Scientia Iranica E (2023) 30(1), 260–272

Sharif University of Technology
Scientia Iranica
Transactions E: Industrial Engineering
http://scientiairanica.sharif.edu

Group decision-making based on last aggregation approach under interval-valued Pythagorean fuzzy environment for sustainable project decision

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Received 24 October 2019; received in revised form 19 December 2020; accepted 25 January 2021

KEYWORDS
Group decision-making;
Interval-Valued Pythagorean Fuzzy Sets (IVPFSs);
Linear Assignment Method (LAM);
Last aggregation;
IVPFSs ranking;
Sustainable project evaluation.

Abstract. Interval-Valued Pythagorean Fuzzy Sets (IVPFSs) as an enhanced type of Pythagorean Fuzzy Sets (PFSs) improve the expression of membership, non-membership, and hesitancy degrees, compared to Intuitionistic Fuzzy Sets (IFSs). In order to benefit from the advantages of IVPFSs, the current research proposes a new group decision-making method based on Linear Assignment Method (LAM). In this approach, both subjective and objective weights of criteria were taken into account. Moreover, the proposed method applied a new ranking method for IVPFSs. To avoid the shortcomings of the first aggregation methods, the introduced decision-making approach focused on the last aggregation approach. Finally, the method was used in a case study of sustainable project evaluation in order to depict the applicability of this method.

1. Introduction

Dealing with uncertainty demands flexible tools [1-3]. Intuitionistic Fuzzy Sets (IFSs) represent one of the applicable tools for modeling uncertainty [4]. These sets are able to model membership, non-membership, and hesitation degrees. It should be noted that despite their novelty, these sets cannot completely model the opinions of Decision-Makers (DMs). One example is a situation where the aforementioned degrees add up into a number larger than 1. Yager and Abbasov [5], and Yager [6] introduced Pythagorean Fuzzy Sets (PFSs) to handle such situations. These sets are developed based on IFSs and they can enhance the flexibility and applicability of IFSs.

PFSs are the improved forms of IFSs. To put it differently, they enjoy the merits of the IFSs besides their ability to offer a wider range of model agreement, disagreement, and hesitancy. For instance, upon comparing PFSs with the classic fuzzy sets, it can be concluded that PFSs have membership, non-membership, and hesitancy degrees while classic fuzzy sets can only include a crisp value of membership at the interval [0, 1]. Not only do the PFSs address the fuzziness of "agreement", but they incorporate the "disagreement" of the experts into the process [7,8].

PFSs have been recently used in several studies. Zhang and Xu [9] employed them to improve the well-known TOPSIS method. Peng and Yang [10] devised some operators for division and subtraction of PFSs. Yager [11] used the geometric mean and Ordered Weighted Geometric (OWG) operator to compare the alternatives using Pythagorean membership grades. Liang e al. [12] extended the VIKOR method using TODIM method under Pythagorean uncertainty.

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doi: 10.24200/sci.2021.54711.3880

Mohagheghi et al. [13] applied the PFSs to investigate the construction project selection, considering the resilience criterion. Garg [14] introduced new relations for correlation coefficient and weighted correlation coefficient to evaluate the PFSs. Garg [1] investigated the accuracy function for Interval-Valued Pythagorean Fuzzy Sets (IVPFSs) and applied the function to decision-making. Of note, IVPFSs have been recently used to extend some of the well-known decision-making methods. Yu et al. [15] proposed a group decision-making method based on the TOPSIS method. Liang et al. [16] established a method based on the MULTIMOORA method under IVPF environments. Ding et al. [17] introduced an IVPFS decision-making method through the prospect theory. Wu et al. [18] developed the DEA in the IVPF environments to address green supplier selection problem. Ilbahar and Kahraman [19] applied the WASPAS method to measure the retail store performance under the IVPF environment.

A review of both PFS- and IVPFS-based methods indicates that these sets are still new in decision-making problems, and their beneficial characteristics help address many uncertain decision-making situations. One of the decision-making methods that is new to these sets is Linear Assignment Method (LAM) [20]. This method applies a criterion-wise ranking approach using a set of criteria weights. The LAM applies linear compensatory discipline to handle the interaction and combination of the decision-making criteria. Another significant point in using PFSs and IVPFSs is the development of proper defuzzification and ranking methods. One approach in this process is how to use the concept of ideal solutions. Mohagheghi et al. [21] proposed a distance-based similarity measure between Interval Type-2 Fuzzy Numbers (IT2FN) to evaluate the performance of IT2FNs in any comparing applications. Despite its simplicity, this approach applies all the values that characterize a fuzzy number. However, this approach is not used to address IVPFSs. Another approach to improving a decision-making process is use of entropy to address weights of criteria. An increasingly popular approach regarding the decision preferences is the application of fuzzy entropies to better obtain the weights of the attributes. One of the main reasons for the practicality of fuzzy entropies is their eligibility for evaluation of fuzziness [22]. However, given the novelty of the IVPFSs in decision-making, this approach is still fresh in this area. Another decision-making approach that is new for IVPFSs and PFSs is the application of last aggregation methods. These methods avoid information loss by causing a delay in the aggregation process. One aggregation method is Weighted Aggregated Sum Product Assessment (WASPAS) method proposed by Zavadskas et al. [23]. One of the advantages of the WASPAS is the ability to prove the possibility of achieving the highest level of accuracy in assessments based on the principles of weighted aggregated functions optimization.

The main motivations behind this study are summarized in the following:

1. In recent years, IVPFSs have been widely used to extend well-known decision-making methods under a new uncertain environment which, in turn, enhanced the reliability of the decision-making methods;
2. The LAM method took into consideration the weights of criteria in its computational process, while a majority of other studies consider the weights given by the experts and do not address them in the form of subjective and objective weights;
3. Most of the proposed group decision-making methods are categorized as the first aggregation, which may lead to loss of information;
4. Sustainability in projects and project portfolios is a practical trend that requires proper consideration as well as application of tools for addressing complex conditions.

Organizational decision-making processes, which often have several uncertain factors, and situation evaluations are often carried out in groups [24–27]. In addition, in Multi-Criteria Group Decision-Making (MCGDM), reaching a preference-based choice over the available alternatives is the main objective [28–31]. The alternatives are investigated based on a number of criteria. A group of experts or DMs facilitate the implementation of the entire process [32].

In order to facilitate the improvement of MCGDM under IVPF uncertainty in this paper, a new decision-making method was introduced. The method was then used in sustainable project selection. In the following, the novelties of this research study are listed:

1. The PFS method borrowed the principles of the well-known method LAM to introduce a last aggregation method for project evaluation;
2. A new ranking method for IVPFSs was introduced, which was also used in the LAM process;
3. IVPFS entropy was presented to determine the importance of each criterion;
4. Score function of IVPFSs was used to address the LAM method;
5. The method was last aggregation and aggregation was carried out through the WASPAS method.

The rest of this paper is organized as follows. Section 2 presents a preliminary introduction of the proposed method. Section 3 discusses the presented method. Section 4 presents the application process of the method. Section 5 presents a comparative analysis. Eventually, Section 6 gives the concluding remarks.

2. Interval-valued Pythagorean fuzzy sets

PFSs are extensions of IFSs, offering greater flexibility and dominance in manifestation of uncertainty than their counterparts. Their capability to represent the degrees of membership, non-membership, and hesitancy is actually beneficial mainly due to their role in increasing the space for setting such values. One typical example of the privilege of PFSs over the IFSs is when a DM assigns the value to a candidate that satisfies a criterion as $\frac{\sqrt{3}}{2}$ and another value to that which does not meet the criterion as $\frac{1}{2}$. Obviously, in this example, the application of IFSs is not possible mainly because $\frac{\sqrt{3}}{2} + \frac{1}{2} \succ 1$. On the contrary, PFSs are able to overcome such uncertainties given their possession of a larger space for expression of such situations and, in this case $(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 \leq 1$. The PFSs can express higher levels of vagueness than the IFSs. In addition, in real-world situations where it is necessary to make a decision under uncertainty, PFSs have higher potentials in dealing with the vague and imperfect information than that of the IFSs. Figure 1 makes a comparison between PFSs and IFSs [6,9].

In the following, some basic definitions of IFS and PFS are given.

A given IFS set such as Q in a universe of discourse (O) is expressed as follows:

$$Q = \{\prec x, \mu_Q(O), \nu_Q(O) \succ | \, o \in O\}, \tag{1}$$

where $\mu_Q : O \to [0,1]$ shows the membership degree, while $\nu_Q : O \to [0,1]$ expresses the non-membership value of element $o \in O$ to the set Q under the following condition:

$$0 \leq \mu_Q(O), \;\; \nu_Q(O) \leq 1. \tag{2}$$

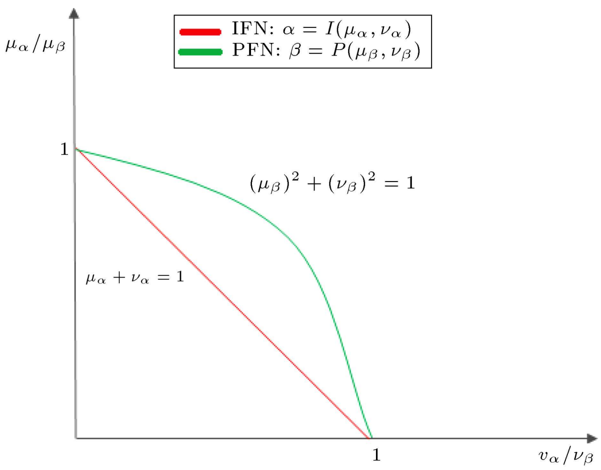

Figure 1. Spaces of PFS and IFS.

The last value in these sets is the degree of indeterminacy $\pi_Q(o)$:

$$\pi_Q(o) = 1 - \mu_Q(O) - \nu_Q(O). \tag{3}$$

IFSs were first proposed by Atanassov [4] and instead of only containing the value of membership, they possessed the values of membership, non-membership, and hesitancy. These sets were applied to address the uncertainties of various real-world situations like sustainable supplier selection [33], renewable energy evaluation [34], sustainable project portfolio selection [35], sustainable material selection [36], and evaluation of product end-of-life scenarios, to name a few [37].

Despite their successful applications, these sets cannot often deal with uncertain elements. In other words, there are conditions, where the degrees a candidate like Q_i satisfies and dissatisfies with respect to attribute P_j, that add up to a value greater than one. Obviously, it is not feasible to show this situation by using the IFSs. To deal with this problem, PFSs are introduced as follows [5,6].

Q as a PFS defined in the universe of discourse (O) is set as:

$$Q = \{\prec x, \mu_Q(O), \nu_Q(O) \succ | \, o \in O\}. \tag{4}$$

In Eq. (4), $\mu_Q : O \to [0,1]$ and $\nu_Q : O \to [0,1]$ set the values of membership and non-membership of element $o \in O$ to the set Q, respectively. The following condition should hold for the expressed values:

$$0 \leq (\mu_Q(O))^2, (\nu_Q(O))^2 \leq 1. \tag{5}$$

$\pi_Q(O)$ shows the value of indeterminacy denoted by the following:

$$\pi_Q(o) = \sqrt{1 - (\mu_Q(O))^2 - (\nu_Q(O))^2}. \tag{6}$$

One way to enhance the PFSs is to use intervals instead of crisp values to address uncertain degrees. Peng and Yang [10] introduced Interval-Valued Pythagorean Fuzzy Sets (IVPFSs). A briefing on these sets is given below.

Interval [0, 1] shows the set of all closed subintervals of [0, 1], and O represents a universe of discourse. As a result, an IVPFS $\tilde{Q}$ in O can be denoted as follows:

$$\tilde{Q} = \left\{ \prec x, \mu_{\tilde{Q}}(O), \nu_{\tilde{Q}}(O) \succ \middle| \, o \in O \right\}. \tag{7}$$

In Eq. (7):

$$\mu_{\tilde{Q}} : O \to Int([0,1])(o \in O \to \mu_{\tilde{Q}}(o) \subseteq [0,1]),$$

and :

$$\nu_{\tilde{Q}} : O \to Int([0,1])(o \in O \to \nu_{\tilde{Q}}(o) \subseteq [0,1]),$$

denote the membership and non-membership values of the element o belonging to O to the set $\tilde{Q}$, respectively. For each $o \in O$, $0 \leq \sup \left\{ \left((\mu_{\tilde{Q}}(o) \right)^2 + \left((\nu_{\tilde{Q}}(o) \right)^2 \leq 1 \right\}$ should hold. In addition, for each $o \in O$, $\mu_{\tilde{Q}}(O)$ and $\nu_{\tilde{Q}}(O)$ are the closed intervals. The lower and upper bounds are shown by $\mu^-_{\tilde{Q}}(O), \mu^+_{\tilde{Q}}(O), \nu^-_{\tilde{Q}}(O), \nu^+_{\tilde{Q}}(O)$, respectively. In the following, $\tilde{Q}$ is expressed as:

$$\tilde{Q} = \{ \prec x, [\mu^-_{\tilde{Q}}(O), \mu^+_{\tilde{Q}}(O)], [\nu^-_{\tilde{Q}}(O), \nu^+_{\tilde{Q}}(O)] \succ | o \in O \}. \tag{8}$$

The following for $\tilde{Q}$ should hold:

$$0 \leq (\mu^+_{\tilde{Q}}(O))^2, (\nu^+_{\tilde{Q}}(O))^2 \leq 1. \tag{9}$$

The degree of indeterminacy is set as:

$$\pi_{\tilde{Q}}(o) = \left[\pi^-_{\tilde{Q}}(o), \pi^+_{\tilde{Q}}(o)\right] = \left[\sqrt{1 - (\mu^+_{\tilde{Q}}(O))^2 - (V^+_{\tilde{Q}}(O))^2}, \sqrt{1 - (\mu^-_{\tilde{Q}}(O))^2 - (V^-_{\tilde{Q}}(O))^2}\right]. \tag{10}$$

The score function of $\tilde{Q}$ is expressed as [10]:

$$s(\tilde{Q}) = \frac{1}{2}[(\mu^+_{\tilde{Q}}(O))^2 + (\mu^-_{\tilde{Q}}(O))^2 - (V^-_{\tilde{Q}}(O))^2 - (V^+_{\tilde{Q}}(O))^2], S(\tilde{Q}) \in [-1, 1]. \tag{11}$$

The accuracy function of $\tilde{Q}$ is obtained as [10]:

$$a(\tilde{Q}) = \frac{1}{2}[(\mu^+_{\tilde{Q}}(o))^2 + (\mu^-_{\tilde{Q}}(o))^2 + (V^-_{\tilde{Q}}(O))^2 + (V^+_{\tilde{Q}}(O))^2], a(\tilde{Q}) \in [0, 1]. \tag{12}$$

The hesitancy degree of $\tilde{Q}$ is calculated as [10]:

$$h(\tilde{Q}) = \frac{1}{2} 1 - (\mu^+_{\tilde{Q}}(o))^2 - (V^+_{\tilde{Q}}(O))^2 + 1 - (\mu^-_{\tilde{Q}}(o))^2 - (V^-_{\tilde{Q}}(O))^2, \ h(\tilde{Q}) \in [0, 1]. \tag{13}$$

The distance between two IVPFNs $\tilde{Q}_1$ and $\tilde{Q}_2$ can be measured as follows [10]:

$$d(\tilde{Q}_1, \tilde{Q}_2) = \frac{1}{4}\Bigg(\left|(\mu^-_{Q1})^2 - (\mu^-_{Q2})^2\right| + \left|(\mu^+_{Q1})^2 - (\mu^+_{Q2})^2\right| + \left|(\nu^-_{Q1})^2 - (\nu^-_{Q2})^2\right| + \left|(\nu^+_{Q1})^2 - (\nu^+_{Q2})^2\right| + \left|(\pi^-_{Q1})^2 - (\pi^-_{Q2})^2\right| + \left|(\pi^+_{Q1})^2 - (\pi^+_{Q2})^2\right| \Bigg). \tag{14}$$

For two PFNs as $\tilde{Q}_1 = ([\mu^-_{Q1}, \mu^+_{Q1}], [\nu^-_{Q1}, \nu^+_{Q1}])$ and $\tilde{Q}_2 = ([\mu^-_{Q2}, \mu^+_{Q2}], [\nu^-_{Q2}, \nu^+_{Q2}])$ and $\rho \succ 0$, the following operations are set as follows [10]:

$$\tilde{Q}_1 \oplus \tilde{Q}_2 = \Bigg(\Bigg[\sqrt{(\mu^-_{Q1})^2 + (\mu^-_{Q2})^2 - (\mu^-_{Q1})^2 (\mu^-_{Q2})^2}, \sqrt{(\mu^+_{Q1})^2 + (\mu^+_{Q2})^2 - (\mu^+_{Q1})^2 (\mu^+_{Q2})^2} \Bigg], \Big[\nu^-_{Q1}\nu^-_{Q2}, \nu^+_{Q1}\nu^+_{Q2}\Big] \Bigg), \tag{15}$$

$$\tilde{Q}_1 \otimes \tilde{Q}_2 = \Bigg([\mu^-_{Q1}\mu^-_{Q2}, \mu^+_{Q1}\mu^+_{Q2}], \Bigg[\sqrt{(\nu^-_{Q1})^2 + (\nu^-_{Q2})^2 - (\nu^-_{Q1})^2 (\nu^-_{Q2})^2}, \sqrt{(\nu^+_{Q1})^2 + (\nu^+_{Q2})^2 - (\nu^+_{Q1})^2 (\nu^+_{Q2})^2} \Bigg] \Bigg), \tag{16}$$

$$\rho\tilde{Q}_1 = \Bigg(\Bigg[\sqrt{1 - (1 - (\mu^-_{Q1})^2)^\rho}, \sqrt{1 - (1 - (\mu^+_{Q1})^2)^\rho} (\nu^-_{Q1})^\rho, (\nu^+_{Q1})^\rho \Bigg] \Bigg), \tag{17}$$

$$\tilde{Q}_1^\rho = \Bigg(\left[(\mu^-_{Q1})^\rho, (\mu^+_{Q1})^\rho\right] \Bigg[\sqrt{1 - (1 - ((\nu^-_{Q1})^2)^\rho}, \sqrt{1 - (1 - ((\nu^+_{Q1})^2)^\rho} \Bigg] \Bigg). \tag{18}$$

The following operations can also be obtained:

$$\tilde{Q}_1 - \tilde{Q}_2 = \Bigg(\Bigg[\sqrt{\frac{(\mu^-_{Q1})^2 - (\mu^-_{Q2})^2}{1 - (\mu^-_{Q2})^2}}, \sqrt{\frac{(\mu^+_{Q1})^2 - (\mu^+_{Q2})^2}{1 - (\mu^+_{Q2})^2}} \Bigg], \left[\frac{\nu^-_{Q1}}{\nu^-_{Q2}}, \frac{\nu^+_{Q1}}{\nu^+_{Q2}}\right] \Bigg),$$

$$\text{if} \quad \mu^-_{Q1} \geq \mu^-_{Q2}, \mu^+_{Q1} \geq \mu^+_{Q2},$$

$$\nu^-_{Q1} \leq \min\left\{\nu^-_{Q2}, \frac{\nu^-_{Q2}\pi^-_{Q1}}{\pi^-_{Q2}}\right\},$$

$$\nu^+_{Q1} \leq \min\left\{\nu^+_{Q2}, \frac{\nu^+_{Q2}\pi^+_{Q1}}{\pi^+_{Q2}}\right\}, \tag{19}$$

$$\frac{Q_1}{Q_2} = \Bigg(\left[\frac{\mu^-_{Q1}}{\mu^-_{Q2}}, \frac{\mu^+_{Q1}}{\mu^+_{Q2}},\right], \Bigg[\sqrt{\frac{(\nu^-_{Q1})^2 - (\nu^-_{Q2})^2}{1 - (\nu^-_{Q2})^2}}, \sqrt{\frac{(\nu^+_{Q1})^2 - (\nu^+_{Q2})^2}{1 - (\nu^+_{Q2})^2}} \Bigg] \Bigg),$$

$$\text{if} \quad \mu^-_{Q1} \geq \nu^-_{Q1} \geq \nu^-_{Q2}, \ \nu^+_{Q1} \geq \nu^+_{Q2},$$

$$\mu^-_{Q1} \le \min\left\{\mu^-_{Q2}, \frac{\mu^-_{Q2}\,\pi^-_{Q1}}{\pi^-_{Q2}}\right\},$$

$$\mu^+_{Q1} \le \min\left\{\mu^+_{Q2}, \frac{\mu^+_{Q2}, \pi^+_{Q1}}{\pi^+_{Q2}}\right\}\Bigg]. \tag{20}$$

3. Proposed methodology

This section discusses the proposed method of IVPFS-LAM. To be specific, this method consists of six main steps. First, the decision matrices are formed; second, they are normalized. To obtain the required matrices for the IVPFS-LAM method, it is necessary to compare the IVPFS values denoting ratings of alternatives. Therefore, a ranking method for IVPFS is introduced. To better address the importance of criteria weights in this method, an IVPFS entropy method is extended and applied. The matrices required for the LAM method are then made, and the introduced IVPFS-LAM is presented. Given the last aggregation nature of the method, the WASPAS method finally aggregates the outcomes.

3.1. Description of making IVPFS decision matrices

In the proposed IVPFS-based method, DMs should express their opinions in the form of IVPFSs. For every value of $o \in O$, $0 \le \sup\left\{(\mu_{\tilde{Q}}(o))^2 + \sup(\mu_{\tilde{Q}}(o))^2 \le 1\right\}$ can be easily confirmed using spreadsheets. As a result, DMT^V will turn into the following matrix:

$$DMT^V = \begin{bmatrix} A^V_{11} & \cdots & A^V_{1h} \\ \vdots & \ddots & \vdots \\ A^V_{g1} & \cdots & A^V_{g\,h} \end{bmatrix}. \tag{21}$$

In this matrix, A_1, A_2, $\cdots$, A_g represents the set of possible alternatives to be evaluated and x_1, x_2, $\cdots$, x_h the set of evaluation criteria. In addition, g shows the number of alternatives, and h the number of criteria. Moreover, $A^V_{g\,h}$ is applied to express the evaluation of candidate A_g according to criterion x_h, which is expressed by the vth DM. In the following, $A^V_{g\,h}$ denotes the following:

$$A^V_{g\,h} = \left(\left[\mu^-_{A^V_{g\,h}}, \mu^+_{A^V_{g\,h}}\right], \left[\nu^-_{A^V_{g\,h}}, \nu^+_{A^V_{g\,h}}\right]\right). \tag{22}$$

The subjective importance value of x_h is expressed as:

$$x^V_h = \left(\left[\mu^-_{x^V_h}, \mu^+_{x^V_h}\right], \left[\nu^-_{x^V_h}, \nu^+_{x^V_h}\right]\right). \tag{23}$$

3.2. Normalization process of IVPFS decision matrices

In this step, a normalization process is employed. The sets of evaluation criteria categorized into benefit and cost are denoted by x_C and x_B, respectively, and is made dimensionless in this step. The matrices are normalized as:

$$A^V_{g\,h} = \begin{cases} A^V_{g\,h};\ h \in x_B \\ (A^V_{g\,h})^C;\ h \in x_C \end{cases} \tag{24}$$

where $(A^V_{g\,h})^C$ denotes the complement of $A^V_{g\,h}$.

3.3. Description of IVPFS ranking and defuzzification step

In order to carry out the LAM, it is necessary to compare and rank the values of $A^V_{g\,h}$. In this regard, this subsection presents a new ranking and comparing method which utilizes all the information expressed by an IVPFS. The studies of Deng [38] and Mohagheghi et al. [21] formed the basis of this method as elaborated in the following:

1. Define the positive ideal solution as $IV\tilde{P}FS_{\max}$ and the negative ideal solution as $IV\tilde{P}FS_{\min}$.
2. Compute the distance-based value of similarity between each *IVPFS* $(A^V_{g\,h})$, $IV\tilde{P}FS_g(g = 1, 2, ..., G)$, and the positive ideal solution (d^+_g) according to Eq. (25):

$$d^+_g(IV\tilde{P}FS_g, IV\tilde{P}FS_{\max}) = \frac{1}{4}$$
$$\Bigg(\left|(\mu^-_{IV\tilde{P}FS_g})^2 - (\mu^-_{IV\tilde{P}FS_{\max}})^2\right| + \left|(\mu^+_{IV\tilde{P}FS_g})^2 - (\mu^+_{IV\tilde{P}FS_{\max}})^2\right| + \left|(\nu^-_{IV\tilde{P}FS_g})^2 - (\nu^-_{IV\tilde{P}FS_{\max}})^2\right| + \left|(\nu^+_{IV\tilde{P}FS_g})^2 - (\nu^+_{IV\tilde{P}FS_{\max}})^2\right| + \left|(\pi^-_{IV\tilde{P}FS_g})^2 - (\pi^-_{IV\tilde{P}FS_{\max}})^2\right| + \left|(\pi^+_{IV\tilde{P}FS_g})^2 - (\pi^+_{IV\tilde{P}FS_{\max}})^2\right|\Bigg). \tag{25}$$

3. Compute the degree of similarity between each *IVPFS* $(A^V_{g\,h})$, $IV\tilde{P}FS_g(g = 1, 2, ..., G)$, and the negative ideal solution (d^-_g) based on Eq. (26):

$$d^-_g(IV\tilde{P}FS_g, IV\tilde{P}FS_{\min}) = \frac{1}{4}$$
$$\Bigg(\left|(\mu^-_{IV\tilde{P}FS_g})^2 - (\mu^-_{IV\tilde{P}FS_{\min}})^2\right|$$

$$+\left|(\mu^{+}_{IV\tilde{P}FS_g})^2-(\mu^{+}_{IV\tilde{P}FS_{\min}})^2\right|$$

$$+\left|(\nu^{-}_{IV\tilde{P}FS_g})^2-(\nu^{-}_{IV\tilde{P}FS_{\min}})^2\right|$$

$$+\left|(\nu^{+}_{IV\tilde{P}FS_g})^2-(\nu^{+}_{IV\tilde{P}FS_{\min}})^2\right|$$

$$+\left|(\pi^{-}_{IV\tilde{P}FS_g})^2-(\pi^{-}_{IV\tilde{P}FS_{\min}})^2\right|$$

$$+\left|(\pi^{+}_{IV\tilde{P}FS_g})^2-(\pi^{+}_{IV\tilde{P}FS_{\min}})^2\right|\Bigg). \quad (26)$$

4. Compute the overall performance of each IVPFS (IVPFSP) through Eq. (27) as shown in Box I.
5. Now given the fact that the values are converted into crisp values, they can be easily compared. IVPFSs can now be ranked in a descending order of $IVPFSP_g$.

This defuzzification is only used in cases where the formation of matrices is required for making the mathematical model of LAM. In other words, this defuzzification will not affect the fuzziness of the decision-making process.

3.4. Description of IVPFS criteria weights

LAM ranks the alternatives based on the criteria weights. To be specific, the weights of criteria play a vital role in the rankings. To date, the subjective weights in the presented method were collected from the DMs. In order to improve this process, an entropy-based criteria weight computation was developed. To put it differently, subjective and objective weights should be taken into consideration in this process. To compute the entropy of importance of the evaluation criteria, the following equation is proposed [39].

$$Ent(X^v)=\frac{1}{v}\sum_{v=1}^{V}\frac{D_1+D_2+\pi^{-}_{x_h^V}+\pi^{+}_{x_h^V}}{V_1+V_2+\pi^{-}_{x_h^V}+\pi^{+}_{x_h^V}}, \quad (28)$$

where $D_1=\min\left\{\mu^{-}_{x_h^V},\mu^{-}_{x_h^V}\right\}$, $D_2=\min\left\{\mu^{+}_{x_h^V},\mu^{+}_{x_h^V}\right\}$, $V_1=\min\left\{\nu^{-}_{x_h^V},\nu^{-}_{x_h^V}\right\}$, $V_2=\min\left\{\nu^{+}_{x_h^V},\nu^{+}_{x_h^V}\right\}$. Then, $Ent(X^v)$ is multiplied by x_h^V to make a novel criteria weight (SOW) which is calculated by Eq. (29) as shown in Box II.

3.5. Description of IVPFS model and solving approach

After computing the values of weights, the evaluation process is introduced as follows:

1. The results from Subsection 3-3 were taken into account to rank the g candidates for each criterion (x_h). A rank frequency matrix T^V is here defined. This non-negative square $(g\times g)$ has elements T^v_{gl} that show the frequency, at which A_g is ranked as the first criterion-wise ranking. The following matrix is the result of this step:

$$T^V=\begin{bmatrix} H^V_{11} & \cdots & H^V_{1g}\\ \vdots & \ddots & \vdots\\ H^V_{g1} & \cdots & H^V_{gg}\end{bmatrix}. \quad (30)$$

To illustrate further, T_{11} shows the frequency at which the alternative A_1 is ranked as the first

$$IVPFSP_g=\frac{\frac{1}{4}\begin{pmatrix}\left|(\mu^{-}_{IV\tilde{P}FS_g})^2-(\mu^{-}_{IV\tilde{P}FS_{\min}})^2\right|+\left|(\mu^{+}_{IV\tilde{P}FS_g})^2-(\mu^{+}_{IV\tilde{P}FS_{\min}})^2\right|+\\ \left|(\nu^{-}_{IV\tilde{P}FS_g})^2-(\nu^{-}_{IV\tilde{P}FS_{\min}})^2\right|+\left|(\nu^{+}_{IV\tilde{P}FS_g})^2-(\nu^{+}_{IV\tilde{P}FS_{\min}})^2\right|+\\ \left|(\pi^{-}_{IV\tilde{P}FS_g})^2-(\pi^{-}_{IV\tilde{P}FS_{\min}})^2\right|+\left|(\pi^{+}_{IV\tilde{P}FS_g})^2-(\pi^{+}_{IV\tilde{P}FS_{\min}})^2\right|\end{pmatrix}}{\frac{1}{4}\begin{pmatrix}\left|(\mu^{-}_{IV\tilde{P}FS_g})^2-(\mu^{-}_{IV\tilde{P}FS_{\min}})^2\right|+\left|(\mu^{+}_{IV\tilde{P}FS_g})^2-(\mu^{+}_{IV\tilde{P}FS_{\min}})^2\right|+\\ \left|(\nu^{-}_{IV\tilde{P}FS_g})^2-(\nu^{-}_{IV\tilde{P}FS_{\min}})^2\right|+\left|(\nu^{+}_{IV\tilde{P}FS_g})^2-(\nu^{+}_{IV\tilde{P}FS_{\min}})^2\right|+\\ \left|(\pi^{-}_{IV\tilde{P}FS_g})^2-(\pi^{-}_{IV\tilde{P}FS_{\min}})^2\right|+\left|(\pi^{+}_{IV\tilde{P}FS_g})^2-(\pi^{+}_{IV\tilde{P}FS_{\min}})^2\right|\end{pmatrix}+\frac{1}{4}\begin{pmatrix}\left|(\mu^{-}_{IV\tilde{P}FS_g})^2-(\mu^{-}_{IV\tilde{P}FS_{\max}})^2\right|+\left|(\mu^{+}_{IV\tilde{P}FS_g})^2-(\mu^{+}_{IV\tilde{P}FS_{\max}})^2\right|+\\ \left|(\nu^{-}_{IV\tilde{P}FS_g})^2-(\nu^{-}_{IV\tilde{P}FS_{\max}})^2\right|+\left|(\nu^{+}_{IV\tilde{P}FS_g})^2-(\nu^{+}_{IV\tilde{P}FS_{\max}})^2\right|+\\ \left|(\pi^{-}_{IV\tilde{P}FS_g})^2-(\pi^{-}_{IV\tilde{P}FS_{\max}})^2\right|+\left|(\pi^{+}_{IV\tilde{P}FS_g})^2-(\pi^{+}_{IV\tilde{P}FS_{\max}})^2\right|\end{pmatrix}},$$

$$g=1,2,...,G. \quad (27)$$

Box I

$$SOW_h^v = \left(\begin{array}{l} \left[\sqrt{1-((1-(\mu^-_{x^V_h})^2)^{\frac{1}{v}\sum_{v=1}^{V}\frac{D_1+D_2+\pi^-_{x^V_h}+\pi^+_{x^V_h}}{V_1+V_2+\pi^-_{x^V_h}+\pi^+_{x^V_h}}}}, \sqrt{1-((1-(\mu^+_{x^V_h})^2)^{\frac{1}{v}\sum_{v=1}^{V}\frac{D_1+D_2+\pi^-_{x^V_h}+\pi^+_{x^V_h}}{V_1+V_2+\pi^-_{x^V_h}+\pi^+_{x^V_h}}}} \right], \\ \left[(\nu^-_{x^V_h})^{\frac{1}{v}\sum_{v=1}^{V}\frac{D_1+D_2+\pi^-_{x^V_h}+\pi^+_{x^V_h}}{V_1+V_2+\pi^-_{x^V_h}+\pi^+_{x^V_h}}}, (\nu^+_{x^V_h})^{\frac{1}{v}\sum_{v=1}^{V}\frac{D_1+D_2+\pi^-_{x^V_h}+\pi^+_{x^V_h}}{V_1+V_2+\pi^-_{x^V_h}+\pi^+_{x^V_h}}} \right] \end{array} \right). \tag{29}$$

Box II

alternative in the appraisal of the vth decision-maker.

2. Followed by the formation of this matrix, it is employed to make a Weighted Rank Frequency Matrix (WRFM). The formation process of the aforementioned matrix is illustrated in Eq. (31):

$$WRFM^V = \begin{bmatrix} WRFM^V_{11} & \cdots & WRFM^V_{1g} \\ \vdots & \ddots & \vdots \\ WRFM^V_{g1} & \cdots & WRFM^V_{gg} \end{bmatrix}, \tag{31}$$

where:

$$WRFM^V_{g1} = SOW^v_{g1} \oplus SOW^v_{g1} \oplus ... \oplus SOW^v_{gT^V_{gl}}.$$

It should be noted that each one of the values of $WRFM^V$ in the weighted rank frequency matrix $WRFM^V$ denotes the value of concordance among all criteria in ranking the ith option lth.

It is possible that ϖ candidates end up with the same scores as those of the relevant criteria. Therefore, the original ranking is departed into $\varpi!$ equalized rankings. In this case, each of the rankings will receive the weight of $1/\varpi!$ [40].

3. Once the aforementioned matrices are formed, A_g for each $l (l = 1, 2, ..., g)$ must be decided in a way that $\sum_{l=1}^{g} s(WRFM^v_{gl})$ is maximized. Here, s denotes the score function which is used to defuzzify the fuzzy values in the model. Obviously, this problem is an $g!$ comparison problem. Let us define $\wp^v_{gl}$ as a binary variable which is equal to 1 if A_g is ranked as l for vth DM; otherwise, it is equal to 0. Consequently, the following linear assignment model can be obtained:

$$\max \sum_{g=1}^{G}\sum_{l=1}^{G}\frac{1}{2}\Big[\Big(\mu^+_{WRFM^v_{gl}}\Big)^2 + \Big(\mu^-_{WRFM^v_{gl}}\Big)^2 - \Big(\nu^-_{WRFM^v_{gl}}\Big)^2 - \Big(\nu^-_{WRFM^v_{gl}}\Big)^2\Big].\wp^v_{gl}, \tag{32}$$

subject to:

$$\sum_{l=1}^{g} \wp^v_{gl} = 1, \; g = 1, 2, ..., G, \; v = 1, 2, ..., V, \tag{33}$$

$$\sum_{g=1}^{G} \wp^v_{gl} = 1, \; l = 1, 2, ..., G, \; v = 1, 2, ..., V, \tag{34}$$

$$\wp^v_{gl} = 0 \,\text{or}\, 1 \,\text{for all}\; g \,\text{and}\; v. \tag{35}$$

Eq. (33) assigns the alternative A_g to only one ranking. Eq. (34) makes each ranking available only for one candidate. This step results in final rankings gathered from each DM. Since this approach is a group decision-making process, an aggregation process is required. In this case, $Rank^v_g$ is used to show the ranking of the alternative g which is carried out according to the opinions of vth DM.

3.6. Description of aggregation based on the WASPAS method

This step involves the use of the WASPAS method to ensure obtaining the highest level of accuracy in approximation through the optimization methodology for the weighted aggregated function. Moreover, it is beneficial to most of other available methods given the ability to improve the accuracy of evaluation [41]. The rankings achieved from the previous step are taken into consideration here to make the aggregated decisions. In this regard, the following equation is proposed:

$$\partial_g = \left(\Im\left(\sum_{v=1}^{V}(Rank^v_g)DW_v\right)\right) + \left((1-\Im)\left(\prod_{v=1}^{V} Rank_g^{v\,DW_v}\right)\right), \tag{36}$$

where DW_v is the weight of the vth expert, and $0 < \Im < 1$ shows the vitality of each part. It should be noted that $0 < DW_v < 1$ and $\sum_{v=1}^{V} DW_v = 1$. Finally, the aggregated rankings (∂_g) can be obtained.

3.7. Step by step algorithm

Step 1. Identify the alternatives $A_g(g = 1, 2, ..., G)$ and evaluation criteria $X_h(h = 1, 2, ..., H)$;

Step 2. Form the decision matrix for each DM using their expertise in the form of IVPFSs;

Step 3. Collect the required information on the subjective importance of decision-making criteria in the form of IVPFSs;

Step 4. Form the normalized decision matrices;

Step 5. Defuzzify the decision matrices based on the instructions given in Section 3.3;

Step 6. Compute the entropy of criteria and form the SOW to address the weights of criteria;

Step 7. Form the rank frequency matrix T^V for each DM;

Step 8. Form the Weighted Rank Frequency Matrix (WRFM) for each DM;

Step 9. Make and solve the linear assignment model according to each DM separately and obtain the values of $Rank_g^v$;

Step 10. Use the WASPAS-based aggregation approach to aggregate the obtained outcomes.

4. Prioritizing projects in a portfolio

Sustainability in project and project portfolio environments has been an essential and applicable research direction for researchers and practitioners for years. Awasthi and Omrani [42], ran a sustainable mobility project evaluation under fuzzy uncertainty. Li et al. [43] investigated the problem of sustainable building project evaluation based on the matter-element theory. In addition, Lei et al. [44] considered sustainability in investment risk evaluation of their renewable energy projects. In this section, the sustainability is considered to evaluate and prioritize projects in a portfolio of projects.

In order to depict the applicability of the presented approach to real-world decision-making problems, this section presents a case study. Projects in a portfolio of an Iranian holding company in gas and oil development are assessed using the method proposed in this paper. The main priority of the studied firm is to invest in the oil, gas, and petrochemical sectors. In addition, the firm is trying to reach an active position in the capital market of the country in order to get to its long-term objective of becoming the largest holding in Iran's petrochemical sector. Moreover, the firm seeks to secure its presence in local, regional, and international markets by obtaining a competitive edge over others. As a result, the firm has kept the data of candidate projects confidential. Given the confidentiality of the data, the authors had to present limited details of the projects.

The projects of this firm (CP_1, CP_2, CP_3, and CP_4) based on the sustainability criteria including economic (SC_1), social (SC_2), environmental (SC_3), cultural (SC_4), and spatial (SC_5) benefits were prioritized. A group of three experts (PME_1, PME_2, and PME_3) was consulted. Each expert reviewed the projects and the criteria and provided their judgment on project and criteria assessments using IVPFSs. To properly achieve IVPFS values, the PMEs were given the preliminary knowledge of IVPFSs and were asked to express their judgments by using membership and non-membership degrees. After gathering the required information, they checked to see if the constraint was not violated. Consequently, the violated values were returned to be adjusted. Employing IVPFSs provides more space to express agreement, disagreement, and hesitancy in comparison with IFSs. In addition to this advantage, IVPFSs carry an advantage in comparison with PFSs, that is, employing intervals to express uncertain elements.

Table 1 depicts the sustainability degree of the projects, and Table 2 shows the values of the evaluation criteria.

Since all the evaluation criteria belong to the

Table 1. Projects sustainability assessments results.

PME_1	SC_1	SC_2	SC_3	SC_4	SC_5
CP_1	([0.55,0.75],[0.25,0.4])	([0.62,0.72],[0.22,0.32])	([0.46,0.49],[0.38,0.4 8])	([0.71,0.75],[0.23,0.32])	([0.65,0.68],[0.23,0.33])
CP_2	([0.35,0.45],[0.6,0.7])	([0.5,0.6],[0.3,0.35])	([0.07,0.15],[0.8,0.9])	([0.69,0.71],[0.25,0.4])	([0.75,0.78],[0.33,0.43])
CP_3	([0.2,0.4],[0.5,0.6])	([0.45,0.48],[0.35,0.39])	([0.12,0.18],[0.74,0.84])	([0.51,0.55],[0.41,0.45])	([0.41,0.45],[0.56,0.6])
CP_4	([0.6,0.8],[0.3,0.4])	([0.72,0.78],[0.15,0.2])	([0.56,0.66],[0.23,0.43])	([0.55,0.65],[0.23,0.33])	([0.45,0.48],[0.55,0.58])
PME_2	SC_1	SC_2	SC_3	SC_4	SC_5
CP_1	([0.4,0.65],[0.32,0.45])	([0.62,0.78],[0.25,0.4])	([0.48,0.52],[0.41,0.45])	([0.65,0.69],[0.28,0.39])	([0.71,0.79],[0.21,0.31])
CP_2	([0.33,0.48],[0.55,0.65])	([0.6,0.65],[0.28,0.39])	([0.12,0.18],[0.71,0.7 5])	([0.55,0.75],[0.29,0.42])	([0.69,0.72],[0.23,0.33])
CP_3	([0.3,0.5],[0.4,0.75])	([0.5,0.55],[0.37,0.47])	([0.18,0.21],[0.7,0.78])	([0.5,0.58],[0.4,0.43])	([0.31,0.42],[0.6,0.69])
CP_4	([0.7,0.85],[0.2,0.4])	([0.79,0.85],[0.12,0.15])	([0.6,0.68],[0.25,0.31])	([0.41,0.48],[0.48,0.55])	([0.85,0.89],[0.17,0.25])
PME_3	SC_1	SC_2	SC_3	SC_4	SC_5
CP_1	([0.41,0.72],[0.2,0.52])	([0.68,0.69],[0.51,0.58])	([0.51,0.53],[0.42,0.48])	([0.6,0.65],[0.3,0.35])	([0.63,0.68],[0.23,0.35])
CP_2	([0.42,0.52],[0.55,0.72])	([0.41,0.45],[0.2,0.25])	([0.11,0.17],[0.61,0.69])	([0.45,0.55],[0.31,0.35])	([0.75,0.8],[0.23,0.3])
CP_3	([0.31,0.51],[0.42,0.62])	([0.39,0.49],[0.55,0.65])	([0.17,0.22],[0.65,0.81])	([0.78,0.52],[0.53,0.56])	([0.39,0.45],[0.5,0.55])
CP_4	([0.62,0.91],[0.18,0.2])	([0.55,0.61],[0.31,0.4])	([0.7,0.71],[0.29,0.35])	([0.55,0.6],[0.2,0.23])	([0.66,0.69],[0.3,0.4])

Table 2. Criteria weights applied in sustainable assessment.

	SC_1	SC_2	SC_3	SC_4	SC_5
PME_1	([0.75,0.8],[0.22,0.35])	([0.62,0.68],[0.23,0.29])	([0.5,0.6], [0.4,0.5])	([0.39,0.49],[0.33,0.38])	([0.5,0.59],[0.33,0.42])
PME_2	([0.68,0.72],[0.12,0.2])	([0.55,0.58],[0.31,0.39])	([0.43,0.49], [0.3,0.39])	([0.45,0.56],[0.33,0.39])	([0.45,0.55],[0.25,0.35])
PME_3	([0.7,0.75],[0.25,0.35])	([0.43,0.49],[0.33,0.38])	([0.5,0.55], [0.33,0.43])	([0.5,0.56],[0.4,0.5])	([0.62,0.75],[0.25,0.28])

$$T^V = \left| \begin{array}{cccc} SOW_4^1 & \begin{array}{c} SOW_1^1 + SOW_2^1 + \\ SOW_3^1 + SOW_5^1 \end{array} & 0 & 0 \\ SOW_5^1 & SOW_4^1 & SOW_1^1 + SOW_2^1 & SOW_3^1 \\ 0 & 0 & SOW_3^1 & \begin{array}{c} SOW_1^1 + SOW_2^1 + \\ SOW_4^1 + SOW_5^1 \end{array} \\ \begin{array}{c} SOW_1^1 + SOW_2^1 \\ +SOW_3^1 \end{array} & 0 & SOW_4^1 + SOW_5^1 & 0 \end{array} \right| . \tag{37}$$

Box III

Table 3. The results of ranking step.

		CP_1	CP_2	CP_3	CP_4
SC_1	d_g^+	0.28	1.23	1.36	0
	d_g^-	1.13	0.66	0	1.36
	$IVPFSP_g$	0.8	0.35	0	1
SC_2	d_g^+	0.34	0.83	1.15	0
	d_g^-	0.8	0.31	0	1.15
	$IVPFSP_g$	0.69	0.27	0	1
SC_3	d_g^+	0.51	2.19	1.8	0
	d_g^-	1.99	0	0.38	2.19
	$IVPFSP_g$	0.79	0	0.17	1
SC_4	d_g^+	0	0.15	0.82	0.53
	d_g^-	0.82	0.67	0	0.43
	$IVPFSP_g$	1	0.81	0	0.45

benefit category, the normalization step is not required. As a result, the ranking and defuzzification steps should be completed to obtained the needed matrices. Initially, the positive and negative ideal solutions are set and then, the computations for obtaining the values of $d_g^+(IV\tilde{P}FS_g, IV\tilde{P}FS_{\max})$ and $d_g^-(IV\tilde{P}FS_g, IV\tilde{P}FS_{\min})$ are done. Finally, the overall performance of each IVPFS is computed. Table 3 presents the results for the first decision-maker.

In order to use both subjective and objective weights in this process, entropy of criteria weights is computed and then, the new weight is constructed. Table 4 presents the entropy and new weights.

In order to establish the linear assignment model, it is required to form the matrix of T^v. Based on the defuzzified values presented in Table 3, the matrix for the first DM can be formed. by Eq. (37) as shown in Box III.

Then, the values of T^v for each DM are calculated. The resulting values are utilized to make the mathematical model of linear assignment method. For instance, the objective function for the first DM is formed as follows:

$$\begin{aligned} \max z = & 0.03x_{11} + 0.8x_{12} + 0x_{13} \\ & + 0x_{14} + 0.039x_{21} + 0.032x_{22} \\ & + 0.62x_{23} + 0.04x_{24} + 0x_{31} \\ & + 0x_{32} + 0.04x_{33} + 0.77x_{34} \\ & + 0.73x_{41} + 0x_{42} + 0.35x_{43} + 0.44x_{44}. \end{aligned} \tag{38}$$

Followed by solving the linear assignment models for each DM in LINGO optimizing software, the following rankings are obtained, the results of which are given in Table 5.

In order to obtain an aggregated result based on the judgments of all experts, the aggregation process should be undergone. Sensitivity analysis of the importance of experts was also carried out, the results of which are given in Table 6. Figure 2 lists the rankings of the alternatives.

Table 4. Entropy and SOW_h^v.

	SC_1	SC_2	SC_3	SC_4	SC_5
PME_1	([0.64,0.69],[0.37,0.5])	([0.62,0.68],[0.23,0.29])	([0.5,0.6], [0.4,0.5])	([0.39,0.49],[0.33,0.38])	([0.5,0.59],[0.33,0.42])
PME_2	([0.57,0.61],[0.25,0.35])	([0.55,0.58],[0.31,0.39])	([0.43,0.49], [0.3,0.39])	([0.45,0.56],[0.33,0.39])	([0.45,0.55],[0.25,0.35])
PME_3	([0.59,0.64],[0.41,0.5])	([0.43,0.49],[0.33,0.38])	([0.5,0.55], [0.33,0.43])	([0.5,0.56],[0.4,0.5])	([0.62,0.75],[0.25,0.28])
Entropy	0.6424	0.822	0.905	0.918	0.8

Table 5. Ranking of each expert.

Expert	CP_1	CP_2	CP_3	CP_4
PME_1	2	3	4	1
PME_2	2	4	3	1
PME_3	2	3	4	1

Table 6. Aggregated results under different weights of experts.

Expert	PME_1	PME_2	PME_3	
Weight of expert	(0.33)	(0.33)	(0.33)	
Project	CP_1	CP_2	CP_3	CP_4
Rank	2	3	4	1
Expert	PME_1	PME_2	PME_3	
Weight of expert	(0.2)	(0.6)	(0.2)	
Project	CP_1	CP_2	CP_3	CP_4
Rank	2	4	3	1
Expert	PME_1	PME_2	PME_3	
Weight of expert	(0.2)	(0.2)	(0.6)	
Project	CP_1	CP_2	CP_3	CP_4
Rank	2	3	4	1

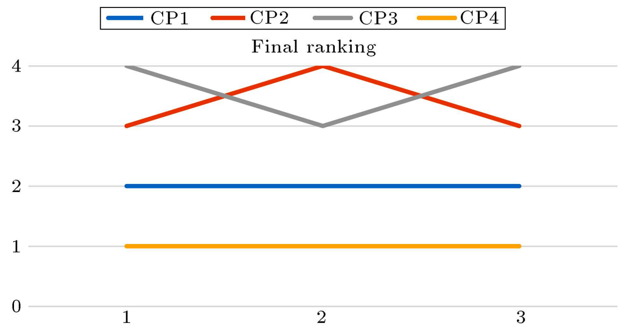

Figure 2. Priority of projects under different weights of DMs

5. Comparative analysis

In order to make a proper comparison and elaborate on the novelties of this paper, this section presents a comparative analysis. To be specific, first, the results are compared with those obtained by Oz et al. [45], as shown in Table 7. Then, a comparative analysis of the proposed method with respect to those methods offered in some of the relevant recent studies was conducted, results of which are given in Table 8.

According to the findings, both methods yielded similar outputs. Therefore, it can be concluded that the results obtained from the proposed method in this study are confirmed with respect to those obtained from the methods proposed by Oz et al. [45]. Table 8 shows the privileges of the proposed method over those in recent studies. This table presents the finer points of the presented method.

Table 7. Comparing the results of the present study with the results obtained by Oz et al. (2019).

	Rank (Oz et al., 2019)	Presented method of this paper
CP_1	3	2
CP_2	2	3
CP_3	4	4
CP_4	1	1

6. Conclusion

Real-world decision-making involves dealing with different uncertain elements. These uncertain elements could change the outcomes of the decision-making process. It is required that this uncertainty be taken into consideration in any real-world cases. Researchers have employed fuzzy sets to enhance the existing decision processes to deal with the vagueness of the decision-making environment. To better address such uncertainty, fuzzy extensions were introduced and applied in decision-making studies. Interval-Valued Pythagorean Fuzzy Sets (IVPFSs) as extensions of Intuitionistic Fuzzy Sets (IFSs) could enhance the presentation of uncertainty by expressing membership, non-membership, and hesitancy in a flexible way. In order to incorporate these sets in the decision-making process, a new Interval-Valued Pythagorean Fuzzy (IVPF) method was proposed. The proposed method was based on the concept of the Linear Assignment Method (LAM). However, this concept was not solely extended to a new environment, and this method could be enhanced from several aspects. Given the importance weights of criteria in the LAM, both subjective and objective weights of criteria were developed to enhance the process. In addition, a new ranking method for IVPFSs was introduced, which served as a step of this process. Another aspect of development was that the introduced method addressed group decision-making through the last aggregation extension of the LAM. This resulted in avoiding information loss. Moreover, the method was further enhanced by considering weights of decision makers in the aggregation process. The WASPAS method was utilized to aggregate the results. In order to investigate the applicability of this method, the data collected from a case study was used to evaluate and prioritize projects in a portfolio based on the sustainable criteria. To better investigate the outcomes of the process, sensitivity analysis of the weights obtained by the experts was carried out, and the results were compared with those obtained from recent studies. Several open questions are left unanswered to be further explored in the future research works. First,

Table 8. Comparing the presented method with similar studies.

Aspect	Description
Uncertainty	Using Pythagorean fuzzy sets that apply the membership, non-membership and hesitancy values with more flexibility in comparison with intuitionistic fuzzy sets.
Weight of criteria	This paper in comparison with other studies that apply linear assignment uses the weight of criteria that consists of subjective and objective weights of criteria.
First and last aggregations	The aggregation in this paper is carried out in the final step of this paper. This would result in a reduction of information loss which is caused when the aggregation is carried out in the initial phases.
Aggregation method	The aggregation is performed based on the WASPAS method. This would provide the step with the benefits of this method.

the impacts of using other concepts rather than entropy could be investigated using this method to highlight the importance of the involved criteria. Second, the subjective and objective data could be used to express the importance of decision-makers in the aggregation part of this paper. The third issue that can be further addressed in detail is consideration of the leniency reduction in the linear assignment method. Fourth, the method can be extended using similar fuzzy sets such as spherical and picture fuzzy sets to investigate the impacts of using different tools. Finally, the proposed method could be employed in other applications to assess the characteristics of the proposed method and IVPFSs in different environments.

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