The analysis of bullwhip effect in supply chain based on hedging strategy compared with optimal order quantity strategy

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Abstract

The bullwhip effect is one of the most important problems in the supply chain management. It can cause large inefficiency in supply chain. Although there are many researches about bullwhip effect, few studies have investigated this phenomenon caused by product price fluctuation. In this paper we consider a two-period supply chain consisting of one supplier, one wholesaler and one retailer. The wholesale price may increase greatly in the beginning of second period. If this happens, a large number of end customers will go to purchase the product from retailer. For managing the end customers' demands in the second period, we consider two ordering strategies available to the retailer including optimal order quantity strategy and hedging strategy with call option. For each strategy, we calculate the bullwhip effect ratio for two periods and compare the results. We found that the lower exercise price in hedging strategy compared with the wholesale price in the optimal order quantity strategy must not contribute to extra product purchase. The research provides new insights into how hedging strategy can reduce bullwhip effect.

Keywords: Bullwhip effect; hedging strategy; price fluctuation; panic buying.

1. Introduction

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A supply chain is defined as a system of suppliers, manufacturers, distributors, retailers, and customers in which material, financial, and information flows connect participants in both directions [1]. In the supply chain system, there are various forms of uncertainty. The “bullwhip effect” is short-hand term for a dynamical uncertainly phenomenon in supply chains [2]. The bullwhip effect is one of the most studied phenomena in supply chain management [3]. It is defined as “the amplification of demand (or order) variance, from customer to factory, as demand information passes back through the supply chain” [4]. The bullwhip effect can cause large inefficiencies in supply chains, for example providing poor service to customers, weak demand forecasting, the loss of income and customers, extra inventory capital in the entire chain of warehouses, can be mentioned [5]. In order to control or eliminate the bullwhip effect we must first understand its causes [6]. Lee et al. [7] introduced four basic causes of this phenomenon: (1) wrong demand forecasting; (2) grouping of orders into batches; (3) fluctuation in the products prices; and (4) corporate policies regarding shortage. In this paper, we focus on the bullwhip effect arising from price fluctuations.

The law of demand is one of the most fundamental concepts in economics. The quantity demanded rises as the price of the products falls [8]. The demand law is an inverse relationship between price and quantity demanded; all things being equal. But there are some exceptions. Namely, the quantity demanded increases with price [9]. They include Giffen goods, speculative goods, conspicuous goods, conspicuous necessities, future expectations about prices, demand for necessities and customer’s irrational behaviour [10]. The supply chain members would be faced with panic buying by these exceptions. People usually buy large amounts of product, known as panic buying or consumer hoarding, to avoid future shortage [11], to confer them a sense of security, comfort, momentarily escape, and alleviate stress [12], although the price has increased. They may be behavioral and emotional responses to scarcity [13], in realistic market, consumers make purchase decisions with respect to price, not only in

the current period, but also in past and future periods [14]. The large fluctuations in price were driven by either large shortages or surpluses in capacity. The shortages were exacerbated by panic buying and over ordering that was followed by a sudden drop in demand [15]. Any panic buying could lead to problems such as bullwhip effect [16].

Panic buying has been frequently observed. For example, fears of a sharp rise in the price of toilet paper have prompted panic buying in Taiwan when toilet paper manufactures were expecting a 10 to 30% price rise [17]. In a short interval of time in September 2008, the price of different types of rice in Iran rise suddenly and simultaneously the demand of rice rises [18]. Also, we can refer to panic buying arising from price fluctuation on cotton market in October 2015 [19], cooking oil and flour in Perak in January 2008 [20], wheat in Middle Eastern and North African countries in August 2007 [21], everything from electronics to wine in Russia in December 2014 [22].

Recently, financial hedging has received growing attention in the operations management literature [23]. Hedging is the act of protecting oneself against futures loss [24]. It is defined as “involves taking counterbalancing actions so that, loosely speaking, the future value varies less over the possible states of nature. These counterbalancing actions involve trading financial instruments, including short-selling, futures, options, and other financial derivatives” [25]. The wise use of derivatives for hedging purposes allows for an effective reduction of price risk exposure [26]. Hedging is known as price insurance, risk shifting or risk transference function [24]. In order to protect against various risks derived from production, demand, and price, option contracts have been extensively used in many industries such as fashion apparel industry, food processing industry, and automobile industry [27]. According to a survey of large US nonfinancial firms, approximately 40% of responding firms routinely purchase options or futures contracts in order to hedge price risks [28].

We know the price fluctuation caused by panic buying and then the bullwhip effect. Panic buying is now a frequent occurrence in many countries after the recent COVID-19 pandemic especially. Also, we are witnessing an increase in the number of panic buying for different reasons in Iran. The number of articles related to panic buying suggests increasing attention towards consumer behaviour under extreme conditions. Financial hedging is an approach to the management of price fluctuations. Many researchers have worked on bullwhip effect and hedging independently. The motivation of this study is to propose a new approach based on hedging to control the bullwhip effect. In this paper, we analyse the bullwhip effect based on hedging strategy through call option contracts compared with optimal order quantity strategy. The organization of this paper is as follows: In section 2, we review the related literature. Section 3 illustrates problem description and section 4 presents the proposed model and describes its details. In section 5, bullwhip effect measures will be addressed and in section 6, they will be compared. Section 7 shows numerical analysis. Finally, in section 8, conclusions and managerial insights are discussed.

2. Literature Review

There have been numerous studies addressing the bullwhip effect in recent years. However, a few have investigated the bullwhip effect due to price fluctuation [29]. Also, the use of financial hedging appears to be increasing over the years. In this research, we consider the hedging role on control of bullwhip effect. Hence, in the following lines, we will only review the literature relevant to bullwhip effect and financial hedging.

2.1. Bullwhip effect

Lee et al. [7] show that price fluctuation can cause the bullwhip effect. Moyaux and McBurney [30] present that some of kinds of speculators are able to stabilize the price in a market and

reduce the price fluctuations caused by the bullwhip effect. Ozelkan and Cakanyildirim [31] analyse the impact of procurement price variability in the upstream of a supply chain on the downstream retail prices. Due to the reverse direction of the price variability propagation (compared to the direction of the bullwhip effect in order variability), they referred this behaviour as the reverse bullwhip effect in pricing (RBP).

Rong et al. [16] study how pricing strategies affect the variability of customers' orders. They show when customer behaviour is sufficiently strategic; the customer order process under one-period correction (1PC) pricing strategy is more volatile than the capacity process.

Bolarin et al. [32] analyse the impact of the price fluctuations on the variability of orders along a traditional multilevel supply chain. They show the bullwhip effect appears. Su and Geunes [33] examine the bullwhip effect results from price fluctuations in a two-echelon supply chain with deterministic and price-sensitive demand. They provide numerical evidence that increased system profit can coexist with the bullwhip effect as a result of price promotions.

Ma et al. [34] present a price-sensitive demand model and first-order autoregressive pricing process. Their findings show the retailer should share its customer demand and price information with its upstream businesses. Also, the wholesaler should adopt end-demand and order information, especially when the product price sensitivity coefficient is large or the demand shocks are low.

Wang et al. [35] investigate the bullwhip effect, from a consumer behaviour perspective. They develop insights into the influence of consumer price forecasting behaviour on the bullwhip effect. Their results demonstrated that consumer price forecasting behaviour can reduce the bullwhip effect, especially when the consumer sensitivity to price changes is medium.

Ma et al. [36] offer insights into how the bullwhip effect in two parallel supply chains with interacting price-sensitive demands is affected in contrast to the situation of single product in a serial supply chain.

Ma and Xie [37] focus on the dynamic pricing game of the duopoly air conditioner market with disturbance in demand. Their results also indicate that the bullwhip effect between the order quantity and the actual demand is weakened gradually along with the price adjustment.

Gao et al. [38] investigate the difference in bullwhip effects in online and offline retail supply chains and offer insights into how frequent price discounts in e-commerce influence the bullwhip effect in the online retail supply chain. Tai et al [39] demonstrate that bullwhip effect can be, under conditions, stronger or weaker than the case where the price is not considered.

Gamasae and Fazel Zarandi [40] investigated the impact of joint demand, orders, lead time and pricing decisions on reducing bullwhip effect. Their paper results show significant reduction in bullwhip effect.

Adnan and Ozelkan [41] investigate the behaviour of the bullwhip effect with respect to the price adjustment speed and historical price discount sensitivity. Their results show that controlling price discount sensitivity is useful for supply chain companies.

Zanddizari et al. [42] model the concept of Distance to Loss (DL) by bullwhip effect. This concept is a function of the retailer's selling price, the manufacture's wholesale price, the end item's salvage value, the retailer's expected demand, and the retailer's variance of demand.

Feng et al. [43] investigate the customer's order variability and the firm's profit under several representative heuristic pricing strategies. They find that the bullwhip effect or reverse bullwhip effect can occur as a consequence of supply dramatic shock and adjusting the prices simultaneously.

Qu and Raff [44] show that a decentralized supply chain may be more resilient to demand shocks than a vertically integrated supply chain. Their results present that adjusting the wholesale price is valuable when the bullwhip effect is most likely to occur and potentially most harmful for manufacturer.

2.2. Financial hedging

Some empirical studies such as Allayannis and Weston [45], Carter et al. [46], Bannai et al. [47], Chen et al. [48], Treanor et al. [49], Brusset and Bertrand [50], Luo et al. [51], Swidan and Merkert [52], Merkert and Swidan [53] demonstrated that financial derivatives enhanced firms' financial performance. Alam and Gupta [54] find that firms engaged in hedging compared to non-hedgers have less volatility in firm's value.

Kallapur and Eldenburg [55] examine that operational hedging policies include strategies such enhancing business operation's flexibility, diversifying production lines and varying the combination of variable and fixed costs. Borensztein et al. [56] use a dynamic optimization model to quantify the potential welfare gains of hedging against commodity price risk for commodity-exporting countries. They show that hedging enhances domestic welfare through reducing export income volatility and decreasing the country's need to hold precautionary reserves.

Liu et al. [57] offer circumstances under which supply chain coordination could be reached. They provide practical insights to the manufacturer and retailer. Tauser and Cajka [58] focus on selected aspects of risk management in agricultural business and compared different hedging methods which are relevant for managing the commodity risk associated with agricultural production.

Turcic et al. [28] has greatly improved researcher's understanding of why and how individual firms should hedge. Yang et al. [59] introduce three coordinating option contracts led by the supplier to reduce the retailer's risk, where the call option contract can reduce the shortage risk, the put option contract can reduce the inventory risk and bidirectional option contract can reduce the bilateral risk.

Park et al. [60] examine a firm's production planning, pricing and financial hedging decisions under exchange-rate and demand uncertainty.

Kouvelis et al. [61] studies the hedging of cash-flow risks in a bilateral supply chain of a supplier and manufacturer. They characterize the interaction of hedging decisions of the supply chain partners and the associated effects of market conditions, production efficiencies, and cash-flow correlation.

Kouvelis et al [62] consider a firm purchasing a storable commodity from a spot market with price fluctuations and access to an associated financial derivatives market. In these circumstance, they survey two types of hedging instruments and compare their performances.

Liu and Wang [63] present a network equilibrium model for supply chain networks with strategic financial hedging. They consider multiple competing firms. The firms are exposed to commodity price risk and exchange rate risk and they can use futures contracts to hedge the risks.

Hu et al. [64] build a simple theoretical model to compare the implications of fuel financial hedge and operational fuel efficiency improvement on airlines' expected profit. They find that financial hedge is more efficient in reducing airlines' profit volatility/risk exposure, while operational improvement would generate a higher expected profit level when its effectiveness is sufficiently high. Hainaut [65] studies hedging strategies of crop harvest incomes with futures and options on indexes of cumulated average temperatures.

March et al. [66] investigate a supply chain in which the vendor can adopt two financial approaches as means for hedging stocks in order to reduce the commodity risk related to the high price fluctuations.

Although in recent years, the number of studies that consider supply chain management as well as financial fields is increasing, none of the published articles has examined the effect of hedging on the bullwhip effect. In fact, what is new to the present study is identifying what will happen to bullwhip effect ratio if the hedging strategy is applied. The current research has

focused on hedging strategy compared with optimal order quantity strategy for calculating bullwhip effect ratio in two echelon supply chain.

3. Problem Description

We consider a two-period supply chain [11, 67] consisting of one supplier, one wholesaler and one retailer. The supplier manufactures a single product selling to the wholesaler. The wholesaler sells the product to the retailer then the retailer sells it to end consumers. We assume that there is a large population of end consumers in market. Also, we presume that the retailer will receive the order at the beginning of each period and the lead time is zero. In the first period, the product price is constant and at the beginning of second period, the product price may increase significantly, which is reasonable in many situations. In each period, the price is independent and identically distributed (i.i.d.) from a normal distribution with average μ and variance σ^2 . If the product price increases greatly, a large number of end customers will purchase the product from the retailer. This is contrary to the law of demand and the reasons of this event were mentioned in the Introduction section. Therefore, demand is a dependent variable on the price of the product.

We assume that in the first period the initial inventory level is zero and the retailer orders the optimal order quantity from the wholesaler. At the end of the first period, the leftover products are carried over to the second period for sale and incur a holding cost. For managing the end customers' demands in the second period, there are two ordering strategies available to the retailer, which are optimal order quantity strategy and hedging strategy. The retailer uses the call option contract for long hedging strategy. This contract is concluded between the wholesaler and the retailer. We suppose shortage is not allowed and in the second period, the retailer can buy additional units from an emergency source at a higher price.

This study aimed to address the following research question:

- "what are the results of hedging on the bullwhip effect ratio?"

According to the conditions listed above, for each period, we will calculate the retailer's optimal order with optimal order quantity strategy and hedging strategy and for these strategies the retailer's bullwhip effect is measured by the ratio of the order quantity variance, encountered by the wholesaler, to the demand variance, faced by retailer. The ratio values are compared to each other. This ratio has been employed by many researchers [39, 68, 69, 70, 71]. We also consider the retailer to be risk neutral. When the retailer is risk neutral, it chooses to maximize its own expected profit [57].

4. The Proposed Model

In this section, the retailer's optimal order quantities are determined by the optimal order quantity and the hedging strategy.

4.1. Notations

To develop the model, notations are summarized as follows.

- **Sets**

- $t = \{1, 2\}$: Time periods; ($t = 1$ shows the first period and $t = 2$ presents the second period)
- $i = \{1, 2\}$: Types of price changes; ($i = 1$ presents the product price is constant or the small change price per unit is occurred and $i = 2$ shows significant increase in price per unit is happened), (for $t = 1, i \neq 2$)
- $j = \{1, 2\}$: Types of retailer's ordering decisions; ($j = 1$ shows the retailer only uses an optimal order quantity strategy and $j = 2$ shows the retailer uses hedging strategy), (for $t = 1, j \neq 2$)

- **Decision Variables**

- q_t^{ij} : The retailer's order quantity in the period t under decision j and price change i
- $(q_t^{ij})^*$: The retailer's optimal order quantity in the period t under decision j and price change i

Also, by assumptions explained in the text, q_1^{12} , q_1^{21} and q_1^{22} are not defined.

- **Parameters**

- p_t : The spot price per unit in the period t
- p_2^k : The exercise price per unit in the second period
- θ_2 : The significant increase in the wholesale price per unit in the second period ($\theta_2 > 0$)
- ε_2 : The small change in the wholesale price per unit in the second period, (ε_2 can be positive or negative or zero)
- w_t : The wholesale price per unit in not-hedging in the period t
- φ : The difference between the wholesale price and the exercise price per unit in the second period
- m_t : The retailer's fixed percentage profit margin in the period t ($m_t > 0$)
- n_2 : The emergency purchasing price per unit by the retailer in the second period
- c_{o2} : The option price per unit in the second period
- h_t : The holding cost per unit in the period t
- c_t : The order cost per unit in the period t
- μ : The average of the product price in the period t
- σ^2 : The variance of the product price in the period t

- σ : The standard deviation of the product price in the period t
- d_1 : The product demand in the first period
- μ_{d_1} : The end customer's average demand in the first period
- $\sigma_{d_1}^2$: The variance of the end customer's demand in the first period
- σ_{d_1} : The standard deviation of the end customer's demand in the first period
- d_2^i : The product demand in the second period under price change i
- $\mu_{d_2^i}$: The end customer's average demand in the second period under price change i
- $\sigma_{d_2^i}^2$: The variance of the end customer's demand in the second period under price change i
- $\sigma_{d_2^i}$: The standard deviation of the end customer's demand in second period under price change i
- r : Consumer sensitivity to price increases in the second period
- a : Basic market demand
- b : The demand curve slope
- $f(x)$: The probability distribution function of the end customer demand to the retailer
- $F(x)$: The cumulated distribution function of the end customer demand to retailer
- $S(q_t^{ij})$: The retailer's expected sales in the period t under decision j and price change i
- $I(q_t^{ij})$: The expected leftover inventory in the period t under decision j and price change i

- $H(q_t^{ij})$: The expected order quantity to the emergency source under decision j and price change i
- $\pi(q_t^{ij})$: The retailer's expected profit in the period t under decision j and price change i
- $\bar{q}_{q_1^{ij}, q_2^{ij}}$: The Total average retailer's order
- $BWE_{q_1^{ij}, q_2^{ij}}$: Bullwhip effect for the retailer's optimal order quantities under decision j and price change i , ($i \neq 1$).
- $p_2^k < w_2 < n_2 < p_2$

4.2. The relation between the wholesale price and the retail price

The wholesaler is selling a product to the retailer at w_t and the retailer is using a fixed percentage profit margin ($m_t > 0$) to identify p_t [72]. The relation between p_t and w_t will be as formula (1).

$$p_t = (1 + m_t) w_t \quad (1)$$

4.3. Types of price Changes

In the first period, w_1 is fixed. We have formula (2):

$$p_1 = (1 + m_1) w_1 \quad (2)$$

At the beginning of the second period, the product price is constant or the small change price per unit (ε_2) or significant increase (θ_2) in price per unit is occurred. Therefore, the relation between w_1 and w_2 will be as formula (3) and formula (4), respectively.

$$w_2 = w_1 + \varepsilon_2 \quad (3)$$

$$w_2 = w_1 + \theta_2 \quad (4)$$

By substituting formula (3) and formula (4) into formula (1), we will have formula (5) and formula (6) for second period:

$$p_2 = (1 + m_2)(w_1 + \varepsilon_2) \quad (5)$$

$$p_2 = (1 + m_2)(w_1 + \theta_2) \quad (6)$$

4.4. Types of demand model

In this paper, the end customers' demand is considered as the product price function and is shown with linear function. For the first period, we consider the linear demand model as formula (7):

$$d_1(p_1) = a - bp_1 \quad (7)$$

For the second period with constant product price or small change price per unit, we consider the linear demand model as formula (8):

$$d_2^1(p_2) = a - bp_2 \quad (8)$$

For the second period, when significant increase in price per unit is happened, the linear demand model can be written as formula (9) [35]:

$$d_2^2(p_2) = (a - bp_2) + rb(p_2 - p_1), r > 1 \quad (9)$$

In the formula (9), the first term on the right-hand side of the equation expresses the underlying demand and is a decreasing function of p_2 , and the second term represents the impact of price behaviour on the demand. p_2 is higher than p_1 , therefore, the customers buy more to reduce their future needs.

4.5. The retailer's first period order quantity

The retailer's first period expected profit is as formula (10):

$$\pi(q_1^{11}) = p_1 S(q_1^{11}) - w_1 q_1^{11} - h_1 q_1^{11} - c_1 q_1^{11} \quad (10)$$

The linear demand model is as formula (11):

$$d_1 = a - b p_1 \quad (11)$$

The inverse demand equation will be as formula (12).

$$p_1 = \frac{1}{b}(a - d_1) \quad (12)$$

Proposition 1. We substitute formula (12) in formula (10) and solve it for q_1^{11} . The retailer's first period optimal order quantity is given by:

$$(q_1^{11})^* = \mu_{d_1} + \sqrt{2\pi}\sigma_{d_1} \left[\frac{1}{2} - \frac{w_1 + h_1 + c_1}{\frac{1}{b}(a - d_1)} \right] \quad (13)$$

Proof: See Appendix 1.

4.6. The retailer's second period order quantity

At the beginning of second period, for purchasing product from wholesaler, the retailer will face one of two options about the product price:

- Constant price or small change price per unit (ε_2)
- Significant increase in price per unit (θ_2).

The retailer can use the optimal order quantity strategy or hedging strategy. The Figure 1 shows product price changes and the retailer ordering decisions in the second period.

[Figure 1]

Therefore, there are four scenarios. Table 1 shows these scenarios.

[Table 1]

In the second period, the retailer's ordering process is as shown in Figure 2.

[Figure 2]

For each scenario, we calculate the retailer's optimal order quantity.

- **Scenario 1**

The retailer's expected profit is as formula (14):

$$\pi(q_2^{11}) = p_2 S(q_2^{11}) - w_2 q_2^{11} - c_2 q_2^{11} - h_2 (q_2^{11} + I(q_1^{11})) - n_2 H(q_2^{11}) \quad (14)$$

The linear demand model is as formula (15):

$$d_2^1 = a - bp_2 \quad (15)$$

The inverse demand equation will be as formula (16).

$$p_2 = \frac{1}{b}(a - d_2^1) \quad (16)$$

Proposition 2. We substitute formula (16) in formula (14) and solve it for q_2^{11} . The retailer's optimal order quantity is given by:

$$(q_2^{11})^* = \mu_{d_2^1} + \sqrt{2\pi}\sigma_{d_2^1} \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\frac{1}{b}(a - d_2^1) + n_2} \right] \quad (17)$$

Proof: See Appendix 2.

- **Scenario 2**

The retailer's expected profit is as formula (18):

$$\pi(q_2^{21}) = p_2 S(q_2^{21}) - w_2 q_2^{21} - c_2 q_2^{21} - h_2 (q_2^{21} + I(q_1^{11})) - n_2 H(q_2^{21}) \quad (18)$$

The linear demand model is as formula (19):

$$d_2^2 = (a - bp_2) + rb(p_2 - p_1) \quad (19)$$

By substituting formula (2) and formula (6) into formula (19), we have formula (20):

$$d_2^2 = (a - bp_2) + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb \quad (20)$$

The inverse demand equation will be as formula (21):

$$p_2 = \frac{1}{b}(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2) \quad (21)$$

Proposition 3. We substitute formula (21) for p_2 in formula (18) and solve it for q_2^{21} .

The retailer's optimal order quantity is given by:

$$\left(q_2^{21}\right)^* = \mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2} \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] \quad (22)$$

Proof: See Appendix 3.

- Scenario 3

The retailer's expected profit is as formula (23):

$$\pi(q_2^{12}) = p_2 S(q_2^{12}) - w_2 q_2^{12} - c_2 q_2^{12} - c_{o2} q_2^{12} - h_2 (q_2^{12} + I(q_1^{11})) - n_2 H(q_2^{12}) \quad (23)$$

The linear demand model is as formula (24):

$$d_2^1 = a - bp_2 \quad (24)$$

The inverse demand equation will be as formula (25):

$$p_2 = \frac{1}{b} (a - d_2^1) \quad (25)$$

Proposition 4. We substitute formula (25) for p_2 in formula (23) and solve it for q_2^{12} .

The retailer's optimal order quantity is given by:

$$\left(q_2^{12}\right)^* = \mu_{d_2^1} + \sqrt{2\pi}\sigma_{d_2^1} \left[\frac{1}{2} - \frac{w_2 + c_2 + c_{o2} + h_2}{\frac{1}{b} (a - d_2^1) + n_2} \right] \quad (26)$$

Proof: See Appendix 4.

- Scenario 4

The retailer's expected profit is as formula (27):

$$\pi(q_2^{22}) = p_2 S(q_2^{22}) - p_2^k q_2^{22} - c_{o2} q_2^{22} - h_2 (q_2^{22} + I(q_1^{11})) - n_2 H(q_2^{22}) \quad (27)$$

The linear demand model is as formula (28):

$$d_2^2 = (a - bp_2) + rb(p_2 - p_1) \quad (28)$$

By substituting formula (2) and formula (6) into formula (28), we have formula (29):

$$d_2^2 = (a - bp_2) + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb \quad (29)$$

The inverse demand equation will be as formula (30):

$$p_2 = \frac{1}{b}(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2) \quad (30)$$

Also, The relation between p_2^k and w_2 will be as formula (31).

$$w_2 = p_2^k + \varphi \quad (31)$$

Proposition 5. We substitute formula (30) for p_2 in formula (27) and solve it for q_2^{22} .

The retailer's optimal order quantity is given by:

$$(q_2^{22})^* = \mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2} \left[\frac{1}{2} - \frac{p_2^k + c_{o2} + h_2}{\frac{1}{b}(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2) + n_2} \right] \quad (32)$$

Proof: See Appendix 5.

Table 2 shows the retailer's optimal quantities for each scenario in the second period.

[Table 2]

5. Bullwhip Effect Measures

In the previous section, the retailer's optimal order quantity under the optimal order quantity strategy and hedging strategy were calculated. In this section, we develop expressions for bullwhip effect under the two strategies. The bullwhip effect ratio will be calculated according to q_1^{11} and q_2^{21} . We then repeat the process for q_1^{11} and q_2^{22} . Next, the results will be compared to each other. It should be noted that for second period, we consider only scenario 2 and scenario 4 because the product price increase occurred in these scenarios.

To quantify the bullwhip effect, we can use the formula (33) that σ_o^2 shows variance of retailer order quantity, and σ_D^2 is variance of end customer demand.

$$BWE = \frac{\sigma_o^2}{\sigma_D^2} \quad (33)$$

Based on the preceding assumption, we can conclude that:

$$Cov(d_1, d_2^i) = 0 \quad (34)$$

Before calculating the bullwhip effect ratio, we have the formula (35) and formula (36) as follow:

$$\bar{q}_{q_1^{ij}, q_2^{ij}} = \frac{1}{2}(q_1^{ij} + q_2^{ij}) \quad (35)$$

$$\sigma_o^2 = \frac{1}{T-1} \sum_{t=1}^T (q_t^{ij} - \bar{q}_{q_1^{ij}, q_2^{ij}})^2 \quad (36)$$

The variance of the market demand during the two periods can be written as formula (37):

$$\sigma_D^2 = Var(d_1, d_2^2) = \sigma_{d_1}^2 + \sigma_{d_2^2}^2 + 2Cov(d_1, d_2^2) = \sigma_{d_1}^2 + \sigma_{d_2^2}^2 \quad (37)$$

According formula (11), we have formula (38) and formula (39) as follow:

$$\sigma_{d_1}^2 = b^2 \sigma^2 \quad (38)$$

$$\sigma_{d_1} = b\sigma \quad (39)$$

According formula (19) and formula (28), we have formula (40) and formula (41) as follow:

$$\sigma_{d_2^2}^2 = (b^2 + 2r^2b^2) \sigma^2 \quad (40)$$

$$\sigma_{d_2^2} = \sqrt{(b^2 + 2r^2b^2)} \sigma \quad (41)$$

Therefore, according to formula (38) and formula (40), we have formula (42) as follow:

$$\sigma_D^2 = (2b^2 + 2r^2b^2) \sigma^2 \quad (42)$$

Also, according to formula (11), formula (15), formula (20) and formula (29), we have formula (43) and formula (44) as follow:

$$\mu_{d_1} = \mu_{d_2^1} = a - b\mu \quad (43)$$

$$\mu_{d_2^2} = a - b\mu + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb \quad (44)$$

5.1. Bullwhip effect ratio for optimal order quantity strategy

With substituting formula (35) and formula (36) in formula (33), we have formula (45)

as follow:

$$BWE_{q_1^{11}, q_2^{21}} = \frac{\sigma_o^2}{\sigma_D^2} = \frac{\left(q_1^{11} - \bar{q}_{q_1^{11}, q_2^{21}}\right)^2 + \left(q_2^{21} - \bar{q}_{q_1^{11}, q_2^{21}}\right)^2}{\sigma_D^2} \quad (45)$$

Theorem 1. The bullwhip effect for optimal order quantity strategy is as follow:

$$BWE_{q_1^{11}, q_2^{21}} = \frac{\frac{1}{2} \left[\left[(m_2 - m_1)w_1 + (1+m_2)\theta_2 \right] rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2}\sigma \right] \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1+m_2)\theta_2 \right] rb - d_2^2 \right) + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{\frac{1}{b} (a - d_1)} \right] \right]^2}{(2b^2 + 2r^2b^2)\sigma^2} \quad (46)$$

Proof: See Appendix 6.

Proposition 6. Bullwhip effect exists (i.e. $BWE_{q_1^{11}, q_2^{21}} > 1$) if formula (47) hold.

$$\begin{aligned} & \left[(m_2 - m_1)w_1 + (1+m_2)\theta_2 \right] rb + 1.25 \left[\sqrt{b^2 + 2r^2b^2}\sigma \right] + 2.5 [b\sigma] \left[\frac{w_1 + h_1 + c_1}{\frac{1}{b} (a - d_1)} \right] > \\ & 2.5 \left[\sqrt{b^2 + 2r^2b^2}\sigma \right] \left[\frac{w_2 + h_2 + c_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1+m_2)\theta_2 \right] rb - d_2^2 \right) + n_2 \right]} \right] + 1.25 [b\sigma] + \sqrt{2(b^2 + 2r^2b^2)}\sigma \end{aligned} \quad (47)$$

Proof. See Appendix 7.

5.2. Bullwhip effect ratio for hedging strategy

According to formula (35) and formula (36), we have formula (48) as follow:

$$BWE_{q_1^{11}, q_2^{22}} = \frac{\sigma_o^2}{\sigma_D^2} = \frac{\left(q_1^{11} - \bar{q}_{q_1^{11}, q_2^{22}}\right)^2 + \left(q_2^{22} - \bar{q}_{q_1^{11}, q_2^{22}}\right)^2}{\sigma_D^2} \quad (48)$$

Theorem 2. The bullwhip effect for hedging strategy is as follow:

$$BWE_{q_1^{11}, q_2^{22}} = \frac{\frac{1}{2} \left[\left[(m_2 - m_1)w_1 + (1+m_2)\theta_2 \right] rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2}\sigma \right] \left[\frac{1}{2} - \frac{p_2^k + c_o + h_2}{\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1+m_2)\theta_2 \right] rb - d_2^2 \right) + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{\frac{1}{b} (a - d_1)} \right] \right]^2}{(2b^2 + 2r^2b^2)\sigma^2} \quad (49)$$

Proof: See Appendix 8.

Proposition 7. Bullwhip effect exists (i.e. $BWE_{q_1^{11}, q_2^{22}} > 1$) if formula (50) hold.

$$\begin{aligned}
& \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + 1.25 \left[\sqrt{b^2 + 2r^2b^2}\sigma \right] + 2.5 \left[b\sigma \right] \left[\frac{w_1 + h_1 + c_1}{\frac{1}{b}(a - d_1)} \right] > \\
& 2.5 \left[\sqrt{b^2 + 2r^2b^2}\sigma \right] \left[\frac{p_2^k + h_2 + c_{o2}}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] + 1.25 \left[b\sigma \right] + \sqrt{2(b^2 + 2r^2b^2)}\sigma
\end{aligned} \tag{50}$$

Proof. See Appendix 9.

6. Comparison of the bullwhip effect ratios under different strategies

In this section, we compare the bullwhip effect ratios for the optimal order quantity strategy and hedging strategy. To compare the bullwhip effect under the two retailer's ordering decisions in the second period, we deduce the following theorem:

Theorem 3. Let $BWE_{q_1^{11}, q_2^{22}}$ ($BWE_{q_1^{11}, q_2^{21}}$) be the bullwhip effect under hedging strategy (optimal order quantity strategy), assuming that the product price for two periods is i.i.d. from normal distribution. If we have $c_{o2} > c_2 + \varphi$, Then

$$BWE_{q_1^{11}, q_2^{22}} < BWE_{q_1^{11}, q_2^{21}} \tag{51}$$

Proof. See Appendix 10.

From Theorem 3, we know that formula (51) depends on the following three parameters: the option price, c_{o2} ; the order cost in the second period, c_2 ; and the difference between the wholesale price and the exercise price in the second period, φ . We can explain Theorem 3 as follows. If the retailer expects that the wholesale price will increase greatly in the beginning of second period and it makes people rush to buy the product, the retailer could hedge against the price fluctuations with call option. But the lower exercise price (p_2^k) compared with the wholesale price (w_2) must not contribute to an extra product purchase by retailer. According to $c_{o2} > c_2 + \varphi$, formula (22) and formula (32), the retailer's optimal order with hedging

strategy will be less than the retailer's optimal order with optimal order quantity strategy. From the theoretical perspective, it has been pointed out that for validity of formula (51), as $c_2 + \varphi$ increases c_{o2} must increase. Totally, under problem description in section 3, when the retailer's optimal order under hedging strategy is less than the retailer's optimal order under optimal order quantity strategy, the bullwhip effect under hedging strategy ($BWE_{q_1^{11}, q_2^{22}}$) is less than the bullwhip effect under optimal order quantity strategy ($BWE_{q_1^{11}, q_2^{21}}$).

7. Numerical analysis

In the preceding sections, we have calculated the retailer's optimal orders and bullwhip effect measures under problem description in section 3 and then we have compared the bullwhip effect ratios for the optimal order quantity strategy and hedging strategy. In this section, we provide numerical experiments to show the results and illustrate the impact of changing the value of parameters on the bullwhip effect measures. This section consists of two parts. First, in section 7.1, we compare d_1, d_2^1 and d_2^2 and show why we considered $r > 1$. Subsequently, in section 7.2, we contrast $BWE_{q_1^{11}, q_2^{22}}$ and $BWE_{q_1^{11}, q_2^{21}}$. We survey the impacts of changing option price (c_{o2}), difference between the wholesale price and the exercise price (φ), customer sensitivity to price increase (r), demand curve slope (b), the significant increase in the wholesale price (θ_2) and the standard deviation of the product price (σ) on $BWE_{q_1^{11}, q_2^{21}}$ and $BWE_{q_1^{11}, q_2^{22}}$ in subsection 7.2.1, subsection 7.2.2, subsection 7.2.3, subsection 7.2.4, subsection 7.2.5 and subsection 7.2.6 respectively. Also, we fixed $a = 200$, $m_1 = 0.25$, $m_2 = 0.3$, $w_1 = 30$, $p_2^k = 29$, $c_1 = 3$, $c_2 = 4$, $h_1 = 1.8$, $h_2 = 2$, $n_2 = 43$ and $\mu = 30$.

7.1. Comparison between d_1, d_2^1 and d_2^2

The end customers' demand in the first period (d_1) and the second period (d_2^1 and d_2^2) are shown in Figure 3. We varied the parameters r and b over the values $r \in \{0, 0.1, 0.2, 0.3, 0.4, \dots, 2\}$ and $b \in \{1, 2, 3, 4, 5, 6\}$. We computed the corresponding demands using formula (11), formula (15) or formula (24) and formula (19) or formula (28). d_2^1 (second period demand in scenario 1 and 3) is slightly smaller than d_1 because the changes of product price was not significant but based on formula (5), the retailer's fixed percentage profit margin in period 2 (m_2) is bigger than period 1 (m_1).

For $r > 1$, d_2^2 (demands in scenario 2 and 4) is bigger than d_1 and d_2^1 which shows that end customer rush to buy due to a significant increase in product price. For $0 < r < 1$, d_2^2 is smaller than d_1 and d_2^1 . For $r = 1$, d_2^2 is equal to d_2^1 . As a result, we only consider $r > 1$.

[Figure 3]

7.2. Comparison between $BWE_{q_1^{11}, q_2^{21}}$ and $BWE_{q_1^{11}, q_2^{22}}$

From Appendix 10, we know $\Delta BWE = BWE_{q_1^{11}, q_2^{22}} - BWE_{q_1^{11}, q_2^{21}}$. In this section, we survey the effect of parameter values changes on ΔBWE .

7.2.1. Option price (c_{o2})

Figure 4 illustrate how ΔBWE changes with c_{o2} for different values of σ when $b = 4$, $r = 1.5$, $\theta_2 = 6$. From Figure 4, it can be observed that for $c_{o2} = 11$, we have $\Delta BWE = 0$. By increasing the option price, c_{o2} , from 11 to 20, ΔBWE will decrease because ΔBWE are negatively correlated with c_{o2} . Also, for $c_{o2} \in \{11, 12, 13, \dots, 20\}$, when the standard deviation of the price rises from 0.5 to 1.5, ΔBWE will continue to

be negative but its value rises. For example, for $c_{o2} = 18$ and $\sigma = 0.5, 1, 1.5$, the value of ΔBWE is -2.0216, -1.0408 and -0.7138 respectively. This means that the lower standard deviation of product price, the hedging strategy is better than the optimal order strategy to decrease bullwhip effect.

[Figure 4]

7.2.2. Difference between the wholesale price and the exercise price (φ)

Figure 5 illustrate how ΔBWE changes with φ for different values of σ when $b = 4$, $r = 1.5$, $\theta_2 = 6$ and $c_{o2} = 20$. We consider $p_2^k \in \{21, 22, 23, 24, \dots, 29\}$, therefore, according formula (31), we have $\varphi \in \{7, 8, \dots, 15\}$. For this value, ΔBWE is negative but it increases while keeping c_{o2} constant. Because with increasing φ , the value of $c_{o2} - (c + \varphi)$ decrease.

Also, for $\varphi \in \{7, 8, \dots, 15\}$, when the standard deviation of the price rises from 0.5 to 1.5, ΔBWE will continue to be negative and its value rises. This means that the lower standard deviation of product price, the hedging strategy is better than the optimal order strategy to decrease bullwhip effect.

[Figure 5]

7.2.3. Consumer sensitivity to price increases (r)

Figure 6 illustrate how ΔBWE changes with r for different values of σ when $b = 4$, $\theta_2 = 6$ and $c_{o2} = 15$. We consider $r \in \{1, 1.1, 1.2, \dots, 2\}$. By raising r , the value of ΔBWE decrease. This means when consumer sensitivity to price increases, the hedging strategy is better than optimal order quantity strategy to decrease bullwhip effect. By increasing standard deviation of product price, σ , and consumer sensitivity to price, r

, although ΔBWE will continue to be negative but its value rises. Namely, by increasing the product price fluctuation and end customer rushing, the hedging strategy is better than the optimal order quantity strategy but it will be less effective.

[Figure 6]

7.2.4. The demand curve slope (b)

Figure 7 illustrate how ΔBWE changes with b for different values of σ when $r=1.5$, $\theta_2=6$ and $c_{o2}=15$. We consider $b \in \{1,2,3,\dots,6\}$. Because from formula (12), formula (21) and formula (30), we know that the end customer demand is correlated with the slope of demand curve, b , negatively. This means that b increases, the number of end customer demand decreases. By raising b , the value of ΔBWE decrease. Also, by raising the standard deviation, σ , and the slope of demand curve, b , the value of ΔBWE is negative, but its values increase. These show that the hedging strategy is better than optimal order strategy to decrease bullwhip effect.

[Figure 7]

7.2.5. The significant increase in the wholesale price (θ_2)

Figure 8 illustrate how ΔBWE changes with θ_2 for different values of σ when $b=4$, $r=1.5$, and $c_{o2}=15$. We consider $\theta_2 \in \{1,2,3,\dots,10\}$. w_2 varies directly as θ_2 according to formula (4). When θ_2 increases, w_2 increases and φ also rises. While keeping c_{o2} constant and based on Theorem 3, the value of $c_{o2} - (\theta_2 + \varphi)$ decreases and the difference between $BWE_{q_1^{11}, q_2^{21}}$ and $BWE_{q_1^{11}, q_2^{22}}$ is reduced. As a result, the value of ΔBWE rises. For $\theta_2=10$, we have $c_{o2} = (\theta_2 + \varphi)$ then $\Delta BWE=0$. Figure 8 presents

it. Also, by rising the standard deviation, σ , the value of ΔBWE is negative, but its values increase.

[Figure 8]

7.2.6. The standard deviation of the product price (σ)

Figure 9 illustrate how ΔBWE changes with σ when $b=4$, $r=1.5$, $\theta_2=6$ and $c_{o2}=15$. We consider $\sigma \in \{0.5, 0.6, \dots, 1.5\}$. By increasing σ , the value of ΔBWE increases. This means that by raising the product price standard deviation, the hedging strategy is better than optimal order quantity strategy to decrease bullwhip effect but it will be less effective.

[Figure 9]

8. Conclusions

This paper introduces the hedging strategy for controlling bullwhip effect and compares it to optimal order quantity strategy. We derive analytical expressions for the bullwhip effect ratio under two strategies, the hedging strategy, and the optimal order quantity strategy. In the following section, we present the results. These results provide some useful managerial insights on the implementation of these strategies.

- (1) When the product price fluctuations cause panic buying and it makes the bullwhip effect, the hedging strategy can help to control it. The retailer may use a long hedge to fix the good price and manage the bullwhip effect.
- (2) If the option price (c_{o2}) is bigger than the sum of the order cost and the difference between the wholesale price and the exercise price ($c + \varphi$), the hedging strategy is better than the optimal order quantity strategy for controlling the bullwhip effect. So in this case, purchasing through a hedging strategy will be more expensive than buying

through an optimal order quantity strategy. It prevents the retailer from buying too much of the product. If the retailer buys over a certain amount of product just because it is cheap, and the intensity of customer demand decreases, the unsold products are carried over the next periods. This increases the bullwhip effect.

- (3) The retailer's ordering behaviour is important when the product price has volatility. In this circumstance, the retailer buys its product by call option contract cheaper than other method and he should be careful about the order quantities. The lower exercise price in hedging strategy compared with the wholesale price in the optimal order quantity strategy must not contribute to the extra product purchase. A large amount of product may protect the retailer against the high fluctuations in the end customers' demand but increases the bullwhip effect ratio. Therefore, it is important to decide about ordering strategy and the order quantities when the product price has fluctuations or we expect it to be.
- (4) The product price is one of the important factors that the end customers pay attention to it. Also, the retailers consider the price as the criterion for the sales strategy. Accurate price forecasting and predicting end customer behavior can help the retailer to choose the right ordering strategy. If the retailer correctly forecasts price increasing, the use of hedging strategy could help to control the bullwhip effect considerably.
- (5) The price standard deviation is a statistical expression that gives an indicator of price fluctuations in the market. High price fluctuations can lead to unstable markets and the end customers' emotional decisions. The reason for these decisions is fear and greed. The high price standard deviation means high price volatility. In this situation, while $(c_{o2} > c + \varphi)$ is established, for controlling the bullwhip effect, the hedging strategy is better than the optimal order quantity strategy but its effectiveness decreases.
- (6) The bullwhip effect is not completely eliminated by hedging strategy.

A summary of our findings indicates that bullwhip effect reduction is important when there are price fluctuations in markets and companies can use hedging strategy to decrease the bullwhip effect.

This paper recommends several future directions to add our understanding of the influence of hedging strategy on the bullwhip effect. First, our model considers only linear demand function; the other demand functions require further study. Second, this paper assesses only the optimal order quantity strategy comparing with hedging strategy; the other ordering strategy can be considered. Finally, extending the two-period supply chain to multi-period chains would be another contribution for the future studies.

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Figure and Table captions:

Figure 1. retailer's ordering decisions in the second period, **(a)** retailer uses an optimal order quantity strategy (O_2), **(b)** retailer uses hedging strategy (C_2).

Figure 2. Retailer's ordering process in the second period.

Figure 3. The influence of r and b on d_1 , d_2^1 and d_2^2 when $\theta_2 = 6$.

Figure 4. The influence of c_{o2} on ΔBWE when $b = 4$, $r = 1.5$, $\theta_2 = 6$.

Figure 5. The influence of φ on ΔBWE when $b = 4$, $r = 1.5$, $\theta_2 = 6$, $c_{o2} = 20$.

Figure 6. The influence of r on ΔBWE when $b = 4$, $\theta_2 = 6$, $c_{o2} = 15$.

Figure 7. The influence of b on ΔBWE when $c_{o2} = 15$, $\theta_2 = 6$, $r = 1.5$.

Figure 8. The influence of θ_2 on ΔBWE when $b = 4$, $r = 1.5$, $c_{o2} = 15$.

Figure 9. The influence of σ on ΔBWE when $b = 4$, $r = 1.5$, $\theta_2 = 6$, $c_{o2} = 15$.

Table 1. Description of scenarios

Table 2. The retailer's optimal order quantity in the second period under decision j and price change i

Figure 1.

Figure 2.

Figure 3.

Figure 4.

Figure 5.

Figure 6.

Figure 7.

Figure 8.

Figure 9.**Table 1.**

Scenarios	Ordering strategy	Price changes
1	Optimal order quantity	$w_2 = w_1 + \varepsilon_2$
2		$w_2 = w_1 + \theta_2$
3	Hedging	$w_2 = w_1 + \varepsilon_2$
4		$w_2 = w_1 + \theta_2$

Table 2.

Scenario	The retailer's optimal order quantity
1	$(q_2^{11})^* = \mu_{d_2^1} + \sqrt{2\pi}\sigma_{d_2^1} \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\frac{1}{b}(a - d_2^1) + n_2} \right]$
2	$(q_2^{21})^* = \mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2} \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\left[\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) \right] + n_2} \right]$
3	$(q_2^{12})^* = \mu_{d_2^1} + \sqrt{2\pi}\sigma_{d_2^1} \left[\frac{1}{2} - \frac{w_2 + c_2 + c_{o2} + h_2}{\frac{1}{b}(a - d_2^1) + n_2} \right]$
4	$(q_2^{22})^* = \mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2} \left[\frac{1}{2} - \frac{p_2^k + c_{o2} + h_2}{\left[\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) \right] + n_2} \right]$

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Appendix 1: Proof of Proposition 1

We substitute formula (12) for p_1 in formula (10). Thus,

$$\pi(q_1^{11}) = \left[\frac{1}{b}(a - d_1) \right] S(q_1^{11}) - w_1 q_1^{11} - h_1 q_1^{11} - c_1 q_1^{11} \quad (\text{A.1})$$

The expected sales in the first period will be as formula (A.2):

$$S(q_1^{11}) = \min(d_1, q_1^{11}) = q_1^{11} - \int_0^{q_1^{11}} F(x) dx \quad (\text{A.2})$$

The expected leftover will be as formula (A.3):

$$I(q_1^{11}) = E(q_1^{11} - d_1)^+ = q_1^{11} - S(q_1^{11}) \quad (\text{A.3})$$

Regarding formula (A.1) and taking the first derivative with respect to q_1^{11} , we obtain formula (A.4).

$$\frac{\partial \pi(q_1^{11})}{\partial q_1^{11}} = \left[\frac{1}{b}(a - d_1) \right] \frac{\partial S(q_1^{11})}{\partial q_1^{11}} - w_1 - h_1 - c_1 \quad (\text{A.4})$$

Result of differentiating $S(q_1^{11})$ is as follow:

$$\frac{\partial S(q_1^{11})}{\partial q_1^{11}} = 1 - F(q_1^{11}) \quad (\text{A.5})$$

Regarding formula (A.2), formula (A.5) and taking the second derivative with respect to q_1^{11} , we obtain formula (A.6).

$$\frac{\partial^2 S(q_1^{11})}{\partial (q_1^{11})^2} = -f(q_1^{11}) \quad (\text{A.6})$$

Regarding formula (A.1) and taking the second derivative with respect to q_1^{11} , we obtain formula (A.7).

$$\frac{\partial^2 \pi(q_1^{11})}{\partial (q_1^{11})^2} = - \left[\frac{1}{b}(a - d_1) \right] f(q_1^{11}) < 0 \quad (\text{A.7})$$

To solve formula (A.1), we consider formula (A.4):

$$\frac{\partial \pi(q_1^{11})}{\partial q_1^{11}} = 0 \quad (\text{A.8})$$

The retailer's optimal order quantity is given by:

$$(q_1^{11})^* = F^{-1} \left(1 - \frac{w_1 + h_1 + c_1}{\frac{1}{b}(a - d_1)} \right) \quad (\text{A.9})$$

We consider to $x \sim N(\mu, \sigma^2)$; we will have formula (A.10), formula (A.11), formula (A.12) and formula (A.13).

$$F^{-1}(x) = \mu + (\sqrt{2}\sigma) \text{erf}^{-1}(2x - 1) \quad (\text{A.10})$$

$$\text{erf}^{-1}(x) = \sqrt{\pi} \left(\frac{1}{2}x + \frac{1}{24}\pi x^3 + \frac{7}{960}\pi^2 x^5 + \dots \right) \quad (\text{A.11})$$

$$\text{erf}^{-1}(x) \sim \frac{\sqrt{\pi}}{2}x \quad (\text{A.12})$$

$$\text{erf}^{-1}(2x - 1) = \frac{\sqrt{\pi}}{2}(2x - 1) \quad (\text{A.13})$$

By substituting (A.13) into formula (A.10), we have formula (A.14).

$$F^{-1}(x) = \mu + \frac{\sqrt{2\pi}\sigma}{2}(2x - 1) \quad (\text{A.14})$$

By considering formula (A.9) and formula (A.14), we have formula (A.15).

$$(q_1^{11})^* = \mu_{d_1} + \sqrt{2\pi}\sigma_{d_1} \left[\frac{1}{2} - \frac{w_1 + h_1 + c_1}{\frac{1}{b}(a - d_1)} \right] \quad (\text{A.15})$$

This completes the proof. $\square$

Appendix 2: Proof of Proposition 2

We substitute formula (16) for p_2 in formula (14). Thus,

$$\pi(q_2^{11}) = \left[\frac{1}{b}(a - d_2^1) \right] S(q_2^{11}) - w_2 q_2^{11} - c_2 q_2^{11} - h_2 (q_2^{11} + I(q_1^{11})) - n_2 H(q_2^{11}) \quad (\text{B.1})$$

The expected sales will be as formula (B.2):

$$S(q_2^{11}) = \min(d_2^1, q_2^{11}) = q_2^{11} - \int_0^{q_2^{11}} F(x) dx \quad (\text{B.2})$$

The expected order quantity to the emergency source will be as formula (B.3):

$$H(q_2^{11}) = H(d_2^1, q_2^{11} + I(q_1^{11})) = E[d_2^1 - q_2^{11} - I(q_1^{11})]^+ = \mu_{d_2^1} - S(q_2^{11}) - I(q_1^{11}) \quad (\text{B.3})$$

Regarding formula (B.1) and taking the first derivative with respect to q_1^{11} , we obtain formula (B.4).

$$\frac{\partial \pi(q_2^{11})}{\partial q_2^{11}} = \left[\frac{1}{b}(a - d_2^1) \right] \frac{\partial S(q_2^{11})}{\partial q_2^{11}} - w_2 - c_2 - h_2 - n_2 \frac{\partial H(q_2^{11})}{\partial q_2^{11}} \quad (\text{B.4})$$

Result of differentiating $S(q_2^{11})$ is as follow:

$$\frac{\partial S(q_2^{11})}{\partial q_2^{11}} = 1 - F(q_2^{11}) \quad (\text{B.5})$$

Result of differentiating $H(q_2^{11})$ is as follow:

$$\frac{\partial H(q_2^{11})}{\partial q_2^{11}} = -\frac{\partial S(q_2^{11})}{\partial q_2^{11}} = F(q_2^{11}) - 1 \quad (\text{B.6})$$

Regarding formula (B.5) and formula (B.6) and taking the second derivative with respect to q_2^{11} , we obtain formula (B.7) and formula (B.8).

$$\frac{\partial^2 S(q_2^{11})}{\partial q_2^{11^2}} = -f(q_2^{11}) \quad (\text{B.7})$$

$$\frac{\partial^2 H(q_2^{11})}{\partial q_2^{11^2}} = f(q_2^{11}) \quad (\text{B.8})$$

Regarding formula (B.1) and taking the second derivative with respect to q_2^{11} , we obtain formula (B.9).

$$\frac{\partial^2 \pi(q_2^{11})}{\partial q_2^{11^2}} = -\left[\frac{1}{b}(a - d_2^1) + n_2 \right] f(q_2^{11}) < 0 \quad (\text{B.9})$$

Therefore, it is clear that $\pi(q_2^{11})$ is concave.

To solve formula (B.1), we consider formula (B.4):

$$\frac{\partial \pi(q_2^{11})}{\partial q_2^{11}} = 0 \quad (\text{B.10})$$

The retailer's optimal order quantity is given by:

$$(q_2^{11})^* = F^{-1} \left(1 - \frac{w_2 + c_2 + h_2}{\frac{1}{b}(a - d_2^1) + n_2} \right) \quad (\text{B.11})$$

By considering formula (A.14) and formula (B.11), we have formula (B.12).

$$(q_2^{11})^* = \mu_{d_2^1} + \sqrt{2\pi}\sigma_{d_2^1} \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\frac{1}{b}(a - d_2^1) + n_2} \right] \quad (\text{B.12})$$

Also, according to formula (16) and $(n_2 < p_2)$ mentioned in subsection 4.1, the denominator of the fraction of formula (B.12) is not zero. Therefore, the boundary conditions are established. This completes the proof. $\square$

Appendix 3: Proof of Proposition 3

We substitute formula (21) for p_2 in formula (18). Thus,

$$\pi(q_2^{21}) = \left[\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) \right] S(q_2^{21}) - w_2 q_2^{21} - c_2 q_2^{21} - h_2 (q_2^{21} + I(q_1^{11})) - n_2 H(q_2^{21}) \quad (C.1)$$

The expected sales will be as formula (C.2):

$$S(q_2^{21}) = \min(d_2^2, q_2^{21}) = q_2^{21} - \int_0^{q_2^{21}} F(x) dx \quad (C.2)$$

The expected order quantity to the emergency source will be as formula (C.3):

$$H(q_2^{21}) = H(d_2^2, q_2^{21} + I(q_1^{11})) = E[d_2^1 - q_2^{21} - I(q_1^{11})]^+ = \mu_{d_2^2} - S(q_2^{21}) - I(q_1^{11}) \quad (C.3)$$

Regarding formula (C.1) and taking the first derivative with respect to q_2^{21} , we obtain formula (C.4).

$$\frac{\partial \pi(q_2^{21})}{\partial q_2^{21}} = \left[\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) \right] \frac{\partial S(q_2^{21})}{\partial q_2^{21}} - w_2 - c_2 - h_2 - n_2 \frac{\partial H(q_2^{21})}{\partial q_2^{21}} \quad (C.4)$$

Result of differentiating $S(q_2^{21})$ is as follow:

$$\frac{\partial S(q_2^{21})}{\partial q_2^{21}} = 1 - F(q_2^{21}) \quad (C.5)$$

Result of differentiating $H(q_2^{21})$ is as follow:

$$\frac{\partial H(q_2^{21})}{\partial q_2^{21}} = -\frac{\partial S(q_2^{21})}{\partial q_2^{21}} = F(q_2^{21}) - 1 \quad (C.6)$$

Regarding formula (C.5) and formula (C.6) and taking the second derivative with respect to q_2^{21} , we obtain formula (C.7) and formula (C.8).

$$\frac{\partial^2 S(q_2^{21})}{\partial q_2^{21^2}} = -f(q_2^{21}) \quad (C.7)$$

$$\frac{\partial^2 H(q_2^{21})}{\partial q_2^{21^2}} = f(q_2^{21}) \quad (C.8)$$

Regarding formula (C.1) and taking the second derivative with respect to q_2^{21} , we obtain formula (C.9).

$$\frac{\partial^2 \pi(q_2^{21})}{\partial q_2^{21^2}} = - \left[\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) + n_2 \right] f(q_2^{21}) < 0, \quad (C.9)$$

Therefore, it is clear that $\pi(q_2^{21})$ is concave.

To solve formula (C.1), we consider formula (C.4):

$$\frac{\partial \pi(q_2^{21})}{\partial q_2^{21}} = 0 \quad (C.10)$$

The retailer's optimal order quantity is given by

$$(q_2^{21})^* = F^{-1} \left(1 - \frac{w_2 + c_2 + h_2}{\left[\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) \right] + n_2} \right) \quad (C.11)$$

By considering formula (A.14) and formula (C.11), we have formula (C.12).

$$(q_2^{21})^* = \mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2} \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\left[\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) \right] + n_2} \right] \quad (C.12)$$

This completes the proof. $\square$

Appendix 4: Proof of Proposition 4

We substitute formula (25) for p_2 in formula (23). Thus,

$$\pi(q_2^{12}) = \left[\frac{1}{b} (a - d_2^1) \right] S(q_2^{12}) - w_2 q_2^{12} - c_2 q_2^{12} - c_{o2} q_2^{12} - h_2 (q_2^{12} + I(q_1^{11})) - n_2 H(q_2^{12}) \quad (D.1)$$

The expected sales will be as formula (D.2):

$$S(q_2^{12}) = \min(d_2^1, q_2^{12}) = q_2^{12} - \int_0^{q_2^{12}} F(x) dx \quad (D.2)$$

The expected order quantity to the emergency source will be as formula (D.3):

$$H(q_2^{12}) = H(d_2^1, q_2^{12} + I(q_1^{11})) = E \left[d_2^1 - q_2^{12} - I(q_1^{11}) \right]^+ = \mu_{d_2^1} - S(q_2^{12}) - I(q_1^{11}) \quad (D.3)$$

Regarding formula (D.1) and taking the first derivative with respect to q_2^{12} , we obtain formula (D.4).

$$\frac{\partial \pi(q_2^{12})}{\partial q_2^{12}} = \left[\frac{1}{b} (a - d_2^1) \right] \frac{\partial S(q_2^{12})}{\partial q_2^{12}} - w_2 - c_2 - c_{o2} - h_2 - n_2 \frac{\partial H(q_2^{12})}{\partial q_2^{12}} \quad (D.4)$$

Result of differentiating $S(q_2^{12})$ is as follow:

$$\frac{\partial S(q_2^{12})}{\partial q_2^{12}} = 1 - F(q_2^{12}) \quad (\text{D.5})$$

Result of differentiating $H(q_2^{12})$ is as follow:

$$\frac{\partial H(q_2^{12})}{\partial q_2^{12}} = -\frac{\partial S(q_2^{12})}{\partial q_2^{12}} = F(q_2^{12}) - 1 \quad (\text{D.6})$$

Regarding formula (D.5) and formula (D.6) and taking the second derivative with respect to q_2^{12} , we obtain formula (D.7) and formula (D.8).

$$\frac{\partial^2 S(q_2^{12})}{\partial q_2^{12^2}} = -f(q_2^{12}) \quad (\text{D.7})$$

$$\frac{\partial^2 H(q_2^{12})}{\partial q_2^{12^2}} = f(q_2^{12}) \quad (\text{D.8})$$

Regarding formula (D.1) and taking the second derivative with respect to q_2^{12} , we obtain formula (D.9).

$$\frac{\partial^2 \pi(q_2^{12})}{\partial q_2^{12^2}} = -\left[\frac{1}{b}(a - d_2^1) + n_2 \right] f(q_2^{12}) < 0 \quad (\text{D.9})$$

Therefore, it is clear that $\pi(q_2^{12})$ is concave.

To solve formula (D.1), we consider formula (D.4):

$$\frac{\partial \pi(q_2^{12})}{\partial q_2^{12}} = 0 \quad (\text{D.10})$$

The retailer's optimal order quantity is given by:

$$(q_2^{12})^* = F^{-1} \left(1 - \frac{w_2 + c_2 + c_{o2} + h_2}{\frac{1}{b}(a - d_2^1) + n_2} \right) \quad (\text{D.11})$$

By considering formula (A.14) and formula (D.11), we have formula (D.12).

$$(q_2^{12})^* = \mu_{d_2^1} + \sqrt{2\pi}\sigma_{d_2^1} \left[\frac{1}{2} - \frac{w_2 + c_2 + c_{o2} + h_2}{\frac{1}{b}(a - d_2^1) + n_2} \right] \quad (\text{D.12})$$

Also, according to formula (25) and $(n_2 < p_2)$ mentioned in subsection 4.1, the denominator of the fraction of formula (D.12) is not zero. Therefore, the boundary conditions are established. This completes the proof.

□

Appendix 5: Proof of Proposition 5

We substitute formula (30) for p_2 in formula (27). Thus,

$$\pi(q_2^{22}) = \left[\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) \right] S(q_2^{22}) - p_2^k q_2^{22} - c_o q_2^{22} - h_2 (q_2^{22} + I(q_1^{11})) - n_2 H(q_2^{22}) \quad (\text{E.1})$$

The expected sales will be as formula (E.2):

$$S(q_2^{22}) = \min(d_2^2, q_2^{22}) = q_2^{22} - \int_0^{q_2^{22}} F(x) dx \quad (\text{E.2})$$

The expected order quantity to the emergency source will be as formula (E.3):

$$H(q_2^{22}) = H(d_2^2, q_2^{22} + I(q_1^{11})) = E[d_2^2 - q_2^{22} - I(q_1^{11})]^+ = \mu_{d_2^2} - S(q_2^{22}) - I(q_1^{11}) \quad (\text{E.3})$$

Regarding formula (E.1) and taking the first derivative with respect to q_1^{11} , we obtain formula (E.4).

$$\frac{\partial \pi(q_2^{22})}{\partial q_2^{22}} = \left[\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) \right] \frac{\partial S(q_2^{22})}{\partial q_2^{22}} - p_2^k - c_o - h_2 - n_2 \frac{\partial H(q_2^{22})}{\partial q_2^{22}} \quad (\text{E.4})$$

Result of differentiating $S(q_2^{22})$ is as follow:

$$\frac{\partial S(q_2^{22})}{\partial q_2^{22}} = 1 - F(q_2^{22}) \quad (\text{E.5})$$

Result of differentiating $H(q_2^{22})$ is as follow:

$$\frac{\partial H(q_2^{22})}{\partial q_2^{22}} = -\frac{\partial S(q_2^{22})}{\partial q_2^{22}} = F(q_2^{22}) - 1 \quad (\text{E.6})$$

Regarding formula (E.5) and formula (E.6) and taking the second derivative with respect to q_2^{22} , we obtain formula (E.7) and formula (E.8).

$$\frac{\partial^2 S(q_2^{22})}{\partial q_2^{22^2}} = -f(q_2^{22}) \quad (\text{E.7})$$

$$\frac{\partial^2 H(q_2^{22})}{\partial q_2^{22^2}} = f(q_2^{22}) \quad (\text{E.8})$$

Regarding formula (E.1) and taking the second derivative with respect to q_2^{22} , we obtain formula (E.9).

$$\frac{\partial^2 \pi(q_2^{22})}{\partial q_2^{22^2}} = -\left[\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) + n_2 \right] f(q_2^{22}) < 0 \quad (\text{E.9})$$

Therefore, it is clear that $\pi(q_2^{22})$ is concave.

To solve formula (E.1), we consider formula (E.4):

$$\frac{\partial \pi(q_2^{22})}{\partial q_2^{22}} = 0 \quad (\text{E.10})$$

The retailer's optimal order quantity is given by:

$$(q_2^{22})^* = F^{-1} \left(1 - \frac{p_2^k + c_{o2} + h_2}{\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) + n_2} \right) \quad (\text{E.11})$$

By considering formula (A.14) and formula (E.11), we have formula (E.12).

$$(q_2^{22})^* = \mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2} \left[\frac{1}{2} - \frac{p_2^k + c_{o2} + h_2}{\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) + n_2} \right] \quad (\text{E.12})$$

This completes the proof. $\square$

Appendix 6. Proof of Theorem 1

$\bar{q}_{q_1^{11}, q_2^{21}}$ is calculated as follow:

$$\begin{aligned} \bar{q}_{q_1^{11}, q_2^{21}} &= \frac{1}{2} (q_1^{11} + q_2^{21}) = \\ &= \frac{1}{2} \left[\left(\mu_{d_1^2} + \sqrt{2\pi}\sigma_{d_1^2} \left[\frac{1}{2} - \frac{w_1 + h_1 + c_1}{\frac{1}{b}(a - d_1)} \right] \right) + \left(\mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2} \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) + n_2} \right] \right) \right] \end{aligned} \quad (\text{F.1})$$

With substituting formula (42) and formula (F.1) in formula (45), we obtain formula (F.2).

$$\begin{aligned} BWE_{q_1^{11}, q_2^{21}} &= \\ &= \frac{\frac{1}{2} \left[\left(\mu_{d_2^2} + \sqrt{2\pi}\sigma_{d_2^2} \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) + n_2} \right] \right) - \left(\mu_{d_1^2} + \sqrt{2\pi}\sigma_{d_1^2} \left[\frac{1}{2} - \frac{w_1 + h_1 + c_1}{\frac{1}{b}(a - d_1)} \right] \right) \right]^2}{(2b^2 + 2r^2b^2)\sigma^2} \end{aligned} \quad (\text{F.2})$$

With substituting formula (39), formula (41), formula (43) and formula (44) in formula (F.2), we have formula (F.3) as follow:

$$BWE_{q_1^{11}, q_2^{21}} = \frac{\frac{1}{2} \left[\left(\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right]rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2}\sigma \right] \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\frac{1}{b} \left(a + [(m_2 - m_1)w_1 + (1 + m_2)\theta_2]rb - d_2^2 \right) + n_2} \right] \right) - \left(\sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)} \right] \right) \right]^2}{(2b^2 + 2r^2b^2)\sigma^2} \quad (\text{F.3})$$

This completes the proof. $\square$

Appendix 7. Proof of Proposition 6

Bullwhip effect exists if formula (G.1) holds.

$$BWE_{q_1^{11}, q_2^{21}} > 1 \quad (G.1)$$

With substituting formula (46) in formula (G.1), we have

$$\frac{\frac{1}{2} \left[\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right]} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)} \right] \right]^2}{(2b^2 + 2r^2b^2)\sigma^2} > 1 \quad (G.2)$$

$$\frac{1}{2(2b^2 + 2r^2b^2)\sigma^2} \left[\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)} \right] \right]^2 > 1 \quad (G.3)$$

$$\left[\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)} \right] \right]^2 > 2(2b^2 + 2r^2b^2)\sigma^2 \quad (G.4)$$

$$\left[\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)} \right] \right]^2 - 2(2b^2 + 2r^2b^2)\sigma^2 > 0 \quad (G.5)$$

We know if $x^2 > y^2$, then $x > y$ and $-x < -y$. Also, We consider $\frac{\sqrt{2\pi}}{2} \square 1.25$, $\sqrt{2\pi} \square 2.5$.

Therefore, we have formula (G.6) and formula (G.7).

$$\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + 1.25 \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] - 2.5 \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{w_2 + c_2 + h_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] - 1.25 [b\sigma] + \quad (G.6)$$

$$2.5 [b\sigma] \left[\frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)} \right] > \sqrt{2(2b^2 + 2r^2b^2)} \sigma$$

$$\left[\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + 1.25 \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] - 2.5 \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{w_2 + c_2 + h_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] - 1.25 [b\sigma] + \right. \quad (G.7)$$

$$\left. - 2.5 [b\sigma] \left[\frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)} \right] \right] < - \left[\sqrt{2(2b^2 + 2r^2b^2)} \sigma \right]$$

After simplification, we have formula (G.8).

$$\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + 1.25 \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] + 2.5 [b\sigma] \left[\frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)} \right] > \quad (G.8)$$

$$2.5 \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{w_2 + c_2 + h_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] + 1.25 [b\sigma] + \sqrt{2(2b^2 + 2r^2b^2)} \sigma$$

Bullwhip effect exists (i.e. $BWE_{q_1^{11}, q_2^{21}} > 1$) if formula (G.8) hold.

This completes the proof. $\square$

Appendix 8. Proof of Theorem 2

$\bar{q}_{q_1^{11}, q_2^{22}}$ is calculated as follow:

$$\bar{q}_{q_1^{11}, q_2^{22}} = \frac{1}{2} (q_1^{11} + q_2^{22}) = \frac{1}{2} \left[\left(\mu_{d_1} + \sqrt{2\pi}\sigma_{d_1} \left[\frac{1}{2} - \frac{w_1 + h_1 + c_1}{b(a-d_1)} \right] \right) + \mu_{d_2} + \sqrt{2\pi}\sigma_{d_2} \left[\frac{1}{2} - \frac{p_2^k + c_{o2} + h_2}{\left[\frac{1}{b} (a + [(m_2 - m_1)w_1 + (1+m_2)\theta_2]rb - d_2^2) \right] + n_2} \right] \right] \quad (H.1)$$

With substituting formula (42) and formula (H.1) in formula (48), we obtain formula (H.2).

$$BWE_{q_1^{11}, q_2^{22}} = \frac{\frac{1}{2} \left[\left(\mu_{d_2} + \sqrt{2\pi}\sigma_{d_2} \left[\frac{1}{2} - \frac{p_2^k + c_{o2} + h_2}{\left[\frac{1}{b} (a + [(m_2 - m_1)w_1 + (1+m_2)\theta_2]rb - d_2^2) \right] + n_2} \right] \right) - \left(\mu_{d_1} + \sqrt{2\pi}\sigma_{d_1} \left[\frac{1}{2} - \frac{w_1 + h_1 + c_1}{b(a-d_1)} \right] \right) \right]^2}{(2b^2 + 2r^2b^2)\sigma^2} \quad (H.2)$$

With substituting formula (39), formula (41), formula (43) and formula (44) in formula (H.2), we have formula (H.3) as follow:

$$BWE_{q_1^{11}, q_2^{22}} = \frac{\frac{1}{2} \left[\left[(m_2 - m_1)w_1 + (1+m_2)\theta_2 \right]rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2}\sigma \right] \left[\frac{1}{2} - \frac{p_2^k + c_{o2} + h_2}{\left[\frac{1}{b} (a + [(m_2 - m_1)w_1 + (1+m_2)\theta_2]rb - d_2^2) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{b(a-d_1)} \right] \right]^2}{(2b^2 + 2r^2b^2)\sigma^2} \quad (H.3)$$

This completes the proof. $\square$

Appendix 9. Proof of Proposition 7

Bullwhip effect exists if formula (I.1) holds.

$$BWE_{q_1^{11}, q_2^{22}} > 1 \quad (I.1)$$

With substituting formula (49) in formula (I.1), we have

$$\frac{\frac{1}{2} \left[\left[(m_2 - m_1)w_1 + (1+m_2)\theta_2 \right]rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2}\sigma \right] \left[\frac{1}{2} - \frac{p_2^k + c_{o2} + h_2}{\left[\frac{1}{b} (a + [(m_2 - m_1)w_1 + (1+m_2)\theta_2]rb - d_2^2) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{b(a-d_1)} \right] \right]^2}{(2b^2 + 2r^2b^2)\sigma^2} > 1 \quad (I.2)$$

$$\frac{1}{2(2b^2 + 2r^2b^2)\sigma^2} \left[\left[(m_2 - m_1)w_1 + (1+m_2)\theta_2 \right]rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2}\sigma \right] \left[\frac{1}{2} - \frac{p_2^k + c_{o2} + h_2}{\left[\frac{1}{b} (a + [(m_2 - m_1)w_1 + (1+m_2)\theta_2]rb - d_2^2) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{b(a-d_1)} \right] \right]^2 > 1 \quad (I.3)$$

$$\left[\left[(m_2 - m_1)w_1 + (1+m_2)\theta_2 \right]rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2}\sigma \right] \left[\frac{1}{2} - \frac{p_2^k + c_{o2} + h_2}{\left[\frac{1}{b} (a + [(m_2 - m_1)w_1 + (1+m_2)\theta_2]rb - d_2^2) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{b(a-d_1)} \right] \right]^2 > 2(2b^2 + 2r^2b^2)\sigma^2 \quad (I.4)$$

$$\left[\left[(m_2 - m_1)w_1 + (1+m_2)\theta_2 \right]rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2}\sigma \right] \left[\frac{1}{2} - \frac{p_2^k + c_{o2} + h_2}{\left[\frac{1}{b} (a + [(m_2 - m_1)w_1 + (1+m_2)\theta_2]rb - d_2^2) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{b(a-d_1)} \right] \right]^2 - 2(2b^2 + 2r^2b^2)\sigma^2 > 0 \quad (I.5)$$

We know if $x^2 > y^2$, then $x > y$ and $-x < -y$. Also, We consider $\frac{\sqrt{2\pi}}{2} \square 1.25$, $\sqrt{2\pi} \square 2.5$.

Therefore, we have formula (I.6) and formula (I.7).

$$\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + 1.25 \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] - 2.5 \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{p_2^k + c_{o2} + h_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] - 1.25 [b\sigma] + \quad (I.6)$$

$$2.5 [b\sigma] \left[\frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)} \right] > \sqrt{2(2b^2 + 2r^2b^2)} \sigma$$

$$- \left[\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + 1.25 \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] - 2.5 \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{p_2^k + c_{o2} + h_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] \right] < - \left[\sqrt{2(2b^2 + 2r^2b^2)} \sigma \right]$$

$$- 1.25 [b\sigma] + 2.5 [b\sigma] \left[\frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)} \right] \quad (I.7)$$

After simplification, we have formula (I.8).

$$\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + 1.25 \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] + 2.5 [b\sigma] \left[\frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)} \right] > 2.5 \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{p_2^k + c_{o2} + h_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] + \quad (I.8)$$

$$1.25 [b\sigma] + \sqrt{2(2b^2 + 2r^2b^2)} \sigma$$

Bullwhip effect exists (i.e. $BWE_{q_1^{11}, q_2^{22}} > 1$) if formula (I.8) hold.

This completes the proof. $\square$

Appendix 10. Proof of Theorem 3

From formula (46) and formula (49), we obtain formula (J.1):

$$\Delta BWE = BWE_{q_1^{11}, q_2^{22}} - BWE_{q_1^{11}, q_2^{21}} =$$

$$\frac{\frac{1}{2} \left[\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{1}{2} - \frac{p_2^k + c_{o2} + h_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)} \right] \right]^2}{(2b^2 + 2r^2b^2) \sigma^2} - \quad (J.1)$$

$$\frac{\frac{1}{2} \left[\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{1}{2} - \frac{w_2 + c_2 + h_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + c_1 + h_1}{\frac{1}{b}(a - d_1)} \right] \right]^2}{(2b^2 + 2r^2b^2) \sigma^2}$$

To prove Theorem 3, we need to indicate that $\Delta BWE < 0$.

According to formula (13), formula (22), formula (32), formula (39) and formula (41), we have formula (J.2) and formula (J.3).

$$\frac{1}{\sqrt{2(2b^2 + 2r^2b^2)} \sigma} \left[\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{1}{2} - \frac{p_2^k + h_2 + c_{o2}}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + h_1 + c_1}{\frac{1}{b}(a - d_1)} \right] \right] > 0 \quad (J.2)$$

$$\frac{1}{\sqrt{2(2b^2 + 2r^2b^2)} \sigma} \left[\left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb + \sqrt{2\pi} \left[\sqrt{b^2 + 2r^2b^2} \sigma \right] \left[\frac{1}{2} - \frac{w_2 + h_2 + c_2}{\left[\frac{1}{b} \left(a + \left[(m_2 - m_1)w_1 + (1 + m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1 + h_1 + c_1}{\frac{1}{b}(a - d_1)} \right] \right] > 0 \quad (J.3)$$

We know if $x^2 - y^2 < 0$ and $x, y > 0$, then $x < y$. Therefore, we have formula (J.4).

$$\begin{aligned}
& \frac{1}{\sqrt{2(2b^2+2r^2b^2)}\sigma} \left[\left[(m_2-m_1)w_1+(1+m_2)\theta_2 \right] rb + \sqrt{2\pi} \left[\sqrt{b^2+2r^2b^2}\sigma \right] \left[\frac{1}{2} - \frac{p_2^k+h_2+c_{o2}}{\left[\frac{1}{b} \left(a + \left[(m_2-m_1)w_1+(1+m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1+h_1+c_1}{\frac{1}{b}(a-d_1)} \right] \right] < \\
& \frac{1}{\sqrt{2(2b^2+2r^2b^2)}\sigma} \left[\left[(m_2-m_1)w_1+(1+m_2)\theta_2 \right] rb + \sqrt{2\pi} \left[\sqrt{b^2+2r^2b^2}\sigma \right] \left[\frac{1}{2} - \frac{w_2+h_2+c_2}{\left[\frac{1}{b} \left(a + \left[(m_2-m_1)w_1+(1+m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \right] - \sqrt{2\pi} [b\sigma] \left[\frac{1}{2} - \frac{w_1+h_1+c_1}{\frac{1}{b}(a-d_1)} \right] \right]
\end{aligned} \tag{J.4}$$

After simplification, formula (J.4) reduces to formula (J.5) and then formula (J.6).

$$\frac{w_2+c_2+h_2}{\left[\frac{1}{b} \left(a + \left[(m_2-m_1)w_1+(1+m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} < \frac{p_2^k+c_{o2}+h_2}{\left[\frac{1}{b} \left(a + \left[(m_2-m_1)w_1+(1+m_2)\theta_2 \right] rb - d_2^2 \right) \right] + n_2} \tag{J.5}$$

$$w_2 + c_2 + h_2 < p_2^k + c_{o2} + h_2 \tag{J.6}$$

As a result, according to the formula (31) and formula (J.6), we obtain formula (J.7).

$$c_{o2} > c_2 + \varphi \tag{J.7}$$

With considering formula (J.7), we can prove Theorem 3.

This completes the proof. $\square$