Bioconvection phenomenon for the boundary layer flow of magnetohydrodynamic Carreau liquid over a heated disk

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Abstract. A numerical examination is conducted for the magnetohydrodynamics steady Carreau fluid flow on the transport of thermal energy and mass specie comprising nanoparticles with gyrotactic microorganisms through heated disk. The role of thermophoresis and Brownian motion are added in this flow problem. Governing equations are achieved by using the boundary layer theory in view of a coupled system of PDEs involving boundary conditions. The highly non-linear system of ODEs is generated using the concept of transformation approach. Since the system of transformed equations is highly nonlinear, so, an approximate solution is estimated via optimal homotopy method. The role of prominent parameters on velocity, thermal energy, mass specie and motile density microorganisms examined graphically. Additionally, graphical observations regarding mass specie, thermal energy and velocities are discussed briefly. It has estimated that motion of fluid particles are diminished because of intensity of magnetic field while mass specie and fluid temperature rise versus enhancement the values of magnetic field.

Keywords: Brownian diffusion; Heat and mass transport; Carreau fluid; Bioconvective process; Rotating heated disk; Boundary layer analysis.

1. Introduction

In the way of extensive applications, researchers have been shown a lot of deliberation regarding consideration of non-Newtonian fluids. The important role behind this study is the
generous existence of those fluids in nature as well. The number of applications of such fluids appears in areas like foodstuffs, extrusion of molten polymers and plastics, synthetic fibers, drilling gas and oil wells, among others. Considerable work regarding domain of non-Newtonian fluids has been completed and phenomenal role have found in such type of fluids (non-Newtonian). Lot of researchers has worked for improvement of thermal conductivity in view of non-Newtonian liquids. The authors have interested due to the diffidence has found in model of power law. But some drawbacks have been investigated in such type of mode (power law model). The drawbacks (in view of limitations, shear rates regarding low and high) have extracted in model of power law while authors have examined viscosity model that is called Carreau rheology. This is special type of Newtonian liquids whereas shear rate is function of viscosity and this phenomenon is useful in view of high shear-rate liquids. Carreau [1-2] have performed the important theory regarding rheological equations based on network molecular models. Griffiths [3] studied the flow behavior of generalized Newtonian liquid through disk by applying Carreau fluid model. Machireddy and Naramgari [4] discussed the role of transport of thermal energy and mass specie with cross-diffusion involving MHD Carreau fluid in stretch surface. An additional notable study on Carreau fluid can be apprehended in [5-8] and various explorations therein. Nanoparticles are tiny particles be made of dense nanoparticles or nanofibers with range (1-100 nm) normally equated through the conventional heat carrying fluids and have a competency of advanced thermal conductivity. Literature depicts that Choi and Eastman [9] initially gave the concept of utilizing fluid which consists of nano-sized particles and base fluid, name it as nanofluid. There are many applications containing: insulation of energy, astronomical, cooling processes, solar amusement and defense, magnetic sticking, mass/heat transport strengthening and medical instruments etc. The applications have required substantial point alter from the conventional fluids. Henceforward nanofluid fluid play vital role for development of interest of authors in current study, some useful studies are discussed in Refs. [10-15].

Magnetohydrodynamic (MHD) is an added working zone of building sciences in present days that includes the effect of magnetic fields. Applications of such type of fluid flow are pumps, power generators, magnetic drug treatment, accelerators, plasma studies and flow meters. Bhatti et al. [16] discussed the mathematical modeling of mass and heat effects on electrically conducting the flow of two-phase peristaltic propulsion through a porous medium with Darcy-Brinkman-
Forchheimer. Many researchers [17-20] interpreted various aspects of non-Newtonian fluid such as Williamson, Micropolar and Carreau fluids etc.

Heat is found in view of energy form which is transformed from one place to another results difference in thermal energy. It is physics that is concerned reading the investigations to produce the difference in energy called heat transfer. Its mathematical form is derived through the Fourier law of conduction. In heat transfer measurement thermal conductivity is a very important factor that is defined as the ability to measure heat conduction. The betterment in thermal conductivity reveals that a material has ability of good conductor and a poor insulator have been found in view low value of thermal conductivity. Similarly, transport of mass specie is related to movement of diffusion of fluid particles from one place to other place. Transport of thermal energy and mass specie for both are the kinetic process that may arise and studied separately or jointly also. The transport of thermal energy and solutal of nanofluid has examined by laws of Fick's and Fourier's. These both movements are modeled by similar mathematical equations in the form of convection and diffusion and both transfers must be considered jointly in some cases i.e. ablation and evaporative cooling. Application regarding transport mass specie and thermal energy in different field are oil transport wonder, dispersion of specific medications in blood, food preparation, cooling of electronic equipment, Manufacturing/materials processing, absorption, drying, precipitation, membrane filtration and evaporation can be seen in [21]. For more mass and heat transfer applications readers are referred to have a look in [22-24] and studies cited therein for detailed understanding.

Rotation is most powerful and useful tool regarding applications (medical equipment’s, gas turbine, food processing and computer operating) while numerous application have found in food of rotating geometries (disk, cylinder and surface. It is evident that rotating disk is considered as important role in research point view. First time, concept regarding rotating disk is developed by Karman [25]. He has found transformation (Von-Karmaan) to compute solution flow problems over heated rotating disk. Several other applications can be found in [26-28] concerning the aspects of rotations.

The particles of such type of impact (bio-convection) have carried out which are not self-boosted microorganisms. The other terminology regarding bio-convection approach that is called boosted microorganism. So, this type of terminology has driven by Platt [29] and he has conducted
that drag force is generated during the movement of microorganism while gravitation torque is produced due to equilibrium position of particles results swimming of cells of microorganism. Chakraborty et al. [30] perfumed the aspects regarding magnetic field and nanoparticles containing via gyrotactic microorganisms. Impact in view of gyrotactic microorganisms with nanoparticles and radiation has estimated by Khan et al. [31]. For further work, readers are referred to the work mentioned in [32-34] and references therein.

The satisfactory of solution has two types of category involving approximately solution and generating accurate approximate solution regarding different parameters. Numerous approaches are applied to find solution of linear type flow problems. In view of analytical technique, (OHAM) analysis approach is found to capture solution non-linear flow problems involving with BCs. Recently, (OHAM) approach is adopted by Marinca and Herisanu [35]. Few appropriate studies concerning the suggested algorithm can be found in [36-39]. Makinde et al. [40] purposed new bouncy induced procedure in view of nanoparticles whereas volume fraction is considered by making variation in thermal conductivity in this flow problem while flow problem is solved by (RK4SM) approach. Makinde et al. [41] captured the flow phenomenon regarding Brownian motion, bouncy force and bio-convection through parabolic surface via microorganism. Mutuku et al. [42] discussed the characteristics of bio-convection subjected to hydro-magnetic considering nanoparticles. Khan et al. [43] conducted aspects of gyrotatic microorganism with nanoparticles on transport of mass specie and heat energy via magnetic field. Makinde et al. [44] developed the flow model regarding the influence of radiations, chemical reaction, Brownian motion and magnetic force over vertical plate. Few important latest contribution dealing the flow problems are reported in [45-47].

The key role of this article to capture the flow phenomenon regarding electrical conducting Carreau rheological fluid considering nanoparticles and gyrostatic microorganism through heated cone is observed. The system of ODEs is obtained from the system of PDEs using Von-Karman transformations adopting approach of OHAM analysis scheme. This current study is designed as follows: section obtains literature review. The mathematical formulation is developed in section two. The formulation of flow problem and numerical solution are captured in section three and four, respectively. The key points regarding flow problem are added in section five. In the end, references are listed.
2. Mathematical formulation and fluid rheology

In the given analysis, we have considered electrically conducting flow for two-dimensional time independent incompressible Carreau fluid and nanoparticles with motile gyrotactic microorganisms, which is induced by rotating disk. The magnetic field strength \(B_0\) for boundary layer is acted along \(z\) direction while motion of fluid particles are generated by movement of wall velocity \(u_r(=r l_0)\), where \(l_0\) is constant rate. Geometrical flow under assumptions is captured by Fig. 1. The influence of induced magnetic field is not taken but features regarding Thermophoresis and ambient motion are observed. The angular velocity \(\Omega_1\) is rotational velocity of rotating disk involving viscous dissipation. The velocity components are based on directions of \((r, \theta, z)\). Initially, disk is heated with \((T_0)\) temperature and after it disk gains ambient temperature \((T_\infty)\). Bio-convective pattern occurs due to movement of motile microorganisms from higher area of microorganisms to low region. The reference and ambient microorganisms concentration are taken as \(n_0\) and \(n_\infty\) respectively. The stress tensor [1] is express as

\[
\tau_j = \left[ \eta_0 \left(1 + \lambda^2 \dot{\gamma}^2 \right)^{\frac{n-1}{2}} \right] \dot{\gamma}, \tag{1}
\]

\[
\dot{\gamma} = \left( \frac{1}{2} \sum_i \sum_j (\gamma_{ij} \gamma_{ij}) \right)^{1/2}, \tag{2}
\]

where \((n)\) power law index. It is estimated that Carreau rheology becomes \((0 < n < 1)\) shear thinning and \((n > 1)\) shear thickening. Governing laws under motile microorganism and nanomaterial are governed by following equations

\[
\nabla \cdot V = 0,
\]

\[
\rho_f [V \cdot \nabla]V = -\nabla P + \nabla \cdot \tau_i + J_1 \times B,
\]

\[
J_1 = \sigma(V \times B),
\]

\[
B = [0,0,B_0],
\]

\[
V \cdot \nabla T - \alpha \nabla^2 T = \tau \left[ D_b \nabla T \cdot \nabla C + \frac{D}{T_\infty} \nabla T \cdot \nabla T \right],
\]
\[(V \cdot \nabla)C = D_b (\nabla^2 C) + D_b (\nabla^2 C) + \left(\frac{D_t}{T_\infty}\right) \nabla^2 T,\]

\[\nabla \cdot J^* = 0,\]

where \((V)\) velocity \((u_i, v_i, w_i)\) flow components \((P)\) pressure, \((\rho_f)\) fluid density, \((T)\) temperature, \((\alpha^*)\) thermal diffusivity, \((C)\) concentration, \((D, D_b)\) thermophoretic diffusion, Brownian numbers and \((J^*)\) microorganisms flux.

\[J^* = nV + n \cdot \hat{V} - D_m \nabla n,\]

\[\hat{V} = \left(\frac{b W_c}{\Delta C}\right) \nabla C.\]

Above equations after the boundary layer approximations are expressed as

\[
\frac{\partial u_i}{\partial r} + \frac{u_i}{r} + \frac{\partial w_i}{\partial z} = 0, \tag{3}
\]

\[
\rho_f \left( u_i \frac{\partial u_i}{\partial r} - \frac{v_i^2}{r} + \frac{w_i}{r} \frac{\partial u_i}{\partial z} \right) = -\frac{\partial p}{\partial r} + \frac{\partial^2 u_i}{\partial z^2} \eta_0 \left[ 1 + \frac{(n-1)}{2} \lambda^2 \left( \frac{\partial v_i}{\partial z} \right)^2 + \left( \frac{\partial u_i}{\partial z} \right)^2 \right] + \frac{(n-1)(n-3)}{4(2)} \lambda^4 \left\{ \left( \frac{\partial v_i}{\partial z} \right)^4 + \left( \frac{\partial u_i}{\partial z} \right)^4 + 2 \left( \frac{\partial v_i}{\partial z} \frac{\partial u_i}{\partial z} \right)^2 \right\} \tag{4}
\]

\[-\sigma B_0^2 u_i,\]

\[
\rho_f \left( u_i \frac{\partial v_i}{\partial r} - \frac{u_i v_i}{r} + \frac{w_i}{r} \frac{\partial v_i}{\partial z} \right) = \frac{\partial^2 v_i}{\partial z^2} \eta_0 \left[ 1 + \frac{(n-1)}{2} \lambda^2 \left( \frac{\partial v_i}{\partial z} \right)^2 + \left( \frac{\partial u_i}{\partial z} \right)^2 \right] + \frac{(n-1)(n-3)}{8} \lambda^4 \left\{ \left( \frac{\partial v_i}{\partial z} \right)^4 + \left( \frac{\partial u_i}{\partial z} \right)^4 + 2 \left( \frac{\partial v_i}{\partial z} \frac{\partial u_i}{\partial z} \right)^2 \right\} \tag{5}
\]

\[-\sigma B_0^2 v_i,\]

\[
(u_i \frac{\partial W_i}{\partial r} - \frac{u_i W_i}{r} \frac{\partial W_i}{\partial z}) = \rho_f \left[ D_b \left( \frac{\partial W_i}{\partial z} \right)^2 + \frac{D_t}{T_\infty} \left( \frac{\partial W_i}{\partial z} \right)^2 \right], \tag{6}
\]
\[
\left( u_i \frac{\partial C}{\partial r} + w_i \frac{\partial C}{\partial z} \right) = D_e \left( \frac{\partial^2 C}{\partial z^2} \right) + \frac{D_t}{T_e} \left( \frac{\partial^2 T}{\partial z^2} \right),
\]

\[
\left( u_i \frac{\partial n}{\partial r} + w_i \frac{\partial n}{\partial z} \right) = D_m \left( \frac{\partial^2 n}{\partial z^2} \right) - \frac{bW s}{\Delta C} \left( \frac{\partial}{\partial z} \left( n \frac{\partial C}{\partial z} \right) \right).
\]

2.1. Boundary conditions

The boundary conditions subjected to fluid flow situation are captured as

\[
u_i = u_s = l_b r, \quad v_i = r \Omega, \quad w_i = 0, \quad T = T_0, \quad n = n_0, \quad C = C_0, \quad \text{at } z = 0,
\]

\[T \rightarrow T_c, \quad v_i = u_i \rightarrow 0, \quad n \rightarrow n_c, \quad C \rightarrow C_c, \quad \text{at } z \rightarrow \infty.
\]

2.2. Similarity analysis

The transformations called selecting Von-Karman are expressed as

\[u_1 = (\Omega r f (\eta)), \quad v_1 = (\Omega r g (\eta)), \quad w_1 = \left( \sqrt{\eta / \Omega r} \right) H(\eta), \quad \eta = \left( \frac{\Omega r}{\sqrt{\nu_f}} \right) z,
\]

\[\theta(\eta) = \frac{T - T_0}{T_0 - T_c}, \quad \phi(\eta) = \frac{C - C_0}{C_0 - C_c}, \quad \xi(\eta) = \frac{n - n_c}{n_0 - n_c}.
\]

The set of dimensionless ODEs are generated as

\[-(f + f^\prime) + H^\prime = 0,
\]

\[H^\prime - HH^\prime - \frac{1}{2} H'(H^\prime) - \frac{g^2}{2} + \frac{n-1}{2} \lambda_1 R_e \left[ -H^\prime(g^\prime) g'' - H^\prime g^2 - H H^\prime \frac{3}{4} H^\prime H^\prime \right] + \frac{(n-3)(n-1)}{4(2)} \left( \lambda_1 R_e \right)^2 \left[ -g^3 H'H^\prime - \frac{5}{16}(H')^3 H^\prime - (H')^2 \frac{3}{2} g^2 H^\prime - g^2 H^\prime (g^\prime)^3 - g^\prime (H^\prime)^3 g^\prime \right] + M H^\prime = 0,
\]

\[g'' - g'H - H g + \frac{n-1}{2} R_e \left[ \frac{g''H^\prime}{4} + 3 g^2 g'' + g' \frac{H^\prime}{2} H^\prime \right] + \frac{(n-3)(n-1)}{4(2)} \left( \lambda_1 R_e \right)^2 \left[ \frac{5 g^4}{16} + \frac{1}{16} g^\prime H^\prime \frac{g''}{4} + \frac{3}{2} H^\prime g^2 g'' + \frac{g''}{4} H^\prime - M g = 0,
\]

\[\theta' + N_i \theta' \phi + N_i \phi'^2 - P'_r \theta = 0,
\]

\[\phi'' + \frac{N_i}{N_b} \phi'' - Sc \phi' = 0,
\]

\[\xi'' - Sc \xi' H + P_i \left[ \phi''(\xi - \Omega) + \xi' \phi' \right] = 0.
\]
BCs regarding flow problem are developed as

\[ \begin{align*}
\text{At } \eta = 0, & \quad H = 0, \quad -2St = H', \quad g = 1, \quad \phi = 1, \quad \xi = 1, \\
\text{At } \eta \rightarrow \infty, & \quad H' = 0, \quad g = 0, \quad \theta = 0, \quad \phi = 0, \quad \xi = 0.
\end{align*} \]  

(17)

Influential dimensionless parameters are listed as

\[ \lambda = r^2 \Omega^2, \quad R_e = \frac{r^2 \Omega^1}{v}, \quad M = \frac{\sigma B^2}{\rho \Omega_2}, \quad P_r = \frac{v_f}{\alpha}, \quad St = \frac{l_0}{\Omega_4}, \]

\[ N_b = \frac{D_b \tau}{\alpha^2} (C_0 - C_x), \quad N_f = \frac{\tau D}{\alpha^2} (T_0 - T_x), \]

\[ P_e = \frac{b \nu}{D_m}, \quad Sc = \frac{\nu_f}{\nu_f}, \quad \Omega = \frac{n_x}{(n_x - n_0)}. \]  

(18)

In which \( \lambda \) Carreau fluid, \( M \) Hartman, \( N_f \) thermophoresis motion, \( Sc \) bio-convection Schmidt number, \( St \) stretching rate, \( Sc \) Schmidt number, \( P_r \) Prandtl, \( R_e \) local Reynolds, \( P_e \) bio-convection Peclet, \( N_b \) Brownian motion numbers.

2.3. Gradient velocity and flux numbers

The gradient velocity \( C_f \), Nusselt \( Nu_r \), Sherwood and motile microorganisms \( Nn_r \) numbers are formulated as:

\[ C_f = \frac{\sqrt{\tau_{rr} + \tau_{\theta \theta}}}{\rho f (r \Omega_4)^2}, \quad Nu_r = \frac{rq_1}{k (T_0 - T_\infty)^\alpha}, \]

\[ Sh_r = \frac{rq_2}{D_m (C_0 - C_x)}, \quad Nn_r = \frac{rq_3}{D_m (n_x - n_0)}, \]

\[ \tau_{rr} |_{z=0} = \frac{\partial u_1}{\partial z} \eta_0 \left[ 1 + \frac{(n-1)}{2} \lambda^2 \left( \left( \frac{\partial n}{\partial z} \right)^2 + \left( \frac{\partial n}{\partial z} \right)^2 \right) \right], \]

\[ \tau_{\theta z} |_{z=0} = \frac{\partial v_1}{\partial z} \eta_0 \left[ 1 + \frac{(n-1)}{2} \lambda^2 \left( \left( \frac{\partial n}{\partial z} \right)^4 + \left( \frac{\partial n}{\partial z} \right)^4 \right) + 2 \left( \frac{\partial n}{\partial z} \right)^2 \right], \]  

(21)

\[ q_1 = -k \frac{\partial T}{\partial z} |_{z=0}, \quad q_2 = -D_b \frac{\partial C}{\partial z} |_{z=0}, \quad q_3 = -D_m \frac{\partial n}{\partial z} |_{z=0}. \]

(23)

By using relations, gradient velocity \( C_f \), Nusselt \( Nu_r \), Sherwood and motile microorganisms
\[ R^\frac{1}{2}(C_f) = (g^{\frac{2}{2}} + f^{\frac{2}{2}})^{1/2} \left[ 1 + \lambda_1 R_c \left( g' g' + f' f' \right) \right]^{\frac{3}{2}}, \quad Nu_{r_c} R_c^{-\frac{3}{2}} = -\theta'(0), \]
\[ Sh_{r_c} R_c^{-\frac{3}{2}} = -\phi'(0), \quad Nn_{r_c} R_c^{-\frac{3}{2}} = -\xi'(0). \]  

\[ (Nn_{r_c}) \text{ numbers in view of dimensionless form are} \]
\[ R^\frac{1}{2}(C_f) = (g^{\frac{2}{2}} + f^{\frac{2}{2}})^{1/2} \left[ 1 + \lambda_1 R_c \left( g' g' + f' f' \right) \right]^{\frac{3}{2}}, \quad Nu_{r_c} R_c^{-\frac{3}{2}} = -\theta'(0), \]
\[ Sh_{r_c} R_c^{-\frac{3}{2}} = -\phi'(0), \quad Nn_{r_c} R_c^{-\frac{3}{2}} = -\xi'(0). \]  

### 3. Computed solution analysis and physical description

In present section we will capture role of fluid flow, heat energy and mass specie curves versus different parameters such as Carreau \((\lambda_1)\), Hartmann \((M)\), Prandtl \((P)\), thermophoresis motion \((N_t)\), Brownian motion \((N_b)\), bioconvective Peclet \((P)\) and power-law index \((n)\) numbers. These graphs (Figs. 2-19) are sketched by applying Optimal homotopy analysis method (OHAM) by using Mathematica 10.0. Bar charts (Figs. 18-21) and numerical values in tabular forms (Tables 1-4) of different parameters are analyzed for gradient velocity \((C_f)\), rate of transfer of heat energy \((Nu_r)\), Sherwood number \((Sh_{r_c})\), motile microorganisms \((Nn_{r_c})\).

The impact of \((n)\) on fluid flow is discussed by Fig. 2. As we see that \((n > 1)\) fluid act as a shear thickening behavior due to this fact \(f(\eta)\) decreases. In Fig. 3, for large values of \((M)\), flow declines because of Lorentz force is enhanced results more resistance is generated during fluid flow particles. Similarly Fig. 4 also shows the decreasing behavior for \(f(\eta)\) on escalating the Carreau parameter \((\lambda_1)\). Fig. 5 is indicated that \(H(\eta)\) velocity slows down versus large values of \((n)\) in view of shear thickening fluid. Similar type behavior is found in Fig. 6 against the values of \((M)\) while fluid particles decay because of resistance force results motion fluid particles slows down. Fig. 7 reveals the characteristics of \((\lambda_1)\) on flow \(H(\eta)\). It can be computed that boost in \((\lambda_1)\) is shows flow phenomenon \(H(\eta)\) decreases. The character of \((n)\) on flow \(H(\eta)\) is estimated by Fig. 8. In this figure, the fluid flow increases for \((n)\) in view of \((n < 1)\). The velocity of fluid \(g(\eta)\) decreases because of increasing values of intensity of \((M)\). It is analyzed in Fig. 9. It is examined that fluid flow becomes speed up versus large values of Carreau liquid number that is found in Fig. 10.

Enhancing the value of \((N_b)\), the heat energy profile \(\theta(\eta)\) outcome is revealed in Fig. 11. It
is invented that heat energy curves $\theta(\eta)$ are increased because of the increment of $(N_b)$ results increase motion of fluid particles. Temperature profile $\theta(\eta)$ boosts up gradually due to collision between particles. TBL is reduced by increasing the values of $(P_r)$. Consequently, fluid temperature $\theta(\eta)$ diminishes that is revealed in Fig. 12. The temperature enhances because of large values thermophores is parameter $(N_r)$. In thermophoresis manner, the heat of fluid is reduced results thermal energy is raised. The role of $(N_r)$ on thermal energy curves $\theta(\eta)$ is performed by Fig. 13. Concentration profile of nanoparticles $\varphi(\eta)$ is increased because of enhancement in momentum diffusivity. Behavior of these parameters on $\varphi(\eta)$ is shown on Fig. 14. Brownian motion parameter $(N_b)$ gives the increasing tendency of mass specie profile $\varphi(\eta)$. (see Fig. 15) and concentration becomes decay due to enhancement in random motion of particles and kinetic energy versus higher $(N_b)$. As an increase $(P_e)$, the speed of swimming cells of microorganisms increased. Through this pattern concentration of motile microorganisms $\xi(\eta)$ in a moving disk will be increased. The graphical impact of $(P_e)$ on concentration of microorganisms $\xi(\eta)$ is demonstrated with the help of Fig. 16. The influence of microorganism’s concentration difference parameter on concentration of motile microorganisms is captured by Fig. 17. For enhancing concentration difference $(\Omega)$, enhances concentration of motile microorganisms for ambient fluid but shows decline in surface of $\xi(\eta)$. Fig. 18 shows the bar chart for gradient velocity for different values of Hartman number $(M)$. It reveals increasing tendency regarding large values of $(M)$ due to high resistance between the fluid particles near the surface. Bar charts for both $(Nu_r)$ and $(Sh_r)$ shows opposite effect with various values of active $(N_b)$ that are captured in Fig. 19 and Fig. 20, respectively. For enhancing values of $(N_b)$ and transport of conductive thermal energy heat is generated because of diffusion of nanoparticles that's decline $(Nu_r)$ while it enhance $(Sh_r)$. Similarly bar chart for $(Nn_r)$ is plotted related to active parameter $(P_e)$ in Fig. 21. The numerical results regarding gradient velocity are computed in Table 1. From
this table, it is mentioned that gradient velocity is reduced via large values of for large magnetic number because enhancement of friction resistance. Further, rate of transport of heat is reduced versus values of \((N_b)\) and \((N_r)\) whereas rate of transport of specie is also reduced because of \((N_b)\) and \((N_r)\) which is observed in Table 2. Table 3 discusses the comportment of numerous influential parameters on rate of mass transportation. In Table 4 displayed the decreasing behavior of density of gyrotactic microorganisms for large values of \((P_e)\).

4. Key findings of performed analysis

The aspects in view of transport of thermal energy and solutal analysis for the magnetohydrodynamic flow of Carreau rheology with nanoparticles with motile microorganisms via heated disk has considered. OHAM is applied to capture the analytic solution of fluid flow phenomenon. The characteristics considering impacts of parameters on flow, heat energy, the concentration of motile microorganisms and nanomaterial observations are done graphically. The valuable observation is summarized as:

- Augmenting values of \((n)\) reveals opposite role on fluid flow in view of shear thickening and shear thinning.
- Escalating values of Hartman and Carreau fluid numbers reduces fluid flow.
- Prandtl and Brownian motion numbers show opposite effect on fluid temperature.
- Large values of Prandtl number reduces thermal energy field and connected layer.
- The maximum heat is generated by increasing values of thermophoresis and Brownian motion numbers.
- Enhance numerical values of \(N_b\) results slow down the transport of mass specie.
- The transport of rate of mass species is also increased versus large values of Prandtl number.
- \(\xi(\eta)\) decays for concentration difference variable of microorganisms \((\Omega)\) and enhances for Peclet number \((P_e)\).
- The gradient velocity is raised against enhancing the values of Hartman number.
- Large values \((N_b)\) leads to reduction in rate of heat energy \((\text{Nu}_r)\) and rate of transport of species becomes speed up due to large values of \(N_b\).
- Augmenting values of Peclet number reduces the density number.
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BIOGRAPHIES

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Wael Al-Kouz is a full professor, goal-driven, project-oriented mechanical engineer, a passionate educator, an entrepreneur, and a recognized CFD and energy expert. He combines seventeen years of academic research and teaching experience, with an exceptional industrial track record. Dr. Al-Kouz received his Ph.D. in Mechanical Engineering in 2009 from Worcester Polytechnic Institute (WPI), USA. In 2013 he joined the Mechanical Engineering Department at the German Jordanian University and later served as Head of the Department of Mechatronics Engineering during the period between 2015 and 2016. In 2016, Dr. Al-Kouz was promoted to an associate professor, and in 2021 to a full professor. Currently, he is affiliated with Mechanical Engineering Department, College of Engineering, Prince Muhammad bin Fahd University, Al-Khobar, Saudi Arabia. Dr. Al-Kouz had published close
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FIGURES AND TABLE CAPTIONS

Fig. 1. Fluid flow geometry.

Fig. 2. Character of \( n \) regarding \( f(\eta) \).

Fig. 3. Character of \( M \) regarding \( f(\eta) \).

Fig. 4. Character of \( \lambda_i \) regarding \( f(\eta) \).

Fig. 5. Character of \( n \) regarding \( H(\eta) \).

Fig. 6. Character of \( M \) regarding \( H(\eta) \).

Fig. 7. Character of \( \lambda_i \) regarding \( H(\eta) \).

Fig. 8. Character of \( n \) regarding \( g(\eta) \).

Fig. 9. Character of \( M \) regarding \( g(\eta) \).

Fig. 10. Character of \( \lambda_i \) regarding \( g(\eta) \).

Fig. 11. Character of \( N_b \) regarding \( \theta(\eta) \).

Fig. 12. Character of \( P_r \) regarding \( \theta(\eta) \).

Fig. 13. Character of \( N_t \) regarding \( \theta(\eta) \).

Fig. 14. Character of \( P_r \) regarding \( \varphi(\eta) \).

Fig. 15. Character of \( N_b \) regarding \( \varphi(\eta) \).

Fig. 16. Character of \( P_c \) regarding \( \zeta(\eta) \).

Fig. 17. Character of \( \Omega \) regarding \( \zeta(\eta) \).

Fig. 18. Bar chart of \( C_f(\eta) \).

Fig. 19. Bar chart of \( Nu_r(\eta) \).

Fig. 20. Bar chart of \( Nn_r(\eta) \).

Fig. 21. Bar chart of \( Nn_r(\eta) \).

Table 1. Numerical values of skin friction coefficient \( R^1_c C_f(0) \) when \( St = 0.09, \lambda_i = 0.9, R_c = 1.2, P_r = 6.7, N_t = 0.1, Sc = 1.2, \Omega = 0.2, P_c = 0.7, N_b = 0.3. \)
Table 2. Numerical study of Nusselt number $R_e^{\frac{1}{2}}Nu_r$ when $n = 2.9, \lambda_i = 0.9, R_e = 1.2, \Omega = 0.2, M = 2.02, P_e = 0.5, St = 0.3$.

Table 3. Numerical results in view of Sherwood number $R_e^{\frac{1}{2}}Sh_r$ when $St = 0.09, M = 2.02, \lambda_i = 0.9, R_e = 1.2, \Omega = 0.2, P_e = 0.7$.

Table 4. Numerical study for $R_e^{\frac{1}{2}}Nn_r(0)$ when $St = 0.09, M = 2.02, \lambda_i = 0.9, R_e = 1.2, N_i = 0.1, P_r = 6.7, N_b = 0.3$.

Fig. 1. Fluid flow geometry.
Fig. 2. Character of $n$ regarding $f(\eta)$.

Fig. 3. Character of $M$ regarding $f(\eta)$.

Fig. 4. Character of $\lambda_1$ regarding $f(\eta)$. 
Fig. 5. Character of $n$ regarding $H(\eta)$.

Fig. 6. Character of $M$ regarding $H(\eta)$.

Fig. 7. Character of $\lambda_1$ regarding $H(\eta)$.  

22
Fig. 8. Character of $n$ regarding $g(\eta)$.

Fig. 9. Character of $M$ regarding $g(\eta)$. 
Fig. 10. Character of $\lambda_i$ regarding $g(\eta)$.

Fig. 11. Character of $N_b$ regarding $\theta(\eta)$. 
Fig. 12. Character of $P_r$ regarding $\theta(\eta)$.

Fig. 13. Character of $N_t$ regarding $\theta(\eta)$.

Fig. 14. Character of $P_r$ regarding $\varphi(\eta)$. 
Fig. 15. Character of $N_b$ regarding $\varphi(\eta)$.

Fig. 16. Character of $P_e$ regarding $\zeta(\eta)$.

Fig. 17. Character of $\Omega$ regarding $\zeta(\eta)$.
Fig. 18. Bar chart of $C_f(\eta)$.

Fig. 19. Bar chart of $Nu_r(\eta)$.

Fig. 20. Bar chart of $Nn_r(\eta)$. 
**Fig. 21.** Bar chart of $N_{n_r}(\eta)$.

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Table 1. Numerical values of skin friction coefficient \( R_e^{\frac{1}{3}} C_f(0) \) when \( St = 0.09, \lambda_1 = 0.9, R_e = 1.2, P_r = 6.7, N_r = 0.1, Sc = 1.2, \Omega = 0.2, P_e = 0.7, N_h = 0.3. \)

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Table 2. Numerical study of Nusselt number \( R_e^{\frac{1}{3}} N_u_r \) when \( n = 2.9, \lambda_1 = 0.9, R_e = 1.2, \Omega = 0.2, M = 2.02, P_e = 0.5, St = 0.3. \)
Table 3. Numerical results in view of Sherwood number $R_e^{-\frac{1}{3}}Sh$ when $St = 0.09$, $M = 2.02$, $\lambda_1 = 0.9$, $R_e = 1.2$, $\Omega = 0.2$, $P_e = 0.7$.

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Table 4. Numerical study for $R_e^{-\frac{1}{3}}Nn_t$ (0) when $St = 0.09$, $M = 2.02$, $\lambda_1 = 0.9$, $R_e = 1.2$, $N_t = 0.1$, $P_r = 6.7$, $N_b = 0.3$. 

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