Hydromagnetic Blasius-Sakiadis flows with variable features and nonlinear chemical reaction

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Abstract

Time-dependent, two-dimensional Sakiadis flow of quiescent fluid is considered. The flow is induced by stationary flat plate via uniform free-stream (Blasius flow). The variable conductivity and viscosity ratio parameters and non-linear chemical reaction are employed in the mathematical equations. Similarity variables are employed in the governing transport expressions to convert into the ordinary differential system. The transformed system is computed numerically by employing Runge-Kutta scheme via shooting criteria. Results of concentration, velocity and temperature distributions are studied through plots. Moreover, mass and heat transfer rates and friction factor have been discussed in detail. The constraint of chemical reaction slow down the friction-factors and heat transportation rates for the Sakiadis-Blasius flow situations and enhances the mass transportation rate in both cases. Rate of mass transportation is smaller in Sakiadis flow as comparative to Blasius flow. The present results of the heat transfer rate are matched with the literature and excellent agreement is noticed.

Keywords: Viscous ratio parameter; non-uniform heat source-sink; variable conductivity; Blasius-Sakiadis flows; chemical reaction

1. Introduction

The fluid flows across a surface or sheet has numerous applications and usages in various branches of science, technology and industry. Therefore, there is a significance to enhance the investigation of this kind of fluid flows with different conditions. The concept of fluid flow over a horizontal
stationary plate under constant velocity is firstly presented by Blasious [1]. Sakiadis [2] proposed another problem in which the plate is moved with some constant velocity. In Blasius-Sakiadis flows, the heat transportation analysis occupies the major part. Hence, the following investigations are focused to different types of heat transfer conditions. The injection or suction on laminar boundary-layer flow induced by continuous movement of sheet is investigated by Pop and Watanabe [3]. Thermal radiation on laminar flow across a moveable plate is executed by Ishak et al. [4]. Yao et al. [5] executed the transportation of heat in Newtonian fluid through a convectively bounded shrinking surface. They noted the different behavior of fluid temperature from prescribed wall temperature cases. Cortell [6] performed a work on boundary-driven quiescent fluid flow under nonlinear radiation. He observed that the temperature ratio and radiation constraints have opposite behavior on temperature distribution. Khan et al. [7] evaluated the Blasius-Sakiadis flow problems of Casson fluid with viscous dissipation. They concluded that the behavior of velocity distribution is reverse for both problems. Olanrewaju [8] analyzed a convective boundary condition on Sakiadis and Blasius flows. They noticed an increment in temperature distribution with Eckert number in Blasius- Sakiadis flows. Sheikholeslami [9] discussed the numerical technique to examine the behavior of entropy in nanofluid flow through the impact of Lorentz force. He discussed the nature of distinct involved parameters on fluid flow quantities by plotting streamlines and contours. In another analysis, Sheikholeslami [10] considered the hydromagnetic water-\text{Al}_2\text{O}_3 nanofluid flow induced by permeable medium. Here, he made a comparative analysis to justify the validity of present numerical technique. Few more novel literature of heat transfer analysis on fluid flow is founded in [11-17].

The investigation of electrically conducting fluids over the Sakiadis and Blasius flow cases plays a crucial role on various sectors like glass painting, food processing and automobiles. The heat transportation in hydromagnetic power-law fluid is reported by Kumari and Nath [18]. They noted that the friction factors and rate of heat transport increased for stronger magnetic field. The MHD nanofluids flow under thermally radiative convective conditions is addressed by Akbar et al. [19]. Devi and Suriyakumar [20] discussed a magnetic field effects on Sakiadis-Blasius flow of nanofluids. They executed that the opposite attitude of velocity in Blasius-Sakiadis flow cases with enhancing values of Hartman parameter. Isa et al. [21] carried the hydro magnetic mixed conduction boundary-driven flow in presence of an exponential temperature distribution. Magneto hydrodynamic stagnation point flow on preamble flat plate is considered and discussed by Hamad
et al. [22]. Ferdows et al. [23] performed the computations to examine the MHD free-convected mass transport in nonlinear fluid flow induced by moving sheet. Ullah et al. [24] described the homotopic asymptotic approach to discuss the Maxwell fluid flow under Lorentz field. Khan et al. [25] disclosed the heterogeneous and homogeneous reactions behavior in magnetized Casson material flow subject to electromagnetic force. Ramli et al. [26] reported the nature of MHD ferro-liquid flow under uniform heat diffusion and second-order slip phenomenon. Kumar et al. [27] addressed the behavior of chemical reactive Williamson fluid flow with magnetic field aspects. They opted the numerical technique to elucidate the solutions of governing problems. Thermal radiative nanofluids flows with magneto-hydrodynamic and accelerated ramped temperature effects are discussed by Hussain et al. [28]. Abbasi et al. [29] examined the second law phenomenon in hydromagnetic peristaltic nanofluid flow under ohmic dissipation and Hall current phenomenon. The solutions are illustrated through numerical procedure. Entropy optimization of electrically conducting nano-material flow induced by the variable thinned sheet is illustrated by Wang et al. [30]. Heat transmission analysis of nonlinear convective flow of Walter’s B material flow under gyrotactic micro-organisms phenomenon is reported by Khan et al. [31]. Abbas et al. [32] demonstrated the second order slip influence in magnetized nanomaterial flow under energy activation processes.

By the inspiration of the above literature, the theme of current study is to find the mixed convective conditions on hydromagnetic Blasius-Sakiadis flows with variable properties and nonlinear chemical reaction. The flow is generated by the time-dependent movement of the surface. Numerical illustrations of the results are reported for the quantities of practical interests. The set of governing equations are evaluated and presented with graphs and tables.

2. Mathematical Formulations

Time-dependent naturally convected Blasius-Sakiadis flow under variable properties and nonlinear chemical reaction is assumed. Brownian movement and thermophoresis force aspects are considered due to nanofluid. The surface is stretched with velocity $U_s(x,t) = \frac{ax}{1-ct}$. The governing expressions for present case are [15-19]:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0, \quad (1)$$

$$\frac{\partial u}{\partial y} + u \frac{\partial u}{\partial x} + \frac{\partial v}{\partial t} = g \left( \beta_1 (C - C_w) + \beta_2 (T - T_w) \right) + \frac{1}{\rho} \left( \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) - \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial y} \right) \right) - \frac{\sigma B^2}{\rho} \mu \quad (2)$$
\[ \rho C_p \left( v \frac{\partial T}{\partial y} + u \frac{\partial T}{\partial x} + \frac{\partial T}{\partial t} \right) = \frac{\partial}{\partial y} \left( K(T) \frac{\partial T}{\partial y} \right) + \frac{16 \sigma^2 T^3}{3k} \frac{\partial^2 T}{\partial y^2} + q^m + \mu(T) \left( \frac{\partial u}{\partial y} \right)^2 + \tau \left( \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right) \] (3)

\[ \frac{\partial C}{\partial y} + u \frac{\partial C}{\partial x} + \frac{\partial C}{\partial t} = D_p \frac{\partial^2 C}{\partial y^2} + D_T \frac{\partial T}{\partial y^2} - k_0 \left( \frac{T}{T_0} \right)^n \left( 1 - \frac{T}{T_0} \right) (C - C_e) \] (4)

with the boundary conditions

i) Blasius problem

\[ \begin{align*}
  v &= 0, u = 0, K, \frac{\partial C}{\partial y} = -h_f(C - C_e), K, \frac{\partial T}{\partial y} = -h_f(T - T_0), \text{at } y = 0, \\
  u &= U_w, C = C_e, T = T_0, \text{as } y \to \infty
\end{align*} \] (5)

ii) Sakiadis problem

\[ \begin{align*}
  v &= 0, u = U_w, K, \frac{\partial C}{\partial y} = -h_f(C - C_e), K, \frac{\partial T}{\partial y} = -h_f(T - T_0), \text{at } y = 0, \\
  u &= 0, T = T_0, C = C_e \text{ as } y \to \infty.
\end{align*} \] (6)

The velocity components are \( u \) and \( v \) in the \( x \) and \( y \)-directions, respectively, \( g \) the acceleration of gravity, \( v \) the kinematic viscosity, \( \beta_c, \beta_T \) the coefficient of concentration and thermal expansions, \( C_e, T_0 \) the ambient concentration and temperature, \( c_p \) the specific heat, \( K(T) \) the variable conductivity, \( \rho \) the density, \( k_0 \) the chemical reaction rate, \( e^{-h_f(\alpha_i\epsilon)}(C - C_e) \left( \frac{T}{T_0} \right)^n \) is Arrhenius function, \( \alpha_i \) is exponent, \( \epsilon \) is activation energy, \( \sigma^* \) the Boltzmann constant, \( k^* \) the Stefan constant, \( h_f, h_i \) the heat and mass transfer coefficients. The variable formula of thermal conductivity and viscosity can be expressed as:

\[ K(T) = K_e \left( 1 + \frac{\epsilon}{\Delta T(T - T_0)} \right), \] (7)

\[ \mu(T) = \frac{\mu_e}{\left( 1 + \omega(T - T_0) \right)^\epsilon}, \] (8)

\[ q^m = \frac{K_e U_w}{\lambda U} \left( A^*(T_w - T_0) + B^*(T - T_w) \right). \]

Here \( \Delta T = (T_w - T_0) \), \( T_w \) the surface temperature, \( K_e, \mu_e \) the thermal conductivity and viscosity of the fluid far away from the surface \( \epsilon \) and \( \omega \) are the smaller parameters namely the conductivity and viscosity variations constraint. \( A^* \) and \( B^* \) are the time and space dependent heat
source or sink. For $A' > 0$ is for heat source and $A' < 0$ for the heat sink. The similarity variables are expressed as

$$
\zeta = \left( \frac{a}{\sqrt{1-c_t}} \right)^{\frac{1}{2}} y, \quad \psi = \left( \frac{va}{(1-c_t)} \right)^{\frac{1}{2}} x f(\zeta), \quad \theta(\zeta) = \frac{(T-T_o)}{(T_u-T_o)} \phi(\zeta) = \frac{(C-C_o)}{(C_v-C_o)}
$$

(9)

In which $\psi(x,y,t)$ the stream function expressed by the relation $u,v = \left( \partial \psi / \partial x, - \partial \psi / \partial y \right)$. The substitution of Eqs. (2)-(4) and making use of Eqs. (7), (8) and (9) we obtain

$$
\frac{1}{(1+E\theta)} f'' - \frac{E}{(1+E\theta)^2} \theta' f'' + f f'' - f'^2 - A \left( f' + \frac{1}{2} \zeta f'^2 \right) - (M + K) f' + \lambda_\theta \theta + \lambda_\phi = 0,
$$

(10)

$$
\frac{A}{2} \zeta \theta' - f \theta' - \frac{1}{Pr} \left( \theta' + \frac{E}{1+E\theta} \theta'' \right) - Nb \theta' \phi - Nt \theta'^2 - Ec f'^2 - \frac{1}{Pr} Nt \theta' - A f' - B \theta = 0,
$$

(11)

$$
\frac{A}{2} \phi' - \phi' - \frac{1}{Le} \left( \phi' + \frac{Nt}{Nb} \theta' \right) - Kr^2 (1+n\gamma\theta) \exp \left( \frac{E_1}{1+\gamma\theta} \right) \phi.
$$

(12)

Boundary conditions can be described as

i) Blasius flow

$$
f(0) = 0, f'(0) = 0, \phi'(0) = -(1 - Bi_1 \phi(0)), \theta'(0) = -(1 - Bi_2 \theta(0)),
$$

$$
f'(\infty) = 1, \theta(\infty) = 0, \phi(\infty) = 0.
$$

(13)

ii) Sakiadis flow

$$
f(0) = 0, f'(0) = 1, \phi'(0) = -(1 - Bi_1 \phi(0)), \theta'(0) = -(1 - Bi_2 \theta(0)),
$$

$$
f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0.
$$

(14)

where $A = c/a$ the unsteady constraint, $M = \frac{\sigma B_o^2 (1-c_t)}{\rho a}$ the magnetic field constraint,

$$
E = \frac{\omega}{(T_u-T_o)}
$$

the viscosity variation constraint, $K = \frac{\nu(1-c_t)}{k_o a}$ the porosity constraint,

$$
Ec = \frac{U_w^2}{C_v(T_u-T_o)}
$$

the Eckert number, $\gamma = \frac{T_u-T_o}{T_v}$ the temperature ratio, $Kr^2 = \frac{k_o^2 \nu}{v}$ the chemical reaction parameter, $E_1 = \frac{E_v}{kT_w}$ the activation energy, $Nr = \frac{16\sigma^2 T_0^3}{3k K}$ the radiation constraint,

$$
Pr = \nu/\alpha_v
$$

the Prandtl number, $Bi_1 = \frac{h_o}{k_o} \sqrt{\frac{\nu(1-c_t)}{ac}}$ the thermal Biot number,

$$
Bi_2 = \frac{h_o}{k_o} \sqrt{\frac{\nu(1-c_t)}{ac}}
$$

the concentration Biot number, $Ni = \frac{\tau D_o AT}{vT_w}$ the thermophoresis,
\[ Nb = \frac{\tau D_{\mu} \Delta C}{\nu} \quad \text{the Brownian motion,} \quad \lambda_v = \frac{g \beta \tau (T_u - T_w)}{aU_w} \quad \text{the thermal buoyancy and} \]

\[ \lambda_c = \frac{g \beta \tau (C_u - C_w)}{aU_w} \quad \text{the concentration buoyancy.} \]

The expressions of friction-factors, local Sherwood and Nusselt numbers can be designed as

\[ C_f = \frac{\tau_w}{\rho U_w^2 / 2}, \quad Sh_i = \frac{x_j u_w}{D_b (C_u - C_w)}, \quad Nu_i = \frac{x_q u_w}{k_u (T_u - T_w)}, \quad (15) \]

where the skin friction \( \tau_w \) and mass and heat transport rates are \( j_u, q_u \) which can be expressed by the relations

\[ \tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad j_u = -D_b \left( \frac{\partial C}{\partial y} \right)_{y=0}, \quad q_u = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}. \quad (16) \]

Substituting Eq. (9) into (15) and (16), we obtained

\[ \text{Re}^{1/2} Cf = \left( 1 + \frac{1}{(1 + E \theta)} \right) f'(0), \quad \text{Re}^{1/2} Sh = -\phi'(0), \quad \text{Re}^{1/2} Nu = -\left( \frac{4}{3} Nr + 1 + \varepsilon + c \theta'(0)^2 \right) \theta'(0), \quad (17) \]

in which \( \text{Re}_s = \frac{U_s x}{\nu} \) is the local Reynolds number.

3. Results and Discussion

Numerical solutions are constructed to evaluate the nature of Eckert number, Biot numbers, chemical reaction parameter, Brownian movement, thermal radiation, magnetic field, thermophoresis and porosity constraints on non-dimensional temperature, velocity and concentration by considering \( (E = 0.2, \varepsilon = 0.2) \) and \( (E = 0, \varepsilon = 0) \). Moreover, the results of local friction-factors coefficient, rate of heat and mass transports are communicated through numerical data for Sakiadis-Blasius flows. The values of non-dimensional constraints are \( K = 0.5, n = 2; Kr = 0.2, \Pr = 0.2, \lambda_v = 0.3, \lambda_c = 0.2, Ec = 0.2, Nb = 0.3, Nr = 0.2, M = 0.5, Bi_i = Bi_f = 0.2, E = 0.2, \varepsilon = 0.2, Nr = 0.5, A' = 0.2, B' = 0.2 \). In this work, the graphs in dashed and solid lines represent the Blasius and Sakiadis flow problems.

Figs. 1-3 exemplify the deviations of porosity constraint on \( f'(\zeta), \theta(\zeta) \) and \( \phi(\zeta) \) fields. From Fig. 1, it is obviously apparent that the participation of porous medium corresponds to high control in flow which slow-down its motion. Therefore, with improve in porosity; the resistance to liquid flow rises hence suppresses the velocity in Blasius-Sakiadis flow cases. The contradictory movement can be perceived in temperature and concentration fields (see Figs. 2 and 3). Fig. 4
demonstrated the nature of Biot number $Bi_1$ on temperature $\theta(\zeta)$ field. Biot number is the ratio of convection and conduction heat transport. Biot number leads to larger temperature at surface which resulted weaker thermal layer. Figs. 5 and 6 describe the part of thermophoretic $Nt$ on $\theta(\zeta)$ and $\phi(\zeta)$ field. The profiles of $\theta(\zeta)$ and $\phi(\zeta)$ are advanced when we offer the augmentation in the values of $Nt$ parameter. We found that the positive thermophoresis represents the cold surface while negative values illustrate the hot surface. Remarkably, the temperature and concentration fields are shown near the surface enhancement far away from the surface it is reduced due to dominance of nonlinear buoyancy forces in the flow.

The characteristics of $Ec$ on $\theta(\zeta)$ in presence and absence of viscosity and conductivity variations constraints are presented in the Fig. 7 for Blasius-Sakiadis flow cases. Here, the thickness of thermal layer grows with an increase in $Ec$. The investigation of Brownian movement $Nb$ on $\theta(\zeta)$ and $\phi(\zeta)$ is made through the Figs. 8 and 9. Fig. 8 describes the nature of $\theta(\zeta)$ for dissimilar $Nb$. The temperature is reduced although the concentration is encouraged with an increment in $Nb$ (see Fig. 9). It is due to fact that in nano-scale systems, the Brownian movement $Nb$ takes place. The significance of magnetic parameter on $f'(\zeta)$, $\theta(\zeta)$ and $\phi(\zeta)$ is discussed in the Figs. 10-12. We saw that the augmentation in $M$ decelerates $f'(\zeta)$ and enriches $\theta(\zeta)$ and $\phi(\zeta)$ fields. Physically, this outcome quantitatively decides with expectations. The executions of magnetic field on electrically-conducting liquid give rise to resistive forces (retardation force) which is called as Lorentz force. This Lorentz force has capability to diminish the motion of boundary layer and to progress the concentration and temperature distributions in Blasius-Sakiadis flows.

The role of different values of $Nr$ on $\theta(\zeta)$ and $\phi(\zeta)$ is classified in the Figs. 13 and 14. Fig. 13 reports that the thicknesses of thermal layer is developed as $Nr$ enhanced. The contradictory phenomena can be pictured in $\phi(\zeta)$ profiles (see Fig. 14). As we expected, the thermal radiation generates heat molecules, it will help to improve the thermal boundary layer.

The participation of chemical reaction constraint is visualized in Fig. 15. This Fig. reports that the larger chemically reactive constraint decelerates the concentration profiles for Blasius-Sakiadis flows. The chemical reaction here resulted in consumption of chemical that leads to decrement in concentration distributions. The most key feature is that chemical reaction has capability to reduce the solutal layer of thickness. The immersion of Biot number $Bi_2$ thrust up the concentration $\phi(\zeta)$
in Blasius-Sakiadis cases for without and with variable properties (see Fig. 16). It is noted that the Blasius flow has more significant concentration profile as compared to Sakiadis flow case. The role of dissimilar values of $A^*$ on $\theta(\zeta)$ and $\phi(\zeta)$ is shown in the Figs. 17 and 18. It is scrutinized that the thicknesses of diffusion layer improved as $A^*$ enhanced. The mixed performance perceived in $\theta(\zeta)$ profiles (see Fig. 17). As we expected, $A^*$ acts as a heat generation or absorption, it will help to improve the thermal boundary layer near the boundary, after sometime the domination of nonlinear chemical reaction (see Fig. 18).

Table 1 illustrates that the current calculated results have worthy match with the results of with Grubka and Bobba [33], Ali [34] and Ishak et al. [35] in limiting circumstances for both Blasius-Sakiadis flow cases. Tables 2 and 3 depict the nature of variation of $Re^{1/2}Cf$, and $Re^{1/2}Nu$ for distinct values of Biot numbers due to thermal and diffusion, chemical reaction, Eckert number, thermophoresis, Brownian movement, non-uniform heat source or sink, thermal radiation, magnetic field and porosity parameters are studied for Blasius-Sakiadis situations in presence $(E = 0.2, \varepsilon = 0.2)$ and absence $(E = 0, \varepsilon = 0)$ of viscosity variation and conductivity variation parameters. The effect of thermal radiation parameter increases the friction factor coefficient, heat transfer in both Blasius and Sakiadis fluid flow cases. While, reverse phenomena can be observed for magnetic field and porosity parameters. Further, it is interesting to note that the Brownian motion parameter diminishes the skin friction coefficient and heat transfer rate for both Blasius and Sakiadis flows, the heat transfer rate enhance in Sakiadis flow and decelerates in Blasius flow. While, opposite trend can be found in thermophoresis parameter. The friction factor coefficient and heat transfer rates increases with increase in Biot numbers due to thermal and diffusion. Finally, it is obvious note that the heat transfer rate is enhanced with the increasing values of porosity, Biot numbers and thermal radiation in both the presence $(E = 0.2, \varepsilon = 0.2)$ and absence $(E = 0, \varepsilon = 0)$ of viscosity variation and conductivity variation parameters for Sakiadis and Blasius flow cases. Similarly, the friction factor is improved with raising values of Biot numbers, Thermophoresis, Eckert number, thermal radiation and non-uniform heat source or sink. Interestingly, we found here that in the presence of variable thermal conductivity and viscosity has more friction values compared to without viscosity and variation parameters in both the Blasius and Sakiadis flow cases. As, we know when we include variable properties conducting nature of the flow is more, due to this we saw aforementioned results.
Table 4 displays the effect of various parameters on Sherwood number for both the Blasius and Sakiadis flows with \((E = 0.2, \varepsilon = 0.2)\) and \((E = 0, \varepsilon = 0)\) cases. The mass transfer rate is boosted with boosting values of porosity, Biot numbers, Brownian motion, magnetic field, thermal radiation and chemical reaction in both the Blasius and Sakiadis flow cases. Non-uniform heat source or sink act as a heat absorption parameter. It is exciting to mention that the mass transfer rate is higher in with inclusion of variable properties compared to without variable properties. This result helps us to conclude that variable properties can be included whenever we need more mass transfer rate in the manufacturing and industrial process.

4. Conclusions

In most of the cases, the authors are neglecting the variable properties due to nonlinear nature of the flow. But, we know all the industrial and manufacturing process has buoyancy force with variable properties. In this investigation we considered free convection with variable properties of the flow to get the exact flow characteristics. We also examine the fluid velocity, temperature and concentration distributions nature corresponding to distinct values of emerging constraints. The solutions are computed through numerical criteria. The following key points are extracted from this research:

1. The rate of mass transportation is weaker in Sakiadis flow situations as comparative to Blasius flow.
2. The Eckert number increases the friction-factors values for Blasius-Sakiadis flow problems.
3. The chemical reaction has ability to diminish the solutal layer of thickness.
4. The porosity parameter decelerates the friction factor coefficient.
5. The thermophoresis constraint enhances the heat transport rates in Blasius-Sakiadis fluid flow cases.

Nomenclature

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>space co-ordinates</th>
<th>((u, v))</th>
<th>velocity components</th>
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<td>(\beta_c, \beta_T)</td>
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<td>$Nu$</td>
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References


Figure captions

Fig. 1: Velocity \( f' (\zeta) \) for distinct values of \( K \).

Fig. 2: Temperature \( \theta (\zeta) \) for different values of \( K \).

Fig. 3: Concentration for distinct values of \( K \).

Fig. 4: Temperature \( \theta (\zeta) \) for distinct values of \( Bi_1 \).

Fig. 5: Temperature \( \theta (\zeta) \) for diverse values of \( Nt \).

Fig. 6: Concentration \( \phi (\zeta) \) for distinct values of \( Nt \).

Fig. 7: Temperature \( \theta (\zeta) \) for distinct values of \( Ec \).

Fig. 8: Temperature \( \theta (\zeta) \) for distinct values of \( Nb \).

Fig. 9: Concentration \( \phi (\zeta) \) for dissimilar values of \( Nb \).

Fig. 10: Velocity \( f' (\zeta) \) for distinct values of \( M \).

Fig. 11: Temperature \( \theta (\zeta) \) for different values of \( M \).

Fig. 12: Concentration \( \phi (\zeta) \) for distinct values of \( M \).

Fig. 13: Temperature \( \theta (\zeta) \) for distinct values of \( Nr \).

Fig. 14: Concentration \( \phi (\zeta) \) for various values of \( Nr \).

Fig. 15: Concentration \( \phi (\zeta) \) for different values of \( Kr \).

Fig. 16: Concentration \( \phi (\zeta) \) for different values of \( Bi_2 \).

Fig. 17: Temperature for different values of \( A^* \).

Fig. 18: Concentration \( \phi (\zeta) \) for dissimilar values of \( A^* \).

Table Captions

| Table 1: Comparative data of \(-\theta(0)\) with existing results in absence of thermal radiation by setting \( Ec = K = Bi_1 = Bi_2 = Kr = 0 \). |
Table 2: The values of $Re_x^{1/2} Cf$, and $Re_x^{1/2} Nu$ for distinct values of involved parameters in Sakiadis and Blasius flow case in $E = 0$, and $\varepsilon = 0$ case.

Table 3: The values of $Re_x^{1/2} Cf$, and $Re_x^{1/2} Nu$ for distinct values of involved parameters in Sakiadis and Blasius flow case in $E = 0.2$, and $\varepsilon = 0.2$ case.

Table 4: The values of $Re_x^{1/2} Sh$ for distinct values of involved parameters in Sakiadis and Blasius flow case in both $E = 0$, $\varepsilon = 0$ and $E = 0.2$, $\varepsilon = 0.2$ case.

Fig. 1: Velocity $f'(\zeta)$ for distinct values of $K$.

Fig. 2: Temperature $\theta(\zeta)$ for different values of $K$. 
Fig. 3: Concentration for distinct values of $K$.

Fig. 4: Temperature $\theta(\zeta)$ for distinct values of $Bi_i$. 
Fig. 5: Temperature $\theta(\zeta)$ for diverse values of $Nt$.

Fig. 6: Concentration $\phi(\zeta)$ for distinct values of $Nt$. 
Fig. 7: Temperature $\theta(\zeta)$ for distinct values of $Ec$.

Fig. 8: Temperature $\theta(\zeta)$ for distinct values of $Nb$. 

Green & Magenta: $E=0; \epsilon=0$
Blue & Red: $E=0.2; \epsilon=0.2$

Solid lines: Sakiadis flow
Dashed lines: Blasius flow
Fig. 9: Concentration $\phi(\zeta)$ for dissimilar values of $Nb$.

Fig. 10: Velocity $f'(\zeta)$ for distinct values of $M$.
Fig. 11: Temperature $\theta(\zeta)$ for different values of $M$.

Fig. 12: Concentration $\phi(\zeta)$ for distinct values of $M$. 

Green & Magenta: $E=0$; $\epsilon=0$
Blue & Red: $E=0.2$; $\epsilon=0.2$

Solid lines: Sakiadis flow
Dashed lines: Blasius flow

$M=0.5,1,1.5$
Fig. 13: Temperature $\theta(\zeta)$ for distinct values of $Nr$.

Fig. 14: Concentration $\phi(\zeta)$ for various values of $Nr$. 
Fig. 15: Concentration $\phi(\zeta)$ for different values of $Kr$.

Fig. 16: Concentration $\phi(\zeta)$ for different values of $Bi_2$. 

Fig. 17: Temperature for different values of $A^*$.

Fig. 18: Concentration $\phi(\zeta)$ for dissimilar values of $A^*$.

**Table 1:** Comparative data of $-\theta'(0)$ with existing results in absence of thermal radiation by setting $Ec = K = Bi_1 = Bi_2 = Kr = 0$.

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Table 2: The values of $\text{Re}^{1/2} \text{Cf}$, and $\text{Re}^{1/2} \text{Nu}$ for distinct values of involved parameters in Sakiadis and Blasius flow case in $E = 0$, and $\epsilon = 0$ case.

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Table 3: The values of $Re^{1/2} Cf$, and $Re^{1/2} Nu$ for distinct values of involved parameters in Sakiadis and Blasius flow case in $E = 0.2$, and $\varepsilon = 0.2$ case.

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Table 4: The values of $Re_x^{1/2} Sh$ for distinct values of involved parameters in Sakiadis and Blasius flow case in both $E = 0$, $\varepsilon = 0$ and $E = 0.2$, $\varepsilon = 0.2$ case.
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Dr. R. L. V. Renuka Devi

Dr. Devi has completed her M. Phil and Ph.D. (Fluid Dynamics) from Sri Venkateswara University, Tirupati, Andhra Pradesh India. She has published 17 research articles in peer reviewed ISI indexed international journals. Her research experience is in Fluid Dynamics, Magnetohydrodynamics, heat mass transfer and non-Newtonian fluids.

Dr. S. V. Siva Rama Raju

Dr. Raju has done his Ph.D. in Mathematics from Andhra University, India. His areas of expertise include graph theory, Lattice theory and fluid dynamics. He has published several papers in international journals of good repute. He received the BEST PAPER PRESENTED award for paper "Semi Complete Graphs" at the XIX Congress of Andhra Pradesh Society for Mathematical Sciences. At present, he is working as Assistant Professor at Abu Dhabi Polytechnic, Abu Dhabi, UAE.

Dr. Chakravarthula SK Raju

Dr. Raju has completed his Doctoral degree from VIT University, Vellore. He has published 3 books (one main author and 2 co-author) and 100 research articles in peer reviewed ISI indexed international journals. His research fields include theoretical simulation of nanofluid, ferrofluid, non-Newtonian fluids with numerical modelling with C, Mathematica, and MATLAB. At present, he is working as an Assistant Professor in GITAM School of Science. Recently, his name was included in top 2% scientists’ of the world data given by the Stanford University, USA.

Dr. Sabir Ali Shehzad

Dr. Sabir Ali Shehzad is working as an Assistant Professor at Department of Mathematics, COMSATS University Islamabad, Sahiwal Pakistan. He has completed his Ph.D in Fluid Mechanics from Quaid-I-Azam University, Islamabad, Pakistan in 2014. His main research interests are Newtonian and non-Newtonian fluids, nanofluids and heat and mass transfer analysis. He has published >300 research articles in various high reputed international journals.

Dr. Fahad Munir Abbasi

Dr. Abbasi is working as an Assistant Professor at Department of Mathematics, COMSATS University Islamabad, Islamabad Campus, Pakistan. He has completed his Ph.D in Fluid Mechanics from Quaid-I-Azam University, Islamabad, Pakistan. His main research interests are peristaltic flow, numerical modeling, Newtonian and non-Newtonian fluids. He has published
>100 research articles in various high reputed international journals. He is also reviewer of more than 50 International journals.