An Availability Evaluation Method for Desalination Process with Three-States Equipment under the Specific Repair Queuing Policy

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Abstract: Oil waste, is one of the most important pollutants in the oil and gas industry. Since the wells' oil has significant saltwater, the effluent amount increases with increasing oil reservoir extraction. Separating the saltwater from the extracted oil before starting the refinery process plays an essential role in reducing the oil costs as well as the useful transfer capacity. This paper presents a new Chapman-Kolmogorov Equation-based (CKEB) method to evaluate a desalination system's availability with three-state equipment and weighted-k-out-of-n configuration. In this system, the equipment is repairable, and each repair facility can repair all equipment types of different sub-systems (pump stations). We consider all failures and repairs to have a constant rate (with Exponential distribution) and use the Chapman-Kolmogorov Equations to drive the system's availability. Then we validate the presented method using a simulation technique. Finally, the elapsed time of both solving techniques is compared. The results show the superiority of the CKEB technique in terms of computational time. Compared with the simulation technique, the computational time ratio for the CKEB method is between 0.0002% to 0.0058% for the small-size problems, between 0.05% to 0.94% for the medium-size problems, and between 1.31% to 5.39% for the large-size problems.

Keywords: Desalination system, Availability, Multi-state equipment, Repairable equipment, Chapman-Kolmogorov Equations, Monte-Carlo Simulation.

1- Introduction

The oil drainage that comes out of the reservoir during oil extraction is one of the most important pollutants in the oil and gas industry. With the increase in extraction of oil reservoirs, the effluent amount is also increased. Over time, by reducing the amount of oil in the well and replacing it with saltwater, saltwater's value increases, which requires separation of the saltwater from the oil. For this purpose, the wells' extracted oil is sent to desalination units for separation of saline waste after decontamination in the units. Suppose the case that waste materials do not separate from the oil before delivering to the refineries. So, the consequence is corrosion of the pipelines and reduce transfer debit. Finally, it causes disruptions during distillation and corrosion in steam boilers and other devices. The presence of these waste materials in the exported oil reduces the cost of oil and reduces transportation's useful capacity. So, the reliability (availability) of the separation systems of this wastewater from oil and injection is the priority of oil companies.

Sewage sludge contains a significant value of heavy metals and various salts, and these materials in nature are one of the major environmental pollutants. Since the sewage sludge will be injected into the wells, it is too essential to separate the environmental pollutant before injection. The presence of these pollutant materials in the sewage sludge is a consequence of incomplete separation in the desalination units. So, there is essential for

wastewater treatment systems to separate oil and particulate matter from the wastewater and prepare it for injection. The desalination plants require a variety of equipment, and many of them are three-state equipment. For example, due to the number of healthy blades, the water pumps have three functional states: working, failed, and low-performance working status.

This paper uses the Chapman-Kolmogorov Equation to calculate a desalination system's availability with three-state equipment. The system's equipment is considered repairable, and all repair facilities in this system can repair all the equipment of different plant sections. Initially, the system's states based on the number of sub-systems (number of equipment), the number of redundant equipment, the number of under-repair equipment in all sub-systems, and the number of repair facilities are determined. Then, the total system availability using the Chapman-Kolmogorov Equation is calculated. To validate the proposed method, the system availability is calculated using a simulation technique. Finally, given the number of sub-systems (equipment), the number of redundant components, and the system's mission time, we propose the condition that the Chapman-Kolmogorov equation-based (CKEB) method has less computational time compared with the simulation method.

This paper consists of seven parts. The second section is the literature review. The third section deals with the system's definitions. In Section Four, the problem definitions are presented. Section five deals with solving methods. Some numerical examples are presented in Section six, and the last section is the conclusion and further studies.

2- Literature review

In this paper, we present a new availability evaluation for a desalination system with three-state components. So, the literature review is presented in three directions. The first direction is related to the multi-state systems (MSS), and discuss the general reliability (availability) evaluation of different MSS systems' configuration. The third direction is related to systems' reliability (availability) evaluation for the redundancy allocation problem (RAP). The last direction regards to reliability evaluation of desalination systems.

In terms of MSS and according to classical reliability models, most of the systems, as well as their components, have two states as workings and failed states. A particular system component can operate at any level of functionality from 0% to 100% with a given probability. These systems are called the MSS. Since increasing the number of the components in an MSS, the system states increase rapidly, and the complexity of the calculations rises astonishingly; calculating the reliability of the MSS using basic reliability methods, if not impossible, is too complicated. In order to reduce the complexity of calculating the reliability of the MSS, Ushakov [1] introduced the Universal Generating Function (UGF) method. The UGF is a well known convenient method to calculate MSSs' reliability with series, parallel, and series-parallel configurations. However, by increasing the system's components, this UGF's computational time increases remarkably.

Wu and Chen [2] proposed the recursive algorithm to estimate the weighted k-out-of-n system's reliability. This technique is another well-known technique for calculating the reliability of the MMSs. Higashiyama [3] proposed instruction to evaluate the reliability of the binary state weighted k-out-of-n system in a shorter time than the proposed recursive

method. Sharifi, Ganjian, and Ghajar [4] worked on a network consisting of two elements with incremental and constant failure rates in real-time situations. Later, they studied the real-time reliability of a k -out-of- n load-sharing system with n identical components [5].

Lisnianski and Ding [6] studied the redundancy of repairable MSS using a combinatorial method of statistical processes and UGF analysis. Sharifi, Memariani, and Noorossana [7] proposed an algorithm for evaluating a k -out-of- n systems' real-time reliability with identical components with fuzzy failure rates. Levitin, Xing, & Dai [8] considered the optimal standby element sequencing problem for k -out-of- n : G heterogeneous cold-standby systems. They optimized the expected system mission cost by selecting the initial sequence of the system's components. Pourkarim, Sharifi, Niaki, et al. [9] proposed a new practical approach for estimating the un-repairable three-state systems' reliability. They first presented a proper definition of system states and then provided differential equations using the Markov process and Chapman-Kolmogorov equation. By solving these differential equations, which calculate the states' probabilities and system reliability, the processing time reduced significantly compared with other available techniques. Lu, Wu, Liu, et al. [10] examined the reliability of the large MSS with repairable components. They reported that some phased-mission systems (PMS) might contain a large number of steps and repairable components in many engineering applications. Levitin, Xing, Dai [11] optimized a 1-out-of- N : G cold standby systems consist of non-repairable components with different productivity or load levels. They first suggested analyzing the system's reliability and expected mission cost, then formulated and solved the completed work-dependent components load's optimal dynamic load distribution. Sharifi and Taghipour [12] considered a k -out-of- n system with non-identical components to optimize the system's inspection interval. In their model, the system was an MSS based on the number of working components during each inspection interval.

Many real-case problems like the RAP deal with calculating the system's reliability. Different system configurations have been considered in RAP, and the systems' reliability is calculated before optimizing the systems' redundancy. The general mathematical model for the RAP was presented by Fyffe, Hines, Lee [13]. The presented model aims to maximize the system's reliability under the system's cost and weight constraints.

After that, many researchers optimized the RAP considering different MSS's configurations. For example, Ramirez-Marquez and Coit [14] considered a series-parallel MSS with capacitated binary components which could provide different MSS performance levels, and calculated the system's reliability using UGF. Tian and Zou [15] optimized a multi-objective RAP for general series-parallel MSS. Later they presented an optimization model for a series-parallel MSS, which jointly determined the optimal component state distribution and optimal redundancy for each stage. The relationship between component state distribution and the component cost was discussed based on an assumption on the components' treatment [16]. Ouzineb, Nourelfath, , Gendreau [17] worked on a RAP with series-parallel MSS and used an efficient Tabu Search (TS) algorithm to solve the presented problem. They considered that the system's performance levels have a continuous range between perfect functioning and complete failure. Mousavi, Alikar, Niaki, et al. [18] investigated a bi-objective RAP for a series-parallel MSS with non-repairable components. They considered that both the performance rates and the availabilities of components are

considered fuzzy due to the uncertainties. They used a fuzzy UGF to evaluate the availability of their considered system.

Wang, Xiong, Xie [19] presented a RAP for cold-standby systems with degrading components and approximated the models' objective function. They used Genetic Algorithm to solve the presented RAP. Lai and Yeh [20] proposed a swarm-based approach called two-stage simplified swarm optimization to solve a RAP for a bridge MSS. Shahriari, Sharifi, and Naserkhaki [21] modeled the system's reliability consisted of a time-dependent continuous performance rate MSS. They considered the system's components might have a continuous performance rate between zero and their maximum performance rate. George-Williams and Patelli [22] evaluated the MSS's reliability with a flow simulation approach and hybrid events. They reported that the system's complexity and multi-state structural features make it challenging to evaluate these systems' reliability and availability. Despite the emergence of diverse techniques for analyzing complex MSS, but simulation seems to be the only proper and doable approach for real systems. Attar, Raissi, and Khalili-Damghani [23] studied the optimization method approach based on simulation for RAP and the availability of the repairable MSS with any statistical distribution.

Some other studies on RAP considering MSS are presented in Table 1 (between 2018-2021).

Insert Table 1 here

In terms of the desalination system's reliability (availability) evaluation, El-Naser [39] presented a method that incorporated equipment reliability considerations into the optimal design of cogeneration systems for power and desalination. Hosseini et al. [40] studied the effects of equipment's reliability on analyzing the thermo-economic of combined power and multi-stage-flash water desalination plant using the Markov process. They expanded their method by considering a multi-objective optimization model and design a combined gas turbine and multi-stage flash desalination plant [41]. Zhou, Chang, and Fane [42] addressed three essential reliability aspects that drive uncertainty in the life cycle assessment of the desalination plants, including the incompleteness of the system boundary, the unrepresentativeness of the database, and the omission of uncertainty analysis. Ailliot, Boutigny, Koutroulis, et al. [43] used the stochastic (Markov-switching auto-regressive model) weather generators for the optimal design and reliability evaluation of hybrid Photovoltaic/Wind-Generator systems providing energy to desalination plants. Wang, Asefa, Wanakule, et al. [44] presented an application of decision-support tools to incorporate uncertainties, seasonal demand forecasts, and system operational constraints of the desalination plants to assist decision-makers in designing more reliable systems using a hidden Markov chain. In all these studies, the effect of considering the system's reliability on the desalination process has been investigated. In this paper, we calculate the system's availability for a system consist of three-state repairable components using CKEB methods. For this reason, we used the procedure which was presented by Pourkarim, Sharifi, Niaki, et al. [9].

A common assumption for simplifying the systems with several sub-systems is that every repair facility may only work on one sub-system's components. This assumption reduces the number of system's states dramatically. However, in real-case problems, the

repair facilities can operate and repair all equipment of different sections (sub-systems) of a plant. This assumption has been violated in this study, and an assumption has been added to the model that allows every repair facility to work on all sub-systems' components. We use the Chapman-Kolmogorov Equation and simulation method to evaluate the system availability. Using the Chapman-Kolmogorov Equation to evaluate system availability, we presented eight different rules. Then the transition probabilities between the system's states, as well as the transition matrix using these eight rules, are calculated. The results (availability) obtained using both methods from the viewpoint of computational time are compared at the end. The novelties of the current paper can be summarized as follows:

- Developing a CKEB method for calculating the availability of systems with multi-tasking repair facilities,
- Presenting a closed-form set of differential equations for calculating the system's availability,
- Comparing the performance of the given model with simulation technique.

The problem raised in this paper is an availability problem. This system has a series-parallel configuration and components with three states of performance. These components are repairable, and every repair facility can offer service to all sub-systems. The mathematical model and the solution offered for the series-parallel system are based on the assumption that redundancy policy is of the active type. The failure and repair rate of the available components are constant. The problem objective is to calculate the availability of the system by using differential equations and state diagrams. Considering the problem conditions, it is a state-dependent system, so Markov analysis in examining this problem is too important.

3- System Definitions

In this section, we present some descriptions for a real-world desalination system.

3-1- The recovery treatment process at a desalination plant

Figure 1 shows the recovery treatment process at a desalination plant. Figure 1 shows that after adding bactericide materials and reverse demulsifiers to the wastewater, it moves to the skimmer reservoir. After a while, a portion of the oil droplets comes to the water's surface, collects by a pipe, and sends it to the reservoir to store contaminated oil. Then it enters the gravity separator, and again a portion of the oil drops is collected, and the remained will be sent to the contaminated oil reservoir. The water is then pumped to under pressure filters, designed to collect the solid particle with more than 10 microns' dimensions, and the dimension of the remaining oil drop to below 25 ppm. Eventually, the wastewater stores in a volatile reservoir and injects into the wells by pressure pumps. At the time of any technical problem in the plant, the wastewater will be sent to the pit, which will cause environmental pollution.

Insert Figure 1 here

3-2- Components of the studied filtration and injection system

3-2-1. Skimmer tank

The dimensions of the skimmer tank are 10 meters in diameter and 5 meters high. The reservoir is equipped with a cochlear plate, which allows the oil to remain on the surface with a lifespan and can be removed by a cut tube that can be changed. Water is also ejected out of the tube through a pipe.

3-2-2. Gravity separator

The weighing separator consists of two cavity piers with a width of 2.5 m, a length of 3.15 m, and a depth of 1.5 m long covered by a steel sheet. The oil floating on the water's surface collects using a rotating tube, and the water is draining from the bottom.

3-2-3. Filter

The utilized sandwich filters have a height of 1.6 to 2.6 meters in diameter and 2 meters in height—filters designed for solid particles up to 10 microns and oil reduction of around 25 ppm.

3-2-4. The dirty oil surge tank

This tank received the oil separated in a skimmer tank and pump the remaining water to the desalination plant. This tank has a diameter of 3 meters, a height of 2 meters, and a design pressure of 0.02 bar.

3-2-5. Disposal water pump

A wastewater pump is a single-stage pump of OH6 designed for working in harsh conditions. This pump receives the wastewater from the disposal water surge tank and injects it into the well. In cases where the required injection pressure is greater than 1200 psi, pre-pressure pumps are used.

4- Problem definition

As shown in Fig. 1, five different types of pumps are functioning at the desalination plant. Failure of each pump will disrupt the desalination process. In this system, considering the number of healthy blades, each pump can be considered three-state equipment. If all the blades are in good condition, the pump will continue to work with its maximum performance. If more than one blade has been damaged, the pump will be deactivated, while if just a blade is failed, the pump will keep working at lower performance. In this paper, the pumps' reduced performance is considered half of the new pump's performance. Moreover, as it is clear, all repair facilities can repair all existing pumps (of any type).

The understudy's system is a weighted-k-out-of-n system with a multi-state component. Since each pump has three states, it should be considered as an MSS. Moreover, for each sub-system (pump-station), when a specific number (k_i) of the pumps were available, the pump station is considered as an operational pump station. For example, if $k_i = 1.5$, it means that the pump station is working with equal or more than one fully-working and one

semi-working pump, or with more than two semi-working pumps. One of the ways to increase the availability of such systems is to use redundant equipment. If the available budget and space for component redundancy are limited, the main issue is to find the best solution for assigning redundant pumps. The first step for optimal allocation is to find a way to calculate the system's availability. We present the following two methods for calculating such systems' availability: CKEB and simulation technique.

4-1- The general procedure of calculating the system's availability

Pourkarim, Sharifi, Niaki, et al. [9] calculated the three-state system's reliability using UGF, Recursive algorithm, and differential equations (using Chapman-Kolmogorov Equations). They reported that the differential equations model's computational time is significantly less than two other techniques, especially for large-scale systems. In this paper, we used the method of differential equations to calculate the system's availability. The general procedure of the model solution in this paper is as follows:

- Step one: determine the system's states,
- Step two: obtain the general state diagram for such a system,
- Step three: determine the relations between the system's states,
- Step four: establish the set of differential equations between the system's states and calculate the system's transition matrix using the Chapman-Kolmogorov equations.
- Step five: calculate the system's availability (which is the main objective of the current study),
- Step six: validate the model using the simulation technique.

4-2- Nomenclatures

The notation (indexes and parameters) used in this paper is presented in Table 2 as follows.

Insert Table 2 here

4-3- System configuration

The general diagram of a RAP is presented in Figure 2. The system consists of S serially connected pump stations, in which the i^{th} ($i = 1, \dots, S$) pump station has n_i redundant pump.

Insert Figure 2 here

Three operational states can be considered for each pump: working, semi-working (or partially failed), and failed. For pump station i , we show the number of working pumps by w_i , the number of semi-working pumps by h_i , and the number of repair facilities which are working on the pump stations' pumps by m_i . We consider that a repair facility can fix a pump when it is completely failed. So, the pump station condition can be determined by (w_i, h_i, m_i) . Using the presented notation, the condition of the system is shown by $\{(w_1, h_1, m_1), \dots, (w_s, h_s, m_s)\}$. The system's states are determined based on the values of w_i , h , and m_i . For example, the first state is $\{(n_1, 0, 0), \dots, (n_s, 0, 0)\}$, which shows that all pumps of all pump stations are fully working, and no semi-working pump is working in pump

stations. Moreover, all repair facilities are available, and none of them are working. For each particular state, the working performance of the pump station calculates using Equation 1 as follows:

$$Po_i = w_j + \frac{h_j}{2}; i = 1, \dots, h \quad (1)$$

and the system is working, if for all pump stations $Po_i \geq k_i$. Since the repair facilities can serve the pump of all pump stations, determining the system's state is very complicated, especially when the number of pump stations increases. The number of possible states dramatically rises. For a particular state, the state is reachable from different other states, shown in Figure 3. This figure illustrates all scenarios that may happen for a particular system's state: partial failure of the functional pump, complete failure of the functional pump, complete failure of a partially failed pump, and repair of a failed pump. For drawing the state-space diagram of the system, only the states with $Po_i \geq k_i$ should be considered.

In general, when a failure happens, two scenarios are desirable: a repair facility is free and starts to repair the failed pump immediately after the failure, or there is no available repair facility in the system, and the failed pump must stay on the waiting repair queue. Likewise, two scenarios are desirable after repairing a failed pump; a failed pump in the repair queue requires a free repair facility, or no pump is on the repair queue.

Insert Figure 3 here

In Figure 3, state 0 is a general state of the system, and states 1 to 8 are all possible states in which state 0 is reachable from them. Figure 3 would be breakdown into eight different rules which may happen for this system. In other words, all possible system states can be expressed by the eight following rules. A general condition for all rules is presented in Equation 2 as follows:

$$0 \leq w_j + h_j \leq n_j; \forall j \neq i \quad (2)$$

4-3-1. Rule one

In Figure 3, the correspondence state of Rule 1 is state 1. Figure 4 illustrates the semi-failure of a working pump in a pump station. In the pump station i , there is $(w_i + 1)$ working pumps $(h_i - 1)$ partially-failed pumps and m_i busy repair facilities. The number of working pumps reduces by one unit by partially-failure of one pump in this pump station, and the number of semi-working pumps increases one unit. Since the state has $(w_i + 1)$ working pumps and partially-failure rate of each pump is λ_{i1} , the transition rate from this state to state 0 is equal to $(w_i + 1) \cdot \lambda_{i1}$. The conditions for using Rule 1 are presented in Equations (3) to (6).

Insert Figure 4 here

$$0 \leq m_i \leq n_i - w_i - h_i \quad (3)$$

$$0 \leq m_j \leq n_j - w_j - h_j; \forall j \neq i \quad (4)$$

$$m_i + \sum_{j \neq i} m_j \leq M \quad (5)$$

$$1 \leq w_i + h_i \leq n_i \text{ \& } 1 \leq h_i \leq n_i \text{ \& } 0 \leq w_i \leq n_i - 1 \quad (6)$$

4-3-2. Rule two

In Figure 3, the correspondence state of Rule 2 is state 2. Figure 5 illustrates the full-failure of a working pump of a pump station. In the pump station i , there is $(w_i + 1)$ working pumps, h_i partially-failed pumps and $(m_i - 1)$ busy repair facilities. By fully-failure of one pump in this pump station, the number of working pumps reduced one unit, and the number of busy repair facilities increases one unit. Since the state has $(w_i + 1)$ working pumps and fully-failure rate of each pump is λ_{i2} , the transition rate from this state to state 0 is equal to $(w_i + 1) \cdot \lambda_{i2}$. The conditions for using Rule 2 are presented in Equations (7) to (10).

Insert Figure 5 here

$$1 \leq m_i \leq n_i - w_i - h_i \quad (7)$$

$$0 \leq m_j \leq n_j - w_j - h_j; \quad \forall j \neq i \quad (8)$$

$$(m_i - 1) + \sum_{j \neq i} m_j \leq M \quad (9)$$

$$0 \leq w_i + h_i \leq n_i - 1 \quad \& \quad 0 \leq w_i \leq n_i - 1 \quad (10)$$

4-3-3. Rule three

In Figure 3, the correspondence state of Rule 3 is state 3. Just like Rule 2, Figure 6 illustrates the full-failure of a working pump in the pump station. But in this rule, there is no available repair facility. So, for this rule, after the full-failure of one pump, the number of working pumps reduces by one unit, but the number of busy repair facilities doesn't change. Since the state has $(w_i + 1)$ working pumps and fully-failure rate of each pump is λ_{i2} , the transition rate from this state to state 0 is equal to $(w_i + 1) \cdot \lambda_{i2}$. The conditions for using Rule 4 are presented in Equations (11) to (14).

Insert Figure 6 here

$$0 \leq m_i \leq n_i - w_i - h_i \quad (11)$$

$$0 \leq m_j \leq n_j - w_j - h_j; \quad \forall j \neq i \quad (12)$$

$$m_i + \sum_{j \neq i} m_j = M \quad (13)$$

$$0 \leq w_i + h_i \leq n_i - 1 \quad \& \quad 0 \leq w_i \leq n_i - 1 \quad (14)$$

4-3-4. Rule four

In Figure 3, the correspondence state of Rule 4 is state 4. Figure 7 illustrates the full-failure of a semi-failed pump in a pump station. In the pump station i , there are w_i working pumps, $(h_i + 1)$ partially-failed pumps, and $(m_i - 1)$ busy repair facilities. By fully-failure of one semi-failed pump, the number of the semi-failed pumps reduces one unit, and the number of the busy repair facilities increases one unit. Since the state has $(h_i + 1)$ semi-failed pumps and fully-failure rate of each pump is λ_{i3} , the transition rate from this state to state 0 is equal to $(h_i + 1) \cdot \lambda_{i3}$. The conditions for using Rule 4 are presented in Equations (15) to (18).

Insert Figure 7 here

$$1 \leq m_i \leq n_i - w_i - h_i \quad (15)$$

$$0 \leq m_j \leq n_j - w_j - h_j; \quad \forall j \neq i \quad (16)$$

$$(m_i - 1) + \sum_{j \neq i} m_j < M \quad (17)$$

$$0 \leq w_i + h_i \leq n_i - 1 \quad \& \quad 0 \leq w_i \leq n_i - 1 \quad (18)$$

4-3-5. Rule five

In Figure 3, the correspondence state of Rule 5 is state 5. Figure 8 illustrates the full-failure of a semi-failed pump in a pump station. But in this rule, there is no available repair facility. So, for this rule, after the full-failure of one semi-failed pump, the number of semi-failed pumps reduces by one unit, but the number of busy repair facilities doesn't change. Since the state has $(h_i + 1)$ semi-working pumps and fully-failure rate of each pump is λ_{i4} , the transition rate from this state to state 0 is equal to $(h_i + 1) \cdot \lambda_{i4}$. The conditions for using Rule 4 are presented in Equations (19) to (22).

Insert Figure 8 here

$$0 \leq m_i \leq n_i - w_i - h_i \quad (19)$$

$$0 \leq m_j \leq n_j - w_j - h_j; \quad \forall j \neq i \quad (20)$$

$$m_i + \sum_{j \neq i} m_j = M \quad (21)$$

$$0 \leq w_i + h_i \leq n_i - 1 \quad \& \quad 0 \leq h_i \leq n_i - 1 \quad (22)$$

4-3-6. Rule six

Rules 6, 7, and 8 are related to pump repair. In Figure 3, the correspondence state of Rule 6 is state 6. Rule 6, illustrated in figure 9, indicates that the one pump's repair is finished, and no failed pump is in the repair queue. In the pump station i , there is $(w_i - 1)$ working pumps, h_i partially-failed pumps, and $(m_i + 1)$ busy repair facilities. The repaired pump starts working, so the number of working pumps increases one unit, and the number of under repair pumps decreases one unit. Since there is $(m_i + 1)$ under repair pumps and the repair rate of each pump is equal μ_i , the transition rate from this state to state 0 is equal $(m_i + 1) \cdot \mu_i$. The conditions for using Rule 6 are presented in Equations (23) to (26).

Insert Figure 9 here

$$m_i = n_i - w_i - h_i \quad (23)$$

$$m_j = n_j - w_j - h_j; \quad \forall j \neq i \quad (24)$$

$$m_i + \sum_{j \neq i} m_j < M \quad (25)$$

$$1 \leq w_i + h_i \leq n_i \quad (26)$$

4-3-7. Rule seven

In Figure 3, the correspondence state of Rule 7 is state 7. Rule 7, illustrated in figure 10, indicates that the one pump's repair is finished, and the pump station has at least one failed pump on the repair queue. In the pump station i , there is $(w_i - 1)$ working pumps, h_i partially-failed pumps, and m_i busy repair facilities. Based on the repair policy, which will be presented in Section 4.4, pump station i has the priority of repair, so, repair facility allocates to one of the failed pump(s) of the pump station i . In this case, the number of the working pumps of the pump station i increases one unit, but the number of the under repair pumps doesn't change. Since there is m_i under repair pumps, and the repair rate of each pump is equal μ_i , the transition rate from this state to state 0 is equal $m_i \cdot \mu_i$. The conditions for using Rule 7 are presented in Equations (27) to (33).

Insert Figure 10 here

$$L = \{j | n_j - w_j - h_j > m_j; j \neq i\} \quad (27)$$

$$w_L = \min_{j \in L} (w_L) \quad (28)$$

$$l = \min_{j \in L, w_j = w_L} (j) \quad (29)$$

$$\text{if } \left\{ \begin{array}{l} (L \neq \emptyset \ \& \ w_i < w_l \ \& \ n_i - w_i - h_i \geq m_i) \\ \text{or} \\ (L \neq \emptyset \ \& \ w_i = w_l \ \& \ n_i - w_i - h_i \geq m_i \ \& \ i < l) \\ \text{or} \\ (L = \emptyset \ \& \ n_i - w_i - h_i \geq m_i) \end{array} \right\} \text{ then } m_i + \sum_{j \neq i} m_j = M \quad (30)$$

$$1 \leq m_i \leq n_i - w_i - h_i \quad (31)$$

$$0 \leq m_j \leq n_j - w_j - h_j; \ \forall j \neq i \quad (32)$$

$$1 \leq w_i + h_i \leq n_i - 1 \quad (33)$$

4-3-8. Rule eighth

In Figure 3, the correspondence state of Rule 8 is state 8. Rule 8, illustrated in figure 11, indicates that the one pump's repair is finished, and the pump station has at least one failed pump on the repair queue. In the pump station i , there is $(w_i - 1)$ working pumps, h_i partially-failed pumps, and $(m_i + 1)$ busy repair facilities. Based on the repair policy, which will be presented in Section 4.4, pump station i has no priority for repair, so the repair facility allocates to one of the failed pump(s) of another pump station. In this case, the number of the working pumps of the pump station i increases one unit, and the number of the under repair pumps decreases one unit. Since there is $(m_i + 1)$ under repair pumps and the repair rate of each pump is equal μ_i , the transition rate from this state to state 0 is equal $(m_i + 1) \cdot \mu_i$. The conditions for using Rule 8 are presented in Equations (34) to (40).

Insert Figure 11 here

$$L = \{j | n_j - w_j - h_j > m_j; j \neq i\} \quad (34)$$

$$w_L = \min_{j \in L} (w_L) \quad (35)$$

$$l = \min_{j \in L, w_j = w_L} (j) \quad (36)$$

$$\text{if } \left\{ \begin{array}{l} (L \neq \emptyset \ \& \ w_i < w_l \ \& \ n_i - w_i - h_i = m_i) \\ \text{or} \\ (L \neq \emptyset \ \& \ w_i = w_l \ \& \ \{ (n_i - w_i - h_i = m_i) \text{ or } (n_i - w_i - h_i > m_i \ \& \ i > l) \}) \\ \text{or} \\ (L \neq \emptyset \ \& \ w_i > w_l) \end{array} \right\} \quad (37)$$

$$\text{then } m_i + \sum_{j \neq i} m_j = M$$

$$0 \leq m_i \leq n_i - w_i - h_i \quad (38)$$

$$0 \leq m_j \leq n_j - w_j - h_j; \ \forall j \neq i \quad (39)$$

$$1 \leq w_i + m_i \leq n_i \quad (40)$$

4-4- Repair queuing policy

Rules 7 and 8 are the same as Rule 6. But in Rules 7 and 8, some pumps are in the repair queue, and a released repair facility must work on one of the failed pumps in the repair queue. In this condition, the important question that should be answered is: which failed pump should be allocated to the available repair facility? To answer this question and solve this problem, we consider a policy that determines the repair priority of the pumps in the repair queue. This policy is defined in four steps:

- Step 1: Determine the R , set of pump stations that have at least one pump in the repair queue,
- Step 2: Determine the working pumps of the pump station(s) of R .
- Step 3: Allocate the repair facility to the failed pump of the pump station with the lowest working pump is R ,
- Step 4: If there is more than one pump station with the lowest working pump in R , allocate the repair facility to the pumps of the pump station with a lower pump station's index (lower value of i).

According to the eight rules mentioned above, the state diagram can be determined for any system with any number of pumps and pump stations. In any situation, one of the above rules will comply. For example, consider a system with three pump stations. Each pump station consists of four pumps, and three repair facility is available. Consider the state $\{(2,1,1), (1,0,1), (2,1,0)\}$. In this state and for the first pump station, two fully-working and one semi-working pump is available, and one pump is under repair, so this station has no pump on the repair queue. One fully-working and no semi-working pump is available in the second pump station, and one pump is under repair, so this station has two pumps on the repair queue. In the third pump station, two fully-working and one semi-working pump are available, and no pump is under repair, so this station has one pump on the repair queue. So, for this state $R = \{2, 3\}$ which is the index of the pump stations with at least one pump on the repair queue. The working power of the second pump station is equal $Po_2 = 1 \times w_2 + 0.5 \times h_2 = 1 \times 1 + 0.5 \times 0 = 1$ and the power of the third pump station is equal $Po_3 = 1 \times w_3 + 0.5 \times h_3 = 1 \times 2 + 0.5 \times 1 = 2.5$. Since we know that one repair facility is available, this repair facility allocates to the pump station with lower working power, the second pump-

station. But for state $\{(1,2,1), (1,2,1), (2,2,0)\}$, the working power of the first and second pump stations are equal ($P_{o1} = P_{o2} = 2$), so the available repair facility allocates to the pump station with the lower index of i , which is the first pump station.

4-5- Transition matrix

The transition matrix is a matrix, which their elements are the transition probabilities between different system states. The state-space diagram of a system with one repairable pump is presented in Figure 12, and the correspondence transition matrix is shown in Figure 13.

Insert Figure 12 here

Insert Figure 13 here

4-6- Differential equations

In memoryless models, using the Markov process and Chapman-Kolmogorov equations, we can obtain differential equations. Lets $P_n(t)$ represents the probability that the system is in state n , at time t . So, $P_n(t)$ is obtained from the differential equation according to Equation (41) as follows:

$$P'_n(t) + \left(\sum_{i \in \text{output follows from state } n} \lambda_i \right) P_n(t) = \sum_{j \in \text{input follows to state } n} \{ \lambda_j P_j(t) \} \quad (41)$$

By solving the set of the differential equation, the probability of each system's state at time t is obtained. The sum of these probabilities is system availability. To do the calculations, all coding is done in MATLAB R2019b software.

4-7- Simulation

One of the most common tools for analyzing the models and real-world systems is the simulation [22, 23, 28, 30, 31]. Particularly in models and systems that have a random nature, this procedure is very efficient. To validate our proposed method, we use a simulation technique. The simulation flowchart of the repairable three-state system is presented in Figure 14.

Insert Figure 14 here

In different periods, the event's times are randomly created concerning their failure rates. Considering that the failure and repair rates are constant, these times calculates using Equation (42).

$$t = \frac{-\text{Ln}\{\text{Rand}(R')\}}{\lambda} \quad (42)$$

Although the simulation technique in this paper is used to validate the proposed model, this method could notably be used to obtain repairable three-state systems with any distribution.

5- Numerical Example

In this section, we solve the instances in three different levels for different purposes. At first, we address a very small-size instance to illustrate how to determine the system's states and calculate the set of differential equations between the system's states. Then we solve a very large-size instance to compare the result of two solving methodologies in detail and demonstrate the presented method's complexity. Finally, we address different 15 instances to show the CKEB technique's superiority compared to the simulation technique.

Due to some restrictions (the company's policy) in reporting the real data, some of the model parameters are estimated by different forecasting techniques using the on-hand information. These estimated data are the model parameters, including transition rates. Data-driven concepts may be used (i.e., statistical and data mining techniques).

The first instance is a very small-size system with two pump stations, while each pump station has two repairable pumps. Moreover, one repair facilities are available to repair the pumps when they entirely failed. The minimum required working performance for each pump station considers as $k_i = 1$, which means that each pump station is considered operational if at least one pump is in fully-working condition or two pumps are in semi-working condition. The system has 43 states, which are divided into two categories: 17 working states and 26 failed states. All system's states for this instant are numbered in Table 3.

Insert Table 3 here

For the instance mentioned above, using Equation (41), the set of differential equations are presented in Equations (43) to (60) as follows.

$$\dot{p}_1(t) + 2 \cdot (\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{21}) \cdot p_1(t) = \mu_2 \cdot p_4(t) + \mu_1 \cdot p_8(t) \quad (43)$$

$$\dot{p}_2(t) + (\lambda_{11} + \lambda_{12} + \lambda_{13} + 2 \cdot \lambda_{21} + 2 \cdot \lambda_{22}) \cdot p_2(t) = 2 \cdot \lambda_{11} \cdot p_1(t) + \mu_2 \cdot p_9(t) \quad (44)$$

$$\dot{p}_3(t) + (2 \cdot \lambda_{11} + 2 \cdot \lambda_{12} + \lambda_{21} + \lambda_{22} + \lambda_{23}) \cdot p_3(t) = 2 \cdot \lambda_{21} \cdot p_1(t) + \mu_1 \cdot p_{12}(t) \quad (45)$$

$$\dot{p}_4(t) + (2 \cdot \lambda_{11} + 2 \cdot \lambda_{12} + \lambda_{21} + \lambda_{22} + \mu_2) \cdot p_4(t) = 2 \cdot \lambda_{22} \cdot p_1(t) + \lambda_{23} \cdot p_3(t) + \mu_1 \cdot p_{13}(t) \quad (46)$$

$$\dot{p}_5(t) + 2 \cdot (\lambda_{11} + \lambda_{12} + \lambda_{23}) \cdot p_5(t) = \lambda_{21} \cdot p_3(t) + \mu_1 \cdot p_{15}(t) \quad (47)$$

$$\dot{p}_6(t) + (\lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{21} + \lambda_{22} + \lambda_{23}) \cdot p_6(t) = 2 \cdot \lambda_{21} \cdot p_2(t) + 2 \cdot \lambda_{11} \cdot p_3(t) \quad (48)$$

$$\dot{p}_7(t) + 2 \cdot (\lambda_{13} + \lambda_{21} + \lambda_{22}) \cdot p_7(t) = \lambda_{11} \cdot p_2(t) + \mu_2 \cdot p_{16}(t) \quad (49)$$

$$\dot{p}_8(t) + (\lambda_{11} + \lambda_{12} + 2 \cdot \lambda_{21} + 2 \cdot \lambda_{22} + \mu_1) \cdot p_8(t) = 2 \cdot \lambda_{12} \cdot p_1(t) + \lambda_{13} \cdot p_2(t) + \mu_2 \cdot p_{14}(t) \quad (50)$$

$$\dot{p}_9(t) + (\lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{21} + \lambda_{22} + \mu_2) \cdot p_9(t) = 2 \cdot \lambda_{22} \cdot p_2(t) + 2 \cdot \lambda_{11} \cdot p_4(t) + \lambda_{23} \cdot p_6(t) \quad (51)$$

$$\dot{p}_{10}'(t) + (\lambda_{11} + \lambda_{12} + \lambda_{13} + 2 \cdot \lambda_{23}) \cdot p_{10}(t) = 2 \cdot \lambda_{11} \cdot p_5(t) + 2 \cdot \lambda_{21} \cdot p_7(t) \quad (52)$$

$$\dot{p}_{11}'(t) + (2 \cdot \lambda_{13} + \lambda_{21} + \lambda_{22} + \lambda_{23}) \cdot p_{11}(t) = \lambda_{11} \cdot p_6(t) + 2 \cdot \lambda_{21} \cdot p_7(t) \quad (53)$$

$$\dot{p}_{12}'(t) + (\lambda_{11} + \lambda_{12} + \mu_1 + \lambda_{21} + \lambda_{22} + \lambda_{23}) \cdot p_{12}(t) = 2 \cdot \lambda_{12} \cdot p_3(t) + \lambda_{13} \cdot p_6(t) + 2 \cdot \lambda_{22} \cdot p_8(t) \quad (54)$$

$$\dot{p}_{13}(t) + (\lambda_{11} + \lambda_{12} + \mu_1 + \lambda_{21} + \lambda_{22}) \cdot p_{13}(t) = 2 \cdot \lambda_{22} \cdot p_8(t) + 2 \cdot \lambda_{23} \cdot p_{12}(t) \quad (55)$$

$$\dot{p}_{14}'(t) + (\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22} + \mu_2) \cdot p_{14}(t) = 2 \cdot \lambda_{12} \cdot p_4(t) + \lambda_{13} \cdot p_9(t) \quad (56)$$

$$\dot{p}_{15}'(t) + (\lambda_{11} + \lambda_{12} + \mu_1 + 2 \cdot \lambda_{23}) \cdot p_{15}(t) = 2 \cdot \lambda_{12} \cdot p_5(t) + \lambda_{13} \cdot p_{10}(t) + \lambda_{21} \cdot p_{12}(t) \quad (57)$$

$$\dot{p}_{16}'(t) + (2 \cdot \lambda_{13} + \lambda_{21} + \lambda_{22} + \mu_2) \cdot p_{16}(t) = 2 \cdot \lambda_{22} \cdot p_7(t) + \lambda_{11} \cdot p_9(t) + \lambda_{23} \cdot p_{11}(t) \quad (58)$$

$$\dot{p}_{17}'(t) + (2 \cdot \lambda_{13} + 2 \cdot \lambda_{23}) \cdot p_{17}(t) = \lambda_{11} \cdot p_{10}(t) + \lambda_{21} \cdot p_{11}(t) \quad (59)$$

$$\begin{aligned} \dot{p}_f(t) = & \lambda_{12} \cdot p_2(t) + \lambda_{22} \cdot p_3(t) + (\lambda_{21} + \lambda_{22}) \cdot p_4(t) + 2 \cdot \lambda_{23} \cdot p_5(t) + (\lambda_{12} + \lambda_{22}) \cdot p_6(t) \\ & + 2 \cdot \lambda_{13} \cdot p_7(t) + (\lambda_{11} + \lambda_{12}) \cdot p_8(t) + (\lambda_{12} + \lambda_{21} + \lambda_{22}) \cdot p_9(t) \\ & + (\lambda_{12} + 2 \cdot \lambda_{23}) \cdot p_{10}(t) + (2 \cdot \lambda_{13} + \lambda_{22}) \cdot p_{11}(t) + (\lambda_{11} + \lambda_{12} + \lambda_{22}) \cdot p_{12}(t) \\ & + (\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}) \cdot p_{13}(t) + (\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}) \cdot p_{14}(t) \\ & + (\lambda_{11} + \lambda_{12} + 2 \cdot \lambda_{23}) \cdot p_{15}(t) + (2 \cdot \lambda_{13} + \lambda_{21} + \lambda_{22}) \cdot p_{16}(t) \\ & + (2 \cdot \lambda_{13} + 2 \cdot \lambda_{23}) \cdot p_{17}(t) \end{aligned} \quad (60)$$

The system's availability is the sum of the probabilities of $p_i(t)$, for all system's working states. The estimated availability for this example is equal to $A(100) = 0.95601$ for a 100 hours system's mission time.

To investigate the computational methods' complexity, a new large-size problem is solved with both methods. We consider different system's mission times from 100 to 450 hours. The system consists of four pump stations with five repair facility. The first pump station has four pumps, the second pump station has two pumps, the second pump station has three pumps, and the fourth pump station has only one pump. the minimum required working performance for all pump stations are considered as $k_i = 0.5$. The presented instance has a 2575 working state and one failed state. The failure rate of the pumps in these four pump stations are $\{(\lambda_{11} = 0.045, \lambda_{12} = 0.028, \lambda_1 = 0.035), (\lambda_{21} = 0.015, \lambda_{22} = 0.043, \lambda_{23} = 0.020), (\lambda_{31} = 0.025, \lambda_{32} = 0.035, \lambda_{33} = .037), (\lambda_{41} = 0.014, \lambda_{42} = 0.044, \lambda_{43} = 0.026)\}$, and the repair rate for all pumps' types is equal $\mu_i = 0.195; i = 1,2,3,4$. The number of simulation technique runs ten times, and the number of simulation iterations for each run is equal to 1,000,000. The results for solving the presented instance using both methods are presented in Table 4. In this table, when system operation time is considered 450 hours, the system's availability calculated with both methods is equal, even though the calculation times vary significantly. Figure 15 shows the system's availability calculated with both methods, and figure 16 illustrates the calculation time of both methods.

Insert Table 4 here

Insert Figure 15 here

Insert Figure 16 here

As shown in Table 4, with an increase in the system's mission time, the computational time of the CKEB method is not significantly changed. In contrast, the simulation method's computational time has increased dramatically, which confirms the CKEB method's efficiency in computational time.

As shown in Figure 15, by increasing the system's mission time, both techniques' availability converges to the same values. The reason is that by increasing the system's mission time, the simulation method approaches its steady-state condition, and the simulation method's availability achieves real system's availability, which is the same as calculated using the CKEB method.

Figure 16 shows that, by increasing on system's mission time, the CKEB method's computational time is not significantly changed. But, the computational time of the simulation method has increased significantly. Thus, by studying both Figures 15 and 16, the following essential points can be concluded:

- The availability obtained from both methods is equal, which is a reason for the proper functioning of the CKEB method.
- The computational time of the CKEB method is significantly less than the computational time of the simulation method, which is a reason for the superiority of the CKEB method

5-1- Managerial insights

To compare the solving methodologies better, we solve 15 different instances in small, medium, and large scales. These instances are designed based on the real pumps' configuration in different under-studied desalination plants. In these instances, the number of pump stations varies from two to five, the number of repair facilities varies from one to seven. So, the number of the systems' states are from 8 to 2576. Other systems' parameters are generated randomly. The results of these 15 instances are presented in Table 5.

Insert Table 5 here

In Table 5, the simulation runs ten times for each instance, the number of the simulation iterations for each run is equal to 1,000,000, and the system mission time equals 200 hours. As is presented in Table 5, for all system configurations, the computational time of the CKEB method is significantly less than the simulation technique's computational time. We used a T-test for comparing the availability and computational time of both methods using Minitab 17. The comparing results for systems' availability and computational time are presented in Table 6.

Insert Table 6 here

As presented in Table 6, both methods reach equal availability, proving the CKEB method's accuracy.

In a desalination plant, considering the workplace constraint, it is essential to optimize the pumps' configuration to keep the plant functional. Moreover, allocating the proper number of multi-tasking maintenance facilities increases the reliability of the pump stations. For this reason, different techniques like the RAP are applicable. The RAP is an Np. Hard problem in nature. So, the lack of an appropriate model to calculate the problem's objective function increases the complexity of the problem. In this paper, we presented a CKEB technique to calculate the system's availability consists of the series sub-systems. The components of each sub-system are considered repairable components, while some multi-tasking maintenance facilities repaired the failed components of all sub-systems.

The computational time ratio for the CKEB method to simulation technique is between 0.0002% to 0.0058% for the small-size problems, between 0.05% to 0.94% for the medium-size problems, and between 1.31% to 5.39% for the large-size problems. So, the presented model can reduce the complexity of the optimization methods used to optimize the components' configuration in such systems.

6- Conclusion and Further Studies

In this paper, a new method based on the Chapman-Kolmogorov equations is presented for evaluating the availability of a desalination system's pump stations with three-state repairable pumps. At the same time, each repair facility can service the pumps of all pump stations. The differential equations were compiled by using the Chapman-Kolmogorov equations. Then by solving the differential equations, the availability of the system was calculated. Finally, to validate the proposed model, we used the simulation method. The results showed that the CKEB technique could calculate the system's availability has significantly less computational time. The computational time ratio for the CKEB method

on simulation technique was between 0.0002% to 0.0058% for the small-size problems, between 0.05% to 0.94% for the medium-size problems, and between 1.31% to 5.39% for the large-size problems. On average, the computational time of the CKEB method for calculating the system's availability was 1.12% of the computational time of the simulation technique, which is the only other technique for calculating such a system's availability. Moreover, considering the proposed CKEB method's steps, the complexity of the proposed method in practice is less than the simulation technique.

For expanding the results of current research, some directions for future studies are as follows: optimizing the components' configuration using the results of the current research, solving the model by considering cold or warm standby policy, to draw the problem nearer to the real-case condition, review, and assessment of the reliability (availability) of repairable continuous-state problems, solving the model with a different repair policy for under-repair equipment, solving the model by considering the repair event for partially failed components, and solving the model with a different repair policy for under-repair equipment.

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Table 1.

Some recent research (between 2018-2020) related to RAP considering MSS.

Name of the researcher(s)	Year	Single or multi-objective(s)?	Repairable components?	The used technique for evaluating the system's availability (reliability)
Essadqi, Idrissi, and Amarir. [24]	2018	Multi	×	UGF
Tavana, Khalili-Damghani, Di Caprio, et al. [25]	2018	Multi	√	Markov process
Sharifi, Moghaddam, and Shahriari [26]	2019	Multi	×	UGF
Sharifi, Shahriari, and Zaretalab [27]	2019	Single	×	Technical & Organizational activities (TOA)
Sun, Li, and Zio [28]	2019	Multi	×	Considering the availability's upper and lower bond
Hadipour, Amiri, and Sharifi [29]	2019	Multi	√	---
Sharifi, Saadvandi, and Shahriari [30]	2020	Single	×	Recursive algorithm
Xu, Pi, Yang, et al. [31]	2020	Single	×	Interval-valued UGF
Borhani-Alamdar and Sharifi [32]	2020	Single	√	UGF
Xiahou, Liu, and Zhang [33]	2020	Multi	×	Evidential network
Sharifi, CheraghDashti Maljail, et al. [34]	2020	Single	×	TOA
Zaretalab, Hajipour, and Tavana [35]	2020	Single	√	UGF
Sharifi and Taghipour [36]	2020	Single	×	UGF-based method
Du and Li [37]	2020	Single	×	UGF
Sharifi, Shahriari, and Khajepour. [38]	2021	Single	×	---

Table 2.

Indexes, parameters, and decision variables.

Indexes:	
i :	Counter of pump stations,
Parameters:	
S :	Number of the system's pump stations,
n_i :	Number of pumps of the pump station i (redundant pump),
λ_{i1} :	Semi-failure rate of a working pump in pump station i ,
λ_{i2} :	The complete-failure rate of a working pump in pump station i ,
λ_{i3} :	The complete-failure rate of a semi-working pump of the pump station i ,
μ_i :	Repair rate of a failed pump of the pump station i ,
M :	Total number of repair facilities,
w_i :	Number of functional pumps in pump station i ,
s_i :	Number of semi-working pumps in sub-system i ,
m_i :	The number of repair facilities which are working in pump station i .
Po_i :	Working power of pump station i ,
k_i :	The minimum required working power for pump station i , to be considered as an operational pump station,

Table 3.

State definition of the presented instant.

Total working power of the system ($P_{O_T} = P_{O_1} + P_{O_2}$)		State
System is working	$P_{O_T} = 4.0$	1: $\{(2,0,0), (2,0,0)\}$,
	$P_{O_T} = 3.5$	2: $\{(1,1,0), (2,0,0)\}$, 3: $\{(2,0,0), (1,1,0)\}$,
	$P_{O_T} = 3.0$	4: $\{(2,0,0), (1,0,1)\}$, 5: $\{(2,0,0), (0,2,0)\}$, 6: $\{(1,1,0), (1,1,0)\}$, 7: $\{(0,2,0), (2,0,0)\}$, 8: $\{(1,0,1), (2,0,0)\}$,
	$P_{O_T} = 2.5$	9: $\{(1,1,0), (1,0,1)\}$, 10: $\{(1,1,0), (0,2,0)\}$, 11: $\{(0,2,0), (1,1,0)\}$, 12: $\{(1,0,1), (1,1,0)\}$,
	$P_{O_T} = 2.0$	13: $\{(1,0,1), (1,0,0)\}$, 14: $\{(1,0,0), (1,0,1)\}$, 15: $\{(1,0,1), (0,2,0)\}$, 16: $\{(0,2,0), (1,0,1)\}$, 17: $\{(0,2,0), (0,2,0)\}$,
System is failed	$P_{O_T} = 2.5$	$\{(2,0,0), (0,1,1)\}$, $\{(0,1,1), (2,0,0)\}$,
	$P_{O_T} = 2.0$	$\{(2,0,0), (0,0,1)\}$, $\{(1,1,0), (0,1,1)\}$, $\{(0,1,1), (1,1,0)\}$, $\{(0,0,1), (2,0,0)\}$,
	$P_{O_T} = 1.5$	$\{(1,1,0), (0,0,1)\}$, $\{(1,0,0), (0,1,1)\}$, $\{(1,0,1), (0,1,0)\}$, $\{(0,2,0), (0,1,1)\}$, $\{(0,0,1), (1,1,0)\}$, $\{(0,1,1), (1,0,0)\}$, $\{(0,1,0), (1,0,1)\}$, $\{(0,1,1), (0,2,0)\}$,
	$P_{O_T} = 1.0$	$\{(1,0,0), (0,0,1)\}$, $\{(1,0,1), (0,0,0)\}$, $\{(0,1,0), (0,1,1)\}$, $\{(0,0,1), (1,0,0)\}$, $\{(0,0,0), (1,0,1)\}$, $\{(0,1,1), (0,1,0)\}$,
	$P_{O_T} = 0.5$	$\{(0,1,0), (0,0,1)\}$, $\{(0,1,1), (0,0,0)\}$, $\{(0,0,0), (0,1,1)\}$, $\{(0,0,1), (0,1,0)\}$,
	$P_{O_T} = 0.0$	$\{(0,0,1), (0,0,0)\}$, $\{(0,0,0), (0,0,1)\}$.

Table 4.

Availability and calculation time of two methods for different system's mission time.

System's mission time (hours)	100	200	350	450
Availability _{sim}	0.82071	0.82022	0.82005	0.81997
Availability _{diff}	0.81899	0.81986	0.81990	0.81990
Elapsed time _{sim}	16992.00	29373.38	54134.43	69562.19
Elapsed time _{diff}	1591.10	1583.63	2128.11	1663.66

Table 5.

The results for 15 sample instants.

Problem Number	System's configuration	Number of the repair facilities	Number of the system's states	System's availability after 200 hours			Elapsed Time (Seconds)		
				Proposed model	Simulation		Proposed model	Simulation	
					Average of 10 runs	The standard deviation of 10 runs		Average of 10 runs	The standard deviation of 10 runs
1	[1,1]	1	8	0.74967	0.74990	0.00067	0.02	7926.05	117.25
2	[1,2]	1	17	0.80665	0.80631	0.00213	0.07	10298.10	235.13
3	[1,2]	2	17	0.84450	0.84461	0.00143	0.46	7892.44	211.63
4	[2,2]	3	35	0.96027	0.96039	0.00086	0.34	9894.69	223.35
5	[2,2]	4	35	0.96027	0.96030	0.00121	0.30	10272.08	271.85
6	[1,2,3]	1	229	0.74543	0.74595	0.00325	10.85	22020.66	370.02
7	[2,2,3]	2	535	0.93804	0.93802	0.00125	56.23	23884.42	460.44
8	[2,2,3]	5	340	0.95702	0.95722	0.00083	23.24	18198.22	213.19
9	[3,3,3]	5	1054	0.99046	0.99057	0.00058	229.00	24487.54	323.54
10	[2,2,2,2]	5	1125	0.91653	0.91735	0.00144	353.31	26755.38	424.01
11	[4,2,3,1]	5	2576	0.81986	0.82022	0.00099	1583.63	29373.38	446.53
12	[2,1,3,2]	3	1101	0.81795	0.81781	0.00112	331.12	25265.04	219.00
13	[2,2,1,4]	5	1380	0.81765	0.81790	0.00188	570.06	27372.54	453.53
14	[3,2,3,1]	7	1557	0.81857	0.81903	0.00066	720.97	26510.92	603.98
15	[2,2,1,2,1]	4	1300	0.72271	0.72212	0.00225	582.96	22370.16	313.13

Table 6.

T-values and P-Values for comparing two solving methodologies.

Problem Number	System's availability after 200 hours		Elapsed Time (Seconds)	
	T-Value	P-value	T-Value	P-Value
1	1.32	0.219	213.77	0.000
2	-0.50	0.626	138.50	0.000
3	0.24	0.813	117.93	0.000
4	0.44	0.669	140.09	0.000
5	0.08	0.939	119.49	0.000
6	0.51	0.625	188.10	0.000
7	-0.05	0.961	163.65	0.000
8	0.76	0.466	269.59	0.000
9	0.60	0.563	237.10	0.000
10	1.80	0.105	196.91	0.000
11	1.15	0.280	196.80	0.000
12	-0.40	0.702	360.04	0.000
13	0.42	0.684	186.88	0.000
14	2.20	0.055	135.03	0.000
15	-0.83	0.428	220.03	0.000

Fig. 1: Recovery treatment process at a desalination plant.

Fig. 2: System configuration.

Fig. 3: General state diagram for the proposed model.

Fig. 4: Diagram of the state related to Rule 1.

Fig. 5: Diagram of the state related to Rule 2.

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Fig. 8: Diagram of the state related to Rule 5.

Fig. 9: Diagram of the state related to Rule 6.

Fig. 10: Diagram of the state related to Rule 7.

Fig. 11: Diagram of the state related to Rule 8.

Fig. 12: State-space diagram of a system with one repairable member.

$$\begin{array}{l}
\textit{Working} (w) \\
\textit{Semi-working} (h) \\
\textit{Failed} (f)
\end{array}
\begin{array}{c}
\begin{array}{ccc}
w & h & f
\end{array} \\
\left[\begin{array}{ccc}
-(\lambda_{i1} + \lambda_{i2}) & 0 & \mu_i \\
\lambda_{i1} & -\lambda_{i3} & 0 \\
\lambda_{i2} & \lambda_{i3} & -\mu_i
\end{array} \right]
\end{array}$$

Fig. 13: Transition matrix related to Figure 12.

Fig. 14: Simulation flowchart of repairable three-state system.

Fig. 15: System Availability for different system operation time obtained with both methods.

Fig. 16: Computational time of both techniques.