Modelling and optimization of robotic manipulator mechanism for computed tomography guided medical procedure

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1. Introduction

Infusion of robots has taken place in almost all the human life sectors, from manufacturing and household to hospitals. Among all the other types of robots, one of the most commonly used robots is robotic arm manipulator. Robotic arms are used in manufacturing industries for tasks like pick and place, painting, welding, etc., while they are used for surgical purposes in the medical field as well.

Industrial robots characterized by three and six Degrees Of Freedom (DOF) have remained widely applicable for many years. The standard 6-DOF robotic manipulator is sufficient for most applications in industrial and home environments. In some instances, redundancy and more DOF are required to achieve the desired target. Some of these applications include robots in medicine, reactors, space, underwater explorations, etc. For such applications that require higher DOF and redundancy, robotic manipulators with more than 6 DOFs are necessary. The first step in designing a robotic manipulator is its link lengths.

Calculations of link lengths for manipulators with more than 6 DOFs are computationally complex, making it difficult to reach just one solution. Development of a redundant manipulator is not as straightforward.
as that of a six or lower DOF manipulator. The main problem of finding the right solution to such issues is that they are multi-objective, multi-constrained, and multivariate. The optimization of parameters such as link length, total reach, and workspace is essential and often contradicting [1].

In the field of robotics, Genetic Algorithm (GA) has been used to optimize various tasks such as path planning of a 3-DOF manipulator to minimize the joint angle change while avoiding obstacles [2]. GA is also used to obtain a solution to inverse kinematics [3]. In the case of a 6-DOF manipulator, the parameter of the PID control system is optimized. This method also exhibited a better result than commonly used techniques such as the empirical approach and trial-error method [4]. To dynamically model a 9-DOF hydraulic manipulator, GA was used. Varying effects of the crossover method, crossover rate, and encoding scheme on the performance of the GA were also analyzed [5]. To reduce the error in reaching the target points of a heavy-duty hydraulic serial robotic manipulator, its structure needs to be optimized. Constraints are applied according to its underground tunnelling task-based workspace. Link lengths were effectively optimized to minimize position and orientation errors [6].

Active research is being conducted to optimize the workspace reach, dexterity, and structure of the serial kinematic robotic manipulator. GA is used to optimize the topology of a modular robot, taking into consideration a specific task [7]. GA was also used to optimize link length and gearbox of a 3-DOF serial manipulator taking into account its dynamic model performance [8].

Optimization of link length, link diameter, and link thickness of a 6-DOF manipulator designed for cleaning a fish tank is also carried out using GA [9]. GA is very versatile and there are many different variants of GA that are used in robotics for optimization of various structural and design parameters. Multi-Objective Genetic Algorithm (MOGA), Multi-Objective Differential Evolution (MODE), and elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) are some of the modified GA used to optimize the design parameters of a robotic gripper [10]. Another variant of GA Multi-Objective Particle Swarm Optimization (MOPSO) is used to optimize the 3-DOF parallel manipulator design [11]. Topology optimization of the robotic link was also attempted using a generative design technique, which is a similar form of GA [12].

Attempts to optimize crane designs have also been made using GA. Cranes can also be considered as a robotic system [13]. Structural optimization of a planar 4-DOF robotic manipulator was obtained using GA to reach a target from a starting point without hitting obstacles. The manipulator had hybrid rotary and prismatic joints [14]. Another attempt is made to optimize the travel time and space of a 3-DOF planar manipulator using GA. The motion planning is optimized in such a way that the robot avoids collision with obstacles within the given torque limits [15]. The trajectory and torque profile of another 3-DOF micro-robot was optimized for surgery. Optimization completed GA, hybrid GA, Pattern Search (PS), and Particle Swarm Optimization (PSO) methods. All the optimization techniques performed almost the same besides hybrid GA, which was slightly better by 3% [16]. Attempts have also been made to design a robotic manipulator with 5-axis for application under Computed Tomography (CT) guidance. Trajectory and path were generated in the same studies [17,18]. The trajectory of a biped robot is optimized for random obstacle avoidance using regression. Ultrasonic distance sensors are used as the sensory input for the regression controller, which then provides the heading angle for the biped robot. Industrial automation benefits from random obstacle avoidance research [19]. Even in the case of flexible robots, adaptive dynamic surface control is implemented to enhance its tracking performance. Parametric uncertainties are overcome using the first-order derivative filter of the inertial parameters [20].

An SCARA robot manipulator is also modeled and simulated as a PRR-type manipulator. Link lengths and DH parameters of this manipulator are calculated mathematically using forward and inverse kinematic equations. The performance of its pick and place was then observed experimentally [21]. Path of a mobile robot is optimized using a GA to obtain a smoother and shorter trajectory. Adaptive penalty factor is used in GA for ensuring the safety of the robot during obstacle avoidance [22]. Trajectory optimization of a 6-DOF manipulator using GA, random average recombination, differential evolution, and linear and geometric cooling strategy was attempted. The goal of this optimization is torque minimization, while a 6-DOF robot is carrying a load from point to point. GA achieved the best result in torque minimization [23].

To minimize the cycle time and vibration of a 6-DOF robot for electronic industries, GA was implemented. It increases the productivity of the electronic PCB manufacturing industry and reduces vibrations by optimizing the velocity of the robot assembly [24]. Much focus has been given to trajectory optimization using GA, regression, and various other algorithms, but research on the optimization of DH parameters based on the required workspace for standard and redundant manipulators has been lacking. Limited research has been conducted for link length optimization of a robot for application in the medical field.

Similar attempts have been made to optimize the structure and trajectory of various DOF robotic manipulators, but there is a limited study on the comparison of link length optimization for the desired workspace reach using GA for 3-DOF, 6-DOF, and 9-
DOF robot manipulators in CT machine environment. Manipulating robots in a CT environment poses quite a challenge, as the body of the CT machine is a major obstacle. The scanning area is the workspace of any manipulator working with a CT machine. The diameter of the scanning region of a standard CT machine is 70 cm [25]; hence, the robotic manipulator must reach the scanning area and maneuver in that region without touching the body of the CT machine.

The novelty of this research is the optimization of link lengths of 3-DOF, 6-DOF, and 9-DOF robotic manipulators. 9-DOF robot is a novel manipulator specifically designed to be mounted on the CT bed with an arc design. Optimization is carried out to determine a better set of link lengths so that the robots can reach and maneuver in the desired three-dimensional CT workspace. Only forward kinematics and error function are used as fitness functions in GA and GA with hybrid functions to optimize the DH parameters including link lengths and joint displacement. Different types of GA coupled with hybrid functions like PS and fmincon are compared for all three robotic manipulators. Different hybrid functions are modified to select the better optimized set of link lengths and joint displacements for each robot.

2. Manipulator designs and DH parameters

Standard rotary 3-DOF, rotary 6-DOF, and a novel 9-DOF hybrid robotic manipulators are designed for this research. The kinematic model of all the robotic manipulators is designed in MATLAB. DH parameters of all the robot manipulator are also determined and mentioned in the respective tables. The red items are the end effector link and the joints including rotary and prismatic, while the blue items denote the rest of the links of the robotic manipulators.

Figure 1 shows the kinematic model of a 3-DOF robot manipulator with rotary joints. The workspace of this robot is spherical when all the joints have full 360° motion.

DH parameters of the 3-DOF manipulator are shown in Table 1 where \( i \) signifies the link number. It is observed that the robot is a standard rotary 3-DOF manipulator.

Figure 2 shows the kinematic model of a 6-DOF robot manipulator with rotary joints. The workspace of this robot is very versatile as it has freedom of motion in every direction and orientation.

DH parameters of the 6-DOF manipulator are shown in Table 2. It is observed that the robot is a standard rotary 6-DOF manipulator.

Figure 3(a) shows the CAD and kinematic model of a novel 9-DOF robot manipulator with rotary and prismatic joints. This robot is designed to be mounted on the CT bed and maneuver in the scanning area. Figure 3(a) shows a CAD model of the robotic manipulator mounted on a CT bed and Figure 3(b) shows the kinematic model of the manipulator. In Figure 3, all the joints of the robotic manipulator are shown at 0° configuration.
Table 2. DH parameters of the 6-DOF robot manipulator.

<table>
<thead>
<tr>
<th>Joint number</th>
<th>Link length displacement $(d_1)$</th>
<th>Joint twist $(\alpha_1)$</th>
<th>Joint angle $(\theta_1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint 1</td>
<td>0</td>
<td>$\pi/2$</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>Joint 2</td>
<td>$l_2$</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>Joint 3</td>
<td>$l_3$</td>
<td>$-\pi/2$</td>
<td>$\theta_3 - \pi/2$</td>
</tr>
<tr>
<td>Joint 4</td>
<td>0</td>
<td>$\pi/2$</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>Joint 5</td>
<td>$l_5$</td>
<td>0</td>
<td>$\theta_5 + \pi/2$</td>
</tr>
<tr>
<td>Joint 6</td>
<td>$l_6$</td>
<td>0</td>
<td>$\theta_6$</td>
</tr>
</tbody>
</table>

DH parameters of the 9-DOF manipulator are given in Table 3. It is observed that the robot is a redundant hybrid 9-DOF manipulator. The arm design of the manipulator constitutes 1-DOF. Arm design is developed for the ergonomics of the patient in a CT machine environment.

3. Mathematical formulation

GA is a heuristic approach to solving an optimization problem [26]. Here, the error functions are minimized to derive the optimum set of link lengths. GA needs a fitness function to optimize these link lengths and for a robotic manipulator, forward kinematic equations serve as a good fitness function because forward kinematic equations relate the DH parameters to the real-world workspace in $X$, $Y$, and $Z$ directions.

The transformation matrix is used to derive the forward and inverse kinematic equations. Mathematical equations for link lengths are derived from the kinematic equations. From these equations, a fitness function for obtaining the link lengths by training the GA is derived. In Eq. (1), shown in Box I, the general

Table 3. DH parameters of the 9-DOF robot manipulator.

<table>
<thead>
<tr>
<th>Joint number</th>
<th>Link length $(l_1)$</th>
<th>Joint displacement $(d_1)$</th>
<th>Joint twist $(\alpha_1)$</th>
<th>Joint angle $(\theta_1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint 1 (vertical)</td>
<td>$l_1$</td>
<td>$d_1$</td>
<td>$-\pi/2$</td>
<td>0</td>
</tr>
<tr>
<td>Joint 1 (horizontal)</td>
<td>$l_2$</td>
<td>$d_2$</td>
<td>$-\pi/2$</td>
<td>$-\pi/2$</td>
</tr>
<tr>
<td>Joint 2</td>
<td>0</td>
<td>$d_3$</td>
<td>$-\pi/2$</td>
<td>$\theta_3 - \pi/2$</td>
</tr>
<tr>
<td>Joint 3</td>
<td>$l_4$</td>
<td>0</td>
<td>$-\pi/2$</td>
<td>$\theta_4 - \pi/2$</td>
</tr>
<tr>
<td>Joint 4</td>
<td>$l_5$</td>
<td>0</td>
<td>0</td>
<td>$\theta_5$</td>
</tr>
<tr>
<td>Joint 5</td>
<td>$l_6$</td>
<td>0</td>
<td>$-\pi/2$</td>
<td>$\theta_6$</td>
</tr>
<tr>
<td>Joint 6</td>
<td>$l_7$</td>
<td>0</td>
<td>$\pi/2$</td>
<td>$\theta_7 + \pi/2$</td>
</tr>
<tr>
<td>Joint 7</td>
<td>0</td>
<td>$d_8$</td>
<td>0</td>
<td>$\theta_8$</td>
</tr>
<tr>
<td>Joint 8</td>
<td>0</td>
<td>$d_9$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Joint 9</td>
<td>0</td>
<td>$d_{10}$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
form of the transformation matrix used for all three-robot manipulators [27] is shown. ‘n’ denotes the number of joints in a robot manipulator. The units of all the link lengths and joint displacements are in cm and the link twist and joint angles are in degrees.

For a 3-DOF manipulator, Eq. (2) shows its transformation matrix formula:

$$T^n_{(n-1)} = \begin{bmatrix}
\cos(\theta_n) & -\sin(\theta_n) \times \cos(\alpha_n) & \sin(\theta_n) \times \sin(\alpha_n) & l_n \times \cos(\theta_n) \\
\sin(\theta_n) & \cos(\theta_n) \times \cos(\alpha_n) & -\cos(\theta_n) \times \sin(\alpha_n) & l_n \times \sin(\theta_n) \\
0 & \sin(\alpha_n) & \cos(\alpha_n) & d_n
\end{bmatrix}$$

(1)

Box I

The desired workspace for the 3-DOF robot ranges from +20 cm to −20 cm in each of X, Y, and Z directions. For a 6-DOF manipulator, Eq. (8) shows its transformation matrix formula:

$$T^n_0 = T^n_1 \times T^n_2 \times T^n_3 \times T^n_4 \times T^n_5 \times T^n_6.$$  

(8)

Eqs. (9) to (12) present the forward kinematic equations of the 6-DOF robot and the fitness function:

$$X_2 = l_2 \times C_1^* \times C_2 - d_4 \times (C_1^* \times S_1 \times S_3 - C_1^* \times C_2^* \times C_3)$$

$$+ l_5 \times S_3 \times (S_1^* \times S_4 - C_4 \times (C_1^* \times C_2^* \times S_3 + C_1^* \times C_3^* \times S_1))$$

$$+ l_6 \times S_3 \times (C_4^* \times S_1 + S_4 \times (C_1^* \times C_2^* \times S_3 + C_1^* \times C_3^* \times S_1))$$

$$+ l_7 \times S_3 \times (C_6^* \times (S_1^* \times S_4 - C_4 \times (C_1^* \times C_2^* \times S_3))$$

$$+ C_1^* \times C_3^* \times S_3)) - C_5^* \times (C_1^* \times S_2 \times S_3 - C_1^* \times C_2^* \times C_3))$$

$$- l_8 \times S_3 \times (C_1^* \times S_2 \times S_3 - C_1^* \times C_2^* \times C_3).$$

(9)

$$Y_2 = l_2 \times C_2^* \times S_1 - l_5 \times S_3 \times (S_1^* \times S_2 \times S_3 - C_2^* \times C_3 \times S_1)$$

$$- l_6 \times C_4^* \times S_5^* \times (C_1^* \times S_4 + C_4 \times (C_2^* \times S_1 \times S_3)$$

$$+ C_5^* \times S_1 \times S_2) + C_6^* \times (S_1^* \times S_2 \times S_3 - C_2^* \times C_3 \times S_1)$$

$$- d_4 \times S_1 \times S_2 \times S_3 - C_4 \times S_1^* \times S_3) - l_5 \times S_6^* \times (C_1^* \times S_4$$

$$+ C_4 \times (C_2^* \times S_1^* \times S_3 + C_3^* \times S_1 \times S_2)) - l_6 \times S_3 \times (C_1^* \times C_4$$

$$- S_4 \times (C_2^* \times S_1 \times S_3 + C_3^* \times S_1 \times S_2))$$

(10)

$$Z_2 = d_1 + d_4 \times S_{23} + l_2 \times S_2 + l_5 \times S^*_{23} \times C_3$$

$$+ l_6 \times C_6^* \times (S_{23} \times C_5 + C_2 \times C_4 \times S_5) + l_5 \times C_{23} \times C_4 \times S_5$$

$$- l_7 \times C_{23} \times S_4 \times S_6.$$  

(11)

3.1. Fitness function for 3-DOF robot

3.1.1. Constraints

In Eqs. (3) to (5) the link length equations of the 3-DOF manipulator are shown. It is observed that all the equations are interdependent, and it is not easy to come to a definitive solution to a range of target coordinate values. Hence, to find an optimum solution, GA is used. Error1 shows the equation for error between all the jth desired coordinates (X, Y, and Z) and derived coordinates:

$$\text{Error}_1 = \sqrt{(X_j - X_1)^2 + (Y_j - Y_1)^2 + (Z_j - Z_1)^2}. \quad (6)$$

Eq. (7) shows the constraints for the 3-DOF robot:

$$l_2 > d_1 \geq l_1.$$  

(7)

Eq. (12) is shown in Box II. X2, Y2, and Z2 are the derived coordinates. δ2, γ2, and θ2 (Eqs. (13)
to (15) are the desired orientation of the approach in
the workspace of the 6-DOF robot and constraints:
\[ \delta_2 = \tan^{-1} \left( \frac{C_{2b}^*}{S_{2b}^* C_0 + C_2^* C_4^* S_0} \right) \]
\[ \left( \frac{S_{2b}^* - C_{2b}^* C_0}{C_{2b}^* S_0 + C_2^* C_4^* C_0} \right) \]
\[ (13) \]
Eq. (14) is shown in Box III.
\[ \beta_2 = -\sin^{-1}(C_0^* S_{2b}^* C_0 + C_{2b}^2 C_4^* S_0 - C_2^* S_{2b}^* S_0). \]
\[ (15) \]
Here, \( C_{2b} \) and \( S_{2b} \) represent cosine \((\theta_2 + \theta_3)\) and sine \((\theta_2 + \theta_3)\), respectively.

3.2. Fitness function for 6-DOF robot
3.2.1. Constraints
Based on Eqs. (9), (11), (13), and (15), forward kinematics and orientations of 6-DOF are complex
and a straightforward solution for link lengths is quite
difficult. Hence, GA is used to determine the link
lengths for the 6-DOF manipulator to cover the desired
workspace. \( \text{Error}_2 \) shows the equation for the error
between all the \( j \)th desired coordinates \((X, Y, \text{and } Z)\)
and orientation of approach \((\text{Euler angles } \delta, \beta, \text{and } \gamma)\)
and derived coordinates and orientation of approach.
\( \gamma_2 \) ranges from \(+60^\circ\) to \(-60^\circ\). \( \text{Error}_2 \) is minimized
using GA, as shown in Eq. (12). The transformation
equation of the novel 9-DOF manipulator is derived using
Eq. (16) [29]:
\[ T_0^{10} = T_0^1 \times T_1^2 \times T_2^3 \times T_3^4 \times T_4^5 \times T_5^6 \times T_6^7 \]
\[ \times T_7^8 \times T_8^9 \times T_9^{10}. \]
(16)
Forward kinematic equations and orientation con-
straints of the novel 9-DOF robotic manipulator are
presented by Eqs. (17)-(19):
\[ X_3 = l_4 + d_3 + d_6 + d_9 + l_5^* C_7 + l_6^* C_7 - d_7^* S_7 \]
\[ - d_8^* S_7 + l_5^* C_7 C_7, \]
\[ (17) \]
\[ Y_3 = l_3 - l_1 + d_2 - l_4^* C_8 - l_9^* C_8 - l_10^* C_8 + l_4^* S_7 \]
\[ + l_6^* S_7 + d_8^* C_7 + d_7^* C_7 + d_9^* S_8 + l_5^* C_7^2 S_7, \]
\[ (18) \]
\[ Z_3 = l_2 + d_4 - d_7 + l_5^* S_0 - l_6^* S_8 - l_7^* S_8 \]
\[ - l_10^* S_8 - d_1^* C_8, \]
\[ (19) \]
\[ \text{Error}_3 = \sqrt{(X_3 - X_3)^2 + (Y_3 - Y_3)^2 + (Z_3 - Z_3)^2}. \]
(20)
\( X_3, Y_3, \) and \( Z_3 \) are the target coordinates in
the workspace of the 9-DOF robot. The desired workspace
for the 6-DOF robot ranges from \(+20 \text{ cm to } -20 \text{ cm}
in each of \( X, Y, \text{and } Z \) directions. Here, \( C_n \) and \( S_n \)
are cosine and sine of the \( n \)th joint, respectively, where
\( n = 1, 2, \ldots, n. \)

3.3. Fitness function for 9-DOF robot
3.3.1. Constraints
Constraints are added in line with Eqs. (21) to (23):
\[ \theta_3 = 0, \]
\[ (21) \]
\[ \theta_4 = \theta_7, \]
\[ (22) \]
\[ \theta_5 = -\theta_6. \]
\[ (23) \]
These constraints provide the desired orientation of the
approach to retrieval of tissue samples while working
on a CT machine. \( \text{Error}_3 \) shows the equation for error
between all the \( j \)th desired coordinates \((X, Y, \text{and } Z)\)
and derived coordinates. As the orientation of the
robotic 9-DOF manipulator is predefined for
the specified tissue retrieval task, the error function in
Eq. (20) does not include the orientation segment, as
included in Eq. (12) for the 6-DOF manipulator. These
equations are used to iterate the GA to find the sets of
link length of 3-DOF, 6-DOF, and novel hybrid 9-DOF
robotic manipulators. The desired workspace for the
9-DOF robot ranges from \(+20 \text{ cm to } +40 \text{ cm in } X, \text{ }
+20 \text{ cm to } -20 \text{ cm in } Y, \text{ and } Z \) directions.

3.4. Boundary conditions
Infinite solutions for link lengths and joint displace-
ments are possible for an open chain robotic manipula-
tor system. Hence, boundary conditions need to be set
for optimization. Upper and lower boundaries for each
robotic manipulator design are shown in Table 4. All
dimensions are in centimeter (cm). These boundary
Table 4. Upper and lower boundaries for 3-DOF, 6-DOF, and 9-DOF robotic manipulators.

<table>
<thead>
<tr>
<th>Link parameter</th>
<th>3-DOF robot Lower</th>
<th>3-DOF robot Upper</th>
<th>6-DOF robot Lower</th>
<th>6-DOF robot Upper</th>
<th>9-DOF robot Lower</th>
<th>9-DOF robot Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>1</td>
<td>15</td>
<td>$d_1$</td>
<td>1</td>
<td>15</td>
<td>$d_3$</td>
</tr>
<tr>
<td>$l_2$</td>
<td>5</td>
<td>25</td>
<td>$l_2$</td>
<td>5</td>
<td>25</td>
<td>$l_4$</td>
</tr>
<tr>
<td>$l_3$</td>
<td>5</td>
<td>25</td>
<td>$l_3$</td>
<td>0</td>
<td>5</td>
<td>$l_5$</td>
</tr>
<tr>
<td>$d_4$</td>
<td>5</td>
<td>20</td>
<td>$d_6$</td>
<td>1</td>
<td>15</td>
<td>$l_7$</td>
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<td>$l_6$</td>
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<td>10</td>
<td>$d_8$</td>
<td>1</td>
<td>5</td>
<td>$d_9$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$d_{10}$</td>
<td>1</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

4. Results and discussion

GA is trained to obtain the optimum set of link lengths for 3-DOF, 6-DOF, and novel 9-DOF robot manipulators by varying the hybrid functions. Optimization of link lengths and joint displacement is carried out by keeping the rotation angles constant. Hence, link lengths and joint displacement are the variables. The hybrid functions used here are PS and finnicn to minimize the error functions. Firstly, an SGA architecture is trained, which has no hybrid function, and the link lengths are found; then, PS and finnicn are applied. All the other parameters are kept constant. The initial parameters include the creation function, initial population size, initial population range, fitness scaling, selection function, crossover fraction, elite count, mutation function, and constraint algorithm. The termination criteria parameters including the number of generations, stall generation, and function tolerance are also kept constant. The creation function is a nonlinear feasible function with an initial population size of 5. The initial population size is kept at 5 to reduce the computational cost. The initial population range is kept between $-10$ and $+10$. Fitness scaling is based on the rank system and stochastic uniform is used as the selection function. The crossover fraction is kept at 0.8 and the elite count is 2.5. The mutation function is selected as adaptive feasible and augmented. The Lagrangian function is taken as the nonlinear constraint algorithm. The formula for Lagrange function is shown in Eq. (24):

$$L = f(x) - \lambda g(x).$$

The variable $f(x)$ is the fitness function and $g(x)$ the equality function according to Eqs. (19)–(21). The variable $\lambda$ is the Lagrange multiplier which is adjusted while training of the GA.

As for the termination condition, the number of generations is 100, stall generations are 50, and function tolerance is kept at $10^{-6}$. The link lengths and joint displacements obtained after optimization
will have a sufficient level of accuracy with a tolerance like $10^{-6}$. Even if one decides to manufacture the links of the robotic manipulator, $10^{-6}$ cm tolerance provides a sufficient amount of accuracy. These parameters are kept constant throughout the study.

Using the parameters mentioned and training with SGA, PS, and fmincon, the overall fitness value and fitness of each individual in the population of a 3-DOF robotic manipulator are plotted in Figure 5. Table 5 shows the optimum link lengths of a 3-DOF manipulator, obtained after iteration using all the three functions. According to the graphs, the training reached global minima.

It is observed in Figure 5 that the best overall fitness value for a 3-DOF robot is obtained using PS while the overall fitness values for SGA and fmincon are 1.16 and 1.38, respectively. Hence, the link lengths determined using PS are optimized better for a 3-DOF robot to reach the desired workspace. The workspaces of this robot are plotted and shown in Figure 6. These workspaces represent the volume covered by the 3-DOF robot with the optimized sets of link length obtained after training the SGA, PS, and fmincon.

Figure 6 shows the desired workspace volume (black) versus the workspace plotted after obtaining the link lengths using SGA (blue), PS (green), and fmincon (red) in two (Figure 6(a)) and three dimensions (Figure 6(b)). It is observed that the SGA produces the workspace closest to the desired one, but the dead zone is very large and, thus, the robot with those link lengths becomes impractical. Dead zones are the regions within the workspace where the end effector of the robotic manipulator cannot be reached. These dead zones result from physical constraints such

![Figure 5](image)

**Figure 5.** (a) Training graph of Standard Genetic Algorithm (SGA). (b) Training graph of Genetic Algorithm (GA) with Pattern Search (PS). (c) Training graph of Genetic Algorithm (GA) with fmincon.

<table>
<thead>
<tr>
<th>Link parameter</th>
<th>SGA (cm)</th>
<th>PS (cm)</th>
<th>Fmincon (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>d1</td>
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<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>l2</td>
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<td>20.00</td>
<td>14.05</td>
</tr>
<tr>
<td>l3</td>
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<td>8.48</td>
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</table>

![Figure 6](image)

**Figure 6.** (a) 2D workspace analysis of 3-DOF robot after using Standard Genetic Algorithm (SGA), Pattern Search (PS) and fmincon. (b) 3D workspace analysis of 3-DOF robot after using Standard Genetic Algorithm (SGA), Pattern Search (PS) and fmincon.
as link lengths and joint displacements. On the other hand, the workspace generated by fmincon does not cover the necessary desired volume, while PS provides better results. The entirety of the workspace is covered with the link lengths obtained by PS. Moreover, the dead zone is relatively small. Hence, PS becomes a better choice to obtain optimized link lengths for the 3-DOF robot.

Now, a 6-DOF manipulator is trained using the same parameters and hybrid functions. The overall fitness value and fitness of each individual in the population of a 6-DOF robotic manipulator are plotted in Figure 7. Table 6 shows the optimum link lengths of a 6-DOF manipulator obtained after iteration using all the three functions. According to the graphs, the training reached global minima.

According to the graphs in Figure 7, the best overall fitness value for a 6-DOF robot is obtained with PS 0.34, while the overall fitness values for SGA and fmincon are 1.94 and 1.54, respectively. Hence, it is observed that the link lengths found using PS are theoretically more optimized to reach the desired workspace. After training all three GAs, the set of link lengths is obtained and the workspace of the 6-DOF robot using this link lengths is plotted in Figure 8.

The constraints and error function ensure that the robotic manipulator reaches the desired position in the workspace at the desired orientation.

Figure 8 shows the workspace analysis of the 6-DOF robot with link lengths obtained after optimization using SGA (blue), PS (green), fmincon (red), and desired workspace (black) in two (Figure 8(a)) and three dimensions (Figure 8(b)). It is observed that the volumes covered using SGA and PS are very similar, while the workspace volume obtained by fmincon is slightly smaller with a smaller dead zone.

### Table 6. Link lengths obtained after optimizing the error function using Standard Genetic Algorithm (SGA), Pattern Search (PS), and fmincon for a 6-DOF robot.

<table>
<thead>
<tr>
<th>Link parameter</th>
<th>SGA (cm)</th>
<th>PS (cm)</th>
<th>Fmincon (cm)</th>
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<tr>
<td>$d_1$</td>
<td>3.058</td>
<td>3.004</td>
<td>3.00</td>
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<tr>
<td>$l_2$</td>
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<td>19.907</td>
<td>19.032</td>
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<td>$l_3$</td>
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<td>0.101</td>
<td>0.610</td>
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<tr>
<td>$d_4$</td>
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<td>13.527</td>
<td>12.845</td>
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<tr>
<td>$l_5$</td>
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<td>6.791</td>
<td>6.951</td>
</tr>
<tr>
<td>$l_6$</td>
<td>0.166</td>
<td>0.117</td>
<td>1.289</td>
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![Figure 7](image_url)

**Figure 7.** (a) Training graph of Standard Genetic Algorithm (SGA). (b) Training graph of Genetic Algorithm (GA) with Pattern Search (PS). (c) Training graph of Genetic Algorithm (GA) with fmincon.
All the methods cover the desired workspace volume and hence, any function can be applied, but Fmincon is more acceptable.

Link lengths for the novel 9-DOF robotic manipulator are optimized using the same parameters and hybrid functions. Here, the desired orientation is fixed as the orientation of approach near the CT machine is limited so as to avoid a collision. The overall fitness value and fitness of each individual in the population of a 9-DOF robotic manipulator are plotted in Figure 9. Table 7 shows the optimum link lengths of a 9-DOF manipulator, obtained after iteration using all the three functions. It is observed in the graphs that the training reached global minima.

Figure 9 shows that the best fitness value for the novel 9-DOF robot designed for tissue retrieval in a CT environment is found as 0.121 using SGA method. Fitness values for PS and Fmincon are 0.124 and 1.36, which are more than those obtained by the SGA method. The set of link lengths obtained from the SGA method should ensure that the robotic manipulator will reach the entire desired target in the desired orientation safely. Lengths of Links 1 and 2 are constant due to CT bed constraints and arm structure design. The workspace of each set of link lengths obtained after training all three GA methods is plotted in Figure 10.

Figure 10 depicts the comparison of workspaces plotted after obtaining link lengths using SGA (blue), PS (green), Fmincon (red), and the desired target workspace (black) in two (Figure 10(a)) and three dimensions (Figure 10(b)). It is observed that the

<table>
<thead>
<tr>
<th>Link parameter</th>
<th>SGA</th>
<th>PS</th>
<th>Fmincon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(cm)</td>
<td>(cm)</td>
<td>(cm)</td>
</tr>
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<td>20.00</td>
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<td>1.002</td>
<td>1.00</td>
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<td>9.990</td>
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Figure 9. (a) Training graph of Standard Genetic Algorithm (SGA). (b) Training graph of Genetic Algorithm (GA) with Pattern Search (PS). (c) Training graph of Genetic Algorithm (GA) with Fmincon.

Figure 10. (a) 2D Workspace analysis of the novel 9-DOF robot manipulator. (b) 3D Workspace analysis of the novel 9-DOF robot manipulator.
desired workspace is cylindrical in shape as the interior scanning area of a CT machine is cylindrical. The cylindrical workspace is solely targeted on the CT bed and scanning area. The advantage of a cylindrical workspace over a spherical one is that the optimization is focused on the desired CT bed and scanning area. Deadzone is not present within the CT scanning workspace because the optimization is carried out only for the cylindrical workspace. Workspace obtained using PS shows better optimization as all the coordinates of the target workspace are covered. A negligible dead zone is observed in the workspace of the novel 9-DOF robot manipulator. SGA and fmincon cover most of the target workspace, but do not cover some coordinates. Hence, it is observed that the PS hybrid optimization function is useful for finding the link lengths of a robotic manipulator designed for specified tasks under CT machine with only forward kinematic equations and primary constraints.

5. Conclusion

In this research, a novel method was proposed to calculate and optimize the DH parameters including link length and joint displacement of a robotic manipulator with 3-DOF, 6-DOF, and 9-DOF using Standard Genetic Algorithm (SGA), Genetic Algorithm (GA) with Pattern Search (PS), and GA with fmincon hybrid functions. Forward kinematics and error function were used as the fitness function for training the GA and GA with hybrid functions. DH parameters obtained after optimization based on the required workspace were then used for simulating the workspace of the robotic manipulator in MATLAB to observe the workspace and dead-zone coverage. Results demonstrated that in the case of a 3-DOF robot, the GA with PS hybrid function provided better-optimized results as it covered the target workspace with minimal dead zones. For a 6-DOF robotic manipulator, GA with fmincon achieved better results covering the desired workspace in the desired orientation as well. The dead zone for the 6-DOF robotic manipulator after using fmincon was smaller than the other two GA functions. As for the novel 9-DOF robot designed to manipulate a needle in the Computed Tomography (CT) machine environment, GA with PS produced better results. The cylindrical workspace required for this task was covered fully by using the PS hybrid function. With a clear lack of significant research on optimization of DH parameters for a medical robotic manipulator, the proposed method shows promising results with different Degree Of Freedom (DOF) robot manipulators by minimizing the error function using forward kinematic equations. The only assumptions this method requires are the constraints for the link lengths. Hence, this proposed method can be successfully implemented for finding the optimum link lengths of open chain robotic systems designed to function in the desired workspace.

Nomenclature

\[ l \] Link length
\[ d \] Joint displacement
\[ \alpha \] Link twist
\[ \theta \] Joint angle
\[ C_n \] Cosine of nth joint
\[ C_{2\theta} \] Cosine \((\theta_2 + \theta_3)\)
\[ S_n \] Sine of nth joint
\[ S_{2\theta} \] Sine \((\theta_2 + \theta_3)\)
\[ \delta \] Euler angle about X axis
\[ \beta \] Euler angle about Y axis
\[ \gamma \] Euler angle about Z axis
\[ n \] Joint number of the robotic manipulator

References


Biographies

Shubham Kamlesh Shah is a PhD Research Scholar at School of Mechanical Engineering, KIIT deemed to be University. He specialized in robotics, machine learning, and machine design. He completed his BSc and MSc in Technology in Mechanical Engineering and Machine Design from KIIT deemed to be University, Bhubaneswar, Odisha India. He has published many research articles in international journals and conferences. He also has three national and international patents published.

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