Performance Improvement of a Grid-connected Voltage Source Converter Controlled by Parabolic PWM Current Control Scheme

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Abstract— Parabolic carrier pulse width modulation (PWM) method is considered as one of the direct current control methods which has been proposed for the voltage-source converters (VSCs). This method has an excellent dynamic response. Besides, it proposes a constant switching frequency by employing a pair of parabolic PWM carriers. However, it suffers from some drawbacks and limitations. The major drawback of this method is its sensitivity to the inductance variations. In other words, in grid-connected applications the exact value of grid inductance should be exactly known to achieve a proper performance from this method. Moreover, it is essential that during each switching cycle the voltage at the point of common coupling remains constant. In grid connected applications such as active power filter these drawbacks may lead to operate at variable or non-expected frequencies. Therefore, this paper concerns the suggestions to deal with the situation. In this paper, by applying the conventional method the aforementioned problems are examined in a grid-connected active power filter. It is shown analytically that by using the proposed method, problems of sensitivity to inductance changes and also necessity to constant voltage at point of common coupling in a switching period will be solved. Finally, simulation and experimental results are presented.

Index Terms— Parabolic PWM current controller, Current controller, Hysteresis control, Switching frequency, Shunt active power filter.

I. INTRODUCTION

Nowadays power electronics converters are used in wide variety of applications [1]-[4]. Among them, the voltage-source converters (VSCs) are used in various applications like electric drives, uninterruptible power supplies and also in active power filters. Due to increased use of VSCs in power electronic systems, current control strategy for them has become one of the topics of interest to researchers [5]-[19].

To implement a current loop with fast response and high tracking precision for voltage source converter, different methods including conventional hysteresis control [1]-[9], adaptive hysteresis control [11], carrier-based pulse width modulation (PWM) control [5]-[7], delta modulation control[5], different types of space vector modulation (SVM) like conventional SVM [6], three-dimension SVM [13],[14], space vector current control on the rotating xy-coordinates [15] etc, have been proposed. All of the mentioned methods can be divided into two below general groups:

- the indirect current control
- the direct tracking error control through PWM

In the first group methods, the current error is imposed on a controller to generate the reference voltage signal. Due to the limited band-width of the current control loop, these methods do not have good dynamic and stable performances in comparison with the second group methods [16].

Among the second group methods, the conventional hysteresis method is known as the most popular one due to its implementation simplicity, current loop fast response, inherent ability of limiting the current peak, and independency of knowing the system parameters. Notwithstanding merits, the hysteresis method major drawback is its switching frequency variation which increases the system switching losses. Variable switching frequency causes problems in designing the passive filter elements in some applications like active power filters as well as some electromagnetic interference (EMI)-related problems [11].

To overcome these problems, different methods have been proposed to generate varying hysteresis band by using the phase-locked loop (PLL) [12], receive feedback of the peak current error or digitally implement the predictive or the adaptive control. However, these methods make the controlling system complicated and deteriorate the dynamic responses [16].

One of the second group methods which is a combination of some features of hysteresis and non-linear carrier-based controlling methods is the current controlling method by using parabolic PWM [16]-[20]. In addition to excellent dynamic response, the
parabolic PWM method proposes a constant switching frequency by utilizing a pair of parabolic PWM carriers. Consequently, it does not need any complicated feedback controlling scheme to make its frequency constant. Besides, this method has a better stability and an easier implementation in comparison with other hysteresis methods [16]. However, this method also has the following drawbacks and limitations which are going to be explained at the next paragraph:

- Sensitivity to the inductance value
- Necessity of constant voltage during a switching period

The major drawback is its sensitivity to the inductance variations which leads to necessity of knowing the exact value of the inductance. Knowing the exact value of the inductance is crucial for adjusting the parabolic carrier amplitude by the controller. Moreover, one of the constraints of this method is the necessity of constant voltage at the point of common coupling (u_s in Fig. 1) in a switching period. The output voltage of a PWM inverter has a switching behaviour in nature. However, the voltage at the point of common coupling can be considered as a sinusoidal one, provided that the output voltage of the inverter is filtered ideally or an ideal AC voltage source is connected to it. Without these two conditions the voltage at the point of common coupling will follow the pulse width modulated voltage waveform and it will not be constant during a switching period. This situation is common in practice, because of the non-ideally performed filtering and the presence of grid impedance. Since the mentioned voltage is varying in a switching period in some applications, for example in active power filters, this problem can disrupt the strong performance of the method and lead to variable frequency in mentioned applications.

To overcome these problems, it is proposed to use a well-known LCL filter between the inverter bridge and the grid. By means of an LCL filter the dependency to the grid side inductance goes away and the necessity to constant voltage is diminished. However, using LCL filter affects the operation of the parabolic PWM controller. Therefore, to be assured about utilizing parabolic PWM control along with LCL filter the relations should be revised completely. The obtained structure is called improved VSC with parabolic PWM control (IVSCPCC) in this paper.

In the remaining part of the paper, for convenience CVSCPCC be used instead of the conventional VSC with parabolic PWM current control. To evaluate the effects of the mentioned drawbacks on the performance of CVSCPCC and IVSCPCC, both of them are utilized to generate compensating current -for a voltage converter three-phase four-wire shunt active power filter (APF) and the related analytical discussion are provided. Before introducing the main issue, it is necessary to give an explanation of the APFs.

The desire to utilize active power filters is increasing due to increasing harmonics pollution in power grids [21] and problems associate with the passive filters [22]. The main task of APFs is to compensate the current or voltage harmonics though nowadays these filters can do some other tasks, as well. In this regard, more information is provided in [23].

Until now, different topologies have been proposed for APFs. Among them, the APFs with the VSCs are highly used to compensate the current harmonics and satisfy the reactive power issue.

The performance of the APF is as follows: at first, it samples the load current, the line voltage or the phase voltage concerning the used algorithm or method; then, by applying the mentioned algorithm, it extracts compensating currents. Finally, the compensating current is injected into the point of common coupling by sending the necessary orders through the current controller into the switches’ gates.

In this paper to extract the compensating signals the synchronous reference frame (SRF) method [24],[25] is used.

2. PARABOLIC PWM PRINCIPLES

To explain parabolic PWM principles, a simplified controlling diagram of the VSC with parabolic PWM current control for single-phased inverter is shown in Fig. 1.

In this figure, \( L_f \) is the converter output inductor, \( U_p \) and \( U_n \) are the voltages at the positive and negative DC rails, respectively, \( S_p \) and \( S_n \) are the upper and lower switches of the inverter leg, \( i \) is the inductor-current, \( i_{ref} \) is the reference current and \( \Delta i \) is the current error which is defined as followings:

\[
\Delta i = i - i_{ref}
\]  

(1)

Moreover, the parabolic PWM current controller (PCC) block is considered as the parabolic PWM modulator. A typical current waveform in one PWM cycle of the mentioned converter is as Fig. 2 in which \( T_p \) and \( T_n \) are the conduction periods of \( S_p \) and \( S_n \) switches, respectively. \( T \) is the switching period which is equal to \( T = T_p + T_n \). In addition, \( \Delta i_{p-peak} \), \( \Delta i_{n-peak} \) and \( \Delta i_{p-p} \) are respectively known as values of the positive peak, negative peak and peak-to-peak of the current error, as well.

Besides, it is supposed that \( U_p, U_n, u_s \) and the \( i_{ref} \) slope are all constant in one switching period and the inverter current \( i \) follows
the reference current $i_{ref}$ symmetrically and continuously, that is:

$$\Delta i_{p,\text{peak}} = -\Delta i_{n,\text{peak}} = \frac{\Delta i_{p-p}}{2} \quad (2)$$

When the $S_p$ switch is on, it is possible to justify the following relationship:

$$L_f \frac{di}{dt} = U_p - u_s \quad (3)$$

Substituting (1) within (3) yields:

$$L_f \frac{d\Delta i}{dt} = U_p - u_s - L_f \frac{di_{ref}}{dt} \approx L_f \frac{\Delta i_{p-p}}{T_p} \quad (4)$$

Besides, when $S_n$ switch is on, the following relationship is satisfied:

$$U_n - u_s - L_f \frac{di_{ref}}{dt} \approx -L_f \frac{\Delta i_{p-p}}{T_n} \quad (5)$$

Subtracting (5) from (4) yields:

$$\Delta i_{p-p} = \frac{T}{L_f} (U_p - U_n) \left[ T_p \left( \frac{T_p}{T} \right)^2 \right]$$

$$= \frac{T}{L_f} (U_p - U_n) \left[ \frac{\Delta i_{p-p}}{T} \right] \quad (6)$$

Consequently, it is possible to write the $\Delta i_{p,\text{peak}}$ and $\Delta i_{n,\text{peak}}$ relationships as the followings:

$$\Delta i_{p,\text{peak}} = -\Delta i_{n,\text{peak}}$$

$$= \frac{T}{2L_f} (U_p - U_n) \left[ \frac{\Delta i_{p-p}}{T} \right] \quad (7)$$

Therefore, if the converter seeks to follow the reference current $i_{ref}$, the instantaneous current errors, must be limited between $\Delta i_{p,\text{peak}}$ and $\Delta i_{n,\text{peak}}$. In fact, in this method, controlling the current error is done by comparing it with a pair of parabolic PWM carriers (a positive carrier and a negative one). Noticeably, when the current error meets these carriers, it can determine the switches states. The positive parabolic function $f_{pa}(t)$ shown in Fig. 3 can be defined as below:

$$f_{PA(t)} = K \left[ \frac{t}{T^*} - \left( \frac{t}{T^*} \right)^2 \right], \quad \text{for} \quad 0 \leq t \leq T^* \quad (8)$$
\[ K = \frac{T_s \times (U_p - U_n)}{2L_f} \] (9)

In the above equations, \( T_s \) is the reference switching period which determines the switching frequency. It is apt to mention that the negative carrier waveform can then be easily generated as \(-f_{nA}(\tau)\).

As seen in Fig. 3, when the current error cuts the positive parabolic waveform in \( t = T_p \) the upper switch of the inverter phase leg \( S_p \) turns off, simultaneously the lower switch \( S_n \) turns on and the negative carrier waveform starts from zero. Consequently, the switches modes are determined in this PWM method. Noticeably, this plan comes true for the ideal mode which has no dead time.

Since the dead time leads to decrease in switching frequency in comparison with the desired quantity, as mentioned in [18] to compensate the dead time effect, the improved \( f_{PA-offset}(t) \) function must be used instead of applying the conventional \( f_{PA}(t) \) function for the carrier waveform (10).

\[
\begin{align*}
\begin{cases}
 f_{PA-offset}(t) &= f_{PA}(t) - \Delta \nu_2 = f_{PA}(t) - f_{PA}(t_{DT}) \\
 f_{PA-offset}(\tau) &= -(f_{PA}(\tau) - \Delta \nu_2) = -(f_{PA}(\tau) - f_{PA}(t_{DT}))
\end{cases}
\end{align*}
\] (10)

where \( f_{PA}(t) \) is the same as (8) and \( t_{DT} \) is the dead time.

\( \Delta \nu_2 \) is the offset of improving \( f_{PA}(t) \) which is calculated from the following relation:

\[ \Delta \nu_2 = K \left[ \frac{t_{DT}}{T_s} \ln \left( \frac{t_{DT}}{T_s} \right) \right] = f_{PA}(t_{DT}) \] (11)

That \( K \) is calculated based on (9).

More comprehensive analysis of the parabolic PWM method and its modes are presented in [16]. Despite the benefits of this method, it also has some drawbacks and limitations, mentioned in introduction. These drawbacks can cause some problems in applications like APFs and they are evaluated in the next sections.

3. APPLYING THE PARABOLIC PWM CURRENT CONTROL IN THE THREE-PHASE FOUR-WIRE SAPF

To investigate the claimed problems of the CVSCPCC, it is applied to generate compensating current for the SAPF with the topology shown in Fig. 4.

For a closer look, single-phase equivalent circuit of the mentioned APF, related to phase \( A \), that is shown in Fig. 5 will be analyzed. To ensure the correct operation of this method here, the governing relationships in this case must be matched with relationships obtained in section II.

From Fig. 5, the following relations can be established:

\[ i_{SA} = i_{LA} - i_A \] (12)

\[ i_n = i_{SA} + i_{SB} + i_{SC} \] (13)

\[ i_s = i_{LA} - i_A + i_{SB} + i_{SC} \] (14)

Considering the paths shown in Fig. 5 and concerning the aforementioned procedure of extracting relationships, when \( S_1 \) is on (Fig. 5a) or \( S_4 \) is on (Fig. 5b), the following relations can be written:
\begin{align}
(L_f + L_S + L_n) & \frac{\Delta i_{p-p}}{T_p} \\
\equiv & \ U_p - u_{SA} + (L_S + L_n) \frac{d i_{LA}}{dt} \\
& -(L_f + L_S + L_n) \frac{d i_{ref -A}}{dt} + L_n \ \frac{d}{dt} (i_{SB} + i_{SC}) \\
\tag{15}
\end{align}

\begin{align}
(L_f + L_S + L_n) & \frac{-\Delta i_{p-p}}{T_n} \\
\equiv & \ U_n - u_{SA} + (L_n + L_n) \frac{d i_{LA}}{dt} \\
& -(L_f + L_S + L_n) \frac{d i_{ref -A}}{dt} + L_n \ \frac{d}{dt} (i_{SB} + i_{SC}) \\
\tag{16}
\end{align}

By comparing the above relations with \((4)\) and \((5)\), it can be deduced that \((L_n + L_n) \frac{d i_{LA}}{dt}\) and \(L_n \ \frac{d}{dt} (i_{SB} + i_{SC})\) terms are added and in the coefficients of \(\frac{\Delta i_{p-p}}{T_p}\) and \(\frac{\Delta i_{p-p}}{T_n}\), \(L_f\) is changed into \((L_f + L_n)\). Here, there are some problems. Concerning the parabolic PWM method sensitivity to the inductance variations, it is necessary to know the precise values of the grid and neutral inductances \((L_n, L_n)\) to set properly the parabolic carrier amplitude. It should be mentioned that finding the precise value of the grid inductance is a difficult task. On the other hand, the time related load current changes \(\frac{d i_{LA}}{dt}\) and also \(i_{SB}\) and \(i_{SC}\) and consequently the voltage at the point of common coupling \((u_{DCVSCPCC}(t))\) are not constant during a switching period and they have some oscillations with the APF switching frequency which is also verified by simulation results. Therefore, by subtracting \((16)\) from \((15)\), same as section II, \((L_n + L_n) \frac{d i_{LA}}{dt}\) and \(L_n \ \frac{d}{dt} (i_{SB} + i_{SC})\) terms are not omitted and the ultimate relationships forms are different from the main parabolic PWM method relationships, so even if \(L_n\) and \(L_n\) are precisely determined \((L_f + L_n + L_n)\) be used instead of \(L_f\) in computation, the oscillation of \(i_{LA}\) causes the switching frequency not be constant.

To get rid of these problems, the IVSCPCC has been proposed in this paper that is shown in Fig. 6.

4. PROPOSED IMPROVED VSC WITH PARABOLIC PWM CURRENT CONTROL (IVSCPCC)

To analyze the IVSCPCC, relationships are derived based on the procedure mentioned in section II. Also, this time, for derivation of relationships the constraint of voltage constancy of \(u_{IVSCPCC}(t)\) that was a limitation for parabolic PWM method, has been omitted. By noticing this fact that the value of \(R_i\) is small, and so its influence is insignificant, below relations can be established for two \(S_p\) on and \(S_p\) off states:

\begin{align}
L_1 \ \frac{di}{dt} = U_p - u_s(t) \\
\tag{17}
\end{align}

\begin{align}
L_1 \ \frac{di}{dt} = U_n - u_s(t) \\
\tag{18}
\end{align}

Now, by substituting \((1)\) in \((17)\) and \((18)\), it is possible to write:
\[ \frac{d\Delta i}{dt} = U_p - U_s(t) - L_u \frac{di_{ref}}{dt} \quad (19) \]

By multiplying two sides of (19) to \( dt \) and by integrating over \((0,T_p)\) and supposing that \( \int u_s(t) dt = U_s(t) \), there is:

\[ \int L_u d\Delta i = \int U_p dt - \int u_s(t) dt - \int L_u di_{ref} \quad (21) \]

and so:

\[ L_u (\Delta i_{p,peak} - \Delta i_{n,peak}) = T_p U_p - U_s(T_p) + U_s(0) - L_u \left[ i_{ref}(T_p) - i_{ref}(0) \right] \quad (22) \]

By dividing two sides of (22) into \( T_p \), it is possible to write:

\[ L_u \left( \frac{\Delta i_{p,p}}{T_p} \right) = U_p - \frac{U_s(T_p)}{T_p} + \frac{U_s(0)}{T_p} - \frac{L_u}{T_p} \left[ i_{ref}(T_p) - i_{ref}(0) \right] \quad (23) \]

By repeating the above procedure for (20) and by noticing the fact that in this mode the integral is calculated over \((T_p,T)\) interval, there is:

\[ -L_u \left( \frac{\Delta i_{n,p}}{T_n} \right) = U_n - \frac{U_s(T)}{T_n} + \frac{U_s(T_p)}{T_n} - \frac{L_u}{T_n} \left[ i_{ref}(T) - i_{ref}(T_p) \right] \quad (24) \]

Since the \( i_{ref} \) slope in a switching period is supposed constant, it is possible to write:

\[ \frac{L_u}{T_p} \left[ i_{ref}(T_p) - i_{ref}(0) \right] = \frac{L_u}{T_n} \left[ i_{ref}(T) - i_{ref}(T_p) \right] \quad (25) \]

Considering (25), by subtracting (24) from (23) there is:

\[ L_u \Delta i_{p,p} \left( \frac{1}{T_p} + \frac{1}{T_n} \right) = U_p - U_n - \frac{U_s(T_p)}{T_p} + \frac{U_s(T)}{T_p} + \frac{U_s(T_p)}{T_n} - \frac{U_s(T_p)}{T_n} \quad (26) \]

By simplifying the (26) and supposing that \( H = \frac{U_s(T_p)}{T_p} + \frac{U_s(0)}{T_p} + \frac{U_s(T)}{T_n} - \frac{U_s(T_p)}{T_n} \), there is:
\[ \Delta i_{p-p} = \frac{T}{L_1} \left[ \frac{T_p}{T} - \left( \frac{T_p}{T} \right)^2 \right] \left( U_p - U_n + H \right) \]  

(27)

By comparing (27) with (6), it is possible to observe that if \( H \) term equals to zero, parabolic PWM principles mentioned in section II can be used yet. \( H \) term will equal to zero in case that:

\[ \frac{U_s(T_p) - U_s(0)}{T_p} = \frac{U_s(T) - U_s(T_p)}{T_n} \]

(28)

In other words, if the slope of the waveform obtained from the integral of the capacitor voltage \( (u_c(t)) \) remains constant in a switching period, it is possible to apply the discussed parabolic PWM method principles.

By choosing proper capacitors \( (C, C_d) \) the \( u_c(t) \) voltage waveform will have quasi-sinusoidal oscillations in a switching period, and these oscillations can be estimated by a sinusoidal function like (29):

\[ u_c(t) = -A \cos(t) \]

(29)

Then by integrating (29), there can be:

\[ U_s(t) = \int u_c(t).dt = -A \sin(t) \]

(30)

Relation (28) is satisfied for the above function, if \( \frac{\sin(T_p) - \sin(0)}{T_p} = \frac{\sin(T) - \sin(T_p)}{T} \). Since \( T_p \) and \( T \) are known as very small quantities, by applying trigonometric equivalent equations it is plausible to write:

\[ \frac{T_p - 0}{T_p} = \frac{T - T_p}{T_n} = 1 \]

(31)

Therefore, in this case, (28) satisfies and it is possible to use the parabolic PWM method principles mentioned in section II.

Generally, by using the \textit{IVSCPCC}, if \( u_c(t) \) voltage of Fig. 6 has a quasi-sinusoidal oscillation during switching or if oscillations behave such that it is possible to make \( H \) term of (27) equal to zero, it is possible to apply parabolic PWM method principles and so there is no need to \( u_{\text{IVSCPCC}}(t) \) or even \( u_c(t) \) voltage constancy in a switching period that was one of the constraints and so a limitation of parabolic PWM method.

Unlike the \textit{CVSCPCC}, in the \textit{IVSCPCC}, to achieve the specific frequency it is sufficient to set inductance value in the controller with the value of \( L_1 \) and the precise value of the inductance that controller sees in the output is not necessary for adjusting the parabolic carrier magnitude.

The \textit{IVSCPCC} is used in mentioned SAPF and relationships have been derived (Fig. 7). If Kirchhoff voltage law (KVL) equations are written for paths shown in Fig. 7 and relationships are derived based on the procedure mentioned in section II, by noticing this fact that the value of \( R_1 \) is small, below relationships can be established for two states of \( S_1 \) on and \( S_1 \) off:

\[ L_1 \frac{\Delta i_{p-p}}{T_p} \approx U_p - u_s(t) - L_1 \frac{di_{\text{ref}}}{dt} \]

(32)

\[ -L_4 \frac{\Delta i_{p-p}}{T_n} \approx U_p - u_s(t) - L_4 \frac{di_{\text{ref}}}{dt} \]

(33)

As observed, some extra terms of (15) and (16) are removed in this case; moreover, the grid and neutral inductances are not used. Also, by choosing proper capacitor for \( C_1, C_d \), voltages across capacitors will be quasi-sinusoidal and IVSCPCC will work as expected.

5. The Elements Designing of the Proposed PCC

Since the output part of the IVSCPCC is actually an LCL filter, so to calculate its elements by means of discussed methods in [26]-[28], it is plausible to act as below.
At first, the total apparent power of system $S_b$ is calculated by the below equation:

$$ S_b = \sqrt{3} V_b I_b $$

(34)

In the above equation, $I_b$ and $V_b$ are the line current and line voltage which are considered as the base values.

The base values of impedance $Z_b$, inductance $L_b$ and capacitance $C_b$ are calculated by the following equations [28]:

$$ Z_b = \frac{V_b^2}{S_b} $$

(35)

$$ L_b = \frac{Z_b}{\omega_b} $$

(36)

$$ C_b = \frac{1}{Z_b \omega_b} $$

(37)

$$ \omega_b = 2\pi(50) \text{ rad/sec} $$

(38)

In the above equation, $\omega_b$ is the line angular frequency. After determining the switching frequency and the resonance frequency which is usually selected between 10 times more than the line frequency and a half of the switching frequency, and by taking the best advantages of the passive damping scheme, per unit quantities of inductance, capacitance and the resistance are measured by the following equations [26]:

$$ L_{pu} = \frac{1}{\omega_{sw} \left| \frac{i_g}{u_i} \right| \left| 1 - \frac{\omega_{sw}^2}{\omega_r^2} \right|} $$

(39)

$$ C_{pu} = \frac{4}{L_{pu} \omega_r^2} $$

(40)

$$ R_{d,pu} = \sqrt{\frac{L_{pu}}{C_{pu}}} $$

(41)

In the above equations, $i_g$ is the switching ripple current at the point of common coupling to the grid at the switching frequency and IEEE-519 recommended that the maximum current distortion for an $\frac{i_{SC}}{i_L} < 20$ for current harmonics ≥35th is 0.3%. $i_{SC}$ refers to short-circuit current, and $i_L$ is the nominal load current. Besides, $u_i$ is the inverter pole voltage ripple at the switching frequency which is considered equal to $\frac{V_{dc}}{4}$, and $\omega_{sw}$ and $\omega_r$ are the switching frequency and resonance frequency, respectively. Note that all of these quantities are per unit [26].

At the end, quantities of $L$, $C$ and $R_d$ are calculated as below:

$$ L = L_{pu} L_b $$

(42)

$$ C = C_{pu} C_b $$

(43)
\[ R_d = R_{d,pu} Z_b \]  

Moreover, in the above relations \( L = L_1 + L_2 \) and \( C = C_1 + C_d \). Noticeably, the best filtering performance is achieved when \( L_1 = L_2 \) and \( C_1 = C_d \). Regarding the capacitance value \( C \), an important thing which must be taken into account is that the capacitance must be big enough to satisfy the condition of \( H = 0 \). Otherwise, the switching frequency is irregular. While designing, if the \( C \) quantity is too small, it is possible to increase it and calculate the new \( L \) quantity by using equation (40) and by keeping \( \omega_r \) constant [26]. Additionally, it must be taken into account that generally due to the voltage drop limitation, the maximum quantity of \( L \) must be smaller than 0.1 per unit. Besides, due to the limitation which relates to the maximum power coefficient decrease, the \( C \) quantity must be smaller than \( 5\% C_b \) [27].

6. Simulation and Experimental Results

In order to verify the mentioned theoretical analysis, simulation and experimental results are presented. Simulation results of using the IVSCPCC and the CVSCPCC to generate compensative current for three-phase four-wire SAPF (Fig.7 and Fig. 4), which were attained using MATLAB-SIMULINK software, are illustrated in this section. In this simulation, a balanced sinusoidal three-phase source with internal impedance is used as the grid and a three-phase rectifier with an inductive-resistive load is used as a non-linear load. The system parameters are shown in Table I.

Fig. 8 illustrates high frequency oscillations of \( i_{LA}, i_{SB} \) and \( i_{SC} \) currents with their switching signals at the state that CVSCPCC with \( L_f \) quantity of 470 \( \mu \)H has been used for SAPF, which all have oscillations and are not constant in switching period. Moreover, the voltage at the point of common coupling of CVSCPCC \( u_{CVSCPCC} \) with switching signal are shown in Fig. 9. As seen in Fig. 9 high frequency oscillation exists at output of CVSCPCC that is in contrast to the voltage constant constraint in a switching period and leads to varying output frequency.

Fig. 10 shows high frequency oscillation of IVSCPCC output voltage. Comparing Fig. 11(a) and (b) shows that despite the high frequency oscillation of IVSCPCC, output frequency in IVSCPCC will be constant while it is not constant for CVSCPCC.

The switching frequency analysis on the \( \Delta i_A \) waveform is done by MATLAB-SIMULINK and the results are shown in Fig. 11 which show that the switching frequency distribution for CVSCPCC and IVSCPCC. It can be seen that unlike the CVSCPCC, the IVSCPCC present a constant switching frequency.

To show ineffectiveness of the quantity and even the changes of the grid inductance \( L_s \), on the correct performance of IVSCPCC, simulation has been done with two different values for the \( L_s \) (\( L_s = 1mH, L_s = 2mH \)) without changing the settings for the inductance value of controller and results are illustrated in Fig.12. As can be seen, despite the doubling of the \( L_s \) value, there is no change at IVSCPCC switching frequency.

The capacitor voltage oscillations of the IVSCPCC with switching frequency are shown in Fig. 13. It can be observed that oscillations are semi-sinusoidal as discussed previously. Also, the integral of capacitor voltage waveform with switching signal are illustrated at Fig. 14. As seen in Fig. 14 the waveform slope obtained from \( u_s \) (\( i \)) capacitor voltage integral is constant in a switching period.

The waveforms of the load current \( i_{LA} \), compensation current \( i_{CA} \) and the source current which all are related to phase \( A \) of SAPF with IVSCPCC are shown in Fig. 15.

Finally, a 40VA prototype hardware of mentioned SAPF, with using IVSCPCC to generate compensating current, and without applying dead time compensation is implemented by TMS320F28335 DSP and ARM STM32 F746ZGT6 Cortex M7 (Fig. 16). STMicroelectronics GW38IH130D IGBTs are used as the main switches in the experiment and prototype parameters are indicated in Table II. As seen in Fig. 17(a), IVSCPCC has been tracked the compensating current properly and it match well with the simulated one, as shown in Fig. 15(b). Also, to check constancy of switching frequency, FFT analysis has been done on switching and current tracking error waveforms separately that results are shown in Fig. 17(b) and 17(c). It can be observed that switching frequency of IVSCPCC, is constant and with respect to effect of dead time on switching frequency it can be said that it is equal to the expected frequency (21.15 kHz).
7. Conclusion

To overcome the problems of conventional VSC with parabolic PWM current control (CVSCPCC), improved VSC with parabolic PWM current controller (IVSCPCC) is presented in this paper. Despite the advantages of CVSCPCC, the sensitivity to inductance variations and necessity of constant voltage at the point of common coupling in a switching period that are drawbacks and limitations of CVSCPCC, can lead to variable or non-expected switching frequency in some applications like APFs. This problem is removed by applying IVSCPCC and it is shown analytically that IVSCPCC has no sensitivity to inductance changes and also it is not necessary for the voltage to be constant at the point of common coupling of IVSCPCC in a switching period and so, has been showed that application domain of parabolic PWM method is too far beyond what has been introduced so far. Finally, presented simulation and experimental results have confirmed discussed theoretical analysis.

REFERENCES


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Fig. 2. Converter output current waveforms.

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Fig. 5. Single-phase equivalent circuit of three-phase four-wire SAPF with CVSCPCC related to phase A: (a) $S_1$ is on and $S_4$ is off; (b) $S_1$ is off and $S_4$ is on.

Fig. 6. Simplified proposed topology of IVSCPCC: (a) General scheme; (b) $S_p$ is on and $S_n$ is off; (b) $S_p$ is off and $S_n$ is on.

Fig. 7. Three-phase four-wire SAPF with IVSCPCC: (a) $S_1$ is on and $S_4$ is off; (b) $S_1$ is off and $S_4$ is on.

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Fig. 9. High frequency oscillation of voltage at the point of common coupling of CVSCPCC with switching signals.

Fig. 10. High frequency oscillation of voltage at the point of common coupling of IVSCPCC with switching signals.

Fig. 11. Spectra of current tracking error $\Delta i_A$: (a) CVSCPCC; (b) IVSCPCC.

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Fig. 13. Capacitor voltage oscillations when IVSCPCC has been used: (a) with low resolution; (b) with high resolution and switching signal.

Fig. 14. Capacitor voltage integral $U_f(t)$ and switching signal when IVSCPCC has been used.

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Fig. 17. Experimental results: (a) compensating current generated by IVSCPCC; (b) Spectra of switching wave form; (c) Spectra of current tracking error $\Delta i_A$.

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<tr>
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</tr>
</thead>
</table>
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### TABLE I
**Simulated System Parameters**

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_L$</td>
<td>Phase to phase rated voltage</td>
<td>380[V]</td>
</tr>
<tr>
<td>$L_s$</td>
<td>Source inductor</td>
<td>1[mH]</td>
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<tr>
<td>$R_s$</td>
<td>Source resistor</td>
<td>0.1[Ω]</td>
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<tr>
<td>$L_n$</td>
<td>Neutral wire inductor</td>
<td>1[mH]</td>
</tr>
<tr>
<td>$R_n$</td>
<td>Neutral wire resistor</td>
<td>0.1[Ω]</td>
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<tr>
<td>$L_f$</td>
<td>Inductor of CPCC</td>
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<tr>
<td>$R_f$</td>
<td>Resistance of inductor of CPCC</td>
<td>0.01[Ω]</td>
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<tr>
<td>$f_s$</td>
<td>System rated frequency</td>
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<tr>
<td>$V_{dc\text{-}ref}$</td>
<td>Dc link voltage</td>
<td>900[V]</td>
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<td>$C_p$</td>
<td>Dc link up capacitor</td>
<td>2200[µF]</td>
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<tr>
<td>$C_n$</td>
<td>Dc link down capacitor</td>
<td>2200[µF]</td>
</tr>
<tr>
<td>$C_1$</td>
<td>Lcl filter capacitor</td>
<td>6.8[µF]</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Lcl filter capacitor</td>
<td>6.8[µF]</td>
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<tr>
<td>$L_1$</td>
<td>Converter side lcl filter inductor</td>
<td>0.47[mH]</td>
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<td>$R_1$</td>
<td>Resistance of converter side lcl filter inductor</td>
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<td>$L_2$</td>
<td>Grid side lcl filter inductor</td>
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<td>Resistance of grid side lcl filter inductor</td>
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<td>Damping resistor of lcl filter</td>
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<td>Smoothing inductor</td>
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<tr>
<td>$L_{L2}$</td>
<td>Inductor of star connection of nonlinear load</td>
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<tr>
<td>$R_{L2}$</td>
<td>Resistance of inductor of star connection of nonlinear load</td>
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<tr>
<td>$L$</td>
<td>Nonlinear load inductor</td>
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<tr>
<td>$R$</td>
<td>Nonlinear load resistor</td>
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<tr>
<td>$f_{sw}$</td>
<td>Switching frequency</td>
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<tr>
<td>$f_r$</td>
<td>Resonance frequency</td>
<td>2.8[kHz]</td>
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<tr>
<td>$t_{DT}$</td>
<td>Dead time</td>
<td>2[µsec]</td>
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### TABLE II
**Prototype System Parameters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_L$</td>
<td>Phase to phase rated voltage</td>
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<td>$f_s$</td>
<td>System rated frequency</td>
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<td>$V_{dc\text{-}ref}$</td>
<td>Dc link voltage</td>
<td>34[V]</td>
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<tr>
<td>$C_p$</td>
<td>Dc link up capacitor</td>
<td>2200[µF]</td>
</tr>
<tr>
<td>$C_n$</td>
<td>Dc link down capacitor</td>
<td>2200[µF]</td>
</tr>
<tr>
<td>$C_1$</td>
<td>Lcl filter capacitor</td>
<td>2.2[µF]</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Lcl filter capacitor</td>
<td>2.2[µF]</td>
</tr>
<tr>
<td>$L_1$</td>
<td>Converter side lcl filter inductor</td>
<td>1.5[mH]</td>
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<tr>
<td>$R_1$</td>
<td>Resistance of converter side lcl filter inductor</td>
<td>1[Ω]</td>
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<tr>
<td>$L_2$</td>
<td>Grid side lcl filter inductor</td>
<td>1.5[mH]</td>
</tr>
<tr>
<td>$R_2$</td>
<td>Resistance of grid side lcl filter inductor</td>
<td>1[Ω]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Value</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td>$R_d$</td>
<td>Damping resistor of LCL filter</td>
<td>27[Ω]</td>
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<tr>
<td>$L_{d1}$</td>
<td>Smoothing inductor</td>
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<tr>
<td>$R_{d1}$</td>
<td>Resistance of smoothing inductor</td>
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<td>Resistance of inductor of star connection of nonlinear load</td>
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<td>$L$</td>
<td>Nonlinear load inductor</td>
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<td>$R$</td>
<td>Nonlinear load resistor</td>
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<tr>
<td>$f_{sw}$</td>
<td>Switching frequency</td>
<td>21.15[kHz]</td>
</tr>
<tr>
<td>$f_r$</td>
<td>Resonance frequency</td>
<td>2.766[kHz]</td>
</tr>
</tbody>
</table>

**Biography:**

**Mohammad Reza Mohammadpour** was born in Kazerun, Iran. He received the B.Sc. degree in electrical engineering from Islamic Azad University, Kazerun Branch, Kazerun, Iran, in 2007 and the M.Sc. degree in electrical engineering from Shiraz University of Technology, Shiraz, Iran, in 2017. His research interests include application of power electronics in industrial and distribution networks, power converters and control of electrical machines.

**Amir Hossein Eslahchi** received the B.S. degree in electrical engineering from Arak University, Arak, Markazy, Iran, in 2015. He is currently working toward the M.S. degree in the Shiraz University of technology. His research interests include the power electronics, impedance network, and power converters.

**Mohammad Mardaneh** received the B.S. degree in Electrical Engineering from Shiraz University, Iran, in 2002, and M.S. and Ph.D. degrees in the same subject from Amirkabir University of Technology, Tehran, Iran, in 2004 and 2008, respectively. He has been Associate Professor at Shiraz University of Technology, Shiraz, Iran, since 2008. His research fields cover: modeling, design and control of electrical machines, and application of power electronics in renewable energy systems and distribution networks.

**Mohammadreza Moslemi** Obtained the B.Sc. degree in Electronic Engineering from Sharif University of Technology in 1999. Later on, he received his M.Sc. degree in Electronic Engineering from Shiraz University in 2001. He received Ph.D. in Electronic Engineering at Islamic Azad University. Since 2005 he has been working as a faculty member of the Electrical Department of Islamic Azad University, Zarghan Branch. His main research interests include power electronics and Nano electronics.

**Zhaleh Hashemi** was born in Isfahan, Iran, in 1981. She received a B.S. degree in Electrical Engineering from Isfahan University of Technology, Iran, in 2002 (as a guest student), and a M.S. degree in Power Engineering from Amirkabir University of Technology, Tehran, Iran, in 2007. She is currently with the Islamic Azad University, Zarghan Branch, Iran.