



Relative agreement method for multiple-criteria decision-making problems with interval numbers

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Abstract. So far, many ways have been provided to solve Multiple-Criteria Decision-Making (MCDM) problems with interval numbers. Most of these methods rank the alternatives according to two criteria, that is, being close to the Positive Ideal Solution (PIS) and far away from the Negative Ideal Solution (NIS). In this paper, a method is presented for solving MCDM problems with interval numbers, such that being close to PIS and being away from NIS have the same effect in alternative ranking. In the proposed method, the first PIS and NIS are determined as interval numbers and distance of each alternative from PIS and NIS is calculated by extension of Euclidean distance. Then, a compromise index is defined to rank the alternatives. Three numerical examples are given to compare the proposed method with other methods presented in the literature.

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1. Introduction

Multiple-Criteria Decision-Making (MCDM) problems belong to an important research field in decision science, management, and operations research. In MCDM problems, a group of alternatives is evaluated and compared based on their performance under several criteria. Based on this assessment, alternatives are ranked and the alternative with the most aggregated performance is selected for implementation. There are many methods to solve MCDM problems [1–6]. Classical methods to solve MCDM problems are as follows: Simple Additive Weighting (SAW) method [7] which uses a simple aggregate function; Analytical

Hierarchy Process (AHP) [1] based on the judgment of Decision-Makers (DMs) to decompose a complex problem into a hierarchy with the goal at the top level of hierarchy; Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [2]; Vlse Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) [8] seeking an alternative, i.e., a compromise solution, which is as close to Positive Ideal Solution (PIS) as possible and as far away from Negative Ideal Solution (NIS) as possible; Complex Proportional Assessment (COPRAS) [3] technique which was initiated by Zavadskas et al. [4] and works similarly to SAW; and ELimination Et Choix Traduisant la REALité (ELECTRE) method has different types. The study of Govindan and Jespen [5] can be mentioned as a comprehensive source of ELECTRE and ELECTRE-based methods. Carneiro et al. [6] considered the effect of cognitive aspects in MCDM. For this purpose, they took credibility, expertise level, and behavior style of DMs into account and proposed a method that links these aspects to the problem. Abdollahi et al. [9]

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proposed a method based on the TOPSIS and entropy to rank multifarious demand response resources.

The above-mentioned classical methods have considered the cases that decision matrix includes exact data. However, it is not always the case; indeed, in most real-world problems, decision information is expressed as either fuzzy or interval numbers. To tackle uncertainty in MCDM problems, an integrated MCDM model based on Fuzzy AHP (FAHP), TOPSIS, and Grey Relational Analysis (GRA) in the paper industry was proposed by Kirubakaran and Ilangkumaran [10]. The authors employed FAHP to determine criteria weights and used TOPSIS-GRA to rank the alternatives. Singh et al. [11] proposed an integrated model based on FAHP and FTOPSIS for a selection of third-Party Logistics (3PL) selection in cold chain management. In their method, authors applied FAHP to rank criteria and used FTOPSIS to select the best 3PL. Mahmoudi et al. [12] presented an extension of fuzzy VIKOR to solve supplier selection problem. They proposed a fuzzy distance measure to rank the suppliers. The alternatives were ranked using the preference ratio method. Hu et al. [13] proposed an MCDM problem with stochastic and intuitionistic data.

Although focusing on interval numbers has been less than fuzzy numbers in uncertain MCDM problems, useful researches have been done in this case. Jahanshahloo et al. [14] developed the TOPSIS method for MCDM problems with interval numbers. In their research, first, the PIS and NIS were determined by a special algorithm and then, separation of each alternative from the PIS and NIS is calculated using interval arithmetic. Finally, alternatives are ranked using the relative closeness of each alternative with respect to the PIS. Jahanshahloo et al. [15] extended the TOPSIS method for MCDM problems with interval numbers. They first identified the PIS and NIS of each alternative, measured the separation of each alternative from its PIS and NIS, and ranked alternatives based on the relative closeness of each alternative with respect to its PIS. Sayadi et al. [16] developed the VIKOR method for MCDM problems with interval numbers. In their method, first, the PIS and NIS were determined. Then, utility and regret measures and VIKOR index were obtained for each alternative by using interval arithmetic. Finally, VIKOR indexes, which are interval numbers, were compared with each other. For this purpose, a coefficient called optimism level of DM was introduced and by using it, interval numbers were compared and alternatives ranked. Dymova et al. [17] developed the TOPSIS method for MCDM problems with interval numbers. Their proposed method is fundamentally different from similar methods. The first difference is that the interval numbers are compared and then, the PIS and NIS are determined. The second difference

is that each alternative is separated from the PIS and NIS by calculating the separation between centers of intervals. Using this measure does not require Euclidean distance or Hamming distance to calculate the separation of each alternative from the PIS and NIS, and the separation of each alternative from the PIS and NIS is easier to calculate. Hafezolkotob et al. [18] developed the multi-MOORA method for MCDM problems with interval numbers. In their method, first, decision matrices were normalized and weighted. Then, in order to rank the alternatives and find the best one, the interval numbers were compared using their degree of possibility. Jahan and Edwards [19] presented a VIKOR method for materials ranking with simultaneous availability of interval data and all types of criteria.

In this paper, a new method is presented to solve interval MCDM problems. In the proposed method, being close to PIS and being away from NIS have the same effect on alternatives ranking. In other words, what matters is achieving a compromise solution, which is as closest to the PIS as possible and as remotest from the NIS as possible simultaneously. For this purpose, the initial decision matrix is normalized first and the PIS and NIS are determined in the interval form. Then, the distance of each alternative from the PIS and NIS is calculated by extension of Euclidean distance and an index called relative agreement is obtained for each alternative. In the end, the alternatives are ranked by sorting these ratios in descending order.

The rest of this paper is organized as follows. Section 2 gives some interval definitions and interval arithmetic. Section 3 is devoted to the proposed method and its algorithm. Section 4 compares the proposed method with the interval TOPSIS method. Section 5 presents three numerical examples to compare the proposed method with those published in the literature. Section 6 concludes the paper with final remarks.

2. Interval numbers and their operation

In this section, some interval definitions and interval operation are presented and these preliminaries will be used in the next section [20–22].

Definition 1. Let $a = [a^l, a^u] = \{x \in \mathbb{R} | a^l \leq x \leq a^u, a^l, a^u \in \mathbb{R}\}$; then, a is called an interval number where a^l, a^u represent the lower and upper bounds of a , respectively. If $a^l = a^u$, then a is a real number. Also, if $a^l > 0$, then a is called a positive interval number and if $a^l \geq 0$, then a is called a non-negative interval number. The set of all interval numbers in \mathbb{R} is represented by $I(\mathbb{R})$.

Let $a = [a^l, a^u]$ and $b = [b^l, b^u]$ be interval numbers and $\lambda \geq 0$ is a real number. Then, we have:

1. $a = b$ if and only if $a^l = b^l$ and $a^u = b^u$;
2. $a + b = [a^l + b^l, a^u + b^u]$;
3. $a - b = [a^l - b^u, a^u - b^l]$;
4. $\lambda a = \lambda[a^l, a^u] = [\lambda a^l, \lambda a^u]$. Specially $\lambda a = 0$ if $\lambda = 0$;
5. $a * b = [\min\{a^l b^l, a^l b^u, a^u b^l, a^u b^u\}, \max\{a^l b^l, a^l b^u, a^u b^l, a^u b^u\}]$;
6. $a^2 = [\max\{0, \min\{a^l a^l, a^l a^u, a^u a^u\}\}, \max\{a^l a^l, a^l a^u, a^u a^u\}]$;
7. If $a^l \geq 0$, then $\sqrt{a} = [\sqrt{a^l}, \sqrt{a^u}]$.

Definition 2. Let $a_i = [a_i^l, a_i^u]$ ($i = 1, 2, \dots, n$) be interval numbers; then, $A = \{a_1, a_2, \dots, a_n\}$ is an interval n -dimensional vector.

Definition 3. Let $a_{ij} = [a_{ij}^l, a_{ij}^u]$ ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$) be interval numbers; then, $A = (a_{ij})_{m \times n}$ is an interval $m \times n$ matrix.

Definition 4. Let $a = [a^l, a^u]$ and $b = [b^l, b^u]$ be interval numbers, $l_a = a^u - a^l$, $l_b = b^u - b^l$. Then, the degree of possibility of $a \geq b$ is defined as follows:

$$p(a \geq b) = \max \left\{ 1 - \max \left(\frac{b^u - a^l}{l_a + l_b}, 0 \right), 0 \right\}, \quad (1)$$

and it satisfies the following properties:

- a. $0 \leq p(a \geq b) \leq 1$;
- b. $p(a \geq b) = 0$ if and only if $a^u \leq b^l$;
- c. $p(a \geq b) = 1$ if and only if $b^u \leq a^l$;
- d. $p(a \geq a) = \frac{1}{2}$;
- e. $p(a \geq b) + p(b \geq a) = 1$.

To rank the interval numbers $a_i = [a_i^l, a_i^u]$ ($i = 1, 2, \dots, n$), first, the interval numbers a_i ($i = 1, 2, \dots, n$) are compared in a pairwise manner using Eq. (1) and let $p_{it} = p(a_i \geq a_t)$, $i, t = 1, 2, \dots, n$. Then, a complementary matrix is constructed as follows:

$$P = (p_{it})_{n \times n}. \quad (2)$$

Summing all elements in each row of matrix P , we have:

$$p_i = \sum_{t=1}^n p_{it} \quad i = 1, 2, \dots, n. \quad (3)$$

Then, the interval numbers a_i ($i = 1, 2, \dots, n$) can be reordered in descending order in accordance with the values of p_i s.

3. The proposed method

In this section, the proposed method and its algorithm are presented. Suppose $X = \{x_1, x_2, \dots, x_I\}$ ($I \geq 2$) as a set of alternatives which we attempt to rank according to their performance in terms of a set of criteria $C = \{c_1, c_2, \dots, c_J\}$. Let w_j ($j = 1, 2, \dots, J$) be the weight of the criterion c_j where $w_j \geq$

0 ($j = 1, 2, \dots, J$), $\sum_{j=1}^J w_j = 1$, and $x_{ij} = [x_{ij}^l, x_{ij}^u]$ ($i = 1, 2, \dots, I, j = 1, 2, \dots, J$) be the performance of the i th alternative under the j th criterion.

First, to normalize the data, we define:

$$y_{ij} = [y_{ij}^l, y_{ij}^u] \quad i = 1, 2, \dots, I; \quad j = 1, 2, \dots, J, \quad (4)$$

where:

$$y_{ij}^l = \frac{x_{ij}^l - x_j^-}{x_j^+ - x_j^-} \quad j \in O,$$

$$y_{ij}^u = \frac{x_{ij}^u - x_j^-}{x_j^+ - x_j^-} \quad j \in O,$$

and:

$$y_{ij}^l = \frac{x_j^+ - x_{ij}^u}{x_j^+ - x_j^-} \quad j \in O',$$

$$y_{ij}^u = \frac{x_j^+ - x_{ij}^l}{x_j^+ - x_j^-} \quad j \in O',$$

where O is associated with benefit criteria (note that the more, the better) and they are considered as output. In addition, O' is associated with cost criteria (the less, the better) and they are considered as input, and:

$$x_j^+ = \max_{i=1,2,\dots,I} \{x_{ij}^u\},$$

$$x_j^- = \min_{i=1,2,\dots,I} \{x_{ij}^l\}.$$

Then, g denotes the PIS which is an interval vector and obtained as follows:

$$g = (g_1, g_2, \dots, g_J)^T, \quad (5)$$

where:

$$g_j = \left[\max_{i=1,2,\dots,I} y_{ij}^l, \max_{i=1,2,\dots,I} y_{ij}^u \right] \quad j = 1, 2, \dots, J.$$

Now, the separation of the alternative x_i ($i = 1, 2, \dots, I$) from the PIS is calculated as follows (using the interval calculations in Section 2):

$$d_i^+ = [d_i^{+(l)}, d_i^{+(u)}] = \sqrt{\sum_{j=1}^J W_j^2 [g_j^l - y_{ij}^u, g_j^u - y_{ij}^l]^2} \quad i = 1, 2, \dots, I. \quad (6)$$

Next, we define:

$$d^+ = \min_{i=1,2,\dots,I} \{d_i^{+(l)}\}. \quad (7)$$

In fact, d^+ shows the least possible separation of an alternative to the PIS. The NIS is represented by $b = (b_1, b_2, \dots, b_J)^T$, where:

$$b_j = \left[\min_{i=1,2,\dots,I} y_{ij}^l, \min_{i=1,2,\dots,I} y_{ij}^u \right] \quad j=1, 2, \dots, J. \quad (8)$$

Similarly, the separation of the alternative $x_i (i = 1, 2, \dots, I)$ from the NIS is computed as follows (using the interval calculation in Section 2):

$$d_i^- = [d_i^{-(l)}, d_i^{-(u)}] = \sqrt{\sum_{j=1}^J w_j^2 [y_{ij}^l - b_j^u, y_{ij}^u - b_j^l]^2}$$

$$i = 1, 2, \dots, I. \quad (9)$$

Besides, let:

$$d^- = \max_{i=1,2,\dots,I} \{d_i^{-(u)}\}. \quad (10)$$

In fact, d^- is the maximum separation to the NIS for all alternatives. Now, the relative agreement index of $x_i (i = 1, 2, \dots, I)$ is defined as follows:

$$R_i = \frac{d_i^-}{d^-} - \frac{d_i^+}{d^+} \quad i = 1, 2, \dots, I. \quad (11)$$

In fact, R_i is an interval number where:

$$R_i = [R_i^l, R_i^u], \quad (12)$$

and:

$$R_i^l = \frac{d_i^{-(l)}}{d^-} - \frac{d_i^{+(u)}}{d^+} \quad \text{and} \quad R_i^u = \frac{d_i^{-(u)}}{d^-} - \frac{d_i^{+(l)}}{d^+}.$$

Remark 1: Generally, the values of d^+ and d^- are positive and they can be zero only in some special cases. In such cases, to obtain $R_i (i = 1, 2, \dots, I)$, Eq. (13) is used instead of Eq. (12):

$$R_i = [R_i^l, R_i^u], \quad (13)$$

where:

$$R_i^l = \frac{d_i^{-(l)} + 1}{d^- + 1} - \frac{d_i^{+(u)} + 1}{d^+ + 1}, \quad \text{and}$$

$$R_i^u = \frac{d_i^{-(u)} + 1}{d^- + 1} - \frac{d_i^{+(l)} + 1}{d^+ + 1}.$$

In fact, through this conversion, the separation of alternatives to the PIS (d_i^+) and NIS (d_i^-) is summed up with the interval number [1,1]. Therefore, d^+ and d^- move one unit to right (see, Example 3). Thus, an index called relative agreement index ($R_i; i = 1, 2, \dots, I$) is assigned to each alternative, where $R_i^u \leq 0$. The higher relative agreement of an alternative means that it is closer to the PIS and farther from the NIS. Thus, $R_i (i = 1, 2, \dots, I)$ shows the satisfactory level for both criteria including the shortest distance from the PIS and the remotest distance from the NIS.

To compare the alternatives, the interval numbers (R_i s) should be compared (as mentioned in Section 2).

For this purpose, the complimentary matrix P is constructed using Eq. (1) as follows:

$$P = (p_{it})_{I \times I} \quad i, t = 1, 2, \dots, I, \quad (14)$$

where:

$$p_{it} = p(R_i \geq R_t).$$

Then, by summing up all elements in each row of matrix P , we have:

$$p_i = \sum_{t=1}^I p_{it} \quad i = 1, 2, \dots, I. \quad (15)$$

Now, all alternatives $x_i (i = 1, 2, \dots, I)$ are ranked according to $p_i (i = 1, 2, \dots, I)$ in descending order. Briefly, the algorithm of the proposed method is summarized in the following steps:

- Step 1:** Normalize the data using Eq. (4);
- Step 2:** Apply Eq. (5) to determine the PIS (g);
- Step 3:** Calculate the separation of the alternative $x_i (i = 1, 2, \dots, I)$ from the PIS (d_i^+) using Eq. (6);
- Step 4:** Obtain d^+ by Eq. (7);
- Step 5:** Apply Eq. (8) to determine the NIS (b);
- Step 6:** Calculate the separation of the alternative $x_i (i = 1, 2, \dots, I)$ from the NIS (d_i^-) using Eq. (9);
- Step 7:** Obtain d^- by Eq. (10);
- Step 8:** Apply Eq. (12) to calculate relative agreement index of $x_i (i = 1, 2, \dots, I) (R_i)$;
- Step 9:** Construct complimentary matrix P using Eq. (14) to compare interval numbers $R_i (i = 1, 2, \dots, I)$;
- Step 10:** Calculate $p_i (i = 1, 2, \dots, I)$ using Eq. (15) and rank all alternatives (x_i s) ($i = 1, 2, \dots, I$) according to $p_i (i = 1, 2, \dots, I)$ in descending order.

4. Comparison

Until now, many methods have been presented to solve the interval MCDM problems [14–19]. The main difference between these methods lies in normalizing decision matrix and aggregating functions. The interval TOPSIS method proposed by Jahanshahloo et al. [15] is one of the most popular methods. In this section, first, the interval TOPSIS method presented by Jahanshahloo et al. [15] is outlined and then, compared by the proposed method. Briefly, the algorithm of the interval TOPSIS method is as follows:

Step 1: Calculate the normalized decision matrix as follows:

$$n_{ij} = [n_{ij}^l, n_{ij}^u] \quad i = 1, 2, \dots, I, \quad j = 1, 2, \dots, J,$$

where:

$$n_{ij}^l = \frac{x_{ij}^l}{\sqrt{\sum_{i=1}^I [(x_{ij}^l)^2 + (x_{ij}^u)^2]}}$$

$$n_{ij}^u = \frac{x_{ij}^u}{\sqrt{\sum_{i=1}^I [(x_{ij}^l)^2 + (x_{ij}^u)^2]}}$$

Step 2: Construct the weighted normalized decision matrix as $v_{ij}^l = w_j n_{ij}^l$ and $v_{ij}^u = w_j n_{ij}^u$ for $i = 1, 2, \dots, I$ and $j = 1, 2, \dots, J$, where w_j ($j = 1, 2, \dots, J$) is the weight of the j th criterion and $\sum_{j=1}^J w_j = 1$.

Step 3: Identify the PIS of the k th alternative as follows:

$$A_k^+ = [A_k^{+l}, A_k^{+u}] \quad k = 1, 2, \dots, I,$$

where:

$$\begin{aligned} A_k^{+l} &= \{(v_1^{+l}, v_2^{+l}, \dots, v_J^{+l})\} \\ &= \left\{ \left(\max_{i \neq k} \{v_{ij}^u, v_{kj}^l\} \mid j \in O \right), \right. \\ &\quad \left. \left(\min_{i \neq k} \{v_{ij}^l, v_{kj}^u\} \mid j \in O' \right) \right\}, \end{aligned}$$

$$\begin{aligned} A_k^{+u} &= \{(v_1^{+u}, v_2^{+u}, \dots, v_J^{+u})\} \\ &= \{(\max v_{ij}^u \mid j \in O), (\min v_{ij}^l \mid j \in O')\}. \end{aligned}$$

Step 4: Identify the NIS of the k th alternative as follows:

$$A_k^- = [A_k^{-l}, A_k^{-u}] \quad k = 1, 2, \dots, I,$$

where:

$$\begin{aligned} A_k^{-l} &= \{(v_1^{-l}, v_2^{-l}, \dots, v_J^{-l})\} \\ &= \{(\min v_{ij}^l \mid j \in O), (\max v_{ij}^u \mid j \in O')\}, \end{aligned}$$

$$\begin{aligned} A_k^{-u} &= \{(v_1^{-u}, v_2^{-u}, \dots, v_J^{-u})\} \\ &= \left\{ \left(\min_{i \neq k} \{v_{ij}^l, v_{kj}^u\} \mid j \in O \right), \right. \\ &\quad \left. \left(\max_{i \neq k} \{v_{ij}^u, v_{kj}^l\} \mid j \in O' \right) \right\}. \end{aligned}$$

Step 5: Calculate the separation of the k th alternative from its PIS as:

$$d_k^+ = [d_k^{+l}, d_k^{+u}] \quad k = 1, 2, \dots, I,$$

where:

$$d_k^{+l} = \left[\sum_{j \in O'} (v_j^{+l} - v_{kj}^l)^2 + \sum_{j \in O} (v_j^{+l} - v_{kj}^u)^2 \right]^{1/2},$$

$$d_k^{+u} = \left[\sum_{j \in O'} (v_j^{+u} - v_{kj}^u)^2 + \sum_{j \in O} (v_j^{+u} - v_{kj}^l)^2 \right]^{1/2}.$$

Step 6: Calculate the separation of the k th alternative from its NIS as:

$$d_k^- = [d_k^{-l}, d_k^{-u}] \quad k = 1, 2, \dots, I,$$

where:

$$d_k^{-l} = \left[\sum_{j \in O'} (v_j^{-l} - v_{kj}^l)^2 + \sum_{j \in O} (v_j^{-l} - v_{kj}^u)^2 \right]^{1/2},$$

$$d_k^{-u} = \left[\sum_{j \in O'} (v_j^{-u} - v_{kj}^u)^2 + \sum_{j \in O} (v_j^{-u} - v_{kj}^l)^2 \right]^{1/2}.$$

Step 7: Define the relative closeness of the k th alternative to its PIS as:

$$R_k = [R_k^l, R_k^u] \quad k = 1, 2, \dots, I,$$

where:

$$R_k^l = \frac{d_k^{-l}}{d_k^{-u} + d_k^{+u}},$$

$$R_k^u = \frac{d_k^{-u}}{d_k^{-l} + d_k^{+l}}.$$

Step 8: Rank the preference order of all the alternatives according to R_k s ($k = 1, 2, \dots, I$).

To make a comparison between the proposed method and the interval TOPSIS method, the normalization step and the used aggregating function are considered in both methods.

4.1. The effect of normalization

Generally, in MCDM problems, normalization (scaling) is employed to remove dimensions of criteria values. Now, let the original data be transformed into the following:

$$x'_{ij} = \alpha x_{ij} + \beta \quad \alpha, \beta \in \mathbb{R} \quad \alpha > 0. \tag{16}$$

If x_{ij} be an interval number, by displaying β in interval form, $[\beta, \beta]$ as mentioned in Section 2, we have:

$$[x'_{ij}, x'_{ij}] = [\alpha x_{ij}^l + \beta, \alpha x_{ij}^u + \beta]$$

$$i = 1, 2, \dots, I, \quad j = 1, 2, \dots, J$$

$$\beta \in I(\mathbb{R}), \quad \alpha \in \mathbb{R}, \quad \alpha > 0. \tag{17}$$

Now, the question is “Does this translation affect the decision results obtained by these two methods?” In

the proposed method, the normalized values of x_{ij} ($i = 1, 2, \dots, I, j = 1, 2, \dots, J$) are calculated using Eq. (4). Therefore, the normalized criterion values of x'_{ij} ($i = 1, 2, \dots, I, j = 1, 2, \dots, J$) are calculated as follows:

$$y'_{ij} = [y'^l_{ij}, y'^u_{ij}], \tag{18}$$

where:

$$y'^n_{ij} = \frac{x'^l_{ij} - x'^-_{ij}}{x'^+_{ij} - x'^-_{ij}} \quad j \in O,$$

$$y'^n_{ij} = \frac{x'^u_{ij} - x'^-_{ij}}{x'^+_{ij} - x'^-_{ij}} \quad j \in O,$$

and:

$$y'^n_{ij} = \frac{x'^+_{ij} - x'^u_{ij}}{x'^+_{ij} - x'^-_{ij}} \quad j \in O',$$

$$y'^n_{ij} = \frac{x'^+_{ij} - x'^l_{ij}}{x'^+_{ij} - x'^-_{ij}} \quad j \in O',$$

where:

$$x'^+_{ij} = \max_{i=1,2,\dots,I} \{x'^u_{ij}\},$$

$$x'^-_{ij} = \min_{i=1,2,\dots,I} \{x'^l_{ij}\}.$$

Through Eq. (17), we have:

$$\begin{aligned} x'^+_{ij} &= \max_{i=1,2,\dots,I} \{\alpha x'^u_{ij} + \beta\} \\ &= \alpha \max_{i=1,2,\dots,I} \{x'^u_{ij}\} + \beta = \alpha x'^+_{ij} + \beta, \end{aligned}$$

$$\begin{aligned} x'^-_{ij} &= \min_{i=1,2,\dots,I} \{\alpha x'^l_{ij} + \beta\} \\ &= \alpha \min_{i=1,2,\dots,I} \{x'^l_{ij}\} + \beta = \alpha x'^-_{ij} + \beta. \end{aligned} \tag{19}$$

Now, using Eqs. (17) and (19), we have:

$$\begin{aligned} y'^n_{ij} &= \frac{\alpha x'^l_{ij} + \beta - \alpha x'^-_{ij} - \beta}{\alpha x'^+_{ij} + \beta - \alpha x'^-_{ij} - \beta} = \frac{x'^l_{ij} - x'^-_{ij}}{x'^+_{ij} - x'^-_{ij}} = y'^l_{ij}, \\ j \in O \quad i &= 1, 2, \dots, I, \end{aligned} \tag{20}$$

$$y'^n_{ij} = \frac{\alpha x'^u_{ij} + \beta - \alpha x'^-_{ij} - \beta}{\alpha x'^+_{ij} + \beta - \alpha x'^-_{ij} - \beta} = \frac{x'^u_{ij} - x'^-_{ij}}{x'^+_{ij} - x'^-_{ij}} = y'^u_{ij}.$$

$$j \in O \quad i = 1, 2, \dots, I.$$

A similar discussion holds for negative criteria ($j \in O'$), as well. Thus, we have:

$$y'_{ij} = y_{ij} \quad i = 1, 2, \dots, I; \quad j = 1, 2, \dots, J. \tag{21}$$

This shows that the proposed method is translation

invariance. That is, the values of the normalized criteria in our method are independent of the measurement units and dimensions of criteria.

In the method proposed by Jahanshahloo et al. [15], the normalized criteria values of x_{ij} ($i = 1, 2, \dots, I, j = 1, 2, \dots, J$) are calculated as follows:

$$\begin{aligned} n_{ij} &= [n^l_{ij}, n^u_{ij}] \\ i &= 1, 2, \dots, I; \quad j = 1, 2, \dots, J, \end{aligned} \tag{22}$$

where:

$$n^l_{ij} = \frac{x^l_{ij}}{\sqrt{\sum_{i=1}^I [(x^l_{ij})^2 + (x^u_{ij})^2]}}$$

$$n^u_{ij} = \frac{x^u_{ij}}{\sqrt{\sum_{i=1}^I [(x^l_{ij})^2 + (x^u_{ij})^2]}}$$

Hence, the normalized criteria values of x'_{ij} ($i = 1, 2, \dots, I, j = 1, 2, \dots, J$) are calculated as follows:

$$\begin{aligned} n'_{ij} &= [n'^l_{ij}, n'^u_{ij}] \\ i &= 1, 2, \dots, I, \quad j = 1, 2, \dots, J, \end{aligned} \tag{23}$$

where:

$$n'^l_{ij} = \frac{x'^l_{ij}}{\sqrt{\sum_{i=1}^I [(x'^l_{ij})^2 + (x'^u_{ij})^2]}}$$

$$n'^u_{ij} = \frac{x'^u_{ij}}{\sqrt{\sum_{i=1}^I [(x'^l_{ij})^2 + (x'^u_{ij})^2]}}$$

Through Eqs. (17) and (23), we have:

$$\begin{aligned} n'^l_{ij} &= \frac{\alpha x'^l_{ij} + \beta}{\sqrt{\sum_{i=1}^I [(\alpha x'^l_{ij} + \beta)^2 + (\alpha x'^u_{ij} + \beta)^2]}} \\ i &= 1, 2, \dots, I; \quad j = 1, 2, \dots, J, \end{aligned} \tag{24}$$

$$n'^u_{ij} = \frac{\alpha x'^u_{ij} + \beta}{\sqrt{\sum_{i=1}^I [(\alpha x'^l_{ij} + \beta)^2 + (\alpha x'^u_{ij} + \beta)^2]}}$$

$$i = 1, 2, \dots, I; \quad j = 1, 2, \dots, J.$$

Thus, $n'^l_{ij} \neq n_{ij}$ if $\beta \neq [0, 0]$, i.e., the normalized criterion values in the interval TOPSIS method could depend on the evaluation units and dimensions of criterion values.

4.2. Aggregating function

The proposed method employs the aggregating function as Eq. (12), which demonstrates the extent that

each alternative is close to the PIS and remote from the NIS. In fact, the selected alternative has the maximum value of R_i ($i = 1, 2, \dots, I$). Evidently, this alternative would like to minimize its distance from the PIS ($d_i^+, i = 1, 2, \dots, I$) and maximize the distance from the NIS ($d_i^-, i = 1, 2, \dots, I$) at the same time. In other words, R_i ($i = 1, 2, \dots, I$) indicates the satisfactory degree that the DM considers a compromise extent between the shortest distance from the PIS and the farthest distance from the NIS.

Thus, in the proposed method, the distances of each alternative from the PIS ($d_i^+, i = 1, 2, \dots, I$) and NIS ($d_i^-, i = 1, 2, \dots, I$) have the same effect in the final ranking, except for the interval TOPSIS method. The interval TOPSIS method [15] is based on the following aggregating function:

$$R_k = [R_k^l, R_k^u] \quad k = 1, 2, \dots, I, \quad (25)$$

where:

$$R_k^l = \frac{d_k^{-l}}{d_k^{-u} + d_k^{+u}},$$

$$R_k^u = \frac{d_k^{-u}}{d_k^{-l} + d_k^{+l}}.$$

Now, consider the k th and j th alternatives. Suppose that R_j and R_k are obtained from Eq. (25) and besides, $R_j^l < R_k^l$ and $R_j^u < R_k^u$, meaning that:

$$\frac{d_j^{-l}}{d_j^{-u} + d_j^{+u}} < \frac{d_k^{-l}}{d_k^{-u} + d_k^{+u}},$$

$$\frac{d_j^{-u}}{d_j^{-l} + d_j^{+l}} < \frac{d_k^{-u}}{d_k^{-l} + d_k^{+l}}. \quad (26)$$

Thus, using the interval TOPSIS method, the k th alternative is ranked higher than the j th alternative. In a special case, Inequality (26) may be satisfied, while $d_j^{+l} < d_k^{+l}$ and $d_j^{+u} < d_k^{+u}$, i.e., the distance of the j th alternative to the PIS may be shorter than the distance of the k th alternative to the PIS, while the k th alternative is higher than the j th alternative in ranking (see, Example 1). This illogical ranking results from this fact that the interval TOPSIS method pays

greater attention to the distance of alternative to the NIS ($d_i^-, i = 1, 2, \dots, I$) than the distance of alternative to the PIS ($d_i^+, i = 1, 2, \dots, I$). Indeed, this is the disadvantage of the interval TOPSIS method.

5. Illustration

In this section, three examples are considered to illustrate the performance of the proposed method and compare the results with those published in the literature.

Example 1. The example adopted from Jahanshahloo et al. [15] is considered here. This example is related to the assessment of six cities in Iran to find the best place for creating a data factory. These cities must be evaluated by four criteria: Distance from border (C_1), cost of creating the factory (C_2), finance (C_3), and product in the region (C_4). C_1 and C_2 are cost criteria; C_3 and C_4 are benefit criteria. Table 1 represents the data. The first criterion is a real number and the others are in interval form and all the criteria have the same importance, i.e., $w_j = 0.25$ ($j = 1, 2, 3, 4$).

By using the proposed method in Step 1, the data are normalized by Eq. (4) (Table 2). In Steps 2 and 3, the PIS (g) is determined by Eq. (5) (Table 3). Then, d_i^+ ($i = 1, 2, \dots, 6$) is calculated for all alternatives using Eq. (6) (Table 4). In Step 4, through Eq. (7), we have $d^+ = 0.1273$. In Steps 5 and 6, the NIS (b) is determined using Eq. (8) (Table 3). Then, d_i^- ($i = 1, 2, \dots, 6$) is calculated for all alternatives using Eq. (9) (Table 4). In Step 7, by using Eq. (10), we have $d^- = 0.3984$. Through Eq. (12) in Step 8, R_i ($i = 1, 2, \dots, 6$) values are calculated (Table 5). In the next steps, to compare interval numbers R_i ($i = 1, 2, \dots, 6$) and rank the alternatives, complementary matrix P is constructed using Eq. (14). Then, p_i values ($i = 1, 2, \dots, 6$) are calculated using Eq. (15) (shown in Table 5) and all alternatives are ranked according to p_i ($i = 1, 2, \dots, 6$) in descending order.

Table 6 shows the ranking obtained from the proposed method and the method of Jahanshahloo et

Table 1. The interval decision matrix for Example 1.

Alternative	C_1	C_2	C_3	C_4
A_1	1451	[2551, 3118]	[40, 50]	[153, 187]
A_2	843	[3742, 4573]	[63, 77]	[459, 561]
A_3	1125	[3312, 4049]	[48, 58]	[153, 187]
A_4	55	[5309, 6488]	[72, 88]	[347, 426]
A_5	356	[3709, 4534]	[59, 71]	[151, 189]
A_6	391	[4884, 5969]	[72, 88]	[388, 474]

Table 2. The interval normalized decision matrix for Example 1.

Alternative	C_1	C_2	C_3	C_4
A_1	0.0000	[0.8560, 1.0000]	[0.0000, 0.2083]	[0.0049, 0.0878]
A_2	0.4355	[0.4864, 0.6975]	[0.4792, 0.7708]	[0.7512, 1.0000]
A_3	0.2335	[0.6195, 0.8067]	[0.1667, 0.3750]	[0.0049, 0.0878]
A_4	1.0000	[0.0000, 0.2995]	[0.6667, 1.0000]	[0.4780, 0.6707]
A_5	0.7844	[0.4963, 0.7059]	[0.3958, 0.6458]	[0.0000, 0.0927]
A_6	0.7593	[0.1318, 0.4074]	[0.6667, 1.0000]	[0.5780, 0.7878]

Table 3. The Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for Example 1.

Ideal	C_1	C_2	C_3	C_4
PIS (g)	1.0000	[0.8560, 1.0000]	[0.6667, 1.0000]	[0.7512, 1.0000]
NIS (b)	0.0000	[0.0000, 0.2995]	[0.0000, 0.2083]	[0.0000, 0.0878]

Table 4. Distance of each alternative from the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for Example 1.

Alternative	d_i^+	d_i^-
A_1	[0.3211, 0.4338]	[0.1391, 0.2563]
A_2	[0.1466, 0.2392]	[0.2148, 0.3767]
A_3	[0.2640, 0.3887]	[0.0990, 0.2310]
A_4	[0.1406, 0.2941]	[0.2918, 0.3984]
A_5	[0.1773, 0.3226]	[0.2075, 0.3102]
A_6	[0.1273, 0.2623]	[0.2533, 0.3843]

Table 5. The relative agreement index and P_i of each alternative for Example 1.

Alternative	R_i	P_i
A_1	[-3.0595, -1.8801]	0.9235
A_2	[-1.3405, -0.2062]	4.2226
A_3	[-2.8054, -1.4946]	1.3501
A_4	[-1.5782, -0.1046]	4.0697
A_5	[-2.0140, -0.6149]	3.1687
A_6	[-1.4251, -0.0354]	4.2654

Table 6. Ranking of alternatives in Example 1.

Alternative	Proposed method	Method of Jahanshahloo et al. [15]
A_1	6	6
A_2	2	4
A_3	5	5
A_4	3	1
A_5	4	3
A_6	1	2

al. [15]. As seen in Table 4, the distance between the second alternative and the PIS is shorter than that for the fifth alternative; however, the proposed method in the mentioned paper puts the fifth and second alternatives in the third and fourth ranks, respectively. Moreover, the distance of the sixth alternative to the PIS is shorter than that for the fourth alternative; however, the method proposed by Jahanshahloo et al. [15] puts the fourth alternative in the first rank and the sixth one in the second rank. The reason for this inappropriate ranking by Jahanshahloo et al. [15] is the same as mentioned before, that is, paying greater attention to distance to the NIS than to the PIS. This drawback causes the alternative with a shorter distance to the PIS to be ranked lower than other alternatives. As seen in Table 6, the proposed method ranks the second alternative as higher than the fifth one and the sixth alternative is ranked higher than the fourth one, which sounds more logical. Meanwhile, in the method proposed by Jahanshahloo et al. [15], the PIS and NIS undergo changes for each alternative such that the relationship between the alternatives is nullified and the final comparison is not relative.

Example 2. Sayadi et al. [16] provided an example which is considered here. In this example, there are three alternatives and two criteria. C_1 is a cost criterion and C_2 is a benefit criterion; both criteria are of similar relative importance, i.e., $w_j = 0.5 (j = 1, 2)$. The interval decision matrix is shown in Table 7.

To solve this problem using the proposed method, data are normalized through Eq. (4) in Step 1 (Table 8).

Table 7. The interval decision matrix for Example 2.

Alternative	C_1	C_2
A_1	[0.75, 1.24]	[2784, 3192]
A_2	[1.83, 2.11]	[3671, 3857]
A_3	[4.90, 5.73]	[4409, 4681]

Table 8. The interval normalized decision matrix for Example 2.

Alternative	C_1	C_2
A_1	[0.9016, 1.0000]	[0.0000, 0.2151]
A_2	[0.7269, 0.7831]	[0.4676, 0.5656]
A_3	[0.0000, 0.1667]	[0.8566, 1.0000]

Table 9. The Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for Example 2.

Ideal	C_1	C_2
PIS (g)	[0.9016, 1.0000]	[0.8566, 1.0000]
NIS (b)	[0.0000, 0.1667]	[0.0000, 0.2151]

Table 10. Distance of each alternative from the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for Example 2.

Alternative	d_i^+	d_i^-
A_1	[0.3208, 0.5024]	[0.3675, 0.5114]
A_2	[0.1571, 0.2992]	[0.3073, 0.4830]
A_3	[0.3675, 0.5051]	[0.3208, 0.5069]

Table 11. The relative agreement index and P_i of each alternative for Example 2.

Alternative	R_i	P_i
A_1	[-2.4798, -1.0420]	1.1749
A_2	[-1.3038, -0.0556]	2.4025
A_3	[-2.5882, -1.3481]	0.9226

Table 12. Ranking of alternatives in Example 2.

Alternative	Proposed method	Sayadi et al. [16] method
A_1	2	2
A_2	1	1
A_3	3	3

In Steps 2 and 3, the PIS (g) is determined using Eq. (5) (Table 9); then, d_i^+ ($i = 1, 2, 3$) is calculated for all alternatives using Eq. (6) (Table 10). In Step 4, we have $d^+ = 0.1571$ through Eq. (7). In Steps 5 and 6, the NIS (b) is determined using Eq. (8) (Table 9); then, d_i^- ($i = 1, 2, 3$) is calculated for all the alternatives via Eq. (9) (Table 10). In Step 7, through Eq. (10), we have $d^- = 0.5114$. In Step 8, R_i values ($i = 1, 2, 3$) are calculated through Eq. (12) (Table 11).

In the next steps, to compare interval numbers R_i ($i = 1, 2, 3$) and rank the alternatives, complementary matrix P is constructed via Eq. (14). Then, p_i ($i = 1, 2, 3$) are calculated using Eq. (15) (shown in Table 11) and all alternatives are ranked according to p_i ($i = 1, 2, 3$) in descending order. Table 12 shows the ranking obtained from the proposed method and the

Table 13. The interval decision matrix for Example 3.

Alternative	C_1	C_2	C_3	C_4
A_1	[6, 22]	[10, 15]	[16, 21]	[18, 20]
A_2	[15, 18]	[8, 11]	[20, 30]	[19, 28]
A_3	[9, 13]	[12, 17]	[42, 48]	[40, 49]

proposed method by Sayadi et al. [16]. It is observed that the ranking obtained from both methods is the same. It should be noted that in the method proposed by Sayadi et al., parameter v is very influential in the final ranking and its optimal value is not known. Moreover, v is considered 0.5 without any justification. Moreover, Indicator α , to compare interval numbers, could significantly affect the final ranking and DM might not be able to determine this value. In fact, it is very difficult to determine the values of both parameters α and v in real-life decision-making situations.

Example 3. The example adopted from Dymova et al. [17] is considered. This example deals with three alternatives and four criteria presented by interval numbers in Table 13, where C_1 and C_2 are benefit criteria; C_3 and C_4 are cost criteria. All the criteria have the same importance, i.e., $w_j = 0.25$ ($j = 1, 2, 3, 4$).

Using the proposed method, Steps 1 to 3 are executed like previous examples. The obtained values are listed in Tables 14 to 16, respectively. In Step 4, we have $d^+ = 0$. Moreover, the obtained values in Steps 5 and 6 are listed in Tables 15 and 16. In Step 7, we have $d^- = 0.4747$. This example is different from the previous ones in Step 8, because $d^+ = 0$ in Step 4. Thus, according to Remark 1, Eq. (12) should be employed instead of Eq. (13) to calculate R_i ($i = 1, 2, 3$) and the obtained values are shown in Table 17. In the next steps, p_i ($i = 1, 2, 3$) are calculated, as shown in Table 17.

Then, all alternatives are ranked in descending order p_i ($i = 1, 2, 3$). The obtained ranking from both of the methods is shown in Table 18. As shown earlier, the two methods are the same in terms of ranking. However, in the method proposed by Dymova et al. [17], the normalized criterion values depend on the evaluation units and dimensions of criterion values and to determine the PIS and NIS and calculate the intervals distance, only the centers of the intervals are taken into account, which may result in partial data loss.

6. Conclusion

Given that determining the exact values of the criteria is difficult in some cases, it is more appropriate to consider them as interval numbers in MCDM problems. In this paper, a method is presented to solve Multiple Criteria Decision Making (MCDM) problems with interval numbers. The proposed method provides a

Table 14. The interval normalized decision matrix for Example 3.

Alternative	C_1	C_2	C_3	C_4
A_1	[0.0000, 1.0000]	[0.2222, 0.7778]	[0.8438, 1.0000]	[0.9355, 1.0000]
A_2	[0.5625, 0.7500]	[0.0000, 0.3333]	[0.5625, 0.8750]	[0.6774, 0.9677]
A_3	[0.1875, 0.4375]	[0.4444, 1.0000]	[0.0000, 0.1875]	[0.0000, 0.2903]

Table 15. The Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for Example 3.

PIS (g)	[0.5625, 1.0000]	[0.4444, 1.0000]	[0.8438, 1.0000]	[0.9355, 1.0000]
NIS (b)	[0.0000, 0.4375]	[0.0000, 0.3333]	[0.0000, 0.1875]	[0.0000, 0.2903]

Table 16. Distance of each alternative from the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for Example 3.

Alternative	d_i^+	d_i^-
A_1	[0.0000, 0.3195]	[0.2301, 0.4747]
A_2	[0.0278, 0.3048]	[0.1383, 0.3853]
A_3	[0.2322, 0.4308]	[0.0278, 0.2862]

Table 17. The relative agreement index and P_i of each alternative for Example 3.

Alternative	R_i	P_i
A_1	[-0.4854, 0.0000]	1.9271
A_2	[-0.5329, -0.0884]	1.7156
A_3	[-0.7338, -0.3600]	0.8573

Table 18. Ranking of alternatives in Example 3.

Alternative	Proposed method	Method of Dymova et al. [17]
A_1	1	1
A_2	2	2
A_3	3	3

balance between closeness to the Positive Ideal Solution (PIS) and being far away from the Negative Ideal Solution (NIS). In other words, these two attributes had the same effect in alternative ranking. As proved in Subsection 4.1, the proposed method was translation invariance. Moreover, in this method, all the calculations were conducted in the interval form unlike the papers in which the computations were not completely based on interval arithmetic.

In the proposed method, the PIS and the NIS were determined as interval numbers and the distance of each alternative to the PIS and NIS by interval extension of the Euclidean norm was calculated. Then, the relative agreement of each alternative was obtained. These cases of relative agreement were compared to-

gether to rank the alternatives. Numerical examples showed the advantages of the proposed method over those in other published studies. For future research, it is suggested that the interval extension of p -norm be employed instead of interval extension of Euclidean norm to obtain the distances of alternatives to the PIS and NIS. Moreover, one can use other methods instead of constructing the complimentary matrix P to compare and sort the interval numbers.

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