A novel exact solution algorithm for a robust product portfolio problem under return uncertainty

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Abstract

This research address the optimization of product portfolio problem under uncertainty using the principles of financial portfolios theory. Since the success of the product portfolio is a strategic decision and it depends on the return's future changes, the return is best to be considered as an uncertain parameter. The innovation of this research is the use of robust optimization approach and providing an exact solution algorithm based on the model of Bertsimas and Sim. Given the assumption of uncertainty in the returns, the product portfolio model is developed based on the robust counterpart formulation of Bertsimas and Sim. An exact solution algorithm is also presented to reduce the solution time. The results obtained by implementing in a real case study of the dairy industry in Iran show that increasing the confidence level decreases the portfolio's total returns and increases its total risk. A comparison between the proposed algorithm and similar methods shows that, on average, it makes 3% improvement in the solution time.

Keywords: Robust optimization, Product portfolio selection, exact solution algorithm, Return,

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Uncertainty.

1. Introduction

In companies that offer multiple products, each product must have a well-defined place in the company's business strategy. Also, investors often show great interest in the companies' choice of product portfolio, that is, the set of products that the management choose to produce and how they are fitted together to ensure optimal returns over a given time period [1,2].

Each product has its own characteristics, demand, and profitability, and requires its own company resources and raw materials. Hence, product portfolios should be designed with careful consideration of all of the above details so as to achieve a really high chance of success in securing optimal returns for the company [3]. The science of portfolio management provides the tools and strategies that assist corporate executives and decision-makers to form optimal product portfolios.

Given the extensive scope of portfolio optimization discussion in the field of economics, Cardoso et al. have recommended that product portfolios are best to be analyzed using the financial portfolio theory [4]. Accordingly, the present work attempts to use the principles of the financial portfolio theory for the formation of optimal product portfolios. In the financial portfolio theory, the primary objectives of portfolios include both risk minimization and return maximization (Biglova, 2004 #295). Therefore, this research also pursues the simultaneous optimization of risk and returns in the formation of product portfolios.

In the remainder of this paper, Section 2 reviews the notable studies on the design of product portfolios, Section 3 introduces the proposed mathematical model, Sections 4 presents the robust counterpart model and an algorithm for exact solution of this model, Section 5 presents the numerical results and an analysis of these results, and Section 6 concludes the paper.

2. Research background

In a research by Gium et al. [5], they developed a systematic framework for improving the efficiency of hospital services with a portfolio approach. In this research, optimal portfolios were created using the Failure Mode and Effect Analysis (FMEA). In another research, Fernandes et al. [6] proposed a new approach for investment in product portfolio when there is some degree of manufacturing flexibility. Hajnoori et al. [7] presented a two-stage approach for stock portfolio selection based on the Markowitz model. In the first stage, a neural network was used to predict stock values based on historical data. Then, the Markovitz model was used to form the optimal stock portfolio. In a study by Solatikia et al. [8], they presented a bi-objective mathematical model for portfolio optimization in fully fuzzy conditions. The objectives of this model were to maximize returns and minimize risk, which was formulated as variance in returns. This model was optimized with the help of a crisp weight method. After solving this mathematical model, the results were evaluated and compared with similar works in the literature.

Takami et al. [9] studied the problem of product portfolio selection in a three-level supply chain where demands for products is predetermined and multiple products can be handled simultaneously. The objective was to form a product portfolio subject to supply chain constraints such that the chain profit is maximized. The numerical results of this study showed that adopting a portfolio management approach will have a significant impact on the supply chain profitability. In another research, Esfahani et al. [10] studied the optimization of project portfolios based on the Markowitz model. In this research, two mathematical models were examined. In one model, the objective was to maximize the portfolio's return by limiting its risk, but in the other model, the objective was to minimize the risk of the portfolio by limiting its total revenue to an optimal level. Finally, these two models were combined into a new mathematical model with the objective of maximizing the weighted sum of returns and risk aversion of the project portfolio. They also developed a harmony search algorithm for solving this mathematical model. The results showed that the proposed algorithm can provide near-optimal solutions for this problem.

Relich and Pawlewski [11] studied the selection of product portfolio under fuzzy conditions. In this research, the importance weights of products were determined by a questionnaire, where respondents were asked to express this importance with the help of linguistic variables. Then, these linguistic variables were transformed into fuzzy numbers. In this work, the objective of product portfolio selection was to achieve the highest fuzzy importance value subject to related production constraints. The numerical results demonstrated the effectiveness of the proposed method and its ability to produce realistic outputs. Goli et al. [12] proposed a multi objective mathematical model for product portfolio design. They solved the proposed model with multi objective invasive weed optimization algorithm. Yevseyeva et al. [13]. proposed several portfolio optimization models which are considered with limited budget provided for buying molecules and the fixed size of the portfolio Finally, Jiang et al. [14] investigated on the performance of NSGA-II and SPEA algorithm in the way of optimizing multi-objective portfolio problem. They founded that NSGA-II has a better efficiency in this problem.

The assumption of uncertainty can help us find more realistic solutions to the product portfolio problem. In this regard, many researchers have focused on the methods that employ scenario-based approaches, fuzzy numbers, and probabilistic numbers. Therefore, there is also a research gap in regard to the use of robust optimization approach in this field. Given the current state of literature, the most important innovation of the present work is the provision of a robust optimization method for designing product portfolio under uncertainty in product returns.

3. Product Portfolio Optimization

The primary goal of any ordinary investment is to gain maximum return with minimum risk. In order to reduce the risk of investment, investors, both natural and legal, prefer to buy a variety of stocks to form a diverse investment portfolio, and hence minimize the effects of periodic fluctuations on their returns. Given the particular importance of risk minimization, investors are always looking for reliable risk assessments to optimize their investment decisions [3]. In this research, the developed Markowitz model is used to optimize product portfolio selection. This model is described below.

i	index of products $i = 1, 2, .., n$
n	The number of products
$\tilde{\mu}_l$	Uncertain return of product i
K	The pre-defined number of products to be placed in the portfolio
LB_i	The investment lower bound of product i if it placed in the portfolio
UB_i	The investment upper bound of product i if it placed in the portfolio
R_i	The risk of producing product i
λ	Return priority coefficient

x_i Percentage of investment that is assigned to product i

u_i Binary variable and equal to 1 if the investment is done on the product i

$$Max\ \lambda \Sigma_{i=1}^{n} \tilde{\mu}_i x_i - (1-\lambda)\Sigma_{i=1}^{n} R_i x_i \tag{1}$$

$$\sum_{i=1}^{n} u_i = k \tag{2}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{3}$$

$$LB_i u_i \le x_i \le UB_i u_i, \forall i = 1,...,n \tag{4}$$

$$u_i \in \{0,1\}, \forall i = 1,...,n \tag{5}$$

$$x_i \ge 0, \forall i = 1,...,n \tag{6}$$

Eq. (1) is the objective function of the mathematical model, which maximizes the weighted sum of risk and returns. Eq. (2) states that exactly K products should be selected for inclusion into the product portfolio. Eq. (3) ensures that the percentages of investment in selected product sum up to 100%. Eq. (4) guarantees that the percentage of investment in any given product remain between (predefined minimum and maximum). Eqs. (5) and (6) define the type of decision variables. Since actual returns are assumed to be uncertain, the parameter $\hat{\mu}_i$ is used to represent the nominal return of product i . This nominal value is assumed to be equal to the mean or expected value of the return of product i . The parameter σ_i represents the standard deviation of the return of product i . The parameter ρ_i is a coefficient that determines how the actual return fluctuates around its nominal value. In other words, the return of product i fluctuate based on Eq. (7).

$$\tilde{\mu}_i \in [\hat{\mu}_i - \rho_i \sigma_i , \hat{\mu}_i + \rho_i \sigma_i] \tag{7}$$

4. Robust Portfolio Optimization

Researchers have proposed several methods, including stochastic programming, feasibility programming, and robust optimization, to address the uncertainty in mathematical modeling. Although robust optimization is not a new method, recent advances in this method have facilitated dealing with uncertainty in optimization problems (Sadjadi, 2012 #279). The robust optimization method proposed by Soyster [15] is too conservative for real-world applications, as it tries to produce optimal solutions that would remain feasible even in the worst-case scenario. Over the past decade, several researchers, including Ben-Tal and Nemirovski [16], and Bertsimas and Sim [17] have proposed less conservative methods with the same purpose. Bertsimas and Sim [17] have introduced a non-deterministic model with several innovations in developing the robust counterpart of the linear programming problem (where the linear program is guaranteed to remain linear). In this model, it is assumed that the returns vary independently in the variation range of the uncertain parameter. They proposed a new term called D-norm, which gives their method many advantages over other robust models.

Where x_j denotes the member of the vector x, t defines the maximization space with the above conditions, and x_t is the element of the vector x that meets the specified maximization conditions.
As a result, the portfolio mathematical model of this research is modified into Eq. (8) with regard to Bertsimas robust counterpart formulation.

Maximize w

Subject to

$$
\begin{aligned}
& w + (1-\lambda)\Sigma_{i=1}^{n} R_i x_i - \lambda(\Sigma_{i=1}^{n} \hat{\mu}_i x_i - z\Gamma - \Sigma_{i=1}^{n} \varphi_i) \le 0 \\
& \rho_i \sigma_i y_i \le z + \varphi_i, \forall i = 1,\ldots,n \\
& -y_i \le x_i \le y_i, \forall i = 1,\ldots,n \\
& \sum_{i=1}^{n} u_i = k \\
& \sum_{i=1}^{n} x_i = 1 \\
& LB_i u_i \le x_i \le UB_i u_i, \forall i = 1,\ldots,n \\
& u_i \in \{0,1\}, \forall i = 1,\ldots,n \\
& z \ge 0, x_i \ge 0, y_i \ge 0, \varphi_i \ge 0, \forall i = 1,\ldots,n
\end{aligned}
\tag{8}
$$

Where φ_i is a variable that is used for maintaining the convexity of the robust counterpart model (Bertsimas, 2004 #52). The parameter Γ controls the probability of violation of constraints and the objective function getting distanced from its ideal value (Bertsimas, 2004 #52). Bertsimas and Sim [17] call Γ the price of robustness. This parameter can take any real value between zero and the number of non-deterministic parameters in the constraint or in other words $\Gamma \in [0, n]$.$\Gamma = 0$ means all parameters maintain their nominal values and $\Gamma = n$ represents the worst possible state, that is, when all input parameters have uncertainty. In the article of Bertsimas and Sim [13], the upper bound of the probability of violation of constraints that involve uncertain data was calculated to $e^{-(\Gamma^2/2n)}$.

4.1. An Exact solution algorithm for robust portfolio model

Bertsimas and Sim [18] showed that if a mathematical model has *n* decision variables and all of these variables have non-deterministic coefficients in the objective function, then instead of solving the robust counterpart model, you can solve $n+1$modified deterministic models with nominal values. Álvarez et al. [19] have proven that at the robustness level of Γ, the robust counterpart model of Bertsimas and Sim can be replaced with $n-\Gamma+2$ deterministic models with nominal values for non-deterministic parameters. The exact solution algorithm presented here is an expansion of the one proposed by Álvarez et al. [19] for the stock portfolio optimization problem. The proof of the optimality of this algorithm is available in [19].

Step 1: Sort the products in the decreasing order of $\rho_i \sigma_i$. In other words, introduce products 1 to n in a way that $\rho_1\sigma_1 \geq \rho_2\sigma_2 \geq ... \geq \rho_n\sigma_n$. Also suppose the product $n+1$with $\rho_{n+1}\sigma_{n+1} = 0$.

Step 2: Determine Γ between 0 to n .

Step 3: For each r which $r \in \{\Gamma, \Gamma+1, ..., n+1\}$, solve the mathematical model presented in Eq.(9). It can be concluded the number of solving this model is equal to $n - \Gamma + 2$.

$$\begin{aligned}
&\text{Maximize } G_r = (1-\lambda)\Sigma_{i=1}^{n} R_i x_i \\
&+\lambda(\sum_{i=1}^{n} \hat{u}_i x_i - \Gamma\rho_r\sigma_r - \Sigma_{j=1}^{r}(\rho_j\sigma_j - \rho_r\sigma_r)x_j) \\
&\sum_{i=1}^{n} u_i = k \\
&\sum_{i=1}^{n} x_i = 1 \\
&LB_i u_i \leq x_i \leq UB_i u_i, \forall i = 1,...,n \\
&u_i \in \{0,1\}, \forall i = 1,...,n \\
&x_i \geq 0, \forall i = 1,...,n
\end{aligned} \tag{9}$$

Step 4: Determine r^* as the best solution found among all the $n - \Gamma + 2$ solved problems. In other words, r^* can be obtained by Eq. (10).

$$r^* = \arg\max\{G_r \mid r = \Gamma, \Gamma+1, ...n+1\} \tag{10}$$

Step 5: The optimal value of the objective function as well as the decision variables at the robust level of Γ can be obtained by Eq. (11).

$$W^* = G_{r^*} \quad x^* = x_{r^*} \quad u^* = u_{r^*} \tag{11}$$

5. Numerical Results

5.1. Case Study

The product portfolio optimization model was implemented for the Pegah Golpayegan Dairy Company in Iran. This company offers a variety of dairy products in different types and shapes. Naturally, the company can produce multiple dairy products simultaneously. One of the main problems of this company is poor decision making in regard to product portfolio formation, which means company spends significant resources on launching products that they fail to yield the expected returns or encounter unacceptable production risks and must be removed from the product portfolio [20-21]. From 2012 to 2017 for example, the company has produced 335 different products, but only 52 of these products have retained their place in the product portfolio, meaning that 283 products have been terminated in this period. Of these 283 products, 195 have been terminated within the first year of launch, mostly because of decline in sales and high market volatility. Hence, this company is in urgent need of a product portfolio with robustness against changes in returns. According to the collected information, currently, 85 products can be considered as potential candidates for inclusion in the company's product portfolio. In this case, the average profit margin over the past two years is assumed to be a good representative of the nominal return of the product ($\widehat{\mu}_i$). The standard deviation of the returns (σ_i) and the parameter ρ_i are considered to be 20%.

5.2. Results of the robust portfolio model

To implement the proposed robust counterpart model, the control parameter Γ should be set. For this purpose, first, there should be a clear definition of confidence level under uncertain conditions. In this study, confidence levels of 99%, 95%, and 90% were examined. Then, control parameters were initialized based on the probability of violation in each model. In this study, control parameters were initialized based on the probability of violation in the model with the confidence level set to 99%, 95%, and 90%. Table 1 shows the values of the control parameter for different confidence levels.

In the portfolio optimization model, the parameter λ, which determines the degree of preference of returns over risk, can significantly affect the optimal solution. Since this parameter can take any value between 0 and 1, the optimal solution was recalculated with λ set to 0, 0.2, 0.4, 0.6, 0.8 and 1. Following the company's recommendation, the size of the product portfolio (K) was set to 50. Using the described settings, the robust counterpart models of Bertsimas was coded in GAMS 14.4 and the globally optimal solution was obtained. Table 2 presents the results of the Bertsimas' robust counterpart models at different confidence levels and λ values.

In Table 2, the Total Return is calculated by multiplying each product's share of portfolio (x_i) by its nominal return ($\widehat{\mu}_i$), the Total Risk is the risk of the entire product portfolio based on the optimal x_i values, and W is the value of the objective function of the robust counterpart model. In Table 2, it can be seen that as λ increases, the total return and the total risk also increase. This is because a higher λ value means investment will be skewed towards those products that have a higher return, which also means a higher risk. Therefore, the optimal solutions obtained with different λ values basically show the effect of a trade-off between returns and risk. Plotting the optimal points obtained for different states of these trade-off results in a curve known as the efficient investment frontier. Figure (1) shows the efficient investment frontier obtained at different confidence levels.

Figure 1 shows that at the efficient investment frontier, as the total return of product portfolio increases, so does its total risk. It can also be seen that as the confidence level increases, the efficient investment frontier moves upward. In other words, at any given level of returns, using a higher confidence level corresponds to expecting a higher total risk for the portfolio. This is because a higher confidence level means that decision accounts for a greater area of uncertainty, but this comes at the expense of a weaker efficient frontier. Therefore, Figure 1 shows not only a trade-off between risk and returns but also a trade-off between the confidence level and the power of the efficient investment frontier. To better understand this trade-off, in Figure 2, W values are plotted against λ.

In Figure 2, it can be seen that as λ increases, the value of the objective function (W) increases too. This is because of higher variations in returns and lower variations in risk as λ value increases. For example, in the model Bertsimas 99%, raising λ from 0 to 1 increases the Total Return by about 0.01 but increases the Total Risk by only 0.007. Hence, the effect of increased returns easily surpasses the effect of increased risk, giving W an ascending trend. This figure also shows that increasing the confidence level from 90% to 99% decreases the value of W at all λ levels. Since the objective is to maximize W, this means that increasing confidence level translates into a lower expected quality for the portfolio. This difference is very small at λ = 0 but increases as we move toward λ = 1. Therefore, it can be concluded that there is always a trade-off between the level of confidence in solutions, decision-makers preference of returns versus risk, and the expected quality of the portfolio; a trade-off that decision-makers have to consider in their analysis.

5.3 Efficiency of proposed Bertsimas solution method

In order to evaluate the efficiency of the proposed algorithm, 16 examples with different dimensions were developed using case study data. These 16 examples are matched in Bertsimas model and optimized using GAMS software, and their solution time has been reported. Then, the algorithm introduced in Section 3.3.2 is implemented in the same software and the solution time is reported. The summary of the results presented in Table 3 is presented.

As shown in Table 3, the exact solution algorithm has been able to reduce the solution time between 0.08% and 6.84%. However, with increasing dimensions, the solution time has increased significantly, and with increasing problem dimensions, the proposed algorithm performs better and provides a higher reduction percentage of solution time. Figure 3 compares the solution time of the two methods.

As shown in Figure 3, with the increase in the dimensions of the examples, the solution time has increased sharply and the greatest time savings have been made in Examples 14-16. Because the choice of the product portfolio is a strategic decision, the exact solution method should be used in large dimensions. However, the robust counterpart model will show a very high solution time by increasing the dimensions of the problem. Therefore, it can be concluded that the proposed method is a very suitable tool for solving the robust product portfolio problem in large dimensions.

6. Conclusion and future researches

In this research, a robust optimization for the product portfolio problem was presented. The goal is to reduce the risk of investment and increase the return on investment in various products. As the portfolio selection problem is a strategic decision and its exact solution is achievable in a short and reasonable time, the numerical results obtained from its exact solution were analyzed.

Since the purpose of the optimization model is the simultaneous risk and return, and the level of confidence is also adjustable in the robust counterpart models, the final output of each model is an investment efficiency boundary under different levels of confidence. The analysis carried out in this research would help decision-makers to choose the most appropriate solution among the possible ones depending on their circumstances. In order to develop this research, it is suggested that the uncertainty in the products' return to be considered as gray and also fuzzy, and the result to be compared to the robust optimization approaches.

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Table 1. Values of the control parameter

	Bertsimas		
	99%	95%	90%
Γ	27.75	22.55	19.77

Table 2. Results of the Bertsimas' robust counterpart model

	λ	0	0.2	0.4	0.6	0.8	1
Bertsimas 99%	Total Return	0.4641	0.4717	0.4737	0.4757	0.4767	0.4769
	Total Risk	0.0014	0.0015	0.0024	0.0041	0.007	0.0083
	W	-0.0014	0.0199	0.0416	0.06413	0.08716	0.1108
Bertsimas 95%	Total Return	0.4641	0.4717	0.474	0.4761	0.4769	0.4771
	Total Risk	0.0014	0.0015	0.002	0.0039	0.0069	0.0082
	W	-0.0014	0.027	0.0558	0.0854	0.1155	0.1435
Bertsimas 90%	Total Return	0.4641	0.4717	0.4741	0.4763	0.477	0.4773
	Total Risk	0.0014	0.0015	0.0019	0.0038	0.0067	0.008
	W	-0.0014	0.0315	0.06489	0.099	0.1336	0.169

Table 3. The results of the proposed method on the test examples

#Example	Dimension			solution time		
	N	K	λ	Robust counterpart model	Exact solution method	Percent reduction
Pr1	55	20	0.3	3.451	3.441	0.29%
Pr2	55	20	0.7	3.419	3.416	0.08%
Pr3	55	50	0.3	3.296	3.285	0.33%
Pr4	55	50	0.7	3.297	3.275	0.67%
Pr5	65	20	0.3	5.776	5.556	3.81%
Pr6	65	20	0.7	5.792	5.674	2.04%
Pr7	65	50	0.3	5.599	5.476	2.20%
Pr8	65	50	0.7	5.612	5.474	2.45%
Pr9	75	20	0.3	9.572	9.309	2.75%
Pr10	75	20	0.7	9.488	9.205	2.98%
Pr11	75	50	0.3	9.125	8.794	3.62%
Pr12	75	50	0.7	9.137	8.636	5.49%
Pr13	85	20	0.3	27.443	25.944	5.46%

#Example	Dimension			solution time		
	N	K	λ	Robust counterpart model	Exact solution method	Percent reduction
Pr14	85	20	0.7	26.094	24.773	5.06%
Pr15	85	50	0.3	26.171	24.842	5.08%
Pr16	85	50	0.7	26.169	24.379	6.84%
Average				11.2151	10.7174	3.07%

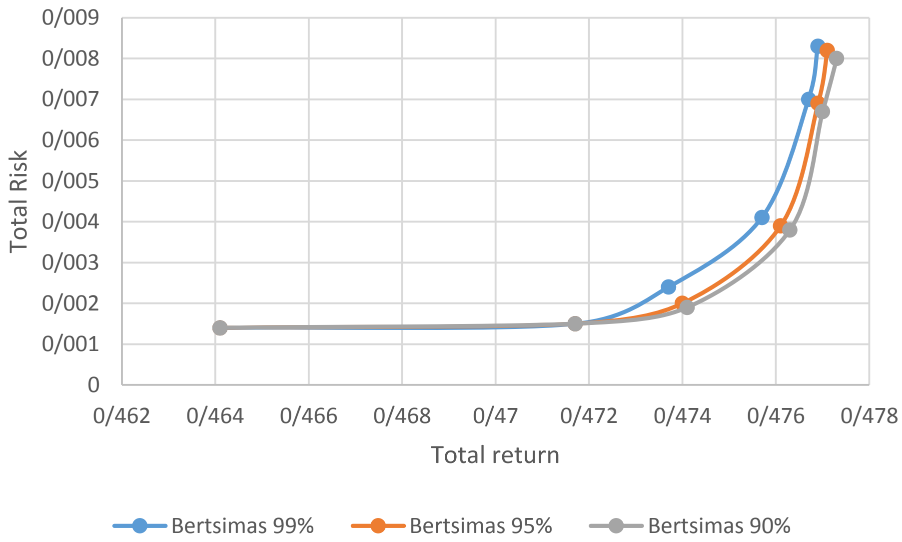

Figure 1. Efficient investment frontier obtained from the Bertsimas' robust counterpart model

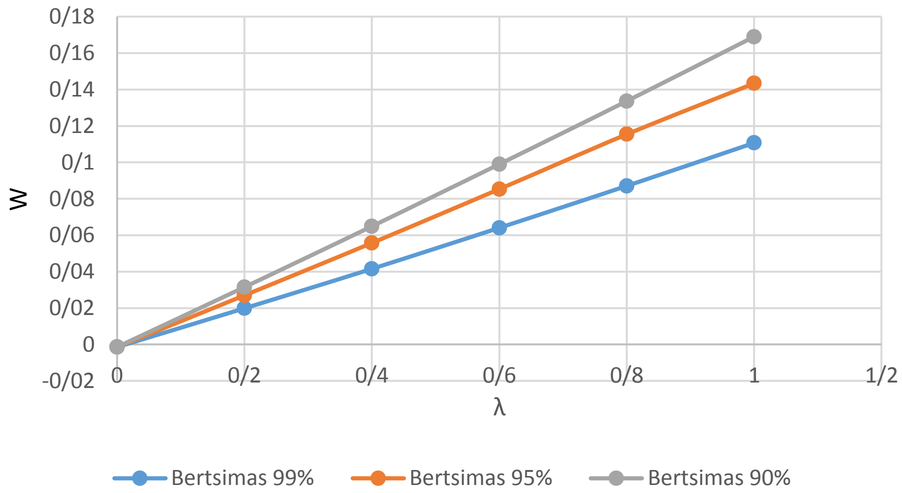

Figure 2. Objective function values of Bertsimas' robust counterpart model at different λ values

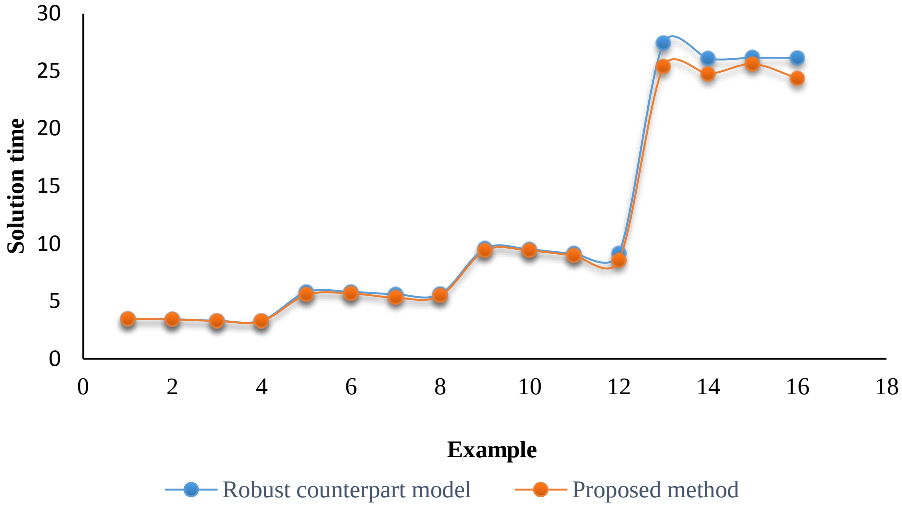

Figure 3. Comparing two methods' solution time

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