Modelling and Optimization of Robotic Manipulator Mechanism for Computed Tomography Guided Medical Procedure

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Abstract: Although industrial robots are common, higher degree of manipulability might be required to expand the applications of manipulators in the field of medicine. Modifying the mechanical design of a robot as per the workspace can be perceived as an optimization problem. Hence, a novel spatial manipulator is designed for a diagnostic apparatus using different optimization algorithms. Standard Genetic Algorithm and Genetic Algorithm (GA) with hybrid functions like pattern search (PS) and fmincon is proposed to optimize the link lengths of a 3 degrees of freedom (DOF), 6-DOF, and novel 9-DOF hybrid redundant manipulator. A 9-DOF robot is designed to manipulate a needle in CT machine environment. The fitness function for all the manipulators is formulated using forward kinematic equations according to their workspace. Limits and constraints of each link is decided beforehand. A comparative study between all the hybrid GA functions is performed. MATLAB is used to solve and train the proposed GA method for optimizing the link lengths. Results show that the GA with PS provide better-optimized link lengths for a 3-DOF and 9-DOF manipulator while fmincon is well suited for a 6-DOF robot manipulator. Workspace and dead zone analysis is also performed using the optimized link lengths obtained.

Keywords: Genetic Algorithm; link length optimization; manipulator workspace; error minimization; robotic manipulator

1. Introduction

Infusion of robots have taken place in almost all the sectors of human life, from manufacturing and household to hospitals. Among all the other types of robots, one of the most commonly used robots are robotic arm manipulators. Robotic arms are used in manufacturing industries for tasks like pick and place, painting, welding, etc., while in the medical field they are used for surgical purposes as well.

Industrial robots that have three and six degrees of freedom (DOF) have been in application for many years. The standard 6-DOF robotic manipulator is sufficient for most applications in industrial and home environments. In some instances, redundancy and more DOF are required for the achievement of the desired target. Some of these applications include robots in medicine, reactors, space, underwater explorations, etc. For such applications, that require more DOF and redundancy, robotic manipulators with more than 6-DOF are necessary. The first step in designing a robotic manipulator is its link lengths.

Calculations of link lengths for manipulators with more than 6-DOF are computationally complex, which makes it difficult to reach just one solution. Development of a redundant manipulator is not as straightforward as that of a six or lower DOF manipulators. The main problem is finding the right solution to such issues is that they are multi-objective, multi-constrained, and multivariate. The optimization of parameters such as link length, total reach, and workspace are essential, and often contradicting. Sun, Liu, Luo, et. al. [1].

In the field of robotics, GA has been used to optimize various tasks such as path planning of a 3-DOF manipulator to minimize the joint angle change while avoiding obstacles, Albert, Koh, Chen, et. al. [2]. GA is also used to obtain the solution for inverse kinematics, Nearchou [3]. In the case of a 6-DOF manipulator, the parameter of the PID control system is optimized. This method also showed a better result than commonly used techniques such as the empirical approach and trial-error method, Situm and Cikovic [4]. To dynamically model a 9 DOF hydraulic manipulator, GA was used. Varying effects of the crossover method, crossover rate, and encoding scheme on the performance of the GA were also analyzed, West, Montazeri, Monk, et. al. [5]. To reduce the error in reaching the target points of a heavy-duty hydraulic serial robotic manipulator, its structure needs to be optimized. Constraints are applied according to its underground tunnelling task-based workspace. Link lengths were effectively optimized to minimize position and orientation errors, Kivelä, Mattila, and Puura [6].

Active research is being conducted to optimize the workspace reach, dexterity, and structure of the serial kinematic robotic manipulator. GA is used to optimize the topology of a modular robot, taking into consideration a specific
task, Chocron and Bidaud [7]. GA was also used to optimize link length and gearbox of a 3-DOF serial manipulator taking into account its dynamic model performance, Jafari, Safavi, and Fadaei [8]. Optimization of link length, link diameter, and link thickness of a 6-DOF manipulator designed for cleaning a fish tank is also carried out using GA, Bjorlykhaug and Egeland [9]. GA is very versatile, and there are many different variants of GA that are used in robotics for optimization of various structural and design parameters. Multi-objective Genetic Algorithm (MOGA), Multi-objective Differential Evolution (MODE), and Elitist Non-Dominated Sorting Genetic Algorithm (NSGA-II) are some of the modified GA used to optimize the design parameters of a robotic gripper, Saravanam Ramabalan, Ebenezer et. al. [10]. Another variant of GA Multi-Objective Particle Swarm Optimization is used to optimize the 3-DOF parallel manipulator design, Wang and Zhang [11]. Topology optimization of the robotic link is also attempted using a generative design technique which is a similar form of GA, Francalanzia Fenech, and Cutajar [12]. Attempt to optimize crane designs have also been made using GA. Cranes can also be considered as a robotic system, Bye, Osen, and Pedersen [13]. Structural optimization of a planar 4-DOF robotic manipulator is attempted using GA to reach a target from a starting point without hitting obstacles. The manipulator had hybrid rotary and prismatic joints, Fires and Oliveira [14]. Another attempt is made to optimize the travel time and space of a 3-DOF planar manipulator using GA. The motion planning is optimized in such a way that the robot avoids collision with obstacles within the given torque limits, Kazem, Mahdi and Oudah [15]. The trajectory and torque profile of another 3-DOF micro-robot was optimized for surgery. Optimization was completed GA, hybrid GA, pattern search (PS), and particle swarm optimization (PSO) methods. All the optimization techniques performed almost the same besides hybrid GA, which was slightly better by 3%, Mohamed, Elgamal and Elshar [16]. Attempts have also been made to design a robotic manipulator with 5-axis for application under CT guidance. Trajectory and path were generated for the same, Shubham and Mishra [17], Shah, Mishra and Mohapatro [18]. The trajectory of a biped robot is optimized for random obstacle avoidance using regression. Ultrasonic distance sensors are used as the sensory input for the regression controller, which then provides the heading angle for the biped robot. Industrial automation will benefit from random obstacle avoidance research, Kumar, Sahu and Parhi [19]. Even in the case of flexible robots, adaptive dynamic surface control is implemented to enhance its tracking performance. Parametric uncertainties are overcome using the first-order derivative filter of the inertial parameters, Li, Cui, Yan, et. al. [20]. A SCARA robot manipulator is also modelled and simulated as a PRR type manipulator. Link lengths and DH parameters of this manipulator is calculated mathematically using forward and inverse kinematic equations. The performance of its pick and place is then observed experimentally, Soyaslan, Uk, Ali, et. al. [21]. Path of a mobile robot is optimized using a genetic algorithm to obtain a smoother and shorter trajectory. Adaptive penalty factor is used in GA for ensuring the safety of the robot during obstacle avoidance, Ma, Liu, Zang, et. al. [22]. Trajectory optimization of a 6-DOF manipulator using GA, random average recombination, differential evolution, linear and geometric cooling strategy is attempted. The goal of this optimization is torque minimization while a 6-DOF robot is carrying a load from point to point. GA showed the best result in torque minimization, Cooper, Griffiths, Andrzejewski, et. al. [23]. To minimize the cycle time and vibration of a 6-DOF robot for electronic industries, GA is implemented. It will increase the productivity of the electronic PCB manufacturing industry and reduce vibrations by optimizing the velocity of the robot assembly[24]. A lot of focus has been given to trajectory optimization using GA, regression, and various other algorithms but research for the optimization of DH parameters based on the required workspace for standard and redundant manipulators has been lacking. Limited research has been conducted for link length optimization of a robot for application in the medical field.

There have been similar attempts to optimize the structure and trajectory of various DOF robotic manipulators, but there is limited study on the comparison of link length optimization for the desired workspace reach using GA for 3-DOF, 6-DOF, and 9-DOF robot manipulator in Computed Tomography (CT) machine environment. Manipulating robots in a CT environment poses quite a challenge, as the body of the CT machine is a major obstacle. The scanning area is the workspace of any manipulator working with a CT machine. The diameter of the scanning region of a standard CT machine is 70 cm [25] hence, the robotic manipulator must reach the scanning area and maneuver in that region without touching the body of the CT machine.

The novelty of this research is the optimization of link lengths of 3-DOF, 6-DOF, and 9-DOF robotic manipulator. 9-DOF robot is a novel manipulator specifically designed to be mounted on the CT bed with an arc design. Optimization is carried out to find a better set of link lengths so that the robots can reach and maneuver in the desired three dimensional CT workspace. Only forward kinematics and error function is used as fitness function in GA and GA with hybrid functions to optimize the DH parameters like link lengths and joint displacement. Different types of genetic algorithm coupled with hybrid functions like pattern search and fmincon, are compared.
for all three robotic manipulators. Different hybrid functions are varied to select the better-optimized set of link lengths and joint displacements for each robot.

2. Nomenclature

$l =$ Link length  
$d =$ Joint displacement  
$\alpha =$ Link twist  
$\theta =$ Joint angle  
$C_n =$ cosine of $n$th joint  
$C_{23} =$ cosine ($\theta_2 + \theta_3$)  
$S_n =$ sine of $n$th joint  
$S_{23} =$ sine ($\theta_2 + \theta_3$)  
$\delta =$ Euler angle about X axis  
$\beta =$ Euler angle about Y axis  
$\gamma =$ Euler angle about Z axis  
n = joint number of the robotic manipulator

3. Manipulator Designs and DH Parameters

A standard rotary 3-DOF, rotary 6-DOF, and a novel 9-DOF hybrid robotic manipulator is designed for this research. The kinematic model of all the robotic manipulators is designed in Matlab. DH parameters of all the robot manipulator are also found out and mentioned in the respective tables. The red items denote the end effector link and the joints including rotary and prismatic. While the blue items denote the rest of the links of the robotic manipulators.

Figure 1 shows the kinematic model of a 3-DOF robot manipulator with rotary joints. The workspace of this robot is spherical when all the joints have full 360° motion. [Figure 1]

DH parameters of the 3-DOF manipulator are shown in table 1 where, i signifies the link number. It is observed that the robot is a standard rotary 3-DOF manipulator.[Table 1]

Figure 2 shows the kinematic model of a 6-DOF robot manipulator with rotary joints. The workspace of this robot is very versatile as it has freedom of motion in every direction and orientation.[Figure 2]

DH parameters of the 6-DOF manipulator are shown in table 2. It is observed that the robot is a standard rotary 6-DOF manipulator.[Table 2]

Figure 3 shows the CAD and kinematic model of a novel 9-DOF robot manipulator with rotary and prismatic joints. This robot is designed to be mounted on the CT bed and maneuver in the scanning area. Figure 3(a) shows a CAD model of the robotic manipulator mounted on a CT bed and figure 3(b) shows the kinematic model of the manipulator. In figure 3, all the joints of the robotic manipulator are shown at 0° configuration.[Figure 3(a)] [Figure 3(b)]

DH parameters of the 9-DOF manipulator are shown in table 3. It is observed that the robot is a redundant hybrid 9-DOF manipulator. The arc design of the manipulator constitutes as 1-DOF. Arc design is developed for the ergonomics of the patient in a CT machine environment.[Table 3]

4. Mathematical formulation

GA is a heuristic approach for solving an optimization problem, Taylor [26]. Here, the error functions are minimized to derive the optimum set of link lengths. GA needs fitness function to optimize these link lengths and for a robotic manipulator forward kinematic equations serve as a good fitness function because forward kinematic equations relate the DH parameters with the realworld workspace in X, Y, and Z direction.

The transformation matrix is used to derive the forward and inverse kinematic equations. Mathematical equations for link lengths are derived from the kinematic equations. From these equations, a fitness function for obtaining the link lengths by training the GA is derived. Equation 1 shows the general form of the transformation matrix used for all three-robot manipulators, Nagrath [27]. ‘n’ is the number of joints in a robot manipulator. The units of all the link lengths and joint displacements are in cm and the link twist and joint angles are in degrees.
\[
T^n_{(n-1)} = \begin{pmatrix}
\cos(\theta_n) & -\sin(\theta_n) \times \cos(\alpha_n) & \sin(\theta_n) \times \sin(\alpha_n) & l_n \times \cos(\theta_n) \\
\sin(\theta_n) & \cos(\theta_n) \times \cos(\alpha_n) & -\cos(\theta_n) \times \sin(\alpha_n) & l_n \times \sin(\theta_n) \\
0 & \sin(\alpha_n) & \cos(\alpha_n) & d_n \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

For a 3-DOF manipulator equation 2 shows its transformation matrix formula. Equation 3 to equation 5 shows the forward kinematic equations of the 3-DOF robot. \(X_1, Y_1,\) and \(Z_1\) are the derived coordinates in the workspace of the 3-DOF robot[28]. Here, \(C_n\) and \(S_n\) are cosine and sine of \(n^{th}\) joint respectively where, \(n = 1, 2, \ldots, n\). \(C_{23}\) and \(S_{23}\) represent cosine \((\theta_2 + \theta_3)\) and sine \((\theta_2 + \theta_3)\), respectively.

### 4.1. Fitness Function for 3-DOF Robot

\[
T^3_0 = T^1_0 \times T^2_1 \times T^3_2
\]

\[
X_i = C_i \times (l_j \times C_{23} + l_2 \times C_2)
\]

\[
Y_i = S_i \times (l_j \times C_{23} + l_2 \times C_2)
\]

\[
Z_i = d_i + l_1 \times S_{23} + l_2 \times S_2
\]

\[
\text{Error}_i = \sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2 + (Z_j - Z_i)^2}
\]

#### 4.1.1. Constraints

\[
l_2 > d_1 \geq l_3
\]

Equations 3 to equation 5 shows the link length equations of 3-DOF manipulator. It is observed that all the equations are interdependent, and it is not easy to come to a definitive solution for a range of target coordinate values. Hence, to find an optimum solution GA is used. \(\text{Error}_i\) shows the equation for error between all the \(j^{th}\) desired coordinates \((X, Y, Z)\) and derived coordinates. Equation 7 shows the constraints for the 3-DOF robot. The desired workspace for the 3-DOF robot is from +20 cm to -20 cm in each X, Y, and Z direction. For a 6-DOF manipulator equation 8 shows its transformation matrix formula. Equation 9 to equation 12 shows the forward kinematic equations of the 6-DOF robot and the fitness function. \(X_2, Y_2,\) and \(Z_2\) are the derived coordinates, \(\delta_2, \gamma_2,\) and \(\beta_2\) (equations 13 to equation 15) are the desired orientation of approach in the workspace of the 6-DOF robot and constraints. Here, \(C_3\) and \(S_3\) represent cosine \((\theta_2 + \theta_3)\) and sine \((\theta_2 + \theta_3)\), respectively.

\[
T^6_0 = T^1_0 \times T^2_1 \times T^3_2 \times T^4_3 \times T^5_4 \times T^6_5
\]

### 4.2. Fitness Function for 6-DOF Robot

\[
X_2 = l_1 \times C_1 \times C_2 - d_1 \times (C_1 \times S_2 \times S_3 - C_1 \times C_2 \times C_3) + l_1 \times S_4 \times (S_1 \times S_4 - C_4 \times (C_1 \times C_2 \times S_3 + C_1 \times C_3 \times S_3))
\]

\[
Y_2 = l_2 \times C_2 \times S_3 + l_1 \times C_5 \times (S_1 \times S_2 \times S_3 - C_2 \times C_3 \times S_4) - l_1 \times S_6 \times (S_1 \times S_4 + C_4 \times (C_2 \times S_1 \times S_3 + C_2 \times S_1 \times S_2))
\]

\[
Z_2 = d_2 + d_2 \times S_{23} + l_1 \times S_{23} + l_2 \times C_4 \times S_3 + l_1 \times S_6 \times (C_1 \times S_3 + C_3 \times S_4 - S_2 \times S_2)
\]

\[
\text{Error}_i = \sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2 + (Z_j - Z_i)^2}
\]
\[
\text{Error}_2 = \sqrt{(X_j - X_2)^2 + (Y_j - Y_2)^2 + (Z_j - Z_2)^2 + (\delta_j - \delta_2)^2 + (\beta_j - \beta_2)^2 + (\gamma_j - \gamma_2)^2}
\]  

(12)

4.2.1. Constraints

\[
\delta_2 = \tan^{-1}\left( \frac{(S_6 * S_2 * S_3 * C_4 + C_2 * S_3 * S_4 + C_2 * C_5 * S_6)}{(S_2 * S_3 * S_5 - C_3 * S_4 * C_6)} \right)
\]  

(13)

\[
\beta_2 = -\sin^{-1}(C_6 * (S_2 * S_3 * C_4 + C_2 * S_4 * S_5) - C_2 * S_3 * S_4 * S_6)
\]  

(15)

It is observed from equation 9 to equation 11 and equations 13 to equation 15, that forward kinematics and orientations of 6-DOF are complex, and a straightforward solution for link lengths is quite difficult. Hence, GA is used to find the link lengths for the 6-DOF manipulator to cover the desired workspace. Error2 shows the equation for error between all the \(j\)th desired coordinates (X, Y, and Z) and orientation of approach (Euler angles \(\delta, \beta, \) and \(\gamma\)) and derived coordinates and orientation of approach \(\gamma_2\) ranges from +60° to -60°. Error2 is minimized using GA, as shown in equation 12. The transformation equation of the novel 9-DOF manipulator is derived using equation 16, Ge [29]. Forward kinematic equations and orientation constraints of the novel 9-DOF robotic manipulator is shown in equations from 17-19. X3, Y3, and Z3 are the target coordinates in the workspace of the 9-DOF robot. The desired workspace for the 6-DOF robot is from +20 cm to -20 cm in each X, Y, and Z direction. Here, \(C_n\) and \(S_n\) are cosine and sine of \(n\)th joint respectively where, \(n = 1, 2, ..., n\).

\[
T_{0}^{10} = T_{0}^{1} \times T_{2}^{3} \times T_{4}^{5} \times T_{6}^{7} \times T_{8}^{9} \times T_{9}^{10}
\]  

(16)

4.3. Fitness Function for 9-DOF Robot

\[
X_3 = l_1 + d_3 + d_8 + d_7 + l_4 * C_7 + l_6 * C_7 - d_5 * S_7 + d_8 * S_7 + l_5 * C_6 * C_7
\]  

(17)

\[
Y_3 = l_3 - l_7 + d_2 - l_8 * C_8 - l_4 * C_8 - l_6 * S_8 + d_5 * C_7 + d_6 * C_7 + d_{10} * S_8 + l_5 * C_6 * S_7
\]  

(18)

\[
Z_3 = l_2 + d_4 - d_8 - l_3 * S_6 - l_8 * S_8 - l_4 * S_8 - l_{10} * S_8 - d_{10} * C_8
\]  

(19)

\[
\text{Error}_r = \sqrt{(X_j - X_3)^2 + (Y_j - Y_3)^2 + (Z_j - Z_3)^2}
\]  

(20)

4.3.1. Constraints

\[
\theta_1 = 0
\]  

(21)

\[
\theta_4 = \theta_1
\]  

(22)

\[
\theta_5 = \theta_6
\]  

(23)

Constraints are added according to equation 21 to equation 23. These constraints provide the desired orientation of approach for retrieval of tissue samples while working on a CT machine. Error2 shows the equation for error between all the \(j\)th desired coordinates (X, Y, and Z) and derived coordinates. As the orientation of the robotic 9-DOF manipulator is predefined for the specified tissue retrieval task, the error function in equation 20 does not include the orientation segment as included in equation 12 for 6-DOF. These equations are used to iterate the GA to find the sets of link length of 3-DOF, 6-DOF, and novel hybrid 9-DOF robotic manipulator. The desired workspace for the 9-DOF robot is from +20 cm to +40 cm in X, +20 cm to -20 cm in Y, and Z direction.

4.4. Boundary Conditions
Infinite solutions for link lengths and joint displacements are possible for an open chain robotic manipulator system. Hence, boundary conditions need to be set for optimization. Upper and lower boundary for each robotic manipulator design is shown in Table 4. All dimensions are in centimeter (cm). These boundary limits are based on the dimensions of the desired workspace. [Table 4]

Using standard GA (SGA) and hybrid GA technique, including pattern search and fminunc. A flowchart depicting the process of GA is shown in figure 4. [Figure 4]

This process is iterated until a termination criterion is reached. After the iterations were completed for SGA, pattern search and fmincon hybrid GA, optimum sets of link lengths for all robot manipulators are obtained.

5. Results and Discussion

GA is trained to obtain the optimum set of link lengths for 3-DOF, 6-DOF, and novel 9-DOF robot manipulator by varying the hybrid functions. Optimization of link lengths and joint displacement is carried out by keeping the rotation angles constant. Hence, Link lengths and joint displacement are the variables. The hybrid functions used here are PS and fmincon to minimize the error functions. Firstly, an SGA architecture is trained, which has no hybrid function, and the link lengths are found than PS and fmincon is applied. All the other parameters are kept constant. The initial parameters include the creation function, initial population size, initial population range, fitness scaling, selection function, crossover fraction, elite count, mutation function, and constraint algorithm. The termination criteria parameters, such as the number of generation, stall generation, and function tolerance, is also kept constant. The creation function is a nonlinear feasible function with an initial population size of 5. The initial population size is kept 5 to reduce the computational cost. The initial population range is kept between -10 and 10. Fitness scaling is based on the rank system, and stochastic uniform is used as the selection function. The crossover function is kept at 0.8, and the elite count is 2.5. The mutation function is selected as adaptive feasible and augmented. The Lagrangian function is taken as the nonlinear constraint algorithm. The formula for lagrange function is shown in equation 24. The variable f(x) is the fitness function, g(x) is the equality function according to the equations 19, 20, and 21. The variable λ is the lagrange multiplier which is adjusted while training of the GA.

\[ L = f(x) - \lambda g(x) \] (24)

As for the termination condition, the number of generations is 100, stall generations are 50, and function tolerance is kept at \(10^{-6}\). The link lengths and joint displacements obtained after optimization will have sufficient level of accuracy with a tolerance like \(10^{-6}\). Even if one decides to manufacture the links of the robotic manipulator \(10^{-6}\) cm tolerance provides a sufficient amount of accuracy. These parameters are kept constant throughout the study.

Using the parameters mentioned and training with SGA, PS, and fmincon, the overall fitness value, and fitness of each individual in the population of a 3-DOF robotic manipulator is plotted in figure 5. Table 5 shows the optimum link lengths of a 3-DOF manipulator, obtained after iteration using all the three functions. It is observed in the graphs that the training reached a global minima. [Figure 5 (a)] [Figure 5 (b)] [Figure 5 (c)][Table 5]

It is observed in Figure 5 that the best overall fitness value for a 3-DOF robot is obtained with PS while the overall fitness value for SGA and fmincon are 1.16 and 1.38, respectively. Hence, it is observed that the link lengths found with PS are better optimized for a 3-DOF robot to reach the desired workspace. The workspaces of this robot are plotted and shown in figure 6. These workspaces represent the volume covered by the 3-DOF robot with the optimized sets of link length obtained after training the SGA, PS, and fmincon. [Figure 6 (a)] [Figure 6 (b)]

Figure 6 shows the desired workspace volume (black) versus the workspace plotted after obtaining the link lengths using SGA (blue), PS (green) and fmincon (red) in two-dimensions (a) and three-dimensions (b). It is observed that the SGA produces the workspace closest to the desired one, but the dead zone is very large, and thus the robot with those link lengths becomes impractical. Dead zones are the regions within the workspace where the end effector of the robotic manipulator is not able to reach. These dead zones are caused due to physical constraints such as link lengths and joint displacements. On the other hand, the workspace generated by fmincon does not cover the necessary desired volume while PS provides better results. The entirety of the workspace is covered with the link lengths obtained from PS also; the dead zone is relatively small. Hence, PS becomes a better choice to obtain optimized link lengths for the 3-DOF robot.
Now a 6-DOF manipulator is trained using the same parameters and hybrid functions. The overall fitness value and fitness of each individual in the population of a 6-DOF robotic manipulator are plotted in figure 7. Table 6 shows the optimum link lengths of a 6-DOF manipulator, obtained after iteration using all the three functions. It is observed in the graphs that the training reached a global minima. [Figure 7 (a)] [Figure 7 (b)] [Figure 7 (c)] [Table 6]

It is observed from the graphs in figure 7 that the best overall fitness value for a 6-DOF robot is obtained with PS 0.34, while the overall fitness value for SGA and fmincon are 1.94 and 1.54, respectively. Hence, it is observed that the link lengths found with PS are theoretically more optimized to reach the desired workspace. After training all three GA, the set of link length is obtained, and using this link lengths workspace of the 6-DOF robot is plotted in figure 8. The constraints and error function ensures that the robotic manipulator will reach the desired position in the workspace at the desired orientation. [Figure 8 (a)] [Figure 8 (b)]

Figure 8 shows the workspace analysis of the 6-DOF robot with link lengths obtained after optimization using SGA (blue), PS (green) and fmincon (red) and desired workspace (black) in two dimension (a) and three dimension (b). It is observed that the volume covered with SGA and PS are very similar while the workspace volume obtained by fmincon is slightly smaller with a smaller dead zone. All the methods cover the desired workspace volume, and hence any function can be applied, but fmincon is more acceptable.

Link lengths for the novel 9-DOF robotic manipulator are optimized using the same parameters and hybrid functions. Here the desired orientation is fixed as the orientation of approach near the CT machine is limited to avoid a collision. The overall fitness value and fitness of each individual in the population of a 9-DOF robotic manipulator are plotted in figure 9. Table 7 shows the optimum link lengths of a 9-DOF manipulator, obtained after iteration using all the three functions. It is observed in the graphs that the training reached a global minima. [Figure 9 (a)] [Figure 9 (b)] [Figure 9 (c)] [Table 7]

Figure 9 shows the best fitness value for the novel 9-DOF robot designed for tissue retrieval in a CT environment is found to be 0.121 using SGA method. Fitness value for PS and fmincon were found to be 0.124 and 1.36, which are more than the SGA method. The set of link lengths obtained from the SGA method should ensure that the robotic manipulator would reach the entire desired target in the desired orientation safely. Link lengths of link 1 and link 2 are constant due to CT bed constraints and arc structure design. The workspace of each set of link lengths obtained after training all three GA methods are plotted in figure 10. [Figure 10 (a)] [Figure 10 (b)]

Figure 10 depicts the comparison of workspaces plotted after obtaining link lengths using SGA (blue), PS (green) and fmincon (red) and the desired target workspace (black) in two dimension (a) and three dimension (b). It is observed that desired workspace is cylindrical in shape as the interior scanning area of a CT machine is cylindrical. The cylindrical workspace is solely targeted on the CT bed and scanning area. The advantage of a cylindrical workspace over a spherical one is that the optimization is focused on the desired CT bed and scanning area. Deadzone is not present within the CT scanning workspace because the optimization is carried out only for the cylindrical workspace. Workspace obtained using PS shows better optimization as all the coordinates of the target workspace is covered. A negligible dead zone is observed in the workspace of novel 9-DOF robot manipulator. SGA and fmincon cover most of the target workspace but do not cover some coordinates. Hence, it is observed that PS hybrid optimization function is useful for finding the link lengths of a robotic manipulator designed for specified tasks under CT machine with just forward kinematic equations and primary constraints.

6. Conclusion

In this research, a novel method is proposed to calculate and optimize the DH parameters like link length and joint displacement of a robotic manipulator with 3-DOF, 6-DOF, and 9-DOF using SGA, GA with PS, and GA with fmincon hybrid functions. Forward kinematics and error function are used as the fitness function for training the GA and GA with hybrid functions. DH parameters obtained after optimization based on the required workspace is then used for simulating the workspace of the robotic manipulator in MATLAB to observe the workspace and dead-zone coverage. Results showed that for a 3-DOF robot GA with PS hybrid function provides better-optimized results as it covered the target workspace with minimal dead zones. For a 6-DOF robotic manipulator GA with fmincon showed better results covering the desired workspace in the desired orientation as well. The dead zone for the 6-DOF robotic manipulator after using fmincon is smaller as compared to the other two GA functions. As for the novel 9-DOF robot designed to manipulate a needle in the CT machine environment, GA with PS showed better results. The cylindrical workspace required for this task is covered fully by using the PS.
hybrid function. With a clear lack of significant research on optimization of DH parameters for a medical robotic manipulator, the proposed method shows promising results with different DOF robot manipulators by minimizing the error function using forward kinematic equations. The only assumptions this method requires are the constraints for the link lengths. Hence, this proposed method can be successfully implemented for finding the optimum link lengths of open chain robotic systems designed to function in the desired workspace.

References

Technical Bibliography

1. **Shubham Kamlesh Shah**
   Shubham Kamlesh Shah is a PhD research scholar at School of Mechanical Engineering, KIIT deemed to be University. He specialized in robotics, machine learning and machine design. He completed his Bachelors in technology and Masters in technology in Mechanical engineering and machine design from KIIT Deemed to be University, Bhubaneswar, Odisha India. He has published many research articles in international journal and conferences. He also has three national and international patents published.

2. **Ruby Mishra** received B.E (Mechanical Engineering) from Utkal University in 1999, MTech (Machine Design) from Jadavpur University, in 2007, and Ph.D. (Machine Design) from Jadavpur University in 2014. She has a teaching and research experience of 21 years. Her research interests include Kinematics of Machine and Mechanism, Biomedical Engineering, Biomedical signal and image processing, robotics, medical image analysis. She is a member of The Institute of Engineers and Indian Science Congress Association. She has published 3 national and international patents. She has published many articles that are indexed in Scopus, Web of Science, and Google Scholar. Currently, she is doing research on Biomedical Instruments like Robotic arm for C.T. Guided Biopsy and Single Lumen Microcatheter for Angiography and Embolization. At present, she is working as an Associate Professor in the School of Mechanical Engineering Department of KIIT Deemed to be University.

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<tr>
<th>Joint Number</th>
<th>Link length ($l_i$)</th>
<th>Joint Displacement ($d_i$)</th>
<th>Link Twist ($\alpha_i$)</th>
<th>Joint Angle ($\theta_i$)</th>
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<tbody>
<tr>
<td>Joint 1</td>
<td>0</td>
<td>$d_1$</td>
<td>$\pi/2$</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>Joint 2</td>
<td>$l_2$</td>
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<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>joint 3</td>
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<td>0</td>
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Table 2. DH parameters of the 6-DOF robot manipulator.

<table>
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<th>Joint Number</th>
<th>Link length ($l_i$)</th>
<th>Joint Displacement ($d_i$)</th>
<th>Link Twist ($\alpha_i$)</th>
<th>Joint Angle ($\theta_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint 1</td>
<td>0</td>
<td>$d_1$</td>
<td>$\pi/2$</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>Joint 2</td>
<td>$l_2$</td>
<td>0</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>Joint 3</td>
<td>$l_3$</td>
<td>0</td>
<td>$-\pi/2$</td>
<td>$\theta_3 - \pi/2$</td>
</tr>
<tr>
<td>Joint 4</td>
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<td>$d_4$</td>
<td>$\pi/2$</td>
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<tr>
<td>Joint 5</td>
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Table 3. DH parameters of the 9-DOF robot manipulator.

<table>
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<tr>
<th>Link Number</th>
<th>Link length ($l_i$)</th>
<th>Joint Displacement ($d_i$)</th>
<th>Link Twist ($\alpha_i$)</th>
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<td>Joint 3</td>
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<td>$-\pi/2$</td>
<td>$\theta_4 - \pi/2$</td>
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<tr>
<td>Joint 4</td>
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<td>0</td>
<td>0</td>
<td>$\theta_5$</td>
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<tr>
<td>Joint 5</td>
<td>$l_6$</td>
<td>0</td>
<td>$-\pi/2$</td>
<td>$\theta_6$</td>
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<td>$\pi/2$</td>
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<td>Joint 9</td>
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Table 4. Upper and lower boundry for 3-DOF, 6-DOF and 9-DOF robotic manipulator.

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<thead>
<tr>
<th>Link Parameter</th>
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<th>6-DOF Robot</th>
<th>9-DOF Robot</th>
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<tr>
<td></td>
<td>Lower</td>
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<td>1</td>
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<td>25</td>
<td>0</td>
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<td>$d_4$</td>
<td>5</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>$l_5$</td>
<td>1</td>
<td>10</td>
<td>1</td>
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<tr>
<td>$l_6$</td>
<td>1</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>$d_9$</td>
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<td>5</td>
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<tr>
<td>$d_{10}$</td>
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Table 5. Link lengths obtained after optimizing the error function using SGA, PS and fmincon for a 3-DOF robot.

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<thead>
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<th>Link Parameter</th>
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<th>PS (cm)</th>
<th>Fmincon (cm)</th>
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<tr>
<td>d₁</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>l₂</td>
<td>20.00</td>
<td>20.00</td>
<td>14.05</td>
</tr>
<tr>
<td>l₃</td>
<td>1.00</td>
<td>8.48</td>
<td>2.54</td>
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</table>
Table 6. Link lengths obtained after optimizing the error function using SGA, PS and fmincon for a 6-DOF robot.

<table>
<thead>
<tr>
<th>Link Parameter</th>
<th>SGA (cm)</th>
<th>PS (cm)</th>
<th>Fmincon (cm)</th>
</tr>
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<tbody>
<tr>
<td>d₁</td>
<td>3.058</td>
<td>3.004</td>
<td>3.00</td>
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<tr>
<td>l₂</td>
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<td>l₃</td>
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<td>d₄</td>
<td>13.808</td>
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<td>l₅</td>
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<tr>
<td>l₆</td>
<td>0.166</td>
<td>0.117</td>
<td>1.289</td>
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Table 7. Link lengths obtained after optimization using SGA, PS and fmincon for novel 9-DOF robot.

<table>
<thead>
<tr>
<th>Link Parameter</th>
<th>SGA (cm)</th>
<th>PS (cm)</th>
<th>Fmincon (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>d₁</td>
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<td>40.00</td>
<td>40.00</td>
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<tr>
<td>d₂</td>
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<td>d₃</td>
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<td>d₁₀</td>
<td>4.957</td>
<td>2.396</td>
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1. **Ruby Mishra*  
Dr. Ruby Mishra is an associate professor at School of Mechanical Engineering, KIIT deemed to be University. She completed her PhD from Jadavpur University, Kolkata, West Bengal, India. She specializes in machine design, kinematics and dynamics of mechanisms.