A mathematical model for the joint planning of maintenance and safety stock in deteriorating imperfect manufacturing systems

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Abstract

In this article, a mathematical model is proposed for the joint planning of maintenance policies and inventory control in a deteriorating production system. A safety stock is maintained to meet the demand during the conduction of maintenance actions and to avoid shortages. The optimal planning of maintenance and inventory improves the productivity of the manufacturing system. In a deteriorating production system, the process has two operational states including in-control and out-of-control states as well as a non-operational state, or failure mode. The time for the transition among the states follows a general continuous distribution. The time duration of maintenance actions is also considered as a random variable. The purpose of this study is to optimize the safety stock level and the time to conduct maintenance actions so that the expected total cost per time unit can be minimized. To verify the efficiency of the model, some numerical examples are solved with a genetic algorithm, and validation is conducted for the solutions. Finally, sensitivity analyses are performed on the critical parameters.

Keywords: Deteriorating production system, maintenance planning, Preventive maintenance, Inventory control, Safety stock

1. Introduction

Maintenance planning, production/inventory control and quality control are the main operational policies that affect the performance of manufacturing systems. Considering these factors jointly can lead to the ideal and optimal planning of production processes and the improvement of productivity in manufacturing systems (Lopes [1]). The role and the conditions of machines and their availability are important factors in production and inventory control (Salmasnia et al. [2]). With respect to the relationship among machinery maintenance, inventory level and product quality, some authors have focused on the joint planning of maintenance, inventory and quality control as important actions for the management of manufacturing systems (Rasay et al. [3]). For years, these actions were taken separately, but some integrated models have recently been developed for their joint planning.

There are interactions and interrelationships among maintenance planning, production/inventory control and quality control, which form the basis of integrated models (Rasay et al. [4]).

Maintenance planning can reduce the downtime of the machines and increase their availability. There are different methods to restore and maintain production processes in an optimal condition. Effective maintenance reduces the overall cost of the company because production capacity is available when needed. Maintenance planning involves the identification of the parts and tools necessary for the job. During performing maintenance actions on the machines, the production process is interrupted and production is stopped. Therefore, a safety stock should be there to meet the demand and avoid possible stock-out during the maintenance operation.

Classical economic manufacturing quantity (EMQ) models ignore deterioration in production processes and assume that machines do not break down (Chakraborty and Giri [5]). In real situations and in most manufacturing systems, however, the production process is imperfect. Defective items may also be produced due to the deterioration of the processes and the failure of the machines (Lopes [1]). Deteriorating production processes generally have two operational states, namely in-control state and out-of-control state, and a failure mode that is non-operational. In the in-control state, the products are produced with a high quality, but, in the out-of-control state, the quality of the produced items may decrease and defective items may be produced. This occurs because the production is done by machines, but an assignable cause pushes the process to deterioration. In the failure state, the machine breaks down and cannot produce the items, and the production process is interrupted. In such systems, the process is initially under control, but, as the time goes on, the machines deteriorate and the process state shifts to an out-of-control state.

Different policies can be implemented to control the quality of products and detect defective items. Sometimes, defective items cannot be detected unless the consumers use them. In our study, a warranty policy is applied to detect these products. It means that all products are sold with a warranty period. If a defective item is detected in this period, it will be repaired.

The impact of deterioration on production planning in manufacturing systems has been examined in different studies. Porteus [6] and Rosenblatt and Lee [7] were the first researchers who studied the impact of deterioration on manufacturing systems. They considered a production process with two operational states, and the process state transition followed an exponential distribution. Groenevelt et al. [8] presented an EMQ model with deterioration taken into account. They considered an exponential distribution for the process state transition and ignored the repair time. Goyal et al. [9] aimed at an imperfect production system and presented an EMQ model on the purpose of minimizing the total cost. Chakraborty et al. [10] developed integrated models for process deterioration, inspection and maintenance.

The interrelations of production/inventory control, maintenance planning and quality control have been studied and analyzed with different policies for production control and quality control. Sana [11] presented a model for an imperfect manufacturing system. He studied the impact of deterioration on a process in which a shift was possible from an in-control state to an out-of-control state. It was assumed that defective products were likely to be produced in the out-of-control state and could be detected by inspection and restored in a high quality by reworking. In this study, it was also assumed that the probability of producing defective items in the out-of-control state would depend on the production rate and the production runtime. The purpose of the model was to determine the optimal production lot size to minimize the total cost. Chakraborty and Giri [5] developed an EMQ model for a deteriorating process. In this study, the process state could transit from an in-control state to an out-of-control state or from an out-of-control state to a failure mode. They considered a general distribution for the time duration of the maintenance actions. The aim of the model was to jointly plan maintenance and determine an optimal safety stock level to minimize the total cost. They proposed a computational algorithm to optimize the model. Sarkar [12] presented an inventory model for imperfect production processes by considering the effect of inflation. The process could transit from an in-control state to an out-of-control state. It was assumed that the defective products were produced in the out-of-control state, they could be detected by inspection and the defective items would be reworked. The purpose of the model was to determine the optimal production lot size to minimize the total cost.

Horenbeek et al. [13] reviewed the models for the integrated optimization of maintenance and inventory. Sarkara et al. [14] developed an economic production quantity (EPQ) model for imperfect production systems. They assumed that defective products would be reworked. They developed three different inventory models for three different distribution functions for the time of the state transition in a production process. Sett et al. [15] studied the joint planning of maintenance, quality and inventory control. They assumed that a safety stock had to be maintained to avoid shortage. The process was likely to transit from an in-control state to an out-of-control state at any random time, and the shift time followed a general distribution. Defective items could be produced in both in-control and out-of-control states. They considered a free repair warranty for non-inspected sold items. Salmasnia et al. [2] studied the joint planning of EMQ, quality and maintenance. They considered the deterioration in a production process with two operational states and conducted preventive maintenance (PM) and corrective maintenance (CM) actions for maintenance. The time duration of the maintenance actions was considered constant. In this study, an X-bar control chart was applied for the process monitoring. The particle swarm optimization (PSO) algorithm was also used for optimization. Lopes [1] developed a mathematical model to plan quality control, maintenance and production simultaneously for an imperfect manufacturing system. He considered PM as the maintenance policy and the PM time duration as a random variable. In this study, it was assumed that a safety stock had to be maintained to avoid shortage during the performance of the PM action. The produced items were inspected, and all the products were sold with a free minimal repair warranty. Duffuaa et al. [16] presented a model to plan production, maintenance and quality simultaneously. At first, they optimized a preventive maintenance schedule. Then, they did production scheduling and inventory control. They assumed three states for the process, and the time of transition among the states followed an exponential distribution. An X-bar control chart was applied to monitor the process and control the quality. The duration of the maintenance actions was considered constant.

Nourelfath et al. [17] presented a model to plan production, maintenance and quality in an imperfect manufacturing system by considering a multi-period multi-product system. It was assumed that the process had two operational states and the process shift would follow a general distribution. The products were inspected during the production cycle to control their quality. The purpose of the model was to determine the optimal safety sock level and production run length to minimize the total cost. Fakher et al. [18] presented a model to plan maintenance, quality and production in a capacitated manufacturing system and performed computational experiments to analyze their interrelationship. The process had three states, and the transition time among the states followed the Weibull distribution. The produced items were inspected to control the quality. The duration of the maintenance actions was constant. The purpose of the model was to determine the production lot size and maximize the profit. Cheng et al. [19] presented a mathematical model to plan production, quality inspection and maintenance with deterioration taken into account. The process had three states, and the transition time among the states followed a gamma stochastic trend. The duration of the maintenance actions was considered as a random variable. A safety stock was maintained to avoid any possible stock-out against uncertainties. In this study, a 100% inspection was done as a quality control policy, and the production lot size was determined with respect to the maintenance planning and the quality control policy. Finally, a simulation technique was practiced for optimization.

Shafiee-Gol et al. [20] developed an EPQ model for imperfect production processes and studied the pricing and production decisions in multi-product single-machine manufacturing systems. Their study involved the inspection of all the produced items and the reworking of the defective ones. Hafidi et al. [21] presented an integrated model of production, maintenance and quality control in deterioration manufacturing systems. They developed a multi-item capacitated lot-sizing problem and considered subcontracting strategies in the integrated model. They used a genetic algorithm for optimization. Gomez et al. [22] proposed an integrated model of production, maintenance and quality control for a continuous production system with quality deterioration. They proposed a dynamic sampling strategy for inspection and quality control and analyzed the interactions among the sampling, production and maintenance strategies. The purpose of the research was to determine an optimal production policy, schedule preventive maintenance and implement a quality control policy to minimize the expected total cost of the system. Salmasnia et al. [23] presented an integrated model of production cycle length, maintenance policy and quality control. They took the time value of money and the stochastic shift size into consideration. The main objective of the model was to determine the production cycle length, maintenance policy and economic-statistical design of a control chart. Wang et al. [24] developed an integrated model of production, maintenance and quality control for a serial production system with stochastic deterioration. To produce each production lot, an inspection was conducted on the process. To determine the situation of the machines, a predictive maintenance policy was practiced based on the predictive failure probability of each machine. The aim of the model was to minimize the total cost of the system. A simulation-based approach was applied for optimization. For comparison purposes, Table 1 summarizes the main features of the most important studies mentioned above.

Chakraborty and Giri [5] devised a mathematical model to jointly plan inventory control and maintenance. They adopted a warranty period policy to detect the defective sold products.

In our study, a mathematical model is presented for the joint planning of maintenance and inventory. For this purpose, the impact of deterioration on the process is taken into account. The process has three states including an in-control state, an out-of-control state and a failure mode. The transitions of the process among these states are based on general continuous random variables. The repair duration is also considered as a random variable. The purpose of the model is to determine an optimal safety stock level and a time to conduct maintenance actions and, thus, to minimize the expected total cost of the process.

The research is structured in several sections. The statement of the problem is presented in Section 2. In Section 3, different scenarios during a production cycle are explained. Section 4 develops the proposed integrated model. Numerical studies and sensitivity analysis are elaborated on in Section 5. The conclusion of the research is presented in Section 6.

2. Problem description

Notations

The following notations have been applied to develop the integrated model.

notation	Description
X_1	The time of transition from the in-control state to the out-of-control state (continuous random variable)
$f_1(x_1)$	Probability density function (p.d.f) of X_1
$F_1(x_1)$	Cumulative distribution function (c.d.f) of X_1
$\overline{F}_1(x_1)$	The survivor function, $1 - F_1$
X_2	The time of transition from the out-of-control state to the failure mode (continuous random variable)
$f_2(x_2)$	p.d.f of X_2
$F_2(x_2)$	c.d.f of X_2
$\overline{F}_2(x_2)$	The survivor function, $1 - F_2$
X_3	The time of transition from the in-control state to the failure mode

	(continuous random variable)
$f_3(x_3)$	p.d.f of X_3
$F_3(x_3)$	c.d.f of X_3
$\overline{F}_3(x_3)$	The survivor function, $1 - F_3$
Z_1	Time duration required for conducting CM action (random variable)
$g_1(z_1)$	p.d.f of Z_1
$G_1(z_1)$	c.d.f of Z_1
Z_2	Time duration required for conducting PM action (random variable)
$g_2(z_2)$	p.d.f of Z_2
$G_2(z_2)$	c.d.f of Z_2
S	Safety stock level (decision variable)
T	Production runtime during a cycle before the conduction of maintenance actions (decision variable)
q_1	Known and constant rate of the production which is equal to the demand rate
q_2 $(q_2 \geq q_1)$	Maximum production rate of the machine
P $(0 < P < 1)$	The probability of producing defective products when the process is in the out-of-control state
C_h	The unit cost of inventory holding
C_s	The unit cost of shortage
C_{CM}	The unit cost of CM action
C_{PM} $(C_{PM} < C_{CM})$	The unit cost of PM action
C_{setup}	The average setup cost
w	A known constant free repair warranty period
C_w	The unit cost of minimal repair during the period w
N	The number of defective products in a cycle
$h(k)$	Hazard rate function for a defective item for $k \geq 0$

Consider a process in which a type of product is produced at a given constant rate equal to the demand rate. The process has two operational states, including an in-control state and an out-of-control state, and a non-operational state, i.e. a failure mode. In this study, the in-control state, the out-of-control state and the failure state are indicated with 0, 1 and F respectively. Each production cycle starts in state 0 to satisfy demand q_1 per a time unit. The process state may transit from state 0 to state 1 at any random time during a production run. Also, the process state may transit from state 0 to the failure state without transition to state 1 or from state 1 to the failure mode. Figure 1 illustrates the transitions among the states of the process.

The items produced in state 1 may be defective with a certain probability, and the items produced in state 0 are all conforming.

If a machine breaks down during the production run, a CM action is conducted on the process. Otherwise, PM is conducted on the process after T unit of time, regardless of the process state. The time durations of the CM action and the PM action are considered as the random variables. The production process is interrupted during the maintenance actions, and the demand is satisfied from the safety stock S to avoid any shortage. Therefore, the safety stock level reduces at the demand rate q_1. The PM and CM actions are perfect. It means that, after the maintenance actions, the machine returns to an as-good-as-new condition and then the production continues at the maximum rate q_2 until the stock level reaches S again. After that, the production rate is set at the normal level q_1 ($q_1 < q_2$). The production cycle in this case is the time duration from the start of the process until the safety stock level reaches S again.

If the maintenance action is completed before $\frac{S}{q_1}$, no shortage occurs. After the maintenance action, the process continues at the maximum rate q_2 until the stock level becomes S again. If the maintenance action is completed after $\frac{S}{q_1}$, there is a shortage to occur. When this shortage occurs after the completion of the maintenance action, the machine starts to produce the products and the safety stock level becomes S again within the time duration of $\frac{S}{q_2 - q_1}$. The demands that are not met during the maintenance actions are lost. In this study, it is assumed that the defective products cannot be detected unless the consumers use them. Therefore, to detect the defective items, all the products are sold with a free minimal repair warranty by considering the warranty period w. Under this policy, if a sold product fails within the period w, it is given a minimal repair at the cost C_w so as to return it to the same condition as before. The procedure described for modeling can be put to practice in real-world situations, such as the automobile industry. Car manufacturers and dealers use warranties to win and retain customers. Some non-conforming items in cars are operational, but they cannot be detected unless the consumers use them. Changing a flat tire on a car, rectifying an ignition or wiring system, changing a broken fan below an engine or any repair of the engine that does not change the overall performance of the car are examples of the minimal repair.

The aim of the model is to plan the maintenance and the inventory control simultaneously to optimize the safety stock level (S) and the time of performing the preventive maintenance action (T) so that the expected total cost of the process per unit of time can be minimized.

3. Possible scenarios during a production cycle

Four scenarios may occur for the process completion during each production cycle.

Scenario 1: In this scenario, the process remains in state 0 until the time point T. In other words, the process state transits neither to state 1 nor to the failure state. Thus, at the time T, the process stops, and PM is conducted. Figure 2 illustrates the evolution of the process under this scenario. Figure 2(a) corresponds to the case where the duration of the PM action is less than $\frac{S}{q_1}$; thus, no shortage occurs. On the other hand, in Figure 2(b), the duration of the PM action is more than $\frac{S}{q_1}$; thus, a shortage happens. After the completion of the PM action, the production starts at the rate q_2 ($q_2 > q_1$) until the safety stock reaches S again. After that, the production continues at a normal rate, q_1. This scenario occurs with the probability in Equation 1.

$$P(S_1) = \overline{F}_1(T)\overline{F}_3(T) \tag{1}$$

Scenario 2: The production process initially operates in state 0 and transits to state 1 after a random time passes as x_1 from the start of the cycle. The process operates under state 1 until T, when the PM action is implemented. Figure 3 illustrates the process under this scenario. Figure 3(a) indicates that the duration of the PM action is less than $\frac{S}{q_1}$; thus, no shortage occurs. On the other hand, Figure 3(b) corresponds to the case where a shortage occurs as the duration of the PM action is bigger than $\frac{S}{q_1}$. This scenario occurs with the probability in Equation 2.

$$P(S_2) = \int_0^T f_1(x)\overline{F}_3(x)\frac{\overline{F}_2(T)}{\overline{F}_2(x)}dx \tag{2}$$

Scenario 3: After a random time passes as x_3 from the beginning of the cycle, the process directly shifts from state 0 to the failure mode without transition to state 1. Once the process state transits to the failure mode, CM is conducted. Figure 4 illustrates the evolution of the process under this scenario. Figure 4(a) indicates that the duration of the CM action is less than $\frac{S}{q_1}$; thus, no shortage occurs. Figure 4(b), however, indicates that a shortage occurs when the duration of the CM action is bigger than $\frac{S}{q_1}$. This scenario occurs with the probability in Equation 3.

$$P(S_3)=\int_0^T f_3(x)\overline{F}_1(x)dx \tag{3}$$

Scenario 4: After a random time passes as x_1 from the beginning of the cycle, the process shifts to state 1. Then, the process at the random time x_2 ($x_1 < x_2 < T$) shifts to the failure state. Once the failure state starts, CM is conducted. After the completion of the CM action, the production starts at a maximum rate (q_2). Production at the rate q_2 continues until the safety stock becomes S again. After that, the production rate is set at q_1. Figure 5 illustrates the evolution of the process under this scenario. Figure 5(a) corresponds to the case where the duration of the CM action is less than $\frac{S}{q_1}$; thus, no shortage occurs. On the other hand, Figure 5(b) corresponds to the case where a shortage occurs as the duration of the CM action is bigger than $\frac{S}{q_1}$. This scenario occurs with the probability in Equation 4.

$$P(S_4)=\int_0^T\int_0^{x_2} f_1(x_1)\overline{F}_3(x_1)\frac{f_2(x_2)}{\overline{F}_2(x_1)}dx_1dx_2 \tag{4}$$

4. Development of the integrated model

The described system consists of stochastic and independent identical cycles. The integrated model is presented based on a renewal reward process and recursive equations. The expected total cost of the system per unit of time is obtained using Equation 5.

$$ECT=\frac{E[TC]}{E[CL]} \tag{5}$$

Where $E[TC]$ is the expected total cost for each cycle and $E[CL]$ is the expected time length of that cycle. $E[TC]$ consists of the expected cost of maintenance ($E[MC]$), the expected holding cost ($E[HC]$), the expected shortage cost ($E[SH]$), the expected warranty cost ($E[WR]$) and the setup cost. Thus, it is calculated through Equation 6.

$$E[TC]=E[MC]+E[HC]+E[SH]+E[WR]+C_{setup} \tag{6}$$

According to the law of total expectation, Equation 7 can be derived to calculate $E[CL]$:

$$E[CL]=P(S_1)E[CL\,|\,S_1]+P(S_2)E[CL\,|\,S_2]+P(S_3)E[CL\,|\,S_3]+P(S_4)E[CL\,|\,S_4] \tag{7}$$

When the i'th scenario occurs with the probability $P(S_i)$, i is equal to 1, 2, 3, 4. Also, $E[CL\,|\,S_i]$ is the average time duration of the cycle.

In scenario 1, $E[CL\,|\,S_1]$ is obtained by Equation 8.

$$E[CL \mid S_1] = \frac{\overline{F}_1(T)\overline{F}_3(T)\int_0^{\frac{s}{q_1}}(T + z_2 + \frac{q_1 z_2}{q_2 - q_1})g_2(z_2)dz_2}{P(S_1)} + \frac{\overline{F}_1(T)\overline{F}_3(T)\int_{\frac{s}{q_1}}^{\infty}(T + z_2 + \frac{s}{q_2 - q_1})g_2(z_2)dz_2}{P(S_1)} \tag{8}$$

In the first term of this equation, the average duration of a cycle is calculated while no shortage occurs. The second term calculates the average time duration of a cycle where a shortage occurs.

In scenario 2, $E[CL \mid S_2]$ is obtained by Equation 9.

$$E[CL \mid S_2] = \frac{\int_0^T \int_0^{\frac{s}{q_1}}(T + z_2 + \frac{q_1 z_2}{q_2 - q_1})g_2(z_2)f_1(x)\overline{F}_3(x)\frac{\overline{F}_2(T)}{\overline{F}_2(x)}dz_2 dx}{P(S_2)} + \frac{\int_0^T \int_{\frac{s}{q_1}}^{\infty}(T + z_2 + \frac{s}{q_2 - q_1})g_2(z_2)f_1(x)\overline{F}_3(x)\frac{\overline{F}_2(T)}{\overline{F}_2(x)}dz_2 dx}{P(S_2)} \tag{9}$$

In the first term of this equation, the average time duration of a cycle is calculated while no shortage occurs. The second term calculates the average time duration of a cycle where a shortage occurs.

Under scenario 3, $E[CL \mid S_3]$ is obtained by Equation 10.

$$E[CL \mid S_3] = \frac{\int_0^T \int_0^{\frac{s}{q_1}}(x + z_1 + \frac{q_1 z_1}{q_2 - q_1})g_1(z_1)f_3(x)\overline{F}_1(x)dz_1 dx}{P(S_3)} + \frac{\int_0^T \int_{\frac{s}{q_1}}^{\infty}(x + z_1 + \frac{s}{q_2 - q_1})g_1(z_1)f_3(x)\overline{F}_1(x)dz_1 dx}{P(S_3)} \tag{10}$$

In the first term of this equation, the average time duration of a cycle is calculated with no shortage to occur. The second term calculates the average time duration of a cycle where a shortage occurs.

Under scenario 4, $E[CL \mid S_4]$ is obtained by Equation 11.

$$E[CL|S_4]=\frac{\int_0^T\int_0^{x_2}\int_0^{\frac{s}{q_1}}(x_2+z_1+\frac{q_1z_1}{q_2-q_1})g_1(z_1)f_1(x_1)\overline{F}_3(x_1)\frac{f_2(x_2)}{\overline{F}_2(x_1)}dz_1dx_1dx_2}{P(S_4)}$$ (11)

$$+\frac{\int_0^T\int_0^{x_2}\int_{\frac{s}{q_1}}^{\infty}(x_2+z_1+\frac{s}{q_2-q_1})g_1(z_1)f_1(x_1)\overline{F}_3(x_1)\frac{f_2(x_2)}{\overline{F}_2(x_1)}dz_1dx_1dx_2}{P(S_4)}$$

In the first term of this equation, the average time duration of a cycle is calculated while no shortage occurs. The second term calculates the average duration of a cycle with a shortage to occur.

4.1. Average cost of the maintenance action in each cycle

The expected cost of the maintenance ($E[MC]$) is the sum of the expected cost of the PM implementation ($E[PM]$) and the expected cost of the CM implementation ($E[CM]$). As the PM action is conducted, if scenario S_1 or S_2 occurs, and the CM action is conducted while scenario S_3 or S_4 occurs, the Equations 12 and 13 are derived:

$$E[PM]=[P(S_1)+P(S_2)-P(S_1).(S_2)]\times C_{PM}\int_0^{\infty}z_2g_2(z_2)dz_2$$ (12)

$$=[P(S_1)+P(S_2)-P(S_1).(S_2)]\times C_{PM}E[z_2]$$

$$E[CM]=[P(S_3)+P(S_4)-P(S_3).(S_4)]\times C_{CM}\int_0^{\infty}z_1g_1(z_1)dz_1$$ (13)

$$=[P(S_3)+P(S_4)-P(S_3).(S_4)]\times C_{CM}E[z_1]$$

4.2.Average inventory holding cost in each cycle

The average holding cost in each cycle is obtained by Equation 14.

$$E[HC]=P(S_1)E[HC|S_1]+P(S_2)E[HC|S_2]+P(S_3)E[HC|S_3]+P(S_4)E[HC|S_4]$$ (14)

Where $E[HC|S_i];i=1,2,3,4$ denotes the average holding costs if the i'th scenario is implemented.

In scenario 1, $E[HC|S_1]$ is obtained by Equation 15.

$$E[HC|S_1]=$$ (15)

$$\frac{C_h[\overline{F}_1(T)\overline{F}_3(T)\int_0^{\frac{s}{q_1}}\{sT+(z_2+\frac{q_1z_2}{q_2-q_1})s-\frac{1}{2}(z_2+\frac{q_1z_2}{q_2-q_1})q_1z_2\}g_2(z_2)dz_2]}{P(S_1)}$$

$$+\frac{C_h[\{sT+\frac{s^2}{2q_1}+\frac{s^2}{2(q_2-q_1)}\}\overline{G}_2(\frac{s}{q_1})\overline{F}_1(T)\overline{F}_3(T)]}{P(S_1)}$$

This equation consists of two terms. The first term calculates the average holding cost in case there is no shortage to occur. The second term calculates the average holding cost if there is a shortage.

In scenario 2, $E[HC \mid S_2]$ is obtained by Equation 16.

$$E[HC \mid S_2] = \frac{C_h[\int_0^T \int_0^{\frac{S}{q_1}} \{sT + (z_2 + \frac{q_1 z_2}{q_2 - q_1})s - \frac{1}{2}(z_2 + \frac{q_1 z_2}{q_2 - q_1})q_1 z_2\} f_1(x)\overline{F}_3(x)\frac{\overline{F}_2(T)}{\overline{F}_2(x)} g_2(z_2)dz_2 dx]}{P(S_2)} + \frac{C_h[\int_0^T \{sT + \frac{s^2}{2q_1} + \frac{s^2}{2(q_2 - q_1)}\}\overline{G}_2(\frac{s}{q_1})f_1(x)\overline{F}_3(x)\frac{\overline{F}_2(T)}{\overline{F}_2(x)}dx]}{P(S_2)} \quad (16)$$

This equation consists of two terms. The first term calculates the average holding cost when there is no shortage. The second term calculates the average holding cost when there exists a shortage.

Under scenario 3, $E[HC \mid S_3]$ is obtained by Equation 17.

$$E[HC \mid S_3] = \frac{C_h[\int_0^T \int_0^{\frac{S}{q_1}} \{sx + (z_1 + \frac{q_1 z_1}{q_2 - q_1})s - \frac{1}{2}(z_1 + \frac{q_1 z_1}{q_2 - q_1})q_1 z_1\} f_3(x)\overline{F}_1(x) g_1(z_1)dz_1 dx]}{P(S_3)} + \frac{C_h[\int_0^T \{sx + \frac{s^2}{2q_1} + \frac{s^2}{2(q_2 - q_1)}\}\overline{G}_1(\frac{s}{q_1})f_3(x)\overline{F}_1(x)dx]}{P(S_3)} \quad (17)$$

This equation consists of two terms. The first term calculates the average holding cost if there is no shortage to occur. The second term calculates the average holding cost if there is a shortage.

Under scenario 4, $E[HC \mid S_4]$ is obtained by Equation 18.

$$E[HC \mid S_4] = \frac{C_h[\int_0^T \int_0^{x_2} \int_0^{\frac{S}{q_1}} \{sx_2 + (z_1 + \frac{q_1 z_1}{q_2 - q_1})s - \frac{1}{2}(z_1 + \frac{q_1 z_1}{q_2 - q_1})q_1 z_1\} g_1(z_1) f_1(x_1)\overline{F}_3(x_1)\frac{f_2(x_2)}{\overline{F}_2(x_1)} dz_1 dx_1 dx_2]}{P(S_4)} + \frac{C_h[\int_0^T \int_0^{x_2} \int_{\frac{S}{q_1}}^{\infty} \{sx_2 + \frac{s^2}{2q_1} + \frac{s^2}{2(q_2 - q_1)}\} g_1(z_1) f_1(x_1)\overline{F}_3(x_1)\frac{f_2(x_2)}{\overline{F}_2(x_1)} dz_1 dx_1 dx_2]}{P(S_4)} \quad (18)$$

This equation consists of two terms. The first term calculates the average holding cost when there is no shortage. The second term calculates the average holding cost when there is a shortage.

4.3. Average shortage cost in each cycle

The average shortage cost in each cycle is obtained by Equation 19.

$$E[SH]=P(S_1)E[SH\,|\,S_1]+P(S_2)E[SH\,|\,S_2]+P(S_3)E[SH\,|\,S_3]+P(S_4)E[SH\,|\,S_4] \tag{19}$$

Where $E[SH\,|\,S_i];i=1,2,3,4$ denotes the average shortage costs in case the i'th scenario is implemented.

In scenario 1, $E[SH\,|\,S_1]$ is obtained by Equation 20.

$$E[SH\,|\,S_1]=\frac{C_s[\overline{F}_1(T)\overline{F}_3(T)\int_{\frac{s}{q_1}}^{\infty}(q_1z_2-s)g_2(z_2)dz_2]}{P(S_1)} \tag{20}$$

In scenario 2, $E[SH\,|\,S_2]$ is obtained by Equation 21.

$$E[SH\,|\,S_2]=\frac{C_s[\int_0^T\int_{\frac{s}{q_1}}^{\infty}(q_1z_2-s)g_2(z_2)f_1(x)\overline{F}_3(x)\frac{\overline{F}_2(T)}{\overline{F}_2(x)}dz_2dx]}{P(S_2)} \tag{21}$$

Under scenario 3, $E[SH\,|\,S_3]$ is obtained by Equation 22.

$$E[SH\,|\,S_3]=\frac{C_s[\int_0^T\int_{\frac{s}{q_1}}^{\infty}(q_1z_1-s)g_1(z_1)f_3(x)\overline{F}_1(x)dz_1dx]}{P(S_3)} \tag{22}$$

Under scenario 4, $E[SH\,|\,S_4]$ is obtained by Equation 23.

$$E[SH\,|\,S_4]=\frac{C_s[\int_0^T\int_0^{x_2}\int_{\frac{s}{q_1}}^{\infty}(q_1z_1-s)g_1(z_1)f_1(x_1)\overline{F}_3(x_1)\frac{f_2(x_2)}{\overline{F}_2(x_1)}dz_1dx_1dx_2]}{P(S_4)} \tag{23}$$

4.4.Average warranty cost in each cycle

The average number of the defective items that may be detected in the warranty period is obtained from Equation 24.

$$\int_0^w E[N]h(k)dk \tag{24}$$

Therefore, the average warranty cost in each cycle can be obtained from Equation 25.

$$E[WR]=C_w\int_0^w E[N]h(k)dk \tag{25}$$

The average number of the defective products in state 1 in a cycle is obtained from Equation 26.

$$E[N]=P(S_2)E[N\mid S_2]+P(S_4)E[N\mid S_4] \tag{26}$$

As mentioned before, defective items may be produced in state 1. Therefore, in Equation 26, $E[N\mid S_i]$ denotes the average number of the defective products in state 1 in case scenarios 2 and 4 are occurred.

Under scenario 2, $E[N\mid S_2]$ is obtained by Equation 27.

$$E[N\mid S_2]=\frac{q_1P[\int_0^T(T-x)f_1(x)\overline{F}_3(x)\frac{\overline{F}_2(T)}{\overline{F}_2(x)}dx]}{P(S_2)} \tag{27}$$

Under scenario 4, $E[N\mid S_4]$ is computed using Equation 28.

$$E[N\mid S_4]=\frac{q_1P[\int_0^T\int_0^{x_2}(x_2-x_1)f_1(x_1)\overline{F}_3(x_1)\frac{f_2(x_2)}{\overline{F}_2(x_1)}dx_1dx_2]}{P(S_4)} \tag{28}$$

5. Numerical study

5.1. Numerical example

In this section, a numerical example is solved to verify the efficiency of the model. Due to the complexity of the equations of the integrated model, the exact methods are not effective to solve the model. Therefore, the model is first solved by a grid search algorithm. Since the runtime of the grid search is not acceptable, a genetic algorithm is coded by the MATLAB software to optimize the model. The genetic algorithm is a meta-heuristic method to solve complex problems. It is inspired by the process of natural selection. The algorithm is commonly used to generate high-quality solutions for optimization problems, and it is also applied in many scientific subjects such as operation research and computer science (Jensen [25]; Chou and Chen [26]).

For the numerical study of this section, it is assumed that the time of process states transitions from the in-control state to the out-of-control state (x_1), from the out-of-control state to machine breakdown (x_2) and from the in-control state to machine breakdown (x_3) are based on the Weibull distribution with the probability density functions presented in Equations 29, 30 and 31.

$$f_1(x_1) = \lambda_1 v_1 (\lambda_1 x_1)^{v_1 - 1} e^{-(\lambda_1 x_1)^{v_1}} ; x_1 \geq 0, \lambda_1 \geq 0, v_1 \geq 1 \quad (29)$$

$$f_2(x_2) = \lambda_2 v_2 (\lambda_2 x_2)^{v_2 - 1} e^{-(\lambda_2 x_2)^{v_2}} ; x_2 \geq 0, \lambda_2 \geq 0, v_2 \geq 1 \quad (30)$$

$$f_3(x_3) = \lambda_3 v_3 (\lambda_3 x_3)^{v_3 - 1} e^{-(\lambda_3 x_3)^{v_3}} ; x_3 \geq 0, \lambda_3 \geq 0, v_3 \geq 1 \quad (31)$$

It is also assumed that the corrective maintenance time (z_1) and the preventive maintenance time (z_2) follow the Weibull distribution with the probability density functions presented in Equations 32 and 33.

$$g_1(z_1) = \gamma_1 \theta_1 (\gamma_1 z_1)^{\theta_1 - 1} e^{-(\gamma_1 z_1)^{\theta_1}} ; z_1 \geq 0, \gamma_1 \geq 0, \theta_1 \geq 1 \quad (32)$$

$$g_2(z_2) = \gamma_2 \theta_2 (\gamma_2 z_2)^{\theta_2 - 1} e^{-(\gamma_2 z_2)^{\theta_2}} ; z_2 \geq 0, \gamma_2 \geq 0, \theta_2 \geq 1 \quad (33)$$

Moreover, the lifetime of non-conforming items is assumed to be based on the Weibull distribution with the probability density function presented in Equation 34.

$$h(k) = \frac{k}{18} e^{-(\frac{k}{6})^2} \quad (34)$$

It is mostly the data in reference [5] that is used to conduct the example. The data used in this study are presented in Table 2.

The values of the Weibull distributions parameters are as follows:

$\lambda_1 = 0.3$, $\lambda_2 = 0.3$, $\lambda_3 = 0.3$, $v_1 = 2$, $v_2 = 2$, $v_3 = 2$

$\gamma_1 = 0.4$, $\gamma_2 = 0.4$, $\theta_1 = 2$, $\theta_2 = 2$

The parameters of the genetic algorithm used for optimization are presented in Table 3. The optimal values of *T*, *S* and *ECT* are given in Table 4. Accordingly, the optimal value of the safety stock level is 131, and it is recommended to conduct a PM action after 2.7467 time units pass from the start of the production cycle. Using this policy, the expected total cost per time unit is optimized, which is 81.4075.

To validate the results obtained by the genetic algorithm, they are compared with those obtained by the optimization algorithm proposed in reference [5]. In this reference, a computational algorithm was proposed so as to reach a global optimum. The integrated model proposed in the present study is solved by the genetic algorithm and with different values of λ_1 and λ_2; the other parameters are constant. The results are compared with those in reference [5]. The obtained solutions are presented in Table 5.

The comparison of the results indicates that the difference between the optimal values of the objective function (*ECT*) in our model and reference [5] is little. Therefore, the solutions obtained by the genetic algorithm in this study are nearly optimal. The comparison of the results also indicates the efficiency of our proposed model and the validity of the solutions obtained by the genetic algorithm. The results of the comparison are indicated in Figure 6.

5.2. Sensitivity analysis

In this section, a sensitivity analysis is performed on the critical parameters to identify their effects on the decision variables and the total cost.

The effects of the change of λ_1 on the decision variables and *ECT* are indicated in Table 6 and Figures 7, 8 and 9. The results obtained by the genetic algorithm indicate that larger values of λ_1 lead to a decrease in the optimal value of *T* and an increase in the optimal values of the safety stock and *ECT*. This is because, an increase of λ_1 increases the probability of the process shift from state 0 to state 1 and increases the probability of producing defective items. Thus, the minimal repair cost increases during the warranty period. Also, an increase of λ_1, increases the safety stock level to avoid shortage. The expected holding cost increases by an increase in the value of *S*. Therefore, when the holding cost and minimal repair cost increase, the *ECT* increases too. For the larger values of λ_1, the optimal value of *T* decreases to reduce the probability of the process shift from state 0 to state 1 and reduce the average number of the defective items in state 1.

The effects of the change of λ_2 and λ_3 on the decision variables and *ECT* are indicated in Table 7 and Figures 10-15. Larger values of λ_2 and λ_3 decrease the value of *T*, but the optimal values of the safety stock and *ECT* increase. With larger values of λ_2 and λ_3, the value of *T* decreases to reduce the probability of machine failure. Also, as the values of λ_2 and λ_3 increase, the safety stock level increases as well to avoid possible stock-out during the performance of maintenance actions. Due to an increase in the failure rate, the maintenance cost increases, and an increase in the safety stock level leads to a rise in the holding cost and, thus, a rise in *ECT* too.

Larger values of $\gamma_1$and $\gamma_2$increase the optimal value of *S* to avoid possible stock-out and meet the demand during the conduction of the CM action and the PM action. The obtained results are presented in Table 8 and Figure 16.

An increase in the PM cost leads to a decrease in the safety stock level, while an increase in the CM cost causes an increase in the safety stock level. Table 9 and Figure 17 indicate the obtained results.

Larger values of the holding cost decrease the value of *S*, and an increase in the shortage cost causes an increase in the value of *S*. Table 10 and Figure 18 indicate the obtained results.

An increase in the value of *P* decreases the optimal value of *T* but increases the optimal values of *S* and *ECT*. With the larger values of *P*, the value of *T* decreases to reduce the number of the defective products in state 1. As the value of *P* rises, the optimal value of *S* rises too so as to avoid shortage. Also, an increase in the value of *P* leads to an increase in the possibility of producing defective products. Therefore, the repair cost during the warranty period increases. Due to the increase of the safety stock level, the expected holding cost increases. These two increased values give a rise to *ECT*. The obtained results are presented in Table 11 and Figures 19 and 20.

6. Conclusion

In this study, a mathematical model was presented for the joint planning of maintenance and inventory control in a deteriorating production system. The process deteriorated and underwent two operational states and a failure mode. The transitions among the states of the process were based on general continuous random variables. PM and CM could be conducted during each production cycle. The time duration to conduct maintenance actions was considered as a continuous random variable. The purpose of the presented model is to integrate the decisions on the determination of the optimal safety stock level and the time to perform maintenance actions and, thus, to minimize the expected total cost of the process. A numerical study was performed, and a series of sensitivity analyses were conducted regarding some important parameters. Due to the complexity of the proposed integrated model, the genetic algorithm was employed for optimization, and the solutions were validated. The results indicated the acceptable performance of the model in real situations; all the products were sold with a free minimal repair warranty. The modeling system described in this study can be used in real-world situations such as the automobile industry. Car manufacturers and car dealers apply warranty periods to win and retain customers because some non-conforming items in cars are operational and can only be detected after a period of use.

The managers in manufacturing systems can make dynamic and ideal plans for their production processes by the joint planning of maintenance, production and quality control. Machinery, human resources and other production aspects can be applied in an optimal manner, and the productivity of the production process can finally be improved by integrated planning. The integrated model can be of benefit in manufacturing systems with the JIT (Just in Time) policy. In such systems, the products are produced with respect to the demand level, and the managers' purpose is to eliminate the inventory. Therefore, the integrated model makes it possible to jointly deal with production plans, maintenance schedules and quality control policies with respect to the demand level and the production capacity. The integrated models can also be applied in continuous manufacturing systems. In such systems, the process should operate without interruptions during the production process. Therefore, optimal planning for production, maintenance scheduling and quality control is important. The efficiency of production processes can, indeed, be improved by the use of an integrated model that enables the joint planning of production, maintenance and quality control.

As for the future research in the field, different quality control policies such as acceptance sampling plans or 100% inspection policies can be formulated to detect defective items during production processes. The insights provided by this study may be utilized to develop an integrated model for different manufacturing systems such as multi-machine manufacturing systems, multi-stage systems and series production systems.

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Table 1

	Number of process states	Process state transition			Failure mechanism	Quality control policy	Repair time	Production/inventory control policy	Inventory shortage consideration
		In-control to out-of-control	Out-of-control to failure	In-control to failure					
[1]	2	✓			Random variable with General distribution	Product inspection and warranty period	Random variable	Determination of safety stock	✓
[2]	2	✓			Random variable with Weibull distribution	X-bar control chart	Constant	EPQ	
[5]	3	✓	✓		Random variable with General distribution	Warranty period	Random variable	Determination of safety stock and production run length	✓
[11]	2	✓			Random variable with Exponential distribution	Product inspection	-	Determination of production lot size	
[12]	2	✓			Random variable	Product	-	Determination of	

				with General distribution	inspection		production lot size	
[15]	2	✓		Random variable with General distribution	Product inspection and warranty period	Random variable	Determination of buffer inventory	✓
[16]	3	✓	✓	Random variable with Exponential distribution	X-bar control chart	Constant	Production scheduling	
[17]	2	✓		Random variable with General distribution	Product inspection	Constant	Determination of safety stock and production run length	✓
[18]	3	✓	✓	Random variable with Weibull distribution	Product inspection	Constant	Determination of production lot size	✓
[19]	3	✓	✓	Random variable with Gamma distribution	100% inspection	Random variable	Determination of production lot size	✓
[20]	-				100% inspection	-	Determination of production lot size	
[21]	2	✓		Random variable with General distribution	Periodic inspection	Constant	Determination of production lot size	✓
[22]	-			Number of repairs	Sampling plan	Random variable	Determination of production lot size	✓
[23]	2	✓		Random variable with Weibull distribution	X-bar control chart	Constant	Determination of production run length and inventory level	
[24]	2	✓		Random variable with	Periodic inspection	Constant	Determination of production run length and inventory	✓

					Gamma distribution			level	
This paper	3	✓	✓	✓	Random variable with General distribution	Warranty period	Random variable	Determination of safety stock and production run length	✓

Table 2

q_1	q_2	P	C_h	C_s	C_{CM}	C_{PM}	C_w	W	C_{setup}
90	160	0.3	0.2	0.8	50	5	20	1	300

Table 3

Generation number	Population size	Crossover rate	Mutation rate	cm
100	30	0.4	0.1	0.2

Table 4

T^*	S^*	ECT
2.7467	131	81.4075

Table 5

	ECT (our model)	ECT (reference [5])
$\lambda_1 = 0.1, \lambda_2 = 0.25$	101.4399	96.8037
$\lambda_1 = 0.3, \lambda_2 = 0.25$	108.0275	97.0267
$\lambda_1 = 0.5, \lambda_2 = 0.25$	114.0605	97.3386
$\lambda_1 = 0.7, \lambda_2 = 0.25$	114.9084	97.6490
$\lambda_1 = 0.5, \lambda_2 = 0.1$	93.1943	88.0799
$\lambda_1 = 0.5, \lambda_2 = 0.2$	98.8950	95.4210
$\lambda_1 = 0.5, \lambda_2 = 0.3$	106.2644	98.5072
$\lambda_1 = 0.5, \lambda_2 = 0.4$	108.90	99.7307

$\lambda_1 = 0.5, \lambda_2 = 0.5$	111.6721	100.295

Table 6

λ_1	T	S	ECT
0.1	7.076	115.765	90.2431
0.2	5.856	126.125	113.975
0.35	5.2825	131.405	122.805
0.4	3.0086	189.462	187.4
0.5	1.8288	194.146	194.111

Table 7

		T	S	ECT
λ_2	0.1	5.8527	45.5386	87.3224
	0.2	4.7798	71.5818	96.706
	0.35	2.7741	138.119	98.7942
	0.4	1.1537	155.811	132.621
	0.5	0.7874	165.61	160.53
λ_3	0.1	9.8834	41.4498	108.383
	0.2	7.1648	81.5902	110.877
	0.35	5.2074	85.6166	120.228
	0.4	4.9811	120.191	128.292
	0.5	2.0696	183.79	193.979

Table 8

	γ_1					γ_2				
	0.1	0.2	0.4	0.5	0.6	0.1	0.2	0.4	0.5	0.6
S	25.03	61.92	131.85	152.88	198.08	45.53	61.12	131.85	170.72	182.56

Table 9

	C_{PM}					C_{CM}				
	2	5	10	15	20	20	30	50	70	100
S	179.75	131.85	119.96	91.85	57.61	61.79	125.56	131.85	152.31	174.87

Table 10

	C_h					C_s					
	0.1	0.3	0.5	0.6	0.7	0.8	1	2	3	4	5
S	181.62	142.74	130.79	113.19	56.57	131.85	166.78	187.07	188.03	188.46	198.16

Table 11

P	*T*	*S*	*ECT*
0.2	3.9819	129.592	80.9773
0.3	2.7467	131.855	81.4075
0.4	2.4187	136.703	86.0378
0.5	1.8222	162.73	89.1744
0.7	1.0937	170.223	93.541

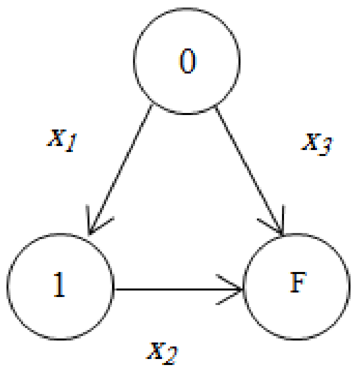
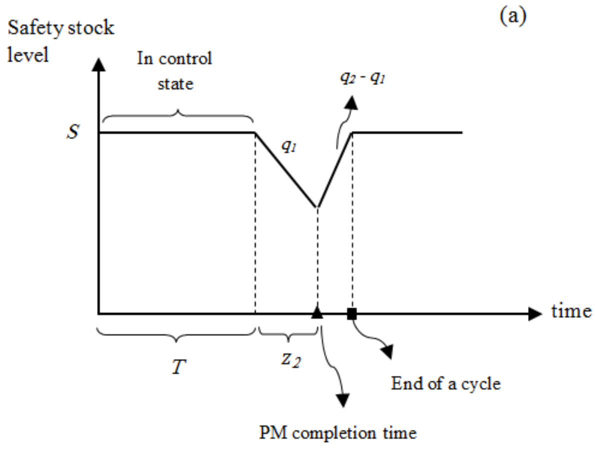


Figure 1


(a)
Safety stock
level
In control
state
S
q1
q2 - q1
time
T
z2
End of a cycle
PM completion time

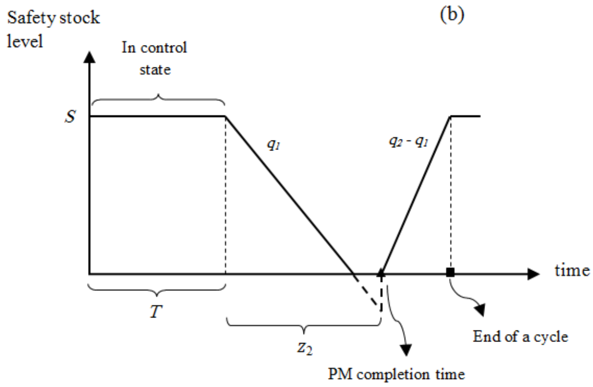
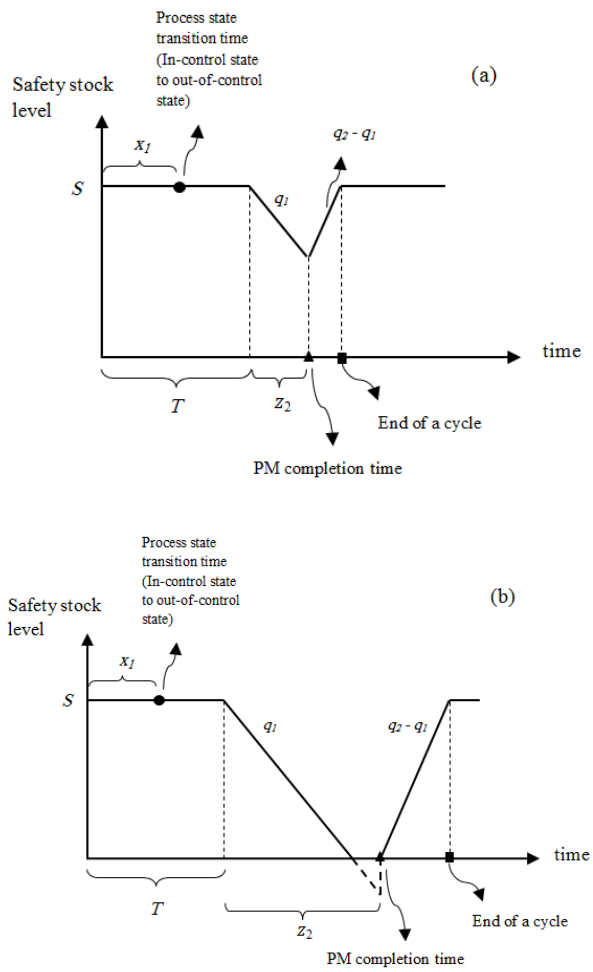
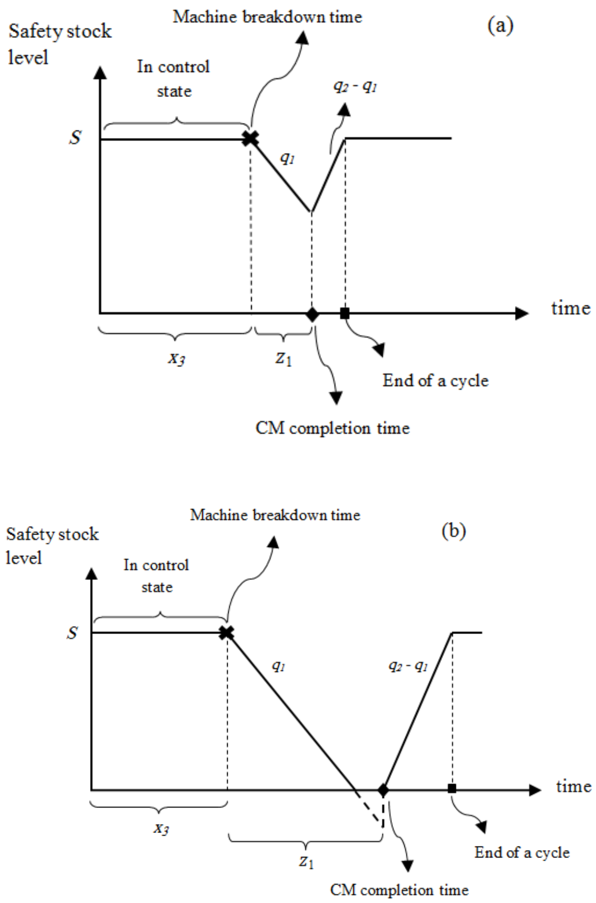

Safety stock
level
(b)
In control
state
S
q1
q2 - q1
time
T
z2
End of a cycle
PM completion time

Figure 2


Process state
transition time
(In-control state
to out-of-control
state)
Safety stock
level
(a)
x1
q2 - q1
S
q1
time
T
z2
End of a cycle
PM completion time
Process state
transition time
(In-control state
to out-of-control
state)
Safety stock
level
(b)
x1
S
q1
q2 - q1
time
T
z2
End of a cycle
PM completion time
Figure 3



(a)
Safety stock level
Machine breakdown time
In control state
S
q_1
$q_2 - q_1$
time
x_3
z_1
End of a cycle
CM completion time
(b)
Safety stock level
Machine breakdown time
In control state
S
q_1
$q_2 - q_1$
time
x_3
z_1
End of a cycle
CM completion time

Figure 4

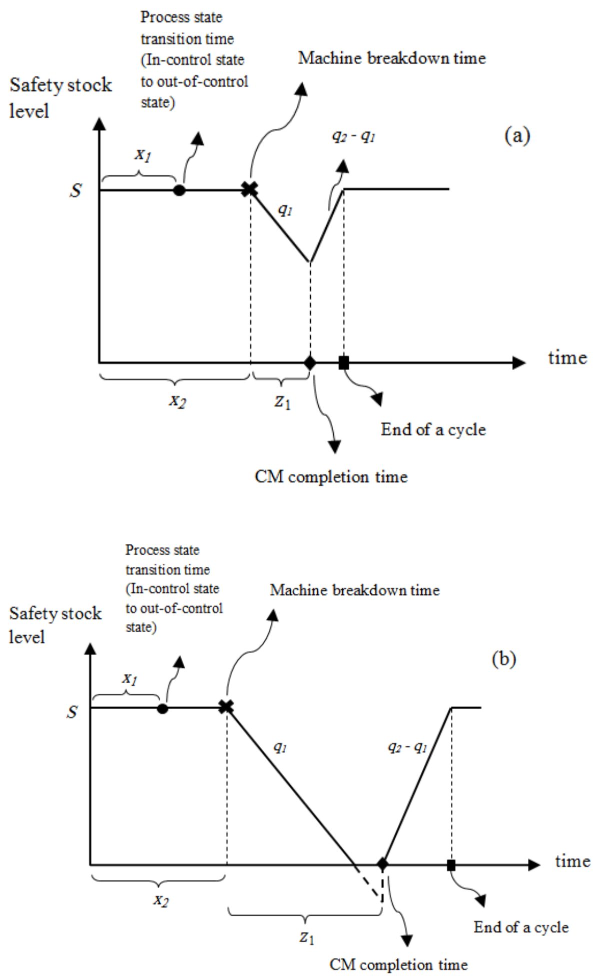


Figure 5

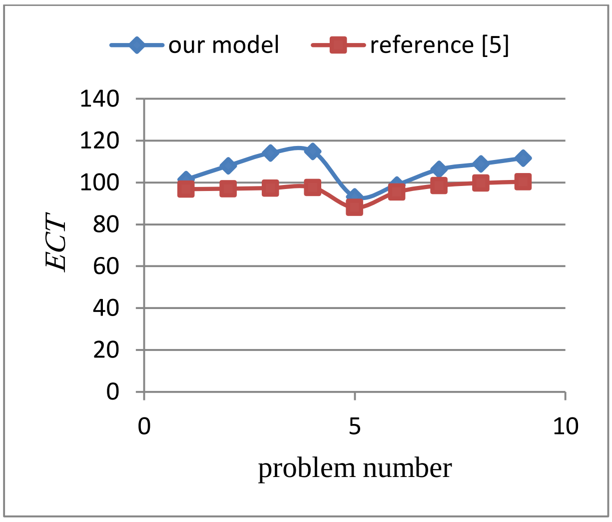


Figure 6

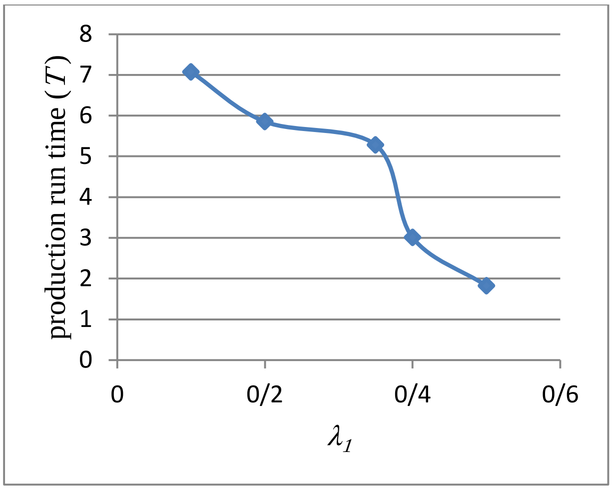


Figure 7

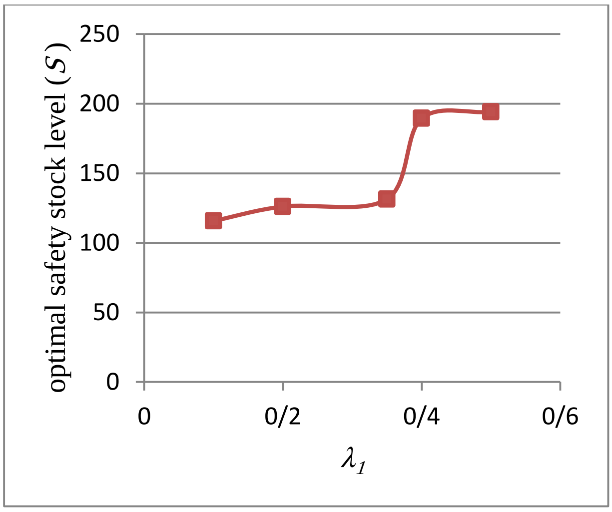


Figure 8

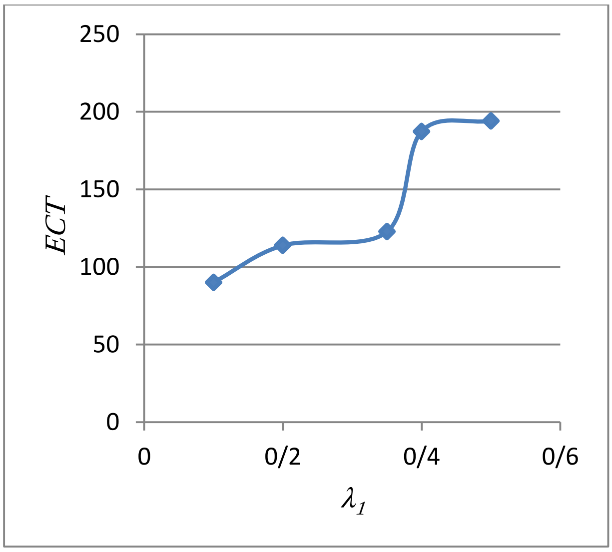


Figure 9

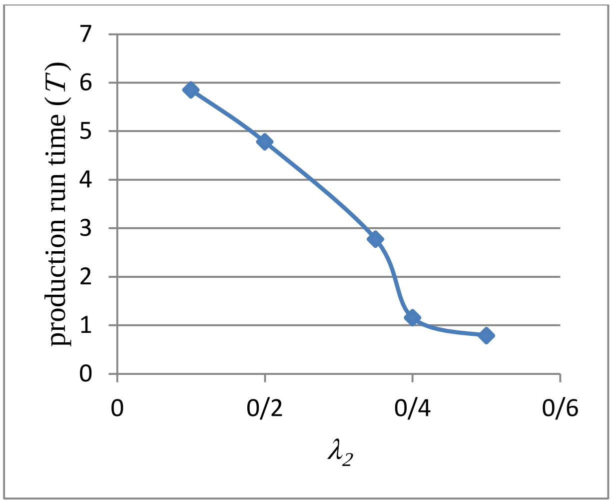


Figure 10

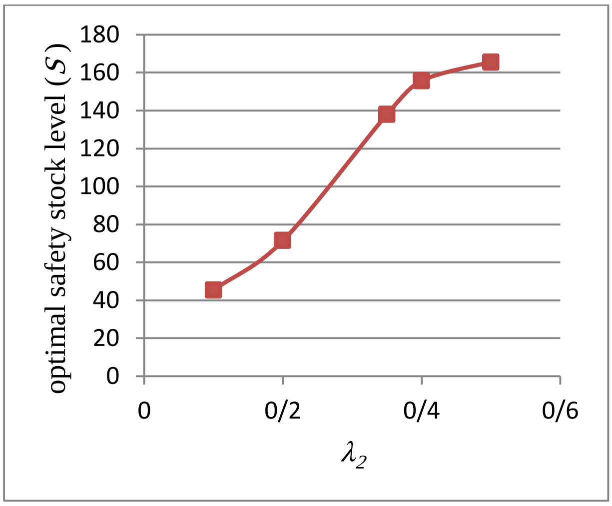


Figure 11

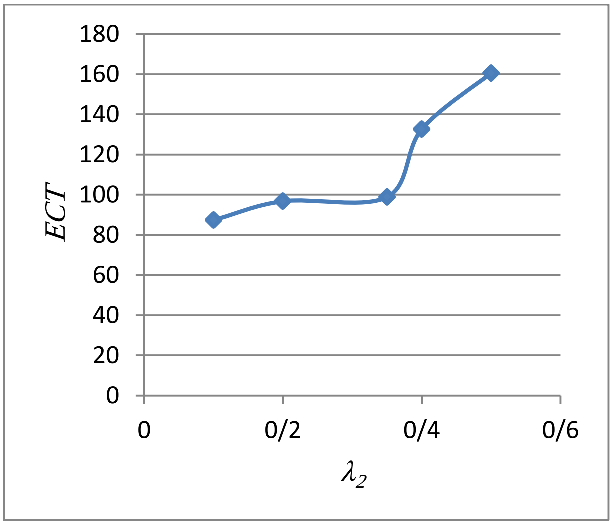


Figure 12

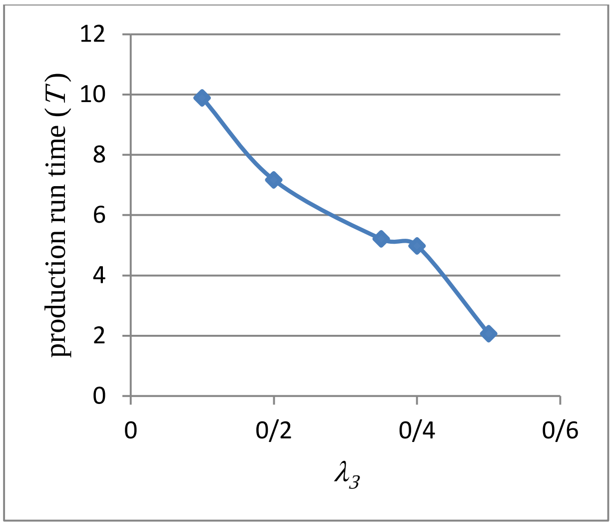


Figure 13

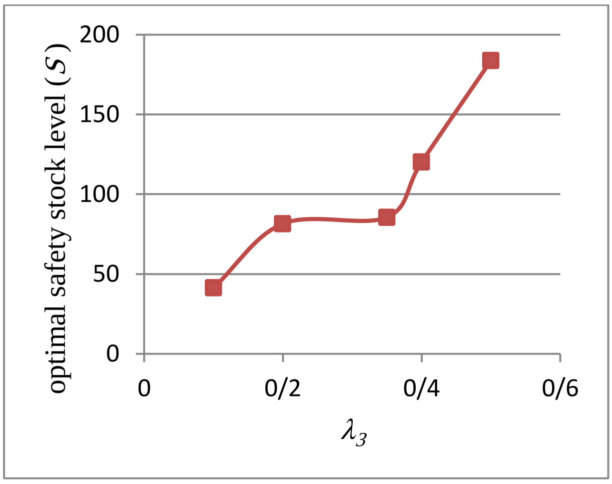


Figure 14

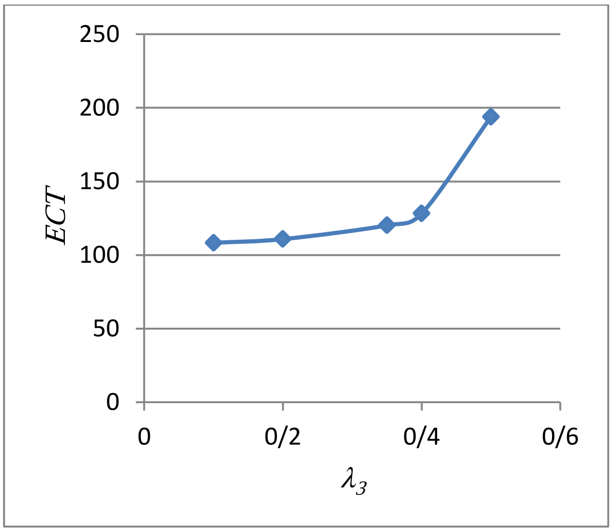
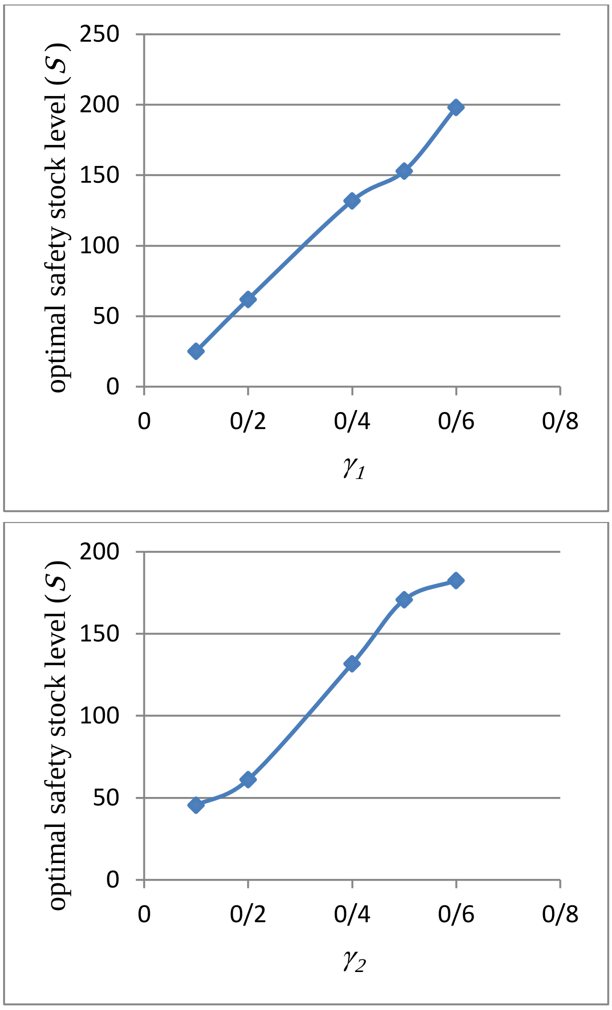
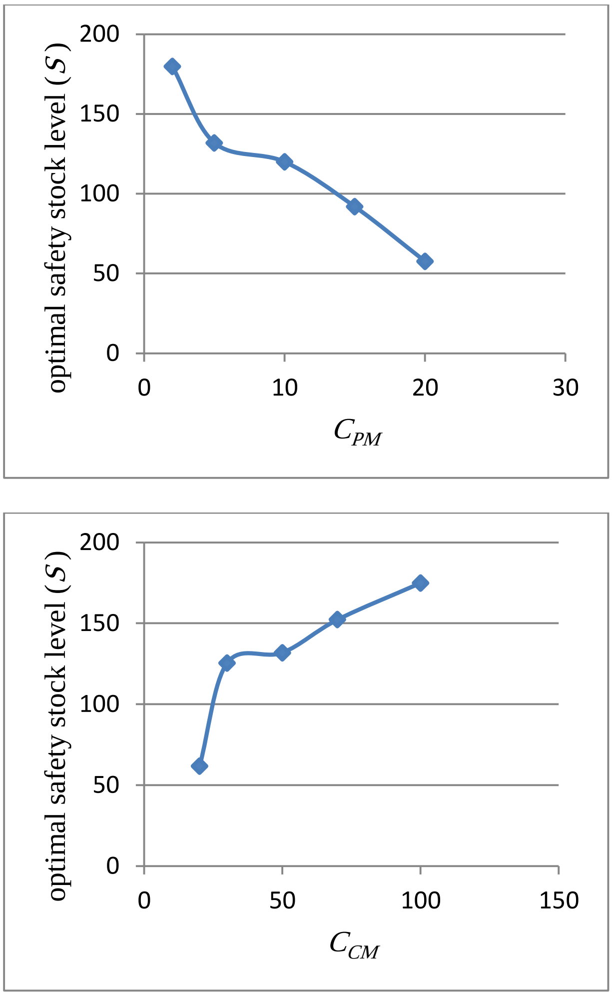


Figure 15



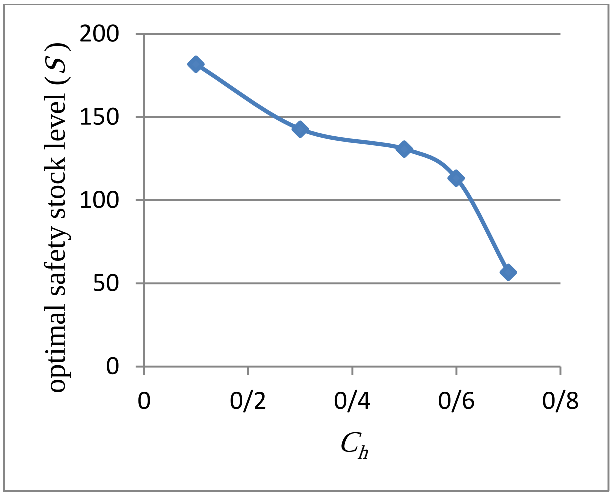
250
200
150
100
50
0
optimal safety stock level (S)
0
0/2
0/4
0/6
0/8
γ_1
200
150
100
50
0
optimal safety stock level (S)
0
0/2
0/4
0/6
0/8
γ_2

Figure 16



200
150
100
50
0
optimal safety stock level (S)
0
10
20
30
C_{PM}
200
150
100
50
0
optimal safety stock level (S)
0
50
100
150
C_{CM}

Figure 17


optimal safety stock level (S)
200
150
100
50
0
0
0/2
0/4
0/6
0/8
C_h

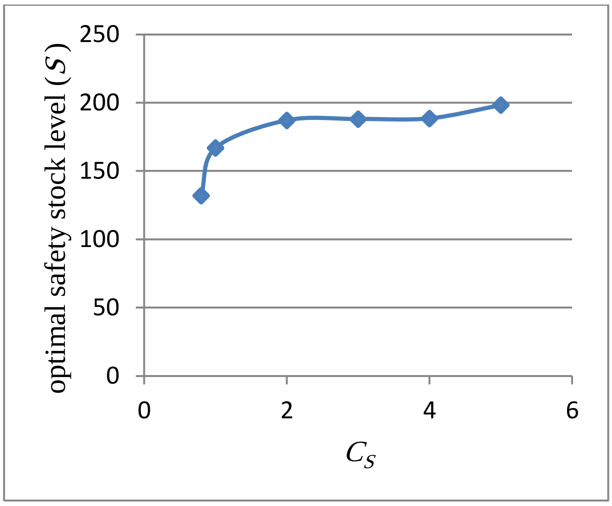
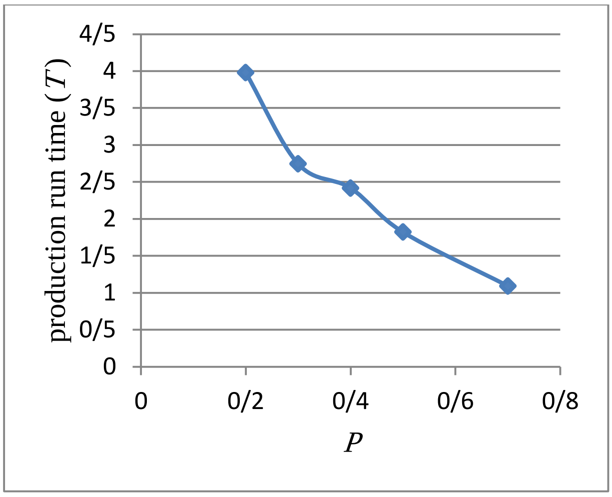

optimal safety stock level (S)
250
200
150
100
50
0
0
2
4
6
C_S

Figure 18



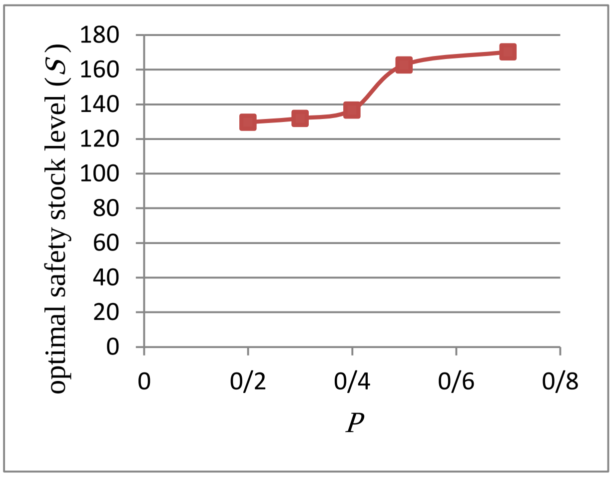


Figure 19

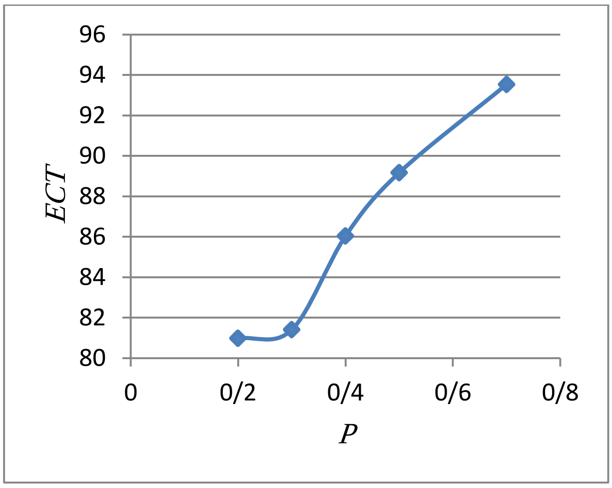


Figure 20