance of the proposed approach and a sensitivity analysis. Finally, Section 5 presents conclusions from the research and future research.

2. Literature review

Due to the rapid advancement of the healthcare industry and the critical nature of scheduling for ORs, numerous studies have been conducted in recent years focusing on the planning and scheduling of ORs. This section reviews relevant findings in this context. Azadeh et al. attempted to investigate the scheduling of emergency department patients based on their treatment priority. They formulated the problem as a flexible open shop problem and developed a MILP model and a genetic algorithm to minimize patients' total waiting time [7]. Li et al. developed a Multi-Objective Integer Linear Programming (MOILP) model to optimally schedule elective surgeries regarding the availability of ORs and surgeons. The selected objective functions comprised minimization of the number of patients waiting for service, underutilization of OR time, maximum expected number of patients in the recovery unit, and expected range of patients in the recovery unit. They developed two Goal Programming (GP) approaches to this problem [8]. Denton et al. parented a stochastic optimization approach to assign surgeries to the ORs. Objective function seeks to minimize the fixed cost of opening ORs and a variable cost of overtime. They proposed a simple heuristic method to solve the problem [9]. Rachuba and Werners considered ORs scheduling regarding uncertain surgery durations and patients' emergency arrivals. In order to consider stakeholders' different objectives, they proposed a multi-objective robust mixed-integer mathematical model [10]. Jebali and Diabat developed a two-stage chance-constrained programming model for ORs scheduling that considered uncertain surgery time, random patient length of stay in the ICU, and random reserved resources for the emergency cases. The selected objective function aims to minimize patients' costs, OR utilization costs, and penalty costs upon exceeding ICU capacity. They proposed a Sample Average Approximation (SAA) algorithm to solve the model [11].

Liu et al. developed a two-step MILP model and an SAA approach for the ORs scheduling problem in which the surgery time is as an uncertain parameter. The proposed approach seeks to increase resource utilization, reduce the ORs cost, and improve surgeons' satisfaction [12]. Molina-Pariente et al. studied a stochastic OR scheduling problem which aimed to minimize the undertime and overtime costs of the ORs and the cost of exceeding the system capacity. They considered different uncertain parameters such as surgery time, surgeons' capacity, and arrivals of emergency surgeries. They proposed a Monte Carlo optimization-based approach, in which the iterative greedy local search method was combined with Monte Carlo simulation [13]. Neyshabouri and Berg studied a two-stage robust optimization model to handle the uncertainty in surgery time and length of stay in the surgical ICU. They developed a column-and-constraint generation approach to generate optimal solutions [14]. Liu et al. developed an iterative auction mechanism to maximize the overall social welfare of patients in the scheduling problem for ORs. They also took into account the eligibility constraints of the ORs [15]. Ravnskjær Kroer et al. developed a stochastic model in which surgery times varied and the emergency patients' arrivals were not deterministic. The proposed approach generated robust OR schedules that minimized overtime work and utilized unused capacity [16]. Koppka et al. dealt with a mathematical model that assigned the ORs available time to enhance the overall performance. The selected objective function maximized the likelihood of a perfect day going by without overtime or cancellations. They considered surgery time and the number of patients during the day to be uncertain parameters [17]. Moosavi and Ebrahimnejad proposed a multi-objective mathematical model for upstream and downstream units of ORs. They suggested a robust counterpart of the mathematical model to consider uncertain parameters, including surgery time, emergency demand, and length of stay. They also developed an MIP-based local search neighborhood approach to solve the problem [18]. Sagnol et al. considered a parallel-machine scheduling problem to formulate the problem of allocating ORs to surgeries, such that surgery time followed a lognormal distribution. They solved the robust counterpart of the proposed model using a cutting-plane approach [19]. Kamran et al. considered patients' allocation to available OR blocks. The authors proposed multi-objective two-stage stochastic and two-stage chance-constrained stochastic programming models. They solved the proposed models by using the SAA approach and Bender's decomposition method [20]. Hamid et al. developed a new and comprehensive MILP model to consider inpatient surgeries in ORs scheduling. They developed two metaheuristic algorithms, namely NSGA-II and MOPSO, to achieve the Pareto solutions and applied the PROMETHEE-II approach to select the best solution among the Pareto solutions [21]. Lin and Chou focused on multifunctional ORs in ORs scheduling problem. The selected objective functions included maximizing ORs utilization, minimizing the overtime cost, and minimizing the wasting cost of unused time. To cope with the problem, they developed some simple heuristic methods, hybrid genetic algorithm, and elite search approach for this problem [22].

Vali-Siar et al. developed an MILP model to plan and schedule ORs with respect to the uncertainty in surgery and recovery duration. They proposed a new heuristic approach to minimize the tardiness in surgeries, as well as over and idle times [23]. Silva and De Souza addressed ORs scheduling problem with comment resources. They considered uncertain surgery times and patients' arrival. They proposed an approximate dynamic programming approach to minimize the total expected cost. The experimental results show that the proposed approach can reduce the total expected cost, significantly [24]. Zhang et al. addressed ORs scheduling problem regarding the downstream resources capacity constraints. They proposed a stochastic programming model, in which surgery duration and length-of-stay were as uncertain parameters. They developed column-generation-based heuristic methods to solve the research problem [25]. Akbarzadeh et al. studied the surgical case in which operation room planners sought to make a balance between the capacity and demand. They developed a three-phase column generation-based heuristic to generate a feasible solution and it was improved via local branching [26]. Nasiri et al. proposed a mathematical model to select and assign elective surgeries on a particular day. They proposed a fuzzy robust optimization approach to maximize the number of surgeries using fixed resources, minimize the total fixed and overtime costs of the ORs, and minimize the maximum completion time of ORs [27]. Najjarbashi and Lim proposed a risk-based solution approach for the ORs scheduling problem regarding the Conditional Value-at-Risk (CVaR) concept. They developed a stochastic MILP model to minimize the CVaR of over- and idle time costs [28]. Atighehchian et al. studied ORs scheduling problem concerning uncertain duration of surgeries in a multi-resource environment. They presented a two-stage stochastic mixed-integer programming model to minimize the ORs' idle and over times [29]. Marchesi et al. proposed a two-stage stochastic programming model with constant recourse to solve ORs staffing and scheduling problem. They developed possible realization scenarios to deal with the demand uncertainty by the SSA algorithm [30]. Barrera et al. proposed a stochastic dynamic mathematical model for the ORs scheduling problem to minimize the cost of referrals to the private sector. They developed a heuristic approach to achieve near-optimal solutions in a reasonable time [31]. Bovim et al. proposed a two-stage stochastic optimization model combined with a simulation-optimization approach to schedule ORs. They considered arrivals of the emergency patients and surgery duration as uncertain parameters [32]. Khaniyev et al. studied next-day ORs scheduling problem regarding the uncertain surgery durations to minimize the weighted sum of the ORs' idle and over times and expected patient waiting times. They proposed some simple heuristics motivated by a real situation to find near-optimal solutions [33]. In order to convince the readers, some of the main findings on the ORs scheduling problem are summarized in Table 1.

2.1. Research contributions

Based on our discussion of the existing research landscape on the topic of ORs scheduling problem and according to the findings summarized in Table 1, the present research makes the following contributions. A central contribution presented in this study is the consideration of shared resources for ORs scheduling. However, clinical practice highlights the relevance of shared resources as a central constraining factor in surgery planning. Thus, embedding it in the optimization approach adjusts existing models to practical realties faced in hospital management and surgery. Based on the literature discussion, this crucial aspect is missing in existing methodological approaches. This is of specific relevance to decision-making concerning critical surgeries in emergencies.

This paper considers two different objective functions for addressing the ORs scheduling problem, including the maximum completion time of the surgical operations and the sum of the tardiness and earliness times. The first objective is of relevance to hospital management to reduce the maximum completion time and consequently, reduce ORs costs. The second objective concerns patients since both the positive deviation (tardiness) and negative deviation (earliness) from the scheduled surgery affect patient satisfaction as well as corresponding costs. As can be seen in the literature, a highly regarded approach to the multi-objective ORs scheduling problem is the GP approach [4,8,15]. In the GP, an aspiration level is defined for each objective function which is determined according to decision-makers' opinions. One of the limitations of utilizing GP to handle multi-objective models is that the preference structure of the decision-makers is not easily considered, which is far from reality. The application of utility function can tackle this difficulty. In this study, for the first time, the MCGP approach with utility function is used to handle the bi-objective ORs scheduling problem. As its key advantage, this approach considers decision-makers' preferences, which affect their approach to maximize utility.

One of the main concerns about the ORs scheduling problem is uncertainty of the input parameters. There are various approaches to cope with this issue, such as stochastic programming, robust optimization approach, and fuzzy approach. Unlike the stochastic programming, the robust optimization approach does not need any information about the probability distribution of the uncertain data. Due to the nature of the data of the ORs scheduling problem, it is quite difficult or impossible to achieve the probability distribution of data in the ORs. Thus, we considered a robust optimization approach in this research. Based on the literature pieces reviewed, the robust optimization approach has been rarely studied in the ORs scheduling problem and the approach of Bertsimas and Sim [34] has not been studied in previous research.

3. Methods

3.1. Problem statement

Suppose that there are several patients that must be assigned to ORs according to a patient allocation matrix. This matrix indicates the allocation of patients to respective surgery rooms and shows which OR is equipped for the patient's surgery. Each OR requires preparation at the beginning of the operation. Accordingly, in between two successive operations, a specific amount of time is allocated to washing and reheating. After surgery, patients are immediately transferred to the recovery room. In this research, a number of shared resources are considered such that if a surgery uses a shared resource, other surgeries cannot use it during the present surgery. According to the problem definition, the following assumptions are made in this research:

- The number of ORs and recovery beds is less than the number of patients;
- Each operation is assigned a weight, which indicates the surgery priority. In this way, the higher the assigned weight to the surgery, the higher the urgency of the surgery;
- There is a time interval for starting a surgery due to the type of surgery and ORs limitation;
- Patients should be assigned to the ORs according to the allocation matrix;
- Each patient will be transferred to the recovery room immediately after surgery in OR;
- Patients who need shared sources are identified by a shared resource matrix.

3.2. Mathematical model

In this section, a bi-objective MILP model is presented to schedule the surgeries in the ORs and recovery room. For this purpose, the following notations are used in the model:

Table 1. Categorization of the related research.

Reference	Year	Objective function(s)	Uncertainty	Shared resources	Solving method
Rachuba and Werners [10]	2017	Minimizing waiting time, staff overtime, and the number of deferrals	Robust mixed-integer programming		Multi-objective robust mixed-integer mathematical model
Liu et al. [12]	2017	Maximizing the social welfare of patients, maximize surgeons' preference values, and minimize the revelation of surgeons' private information	Stochastic mixed-integer programming		Sample Average Approximation (SAA) algorithm
Neyshabouri & Berg [14]	2016	Minimizing total costs	Two-stage robust optimization		Column-and-constraint generation method
Molina-Pariente [13] et al.	2018	Minimizing the total expected cost of the surgical resources	Stochastic mixed-integer programming		Monte Carlo simulation & Iterative greedy local search method
Sagnol et al. [19]	2018	Minimizing the fixed cost and the overtime cost	Robust optimization approach		Exact solution methods
Kamran et al. [20]	2018	Minimizing waiting time of patients, tardiness, cancellation, block overtime, and the number of surgery days of surgeons	Two-stage stochastic programming & Two-stage chance-constrained stochastic programming		SAA & Benders decomposition
Nasiri et al. [27]	2019	Maximizing the number of surgeries, minimize the total fixed and overtime costs, and the maximum of completion time of ORs	Fuzzy robust optimization		Multi-Objective Goal Programming (MOGP) approach
Najjarbashi & Lim [28]	2019	Minimizing the CVR of over and idle time costs	Stochastic mixed-integer programming		CPLEX
Atighehchian et al. [29]	2019	Sum of the expected ORs over and idle time costs	Two-stage stochastic programming		L-shaped algorithm
Marchesi et al. [30]	2020	Minimizing the total number of waiting patients	Two-stage stochastic programming		SAA
Silva and De souza [24]	2020	Minimizing total expected cost	Stochastic dynamic programming		Dynamic programming
Zhang et al. [25]	2020	Minimizing patient-related and hospital-related costs	Stochastic programming		Column-generation-based heuristic
Barrera et al. [31]	2020	Minimizing the cost of referrals to the private sector	Stochastic dynamic programming		Heuristic algorithms
Khaniyev et al. [33]	2020	Minimizing the weighted sum of the room idle and overtime, and expected patient waiting times	Scenario-based programming		Heuristic algorithms
Present research	2021	Minimizing the maximum completion time and the sum of the earliness and tardiness time of the surgical operations	Robust optimization approach	√	Multi-Choice Goal Programming (MCGP) with utility function

Index

r	Index of ORs ($r = 1, 2, \ldots, R$)
a, b	Index of patients ($a, b = 1, 2, \ldots, A$)
t	Index of beds in the recovery room ($t = 1, 2, \ldots, N$)

Parameters

M	Big positive number
R	Number of ORs
A	Number of patients
N	Number of beds in the recovery room
P^1_{ar}	Surgery time of patient a in OR r
ST_r	Initial preparation time of OR r
STT_r	Preparation time between two successive surgeries in OR r
P^2_{ar}	Time of hospitalization in the recovery room for patient a on bed t
Per_{ar}	Allocation matrix; 1 if patient a is permitted to assign OR r, 0 otherwise
fac_a	Shared resources matrix; 1 if patient a needs the shared resource, 0 otherwise
$[l_a, u_a]$	The time interval for starting the surgery of patient a
w_a	The weight of surgery of patient a
SST_t	Preparation time of bed t in the recovery room
AR	Number of available shared resources

Decision variables

v_{ar}	1 if the patient a is assigned to operating room r; 0 otherwise
z_{abr}	1 if the patient a is assigned to OR r before patient b; 0 otherwise
c^1_a	Completion time of surgery for patient a in the OR
c^2_a	Completion time of recovery operation for patient a in the recovery room
ρ_{ab}	1 if surgery times of patient a and patient b have overlap; 0 otherwise
O_{at}	1 if patient a is assigned to bed t in the recovery room; 0 otherwise
S_{abt}	1 if patient b is assigned to bed t after patient a in the recovery room; 0 otherwise
$startT_a$	Start time of surgery for patient a
T_a	Tardiness of surgery for patient a
E_a	Earliness of surgery for patient a
$C_{\max}$	Maximum completion time of the surgical operations

Using the above notations, the proposed model can be stated as follows:

$$\text{Min} Z_1 = C_{\max}, \tag{1}$$

$$\text{Min} Z_2 = \sum_{a}^{A} w_a (T_a + E_a), \tag{2}$$

s.t.:

$$\sum_{r=1}^{A} v_{ar}.Per_{ar} = 1 \qquad a = 1, 2, ..., A, \tag{3}$$

$$c^1_a \geq \sum_{r=1}^{R} (P^1_{ar} + ST_r) \times v_{ar} \qquad a = 1, 2, ..., A, \tag{4}$$

$$c^1_a + M(2 + z_{abr} - v_{ar} - v_{br}) \geq c^1_b + (P^1_{ar}.v_{ar}) + (STT_r.v_{ar}) \qquad a, b = 1, 2, ..., A \ \& \ a \neq b \quad r = 1, 2, ..., R, \tag{5}$$

$$c^1_b + M(3 - z_{abr} - v_{ar} - v_{br}) \geq c^1_a + (P^1_{br}.v_{br}) + (STT_r.v_{br}) \qquad a, b = 1, 2, ..., A \ \& \ a \neq b \quad r = 1, 2, ..., R, \tag{6}$$

$$M.\rho_{ab} \geq c^1_a - (c^1_b - \sum_{r=1}^{R} P^1_{br}.v_{br}) \qquad a, b = 1, 2, ..., A \ \& \ a \neq b, \tag{7}$$

$$fac_a + \sum_{b=1}^{A} fac_b.(\rho_{ab} + \rho_{ba} - 1) \leq AR \qquad a = 1, 2, ..., A, \tag{8}$$

$$\sum_{t=1}^{N} o_{at} = 1 \qquad a = 1, 2, ..., A, \tag{9}$$

$$c^2_a = c^1_a + \sum_{t=1}^{N} P^2_{at}.o_{at} \qquad a = 1, 2, ..., A, \tag{10}$$

$$c^2_a + M.(2 + s_{abt} - o_{at} - o_{bt}) \geq c^2_b + (P^2_{at}.o_{at}) + (SST_t.o_{at}) \qquad a, b = 1, 2, ..., A \ \& \ a \neq b \quad t = 1, 2, ..., N, \tag{11}$$

$$c^2_b + M.(3 - s_{abt} - o_{at} - o_{bt}) \geq c^2_a + (P^2_{bt}.o_{bt}) + (SST_t.o_{bt}) \qquad a, b = 1, 2, ..., A \ \& \ a \neq b \quad t = 1, 2, ..., N, \tag{12}$$

$$v_{ar} \leq Per_{ar} \qquad a = 1, 2, ..., A \quad r = 1, 2, ..., R, \tag{13}$$

$$C_{\max} \geq c_a^2 \qquad a = 1,2,...,A, \tag{14}$$

$$starT_a \geq c_a^1 - \sum_{r=1}^{R} P_{ar}^1.v_{ar} \qquad a = 1,2,...,A, \tag{15}$$

$$T_a \geq starT_a - u_a \qquad a = 1,2,...,A, \tag{16}$$

$$E_a \geq l_a - starT_a \qquad a = 1,2,...,A, \tag{17}$$

$$c_a^1, c_a^2, start_a, T_a, E_a, C_{\max} \geq 0$$

$$a = 1,2,...,A, \;\; a,b = 1,2,...,A \;\&\; a \neq b, \tag{18}$$

$$z_{abr}, v_{ar}, s_{abt}, o_{at}, \rho_{ab} \in \{0,1\}$$

$$r = 1,2,...,R, \;\; t = 1,2,...,N. \tag{19}$$

The primary objective function aims to minimize the maximum completion time of surgical operations, while the secondary objective function seeks to minimize the sum of earliness and tardiness. Constraint set (3) indicates that each patient must be assigned to one OR regarding the allocation matrix. Constraint set (4) shows the start time of surgery for a patient due to initial preparation time. Constraint sets (5) and (6) represent the relation between the completion times of two successive surgeries. Constraint set (7) is incorporated into the model to show the overlap between surgeries. Constraint set (8) indicates that surgeries requiring the shared resources should not be done, simultaneously. Constraint set (9) indicates that each patient must be assigned to only one recovery bed. Constraint set (10) shows the relationship between completion time of surgery in the ORs and recovery room. Constraint sets (11) and (12) express the completion times of two successive surgeries in the recovery room. Constraint set (13) prevents the allocation of patients to the unrelated ORs. Constraint set (14) shows the linearization of the first objective function, while Constraint set (15) calculates the start time of the patient's surgery. Constraint sets (16) and (17) calculate tardiness and earliness times and finally, Constraint sets (18) and (19) show the type of decision variables.

3.3. Robust optimization approach

Robust optimization is one of the new ways in mathematical programming that has attracted much attention recently [35]. The main objective of the robust optimization approach is to select solutions that are able to cope with the uncertain data. It is assumed that uncertain data are bounded, but unknown, and in most research studies, uncertainty space is assumed to be convex. Unlike the stochastic programming, the robust optimization approach does not need any information about the probability distribution of the uncertain data. In order to tackle the uncertainty, the optimization problem with uncertainty parameters is transformed into a robust counterpart [36]. During the last years, many researchers have tried to develop new and efficient approaches regarding data uncertainty. One of the main developments in this context is the approach proposed by Bertsimas and Sim [34]. They proposed a robust approach for a mathematical model by incorporating an uncertainty budget. The main advantage of this approach over other versions of the robust optimization approach is that it allows for the control of the conservatism of the solutions through this parameter. In the robust optimization formulation, $\Gamma_i \in [0, |J_i|]$, as a budget parameter, is inserted into the model, where $|J_i|$ represents the number of uncertain coefficients in constraint set i. The proposed methodology ensures that constraints will be satisfied despite data variation and the protection level does not depend on the solution of the robust model [34]. To develop the robust counterpart of the proposed MILP model, the following parameters and decision variables are considered:

Parameters

$\Gamma 1_b$	Uncertainty budget of Constraint set (4)
$\Gamma 2_{abr}$	Uncertainty budget of Constraint sets (5) and (6)
$\Gamma 3_{ab}$	Uncertainty budget of Constraint set (7)
$\Gamma 4_a$	Uncertainty budget of Constraint set (10)
$\Gamma 5_{abt}$	Uncertainty budget of Constraint sets (11) and (12)
$\hat{P}_{ar}^1$	Measure of uncertainty for the surgery time of patient a in OR r
$\hat{P}_{ta}^2$	Measure of uncertainty for the hospitalization time of patient a in the recovery room on bed t

Decision variables

$G1_b, Be1_{ar}$	Auxiliary dual variables of Constraint set (4)
$G2_{abr}, Be2_{ar}$	Auxiliary dual variables of Constraint sets (5) and (6)
$G3_{ab}, Be3_{ar}$	Auxiliary dual variables of Constraint set (7)
$D1_a, B1_{at}^2$	Auxiliary dual variables of Constraint set (10)
$D2_{abt}, B2_{at}^2$	Auxiliary dual variables of Constraint sets (11) and (12)

It is worth noting that the robust counterpart model by Bertsimas and Sim [34] is designed for inequality constraints. On the other hand, the Constraint set (10) consists of an equal equation. When attempting to replace the equality constraint with two inequality constraints (less than or equal and greater than or equal), it may result in infeasibility of the model. Thus, we applied the proposed approach by Lin et al. [37]. Regarding the Constraint set (10), if the term c_a^2 exists, it is replaced by $c_a^1 + \sum_{t=1}^{N} P_{at}^2.o_{at}$. The robust counterpart of the model is given as follows:

$$\text{Min} Z_1 = C_{\max}, \tag{20}$$

$$\text{Min} Z_2 = \sum_{a}^{A} w_a (T_a + E_a), \tag{21}$$

s.t.:

$$c_b^1 \geq \sum_{r=1}^{R} P_{br}^1.v_{br} + G1_b \Gamma 1_b + \sum_{r=1}^{R} Be1_{br}.v_{br} + \sum_{r=1}^{R} ST_r.v_{br} \qquad b = 1,2,..,A, \tag{22}$$

$$G1_b + Be1_{br} \geq \hat{P}_{br}^1.v_{br} \qquad b = 1,2,...,A, \quad r = 1,2,...,R, \tag{23}$$

$$c_a^1 + M(2 + z_{abr} - v_{ar} - v_{br}) \geq c_b^1 + (P_{ar}^1.v_{ar}) + (STT_r.v_{ar}) + (G2_{abr}.\Gamma 2_{abr}) + Be2_{ar} \qquad a,b = 1,2,\ldots,A \ \& \ a \neq b, \quad r = 1,2,...,R, \tag{24}$$

$$c_b^1 + M(3 - z_{abr} - v_{ar} - v_{br}) \geq c_a^1 + (P_{br}^1.v_{br}) + (STT_r.v_{br}) + (G2_{abr}.\Gamma 2_{abr}) + Be2_{br} \qquad a,b = 1,2,\ldots,A \ \& \ a \neq b, \quad r = 1,2,...,R, \tag{25}$$

$$G2_{abr} + Be2_{br} \geq \hat{P}_{br}^1.v_{br} \qquad a,b = 1,2,...,A, \quad r = 1,2,...,R, \tag{26}$$

$$M.\rho_{ab} \geq c_a^1 - \left(c_b^1 - \sum_{r=1}^{R} P_{br}^1.v_{br} - G3_{ab}.\Gamma 3_{ab} - \sum_{r=1}^{R} Be3_{br}.v_{br} \right) \qquad a,b = 1,2,...,A \ \& \ a \neq b, \tag{27}$$

$$G3_{ab} + Be3_{br} \geq \hat{P}_{br}^1.v_{br} \qquad a,b = 1,2,...,A, \quad r = 1,2,...,R, \tag{28}$$

$$c_a^2 + M.(2 + s_{abt} - o_{at} - o_{bt}) \geq \left(c_b^2 + \sum_{t=1}^{N} P_{at}^2.o_{at} + (D1_a.\Gamma 4_a) + \sum_{t=1}^{N} B1_{at}^2.o_{at} \right) + (P_{at}^2.o_{at}) + (SST_t.o_{at}) + (D2_{abt}.\Gamma 5_{abt}) + B2_{at}^2 \qquad a,b = 1,2,...,A \ \& \ a \neq b, \ t = 1,2,...,N, \tag{29}$$

$$c_b^2 + M.(3 - s_{abt} - o_{at} - o_{bt}) \geq \left(c_a^1 + \sum_{t=1}^{N} P_{at}^2.o_{at} + (D1_a.\Gamma 4_a) + \sum_{t=1}^{N} B1_{at}^2.o_{at} \right) + (P_{bt}^2.o_{bt}) + (SST_t.o_{bt}) + (D2_{abt}.\Gamma 5_{abt}) + B2_{bt}^2 \qquad a,b = 1,2,...,A \ \& \ a \neq b, \ t = 1,2,...,N, \tag{30}$$

$$D1_a + B1_{at}^2 \geq \hat{P}_{at}^2.o_{at} \qquad a = 1,2,...,A, \quad t = 1,2,...,N, \tag{31}$$

$$D2_{abt} + B2_{bt}^2 \geq \hat{P}_{bt}^2.o_{bt} \qquad a,b = 1,2,...,A \ \& \ a \neq b, \quad t = 1,2,...,N, \tag{32}$$

$$G1_b, G2_{abr}, G3_{ab}, D1_a, D2_{abt} \geq 0 \qquad a,b = 1,2,...,A \ \& \ a \neq b,$$

$$Be1_{ar}, Be2_{ar}, Be3_{ar}, B1_{at}^2, B2_{at}^2 \geq 0 \qquad r = 1,2,...,R \ \ t = 1,2,...,N, \tag{33}$$

and Constraint sets (3),(8),(9),(13)–(19).

3.4. MCGP with utility function

GP approach is one of the highly regarded approaches to tackling multi-objective optimization models. It seeks to minimize unfavorable deviations of the objective functions from the goals. GP seeks to minimize the sum of the deviations from the expectation (anticipation) level for the objective functions [38]. In this paper, the MCGP approach considering utility function is applied to tackle the proposed model. In the classic GP approach, it is necessary to define an aspiration level for each objective function with respect to the decision-maker's opinion. On the other side, his/her preference structure is not considered easily and may be far from reality. Thus, Chang [39] defined utility function to incorporate the decision-maker's preference values into the model. One of the main advantages of the MCGP with utility function rather than other versions of GP is to consider the decision maker's preference value in which the decision-maker attempts to optimize her/his expected utility [40]. In order to present the mathematical model, the following parameters and decision variables are considered:

Parameters

$[U_{k,\min}, U_{k,\max}]$ The range of the kth aspiration level

β_k^d The weight of normalized deviation

β_k^δ The weight of positive and negative deviations

Decision variables

y_k The continuous decision variable

d_k^+ The positive deviations of $f_k(X)$ from y_k

d_k^- The negative deviations of $f_k(X)$ from y_k

δ_k^- The normalized deviation of y_k from $U_{k,\min}$

λ_k The utility value

The proposed model is presented as follows:

$$\text{Min} \sum_k [\beta_k^d(d_k^+ + d_k^-) + \beta_k^\delta \delta_k^-], \tag{34}$$

s.t.:

$$\lambda \leq \frac{U_{k,\max} - y_k}{U_{k,\max} - U_{k,\min}} \qquad \forall k, \tag{35}$$

$$f_k(X) + d_k^- - d_k^+ = y_k \qquad \forall k, \tag{36}$$

$$\lambda_k + \delta_k^- = 1 \qquad \forall k, \tag{37}$$

$$U_{k,\min} \leq y_k \leq U_{k,\max} \qquad \forall k, \tag{38}$$

$$d_k^-, d_k^+ = 0 \qquad \forall k, \tag{39}$$

$$d_k^-, d_k^+, \delta_k^-, \lambda_k \geq 0 \qquad \forall k. \tag{40}$$

As a result, the MCGP with utility function for the proposed robust model is as:

$$\text{Min} \beta_1^d(d_1^+ + d_1^-) + \beta_2^d(d_2^- + d_2^+) + \beta_1^\delta \delta_1^- + \beta_2^\delta \delta_2^-, \tag{41}$$

s.t.:

$$\lambda_1 \leq \frac{C_{\max}^+ - C'_{\max}}{C_{\max}^+ - C_{\max}^-}, \tag{42}$$

$$\lambda_2 \leq \frac{ET^+ - ET'}{ET^+ - ET^-}, \tag{43}$$

$$C_{\max} + d_1^- - d_1^+ = C'_{\max}, \tag{44}$$

$$\sum_{a=1}^{A} w_a(T_a + E_a) + d_2^- - d_2^+ = ET' \tag{45}$$

$$\lambda_1 + \delta_1^- = 1, \tag{46}$$

$$\lambda_2 + \delta_2^- = 1, \tag{47}$$

$$C_{\max}^- \leq C'_{\max} \leq C_{\max}^+, \tag{48}$$

$$ET^- \leq ET' \leq ET^+, \tag{49}$$

and Constraint sets (3),(8),(9),(13)–(31).

4. Results and discussion

This section examines the quality of the mathematical model and the solution approach presented for the ORs scheduling problem. Subsection 4.1 introduces 36 random test problems in order to evaluate the performance of the robust MILP model and analyze the obtained results. Then, a real case study is presented and the quality of the proposed approach is analyzed based on real data in Subsection 4.2. Finally, a sensitivity analysis is carried out to investigate the impact of various parameters on the research problem.

4.1. Random test problems

This section aims to investigate the performance of the proposed approach in resolving the random test problems. A total of 36 test problems were generated and four characteristics with different levels were used to typify the test problems. It should be noted that the proposed model was solved by GAMS24 software using a PC featuring i7, 2.67 GHz, 6 GB RAM. Test problems characteristics are summarized in Table 2.

To solve the test problems through MCGP approach with utility function, three steps are considered as follows:

1. The negative ideal point of the maximum completion time and positive ideal point of the sum of the earliness and tardiness of the surgeries are computed. The objective is to minimize the maximum completion time of the surgeries while the sum of the earliness-tardiness of the surgical operations time is considered as a constraint;
2. The positive ideal point of the maximum completion time and negative ideal point of the sum of the earliness and tardiness of the surgeries is calculated. The objective is to minimize the sum of the earliness

Table 2. Test problems characteristics of the random test problems.

Parameters	Values
Number of ORs	5,10
Number of beds in the recovery room	5,10
Number of patients	10,20,30
Number of shared resources	1,2,3

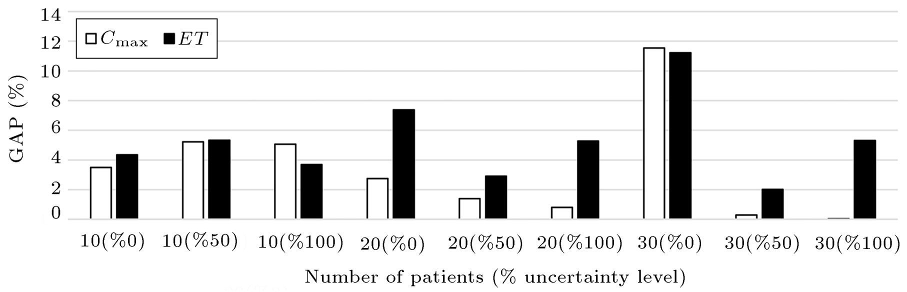

Figure 1. The gap between the objective functions and the ideal negative values regarding the number of patients.

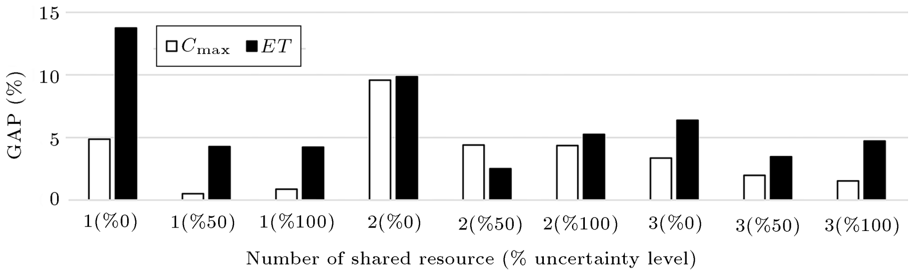

Figure 2. The gap between the objective functions and the ideal negative values regarding the number of shared resources.

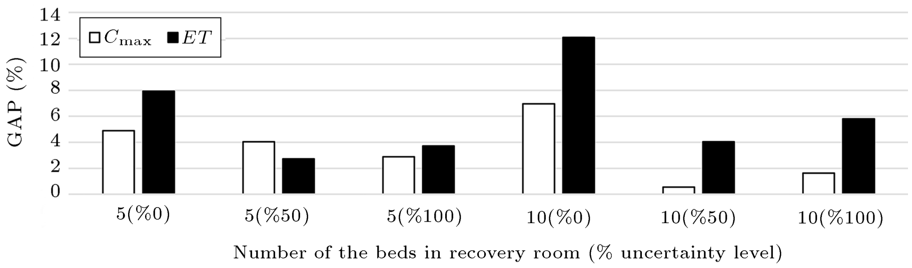

Figure 3. The gap between the objective functions and the ideal negative values regarding the number of beds in the recovery room.

and tardiness of the surgeries while the maximum completion time of the surgeries is considered as a constraint;

3. Regarding the negative and positive ideal points of the objective functions in these two steps, the MCGP problem with utility function is solved and the final solution is obtained.

In order to solve the test problems, three levels of uncertainty in surgery times and hospitalization duration in the recovery room were considered. The obtained results are summarized in Table 3.

To analyze the results presented in Table 3, the difference between the objective functions and the ideal negative value is examined with respect to the various characteristics of the test problems.

Figure 1 shows the gap between the objective functions and the ideal negative values regarding the number of patients and uncertainty levels.

As can be seen, by increasing the number of patients, the gap for the objective function $C_{\max}$ generally decreases. However, when the number of patients increases significantly, we can see a general increasing trend in the gap of ET objective function.

Figure 2 depicts the discrepancy between the objective functions and the ideal negative values, taking into account the shared resources and uncertainty levels.

Regarding the varying number of shared resources, the gap for $C_{\max}$ objective function is generally constant. In the ET objective function, the gap generally narrows as the number of shared resources increases.

Figure 3 represents the gap between the objective functions and the ideal negative values concerning the number of recovery room beds and levels of uncertainty.

According to Figure 3, there is no general trend with respect to the number of beds in the recovery room. However, the average gap for ET objective function is almost constant at varying numbers of beds in the recovery room.

Finally, Figure 4 represents the gap between the objective functions and the ideal negative values regarding the number of ORs and uncertainty levels.

Table 3. Results of the test problems.

TP	NORs	NBRR	NP	NSR	$[\Gamma_1, \Gamma_2]$	$C_{\max}^-$	$C_{\max}^+$	$C'_{\max}$	GAP	ET^-	ET^+	ET'	GAP	Time (second)
1	5	5	10	1	(0%, 0%)	300	385	315	5.0%	389	577	395	1.5%	5.3
					(50%, 50%)	350	465	370	5.7%	468.5	738	488.5	4.2%	6.4
					(100%, 100%)	385	465	385	0.0%	468.5	639.5	468.5	0.0%	5.5
2	5	5	10	2	(0%, 0%)	205	255	255	24.3%	317	375	375	18.2%	5.1
					(50%, 50%)	245	360	360	46.9%	378.5	399	399	5.4%	9.2
					(100%, 100%)	245	370	370	51.0%	378.5	429	429	13.3%	5.7
3	5	5	10	3	(0%, 0%)	205	240	205	0.0%	306.5	379	324	5.7%	6.6
					(50%, 50%)	245	350	245	0.0%	364.5	524.5	364	0.1%	6.0
					(100%, 100%)	245	310	245	0.0%	364.5	401	364.5	0.0%	6.8
4	5	5	20	1	(0%, 0%)	680	715	690	1.4%	2077	3421	2104	1.2%	3000.0
					(50%, 50%)	795	925	800	0.6%	2982.5	4111	2982.5	0.0%	3001.0
					(100%, 100%)	815	840	815	0.0%	2784.5	4236.5	2784.5	0.0%	2240.0
5	5	5	20	2	(0%, 0%)	590	620	590	0.0%	1731.5	2885	1814.5	7.7%	3000.0
					(50%, 50%)	670	835	670	0.0%	2356	3754.5	2356	0.0%	1985.0
					(100%, 100%)	670	790	670	0.0%	2044	3065	2188	7.0%	3005.0
6	5	5	20	3	(0%, 0%)	590	660	590	0.0%	1693.5	2910	1758	3.8%	2208.0
					(50%, 50%)	670	700	670	0.0%	2104.5	3223	2104.5	0.0%	3000.0
					(100%, 100%)	670	745	670	0.0%	2006.5	3666.5	2059	2.6%	3001.0
7	5	5	30	1	(0%, 0%)	895	10065	980	9.4%	3996	6172.5	4836.5	21.0%	3004.0
					(50%, 50%)	1260	1375	1260	0.0%	5920	9599	6441.5	8.8%	3004.0
					(100%, 100%)	1110	1125	1110	0.0%	7692	8304.5	7692	0.0%	3005.0
8	5	5	30	2	(0%, 0%)	825	10075	915	10.9%	4002.5	5814.5	5343	33.4%	3005.0
					(50%, 50%)	980	1225	980	0.0%	4741	7365.5	4741	0.0%	3006.0
					(100%, 100%)	985	1260	985	0.0%	5677.5	7209	5677.5	0.0%	3007.0
9	5	5	30	3	(0%, 0%)	825	10075	860	4.2%	3523.5	5883	3664	3.9%	3003.0
					(50%, 50%)	950	1025	950	0.0%	4464.5	6202.5	4690.5	5.0%	3007.0
					(100%, 100%)	950	1115	950	0.0%	4218	6365.5	5161	22.3%	3004.0
10	5	10	10	1	(0%, 0%)	300	325	305	1.6%	389	633.5	428	10.0%	13.6
					(50%, 50%)	350	385	350	0.0%	468.5	763.5	578	23.4%	14.5
					(100%, 100%)	350	465	385	10.0%	468.5	601.5	468.5	0.0%	15.8

TP: Test Problem; NORs: Number of ORs; NBRR: Number of Beds in the Recovery Room; NP: Number of Patients; NSR: Number of Shared Resources; $[\Gamma_1, \Gamma_2]$: Uncertainty levels; $C_{\max}^-$: The ideal negative value of the first objective function; $C_{\max}^+$: The ideal positive value of the first objective function; $C'_{\max}$: The obtained value of the first objective function; ET^-: The ideal negative value of the second objective function; ET^+: The ideal positive value of the second objective function; ET': The obtained value of the second objective function; GAP: The gap between the objective function and the ideal negative value.

Table 3. Results of the test problems (continued).

TP	NORs	NBRR	NP	NSR	$[\Gamma_1, \Gamma_2]$	$C^-_{\max}$	$C^+_{\max}$	$C'_{\max}$	GAP	ET^-	ET^+	ET'	GAP	Time (second)
					(0%, 0%)	295	345	300	1.6%	441.5	611	441.5	0.0%	3.9
11	5	10	10	2	(50%, 50%)	340	380	340	0.0%	557.5	795	559.5	0.3%	12.8
					(100%, 100%)	340	430	340	0.0%	557.5	683.5	559.5	0.3%	11.0
					(0%, 0%)	295	345	295	0.0%	437.5	520.5	439.5	0.4%	4.1
12	5	10	10	3	(50%, 50%)	340	460	340	0.0%	553.5	791	557.49	0.7%	9.8
					(100%, 100%)	340	395	340	0.0%	553.5	708.5	557.5	0.7%	44.5
					(0%, 0%)	680	715	685	0.7%	2140.5	2860	2763	29.0%	3000.0
13	5	10	20	1	(50%, 50%)	800	955	800	0.0%	3050.5	4529	3076	0.8%	3000.1
					(100%, 100%)	790	965	795	0.6%	2627	4064	2991	13.8%	3002.1
					(0%, 0%)	590	620	590	0.0%	2014	2675	2014	0.0%	2115.3
14	5	10	20	2	(50%, 50%)	670	680	670	0.0%	2124.5	2915	2124.5	0.0%	2933.4
					(100%, 100%)	670	705	670	0.0%	2076	3227	2152.5	3.6%	3000.0
					(0%, 0%)	745	770	745	0.0%	2189	3703.5	2279	4.1%	2077.5
15	5	10	20	3	(50%, 50%)	840	935	840	0.0%	2531.5	4296	2651.5	4.7%	2831.3
					(100%, 100%)	840	860	840	0.0%	2531.5	4208	2703	6.7%	3000.0
					(0%, 0%)	1095	68618.5	1360	24.0%	3996	6621	6272	56.9%	3007.1
16	5	10	30	1	(50%, 50%)	1465	1695	1465	0.0%	8523	9628.5	8523	0.0%	2315.4
					(100%, 100%)	1375	1665	1375	0.0%	8277	8504	8277	0.0%	3004.1
					(0%, 0%)	1010	60239.2	1260	24.7%	5363	6498	6498	21.1%	3005.0
17	5	10	30	2	(50%, 50%)	1350	1400	1350	0.0%	7829.5	9463	7829.5	0.0%	2294.9
					(100%, 100%)	1150	2000	1150	0.0%	6000	8742.5	6000	0.0%	2505.7
					(0%, 0%)	1010	53832.5	1045	3.4%	3979	7541.5	4145.5	4.1%	3004.0
18	5	10	30	3	(50%, 50%)	1145	1405	1145	0.0%	7430	8805	7430	0.0%	2027.1
					(100%, 100%)	1135	1535	1140	0.4%	7412	8265	7412	0.0%	3006.1
					(0%, 0%)	390	395	390	0.0%	483	779	494	2.2%	7.3
19	10	5	10	1	(50%, 50%)	445	505	445	0.0%	599	1053.5	642	7.1%	15.3
					(100%, 100%)	445	500	445	0.0%	599	982	642	7.1%	12.2
					(0%, 0%)	305	385	305	0.0%	359.5	460	359.5	0.0%	3.4
20	10	5	10	2	(50%, 50%)	340	440	340	0.0%	458.5	724.5	458.5	0.0%	6.0
					(100%, 100%)	340	485	340	0.0%	458.5	668.5	458.5	0.0%	6.6

TP: Test Problem; NORs: Number of ORs; NBRR: Number of Beds in the Recovery Room; NP: Number of Patients; NSR: Number of Shared Resources; $[\Gamma_1, \Gamma_2]$: Uncertainty levels; $C^-_{\max}$: The ideal negative value of the first objective function; $C^+_{\max}$: The ideal positive value of the first objective function; $C'_{\max}$: The obtained value of the first objective function; ET^-: The ideal negative value of the second objective function; ET^+: The ideal positive value of the second objective function; ET': The obtained value of the second objective function; GAP: The gap between the objective function and the ideal negative value.

Table 3. Results of the test problems (continued).

TP	NORs	NBRR	NP	NSR	$[\Gamma_1, \Gamma_2]$	$C^-_{\max}$	$C^+_{\max}$	$C'_{\max}$	GAP	ET^-	ET^+	ET'	GAP	Time (second)
21	10	5	10	3	(0%, 0%)	545	650	550	0.9%	330	693	334	1.2%	2.1
					(50%, 50%)	575	580	575	0.0%	428	1057	428	0.0%	4.7
					(100%, 100%)	575	580	575	0.0%	428	1162	428	0.0%	3.3
22	10	5	20	1	(0%, 0%)	670	795	685	2.2%	2293	3408.5	2293	0.0%	3000.4
					(50%, 50%)	855	1060	855	0.0%	2558	3798	2558	0.0%	2365.1
					(100%, 100%)	865	910	865	0.0%	2438.5	4254	2598	6.5%	3001.2
23	10	5	20	2	(0%, 0%)	440	480	445	1.1%	1559.5	1924.5	1600.5	2.6%	3002.4
					(50%, 50%)	505	880	530	4.9%	2059.5	2222.5	2059.5	0.0%	2990.1
					(100%, 100%)	530	580	530	0.0%	1807	2546	1871	3.4%	3003.5
24	10	5	20	3	(0%, 0%)	365	585	420	15.0%	1199	1592.5	1202.5	0.2%	3000.0
					(50%, 50%)	435	620	485	11.4%	1384	2067	1499.5	8.3%	3000.1
					(100%, 100%)	430	650	435	1.1%	1496.5	2167	1569	4.8%	3000.7
25	10	5	30	1	(0%, 0%)	1000	10065	1045	4.5%	3152.5	6545	3728	18.2%	3003.0
					(50%, 50%)	1290	1615	1290	0.0%	5328	7549.5	5328	0.0%	3006.1
					(100%, 100%)	1190	2060	1190	0.0%	5450.5	6545	5450.5	0.0%	3007.9
26	10	5	30	2	(0%, 0%)	740	10115	770	4.0%	3099.5	5019	3505.5	0.0%	3003.2
					(50%, 50%)	1045	1440	1045	0.0%	3829.5	5913	4068	6.2%	3004.3
					(100%, 100%)	965	1355	965	0.0%	4835	6342.5	4835	0.0%	2235.7
27	10	5	30	3	(0%, 0%)	650	10065	685	5.3%	2190	3740	2672	22.0%	3004.2
					(50%, 50%)	725	850	750	3.4%	2979	5223	3106	4.2%	3006.0
					(100%, 100%)	725	845	725	0.0%	2894.5	4447.5	2894.5	0.0%	3007.5
28	10	10	10	1	(0%, 0%)	390	480	395	1.2%	483	602.5	483	0.0%	12.3
					(50%, 50%)	445	455	445	0.0%	599	1145	642	7.1%	20.3
					(100%, 100%)	445	475	445	0.0%	599	1060	642	7.1%	19.3
29	10	10	10	2	(0%, 0%)	350	485	365	4.2%	456	622.5	456	0.0%	11.7
					(50%, 50%)	405	460	410	1.2%	577.5	1069.5	638	10.4%	22.2
					(100%, 100%)	405	425	410	1.2%	577.5	990.5	638.5	10.5%	17.8
30	10	10	10	3	(0%, 0%)	300	415	310	3.3%	377	513	426.5	13.1%	10.8
					(50%, 50%)	335	550	365	8.9%	490.5	737	516.5	5.3%	16.1
					(100%, 100%)	335	585	365	8.9%	490.5	740.5	516.5	5.3%	15.6

TP: Test Problem; NORs: Number of ORs; NBRR: Number of Beds in the Recovery Room; NP: Number of Patients; NSR: Number of Shared Resources; $[\Gamma_1, \Gamma_2]$: Uncertainty levels; $C^-_{\max}$: The ideal negative value of the first objective function; $C^+_{\max}$: The ideal positive value of the first objective function; $C'_{\max}$: The obtained value of the first objective function; ET^-: The ideal negative value of the second objective function; ET^+: The ideal positive value of the second objective function; ET': The obtained value of the second objective function; GAP: The gap between the objective function and the ideal negative value.

Table 3. Results of the test problems (continued).

TP	NORs	NBRR	NP	NSR	$[\Gamma_1, \Gamma_2]$	$C^-_{\max}$	$C^+_{\max}$	$C'_{\max}$	GAP	ET^-	ET^+	ET'	GAP	Time (second)
31	10	10	20	1	(0%, 0%)	670	720	685	2.2%	2067	3494	2487	20.3%	3003.1
					(50%, 50%)	910	1000	910	0.0%	2838.5	5112.5	2838.5	0.0%	2104.4
					(100%, 100%)	860	955	860	0.0%	2761	4555.5	2761	0.0%	2152.1
32	10	10	20	2	(0%, 0%)	435	510	445	2.2%	1527.5	2022	1621	6.1%	3003.2
					(50%, 50%)	545	655	545	0.0%	1876.9	2433.5	2024	7.8%	3003.1
					(100%, 100%)	525	1035	525	0.0%	2025.5	2618	2025.5	0.0%	2562.0
33	10	10	20	3	(0%, 0%)	365	455	395	8.2%	1166	1695	1313.5	13.6%	3003.3
					(50%, 50%)	430	495	430	0.0%	1413	2135.5	1602	13.3%	3007.1
					(100%, 100%)	429.9	545	465	8.1%	1405.5	2000.9	1616	14.9%	3003.2
34	10	10	30	1	(0%, 0%)	805	60013.5	855	6.2%	3233	4878.5	3386	4.7%	3008.4
					(50%, 50%)	1010	1375	1010	0.0%	4786	4878.5	4786	0.0%	3008.2
					(100%, 100%)	985	1135	985	0.0%	3884.5	4878.5	4523.5	16.4%	3007.6
35	10	10	30	2	(0%, 0%)	535	83550.6	760	42.0%	2633	3565.5	3409.5	29.4%	3006.1
					(50%, 50%)	735	1265	735	0.0%	3375.5	5820.5	3375.5	0.0%	2387.4
					(100%, 100%)	1070	83550.6	1070	0.0%	3940	5000	4929	25.1%	3006.1
36	10	10	30	3	(0%, 0%)	490	73969.8	490	0.0%	2061	3245.5	2154.5	4.5%	3000.2
					(50%, 50%)	540	995	540	0.0%	2988	3837.5	2988	0.0%	2054.3
					(100%, 100%)	540	935	540	0.0%	3571	4075.5	3571	0.0%	2395.4

TP: Test Problem; NORs: Number of ORs; NBRR: Number of Beds in the Recovery Room; NP: Number of Patients; NSR: Number of Shared Resources; $[\Gamma_1, \Gamma_2]$: Uncertainty levels; $C^-_{\max}$: The ideal negative value of the first objective function; $C^+_{\max}$: The ideal positive value of the first objective function; $C'_{\max}$: The obtained value of the first objective function; ET^-: The ideal negative value of the second objective function; ET^+: The ideal positive value of the second objective function; ET': The obtained value of the second objective function; GAP: The gap between the objective function and the ideal negative value.

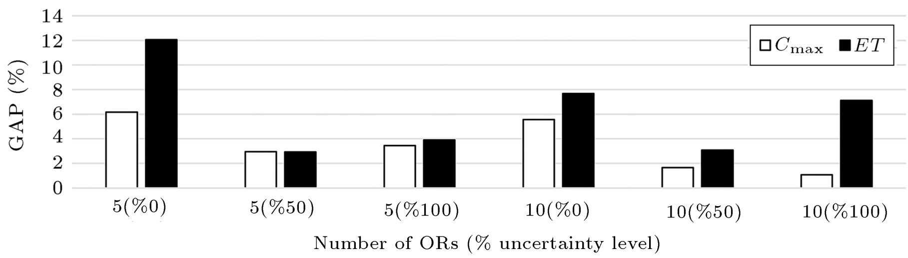

Figure 4. The gap between the objective functions and the ideal negative values regarding the number of ORs.

Based on Figure 4, the gap for both objective functions narrows when the number of ORs decreases.

4.2. Case study

4.2.1. Description and data

This article includes a real-world case study to demonstrate the approach. The case study focuses on Shahid Beheshti hospital, a public hospital located in Babol, in northern Iran. Established in 1974, the hospital's OR unit comprises nine OR and is spread across two floors. The types of surgeries on each floor are presented in Table 4. The OR unit at the hospital is operational from Saturday to Wednesday, while Thursdays are reserved for the OR and providing emergency services. The hospital's recovery room has a total of seven beds available.

Note that the OR allocation is only taken into account for non-emergency cases. In the event of an

Table 4. Types of surgeries on each floor.

Floor	Types of surgeries	Number of ORs
First floor	Orthopedics	3
	General surgery	2
	Urology	1
	Neurosurgery	1
Second floor	General surgery	1
	Kidney transplant	1

Table 5. Prioritization of patients in the OR unit.

Patient status	Weight
Patients with unstable condition	1
Elderly patients and pediatric patients	[0.5,1]
Patients with stable condition	Less than 0.5

emergency, the first available OR is allocated to the emergency case. The working hours of the OR unit are divided into three working shifts: morning, evening, and night. There is no waiting list at Shahid Beheshti hospital and surgeries are performed based on a defined prioritization system outlined in Table 5. Each patient is assigned a specific weight within the range of [0,1], indicating their priority level for surgery.

Every patient is transferred to the waiting room for surgery. In the waiting room, the hospitalization records of patients are checked and they are subsequently prepared for surgery. Afterward, the patient is transferred to the assigned operation room. After the surgery is completed, the patient is promptly moved to the recovery room for stabilization. Finally, the patient's overall condition is assessed to determine whether transfer to the ICU or general unit is necessary.

Prior to the initial surgery, as well as between subsequent surgeries, and following the final surgery, various preparatory and cleaning procedures must be performed. On average, the equipment preparation and sterilization in the ORs require about 45–60 minutes initially. In addition, between successive surgeries, a period of approximately 10–25 minutes is required for the same purpose, while after the last surgery, about 45–60 minutes are required to perform cleaning operations, on average. As mentioned earlier, the surgery and hospitalization time durations in the recovery room are considered uncertain parameters. According to the data derived from our case study, minimum and maximum times are presented in Table 6.

In the OR unit at the Shahid Beheshti hospital, radiology equipment is a shared source for orthopedic surgeries and some general surgeries. Due to budget limitation, only one radiology equipment is available and the surgeries that require this equipment cannot be performed, simultaneously.

4.2.2. Results

In order to compare the proposed approach with the current approach and to demonstrate the superiority of the proposed approach, the real data of five different days are gathered from the ORs at the Shahid Beheshti hospital. Of note, regarding the real and fixed data sets from the case study, uncertainty budgets (Γ_1 and Γ_2) are equal to 0. As a result, according to the solution obtained by the MCGP method, values of the first and second objective functions are presented in Table 7.

Based on the information presented in Table 7, there appears to be a 10.28% improvement in the first objective function and a 10.04% improvement in the second objective function. Thus, the obtained results show the superiority of the proposed approach to solve the research problem. Besides, we statistically compared the proposed approach with current approaches at a confidence interval of 95%. The hypotheses are tested as follows:

$$C_{\max_{proposed.approach}} = C_{\max_{current.approach}}$$
$$C_{\max_{proposed.approach}} < C_{\max_{current.approach}}$$

and:

$$ET_{proposed.approach} = ET_{current.approach}$$
$$ET_{proposed.approach} < ET_{current.approach}$$

The P-values of the first and second tests are 0.098

Table 6. Minimum and maximum times for different surgeries and the recovery room (minute).

	Type of surgery	Minimum surgery time	Maximum surgery time
Surgery times in ORs	Orthopedics	30	240
	Urology	15	270
	Neurosurgery	30	360
	General surgery	30	300
	Kidney transplant	180	300
Hospitalization time in the recovery room	**Type of surgery**	**Minimum time**	**Maximum time**
	All surgeries	10	120

Table 7. Results of the case study.

Day	Number of patients	Number of patients that need shared resource	$C'_{\max}$			ET'		
			Proposed approach	Current approach	Improvement	Proposed approach	Current approach	Improvement
1	30	4	765	820	7.8%	3308	3774	12.33%
2	20	3	665	780	14.74%	1699	1990	14.62%
3	17	3	630	700	10%	1184	1250	5.28%
4	12	3	615	675	7.4%	890	950	6.31%
5	10	2	540	610	11.47%	477	540	11.66%

Table 8. Different situations evaluated through sensitivity analysis.

Case	Description	NP	NBRR	NSR	NORs
1	Base case	30	7	1	9
2	Sensitivity analysis on the number of shared resources	30	7	2	9
3		30	7	3	9
4	Sensitivity analysis on the ORs (use lower-load OR to help other ORs)	30	7	1	9
5		30	7	1	9

Table 9. Sensitivity analysis of the number of shared resources.

Case	NOR	NBRR	NP	NSR	$[\Gamma_1, \Gamma_2]$	$C'_{\max}$	Percentage of change	ET'	Percentage of change
1	9	7	30	1	(50%, 50%)	1030	–	4664.5	–
2	9	7	30	2	(50%, 50%)	695	–32.5%	3636	–22%
3	9	7	30	3	(50%, 50%)	590	–42.7%	3359.5	–27.9%

and 0.404. H_0 is rejected in both tests, which indicates that the proposed approach outperforms the current approach, statistically.

4.2.3. Sensitivity analysis

This section presents a sensitivity analysis that takes into account the influence of each parameter on the obtained results. Two key parameters, (i) the number of shared resources and (ii) a new policy for assigning the surgeries to the ORs, are considered here. Table 8 displays the various scenarios analyzed in the sensitivity analysis.

As can be seen, Case 1 is related to the basic model while Cases 2 and 3 show the number of shared resources. Finally, Cases 4 and 5 take into account different policies on the assignment of surgeries to the ORs.

(i) ***Sensitivity analysis of the number of shared resources.*** This section attempts to evaluate the effect of the number of shared resources on both objective functions. To this end, in addition to the base case, we consider two new values of 2 and 3 for the shared resources. The obtained results are given in Table 9.

With reference to Table 9, it can be observed that the shared resource serves as a bottleneck in enhancing the productivity of the ORs with respect to the selected objective functions. Increasing the number of shared resources to 2 and 3 results in a reduction of about -32.5% and -42.7%, respectively, in the value of $C_{\max}$. A similar trend can be seen for the second objective function, where there is a decrease in ET of about –2% and -27.9% relative to the base case.

(ii) ***Sensitivity analysis on the ORs assigning policy.*** To conduct a sensitivity analysis on the ORs assigning policy, two different policies are taken into account:

- The first policy allows patients whose surgery time is less than 60 minutes to be assigned to other ORs that are capable of performing the same surgery;
- The second policy allows patients who need to

Table 10. Sensitivity analysis of the ORs assigning policy.

Case	NOR	NBRR	NP	NSR	$[\Gamma_1, \Gamma_2]$	$C'_{\max}$	Percentage of change	ET'	Percentage of change
1	9	7	30	1	(50%, 50%)	1030	–	4664.5	–
4	9	7	30	1	(50%, 50%)	1025	–0.4%	4478	–3.9%
5	9	7	30	1	(50%, 50%)	985	–4.3%	4508	–3.3%

undergo surgeries in higher work-load ORs to be assigned to a lower work-load OR if it is available.

As can be seen in Table 10, both of the proposed policies enhance the performance of the ORs scheduling. The first policy appears to be more effective than the second one in terms of $C_{\max}$. However, there is no significant difference between the two policies in terms of ET.

5. Conclusion

The focus of this study is on the scheduling of Operating Rooms (ORs), taking into account shared resources, uncertainty in surgery time, and the time required for patients to recover in beds in the recovery room. To address this problem, a bi-objective robust Mixed-Integer Linear Programming (MILP) model is proposed. The first objective function minimized the maximum completion time of the surgical operations, while the second objective function minimized the sum of the earliness and tardiness of the surgical operations. To solve the proposed model, the Multi-Choice Goal Programming (MCGP) approach with a utility function was utilized, using both random test problems and real data obtained from Shahid Beheshti hospital in Babol, Iran. Additionally, a sensitivity analysis was conducted on selected parameters. The results of the study demonstrated the effectiveness of the proposed method in solving the OR scheduling problem, and showed that it outperformed existing methods.

For future research, it is recommended that a surgical team be set up and all working shifts and relative costs of each surgery be considered. Emergency cases ought to be be incorporated in the ORs scheduling problem. To solve the problem, we can develop exact approaches such as Branch and Bound method, Bender decomposition algorithm, or Lagrangian relaxation algorithms.

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