STUDY OF THERMALLY DEVELOPED FLOW OF VISCOUS FLUID OVER A POROUS STRETCHING SURFACE CONTACTING GYROTACTIC MICROORGANISMS USING BUONGIORNO MODEL

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\textbf{Abstract:} Recent trend in advanced nanotechnology has developed the thermal consequences of nanoparticles due to increasing significance in various engineering and thermal extrusion systems. In this continuation, two-dimensional flow of viscous nanoliquid in the presence of gyrotactic micro-organisms encountered by a porous stretched surface is addressed numerically. The novel aspects of Brownian diffusion and thermophoresis are studied by using Buongiorno model. The thermal radiation impact is imposed in the energy equation. A set of pertinent transformations has been suggested to transform the governing non-linear partial differential equations into system of non-linear ordinary differential equations. A famous numerical method, finite difference technique, is engaged to acquire the numerical solution of modeled dimensionless equations. The flow analysis for effects of numerous prominent parameters on velocity, temperature, concentration and motile micro-organisms profiles is presented
graphically. In the presence of thermal radiation, velocity profiles detract with augment of bioconvection Rayleigh number and buoyancy ratio parameter, while opposite trend is observed for boosting the Grashoff number. The porous medium as well as the radiation enhance the fluid temperature.

**Keywords:** Bioconvection flow, viscous fluid, gyrotactic micro-organisms, heat generation or absorption, porous medium.

1. **Introduction**

Due to inclusive practical applications of nanoparticles in recent decade, a foremost attention has been devoted by scientists towards the study of nanoparticles which encountered a diverse physical significance in many industrial, engineering and real problems. Recent developments in the industrial systems insisted the researchers to discover new and cheap resources of energy. Although many traditional techniques have been developed to improve such energy resources and thermal efficiency processes but many environmental problems like global warming, the emission of carbon dioxide and depletion of ozone layer are resulted from such physical processes. The energy resources based on the utilization of such nanoparticles are suggested as most fascinating and improved heat transportation mechanisms. Beside this, the implication of nanoparticles is also encountered in the area of bio-medical sciences like lasers, diagnoses of different diseases, annihilation of cancer tissues, brain tumor, artificial lungs etc. Further, some more applications of nanoparticles in engineering processes involve cooling processes, enhancement of extrusion processes, solar system etc. The nanoparticles with high thermal conductivity are small sized solid particles such as carbides (SiC), stable metals (Ag, Al, Cu), and non-metals like carbon nano-tubes and oxides (SiO₂, CuO, and Al₂O₃), which are merged in the base fluids like ethylene glycol, oil, water and bio-fluids to enhance the temperature of the
The primary attention regarding the interaction of such enhanced thermo-physical particles was initiated by Choi [1] in 1995 which was later on, directed to numerous investigators to the flow of such tiny sized particles. For instance, Buongiorno’s [2] suggested the most important slip parameters namely thermophoresis parameter effects and Brownian motion aspects associated with the movement of nanoparticles. Khan et al. [3] established a series solution regarding the Maxwell nanofluid configured by a stretched surface with additional effect of heat source/sink and radiative heat flux features. The series solution based on the famous convergent technique was developed by confirming the accuracy of solution. The numerical investigation for natural convection flow of nanoparticles by following Lattice Boltzman technique was carried out by Rasul et al. [4]. Mashaei et al. [5] performed the numerical simulation in order to investigate the effects of discrete heat source and sink in the laminar force convection of nanofluid flow across the channel. The homotopic approach was used by Kumam et al. [6] in order to discuss the magnetohydrodynamic and radiative effects in cross nanofluid associated with rotating channels. Hassan et al. [7] used Forchheimer porous medium model to explore the permeability prospective in nanofluid flow over a wavy surface. Guha and Nayek [8] pointed out thermal consequences in natural convection flow of nanofluid over a vertical plate. Numerical approach known as finite volume method was employed by Sheikholeslami et al. [9, 10] to investigate the enhancement in heat transfer rate of nanomaterials. Hakeem et al. [11] reported the thermal features in the nanoparticles encountered by permeable configuration. Further recent studies regarding nanofluid flow can be seen in [12-18].

The practical applications (including crystal growing, extrusion process, glass blowing and manufacture of foods and papers) of the phenomena of fluid flow over stretching surfaces make its study significant. Turkyilmazoglu [19] provided the multiple analytical solutions for fluid
flows over the stretching surfaces. Series solution was established by Khan [20] to investigate the problem of visco-elastic fluid flow over stretching sheet. Further recent studies related to applications of fluid flow over stretching surfaces can be cited [21-27].

In recent years, a diverse attention is signified by researchers towards the phenomenon of bioconvection as it involves some miscellaneous applications in modern bio-technology. The bioconvection reveals the microscopic convection of fluid particles which is associated with the variation in the density distribution. The bioconvection process is the collective motion of motile microorganism in some specified direction. Such self-oriented movement of microorganism enhances the liquid density effectively. Bioconvection appears when microorganisms which are less dense than water are swimming upward on average. The different species have different reasons for upward swimming. Oxygen concentration gradient acts as stimulator, which is responsible for forcing the microorganism to swim in a specific direction by self-propelling. The motion of microorganisms due to such stimulators referred to taxis. It is remarked that movements of microorganisms may be comprised as chemotaxis, phototaxis, gravitaxis and gyrotaxis. Moreover, the microorganism’s movement is also responsible due to chemical gradient, towards or away from the light, directed opposite to gravity and due to gravitational viscous effects moves towards down welling fluid. On this end, viscous shear forces and gravity effects near the bottom-heavy wall, produce torque to control the gyrotaxis. Therefore, the bottom heavy algae/bacteria are involved in bioconvection. The basic difference between bioconvection and nanofluid revealed that bioconvection phenomenon is self-propelled while the movement of nanoparticles is characterized by thermophoresis or Brownian motion. It is experimentally proved that the utilization of gyrotactic microorganisms is more effective to enhance stability and mass transportation of nanoparticles. The primary investigation on
bioconvection phenomenon with evaluation of nanoparticles has been reported by Kuznetsov [28, 29]. Later on, due to reported significance of this phenomenon in various industrial and biotechnology, authors have presented much useful continuation on this topic. For instance, bioconvection study of natural convection fluid flow of nanofluid incorporating gyrotactic microorganism past a truncated cone with utilization of convective boundary conditions was presented by Khan et al. [30]. Hayat et al. [31] deals with the mixed convective flow of viscoelastic nanofluid with gyrotactic microorganisms induced by stretched cylinder. Mehryan et al. [32] numerically presented the thermo-physical characteristics of nanofluid featuring the gyrotactic microorganisms configured by nonlinear surface. Akbar [33] inspected the bioconvection phenomenon in the peristaltic transport of nanofluid in asymmetric channel. Another theoretical bioconvection for the micropolar nanofluid was numerically performed by Atif et al. [34]. The numerical investigation associated with the bioconvection of nanofluid induced by moving plate was explained by Zuhra et al. [35]. Atif et al. [36] discussed the problem of magnetohydrodynamics Carreau nanofluid over a stretched surface. Nawaz et al. [37] used a finite element method to discuss the Brownian motion of nanofluid particles subject to hydro thermal effects. Further, latest studies regarding the bioconvection can be seen in [38-40].

The literature survey reveals that no work scrutinizing the heat absorption and generation features in thermally developed flow of viscous nanofluid contacting gyrotactic microorganisms over a porous stretching surface using our proposed numerical method is available yet. This is the motivation to present study.

A mathematical model for the flow, heat and concentration of a viscous nanofluid over a permeable stretching sheet taking into account the role of porosity, heat source/sink and thermal radiation is developed. Moreover, the bioconvection features of nanoparticles are considered in
the presence of motile microorganisms. The discretized mathematical equations together with boundary conditions are treated numerically via an efficient finite difference technique along with MATLAB software.

2. Problem statement

Consider a steady, incompressible and two-dimensional boundary layer nanofluid containing gyrotactic microorganism induced by a porous stretched and shrinked sheet (Figure 1.). Newtonian fluid obeying Boussinesq’s approximation is under consideration. Stretched sheet is considered to be permeable with suction/injection features. The thermophoresis and Brownian movement aspects are described by Buongiorno’s nanofluid. Heat transfer (due to thermal conduction) takes place according to Fourier’s Law. Due to thermally developed surface, the consequence of thermal radiation is considered by Rosseland approximation and heat absorption/generation prospective is also carried out in the energy equation. It is further assumed that the direction of microorganism’s swimming along with swimming velocity is not affected by nanoparticles and the suspension of nanoparticles is assumed to be diluted and stabled. The stretched/shrinked surface is retained by uniform velocity \( U_w = ax \), where \( a \) and \( x \) being positive constants and coordinate along the stretching/shrinking surface respectively. The rate of mass transportation (due to molecular diffusion) takes place according to Fick’s law with injection \( (v_o > 0) \) and suction \( (v_o < 0) \). Following these assumptions and (Aman et al. [41], Ahmad et al. [42], Sheikholeslami et al. [43] and Wahid et al. [44]), the governed equations in current flow situation may lead to following forms:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]
\[
\begin{align*}
\vec{u} \frac{\partial \vec{u}}{\partial x} + \vec{v} \frac{\partial \vec{u}}{\partial y} &= g \frac{\partial^2 \vec{u}}{\partial y^2} + g \beta \left(1 - \vec{C}_\infty\right) \left(\bar{T} - \bar{T}_\infty\right) - \frac{g}{\rho_f} \left(\rho_p - \rho_f\right) \left(\bar{C} - \bar{C}_\infty\right), \\
- \frac{g\gamma}{\rho_f} \left(\rho_m - \rho_f\right) \left(\bar{N} - \bar{N}_\infty\right) - \frac{g}{k^*} \vec{u}, \\
\end{align*}
\]

where \(\vec{u}\) and \(\vec{v}\) respectively denote the horizontal and vertical velocity components. Further, \(\beta\), \(g\), \(\bar{T}\), \(k^*\), \(\alpha\), \(Q_0\), \(D_B\), \(D_T\), \(D_n\), \(b\), \(W_c\), \(\bar{C}\), \(\bar{N}\), \(\tau\), \(\rho_f\), \(\rho_p\), \(c_f\) and \(c_p\) symbolize the kinematics viscosity, gravity vector, temperature, Darcy permeability, fluid volume expansion coefficient, thermal diffusivity, thermal coefficient, Brownian diffusion coefficient, thermophoresis constant, microorganism diffusivity, chemotaxis constant, maximum cell swimming speed, nanoparticles volume fraction, microorganism concentration, ratio of effective heat capacity of fluid, fluid density, nanoparticles density and specific heat constant respectively.

The terms regarding bioconvection and porosity are included in the momentum equation.

Generally, movements of nano particles are stabilized by use of the microorganisms in base fluid. Bioconvection has some principle uses in other fields like agriculture, chemical, biotechnology and wastewater management. While the convective heat and mass transfer in fluid- imbued porous media have inclusive range of applications including water movements in geothermal reservoirs, thermal insulation engineering, heat pipes, nuclear waste repository,
underground spreading of chemical waste, grain storage, geothermal engineering and higher recovery of petroleum reservoirs. Furthermore, the heat equation is entrenched by the effects of heat source and radiation. There is basically high temperature difference between ambient fluid and surface which caused thermal convection having a significant heat sink. There are lots of convection applications in the framework of endothermic and exothermic reactions of chemical compounds. Usage of radiations are in the fields of academics, medicine, generating electricity and industry. Moreover, radiation has valuable applications in the areas like archaeology, agriculture, law enforcement, geology, space exploration and many others.

The appropriate boundary conditions for considered flow problem are as follows:

\[
\begin{align*}
\tilde{v} & = v_0, \quad \tilde{u} = \lambda U_w, \quad \tilde{T} = \tilde{T}_w, \quad \tilde{C} = \tilde{C}_w, \quad \tilde{N} = \tilde{N}_w \quad \text{at} \quad y = 0, \\
\tilde{u} & \to 0, \quad \tilde{T} \to \tilde{T}_w, \quad \tilde{C} \to \tilde{C}_\infty, \quad \tilde{N} \to \tilde{N}_\infty \quad \text{as} \quad y \to \infty,
\end{align*}
\]  

(6)

where, \( \lambda \) is a parameter with stretching \( (\lambda > 0) \) and shrinking \( (\lambda < 0) \). Moreover, \( \omega \) and \( \infty \) depict the values at solid surface and at far away the surface, respectively.

2.1 Dimensionless quantities

In order to transmute the flow system in dimensionless form, following transformations are utilized

\[
\psi = (a \theta)^\frac{1}{2} \tilde{\psi}(\eta), \quad \eta = \left(\frac{a}{\theta}\right)^\frac{1}{2} y, \quad \theta(\eta) = \frac{\tilde{T} - \tilde{T}_\infty}{\tilde{T}_w - \tilde{T}_\infty}, \quad \phi(\eta) = \frac{\tilde{C} - \tilde{C}_\infty}{\tilde{C}_w - \tilde{C}_\infty}, \quad \chi(\eta) = \frac{\tilde{N} - \tilde{N}_\infty}{\tilde{N}_w - \tilde{N}_\infty}
\]  

(7)

The governing Eqs. (2)-(5) converted into dimensionless forms while employing the transformations (7) are:

\[
f'' + ff' = f'^2 - \varepsilon f' + Gr \left( \theta - Nr\phi - Rb\chi \right) = 0,
\]  

(8)

\[
(1 + Rd)\theta' + Pr f \theta' + Pr \left( Nb\theta'\phi' + Nt\theta'^2 + \delta \theta \right) = 0,
\]  

(9)
\( \phi'' + \frac{Nt}{Nb} \theta'' + \text{Lef} \phi' = 0, \) \hspace{1cm} (10) \\
\( \chi'' + Lbf \chi' - Pe[\chi' \phi' + \phi'(\chi + \sigma)] = 0, \) \hspace{1cm} (11)

Where continuity Eq. (1) is identically satisfied, represents the possible fluid motion. Similarly, boundary conditions get form:

\[
\begin{align*}
&f(0) = S, \quad f'(0) = \lambda, \quad \theta(0) = 1, \quad \phi(0) = 1, \quad \chi(0) = 1, \\
&f' = 0, \quad \theta = 0, \quad \phi = 0, \quad \chi = 0, \quad \text{as} \quad \eta \to \infty
\end{align*}
\]  \hspace{1cm} (12)

where \( Gr \) is Grashoff number, \( Nr \) notifies the buoyancy number, \( Rb \) represents bioconvection Rayleigh number, \( \varepsilon \) denotes porosity parameter, \( Pr \) indicates Prandtl number, \( Nb \) being the Brownian motion, \( Nt \) represents thermophoresis parameter, \( \delta \) is the source/ sink parameter, \( Le \) is Lewis number, \( Lb \) is bioconvection Lewis number, \( \sigma \) is the dimensionless parameter, \( Pe \) being the bioconvection Peclet number while \( S \) is the mass flux parameter with suction \( (S > 0) \) and injection \( (S < 0) \). Mathematically theses parameters are expressed as:

\[
Gr = \frac{g \beta (1 - \tilde{C}_w) (\tilde{T}_w - \tilde{T}_\infty)}{a \tilde{u}}, \quad Nr = \frac{(\rho_p - \rho_f) (\tilde{C}_w - \tilde{C}_\infty)}{\beta \rho_f (1 - \tilde{C}_\infty) (\tilde{T}_w - \tilde{T}_\infty)}, \quad Rb = \frac{\gamma (\rho_n - \rho_f) (\tilde{N}_w - \tilde{N}_\infty)}{\beta \rho_f (1 - \tilde{C}_\infty) (\tilde{T}_w - \tilde{T}_\infty)},
\]

\[
Nb = \frac{\tau D_b (\tilde{C}_w - \tilde{C}_\infty)}{\partial}, \quad Nt = \frac{\tau D_l (\tilde{T}_w - \tilde{T}_\infty)}{\partial \tilde{T}_\infty}, \quad \delta = \frac{Q_0}{a \rho c_f}, \quad Le = \frac{\partial}{D_n}, \quad Pr = \frac{\partial}{\alpha}, \quad \varepsilon = \frac{\partial}{ak'}, \quad Lb = \frac{\partial}{D_n},
\]

\[
\sigma = \frac{\tilde{N}_w}{\tilde{N}_\infty}, \quad Pe = \frac{b W}{D_n}.
\]

2.2. Physical quantities

In present study, the physical quantities of practical interest are the skin friction coefficient, local Nusselt number, local Sherwood number and the local density of motile microorganisms which have been justified in following forms:
\[ C_f = \frac{\tau_w}{\rho U_w^2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \quad Sh_x = \frac{xq_m}{D_B (\bar{C}_w - \bar{C}_\infty)}, \quad \tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0} \]
\[ q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}, \quad q_m = -D_B \left( \frac{\partial \bar{C}}{\partial y} \right)_{y=0}, \quad q_n = -D_n \left( \frac{\partial \tilde{N}}{\partial y} \right)_{y=0} \]

where \( \tau_w, q_w, q_m \) and \( q_n \) are surface shear stress, wall heat flux, wall mass flux and the motile microorganisms density, respectively.

The physical quantities defined in Eq. (13) may be written as
\[ C_f \left( \frac{\text{Re}_x}{\text{Re}_x} \right)^{1/2} = f''(0), \quad Nu_x \left( \frac{\text{Re}_x}{\text{Re}_x} \right)^{1/2} = -\theta'(0), \quad Sh_x \left( \frac{\text{Re}_x}{\text{Re}_x} \right)^{1/2} = -\phi'(0), \quad Nn_x \left( \frac{\text{Re}_x}{\text{Re}_x} \right)^{1/2} = -\chi'(0) \]
(14)

where \( \text{Re}_x = U_w x / \theta \) is the local Reynolds number.

3. Numerical Solution

Mathematicians, scientists and researchers are paying great intention in solving nonlinear physical models as most of the nonlinear physicals models exist in nature. Some researchers provided the analytical solutions for such physical phenomena under certain conditions (Wahid et al. [45-46], Turkyilmazoglu [47] and Khan et al. [48]). The governing equations of present physical model are highly nonlinear. It is difficult to solve such differential equations analytically. So, we apply numerical technique known as finite difference method to solve such nonlinear coupled equations. As Eq. (8) is of 3rd order, firstly we reduce its order by substituting \( f' = q \), so the equations and corresponding boundary conditions get the form
\[ q'' + fq' - q^2 - \varepsilon q = Gr \left( Nr \phi + Rb \chi - \theta \right), \]
(15)
\[ (1 + Rd)\theta'' + Pr f \theta' = -Pr \left( Nb \theta' \phi' + Nt \theta'^2 + \delta \theta \right), \]
(16)
\[ \phi'' + Lef \phi' = -\frac{Nt}{Nb} \theta'', \]
(17)
\[ \chi'' + (Lbf - Pe \phi') \chi' - Pe \chi \phi'' = Pe \sigma \phi'', \]
(18)
with corresponding boundary conditions get form,

\[
\begin{align*}
  f(0) &= S, \quad q(0) = \lambda, \quad \theta(0) = 1, \quad \phi(0) = 1, \quad \chi(0) = 1, \\
  q &= 0, \quad \theta = 0, \quad \phi = 0, \quad \chi = 0, \quad \text{as} \quad \eta \to \infty
\end{align*}
\]  

(19)

In order to perform the numerical computation for above formulated equations, we first discretize the flow domain \([0, \infty)\) with step size \(h\). Famous Simpson’s numerical iterative procedure is applied to integrate the equation \(f' = q\). Further, famous central difference approximation is employed to discretize the involved derivatives in Eqs. (15)-(19) and then iterative solution is obtained by employing succession over relaxation method (SOR). This iterative method is stopped when

\[
\max \left( \| q_{i+1} - q_i \|_2, \| f_{i+1} - f_i \|_2, \| \theta_{i+1} - \theta_i \|_2, \| \phi_{i+1} - \phi_i \|_2, \| \chi_{i+1} - \chi_i \|_2 \right) < Tol_{\text{err}}
\]

is satisfied up to four consecutive iterations.

4. Results and discussion

In this section, we exploit the interesting features of involved physical parameters on velocity, temperature, concentration and motile microorganism distributions. These parameters have material and flow properties. They may characterize the geometric dimensions. Instead of allocating the specific domain dimensions and fluid properties to these parameters, a better approach is to allocate the fix values to these parameters to analyze the flow and thermal characteristics of fluid dynamics problems (Ahmad et al. [42] and Wahid et al. [44]). The range of the parameters for the study under consideration may be chosen in such a manner that a further increment in the upper limit does not affect the qualitative behavior of the profiles. A detailed discussion of numerical results, describing the effects of the various governing parameters, such as traditional Lewis number \(Le\), bioconvection Lewis number \(Lb\), bioconvection Péclet number \(Pe\), the suction/injection parameter \(S\), bioconvection Rayleigh
number $R_b$, buoyancy ratio parameter $N_r$, Brownian motion parameter $N_b$, thermophoresis parameter $N_t$, permeability parameter $\varepsilon$, Grashof number $G_r$, Prandtl number $P_r$, the microorganisms concentration difference parameter $\chi$, heat source/sink parameter $\delta$ and dimensionless parameter $\sigma$ are shown by Figures 2-18.

The importance of buoyancy ratio $N_r$, permeability parameter $\varepsilon$, mixed convection $G_r$, bioconvected Rayleigh number $R_b$ and suction/injection parameter $S$ on velocity are shown in Figures 2-6. From the velocity profiles in (Figures 2-6), it has been detected that at the plate surface, the velocity is maximum but in the region beyond the plate surface at the free stream, it is exponentially decreased to zero.

Figure 2 illustrates the effects of porosity parameter $\varepsilon$ on velocity field $f'(\eta)$. The reduction in velocity field occurs by increasing the porosity of the medium. Physical justification for such declining trend is associated with the sizes of pores inside the porous medium which become higher. In this case, a resistive force from the opposite direction of the flow acts on the fluid, and as a result velocity boundary layer thickness decreases. The influence of suction/injection parameter $S$ on velocity distribution $f'(\eta)$ is shown in Figure 3. The visualized results claimed that $f'(\eta)$ decreases gradually. Physically, the escalating behavior of dragging of nanofluid particles through the stretching surface is observed by enlarging the suction parameter $S$, which demises the velocity profiles. It is observed from Figure 4 that a declining velocity profile has been predicted with evaluation of $N_r$. In fact, resistive buoyancy forces (act opposite to pressure gradient) are resulted by strengthening $N_r$, which reduce the speed of fluid flow. The impact of Rayleigh number $R_b$ on velocity $f'(\eta)$ is shown by Figure 5. It is predicted that the velocity profile decreases due to increase in Rayleigh number. The physical traits of such type of
occurrence may comprise that the bioconvection Rayleigh number $Rb$ includes the buoyancy ratio (resistive) forces, which are responsible to resist the movement of fluid particles in the entire flow domain. Physically, the effects of buoyancy forces become more significant by boosting up the values of $Rb$, which are resulted in the form of decline in the fluid velocity. While on the other hand, opposite trend is observed in Figure 6 for $Gr$. It is cleared that velocity of the fluid $f'(\eta)$ increases by increasing the Grashoff number $Gr$, which is related to the involvement of buoyancy ratio forces versus viscous forces. Enlarging the Grashoff number, may weaken the effect of viscous forces, which augment the fluid velocity.

The graphical consequences for buoyancy ratio parameter $Nr$, bioconvection Rayleigh number $Rb$, radiation parameter $Rd$, Brownian motion parameter $Nb$, thermophoresis parameter $Nt$ and heat source/sink parameter $\delta$ on temperature $\theta(\eta)$ are claimed in Figures 7-12. It is observed through Figures 7 and 8, that the temperature of the nanofluid increases, by enhancing the bioconvection Rayleigh number $Rb$ and buoyancy ratio parameter $Nr$. The physical aspect of strengthen of temperature profiles is due to the involvement of the buoyancy forces. The influence of radiation parameter $Rd$ over temperature is depicted through Figure 9. By increasing the radiation parameter, the temperature profiles rise. Physically, by enhancing the radiation parameter, the mean absorption coefficient decreases, which is responsible to increase the rate of radiative heat transfer to the liquid. Therefore, the temperature of the fluid is enhanced. The behavior of $Nb$ and $Nt$ on $\theta(\eta)$ are depicted through Figure 10 and 11. It is predicted that thermal boundary layer thickness and temperature are increasing function of $Nb$ and $Nt$. Random motion of the fluid particles is enhanced by increasing $Nb$, due to which a large amount of heat is produced. As a result, temperature of the fluid is enhanced. On the other
hand, due to thermophoresis, large amount of nanoparticles are pulled away from hot surface to cold surface, which causes to raise the temperature of the fluid.

The effects of heat source/sink parameter $\delta$ on temperature distribution are elaborated in Figure 12, which indicate that enhancement in temperature occurs due to increase in heat source strength. As by increasing the heat source strength, a large amount of heat is entered to the system, due to which temperature of the fluid rises and thermal boundary layer thickness is increased. This result has basic importance for the heat transfer flow problems. Figure 13 is sketched to show the effects of Brownian motion parameter $Nb$ on concentration profiles. Collision between the particles increases due to increase in $Nb$. Hence concentration profiles decrease with the increase in $Nb$. Impact of thermophoresis parameter $Nt$ on concentration profile is shown in Figure 14. It is noted that concentration profiles are increasing function of $Nt$. Increase in $Nt$, causes the enhancement in thermal conductivity of fluid, which is responsible for the enhancement in the concentration profiles. Figure 15 is designed to examine the influence of Lewis number $Le$ on concentration. It can be observed that concentration boundary layer thickness and concentration profiles decrease by boosting the values of Lewis number $Le$. Since, Brownian diffusion coefficient is inversely proportional to the Lewis number, so small diffusivity occurs, for larger values of Lewis number $Le$. Hence, concentration distributions decrease by increasing the Lewis number $Le$. The effect of Peclet number $Pe$ on motile density profiles is shown in Figure 16. The increase in $Pe$ is responsible for decrease in diffusivity of microorganisms, which causes to decrease the motile density of the fluid. The impacts of microorganism concentration parameter and bioconvection Lewis number on motile density are disclosed in Figures 17 and 18. The deterioration in motile density occurs by increasing microorganism concentration difference parameter and bioconvection Lewis number.
Physically, due to increase in $\sigma$, the surface concentration decreases, therefore, the strength of particles decays and hence mass also reduces. And as a result, decrement in density profiles occurs. Also, augmenting in $Lb$ may reduce the microorganism’s diffusivity, which is responsible for reduction of motile density of microorganisms.

Table 1 illustrates the comparison values of $-\theta'(0)$ with those reported work by Aman et al. [41], by neglecting the existence of effects of porosity parameter, radiation parameter, mixed convection, magnetic field and heat absorption/generation parameters in Eqs. (9) and (10). It is comprehended that the comparison is in excellent agreement and thus ensures the accuracy of numerical results.

The impact of various parameters like $Gr$, $Nr$, $Rb$ and $\varepsilon$ on $f''(0)$ is illustrated in Table 2. It is clear from table that $f''(0)$ increases with the increase of $Nr$, $Rb$ and $\varepsilon$, in contrast to $Gr$.

Table 3 captures the numerical variation of $-\theta'(0)$ against various parameters $Nr$, $Rb$, $Nb$, $Nt$, $Rd$, $Pr$ and $\delta$. It is observed that local Nusselt number $-\theta'(0)$ has got maximum values via $\delta$ and $Pr$, while opposite trend is observed for other ones. The effect of diverse values of $Nr$, $Rb$, $Nb$, $Nt$ and $Le$ over $-\phi'(0)$ is shown in Table 4. The local Sherwood number $-\phi'(0)$ enhances by increasing $Nb$ and $Le$, while decreasing trend is observed for other parameters. Finally, Table 5 presents the numerical variation of $-\chi'(0)$ against values of $Nr$, $Rb$, $Pe$ and $Lb$. Carefully observing, from table that motile density of microorganisms decreases by increasing $Nr$ and $Rb$, while it is enhanced by the effects of $Pe$ and $Lb$. Note that the multiple solutions may exist for shrinking of the sheet and suction (Turkyilmazoglu [19], Lund et al. [49] and Mustafa [50]), and may be presented in a subsequent paper.
5. Conclusions

The reported contribution deals with the bioconvection aspects in the flow, heat and concentration of nanofluid over a porous trenched configuration embedded in a resistive porous medium. The numerical based results are evaluated by employing an efficient method based on finite difference scheme. This method gives fast convergence. Major observations are given as follows:

- The porosity of the medium and suction through the sheet tend to slow the flow velocity.
- The temperature of nanoparticles is improved with proper variation of both thermophoresis and Brownian motion constants.
- The heat transfer rate increases by increasing the Prandtl number.
- The variation in Lewis number decreases the concentration.
- The Peclet and bioconvection Rayleigh numbers decrease the gyrotactic microorganism’s distribution.
- The porous medium causes an increase in the fluid temperature.

Comparison of our present results with the results of [41], indicates that the resistive forces of porous medium and thermal radiations are more capable to handle the bioconvection in flows with larger shear stresses and temperature in comparison to the flow over a stretching sheet embedded in a non-porous medium without radiation. The reported results may be helpful in the bio-fuels applications and thermal extrusion phenomenon.
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References


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<td>Figure 4. The variation of velocity for $S = Rb = Nb = Nt = Lb = 0.2$, $\varepsilon = 6$, $Gr = 1.8$, $Pr = 2.5$, $\sigma = \delta = 0.1$, $Le = 1.6$, $Pe = 2$, $Rd = 2.3$, $\lambda = 0.9$ and various $Nr$.</td>
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<td>Figure 5. The variation of velocity for $S = 0.1$, $\varepsilon = 3$, $Gr = \lambda = 0.9$, $Nr = 0.2$, $Nb = Nt = Lb = 0.3$, $Pr = 2$, $\sigma = \delta = 0.1$, $Le = 4.9$, $Pe = 1.1$, $Rd = 0.8$ and various $Rb$.</td>
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**Figure 6.** The variation of velocity for $S = Rb = \sigma = 0.3$, $\varepsilon = Pr = 10$, $Nr = Nb = Nt = \delta = 0.1$, $Le = 5.5$, $Pe = 0.5$, $Lb = 0.9$, $Rd = 3.5$, $\lambda = 1$ and various $Gr$.

**Figure 7.** The variation of temperature for $S = 0.3$, $\varepsilon = 6$, $Gr = 5.8$, $Rb = \sigma = 0.2$, $Pr = 6.8$, $Nb = Nt = Rd = 0.3$, $Lb = 1.5$, $\delta = 0.1$, $Le = Pe = 2$, $\lambda = 1$ and various $Nr$.

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**Figure 9.** The variation of temperature for $S = Nr = \sigma = 0.2$, $\varepsilon = Le = 5.5$, $Gr = 4$, $Rb = 0.4$, $Pr = 10$, $Nb = Nt = \delta = 0.1$, $Lb = 0.8$, $Pe = 1.3$, $\lambda = 1$ and various $Rd$.

**Figure 10.** The variation of temperature for $S = 0.2$, $\varepsilon = 4$, $Gr = 5$, $Nr = 0.4$, $Rb = 0.3$, $Pr = 7$, $Rd = 0.9$, $Nt = Lb = Pe = \sigma = \delta = 0.1$, $Le = 5.5$, $\lambda = 1$ and various $Nb$.

**Figure 11.** The variation of temperature for $S = Nb = \sigma = Rd = Lb = \delta = 0.1$, $\varepsilon = Le = 4.5$, $Gr = 3.5$, $Nr = 0.2$, $Rb = 0.3$, $Pr = 7.5$, $Pe = 1.1$, $\lambda = 1$ and various $Nt$.

**Figure 12.** The variation of temperature profiles for $Nt = Nb = \sigma = 0.1$, $S = Nr = Rb = 0.2$, $\varepsilon = 0.7$, $Gr = 1.7$, $Pr = 5.2$, $Le = 4.9$, $Pe = Rd = 0.9$, $Lb = 1.2$, $\lambda = 1$ and various $\delta$.

**Figure 13.** The variation of concentration for $S = Nr = Rb = Nt = Lb = \sigma = \delta = 0.1$, $\varepsilon = 4.5$, $Gr = 4$, $Pr = 7.5$, $Le = 5.5$, $Pe = 1.1$, $Rd = 2.2$, $\lambda = 1$ and various $Nb$.

**Figure 14.** The variation of concentration for $S = Rb = Nr = \delta = Pe = Rd = Lb = 0.1$, $\varepsilon = Gr = Le = 4.5$, $Pr = 1.5$, $Nb = 0.2$, $\sigma = 0.3$, $\lambda = 1$ and various $Nt$.

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**Figure 16.** The variation of density of motile microorganism for $S = 0.3$, $\varepsilon = 1.5$, $Gr = 2$, $Nr = Nt = Rb = 0.1$, $Pr = \sigma = 0.5$, $Nb = 1.3$, $\delta = 0.7$, $Le = 4.5$, $Lb = 1.8$, $Rd = 0.2$, $\lambda = 1$ and various $Pe$.

**Figure 17.** The variation of density of motile microorganism for $S = 0.2$, $\varepsilon = 3$, $Gr = 4$, $Rb = 0.4$, $Pr = 3.5$, $Nr = Nb = Nt = Lb = \delta = 0.1$, $Le = 4.5$, $Pe = Rd = 1.2$, $\lambda = 1$ and various $\sigma$.

**Figure 18.** The variation of density of motile microorganism for $S = Nr = Rb = 0.2$, $\varepsilon = 0.7$, $Nt = Nb = \sigma = 0.1$, $Gr = 1.7$, $Pr = 5.2$, $Le = 4.9$, $Pe = Rd = 0.9$, $\delta = 1.2$, $\lambda = 1$ and various $Lb$. 

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### List of Tables

<table>
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<th>The comparison values of $-\theta'(0)$, $(Rd = \delta = Gr = Rb = Nr = \varepsilon = M = 0)$ with [41].</th>
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<td>Table 2.</td>
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Figure 1. Physical Configuration.

Figure 2. The variation of velocity for $S = 0.2$, $Gr = 5$, $Nr = 0.4$, $Rb = 0.6$, $Pr = 4.5$, $Nb = Nt = \sigma = \delta = 0.1$, $Le = 4.5$, $Pe = 1.2$, $Lb = 0.9$, $Rd = 0.9$, $\lambda = 1$ and various $\varepsilon$. 
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Figure 4. The variation of velocity for $S = Rb = Nb = Nt = Lb = 0.2$, $\varepsilon = 6$, $Gr = 1.8$, $Pr = 2.5$, $\sigma = \delta = 0.1$, $Le = 1.6$, $Pe = 2$, $Rd = 2.3$, $\lambda = 0.9$ and various $Nr$. 
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Figure 14. The variation of concentration for $S = Rb = Nr = \delta = Pe = Rd = Lb = 0.1$, $\varepsilon = Gr = Le = 4.5$, $Pr = 1.5$, $Nb = 0.2$, $\sigma = 0.3$, $\lambda = 1$ and various $Nt$. 
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Figure 16. The variation of density of motile microorganism for $S = 0.3$, $\varepsilon = 1.5$, $Gr = 2$, $Nr = Nt = Rb = 0.1$, $Pr = \sigma = 0.5$, $Nb = 1.3$, $\delta = 0.7$, $Le = 4.5$, $Lb = 1.8$, $Rd = 0.2$, $\lambda = 1$ and various $Pe$. 
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Figure 18. The variation of density of motile microorganism for \( S = Nr = Rb = 0.2, \varepsilon = 0.7, Nt = Nb = \sigma = 0.1, Gr = 1.7, Pr = 5.2, Le = 4.9, Pe = Rd = 0.9, \delta = 1.2, \lambda = 1 \) and various \( Lb \).
Table 1. The comparison values of $-\theta'(0)$, ($Rd = \delta = Gr = Rb = Nr = \varepsilon = M = 0$) with [41].

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Biographies

Muhammad Ashraf is working as a Professor in the Centre for Advanced Studies in Pure and Applied Mathematics, Bahauddin Zakariya University, Multan, Pakistan. He has a teaching experience of more than 25 years. His research interests include flow, heat and mass transfer analysis of micropolar, Casson, Nano, hybrid-Nano, and dusty fluids using numerical methods. Moreover, his PhD students are working on the topics such as flow through Darcy-Forchheimer medium and fluid flows contacting gyrotactic microorganisms.

Shaheen Akhter is PhD Scholar under the supervision of Professor Dr. Muhammad Ashraf in the Centre for Advanced Studies in Pure and Applied Mathematics, Bahauddin Zakariya University, Multan, Pakistan. She is Lecturer at COMSATS University Islamabad, (Sahiwal Campus), Pakistan.