A Combinatorial Optimization Solution for Activity Prioritizing Problem

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Abstract: This paper discusses a special situation in project management in which an analyst wants to prioritize several independent activities to handle all them one after another, in such a way that there are no precedence relationships over the activities. As a novel idea, in this research, the notion is that the structure of prioritized activities is a linear arrangement, and therefore it can be taken into account as a combinatorial optimization problem. The paper formulates a mathematical model, and employs it in two real cases from oil and gas industry. In addition, the paper develops a row generation procedure to solve large scale problems, and reports the computational results for the problem instances of size up to 300 activities. The results demonstrate the applicability and efficiency of the proposed methodology.

Keywords: Activity Prioritizing Problem; Project scheduling; Mathematical programming; Row generation; Combinatorial optimization; Branch-and-cut, Oil and gas industry.

1. Introduction

As Gerardi [1] states in his book, project managers generally list several explanations as to why they love project management, such as they get to learn new things, work with new people, gain new skills, and are on the leading edge of innovation. In line with this view, this paper concentrates on an interesting challenge in project planning named Activity Prioritizing Problem (APP) which is a branch of project scheduling problem. Project scheduling has attracted considerable attention because of its critical role in project resource management [2, 3]. Consider a situation in which there are project activities with no precedence relationships over them. For instance, after scheduling a lot of project activities, a subset of them (with no inter-related dependencies) should be done in a limited period of time. In this situation, the problem simply is to assign priorities to such activities. It is clear that this challenge is not affected by duration of activities. Instead, several factors likely influence the priorities, such as project' owner view, program and project manager' ideas, risk factors [4], etc. In this regard, the role of project manager and project experts have been addressed as fundamental dimensions for the project success [5], although this fact has been ignores by the literature [6, 7]. On the other hand, Greek and Pullin [8] argue that many project managers do not focus on the criticality and urgency of project activities. Whereas a major concern in project management is the determination of a priority ranking for all activities that are candidates for assignment [9].

With the APP, there are many descriptive solutions and a few analytical models in the relevant literature. Among various descriptive guidelines (6-step process, Eisenhower matrix, Brian Tracy's method, ABCDE method, Bubble sort technique, Moscow method, Pareto principle, Zen habit, Rag rating system, Scrum prioritization, 1-3-9 method, and so on) let have little explanations for only two methods. According to the Zen habit method

[10], as a really simple prioritization guideline, each time the manager writes down one to three most important activities that he/she wants to complete on a limited period of time, and get on with completing them. The Eisenhower matrix, popularized by Covey [11], works by making segregate activities into four quadrants based on urgency and importance: urgent and important, important but not urgent, urgent but not important, and neither urgent nor important.

Despite the importance of the weighting and prioritizing project activities, analytical models have been rarely addressed in the related literature [7, 12]. In addition, most of the analytical studies use Multiple Attribute Decision Making (MADM) techniques to get a rank or weight vector of activities. Chang and Ibbs [9] introduced an expert system that uses possibility theory and the generalized modus ponens logic inference rule to prioritize activities in building construction projects. Alencar et al. [13] employed ELECTRE III technique to prioritize activities in a real-world construction project. Jha and Misra [14] made a survey to identify major factors in classifying and ranking of construction coordination activities. An analytical effort was performed by Mota et al. [15] who presented a model for supporting project managers to focus on the main activities of a project network using a MADM technique, taking into account several, often contradictory, points of view. That model aims to assign project activities into three classes of managerial practice. In an article by Baykasoglu et al. [12], a practical fuzzy rating and ranking technique was proposed to prioritize project activities with fuzzy attributes. They considered prioritization of activities instead of their classification. In addition, their technique was in some aspects similar to Mota et al.'s model [15], with substantial differences. Vanhoucke [16] introduced a scheme to measure the activities sensitivity and network topological information to monitor the project time performance. Mota and de Almeida [17] proposed an ELECTRE-based model to help project managers taking decisions about the key activities of project. Golpira [18] made a comparison between conventional MADM and Fuzzy MADM to determine activities weights and priorities using Analytical Hierarchy Process (AHP) and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) methods. Hadad et al. [19] prioritized the project activities attributes using an MADM model. These attributes include activities expected time, activities standard time deviation, activities expected cost, activities standard cost deviation and the activities time and cost obtained through simulation. Hadad [20] proposed a technique to prioritize project activities using AHP and Data Envelopment Analysis (DEA) as the research tools to select the proper influencing factors. Kalayathankal et al. [21] applied a fuzzy technique to prioritize activities in a software development project with the analogous activities in the electrical substation construction studied by Mota et al. [15]. They claimed brought out better project output in terms of finance, personnel, equipment, space, time etc. Berjis et al. [7] proposed an approach to determine the weights of project activities using DEA. They identified the parameters affecting the importance of activities through a review of the related literature and experts' opinions. The parameters included activity duration, activity cost, activity importance, activity difficulty, safety, communication rate, intellectual effort, physical effort, unfavourable work conditions and work related hazards.

In the current article, a novel optimization solution is proposed for a special situation of the APP. The idea in the proposed approach is completely different from previous works. The organization of the paper is as follows. Section 2 points out the problem characterization and related definitions. Section 3 develops a mathematical

model to solve the problem. Section 4 includes two real cases studied from oil and gas industry. In section 5, the paper discusses how to deal with large scale problems followed by experimental analysis in section 6. Finally, section 7 summarizes the results achieved, and provides future efforts possibilities.

2. Characterization of the problem

Herein, a special situation of the APP is introduced. This situation is an interesting issue which the authors encountered it in real projects. Indeed, real cases (described in section 4) were the root of the idea of the problem discussed in this paper. As a result, the need for the proposed model (described in section 3) originates from real-world projects, and thus can be employed in actual challenges.

Suppose a situation in which there are a variety of project activities and the aim is to prioritize activities to handle all them one after another. Indeed, all activities need a special unique resource which is available only for one activity in any point of the time. Taking this fact into account, we want to sequentially perform activities from the most important to the least important. Thus an arrangement structure of activities needs to be formed. That special resource may be a unique montage platform, a unique consultant, a unique working front, etc. The solution is influenced by some different “adjacency factors”. An adjacency factor indicates a reason whereby some activities need to be assigned the locations in the arrangement structure close to each other. Obviously, any linear permutation including all activities is a valid solution for the problem. Thus, the idea behind the current paper arises from this point of view that in the prioritizing challenge explained above is a linear arrangement actually; and therefore it is a combinatorial optimization problem in which activities should be arranged from the most important to the least important.

Let us define the problem in a general form. The problem has two groups of elements: “activities” and “adjacency factors” (henceforth referred to as factors). There are m activities $T_1, T_2, \dots, T_m$ to be prioritized and n factors having positive weights $\gamma_1, \gamma_2, \dots, \gamma_n$. In view of an assumed factor, the relevant activities should be placed one after another in the overall arrangement structure. The activities respecting j th factor are shown by subset $f_j \subset \{T_1, \dots, T_m\}$. An assumed factor may be proposed by any of o experts ($k = 1, 2, \dots, o$). Additionally, each expert may suggest various factors. The factors are actually derived from similarity of the complexity of activities, relationships among activities, equality in urgency of activities, etc. The weight of a given factor is calculated as Equation (1) which is weighted geometric mean stressed by many authors to aggregate the individual weights [22-25]. In this equation w_{jk} is the weight of j th factor preferred by k th expert. This individual weight is a number between 1 and 10. If an expert has no idea about a factor, the respected items should be dropped out from Equation (1). The power u_k , a number between 0 and 1, indicates the importance of k th expert’s comments. This power depends on the expert’s skills, experiences, abilities, or his/her area of educations.

$$\gamma_j = \left(\prod_{k=1}^o w_{jk}^{u_k} \right)^{1/\sum_{k=1}^o u_k}, \quad j = 1, \dots, m. \quad (1)$$

In view of jth factor, the best case is comprised of a contiguous chain of all the activities in f_j . In such a case, complete weight γ_j will be added to the objective value of the problem mathematical model; otherwise, a proportion of γ_j would be met. Taking this into consideration, the problem is to arrange activities so as to maximize the sum of proportions of factors weights. The value of each proportion depends on the adjacency of relevant activities in the overall arrangement structure.

Truly, the introduced challenge falls into combinatorial optimization problem. Although a wide range of models and algorithms exists to deal with famous graph-theoretic and combinatorial optimization problems, the introduced issue is not like any well-known problems. In particular, it is completely different from Cell Formation Problem [26, 27], Job-Shop Scheduling Problem [28], Project Scheduling Problem [2], Single Row Facility Layout Problem [29], Linear Ordering Problem [30], Minimum Linear Arrangement Problem [31, 32], Adjacency Problem [33], MADM problem [22, 34], and Linear Assignment Problem [35].

3. Modeling the problem

First, let us define:

- T_i refers i th activity ($i = 1, \dots, m$).
- f_j stands for the set of activities respecting j th factor ($j = 1, \dots, n$).
- γ_j indicates the weight of j th factor ($j = 1, \dots, n$).
- θ_{pq} ($p < q$), a binary variable, equals one if activity T_p is placed exactly adjoining activity T_q .
- $s()$ shows size of a set.

Now, we write formulation of the problem as Model (2)-(5).

$$Max \sum_{j=1}^n \left(\frac{\sum_{T_p \& T_q \in f_j | p < q} \theta_{pq}}{s(f_j) - 1} \right) \gamma_j \quad (2)$$

S.t.

$$\sum_{q=1, \dots, m | i < q} \theta_{iq} + \sum_{p=1, \dots, m | p < i} \theta_{pi} \leq 2, \quad i = 1, \dots, m \quad (3)$$

$$\sum_{T_p \& T_q \in B | p < q} \theta_{pq} \leq s(B) - 1, \quad B \{T_1, \dots, T_m\}, \quad 2 < s(B) \quad (4)$$

$$\forall \theta_{pq} \in (0, 1) \quad (5)$$

For the sake of better understanding, an explicit formulation for a small problem with corresponding LINGO code was provided in Appendix.

The rule to formulate Function (2) is “If two activities of f_j is placed right beside each other in the overall string of activities, then a “1” is added to objective function coefficient of γ_j ”. Let us consider some cases. As an instance, suppose that there are 4 activities and 3 factors, and we want to determine the objective function coefficients of γ_1, γ_2 and γ_3 , for given string $T_4T_1T_3T_2$. The set of activities respecting the factors are $f_1 = \{T_1, T_3, T_4\}$, $f_2 = \{T_2, T_4\}$, and $f_3 = \{T_1, T_2, T_4\}$. For a given factor f_j , at the best case, in which each activities of f_j has at least one adjacent activity from f_j , $\sum_{T_p \& T_q \in f_j | p < q} \theta_{pq}$ equals $s(f_j) - 1$, as a result, objective function coefficient of γ_j equals 1. For example, in the above instance, considering f_1 , T_4 is located next to T_1 , and T_1 is adjacent to T_3 , thus $\sum_{T_p \& T_q \in f_1 | p < q} \theta_{pq} = 2$ and coefficient of γ_1 equals 1. At the worst case, none of activities of f_j is placed right next to another activity of f_j , consequently $\sum_{T_p \& T_q \in f_j | p < q} \theta_{pq} = 0$, thus the objective function coefficient for γ_j will be equal to zero. For example, considering the second factor in the above instance, T_2 is not located next to T_4 , thus $\sum_{T_p \& T_q \in f_2 | p < q} \theta_{pq} = 0$ and thus coefficient of γ_2 equals 0 too. As another example, let's consider the third factor. Because T_4 and T_1 are adjoining, but T_2 is not located next to T_4 or T_1 , this results in $\sum_{T_p \& T_q \in f_3 | p < q} \theta_{pq} = 1$, i.e. coefficient of γ_3 equals 0.5.

Each activity might have at most two neighbourhoods in any solution. This limitation is observed by Inequalities (3). From the point of view of mathematical field of graph theory, these constraints cause the Model (2)-(5) generate only simple trees instead of unacceptable trees such as Figure 1-a.

The solution of the problem should be a contiguous chain of all the activities. As a result, no cycle is accepted. Constraints (4) prevent generating cyclic patterns such as Figure 1-b.

After having solved the Model (2)-(5), two issues should be handled. First, the solution may be non-connected trees such as Figure 1-c in which we need to join them to make a linear arrangement of all activities. Of course, it is possible to add Constraint (6) to the Model (2)-(5) to prevent generation of non-connected trees. But, adding this constraint is not advised, because it causes the model randomly connects the parts of non-connected tree. Instead, we advise the experts compare the tail-end activities of non-connected parts of trees, and then determine the most similar activities as joining points. As the second issue, we need to determine the direction of the arranged structure of activities from the most important to the least important. This aim also can be reached by comparing two tail-end activities of the structure using the expert's opinions.

$$\sum_{p=1}^m \sum_{q=1}^m \theta_{pq} = m - 1. \quad (6)$$

Insert Figure 1 here

4. Cases studied

4.1. Transformer oil commercialization

The focus of this case concerns a project of the know-how commercialization of a technology known in the industry as transformer oil. This technology is a highly-refined mineral oil that is stable at high temperatures and has excellent electrical insulating properties. It is used in oil-filled transformers, some high voltage capacitors, fluorescent lamp ballasts, and some types of high voltage switches and circuit breakers. Transformer oil is used in oil-immersed transformers, high voltage capacitors, tap changers, fluorescent lamp ballasts, and some switches and circuit breakers.

In one of the project modules, it was needed to prioritize seven major independent activities as in Table 1. Two experts were involved in this case. The importance of the experts was identified as $u_1 = 1$ and $u_2 = 0.5$. The experts recommended five factors #1 to #5. The first expert proposed the factors weights 8, 8, 5, 4, and 7 respectively. The latter gave only ideas on the factors #2 and #5 with $w_{22} = 3.5$ and $w_{52} = 9$. The final weights of the factors were obtained as $\gamma_1 = (8^1)^{1/1} = 8$, $\gamma_2 = (8^1 \times 3.5^{0.5})^{1/1.5} = 6.07$, $\gamma_3 = (5^1)^{1/1} = 5$, $\gamma_4 = (4^1)^{1/1} = 4$, and $\gamma_5 = (7^1 \times 9^{0.5})^{1/1.5} = 7.61$. The model (2)-(5) was formulated with 21 variables and 105 constraints. This model was solved with the use of the LINGO release 8.0 on a personal computer. The optimal arrangement of the activities was obtained as $T_7 T_5 T_4 T_1 T_2 T_3 T_6$ corresponding to the objective value 26.68. Therefore, factor #4 is the only one whose activities include a gap. The experts determined T_6 as the most important activity.

Insert Table 1 here

4.2. Planning at NIOC

National Iranian Oil Company (NIOC), according to its mission on oil production from the country's reservoirs, compiles five-year productions and investment plans for all oil and gas fields in the country. This task is the responsibility of the NIOC's project planning management deputy. In this regard, this deputy receives final reports of several studies made by other deputies (such as deputy for hydrocarbon reservoirs, deputy for economic feasibility and investment, deputy for oil and gas projects, and so on), and analyses them to make investment plan.

The current case studied, reports a real project at the NIOC in which the output of eight studies was received to be analysed by the relevant team. Therefore, eight activities were defined as:

- T_1 : Analysing reservoir study.
- T_2 : Analysing surface and downstream study.
- T_3 : Analysing the study of target markets.

- T_4 : Analysing the study of energy capital financing.
- T_5 : Analysing the study of the type of development and maintenance contracts.
- T_6 : Analysing of oil transfer and transportation study.
- T_7 : Analysing the evaluation of similar projects.
- T_8 : Analysing the production of crude oil.

Due to the limited human resources in the team (i.e., technical and planning experts), the activities had to be done sequentially. Moreover, based on four factors, defined by the planning management of the NIOC, the following groups of activities were determined to be done continuously:

- Activities T_1 and T_2 (due to engineering and reservoir factor, with weight 100).
- Activities T_3 , T_6 , and T_8 (due to international market factor, with weight 40).
- Activities T_4 and T_5 (due to economic feasibility factor, with weight 70).
- Activities T_4 and T_7 (due to technology factor, with weight 40).

Using the proposed model, the optimal solution was obtained including three non-connected trees $T_5T_4T_7$, T_1T_2 and $T_6T_3T_8$. Finally, according to the expert's opinions, the linear arrangement of activities was determined as $T_1T_2T_7T_4T_5T_6T_3T_8$ with the best objective value 250.

5. Dealing with large scale problems

The Model (2)-(5) includes $m^2 / 2 - m / 2$ binary variables and $2^m - m^2 / 2 + m / 2$ constraints ($m > 2$). Hence, the constraints exponentially grow in number by increasing m . For example, there are 22 constraints for $m = 5$, 979 constraints for $m = 10$, and 32663 constraints for $m = 15$. The problem was modelled as binary programming that is a subset of integer programming. Since integer programming is NP-hard [36, 37], the problem must also be hard. Due to the involved NP-hardness of the problem, heuristic and meta-heuristic approaches may strike us. Despite this, the current paper aims at looking for a method that provides global optimal solution. To solve large scale models, the paper suggests a relaxation that is removing Constraints (4) from the Model (2)-(5), resulting in a relaxed model with only m constraints. Needless to say, the solution pattern of the relaxed model may include cycles and non-connected sub-graphs. Definitely, the output pattern may be one of two cases: a solution pattern i.e. a linear layout of all of activities, or a non-connected graph which includes at least one cycle. Accordingly, at first, we should identify any cycles, if those exist. With this purpose in mind, a pseudo code is designed as Figure 2. This code, in each round, arbitrarily chooses an initial vertex named "root", afterward, tries to search the next connected vertex to the initial vertex. The algorithm continues to search for the next vertex connected to the previous found one until getting the root or not finding the next connected vertex. The former shows a cycle and the latter indicates a chain.

Insert Figure 2 here

Obviously, if there were no cycles (i.e., $t = 0$), the current pattern is the optimal solution; otherwise, there would be one or more cycles. Therefore, in order to prevent generating the cycles, under a row generation approach, new Constraints (7) will be added to the Model (2)-(5), where V_r is the set of vertices (i.e. activities) for the r th cycle. This approach is often called branch-and-cut algorithm. Moreover, it should be noted that the row generation approach is a dual form of column generation approach [38].

$$\sum_{T_p \& T_q \in V_r | p < q} \theta_{pq} \leq s(V_r) - 1, \quad \forall r = 1, 2, \dots, t. \quad (7)$$

Let us, in summing up, review the proposed procedure. At the first iteration, the Model (2)-(5) is solved without Constraints (4) (i.e., $t = 0$). Now, subject to the presence of any cycle in the generated pattern, the relevant Constraint (7) would be added into the model. Admittedly, this version of model includes Constraints (7) instead of Constraints (4). This model should be solved and rechecked for presence of cycles again. This iterative process continues until no cycle can be found.

6. Experimental evaluation

As a preliminary assumption, we assume that there are three parameters affecting running time: the number of activities (m), the number of factors (n), and the size of factors (s). Consequently, a typical problem is addressed by (m, n, s) i.e., a problem with m activities, and n factors by size s . By dint of the fact that the current study is the first research to consider the introduced problem, naturally there was no benchmark instance in the literature. Hence, the instance problems (i.e. data set) were randomly generated by a macro module program in Excel software. LINGO software version 8.0 was used to solve the mathematical models along with an Excel macro to identify cycles. The procedure was run on an ASUS Core (i7-7500) 2.7 GHz with 12.00 GB memory.

6.1. Sensitivity analysis

For the purpose of sensitivity analysis, the study is conducted on several instance problems with the number of activities ranging from $m = 25$ to $m = 300$, the number of factors ranging from $n = 100$ to $n = 2500$, and the sizes of factors ranging from $s = 3$ to $s = 75$. Figures 3-a, 3-b, 3-c, 3-d, 3-e and 3-f portray running time plots resulting from solving instance problems. All reported times are average of ten times running the algorithm on each level of the parameters.

Figures 3-a, 3-b and 3-c show variation in running time with s , each curve under a fixed m and n . These figures indicate that increasing s often results in greedy growing running time. Interestingly, we found out that for $n = 500$ and $n = 1000$ a peak is occurred for s around 10. Figures 3-d and 3-e present how running time varies as n is grown. These figures are made with fixed m (i.e., $m = 100$ and $m = 150$), and three levels $s = 10$, $s = 20$, and $s = 30$. The curves in Figure 3-d show a slow incremental effect of n on the running time of

instance problems even up to 2500 factors. On the contrary, Figure 3-e does not follow the same trends as curves in Figure 3-d. The curves have increasing trend up to around $n = 1550$, followed by an abrupt drop, such that there is a rather uniform trend after $n = 1600$. Figure 3-b confirms this phenomenon, too; where the curve of $n = 2000$ is very under the curve of $n = 1500$. Figure 3-f shows the variation in running time with n . This figure is drawn based on instance problems by $n = 500$ and four levels of s . The results of analysis illustrated that the curves as in Figure 3-f are the only ones with exponential growth. Almost all the curves show exponential growth in running time, when m increases. Except curve with $s = 3$, on the other cases there were large scale instance problems (about $m = 250$ to $m = 300$ activities) with running time even more than 24 hours, only for the first iteration of the solving procedure.

Insert Figure 3 here

Regarding unusual variations in Figures 3-c and 3-e, we could not find out a definite interpretation except that it may be because of intrinsic connection among factors. It should be noted that all the solved instance problems obtained the optimal solution through between 3 to 12 iterations. Further, as the figures show, running times range between 1 and 3500 seconds. In sum, the running times are very reactive (i.e., alert) to m , and slightly sensitive to s and n .

6.2. Efficiency analysis

For the sake of demonstrating the efficiency of the developed row generation algorithm, the employed approach would be as try to solve the test problems using the un-relaxed Model (2)-(5) as much as possible depending on running time or computer memory limitations, and then solve the same test problems using the row generation algorithm. This will show the effectiveness of the row generation algorithm over the un-relaxed Model (2)-(5). In order to determine the levels of parameters (m , n , and s) to generate the instance problems, the major result of the previous section, sensitivity analysis, was used. Correspondingly, the number of activities (m) is considered as the main parameter to be varied, with two levels of the other parameters as low ($n = m$ and $s \cong 0.25m$) and high ($n = 3m$ and $s \cong 0.75m$). Table 2 presents the characteristics and results of 10 test problems with the number of activities ranging from $m = 5$ to $m = 19$ for the un-relaxed Model (2)-(5) compared with the relaxed version. It was not possible to bring in the instances with more activities due to computer memory limitations (after verifying that using greater number of activities did not affect the obtained results). For comparison, the two last columns in Table 2 were considered. The former, decreasing in constraints, is defined as one minus the ratio of the total number of constraints in the un-relaxed Model (2)-(5) to that in the relaxed model. The latter, decreasing in running time, is similar to the former except that it concerns running time instead of the total number of constraints. Unquestionably, the results confirm the efficiency of the proposed algorithm.

Insert Table 2 here

6.3. Running time analysis

This section answers this question: “What size of the problems can be solved with the use of the proposed branch-and-cut algorithm in a logical time?” Among many solved problems, the results for average running time of 45 typical tested problems are shown in Table 3, sorted by running time. Problems P43 to P45 are typical instances with very high running time. We can conclude that the proposed procedure is undeniably useful to get the optimal solution for the problems including even up to 250 or some more activities in at most 60 minutes. But, when the number of activities is going on around 300 activities or more, the running time may unexpectedly jump to even more than 24 hours.

Insert Table 3 here

7. Concluding remarks

To start with, this paper defined a special problem of prioritizing activities (named the APP) in which the aim is to handle activities one after another, by a serial form. Later, the notion of making a solution for the problem with the use of a mathematical model was explained. Studying on the state-of-the art of optimization and decision analysis, it is found out that this idea has not been studied. Taking this fact into considerations, the first contribution of the research was in introducing the idea of prioritizing activities as an optimization problem. Additionally, the challenge of large scale problems was also discussed, and next, a branch-and-cut procedure to deal with such situations was developed. Thereby, another contribution of the research is in demonstrating how a row generation approach can be used to get the optimal solution of the APP. The evaluations and discussions presented the efficiency and usability of the methodology for large scale problems even for instances of up to 250 activities. In conclusion, the proposed methodology has a promising future and practical applications in project decision making challenges in a productive manner. Further researches can be undertaken in (I) applying the proposed method in complex and large size real-world cases, and (II) developing heuristic algorithms to solve the problem.

Appendix

In order to illustrate how the Formulation (2)-(5) is written, this appendix provides a numerical example for a small problem. This problem includes five activities T_1 to T_5 , and six factors with relevant sets as $f_1 = \{T_3, T_4\}$, $f_2 = \{T_1, T_3\}$, $f_3 = \{T_1, T_5\}$, $f_4 = \{T_1, T_2\}$, $f_5 = \{T_1, T_3, T_4, T_5\}$, and $f_6 = \{T_1, T_2, T_4\}$. The weights of factors are $\gamma_1 = 9, \gamma_2 = 7, \gamma_3 = 8, \gamma_4 = 2, \gamma_5 = 7$, and $\gamma_6 = 3$. The mathematical Model (2)-(5) for this problem is:

$$\text{Max } 9(\theta_{34}) + 7(\theta_{13}) + 8(\theta_{15}) + 2(\theta_{12}) + 7\left(\frac{\theta_{13} + \theta_{14} + \theta_{15} + \theta_{34} + \theta_{35} + \theta_{45}}{3}\right) + 3\left(\frac{\theta_{12} + \theta_{14} + \theta_{24}}{2}\right)$$

$$\theta_{12} + \theta_{13} + \theta_{14} + \theta_{15} \leq 2$$

$$\theta_{12} + \theta_{23} + \theta_{24} + \theta_{25} \leq 2$$

$$\theta_{13} + \theta_{23} + \theta_{34} + \theta_{35} \leq 2$$

$$\theta_{14} + \theta_{24} + \theta_{34} + \theta_{45} \leq 2$$

$$\theta_{15} + \theta_{25} + \theta_{35} + \theta_{45} \leq 2$$

$$\theta_{12} + \theta_{13} + \theta_{23} \leq 2$$

$$\theta_{12} + \theta_{14} + \theta_{24} \leq 2$$

$$\theta_{12} + \theta_{15} + \theta_{25} \leq 2$$

$$\theta_{13} + \theta_{14} + \theta_{34} \leq 2$$

$$\theta_{13} + \theta_{15} + \theta_{35} \leq 2$$

$$\theta_{14} + \theta_{15} + \theta_{45} \leq 2$$

$$\theta_{23} + \theta_{24} + \theta_{34} \leq 2$$

$$\theta_{23} + \theta_{25} + \theta_{35} \leq 2$$

$$\theta_{24} + \theta_{25} + \theta_{45} \leq 2$$

$$\theta_{34} + \theta_{35} + \theta_{45} \leq 2$$

$$\theta_{12} + \theta_{13} + \theta_{14} + \theta_{23} + \theta_{24} + \theta_{34} \leq 3$$

$$\theta_{12} + \theta_{13} + \theta_{15} + \theta_{23} + \theta_{25} + \theta_{35} \leq 3$$

$$\theta_{12} + \theta_{14} + \theta_{15} + \theta_{24} + \theta_{25} + \theta_{45} \leq 3$$

$$\theta_{13} + \theta_{14} + \theta_{15} + \theta_{34} + \theta_{35} + \theta_{45} \leq 3$$

$$\theta_{23} + \theta_{24} + \theta_{25} + \theta_{34} + \theta_{35} + \theta_{45} \leq 3$$

$$\theta_{12} + \theta_{13} + \theta_{14} + \theta_{15} + \theta_{23} + \theta_{24} + \theta_{25} + \theta_{34} + \theta_{35} + \theta_{45} \leq 4$$

$$\theta_{12}, \theta_{13}, \theta_{14}, \theta_{15}, \theta_{23}, \theta_{24}, \theta_{25}, \theta_{34}, \theta_{35}, \theta_{45} \in (0,1)$$

To this example, the LINGO code and the solution are presented as Figure 4, which result in the optimal string as $T_5 T_1 T_3 T_4 T_2$. This corresponds to the objective value 32.5.

Insert Figure 4 here

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Figure 1. Infeasible patterns containing: (a) unacceptable tree, (b) cycle, and (c) non-connected tree

Inputs: Set of θ_{pq} values, came from solution of the relaxed mathematical model

Method:

```

{
   $CT \leftarrow \{T_1, \dots, T_m\}$     !the activities to be checked
   $t \leftarrow 0$ 
  Next cycle:
    Take an element with smallest numerator from  $CT$  , named  $root$ 
     $x \leftarrow$  numerator of  $root$ 
     $Cycle \leftarrow \{T_x\}$ 
     $previous \leftarrow \{\}$ 
     $CT \leftarrow CT - \{T_x\}$ 
    Next search:
      Search  $(\{T_1, \dots, T_m\} - \{previous\})$  for an element  $T_y$  in the way that
       $\theta_{xy} = 1$  (if  $x < y$ ) or  $\theta_{yx} = 1$  (if  $y < x$ ); then:
        On success do:    !an activity connected  $T_x$  to was found
          {
             $CT \leftarrow CT - \{T_y\}$ 
             $Cycle \leftarrow Cycle + \{T_y\}$ 
            If  $T_y = root$  then put  $t \leftarrow t + 1$  &  $V_t \leftarrow Cycle$  ; after that go to Next cycle    !a cycle was found
             $previous \leftarrow T_x$ 
             $x \leftarrow y$ 
            Go to Next search
          }
        On failure, if  $CT = \{\}$  then stop; otherwise go to Next cycle
    }
}

```

Outputs: Sets of cycles: there are t cycles ($r = 1, 2, \dots, t$); V_r : Set of vertices in the r th cycle.

Figure 2. The pseudo code for cycle identification

(a)

(b)

(c)

(d)

(e)

Figure 3. Comparative analysis of running times (in seconds)

```

LINGO Model - Lingo sample

Sets:
Activity/1..5/;
Pair (Activity, Activity)|&2 #GT# &1:Teta;

Endsets

Max=9*Teta(3,4)+7*Teta(1,3)+8*Teta(1,5)+2*Teta(1,2)+(7/3)*(Teta(1,3)+Teta(1,4)+Teta(1,5)+Teta(3,4)+Teta(3,5)+Teta(4,5))+
(3/2)*(Teta(1,2)+Teta(1,4)+Teta(2,4));

@for(Activity(i): @sum(Pair(p,q)|p #EQ# i #OR# q #EQ# i: Teta(p,q)<=2);

@for(Activity(i1): @for(Activity(i2) | i2 #GT# i1: @for(Activity(i3) | i3 #GT# i2: +Teta(i1, i2)+Teta(i1, i3)+Teta(i2, i3)<= 2;)));

@for(Activity(i1): @for(Activity(i2) | i2 #GT# i1: @for(Activity(i3) | i3 #GT# i2: @for(Activity(i4) | i4 #GT# i3:
Teta(i1, i2)+Teta(i1, i3)+Teta(i1, i4)+Teta(i2, i3)+Teta(i2, i4)+Teta(i3, i4)<= 3;)))));

Teta(1,2)+Teta(1,3)+Teta(1,4)+Teta(1,5)+Teta(2,3)+Teta(2,4)+Teta(2,5)+Teta(3,4)+Teta(3,5)+Teta(4,5)<=4;

@for(Pair(p,q): @Bin(Teta));

```

Solution Report - Lingo sample

Global optimal solution found at iteration: 4
Objective value: 32.50000

Variable	Value	Reduced Cost
Teta(1, 2)	0.000000	-3.500000
Teta (1, 3)	1.000000	-9.333333
Teta (1, 4)	0.000000	-3.833333
Teta (1, 5)	1.000000	-10.33333
Teta (2, 3)	0.000000	0.000000
Teta (2, 4)	1.000000	-1.500000
Teta (2, 5)	0.000000	0.000000
Teta (3, 4)	1.000000	-11.33333
Teta (3, 5)	0.000000	-2.333333
Teta (4, 5)	0.000000	-2.333333

Figure 4. The LINGO code and the related solution report for the numerical example

Table 1. The activity-factor matrix for the case studied

Activities		Factors				
		1	2	3	4	5
T₁	Specifying electrically insulate requirements.	*	*			
T₂	Specifying suppress corona and arcing equipment.		*	*	*	
T₃	Studying on coolant features.		*	*		*
T₄	Determining resist oxidation items.	*	*			
T₅	Determining deposit formation items.	*				
T₆	Studying on the overall system performance.		*		*	*
T₇	Making the manual for installation of the system.	*				

Table 2. The characteristics and results of 10 instances for both the un-relaxed and relaxed forms

Problem	(m, n, s)	The un-relaxed form				The relaxed form					Comparison	
		Variables	Constraints	Optimal objective function value	Running time (seconds)	Variables	Constraints	Added constraints for loops	Final objective function value	Running time (seconds)	Decreasing in constraints (percentage)	Decreasing in running time (percentage)
P1	(5, 5, 3)	10	22	237.5	0.06	10	5	1	237.5	0.05	72.727%	16.667%
P2	(5, 15, 4)	10	22	692.66	0.07	10	5	3	692.66	0.05	63.636%	28.571%
P3	(10, 10, 3)	45	979	355	0.1	45	10	0	355	0.05	98.979%	50.000%
P4	(10, 30, 7)	45	979	1319.5	0.1	45	10	1	1319.5	0.08	98.876%	20.000%
P5	(15, 15, 4)	105	32663	536.66	3.32	105	15	4	536.66	0.07	99.942%	97.892%
P6	(15, 45, 11)	105	32663	1949.5	6.04	105	15	3	1949.5	0.07	99.945%	98.841%
P7	(17, 17, 4)	136	130936	467.33	66.13	136	17	9	467.33	0.1	99.980%	99.849%
P8	(17, 51, 13)	136	130936	2290.583	23.9	136	17	4	2290.583	0.09	99.984%	99.623%
P9	(19, 19, 5)	171	524117	627	121.3	171	19	2	627	0.08	99.996%	99.934%
P10	(19, 57, 14)	171	524117	2259	693	171	19	4	2259	0.12	99.996%	99.983%

Table 3. The running time (in seconds) of the selected instances

Problem	m	(m, n, s)	Running time	Average
P1	150	(150,100,30)	98	946.61
P2		(150,500,30)	126	
P3		(150,500,10)	567	
P4		(150,1800,10)	651	
P5		(150,1900,10)	798	
P6		(150,200,20)	896	
P7		(150,2500,10)	917	
P8		(150,2000,20)	1064	
P9		(150,2200,10)	1099	
P10		(150,1500,30)	1134	
P11		(150,2100,10)	1295	
P12		(150,2500,10)	1526	
P13		(150,2000,30)	2135	
P14	200	(200,500,10)	105	443
P15		(200,500,3)	112	
P16		(200,500,20)	280	
P17		(200,500,30)	294	
P18		(200,500,30)	427	
P19		(200,500,20)	623	
P20		(200,500,10)	1260	
P21	225	(225,1500,5)	91	1726
P22		(225,500,5)	189	
P23		(225,2000,5)	217	

Problem	m	(m, n, s)	Running time	Average
P24		(225,1000,10)	266	
P25		(225,2000,5)	322	
P26		(225,500,40)	378	
P27		(225,1000,20)	476	
P28		(225,1000,40)	910	
P29		(225,1500,20)	959	
P30		(225,2000,20)	1715	
P31		(225,500,5)	2065	
P32		(225,2000,30)	2163	
P33		(225,1000,10)	6636	
P34		(225,1000,10)	7777	
P35	250	(250,500,3)	105	1300.83
P36		(250,500,20)	252	
P37		(250,500,10)	693	
P38		(250,500,20)	1267	
P39		(250,500,10)	2492	
P40		(250,500,20)	2996	
P41	300	(300,500,3)	413	-
P42		(300,500,30)	651	
P43		(300,500,10)	More than 24 hours	
P44		(300,500,20)	More than 24 hours	
P45		(300,500,30)	More than 24 hours	