A Sustainable Production-Inventory Model Joint with Preventive Maintenance and Multiple Shipments for Imperfect Quality Items

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Abstract.

The quality of products, maintenance activities, and transportation policies are primary concerns of managers in inventory and production planning problems. Environmental issues and regulations are growing increasingly and attracted much attention to sustainable production. Previous authors conducted a wide range of research on these problems separately. Regarding the gap of an integrated framework, a sustainable economic production quantity model is formulated by considering preventive maintenance and multiple shipments policy where a portion of produced items is defective. Two special cases are studied. In Case I, the production period's demand is satisfied by the produced items in the previous cycle. In Case II, simultaneous production and consumption during the production period is considered and mathematically formulated. An analytical method is presented for solving the models, and a numerical example is discussed for both cases. Comparing two cases proves that Case II is more beneficial and decreases the overall cost of the inventory system. Sensitivity analysis of the model is performed, and some insights are derived by changing some of the parameters.

Keywords: Sustainable Inventory, Economic Production Quantity, Preventive Maintenance, Multiple Shipments, Imperfect Quality Items

1. Introduction

Inventory can be defined as all items and materials which are held for satisfying future demand. There are various reasons for holding inventory; some of the reasons are: deal with market volatility, shortage prevention, and balancing the production processes. These reasons drew the attention of operations research experts to design and optimize inventory systems. As a result, Harris[1] presented the first classic inventory model, economic order quantity (EOQ). The objective was to minimize the system's overall cost, including procurement cost, ordering cost, and holding cost. Although the development of EOQ was an important advancement in inventory systems, the model was not consistent with items in some production settings. Regarding that, the economic production quantity (EPQ) model was developed[2]. The primary difference between EOQ and EPQ was relaxing the assumption about the instant receiving of orders. After that, several researchers investigated more advanced inventory systems and tried to incorporate real-world situations.

According to the Intergovernmental Panel on Climate Change (IPCC) reports, the global warming problem poses a severe threat to the world. Greenhouse gases emission, particularly carbon, can be realized as the principal cause of global warming. The last statistic confirmed that a considerable portion of emission is related to the industrial and transportation sector[3]. Due to this role, the concept of sustainable production has emerged. Sustainable production can be defined as producing desired items by responsible production processes that try to take into account environmental anxieties. One of the most effective methods to curb emission is making operational adjustments and optimizing the production and inventory system variables, considering the environmental initiatives.

The inventory system's managers are interested in reducing the system's cost as much as possible. Nevertheless, attaining this goal is not possible without paying attention to the quality of products. Quality becomes one of the substantial factors that can affect business success. At first, the formulation of classic inventory models was performed without quality criteria. However, in recent years, the indicated rationales drew the researchers' concentration on this crucial factor.

A wide range of activities and parameters can influence the quality of a production system. Maintenance plans are one of these high-impact activities. A maintenance plan commonly compromises

different technical activities such as checking, service, repair, or, if needed, replacing the equipment and parts. Maintenance plans can fall into different categories. Preventive maintenance, briefly known as (PM), is one of the first and effective maintenance plans. PM deals with a group of prescheduled activities that should be performed to keep the production state at the desired level. Production plants need inventory models that can fulfill the need for an appropriate maintenance plan.

The classic inventory models have another unrealistic assumption for simplicity, which is about the continuous delivery policy. In the real-world states, vehicles are used for item delivery, and a continuous replenishment procedure is usually unenforceable. Several parameters are included in multiple shipments delivery, such as the vehicle capacity, transportation time to demand point, etc. Consideration of these parameters can significantly change an inventory system's performance.

As will be seen in the literature review, no work presents PM, multiple shipments policy, and carbon emission in an integrated production-inventory framework. Therefore, we try to cover this gap by presenting a joint sustainable EPQ model with imperfect quality items under PM and multiple shipments assumptions.

The rest of this paper is continued as follows. In section 2, we attempt to review the previously published works. In Section 3, we develop two cases for a sustainable EPQ model with imperfect quality items, multiple deliveries of goods, and PM policy. We solve a numerical example for the presented models and provide sensitivity analysis for some parameters of models in section 4. We explain the conclusion and future directions of the current research in section 5.

2. Literature review

After the classic inventory models, researchers tried to present the extensions by considering the real-world situations [4-8]. In this section, a literature review is presented with a focus on the features of this work.

2.1. Inventory systems with carbon emission consideration

Hua, et al. [9], for the first time, examined a market-based carbon emission trading mechanism in the EOQ model. A fixed carbon emission cap was considered for the system, and the shortage or surplus quota could be bought or sold through, respectively. Bouchery, et al. [10] tried to integrate the economic, social, and environmental factors in the classic EOQ by developing a multi-objective replenishment model. Chen, et al. [11] addressed EOQ models with four carbon emission policies: strict cap, cap-and-offset, cap-and-price, and carbon tax. A new variant of the sustainable EOQ model was studied by Battini, et al. [12] where the life

cycle analysis (LCA) and direct accounting method were used to attain the carbon emission factors. [Hovelaque and Bironneau \[13\]](#) modeled a replenishment system for which price and carbon emission depended on the demand. An inventory EOQ model for growing items with an emphasis on environmental concerns was developed by [Zhang, et al. \[14\]](#). [Kazemi, et al. \[15\]](#) worked on the EOQ model with low-quality items and extended this model by considering the warehousing and disposal emission cost. [Taleizadeh, et al. \[16\]](#) formulated four EPQ models according to various shortage scenarios. The direct accounting approach was used to consider the carbon footprints. The possibility of investment in carbon emission reduction was studied in the [Lee \[17\]](#) paper. [Mishra, et al. \[18\]](#) suggested a sustainable inventory model for non-instantaneous deteriorating items where ordering cost, carbon emission, and the deterioration rate depended on the investment.

2.2. Inventory systems with imperfect quality items

As an important extension of inventory models with defective items, [Salameh and Jaber \[19\]](#), supposed that all items in an EOQ system do not have a perfect quality. [Chang \[20\]](#) proposed two EOQ models by considering demand and defective rate of system as fuzzy parameters. The contribution of [Rezaei \[21\]](#) paper was considering the backorder shortage in the EOQ model with imperfect items. [Lo, et al. \[22\]](#) suggested a deteriorating production-inventory model where the products' deterioration rate was a random variable. Partial backorder, inflation, and considering both retailer and manufacturer viewpoints were other aspects of this work. [Chung, et al. \[23\]](#) studied the imperfect quality items in the two-warehouse inventory model for the first time. An EPQ model with stochastic demand considering inflation and defective items was presented by [Sarkar and Moon \[24\]](#). [Konstantaras, et al. \[25\]](#) analyzed the effect of learning during the inspection process in two cases of the EOQ model with faulty items. [Wee, et al. \[26\]](#) considered some inspection constraints in their EPQ model with defective items. Unlike the many previous assumptions, the inspection rate can be less or equal to the production rate. [Chang, et al. \[27\]](#) presented an EOQ model with imperfect items where the inspection error and delay in payment were taken into account in the inspection operation. A production-inventory system was developed by [Mokhtari \[28\]](#). In this work, the raw materials for a production system were supplied by order issues, and the final goods were produced by implementing the production process. In Mokhtari's paper [28], the imperfect quality was considered for raw material and final products. [Khakzad and Gholamian \[29\]](#) presented a deteriorating inventory model in which the average deterioration rate depended on the inspection process. Stock-dependent demand production-inventory model with imperfect quality goods and permissible delay in payment was formulated by [Dhaka, et al. \[30\]](#).

2.3. Inventory systems with multiple shipments policy

The first appearance of multiple shipments policy in inventory systems literature was [Chiu, et al. \[31\]](#) work. In addition to the multiple shipment concept, they also incorporated rework of defective items in their model. [CáRdenas-Barrón, et al. \[32\]](#) relaxed the assumption of [Chiu, et al. \[31\]](#) model, which was about the fixed number of shipments. [Ritha and Martin \[33\]](#) proposed the packaging and switching cost in the replenishment model under multiple shipments policy. [Taleizadeh, et al. \[34\]](#) modeled a production-inventory system considering multiple shipments policy where Lot-size, the selling price, and the number of shipments were the decision variables of model. [Taleizadeh, et al. \[35\]](#) worked on an EPQ model with imperfect items for joint optimization of selling price and lot-size considering the multiple shipments assumptions. An inventory system with capital investment dependent setup cost, defective items, and rework process was studied by [Priyan and Uthayakumar \[36\]](#). Multiple deliveries policy was one of the components of the formulated model. The EPQ model for deteriorating items, where the delivery of products is done under multiple shipments policy, was suggested by [Kalantari and Taleizadeh \[37\]](#).

2.4. Inventory systems with maintenance plans

[Rezg, et al. \[38\]](#) investigated a production system with maintenance plans where there was a minimum availability level constraint in the proposed model. [Liao, et al. \[39\]](#) proposed a deteriorating production system. Rework and maintenance can be perfect or imperfect in their system. Repair time and machine breakdown were assumed as random variables in [Widyadana and Wee \[40\]](#) article for a deteriorating production-inventory model. An inventory model with stochastic maintenance and rework time was the result of [Wee and Widyadana \[41\]](#) work. An extension of EPQ models with maintenance activities was [Liao \[42\]](#) work, where the warranty program was also considered in the mathematical model. [Jafari and Makis \[43\]](#) formulated an EPQ model with PM and condition monitoring assumptions. They formulated the problem by using Semi-Markov decision. [Peng and van Houtum \[44\]](#) tried to optimize condition-based maintenance and lot-sizing policies simultaneously. An imperfect inventory control system was proposed by [Nasr, et al. \[45\]](#) to find optimal maintenance scheduling and lot-size policies where the quality of items correlates with each other. [Taleizadeh \[46\]](#) studied a multi-item manufacturing model with a partial backorder shortage. PM can be performed whether the stock level is positive or negative. [Lai, et al. \[47\]](#) presented hybrid planning of PM and emergency maintenance in a defective EPQ model. [Tsao, et al. \[48\]](#) considered the predictive maintenance plan for EPQ with defective items and reworks. An EPQ model with PM activities was analyzed by [Mokhtari and](#)

Asadkhani [49] where the production of defective items was considered. Table 1 represents a summary of our literature review results.

[Please insert Table 1 about here.]

We conclude that there is a research opportunity for developing a sustainable EPQ based model that integrates PM, quality of items, and multiple shipments policy. Due to everyday real-world situations, two cases are presented for this structure.

3. Problem description and formulation

In this section, we explain the detail of the proposed inventory system and then present a mathematical formulation.

The main components of our work are:

- Production of defective items occurs with a known probability.
- A 100% inspection policy is performed to recognize the defective items.
- A disposal policy is considered for the identified defective items.
- The delivery policy is multiple shipments.
- PM plan is a part of the inventory system.
- Carbon emission is formulated by the direct accounting method.

Assume that the lead time parameter is zero in this system, and an order is placed at the beginning of the cycle. The shortage of items is not permitted. When the PM activities are performed, the production operation is suspended, and after that, it will be continued. The production period consists of sequential s -cycles: a production sub-cycle t_p is started, and after the production of items in size of Q_{sc} , maintenance activities are implemented during a maintenance sub-cycle t_m . x is a decision variable that represents the number of implementing maintenance activities. PM activities impose a cost on the system. We define K_m as the maintenance cost per activity. There are two setup costs in this model where K is the setup cost for the overall cycle and, K_s is the setup cost for a production sub-cycle. It is assumed that a fixed θ percentage of a Q size order is defective. These defective items are discovered by performing a 100% inspection at the end of the production period. The items are disposed of at the cost of C_s . The number of shipments is considered as a fixed parameter n . The delivery is done by one vehicle with travel time t_T , and the capacity for this

vehicle is $Q_T \cdot K_T$ is the system delivery cost per shipment. According to the tangibility of the direct accounting approach [16], carbon emission is formulated by this method. The carbon emissions from various activities of the system are translated to an evident cost parameter. The parameters C_{ep} , C_{ei} , C_{et} and C_{mt} respectively show the production emission cost, the inventory holding emission cost, the transportation emission cost, and the maintenance emission cost.

We investigate two cases for the proposed structure:

- Case I: There is no demand satisfaction during the production period from the produced items of the same period. The produced items in one production period are used in the demand period of the same cycle and production period of the next cycle.
- Case II: There is demand satisfaction during the production cycle. The produced items in one cycle are used in the demand period and the production period of the same cycle.

3.1. Notations

The notations of proposed models can be summarized as follow:

3.1.1 Parameters:

P	The production rate per unit of time
D	The demand rate per unit of time
K	The setup cost per unit of cycle
I_{i1}	The inventory level of Case II before demand satisfaction in PM time
I_{i2}	The inventory level of Case II after demand satisfaction in PM time
K_s	The setup cost per unit of production sub-cycle
K_m	The PM cost per activity
h	The holding cost per unit of item per unit of time
C	The procurement cost per unit of item
C_s	The disposal cost per unit of a defective item
K_T	The delivery cost per unit of shipment
C_{ep}	The production emission cost per unit of item
C_{ei}	The inventory holding emission cost per unit of item per unit of time
C_{et}	The transportation emission cost per unit of shipment

C_{em}	The PM emission cost per unit of activity.
t_p	The production time of the machine in one sub-cycle
t_m	The PMactivity time in one sub-cycle
θ	The percentage of defective items per batch
n	Number of shipments to deliver the batch to customer

3.1.2 Variables:

Q	The production quantity in one cycle
Q_{sc}	The production quantity in one sub-cycle
Q_T	The capacity of the vehicle to carry items
T_p	The production period in one cycle
T_D	The demand period in one cycle
x	The number of implementing maintenance activities in one cycle
t_T	The travel time of the vehicle

3.1.3 Abbreviations:

OC	The total ordering cost
PMC	The total PM cost
PC	The total procurement cost
DC	The total disposal cost
TrC	The total transportation cost
HC	The total holding cost
$PCEC$	The total production emission cost
$HCEC$	The total inventory holding emission cost
$TCEC$	The total transportation emission cost
$MCEC$	The total PM emission cost

3.2. Case I: Asynchronous production and consumption

The produced items in the production period of a cycle are used for demand satisfaction in the demand period of the same cycle and the production period of the next cycle. In this case, the goal is to determine the economic production quantity Q while considering the above assumption to minimize the total cost of the inventory system. The production period is divided into serial production and maintenance sub-

cycles. At first, production is performed to produce Q_{sc} during t_p time, and after that, the maintenance activities are performed during t_m . After the end of the last sub-cycle, $Q(1-\theta)$ defective items are recognized by a 100% inspection process and disposed of at the cost of C_s . The number of shipments is a specified parameter n according to company policy. The travel time of the vehicle is indicated by t_T . Also, system management should decide the vehicle capacity Q_T . Figure 1 depicts the inventory level for Case I.

[Please insert Figure 1 about here.]

A total cycle composed of a production cycle and demand cycle due to equation (1):

$$T = T_p + T_D \quad (1)$$

Regarding the line slope equation (2):

$$\tan \alpha = p = \frac{Q_{sc}}{t_p} \quad (2)$$

So, the production quantity in one production sub-cycle can be calculated as follow:

$$Q_{sc} = P t_p \quad (3)$$

The number of production and maintenance activities can be denoted by dividing the overall order quantity by the batch size in one sub-cycle:

$$x = \frac{Q}{Q_{sc}} = \frac{Q}{P t_p}. \quad (4)$$

The production period in one cycle can be indicated as:

$$T_p = x (t_p + t_m) = \frac{Q}{P t_p} (t_p + t_m) \quad (5)$$

Regarding the line slope equation:

$$\tan \beta = D = \frac{(1-\theta)Q}{(T_D + T_p)} \quad (6)$$

By simplifying equation (6):

$$T_D = \frac{(1-\theta)Q}{D} - T_p \quad (7)$$

Since in equation (5), the production period is calculated by the model's parameters, this equation is placed in equation (7) to calculate the demand period as:

$$T_D = \frac{(1-\theta)Q}{D} - \frac{Q}{Pt_p} (t_p + t_m) \quad (8)$$

And the transportation time of the vehicle is calculated as:

$$t_T = \frac{T_D}{n} = \frac{1}{n} \left[\frac{(1-\theta)Q}{D} - \frac{Q}{Pt_p} (t_p + t_m) \right] \quad (9)$$

Finally, the vehicle capacity can be expressed as:

$$Q_T = \frac{(1-\theta)Q}{n} \quad (10)$$

There are two types of setup costs in this model. One is related to the complete cycle, and another is for operation runs in each sub-cycle. Equation (11) shows the total set up cost of the system for one cycle of Case I:

$$OC = K + xK_s = K + \frac{Q}{Q_{sc}} K_s = K + \frac{Q}{Pt_p} K_s \quad (11)$$

The PMcost for a particular cycle can be obtained as:

$$PMC = xK_m = \frac{Q}{Q_{sc}} K_m = \frac{Q}{Pt_p} K_m \quad (12)$$

Procurement of a batch with size Q incurs the purchasing cost for inventory system as:

$$PC = CQ \quad (13)$$

The disposal of produced defective items by any method or policy imposes a disposal cost to the system, which can be indicated as:

$$DC = C_s (\theta Q) \quad (14)$$

The number of shipments is assumed to be the only and the only parameter that can influence the transportation cost of non-defective items. So, we computed the transportation cost of the system in one cycle by equation (15):

$$TrC = nK_T \quad (15)$$

We separate the holding cost of the inventory system. The inventory holding cost in the production period (T_p) can be written as follow:

$$HC_1 = h \left[\frac{Qt_p}{2} + Qt_m + \frac{Q(x-1)(t_p+t_m)}{2} \right] = h \left[\frac{Qt_p}{2} + Qt_m + \frac{Q \left(\frac{Q}{Pt_p} - 1 \right) (t_p+t_m)}{2} \right] = \frac{h}{2} \left[Qt_m + \frac{Q^2(t_p+t_m)}{Pt_p} \right] \quad (16)$$

A detailed description of the calculation procedure is presented in appendix A.

The holding cost of items during the demand period (T_D) can be achieved as:

$$HC_2 = h \left(\frac{n-1}{2n} \right) (1-\theta) QT_D = h \left(\frac{n-1}{2n} \right) \left[\left(\frac{(1-\theta)^2 Q^2}{D} \right) - \left(\frac{Q^2}{pt_p} (t_p+t_m) (1-\theta) \right) \right] \quad (17)$$

We also explain the extraction of the average inventory level during the demand period through appendix B.

The production emission cost is calculated as:

$$PCEC = C_{ep} D \quad (18)$$

The inventory holding emission cost is formulated as:

$$HCEC = C_{ei} \left[\frac{Qt_m}{2} + \frac{Q^2(t_p+t_m)}{2Pt_p} + \left(\frac{n-1}{2n} \right) \left[\left(\frac{(1-\theta)^2 Q^2}{D} \right) - \left(\frac{Q^2}{pt_p} (t_p+t_m) (1-\theta) \right) \right] \right]$$

And the transportation carbon emission is:

$$TCEC = C_{et} n \quad (20)$$

We model the PM emission cost as:

$$MCEC = C_{em} x = C_{em} \frac{Q}{P t_p} \quad (21)$$

The total cost objective function per unit of a cycle for Case I can be determined as:

$$\begin{aligned} TC(Q) = & K + CQ + \frac{Q}{P t_p} K_s + C_s \theta Q + \frac{Q}{P t_p} K_m + n K_T \\ & + \frac{h}{2} \left[Q t_m + \frac{Q^2 (t_p + t_m)}{P t_p} \right] + h \left(\frac{n-1}{2n} \right) \left[\left(\frac{(1-\theta)^2 Q^2}{D} \right) - \left(\frac{Q^2 (t_p + t_m)(1-\theta)}{P t_p} \right) \right] + C_{ep} D \\ & + C_{ei} \left[\frac{Q t_m}{2} + \frac{Q^2 (t_p + t_m)}{2 P t_p} + \left(\frac{n-1}{2n} \right) \left[\left(\frac{(1-\theta)^2 Q^2}{D} \right) - \left(\frac{Q^2 (t_p + t_m)(1-\theta)}{P t_p} \right) \right] \right] + C_{et} n + C_{em} \frac{Q}{P t_p} \end{aligned} \quad (22)$$

To computing the total cost during the planning horizon, we multiply $\frac{1}{T}$ and $TC(Q)$ and simplify the obtained formula, which results in equation (23) as:

$$\begin{aligned} TCU(Q) = & TC(Q) \times \frac{1}{T} = \frac{KD}{(1-\theta)Q} + \frac{CD}{(1-\theta)} + \frac{DK_s}{P t_p (1-\theta)} + \frac{C_s \theta D}{(1-\theta)} + \frac{DK_m}{P t_p (1-\theta)} \\ & + \frac{n K_T D}{(1-\theta)Q} + \frac{h}{2} \left[\frac{t_m D}{(1-\theta)} + \frac{Q (t_p + t_m) D}{P t_p (1-\theta)} \right] + h \left(\frac{n-1}{2n} \right) \left[(1-\theta)Q - \left(\frac{QD (t_p + t_m)}{P t_p} \right) \right] + \frac{C_{ep} D^2}{(1-\theta)Q} \\ & + C_{ei} \left[\frac{t_m D}{2(1-\theta)} + \frac{Q (t_p + t_m) D}{2 P t_p (1-\theta)} + \left(\frac{n-1}{2n} \right) \left[(1-\theta)Q - \left(\frac{QD (t_p + t_m)}{P t_p} \right) \right] \right] + \frac{n C_{et} D}{(1-\theta)Q} + \frac{D C_{em}}{P t_p (1-\theta)} \end{aligned} \quad (23)$$

We optimize the obtained function by an analytical method as follow:

In the following, it will be proved that the present function is convex, and there is a global minimum for this function. By Differentiation from the total cost function, equation (24) is elicited as follow:

$$\begin{aligned} \frac{d(TCU(Q))}{dQ} &= \frac{-KD}{(1-\theta)Q^2} + \frac{-nK_T D}{(1-\theta)Q^2} + \frac{hD(t_p+t_m)}{2Pt_p(1-\theta)} + h\left(\frac{n-1}{2n}\right) \left[(1-\theta) - \left(\frac{D(t_p+t_m)}{pt_p} \right) \right] \\ &\quad - \frac{C_{ep}D^2}{(1-\theta)Q^2} + C_{ei} \left[\frac{D(t_p+t_m)}{2Pt_p(1-\theta)} + \left(\frac{n-1}{2n} \right) \left[(1-\theta) - \left(\frac{D(t_p+t_m)}{pt_p} \right) \right] \right] - \frac{nC_{et}D}{(1-\theta)Q^2} \end{aligned} \quad (24)$$

The second derivation of this function is attained as:

$$\frac{d^2(TCU(Q))}{d^2Q} = \frac{KD}{(1-\theta)Q^3} + \frac{nK_T D}{(1-\theta)Q^3} + \frac{C_{ep}D^2}{(1-\theta)Q^3} + \frac{nC_{et}D}{(1-\theta)Q^3} \quad (25)$$

The second derivation of the objective function is always positive, so the convexity of the total cost function is approved, and we can calculate the EPQ by setting the first derivation equal to zero:

$$\begin{aligned} \frac{d(TCU(Q))}{dQ} &= \frac{-KD}{(1-\theta)Q^2} + \frac{-nK_T D}{(1-\theta)Q^2} + \frac{hD(t_p+t_m)}{2Pt_p(1-\theta)} + h\left(\frac{n-1}{2n}\right) \left[(1-\theta) - \left(\frac{D(t_p+t_m)}{pt_p} \right) \right] - \frac{C_{ep}D^2}{(1-\theta)Q^2} \\ &\quad + C_{ei} \left[\frac{D(t_p+t_m)}{2Pt_p(1-\theta)} + \left(\frac{n-1}{2n} \right) \left[(1-\theta) - \left(\frac{D(t_p+t_m)}{pt_p} \right) \right] \right] - \frac{nC_{et}D}{(1-\theta)Q^2} = 0 \end{aligned} \quad (26)$$

By simplifying the above equation, the Q^* is specified as:

$$Q^* = \sqrt{\frac{KD + nK_T D + C_{ep}D^2 + nC_{et}D}{\frac{hD(t_p+t_m)}{2Pt_p} + h\left(\frac{n-1}{2n}\right) \left[(1-\theta)^2 - \left(\frac{D(t_p+t_m)(1-\theta)}{pt_p} \right) \right] C_{ei} \left[\frac{D(t_p+t_m)}{2Pt_p} + \left(\frac{n-1}{2n} \right) \left[(1-\theta)^2 - \left(\frac{D(t_p+t_m)(1-\theta)}{pt_p} \right) \right] \right]}} \quad (27)$$

3.3.Case II: Simultaneous production and consumption

All of the explained assumptions for Case I are valid in this case except that we assume the produced items in a particular cycle are consumed in the same cycle, and a portion of produced items are consumed in the production period. Figure 2 shows the inventory level of Case II.

[Please insert Figure 2 about here.]

There is the assumption about the imperfect produced items in the system wherein they are accumulated during the production period T_p and finally disposed after the end of the production process. Regarding this situation, an extra hypothesis is needed to prevent the shortage during the production and maintenance time. If θQ_s be defective in each production sub-cycle, then the demand during maintenance and production is satisfied by consuming $(1-\theta)Q_s$, and the following equation in each sub-cycle should be valid:

$$(1-\theta)Q_s - Dt_m > 0 \quad (28)$$

A complete cycle is composed of the production period and demand period:

$$T = T_p + T_D \quad (1)$$

By using the line slope formula:

$$\tan \alpha = P - D = \frac{Q_{sc}}{t_p} \quad (29)$$

the Q_{sc} value is calculated as follow:

$$Q_{sc} = (P - D)t_p \quad (30)$$

Similar to Case II, the number of PM activities is calculated as:

$$x = \frac{Q}{Q_{sc}} = \frac{Q}{(P - D)t_p} \quad (31)$$

The production period of Case II is:

$$T_p = x(t_p + t_m) = \frac{Q}{(P - D)t_p}(t_p + t_m) \quad (32)$$

By considering the line slope equation:

$$\tan \gamma = D = \frac{(1-\theta)I_{max}}{T_D} \quad (33)$$

The demand period can be calculated as:

$$T_D = \frac{(1-\theta)I_{max}}{D} = \frac{x \left[(1-\theta)(P-D)t_p - (1-\theta)Dt_m \right]}{D} \quad (34)$$

The transportation time can be expressed as:

$$t_T = \frac{T_D}{n} = \frac{1}{n} \left[\frac{x \left[(1-\theta)(P-D)t_p - (1-\theta)Dt_m \right]}{D} \right] \quad (35)$$

We can calculate the vehicle's capacity as:

$$Q_T = \frac{(1-\theta)I_{max}}{n} = \frac{(1-\theta) \left[x(P-D)t_p - xDt_m \right]}{n} \quad (36)$$

The setup cost is imposed on the inventory system as:

$$K + xK_s = K + \frac{Q}{Q_{sc}} K_s = K + \frac{Q}{(P-D)t_p} K_s \quad (37)$$

The PM cost is expressed as:

$$xK_m = \frac{Q}{Q_{sc}} K_m = \frac{Q}{(P-D)t_p} K_m. \quad (38)$$

The purchasing cost for a batch with the size of Q can be calculated by multiplying the purchasing cost of an item and the batch size as:

$$CQ \quad (39)$$

As mentioned earlier, the defective items are accumulated during the production period, and then disposal of them should be done after the production operation. If C_s be the disposal cost per unit of an item, then the total disposal cost for a particular cycle can be formulated as:

$$DC = C_s \theta I_{max} = C_s \theta \frac{Q}{(P-D)t_p} \left[(P-D)t_p - Dt_m \right] \quad (40)$$

The transportation cost of Case II is as below:

$$Trc = n K_T \quad (41)$$

To formulate the holding cost, similar to Case I, the inventory holding cost for the production period and the demand period are obtained separately. In Case II and its production period, production and demand satisfaction occurred at the same time. To prevent the shortage, $(1-\theta)Q_{sc} - dt_m > 0$ should be valid. In continue, the inventory level before the implementation of maintenance activities in sub-cycle i is defined as I_{i1} and after the end of this activity as I_{i2} where:

$$I_{i1} = i(P-D)t_p - (i-1)Dt_m, \forall i = 1, \dots, x \quad (42)$$

$$I_{i2} = i(P-D)t_p - iDt_m, \forall i = 1, \dots, x \quad (43)$$

It is obvious that $I_{02} = 0$. By using the above equations, the average stock level in the production period is as follow:

$$A_1 = \sum_{i=1}^x \left[\left(\frac{I_{i1} + I_{(i-1)2}}{2} \right) t_p + \left(\frac{I_{i1} + I_{i2}}{2} \right) t_m \right] \quad (44)$$

By expanding the equation (44):

$$A_1 = \frac{1}{4} \left[\frac{2Q^2}{(P-D)} - \frac{2QDt_m(Q - (P-D)t_p)}{(P-D)^2 t_p} + \frac{2Qt_p t_m(Q + (P-D)t_p)}{(P-D)t_p} - \frac{2Q^2 Dt_m^2}{(P-D)^2 t_p^2} \right] \quad (45)$$

So, equation (46) shows the inventory holding cost in the production period:

$$HC_1 = h \left[\frac{2Q^2}{(P-D)} - \frac{2QDt_m(Q - (P-D)t_p)}{(P-D)^2 t_p} + \frac{2Qt_m(Q + (P-D)t_p)}{(P-D)t_p} - \frac{2Q^2 Dt_m^2}{(P-D)^2 t_p^2} \right] \quad (46)$$

The inventory holding cost during the demand period is obtained similar to Case I according to equation (47):

$$HC_2 = h \left(\frac{n-1}{2n} \right) \left(\frac{(1-\theta) \left[Q - \frac{Q}{(P-D)t_p} Dt_m \right]^2}{D} \right) \quad (47)$$

The detailed procedure is clarified in Appendix C.

The carbon emission cost for Case II can be formulated as:

$$PCEC = C_{ep} D \quad (48)$$

The inventory holding emission cost is:

$$C_{ei} \left(\frac{Q^2}{2(P-D)} - \frac{QDt_m(Q-(P-D)t_p)}{2(P-D)^2 t_p} + \frac{Qt_m(Q+(P-D)t_p)}{2(P-D)t_p} - \frac{Q^2 Dt_m^2}{2(P-D)^2 t_p^2} + \left(\frac{n-1}{2n} \right) \left(\frac{(1-\theta) \left[Q - \frac{Q}{(P-D)t_p} Dt_m \right]^2}{D} \right) \right) \quad (49)$$

The transportation emission cost is as follow:

$$PCEC = C_{et} n \quad (50)$$

The PM emission cost can be written as:

$$MCEC = C_{em} x = C_{em} \frac{Q}{(P-D) t_p} \quad (51)$$

The total cost of Case II per unit of the cycle is as given below:

$$\begin{aligned} TC(Q) = & K + \frac{Q}{(P-D)t_p} K_s + \frac{Q}{(P-D)t_p} K_m + CQ + C_s \theta \frac{Q}{(P-D)t_p} [(p-D)t_p - Dt_m] + n K_T \\ & + h \left[\frac{Q^2}{2(P-D)} - \frac{QDt_m(Q-(P-D)t_p)}{2(P-D)^2 t_p} + \frac{Qt_m(Q+(P-D)t_p)}{2(P-D)t_p} - \frac{Q^2 Dt_m^2}{2(P-D)^2 t_p^2} \right] \\ & + h \left(\frac{n-1}{2n} \right) \left(\frac{(1-\theta) \left[Q - \frac{Q}{(P-D)t_p} Dt_m \right]^2}{D} \right) + C_{ep} D + C_{ei} \left(\frac{Q^2}{2(P-D)} - \frac{QDt_m(Q-(P-D)t_p)}{2(P-D)^2 t_p} \right. \\ & \left. + \frac{Qt_m(Q+(P-D)t_p)}{2(P-D)t_p} - \frac{Q^2 Dt_m^2}{2(P-D)^2 t_p^2} + \left(\frac{n-1}{2n} \right) \left(\frac{(1-\theta) \left[Q - \frac{Q}{(P-D)t_p} Dt_m \right]^2}{D} \right) \right) \end{aligned} \quad (52)$$

Finally, to obtain the total cost over the planning horizon, we should multiply $\frac{1}{T}$ and the total cost of the model as:

$$\begin{aligned}
TCU(Q) = TC(Q) \times \frac{1}{T} = & \frac{1}{[pt_p - \theta pt_p + \theta Dt_p + \theta Dt_m]} \left\{ \frac{KDt_p(P-D)}{Q} + K_s D + K_m D \right. \\
& + CDt_p(P-D) + \frac{nK_T Dt_p(P-D)}{Q} + C_s \theta Dt_p(P-D) \left(1 - \frac{Dt_m}{t_p(P-D)} \right) + \frac{nC_{et} Dt_p(P-D)}{Q} + C_{em} D \\
& + h \left[\frac{QDt_p}{2} - \frac{D^2 t_m (Q - (P-D)t_p)}{2(P-D)} + \frac{Dt_m (Q + (P-D)t_p)}{2} - \frac{QD^2 t_m^2}{2t_p(P-D)} \right] + \frac{C_{ep} D^2 t_p(P-D)}{Q} \\
& + h \left(\frac{n-1}{2n} \right) (1-\theta)(P-D)Qt_p \left(1 - \frac{Dt_m}{t_p(P-D)} \right)^2 + C_{ei} \left[\frac{QDt_p}{2} - \frac{D^2 t_m (Q - (P-D)t_p)}{2(P-D)} + \frac{Dt_m (Q + (P-D)t_p)}{2} \right. \\
& \left. \left. - \frac{QD^2 t_m^2}{2t_p(P-D)} + \left(\frac{n-1}{2n} \right) (1-\theta)(P-D)Qt_p \left(1 - \frac{Dt_m}{t_p(P-D)} \right)^2 \right] \right\}
\end{aligned} \tag{53}$$

It will be approved that the presented total cost (53) is convex, and there is a global minimum for this function. Similar to Case I, the first derivation of the objective function is calculated as:

$$\begin{aligned}
\frac{d(TCU(Q))}{dQ} = & \frac{1}{[pt_p - \theta pt_p + \theta Dt_p + \theta Dt_m]} \left\{ \frac{-KDt_p(P-D)}{Q^2} - \frac{nK_T Dt_p(P-D)}{Q^2} \right. \\
& - \frac{C_{ep} D^2 t_p(P-D)}{Q^2} \\
& + h \left[\frac{Dt_p}{2} - \frac{D^2 t_m}{2(P-D)} + \frac{Dt_m}{2} - \frac{D^2 t_m^2}{2t_p(P-D)} \right] + h \left(\frac{n-1}{2n} \right) (1-\theta)(P-D)t_p \left(1 - \frac{Dt_m}{t_p(P-D)} \right)^2 \\
& + C_{ei} \left[\frac{Dt_p}{2} - \frac{D^2 t_m}{2(P-D)} + \frac{Dt_m}{2} - \frac{D^2 t_m^2}{2t_p(P-D)} + \left(\frac{n-1}{2n} \right) (1-\theta)(P-D)t_p \left(1 - \frac{Dt_m}{t_p(P-D)} \right)^2 \right] \\
& \left. - \frac{nC_{et} Dt_p(P-D)}{Q^2} \right\}
\end{aligned} \tag{54}$$

And the second derivation as:

$$\frac{d^2(TCU(Q))}{d^2Q} = \frac{1}{Q^3 [pt_p - \theta pt_p + \theta Dt_p + \theta Dt_m]} \quad (55)$$

$$\{2KDt_p(P-D) + 2nK_T Dt_p(P-D) + 2C_{ep} D^2 t_p(P-D) + 2nC_{et} Dt_p(P-D)\}$$

Since the equation (55) is always positive, the convexity of the objective function for Case II is approved, and the EPQ is obtained by setting the first derivation equal to zero value as:

$$Q^* = \sqrt{\frac{Dt_p(P-D)[K + nK_T D + C_{ep} D + nC_{et}]}{(C_{ei} + h) \left[\left(\frac{n-1}{2n} \right) (1-\theta)(P-D)t_p \left(1 - \frac{Dt_m}{t_p(P-D)} \right)^2 \right] + (C_{ei} + h) \left[\frac{Dt_p}{2} - \frac{D^2 t_m}{2(P-D)} + \frac{Dt_m}{2} - \frac{D^2 t_m^2}{2t_p(P-D)} \right]}} \quad (56)$$

The optimal value of the dependent variable such as x , Q_T and etc. are calculated regarding the Q^* . There is an important note about the value of x^* which is shown the number of maintenance activities during the production period. It is possible that this variable does not get an integer value, and this is not a feasible solution. A simple algorithm is presented for calculating the optimal number of maintenance activities when there is an infeasible solution. The algorithm includes eight steps as follow and can be used for both cases:

- 1- Find the x^* regarding the Q^*
- 2- If x^* is an integer, set $x^* = x^*$ and stop the algorithm.
- 3- If x^* is not integer, set $x_l = \lceil x^* \rceil$ and $x_u = \lfloor x^* \rfloor + 1$.
- 4- Regarding x_l and Q_{sc} , calculate the Q_l by equation (4) for Case I and by equation (31) for Case II.
- 5- Calculate the TCU_l for Case I by equation (23) and for Case II by equation (53).
- 6- Regarding x_l and Q_{sc} , Calculate the Q_u by equation (4) for Case I and by equation (31) for Case II.
- 7- Calculate the TCU_u for Case I by equation (23) and for Case II by equation (53).
- 8- Find the minimum total cost as $TCU^* = \min \{TCU_l, TCU_u\}$ and set the related Q as Q^* and the related x as x^* .

Figure 3 shows the flowchart of the proposed algorithm.

[Please insert Figure 3 about here.]

Computational experiments

To investigate the performance of the proposed models, a numerical example is presented in this section. After that, a sensitivity analysis is implemented to see how the model may be affected by changing the key parameters.

4.1.Numerical example

An inventory system is considered with $P = 12000$, $D = 6000$, $C = 50$, $C_s = 3$, $K = 1500$, $K_s = 3$, $K_m = 200$, $h = 0.15$, $K_T = 500$, $t_p = 0.5$, $t_m = 0.02$, $\theta = 0.07$, $n = 5$, $C_{ep} = 30$, $C_{ei} = 10$, $C_{et} = 40$ and $C_{em} = 25$. [Figure 4](#) and [Figure 5](#) show the convex behavior of objective function for Case I and Case II, respectively.

[Please insert Figure 4 about here.]

[Please insert Figure 5 about here.]

The computational results are presented in [Table 2](#).

[Please insert Table2 about here.]

The better performance of Case II is validated according to this table. However, the number of maintenance activities is infeasible, so we need to execute the algorithm. The below results in [Table 3](#) are achieved after the implementation of the algorithm:

[Please insert Table3 about here.]

The better performance of Case II is also approved after the implementation of the algorithm, based on [Table 3](#). Regarding that, when there is demand satisfaction during the overall cycle (Case II), the inventory system's total cost would be fewer (nearly 46%) than when there is demand satisfaction just in the demand period (Case I).

4.2.Sensitivity analysis

The example's parameters may not always be fixed, and various factors can influence them. The parameters are changed in a range -30 to $+30$ percentage, and both presented models are solved to see how the optimal solutions are affected.

Figure 6 and Figure 7 are drawn to show that as the defective item's rate is increased, more disposal cost is imposed on the system, which raises the system's overall cost. We can conclude from Table 4 that the systems with a higher defective rate should place large order sizes to prevent the shortage.

[Please insert Table 4 about here.]

[Please insert Figure 6 about here.]

[Please insert Figure 7 about here.]

Table 5 illustrates that when maintenance time is increased, the optimal order quantity of Case I and the optimal total cost of Case II are decreased. Interestingly, increasing in the maintenance activities time results in an increasing trend in the imposed on Case I and a decreasing trend in the imposed on Case II. The adverse behavior of cost functions is depicted in Figure 8 and Figure 9.

[Please insert Table 5 about here.]

[Please insert Figure 8 about here.]

[Please insert Figure 9 about here.]

It was predictable that when the delivery cost is increased, the system needs to decrease total transportation costs by delivering more items to the demand point in one shipment. Due to the correlation between Q and Q_T , the batch size should be increased logically. We proved these hypotheses by increasing delivery cost, solving the related models, and presenting the results in Table 6. It is obvious that K_T is a cost parameter, and increasing a cost parameter will increase the total cost of cases. The trends of change in cost functions are shown in Figure 10 and Figure 11.

[Please insert Table 6 about here.]

[Please insert Figure 10 about here.]

[Please insert Figure 11 about here.]

We analyzed the impact of maintenance activities cost on the cases's performance. From Table 7 results, we found that K_m does not affect the optimal batch size. Increasing the total cost values of Case I and Case II is occurred when the maintenance cost is increased. These results are presented in Figure 12 and Figure 13.

[Please insert Table7 about here.]

[Please insert Figure 12 about here.]

[Please insert Figure 13 about here.]

The considered cost parameters for the emission of the system may not be stable due to some reasons such as regulations change. By increasing the production emission cost parameter, we concluded that the management should attempt to compensating increase by producing fewer items. An increasing trend in total costs is inevitable in this state. The results are presented in [Table 8](#), [Figure 14](#), and [Figure 15](#). As presented below, Case I's optimal order size and total cost are more sensitive to change in this parameter than Case II ones.

[Please insert Table8 about here.]

[Please insert Figure 14 about here.]

[Please insert Figure 15 about here.]

5. Conclusions and feature directions

Equipmentmaintenance and related activities, transportation policies, and quality aspects are among the competitive business environment challenges. On the other hand, global warming problems cause different pressure from governments on companies to consider these concerns in their operations. We found that nobody studiesa sustainableproduction-inventory model in a manner that imperfect quality of items, PMplan, and multiple shipments policy be considered.Twocases werestudied and mathematically formulated. By proving the convexity of function, the optimization was done by derivation methods. We comparedthe numerical example of cases and concluded that simultaneous production and consumption (Case II) imposes less cost (less than half) to the system than asynchronous production and consumption (Case I).Also, the EPQwill also be lower in Case II. Extensive sensitivity analyses are also performed to get insightsinto the performances of models.

Some related real-world assumptions can make the model more realistic. Other maintenance categories such as condition-based maintenance can be incorporated in the models in addition to preventive ones. We assumed that there is only one vehicle for the transformation of items where this is may not be valid in practice. Formulating the problem by considering the percentage of defective items as a random variable and consideringthe rework process can be another extension.

References

- [1] Whitmann, H., "How many parts to make at once," *The magazine of management*, pp. 135-136, 1913.
- [2] Taft, E., "The most economical production lot," *Iron Age*, vol. **101**, no. 18, pp. 1410-1412, 1918.
- [3] Tiwari, S., Daryanto, Y., and Wee, H. M., "Sustainable inventory management with deteriorating and imperfect quality items considering carbon emission," *Journal of Cleaner Production*, vol. **192**, pp. 281-292, 2018.
- [4] Mokhtari, H., Naimi-Sadigh, A., and Salmasnia, A., "A computational approach to economic production quantity model for perishable products with backordering shortage and stock-dependent demand," *Scientia Iranica. Transaction E, Industrial Engineering*, vol. **24**, no. 4, pp. 2138-2151, 2017.
- [5] Hemmati, M., Fatemi Ghomi, S., and Sajadieh, M. S., "Inventory of complementary products with stock-dependent demand under vendor-managed inventory with consignment policy," *Scientia Iranica*, vol. **25**, no. 4, pp. 2347-2360, 2018.
- [6] Mokhtari, H. and Asadkhani, J., "Economic Order Quantity for Imperfect Quality Items under Inspection Errors, Batch Replacement and Multiple Sales of Returned Items," *Scientia Iranica*, 2019.
- [7] Mokhtari, H. and Fallahi, A., "THE ECONOMIC PRODUCTION QUANTITY (EPQ) MODEL CONSIDERING INFLATION, TIME VALUE OF MONEY AND INVESTMENT VARIABLE IN PRODUCTION CAPACITY," *Sharif Journal of Industrial Engineering & Management*, vol. **35.1**, no. 2.2, pp. 53-67, 2020, doi: 10.24200/j65.2019.51273.1902.
- [8] Mokhtari, H., "Joint ordering and reuse policy for reusable items inventory management," *Sustainable Production and Consumption*, vol. **15**, pp. 163-172, 2018.
- [9] Hua, G., Cheng, T., and Wang, S., "Managing carbon footprints in inventory management," *International Journal of Production Economics*, vol. **132**, no. 2, pp. 178-185, 2011.
- [10] Bouchery, Y., Ghaffari, A., Jemai, Z., and Dallery, Y., "Including sustainability criteria into inventory models," *European Journal of Operational Research*, vol. **222**, no. 2, pp. 229-240, 2012.
- [11] Chen, X., Benjaafar, S., and Elomri, A., "The carbon-constrained EOQ," *Operations Research Letters*, vol. **41**, no. 2, pp. 172-179, 2013.
- [12] Battini, D., Persona, A., and Sgarbossa, F., "A sustainable EOQ model: Theoretical formulation and applications," *International Journal of Production Economics*, vol. **149**, pp. 145-153, 2014.
- [13] Hovelaque, V. and Bironneau, L., "The carbon-constrained EOQ model with carbon emission dependent demand," *International Journal of Production Economics*, vol. **164**, pp. 285-291, 2015.
- [14] Zhang, Y., Li, L.-y., Tian, X.-q., and Feng, C., "Inventory management research for growing items with carbon-constrained," in *2016 35th Chinese Control Conference (CCC)*, 2016: IEEE, pp. 9588-9593.
- [15] Kazemi, N., Abdul-Rashid, S. H., Ghazilla, R. A. R., Shekarian, E., and Zanoni, S., "Economic order quantity models for items with imperfect quality and emission considerations," *International Journal of Systems Science: Operations & Logistics*, vol. **5**, no. 2, pp. 99-115, 2018.
- [16] Taleizadeh, A. A., Soleymanfar, V. R., and Govindan, K., "Sustainable economic production quantity models for inventory systems with shortage," *Journal of Cleaner Production*, vol. **174**, pp. 1011-1020, 2018.
- [17] Lee, J.-Y., "Investing in carbon emissions reduction in the EOQ model," *Journal of the Operational Research Society*, pp. 1-12, 2019.

- [18] Mishra, U., Wu, J.-Z., Tsao, Y.-C., and Tseng, M.-L., "Sustainable inventory system with controllable non-instantaneous deterioration and environmental emission rates," *Journal of Cleaner Production*, vol. **244**, p. 118807, 2020.
- [19] Salameh, M. and Jaber, M., "Economic production quantity model for items with imperfect quality," *International journal of production economics*, vol. **64**, no. 1-3, pp. 59-64, 2000.
- [20] Chang, H.-C., "An application of fuzzy sets theory to the EOQ model with imperfect quality items," *Computers & Operations Research*, vol. **31**, no. 12, pp. 2079-2092, 2004.
- [21] Rezaei, J., "Economic order quantity model with backorder for imperfect quality items," in *Proceedings. 2005 IEEE International Engineering Management Conference, 2005.*, 2005, vol. **2**: IEEE, pp. 466-470.
- [22] Lo, S.-T., Wee, H.-M., and Huang, W.-C., "An integrated production-inventory model with imperfect production processes and Weibull distribution deterioration under inflation," *International Journal of Production Economics*, vol. **106**, no. 1, pp. 248-260, 2007.
- [23] Chung, K.-J., Her, C.-C., and Lin, S.-D., "A two-warehouse inventory model with imperfect quality production processes," *Computers & Industrial Engineering*, vol. **56**, no. 1, pp. 193-197, 2009.
- [24] Sarkar, B. and Moon, I., "An EPQ model with inflation in an imperfect production system," *Applied Mathematics and Computation*, vol. **217**, no. 13, pp. 6159-6167, 2011.
- [25] Konstantaras, I., Skouri, K., and Jaber, M., "Inventory models for imperfect quality items with shortages and learning in inspection," *Applied mathematical modelling*, vol. **36**, no. 11, pp. 5334-5343, 2012.
- [26] Wee, H.-M., Wang, W.-T., and Yang, P.-C., "A production quantity model for imperfect quality items with shortage and screening constraint," *International Journal of Production Research*, vol. **51**, no. 6, pp. 1869-1884, 2013.
- [27] Chang, C.-T., Cheng, M.-C., and Soong, P.-Y., "Impacts of inspection errors and trade credits on the economic order quantity model for items with imperfect quality," *International Journal of Systems Science: Operations & Logistics*, vol. **3**, no. 1, pp. 34-48, 2016.
- [28] Mokhtari, H., "Optimal lot size in a manufacturing system with imperfect raw materials and defective finished products," *Scientia Iranica*, vol. **26**, no. 4, pp. 2561-2578, 2019.
- [29] Khakzad, A. and Gholamian, M. R., "The effect of inspection on deterioration rate: An inventory model for deteriorating items with advanced payment," *Journal of Cleaner Production*, p. 120117, 2020.
- [30] Dhaka, V., Pareek, S., and Mittal, M., "Stock-Dependent Inventory Model for Imperfect Items Under Permissible Delay in Payments," in *Optimization and Inventory Management*: Springer, 2020, pp. 181-194.
- [31] Chiu, Y.-S. P., Liu, S.-C., Chiu, C.-L., and Chang, H.-H., "Mathematical modeling for determining the replenishment policy for EMQ model with rework and multiple shipments," *Mathematical and Computer Modelling*, vol. **54**, no. 9-10, pp. 2165-2174, 2011.
- [32] Cárdenas-Barrón, L. E., Taleizadeh, A. A., and Treviño-Garza, G., "An improved solution to replenishment lot size problem with discontinuous issuing policy and rework, and the multi-delivery policy into economic production lot size problem with partial rework," *Expert Systems with Applications*, vol. **39**, no. 18, pp. 13540-13546, 2012.
- [33] Ritha, W. and Martin, N., "Replenishment policy for EMQ model with rework, multiple shipments, switching and packaging," *Int. J. Eng. Res. Technol*, vol. **2**, no. 1, pp. 1-8, 2013.
- [34] Taleizadeh, A. A., Kalantari, S. S., and Cárdenas-Barrón, L. E., "Determining optimal price, replenishment lot size and number of shipments for an EPQ model with rework and multiple

- shipments," *Journal of Industrial and Management Optimization*, vol. **11**, no. 4, pp. 1059-1071, 2015.
- [35] Taleizadeh, A. A., Kalantari, S. S., and Cárdenas-Barrón, L. E., "Pricing and lot sizing for an EPQ inventory model with rework and multiple shipments," *Top*, vol. **24**, no. 1, pp. 143-155, 2016.
 - [36] Priyan, S. and Uthayakumar, R., "Setup cost reduction EMQ inventory system with probabilistic defective and rework in multiple shipments management," *International Journal of System Assurance Engineering and Management*, vol. **8**, no. 2, pp. 223-241, 2017.
 - [37] Kalantari, S. S. and Taleizadeh, A. A., "Mathematical modelling for determining the replenishment policy for deteriorating items in an EPQ model with multiple shipments," *International Journal of Systems Science: Operations & Logistics*, pp. 1-8, 2018.
 - [38] Rezg, N., Dellagi, S., and Chelbi, A., "Joint optimal inventory control and preventive maintenance policy," *International Journal of Production Research*, vol. **46**, no. 19, pp. 5349-5365, 2008.
 - [39] Liao, G.-L., Chen, Y. H., and Sheu, S.-H., "Optimal economic production quantity policy for imperfect process with imperfect repair and maintenance," *European Journal of Operational Research*, vol. **195**, no. 2, pp. 348-357, 2009.
 - [40] Widyadana, G. A. and Wee, H. M., "Optimal deteriorating items production inventory models with random machine breakdown and stochastic repair time," *Applied Mathematical Modelling*, vol. **35**, no. 7, pp. 3495-3508, 2011.
 - [41] Wee, H. M. and Widyadana, G. A., "A production model for deteriorating items with stochastic preventive maintenance time and rework process with FIFO rule," *Omega*, vol. **41**, no. 6, pp. 941-954, 2013.
 - [42] Liao, G.-L., "Production and maintenance policies for an epq model with perfect repair, rework, free-repair warranty, and preventive maintenance," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. **46**, no. 8, pp. 1129-1139, 2015.
 - [43] Jafari, L. and Makis, V., "Joint optimal lot sizing and preventive maintenance policy for a production facility subject to condition monitoring," *International Journal of Production Economics*, vol. **169**, pp. 156-168, 2015.
 - [44] Peng, H. and van Houtum, G.-J., "Joint optimization of condition-based maintenance and production lot-sizing," *European Journal of Operational Research*, vol. **253**, no. 1, pp. 94-107, 2016.
 - [45] Nasr, W. W., Salameh, M., and Moussawi-Haidar, L., "Economic production quantity with maintenance interruptions under random and correlated yields," *International Journal of Production Research*, vol. **55**, no. 16, pp. 4544-4556, 2017.
 - [46] Taleizadeh, A. A., "A constrained integrated imperfect manufacturing-inventory system with preventive maintenance and partial backordering," *Annals of Operations Research*, vol. **261**, no. 1-2, pp. 303-337, 2018.
 - [47] Lai, X., Chen, Z., and Bidanda, B., "Optimal decision of an economic production quantity model for imperfect manufacturing under hybrid maintenance policy with shortages and partial backlogging," *International Journal of Production Research*, vol. **57**, no. 19, pp. 6061-6085, 2019.
 - [48] Tsao, Y.-C., Lee, P.-L., Liao, L.-W., Zhang, Q., Vu, T.-L., and Tsai, J., "Imperfect economic production quantity models under predictive maintenance and reworking," *International Journal of Systems Science: Operations & Logistics*, pp. 1-14, 2019.
 - [49] Mokhtari, H. and Asadkhani, J., "Extended Economic Production Quantity Models with Preventive Maintenance," *Scientia Iranica*, 2019.

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Appendix A: Derivation of inventory holding cost during production period for Case I

If $x = 1$:

$$HC_1 = h \left[1 \left(\frac{Qt_p}{2} + Qt_m \right) \right] \quad (A1)$$

If $x = 2$:

$$HC_1 = h \left[2 \left(\frac{Qt_p}{4} + \frac{Qt_m}{4} \right) + 1 \left(\frac{Q}{2} (t_p + t_m) \right) \right] \quad (A2)$$

If $x = 3$:

$$HC_1 = h \left[3 \left(\frac{Qt_p}{6} + \frac{Qt_m}{3} \right) + (1+2) \left(\frac{Q}{3} (t_p + t_m) \right) \right] \quad (A3)$$

If $x = x$:

$$HC_1 = h \left[x \left(\frac{Qt_p}{2x} + \frac{Qt_m}{x} \right) + \left(\frac{x(x-1)}{2} \right) \left(\frac{Q}{x} (t_p + t_m) \right) \right] = h \left[\left(\frac{Qt_p}{2} + Qt_m \right) + \left(\frac{Q(x-1)(t_p + t_m)}{2} \right) \right] \quad (A4)$$

The production period in Case I for different value of x is represented in Figure A1:

[Please insert Figure A1 about here.]

Appendix B: Derivation of inventory holding cost during demand period for Case I

If $n = 1$:

$$HC_2 = 0 \quad (B1)$$

If $n = 2$:

$$HC_2 = h \left(\frac{(1-\theta)Q}{2} \times \frac{T_D}{2} \right) = h \left(\frac{1}{2^2} \right) (1-\theta)QT_D \quad (B2)$$

If $n = 3$:

$$HC_2 = h \left(\frac{2(1-\theta)Q}{3} \times \frac{T_D}{3} + \frac{(1-\theta)Q}{3} \times \frac{T_D}{3} \right) = h \left(\frac{1+2}{3^2} \right) (1-\theta)QT_D \quad (B3)$$

If $n = n$

$$HC_2 = h \left(\frac{\sum_{i=1}^{n-1} i}{n^2} \right) (1-\theta)QT_D = h \left(\frac{\frac{n(n-1)}{2}}{n^2} \right) (1-\theta)QT_D = h \left(\frac{n-1}{2n} \right) (1-\theta)QT_D \quad (B4)$$

The demand period in Case I for different value of n is represented in Figure B1:

[Please insert Figure B1 about here.]

Appendix C: Derivation of inventory holding cost during Demand period for Case II

If $n = 1$:

$$HC_2 = 0 \quad (C1)$$

If $n = 2$:

$$HC_2 = h \left(\frac{(1-\theta)I_{max}}{2} \times \frac{T_D}{2} \right) = h \left(\frac{1}{2^2} \right) (1-\theta)I_{max}T_D \quad (C2)$$

If $n = 3$:

$$HC_2 = h \left(\frac{2(1-\theta)I_{max}}{3} \times \frac{T_D}{3} + \frac{(1-\theta)I_{max}}{3} \times \frac{T_D}{3} \right) = h \left(\frac{1+2}{3^2} \right) (1-\theta) I_{max} T_D \quad (C3)$$

If $n = n$:

$$HC_2 = h \left(\frac{\sum_{i=1}^{n-1} i}{n^2} \right) (1-\theta) I_{max} T_D = h \left(\frac{\frac{n(n-1)}{2}}{n^2} \right) (1-\theta) I_{max} T_D = h \left(\frac{n-1}{2n} \right) (1-\theta) I_{max} T_D \quad (C4)$$

We show that $T_D = \frac{I_{max}}{D}$, $I_{max} = x(P-D)t_p - xDt_m$ and $x = \frac{Q}{Q_{sc}} = \frac{Q}{(P-D)t_p}$ so:

$$HC_2 = h \left(\frac{n-1}{2n} \right) \frac{(1-\theta)I_{max}^2}{D} = h \left(\frac{n-1}{2n} \right) \left(\frac{(1-\theta)[x(P-D)t_p - xDt_m]^2}{D} \right) = h \left(\frac{n-1}{2n} \right) \left(\frac{(1-\theta) \left[\frac{Q}{(P-D)t_p} (P-D)t_p - \frac{Q}{(P-D)t_p} Dt_m \right]^2}{D} \right) \quad (C5)$$

The demand period in Case II for different value of n is represented in Figure C1:

[Please insert Figure C1 about here.]

Figure captions

Figure 1: Inventory level of Case I

Figure 2: Inventory level of Case II

Figure 3: The flowchart of the proposed algorithm

Figure 4: Total cost function behavior of Case I

Figure 5: Total cost function behavior of Case II

Figure 6: Sensitivity analysis of Case I total cost to θ

Figure 7: Sensitivity analysis of Case II total cost to θ

Figure 8: Sensitivity analysis of Case I total cost to t_m

Figure 9: Sensitivity analysis of Case II total cost to t_m

Figure 10: Sensitivity analysis of Case I total cost to K_T

Figure 11: Sensitivity analysis of Case II total cost to K_T

Figure 12: Sensitivity analysis of Case I total cost to K_m

Figure 2: Inventory level of Case II

Figure 3: The flowchart of the proposed algorithm

Figure 4: Total cost function behavior of Case I

Figure 5: Total cost function behavior of Case II

Figure 6: Sensitivity analysis of Case I total cost to θ

Figure 7: Sensitivity analysis of Case II total cost θ

Figure 8: Sensitivity analysis of Case I total cost to t_m

Figure 9: Sensitivity analysis of Case II total cost to t_m

Figure 10: Sensitivity analysis of Case I total cost to K_T

Figure 11: Sensitivity analysis of Case II total cost to K_T

Figure 12: Sensitivity analysis of Case I total cost to K_m

Figure 13: Sensitivity analysis of Case II total cost to K_m

Figure 14: Sensitivity analysis of Case I total cost to C_{ep}

Figure 15: Sensitivity analysis of Case II total cost to C_{ep}

Figure A1: The production period in Case I

Figure B1: The demand period in Case I

Figure C1: The demand period in Case II

Tables

Table 1: The features of the related inventory systems in the literature

Paper	Year	Model type		Features			
		EOQ	EPQ	Maintenance plan	Imperfect quality	Multiple shipments	Carbon emission
Salameh and Jaber [19]	2000	*			*		
Chang [20]	2004	*			*		
Rezaei [21]	2005	*			*		
Lo, et al. [22]	2007	*	*		*		
Rezg, et al. [38]	2008		*	*			
Chung, et al. [23]	2009	*			*		
Liao, et al. [39]	2009		*	*	*		
Hua, et al. [9]	2011	*					*
Chiu, et al. [31]	2011		*		*	*	
Widyadana and Wee [40]	2011		*	*			
Sarkar and Moon [24]	2011		*		*		
Bouchery, et al. [10]	2012	*					*
Konstantaras, et al. [25]	2012	*			*		
CáRdenas-BarróN, et al. [32]	2012		*		*	*	
Chen, et al. [11]	2013	*					*
Wee, et al. [26]	2013		*		*		
Ritha and Martin [33]	2013		*		*	*	
Wee and Widyadana [41]	2013		*	*	*		

Paper	Year	Model type		Features			
		EOQ	EPQ	Maintenance plan	Imperfect quality	Multiple shipments	Carbon emission
Battini, et al. [12]	2014	*				*	*
Taleizadeh, et al. [34]	2015		*		*	*	
Liao [42]	2015		*	*	*		
Jafari and Makis [43]	2015		*	*			
Chang, et al. [27]	2016	*			*		
Taleizadeh, et al. [35]	2016		*		*	*	
Peng and van Houtum [44]	2016		*	*			
Zhang, et al. [14]	2016	*					*
Priyan and Uthayakumar [36]	2017		*		*	*	
Nasr, et al. [45]	2017		*	*	*		
Kazemi, et al. [15]	2018	*			*		*
Taleizadeh [46]	2018		*	*	*		
Kalantari and Taleizadeh [37]	2018		*			*	
Taleizadeh, et al. [16]	2018		*				*
Mokhtari [28]	2019	*	*		*		
Lai, et al. [47]	2019		*	*	*		
Lee [17]	2019	*					*
Tsao, et al. [48]	2019		*	*	*		
Mokhtari and Asadkhani [49]	2019		*	*	*		
Khakzad and Gholamian [29]	2020	*			*		

Paper	Year	Model type		Features			
		EOQ	EPQ	Maintenance plan	Imperfect quality	Multiple shipments	Carbon emission
Mishra, et al. [18]	2020	*					*
Dhaka, et al. [30]	2020	*			*		

Table 2: The computational results of the numerical example

	Q^*	TCU^*	Q_{sc}^*	Q_T^*	T_P^*	T_D^*	T^*	t_T^*	x^*
Case I	16246.69	471128.30	6000.00	3021.88	1.41	1.11	2.52	0.22	2.71
Case II	11371.62	257275.59	3000.00	2030.52	1.97	1.69	3.66	0.34	3.79

Table 3: The computational after the execution of the algorithm

	Q^*	TCU^*	Q_{sc}^*	Q_T^*	T_P^*	T_D^*	T^*	t_T^*	x^*
Case I	18000.00	471897.21	6000.00	3348.00	1.56	1.23	2.79	0.25	3.00
Case II	12000.00	257421.09	3000.00	2142.72	2.08	1.79	3.87	0.77	4.00

Table 4: Sensitivity analysis of Cases to the portion of defective items

		Optimal value				Change percentage			
		Q		Tc		Q		Tc	
Parameter	Change	Case I	Case II	Case I	Case II	Case I	Case II	Case I	Case II
θ	−30%	16026.14	11319.70	462296.20	254890.46	−1.36	−0.46	−1.87	−0.93
	−20%	16099.41	11336.93	465192.79	255680.24	−0.91	−0.31	−1.26	−0.62
	−10%	16172.93	11354.23	468136.46	256475.27	−0.45	−0.15	−0.64	−0.31
	0%	16246.69	11371.62	471128.30	257275.59	0.00	0.00	0.00	0.00
	+10%	16320.69	11389.08	474169.49	258081.26	+0.46	+0.15	+0.65	+0.31

+20%	16394.92	11406.63	477261.19	258892.33	+0.91	+0.31	+1.30	+0.63
+30%	16469.37	11424.26	480404.64	259708.85	+1.37	+0.46	+1.97	+0.95

Table 5: Sensitivity analysis of Cases to the maintenance activities time

Parameter	Change	Optimal value				Change percentage			
		Q		T_c		Q		T_c	
		<i>Case I</i>	<i>Case II</i>	<i>Case I</i>	<i>Case II</i>	<i>Case I</i>	<i>Case II</i>	<i>Case I</i>	<i>Case II</i>
t_m	−30%	16261.84	11311.11	470795.61	257744.33	+0.09	−0.53	−0.07	+0.18
	−20%	16256.78	11331.15	470906.52	257588.60	+0.06	−0.36	−0.05	+0.12
	−10%	16251.73	11351.32	471017.42	257432.35	+0.03	−0.18	−0.02	+0.06
	0%	16246.69	11371.62	471128.30	257275.59	0.00	0.00	0.00	0.00
	+10%	16241.65	11392.06	471239.17	257118.30	−0.03	+0.18	+0.02	−0.06
	+20%	16236.62	11412.64	471350.03	256960.49	−0.06	+0.36	+0.05	−0.12
	+30%	16231.59	11433.36	471460.87	256802.15	−0.09	+0.54	+0.07	−0.18

Table 6: Sensitivity analysis of Cases to the transportation cost of items

Parameter	Change	Optimal value				Change percentage			
		Q		T_c		Q		T_c	
		<i>Case I</i>	<i>Case II</i>	<i>Case I</i>	<i>Case II</i>	<i>Case I</i>	<i>Case II</i>	<i>Case I</i>	<i>Case II</i>
K_T	−30%	16213.58	11348.44	470830.17	257070.64	−0.20	−0.20	−0.06	−0.08
	−20%	16224.63	11356.17	470929.62	257139.00	−0.14	−0.14	−0.04	−0.05
	−10%	16235.66	11363.90	471028.99	257207.32	−0.07	−0.07	−0.02	−0.03
	0%	16246.69	11371.62	471128.30	257275.59	0.00	0.00	0.00	0.00
	+10%	16257.71	11379.33	471227.55	257343.81	+0.07	+0.07	+0.02	+0.03

	Optimal value				Change percentage			
	Q		Tc		Q		Tc	
+20%	16268.73	11387.04	471326.72	257411.99	+0.14	+0.14	+0.04	+0.05
+30%	16279.73	11394.75	471425.83	257480.12	+0.20	+0.20	+0.06	+0.08

Table 7: Sensitivity analysis of Cases with respect to the PM cost

		Optimal value				Change percentage			
		Q		Tc		Q		Tc	
Parameter	Change	<i>Case I</i>	<i>Case II</i>	<i>Case I</i>	<i>Case II</i>	<i>Case I</i>	<i>Case II</i>	<i>Case I</i>	<i>Case II</i>
K_m	-30%	16246.69	11371.62	471063.79	257213.50	0.00	0.00	-0.01	-0.02
	-20%	16246.69	11371.62	471085.29	257234.20	0.00	0.00	-0.01	-0.02
	-10%	16246.69	11371.62	471106.80	257254.89	0.00	0.00	-0.00	-0.01
	0%	16246.69	11371.62	471128.30	257275.59	0.00	0.00	0.00	0.00
	+10%	16246.69	11371.62	471149.81	257296.28	0.00	0.00	+0.00	+0.01
	+20%	16246.69	11371.62	471171.31	257316.98	0.00	0.00	+0.01	+0.02
	+30%	16246.69	11371.62	471192.82	257337.67	0.00	0.00	+0.01	+0.02

Table 8: Sensitivity analysis of cases with respect to the production carbon emission cost

		Optimal value				Change percentage			
		Q		Tc		Q		Tc	
Parameter	Change	<i>Case I</i>	<i>Case II</i>	<i>Case I</i>	<i>Case II</i>	<i>Case I</i>	<i>Case II</i>	<i>Case I</i>	<i>Case II</i>
C_{ep}	-30%	13659.21	9560.55	447829.42	241258.86	-15.93	-15.93	-4.95	-6.23
	-20%	14572.84	10200.03	456056.17	246914.31	-10.30	-10.30	-3.20	.03

	Optimal value				Change percentage			
	Q		T_c		Q		T_c	
−10%	15432.48	10801.72	463796.73	252235.53	−5.01	−5.01	−1.56	−1.96
0%	16246.69	11371.62	471128.30	257275.59	0.00	0.00	0.00	0.00
+10%	17022.00	11914.29	478109.58	262074.84	+4.77	+4.77	+1.48	+1.87
+20%	17763.51	12433.29	484786.45	266664.82	+9.34	+9.34	+2.90	+3.65
+30%	18475.28	12931.48	491195.55	271070.74	+13.72	+13.72	+4.26	+5.36