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A novel robust model for location-allocation of healthcare facilities considering pre-disaster and post-disaster characteristics

M. Alinaghian[a,*], S.R. Hejazi[a], N. Bajoul[a], and K. Sadeghi Velni[b]

a. *Department of Industrial and Systems Engineering, Isfahan University of Technology, 84156-83111 Isfahan, Iran.*
b. *Department of Industrial Engineering, Fouman Faculty of Engineering, University of Tehran, Fouman, Iran.*

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Abstract. Healthcare centers are one of the most important municipal facilities that are responsible for providing personal and social health. Locating medical centers is an important strategy to improve the performance of these facilities and reduce any delay in the relief process. During disasters, these centers play a critical role in reducing the risk of human casualties. In this regard, the current study aims to establish a new robust mathematical model to simultaneously locate and allocate healthcare facilities, including different levels of medical care services, considering the characteristics in normal and disaster situations. Further, to help victims and prevent overcrowding at medical centers during disasters, establishment of temporary and outpatient centers was suggested for emergency provision of basic services. In addition, the possibility of delivering the needed equipment and medical teams to these centers was incorporated in the proposed modeling. To solve the problem, two metaheuristic algorithms, harmony search algorithm and hybrid tabu search combined with variable neighborhood search algorithm, and a lower bound based on Lagrangian relaxation method were employed. To assess the performance of the proposed algorithms, sample problems were randomly generated. The results obtained from the exact solutions and lower bound from the Lagrangian relaxation method were compared with those from the meta-heuristic algorithms, confirming the good performance of the proposed algorithms.

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1. Introduction

Healthcare centers are one of the most important urban facilities that are directly involved in providing personal and social health. In this regard, fast, timely, and cheap access to these centers in every community gains significance, especially in urban communities. Every year, natural disasters such as earthquakes, floods, storms, etc. strike different parts of the world. Iran is no exception in this regard and according to the studies conducted from 1980 to 2008, 138 similar disasters occurred in this country. As a result, an annual average number of 2689 deaths were recorded, and the consequent damages incurred a huge cost of about $737 million damage [1]. The increasing number of disasters and their destructive impacts, on the one hand, and population growth, on the other hand, lead to an increase in both material damage and human loss in such catastrophes. Despite the technological

*. *Corresponding author. Tel.: 031-33915511*
E-mail addresses: alinaghian@iut.ac.ir (M. Alinaghian); rehejazi@iut.ac.ir (S.R. Hejazi); n.bajool@gmail.com (N. Bajoul); Sadeghi.kosar@ut.ac.ir (K. Sadeghi Velni)

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advances, the repercussions caused by such disasters will always remain one of the main obstacles to the sustainable development of countries. Although the damages cannot be compensated in most cases, preconsideration of preventive measures and proper planning for dealing with these events can help reduce losses to a minimum. One of the most important measures is proper planning during the establishment of healthcare facilities. Since the severity and magnitude of these events are often large, demands for rescue operation after disaster are quite extensive and the relief centers that are supposed to satisfy the needs at urban centers often fail to respond quickly to the demands. It should be noted that dealing with victims, providing medical aids, and transferring the injured people to the relief centers at the right time, especially in the first 72 hours after the disaster called the relief golden hours, play a substantial role in reducing the death and disability rates [2]. Healthcare facilities are generally hierarchical so that in any hierarchy, a series of services are provided where the centers are categorized into different levels based on the type and quantity of such services. In this paper, it is assumed that higher levels of health-care providers cover all services provided at the lower levels. Due to the random and unpredictable nature of natural disasters, especially earthquakes, a comprehensive disaster management plan is required to reduce the risks and attenuate the difficulties caused by disasters. In fact, the main objective of the responses and relief reactions is to plan to better deal with such phenomena and public awareness and reduce deaths, injuries, and loss of property [3]. In addition, given the mentioned nature of the disaster, data uncertainty should be taken into consideration while planning to locate treatment centers. In case disaster uncertainty is neglected or taken for granted, improper planning would occur as a result of which a lot of facilities will be damaged during a disaster or their long distance from disaster centers will become problematic. This in turn makes it difficult to provide services to the victims. For this reason, at times of crisis, establishment of temporary relief centers is highly recommended mainly because these centers can not only reduce the time of providing initial relief services to the injured people but also serve those in need of higher levels of treatment who can be then transferred to healthcare facilities at higher levels. As a result, the transfer rate which is of high importance during disasters can be significantly reduced due to the restrictions on transferring injured people. The higher levels reduce congestion in permanent healthcare facilities, thus offering better services in the centers. It should be mentioned that according to the human resources, while the capacity relief of the temporary relief centers is important, the possibility to send relief teams to the temporary centers through hospitals and other permanent centers should also be considered. The innovations of the present research are summarized in the following:

1. Establishment of a robust model to locate hierarchical healthcare facilities in disaster conditions;
2. Consideration of the possibility of constructing temporary relief centers and transferring medical service teams from the permanent medical centers to the temporary ones;
3. Consideration of the possibility of transferring patients from medical centers of lower level to those of higher level (to be specific, the model has a referral property);
4. Proposal of two proposed meta-heuristic methods based on the harmony search algorithm and tabu search hybrid algorithm using the variable neighborhood search algorithm;
5. Provision of a lower bound based on the Lagrangian relaxation method for the raised problem;
6. Random generation of a number of sample problems in small and large sizes to examine the performance of the proposed algorithms and to evaluate and compare the results of the exact solutions and lower bound obtained from the Lagrangian relaxation method with those from the meta-heuristic algorithms. The results confirm the good performance of the proposed algorithms.

2. Literature review

This section presents a survey of the studies already conducted in three areas: locating healthcare facilities regardless of hierarchy, locating hierarchical healthcare facilities, and locating facilities at times of crisis.

2.1. Location models of healthcare facilities without considering hierarchy

In the literature on determining the healthcare locations, three major topics are highlighted, namely accessibility, adaptability, and reliability.

2.1.1. Accessibility

It is defined as the ability of the patients and customers to access healthcare centers or, in the case of emergency services, ability of the relief workers and relief services to access patients. Generally, input parameters in this type of model such as cost, demand, and travel distance or time are considered as the fixed non-random parameters [4]. Eaton et al. employed a maximal covering model to help planners in Austin, TX select permanent bases for their emergency medical services. This model is solved using the substitution and greedy adding algorithms [4]. Jacobs et al. used the P-median model with a limited capacity of facilities to optimize the collection, testing, and distribution of blood products in North Virginia and Carolina [5]. McAleer and Naqvi used the P-median model for relocating the ambulances in Belfast, Ireland. The problem was how to establish four facilities to serve 54 demand points. To solve this problem, they used a heuristic approach, divided the demand points into four parts, and placed each facility in the possible locations. Consequently, they reached a number of acceptable solutions in each part. They then examined all possible combinations of the acceptable sites using these 54 demand points [6].

Osinuga et al. developed an integrated methodology with the main focus on determining the healthcare location that could minimize the weighted average Euclidean distance between the existing facilities and the new ones to eliminate the need for the demands to travel a very long distance in order to access medical facilities. Geographic Information System (GIS) was also used to store and handle the coordinates and weights of the existing facilities. However, the Weber model and Weiszfeld's algorithm were employed to determine the new healthcare location. This technique was applied to a case study in one of the local government areas in Nigeria [7].

Cardoso et al. proposed a multi-objective and multi-period mathematical programming model to support the planning decisions in the Long-Term Care (LTC) sector. Their proposed model succeeded in providing the needed information to plan for the delivery of institutional LTC services in the medium term both in terms of location selection and capacity planning when tending to move towards an equitable provision of care. For this purpose, they considered three equity-related objectives: equity of access, geographical equity, and socioeconomic equity. They also took into account the costs in their model. The model objective function included multiple objectives. They further pointed out the applicability of their model through the resolution of a case study in the Great Lisbon region in Portugal [8].

Dogan et al. focused on the problem of locating Preventive Healthcare Facilities (PHC). The most important factors that increase participation rate in the PHC programs are establishment of an appropriate infrastructure and provision of satisfactory healthcare. To this end, the mentioned authors used a multi-objective mixed-linear programming model to locate PHC facilities, ensure maximum participation, and provide timely services to potential customers. They applied a model to a case study to determine the location of the cancer early detection, screening, and education centers in Istanbul, Turkey [9].

2.1.2. Adaptability

This feature suggests that location decisions be made with respect to uncertain conditions in the future, especially in the case of facilities such as hospitals which can be difficult, if not impossible, to relocate as conditions change [4]. One of the common measures to take in this area is scenario-based planning based on which some of the decisions are made just before determining the right scenario while other decisions are made when the information has already been obtained from the right scenario. In terms of location, the location of facilities is generally determined before knowing what scenario to choose. The demand points are normally allocated to these places after specifying a scenario and knowing which scenario is currently running. Three performance measures are employed in the scenario-based planning:

- Optimizing the expected performance;
- Minimizing the worst-case performance;
- Minimizing the worst-case regret.

To deal with these issues, Daskin et al. designed a model to minimize the maximum regret in a subset of scenarios that have already been identified with a probability of at least α [10]. Carson and Batta studied the problem of locating an ambulance for State University at Buffalo. In their study, the model was scheduled for four different time periods, considering the variations in the daily conditions and population at various time intervals [11]. Revelle et al. suggested a number of coverage models so as to ensure that the demand nodes and any established facility could be covered by different facilities and the model. The logic of the models is that in case a disaster occurs, emergency services in the area would be out of order or inaccessible and consequently, they should be covered by facilities in other areas [12].

2.1.3. Reliability

Reliability differs from adaptability in that adaptability is defined as the ability to function properly under uncertain conditions in the future. Some inputs such as demand and costs are generally susceptible to uncertainty. On the contrary, reliability is defined as the ability to function properly when a part of a system fails [13]. In other words, adaptability is related to the system environment while reliability is the uncertainty concerning the system function. For example, a change in the capacity of a facility due to failure is attributed to reliability. A sudden disaster may occur due to the capacity restrictions or closure of a number of facilities. Berman et al. described reliability issues for hospitals in Toronto. They located the facilities when the infliction of damage to facilities was quite likely. They also pointed to the closure of some hospitals due to the SARS outbreak and high demand for urban emergency services and facilities mentioned in other locations [14].

Snyder considered two developed P-median models concerning the reliability while Schneider formulated and solved a variety of developed locating models considering reliability [13]. Torres et al. developed a two-objective model to minimize the distance between the health and emergency centers as well as the costs while determining the number of treatment centers that could provide 24-hour emergency services for people. They also presented a harmony meta-heuristic algorithm to solve their model and suggested an approach to reduce the search space and increase the efficiency of their introduction [15].

Hoseinpour and Ahmadi-Javid investigated the design of an immobile service system in which the service process of each facility was subject to the risk of interruptions. They maximized the difference between the service provider's profit and the sum of customers' transportation and waiting costs. To solve large-size instances, they developed an efficient Lagrangian-based solution algorithm [16].

Shishebori and Yousefi Babadi provided an efficient mixed integer linear programming model for a Robust and Reliable Medical Service (MS) center Location Network Design Problem (RR/MSL/NDP) which simultaneously took into account uncertain parameters, system disruptions, and investment budget constraint [17].

2.2. Hierarchical Facility Location Problem (HFLP)

A hierarchical system refers to a system in which the facilities are unilaterally interrelated from top-down or bottom-up at different levels of service. The level of service (consisting of various facilities) is determined in such a way that the lowest and highest levels are referred to as Level 1 and Level k, respectively. Among the important typical applications of HFLP modeling are healthcare systems, emergency medical services, education systems, production-distribution networks, and telecommunication systems. Şahin and Süral classified the hierarchical models based on the flow patterns, objectives functions, service type, and space configuration [18].

- Flow pattern describes how products/services flow among the network nodes. In a network with a single-flow model, demand is formed at zero levels and, based on the priority of the levels of facilities with consideration of the type of service, ends at the highest level (or vice versa). In addition, in a multi-flow network, the demand may be produced at any level such as k ($k = 1 \cdots m$) production and responded at higher (or lower) levels [19];
- *Service availability:* Given the availability of a variety of services, every system is classified into successively inclusive and successively exclusive systems at any hierarchy level [20].

  In a successively inclusive system, each facility at a higher level presents the services provided at the lower-level facility together with a range of different services to customers (e.g., healthcare system) while in a successively exclusive system, facilities at each hierarchy level provide special services to customers being distinct from other levels;
- *Spatial configuration:* If any subset of a large set is considered as a hierarchy, the relationship between these subsets is formed in two types of coherent and incoherent. In a coherent system, all application areas allocated to a particular facility are assigned with an identical facility of higher level while the incoherent systems have no restriction on the spatial configuration of levels. In the following, medical care models among the hierarchical location models is to be discussed. The model proposed by Calvo and Marks is among the first hierarchical location models of health centers allocation. In this continuous multi-stream model, k levels are considered for facilities (with limited capacity) and the number of facilities is determined by the model [20]. Narula and Ogbu presented a two-level model considering the capacity limitations of the facilities. In their model, Level 1 is attributed to the health centers and Level 2 to the hospitals. In addition, five heuristic methods are proposed to solve the model and report the results of some calculations [21]. Parr provided a successively inclusive and multi-flow incoherent model. He briefly pointed to a set of guidelines to fabricate a more realistic model [22]. Tien et al. solved the model presented by Kahlo and Marx which was, in fact, a local successively inclusive model. Then, they presented a successively inclusive model which was the extended version of Kalu and Marx's model and began to solve their model [23].

Gerrard and Church [24] provided a two-level model, and Boffey et al. [25] presented a three-level model considering the possibility of transferring patients from treatment centers at lower levels to those at higher levels. Galvão et al. developed a three-level model that, in addition to considering the transfer of patients to a facility of higher level, incorporated the capacity restrictions of the facilities at high levels and solved the problem using Lagrangian Relaxation (LR) [26]. Yasenovskiy and Hodgson offered a three-level model based on the assumption that people might not always go to the nearest facility and for some reasons, they might instead prefer facilities of higher levels that provide better services at the farthest distance compared to those of lower level with limited services at a nearer distance [27]. Hodgson and Jacobs developed the previous model based on the patient behavior considering different possibilities to transfer the patients to a relief center of higher level according to the patients' required service level [28]. Consideration of uncertainty in health location gains significance for two main reasons.

Pouraliakbarimamaghani et al. proposed a location-allocation model for a capacitated healthcare system. They developed a discrete modeling framework to determine the optimal number of facilities among the candidates and optimal allocation of the existing customers for operating health centers at a coverage distance to ensure that the total sum of customer and operating facility costs could be minimized. The setup costs of the hospitals were based on the costs of customers, fixed costs of establishing healthcare centers, and costs based on the available resources at each level of hospitals. The idea of a hierarchical structure was considered in this study. Two levels of service were considered in the hospitals including low and high levels and sections at different levels that provide different types of services. To solve the model, they proposed two meta-heuristic algorithms including genetic algorithm, simulated annealing, and their combination [29]:

1. Decisions about location determination are long-term decisions and the possibility of making a change in the decision after the locating process is very costly;
2. These facilities play a key role in managing disasters and lack of attention to the uncertainty of such disasters leads to inefficient solutions.

2.3. Facility location models in disasters

Since the extent and intensity of natural disasters are on the rise due to several factors such as population growth, climate change, and global connectivity, it is predicted that the current aids will be insufficient [30]. It should be noted that the nature of natural disasters demands quick responses in a very short time. In such emergency and sophisticated conditions, the decision-maker must respond quickly and effectively to the problems and transfer the wounded from damaged areas to the designated centers. Toregas et al. first studied the relief facilities in 1971. They raised this issue in the envelope form and, then, used linear programming methods to solve it [31]. In another study, Chang et al. modeled how to locate facilities and distributed relief supplies to flood relief, according to different flood scenarios using a random two-step plan, considering the demand uncertainty for the relief [32]. Najafi et al. proposed a randomized multi-objective, multi-product, multi-course model and another model with few types of vehicles to distribute relief tools and transport the wounded after the earthquake and optimization under uncertainty. To ensure the proper function of the distribution program after the earthquake, a robust method was proposed in this study [33]. Bozorgi et al. suggested a robust planning approach to designing the relief logistics services under uncertainty conditions. They considered the parameters of supply, demand, and cost of production and transportation of relief goods at the same time as the inaccurate parameters of the problem. This was the first time that these three factors of uncertainty were taken into account at the same time for robust modeling [34]. Shen et al. formulated the problem of locating facilities in critical conditions and solved the model using a number of heuristic methods. According to their assumptions, when a facility fails, customers' demands will be assigned to other facilities [35].

Shavandi and Mahlooji developed hierarchical location-allocation models for congested systems, for example, in healthcare systems using a queueing theory in a fuzzy framework. The parameters of models were approximately evaluated and later regarded as the fuzzy numbers. Coverage of the demand nodes was approximately considered, which was later regarded as the degree of membership. Using queueing theory and fuzzy conditions, both referral and nested hierarchical models were developed for the Location Set Covering Problem (LSCP) [36].

Motallebi Nasrabadi et al. considered a problem consisting of both patients' and service providers' requirements (i.e., accessibility versus costs) for locating healthcare facilities, allocating service units to those facilities, and determining the facilities' capacities. To this end, they used both short-term and long-term uncertainties in the modeling stage. The queuing theory was employed to consider the stochastic demand and service time as a short-term uncertainty and a service level measurement. They found a way to make the linearized model more efficient by eliminating the excessive binary variables when the service level constraints were approximated using their properties. Additionally, the long-term demographic variations were captured through robust optimization to develop a robust model. To solve the problem under investigation, an evolutionary solution method was designed and its performance was investigated in different settings [37].

Mestre et al. proposed two location-allocation models to deal with uncertainty in the strategic planning of hospital networks. The main objective of their proposed models was to identify the hospital network system when the decision-maker sought to improve local access while minimizing the costs. The key features in the design of hospital networks, including the hospitals that provide multiple services in a hierarchical structure, were modeled on a planning horizon where the network changes might occur. These models function under some assumptions regarding the decisions that should be made without complete knowledge of the uncertain parameters and also decisions that should be made after identifying the uncertainties.

The demand uncertainties were modeled through a set of discrete scenarios. Both models were applied to the case study of the Portuguese National Health Service [38].

Ghezavati et al. proposed a hierarchical location model in the disaster relief chain under uncertainty to determine the timing of customer services. They considered the possibility of closing the roadways for relief operations in a disaster. In the surveyed network, a higher-level relief center offered all services provided by lower-relief facilities, and a robust optimization method and chance-constrained programming were employed [39].

As an option for controlling different types of uncertainty, the fuzzy set theory is recommended. For example, Canós et al. [40], Darzentas [41], and Rao and Saraswati [42] addressed the problem of fuzzy locating. However, all these models assumed that the problem parameters were definite.

On the contrary, some researchers including Zhou and Liu [43] investigated the problem of locating facilities and assigning the demand points to them according to the capacity restrictions of the facilities, considering the demand as a fuzzy case. Table 1 presents a brief review of the recent research studies.

The novelties of this paper are listed below:

- Establishing a robust model to locate hierarchy healthcare facilities in disaster conditions, taking into account the possibility of constructing temporary relief centers and transferring the medical service teams from the permanent medical centers to the temporary ones as well as the possibility of transferring patients from the lower-level centers to the higher-level medical care centers;
- Proposing two meta-heuristic methods based on the harmony search algorithm and tabu search hybrid algorithm with variable neighborhood search algorithm;
- Offering a lower bound based on the Lagrangian relaxation method for the given problem.

3. Robust optimization approach

3.1. Robust optimization method based on scenario

In 1995, Mulvey et al. proposed two important definitions about two types of robustness: solution robustness and model robustness [44].

In an optimization model, a solution is robust if the model remains nearly optimal in all scenarios while a model is robust when the solution is nearly feasible in all scenarios. Mulvey et al. presented a robust optimization model considering cost-benefit analysis concerning both solution and model robustness. In the robust optimization model, there are two types of variables: control and design variables. Design variables are determined before identifying the possible parameters that cannot change after identifying the possible parameters. Control variables are moderated based on a certain understanding of the uncertainty parameters. The robust optimization model provided by Malloy et al. is formulated in the following. Initially, a number of symbols associated with the model are introduced. Consider the following linear programming model that contains some random parameters:

$$\min \; f(x,y) = cx + dy, \tag{1}$$

$$\text{s.t.:} \quad Ax = b, \tag{2}$$

$$Bx + Cy = e, \tag{3}$$

$$x, y \geq 0, \tag{4}$$

where x is the vector of the design variables, and y the vector of the control variables. In addition, A, B, and C are the coefficients of the parameters, and b and e the vectors of the parameters (Right hand values). Moreover, A and b are definite values while B, C, and e have some degrees of uncertainty. A specific understanding of the uncertainty parameter is called scenario to which symbol s is allocated and whose possibility is specified by ρ_s to show a set of scenarios Ω used in this study. The coefficients of uncertainty are allocated as e_s, B_s, and C_s for each scenario $s \in \Omega$. Since the control variable y is moderated after understanding the scenario, y_s can be allocated to scenario s. Due to the uncertainty of the parameters, the model is likely to be unjustified for a number of scenarios. Therefore, η_s represents the infeasibility of the model under scenario s. If the model is feasible, η_s is equal to zero; otherwise, it receives a positive value from the following equations. The model proposed by Mulvey et al. is formulated in the following:

$$\min \; \sigma(x, y_1, y_2, \cdots, y_s) + \gamma\rho(\eta_1, \eta_s, \cdots, \eta_s), \tag{5}$$

$$\text{s.t.:} \quad Ax = b, \tag{6}$$

$$B_s x + C_s y_s + \eta_s = e_s, \tag{7}$$

$$x_s, y_s, \eta_s \geq 0, \qquad \forall \; s \in \Omega. \tag{8}$$

There are two terms in the objective function: The first one indicates the solution robustness while the second shows the model robustness by weight γ. Then, the two terms are discussed. High variance $f_s = (x, y_s)$ is indicative of the high risk of the decision. It should be noted that a small change in parameters with uncertainty can yield a great change in the value of the measuring function. Malloy et al. used this term to show the solution robustness. In the following formula,

Table 1. Summary of the reviewed articles.

Refs.	Hierarchical facilities	Disaster	Uncertainty		Displacement equipment	Temporary relief centers	Level		System type			Solution method		
			Probability	Robust			Two level	Multi Level	Inclusive	Exclusive	Multi flow pattern	Exact	Lower bound	Hiuristic/ metahiuristic
[4]												*		
[5]												*		
[6]														*
[7]												*		
[8]														*
[9]														*
[10]			*											*
[11]			*											*
[12]			*											*
[14]			*											*
[15]			*											*
[16]			*											*
[17]			*											*
[18]				*										*
[21]	*							*		*				*
[22]	*						*			*				*
[23]	*							*	*		*			*
[24]	*							*	*					*
[25]	*						*		*					*
[26]	*							*	*					*
[27]	*							*	*				*	
[28]	*							*	*					*
[29]	*							*	*		*			*
[30]	*						*		*					*
[32]		*	*									*		
[33]		*	*											*
[34]		*		*										*
[35]		*		*										*
[36]		*	*											*
[37]	*	*	*				*			*		*		
[38]	*	*		*			*		*					*
[39]	*	*	*					*	*					*
[40]	*	*	*					*	*					*
This study	*	*		*	*	*		*	*		*		*	*

δ is the weight allocated to the solution variance:

$$\min\ Z = \sum_{s\in\Omega} \rho_s f_s + \delta \sum_{s\in\Omega} \rho_s \left(f_s - \sum_{s\in\Omega} \rho_s f_s \right)^2,$$

$$\sum_s \rho_s = 1. \tag{9}$$

As observed, a second-order term exists in the above formula.

Yu and Li suggested that the formula proposed by Mulvey et al. [44] needs complex and numerous calculations due to non-linearity. Instead, they offered the following formulation [45]:

$$\min\ Z = \sum_{s\in\Omega} \rho_s F_s + \delta \sum_{s\in\Omega} \rho_s \left[\left(F_s - \sum_{s\in\Omega} \rho_s F_s \right) + 2\theta_s \right], \tag{10}$$

$$F_s - \sum_{s\in\Omega} \rho_s F_s + \theta_s \geq 0 \qquad \forall\ s \in \Omega, \tag{11}$$

$$\theta_s \geq 0 \qquad s \in \Omega, \tag{12}$$

where θ_s is the linearizing variable. Eqs. (11) and (12) are used for making the variance term linear.

In addition, in the Mulvey's model objective function, we can add another term, indicative penalty for non-compliance of some model constraints, to some scenarios.

$$\min\ Z = \sum_{s\in\Omega} \rho_s F_s + \delta \sum_{s\in\Omega} \rho_s \left[\left(F_s - \sum_{s\in\Omega} \rho_s F_s \right) + 2\theta_s \right] + \gamma \sum_{s\in\Omega} \rho_s \eta_s. \tag{13}$$

In Eq. (13), γ is the weight considered for the violation of the model constraints that shows the cost-benefit analysis between the model and solution robustness. Moreover, violation of the model from the restrictions in scenario s is denoted by η_s.

4. Problem statement

Healthcare facilities are the most important city facilities that are directly responsible for ensuring individuals' health in any society. Locating healthcare facilities is among the most important strategies for improving their performance and reducing the delay in a normal situation. Moreover, during disasters, these centers gain significance mainly because one of the most important measures that needs to be considered at times of disaster is the optimal allocation of the injured people. Therefore, considering the appropriate location of such facilities in normal and critical conditions can simultaneously increase the level of satisfaction with the facilities and reduce losses. Under such conditions, one of the most important issues is how to deal with the wounded and provide them with the needed health services in the shortest time possible. One of the actions that directly concerns the governments in disasters is proper planning during the establishment of relief and treatment centers. The most important issue in this regard is to select the best location for medical centers. These facilities should be available to the public so as to provide proper services for people even prior to the disaster ocuurrence. During disasters, these centers are also expected to exhibit a good performance. Choosing the wrong location for such centers, on the one hand, can cause downtime and out-of-service conditions during disasters in the case of proximity to the center of disasters. On the other hand, if the mentioned location is too far away from the center of the disaster, the transfer of patients and mortality rate would increase.

During disasters, establishment of outpatient centers can help classify the wounded based on the injury level, and the patients with minor injuries can be treated in the relief centers, and only the highly injured ones will be transferred to the permanent centers.

If establishment of the temporary relief centers is either ignored or postponed, the wounded will be transferred to the permanent centers without prioritizing; hence, the transfer of casualties increases due to limited resources.

Moreover, congestion and chaos at the centers are likely to happen as a result of the transfer of patients with low injury rates to the permanent treatment centers. In addition, due to the lack of accommodation camps as well as permanent treatment centers, patients will encounter low levels of damage after treatment, hence confused in this regard wondering what to do. However, during the establishment of temporary relief centers, considerable attention should be drawn to the capacity of human resources services, and equipment and medical teams should be sent to the temporary centers.

Assumptions

The assumptions of the proposed robust model are illustrated in the following:

- This model takes into account the K types of facility and K levels of service;
- In the k-type health centers, services are provided at the service levels of $c = 1, 2, \cdots, k$ while the service levels of $c = k+1, \cdots, K$ are not provided;
- Each of the demand points can be a candidate point to be considered in a facility establishment;
- The higher-level facilities cover all facility services of lower level; in other words, the model is successively inclusive;
- In each node, you can build at least one facility

and the capacity for each facility is limited for any specific service;

- If the demand for each service of a facility is more than the capacity, a certain amount of penalty is determined;
- In this model, the service reception can begin at any level; in other words, the model has a multi-flow pattern;
- There is the possibility of transferring patients from any level to a higher one, if needed.

In other words, the model enjoys the referral property:

- The cost of establishing all the facilities of a kind is equal;
- The facilities of the same type have an equal capacity at each service level;
- In the first scenario, it is assumed that normal conditions prevail with no disaster occurrence while in other scenarios, an event occurs in different regions with different intensities;
- At times of crisis, in addition to the permanent hierarchical facilities, the temporary facilities to be established, if necessary, are also taken into consideration, which can only provide low-level services;
- Permanent facilities can transfer part of the low-level facilities to the temporary facilities, if necessary;
- In case of failure, permanent facilities are not able to provide any level of services; yet, they can send basic equipment and medical teams to the temporary facilities.

Collections

cu	Set of demand points (candidate), $cu = \{1, 2, \cdots, n\}$, and i, j, h as the corresponding indices
Ks	Levels of medical facilities and provided services, $k = 1, 2, \cdots, K$ and k, c are the corresponding indices
S	Set of provided scenarios, $s = 1, 2, \cdots, S$

Parameters:

Ra_{cc+l}	The transition rate from the service level c to the service level $c+l$
Q_{kc}	The capacity of the facility type k at the service level c
u_{ic}	Percentage of the demanded service type c at the demand point i in scenario s
dam_{js}	Failure probability of the facility j in scenario s
w_{is}	Demand of the node i in scenario s
Bud_1	The total budget available for permanent facilities (Currency)
Bud_2	The total budget available for temporary facilities in each scenario (Currency)
cap	The capacity of temporary facility
CO_k	The cost of establishing permanent facility
Ct	The cost of establishing temporary facility
d_{ijs}	The distance between the demand point i and the facility j in scenario s
d_{jhs}	The distance between the facilities j and h in scenario s
ρ_s	The occurrence probability in scenario s
λ	The weight allocated to solution variance
M	A large number

Decision variables

x_{ijkcs}	Demand (number of patients) in point i with the level of demanded service equal to c from a facility located in point j at level k in scenario s
r_{jhkcs}	Population referred from facility j to the facility h of type k to provide service level c in scenario s
eq_{jkhs}	Amount of low-level service that is transferred from the facility j of type k to the temporary facilities in scenario s
y_{jk}	Equal to 1 if a facility type k to be established in point j; otherwise, it is 0
$temp_{js}$	Equal to 1 if a temporary facility in scenario s to be established in point j; otherwise, it is 0
sh_{jcs}	Shortage cost related to lack of capacity of facility j at the service level c in scenario s

The robust mathematical model is expressed as follows:

$$\text{Minimize} \quad Z = \sum_{j=1}^{n} \sum_{k=1}^{K} CO_k \times y_{jk}$$

$$+ \sum_{s=1}^{S} \rho_s \Bigg(\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{K} \sum_{c=1}^{K} d_{ijs}$$

$$\times x_{ijkcs} + \sum_{j=1}^{n} Ct \times temp_{js}$$

$$+ \sum_{j=1}^{n} \sum_{h=1}^{n} \sum_{k=1}^{K} \sum_{c=1}^{K} d_{jks} r_{jhkcs}$$

$$+\sum_{j=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{K} d_{jks} eq_{jkhs}\Bigg)$$

$$+\lambda\cdot\sum_{s=1}^{S}\rho_s\Bigg[\Bigg(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{c=1}^{K} d_{ijs}$$

$$\times x_{ijkcs}+\sum_{j=1}^{n} Ct\times temp_{js}$$

$$+\sum_{j=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{K}\sum_{c=1}^{K} d_{jks} r_{jhkcs}$$

$$+\sum_{j=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{K} d_{jks} eq_{jkhs}\Bigg)$$

$$-\sum_{s'}\rho_{s'}\Bigg(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{c=1}^{K} d_{ijs}$$

$$\times x_{ijkcs'}+\sum_{j=1}^{n} Ct\times temp_{js}$$

$$+\sum_{j=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{K}\sum_{c=1}^{K} d_{jks} r_{jhkcs'}$$

$$+\sum_{j=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{K} d_{jks} eq_{jkhs'}\Bigg)+2\theta_s\Bigg]$$

$$+\gamma\sum_{j=1}^{n}\sum_{c=1}^{K}\sum_{s=1}^{S}\rho_s Sh_{jcs}. \qquad (14)$$

Subject to:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{K-1}\sum_{c=k+1}^{K}\sum_{s=1}^{S} x_{ijkcs}=0, \qquad (15)$$

$$\sum_{j=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{K-1}\sum_{c=k+1}^{K}\sum_{s=1}^{S} r_{jhkcs}=0, \qquad (16)$$

$$\sum_{k=c}^{K}\sum_{j=1}^{n} x_{ijkcs}=u_c\times w_{is}$$

$$c=1,2,3,\cdots,K, \quad i\in cu, \quad s\in S, \qquad (17)$$

$$\sum_{j=1}^{n} temp_{j1}=0, \qquad (18)$$

$$\sum_{k=1}^{K} y_{jk}\times dam_{js}+\sum_{j=1}^{n}\sum_{s=1}^{S} temp_{js}\leq 1, \qquad j\in cu, \qquad (19)$$

$$\sum_{i=1}^{n}\sum_{c=1}^{K} x_{ijkcs}\leq M\times y_{jk}\times dam_{js}$$

$$j\in cu, \quad k=2,3,\cdots,K, \quad s\in S, \qquad (20)$$

$$\sum_{i=1}^{n}\sum_{c=1}^{K} x_{ijkcs}\leq M\times (y_{jk}\times dam_{js}+temp_{js})$$

$$j\in cu, \quad k=1, \quad s\in S, \qquad (21)$$

$$\sum_{h=1}^{n}\sum_{k'=c+l}^{K} r_{jhk'(c+l)s}=\sum_{i=1}^{n} Ra_{c(c+l)} x_{ijkcs}$$

$$j\in cu, \quad c=1,2,3,\cdots,K-1, \quad c+l\in Ks, \quad s\in S, \qquad (22)$$

$$\sum_{j=1}^{n} r_{jhkcs}\leq M\times y_{hk}\times dam_{hs}$$

$$h\in cu, \quad j\neq h, \quad k,c\in Ks, \quad s\in S, \qquad (23)$$

$$\sum_{h=1}^{n} eq_{jkhs}\leq Q_{kc}\times y_{jk}$$

$$j\neq h, \quad j\in cu, \quad k=1,2,\cdots,K, \quad s\in S, \quad c=1, \qquad (24)$$

$$\sum_{j=1}^{n} eq_{jkhs}\leq \beta\times cap\times temp_{hs}$$

$$j\neq h, \quad h\in cu, \quad k=Ks, \quad s\in S, \qquad (25)$$

$$\sum_{i=1}^{n}\sum_{k=1}^{K} x_{ijkcs}+\sum_{h=1}^{n}\sum_{k=1}^{K} r_{hjkcs}+\sum_{h=1}^{n}\sum_{k=2}^{K} eq_{jkhs}$$

$$-\sum_{i=1}^{n}\sum_{k=1}^{K} Ra_{c(c+l)} x_{ijkcs}-\sum_{k=1}^{K} Q_{kc}\times y_{jk}$$

$$\times dam_{hs}-\Bigg(cap\times temp_{js}+\sum_{h=1}^{n}\sum_{k=1}^{K} eq_{hkjs}\Bigg)=Sh_{jcs}$$

$$j\in cu, \quad s\in S, \quad c\in Ks, \quad c+l\in Ks, \qquad (26)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K} CO_k\times y_{jk}\leq Bud1, \qquad (27)$$

$$\sum_{j=1}^{n} Ct\times temp_{js}\leq Bud2_s, \qquad s\in S, \qquad (28)$$

$$\Bigg(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{c=1}^{K} d_{ij}\times x_{ijkcs}$$

$$+\sum_{j=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{K}\sum_{c=1}^{K} d_{jk} r_{jhkcs}$$

$$\left.+\sum_{j=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{K} d_{jk} eq_{jkhs}\right)$$

$$-\sum_{s'} \rho_{s'} \left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{c=1}^{K} d_{ij} \times x_{ijkcs'}\right.$$

$$+\sum_{j=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{K}\sum_{c=1}^{C} d_{jk} r_{jhkcs'}$$

$$\left.+\sum_{j=1}^{n}\sum_{h=1}^{n}\sum_{k=1}^{K} d_{jk} eq_{jkhs'}\right) + \theta_s \geq 0,$$

$$s \in S, \tag{29}$$

$$x_{ijkcs}, eq_{jkhs}, Sh_{jcs}, r_{hjkcs} \geq 0$$

$$i, j, h \in cu, \quad c, k \in Ks, \quad s \in S, \tag{30}$$

$$y_{jk}, temp_{js} \in \{0, 1\}, \qquad j \in cu, \quad k \in Ks, \quad s \in S. \tag{31}$$

The objective function of the model is based on the Malloy's robust objective function model, and the description of the other parts is already given. Eqs. (15) and (16) ensure that no facility can provide services above its level. Eq. (17) ensures that all demands at each demand point i are assigned to the facilities. Eq. (18) states that in the case of normal situations, there is no temporary facility. Eq. (19) indicates that at any point, one permanent or temporary facility can be found at most (However, in any scenario, subject to the failure of permanent facility in one area, a temporary facility can be found too). Eqs. (20) and (21) state that the services can be received at any particular level only if a facility of the same level or a higher level (non-exclusive) is available at this point. Eq. (22) determines the number of people transferred to the service at the higher level according to the defined rate. In fact, this service cannot meet the requirements for this group of people based on the type of their need. Eq. (23) guarantees that the patients are transferred to wherever a facility is established. Eq. (24) confirms that the permanent facilities, if established, can send equipment from the Service Level 1 (low-level) to the temporary facilities. Eq. (25) expresses that the temporary facilities, if established, can receive up to β% of their capacity from a higher-level facility (permanent facility). Eq. (26) calculates the capacity shortage for all services provided by each center. For this purpose, the shortage of the center is calculated by the number of people in need of service who directly refer to the center besides those who were already transferred to the specific service providers at the center plus the equipment transferred from the desired permanent center to another temporary centers minus the number of people who referred to other services and minus the capacity of the center in the desired service. As observed, for each service, if the number of referred patients exceeds capacity, the corresponding penalty will be considered in the objective function. Eqs. (27) and (28) name the budget restrictions, and Eq. (29) is the linearization constraint in the Malloy's model. Eqs. (30) and (31) determine the type of decision variables.

5. Solution methods

Locating facilities is a complex and difficult problem [46]. Therefore, to solve this problem in higher orders, two meta-heuristic methods as well as a lower-bound method are proposed, which will be later described in this section.

5.1. Harmony search algorithm

In recent years, with the development of computers, meta-heuristic methods have received considerable attention. Among efficient meta-heuristic algorithms, harmony search algorithm is a significant one which is inspired by the music-composing method [47]. Due to its applicability to discrete and continuous optimization problems, simple concept, small number of parameters, and easy implementation, this algorithm has been recognized as one of the most common optimization algorithms in recent years [48]. Any musician or musical instrument is representative of a decision variable in this method. During the algorithm, each musician plays a note and in fact, each decision variable is given a value. The objective of the algorithm iteration is to find the best harmony among the musicians or the global optimal point. The steps of harmony search algorithm can be expressed as follows:

Step 1: Set algorithm parameters

In this algorithm, like any other meta-heuristic algorithm, the algorithm parameters need to be set (note that our problem space is discrete and thus, we explain the algorithm in the discrete space). The harmony search algorithm parameters include the number of vectors in Harmony Memory Size (HMS), probability of selecting Harmony Memory Consideration Rate (HMCR), probability of setting and changing Pitch Adjustment Rate (PAR), and Band-Width (BW) distance used for the problems in continuous spaces. Finally, the maximum number of algorithm iterations is indicated by MaxIt. In this algorithm, each solution is called harmony and represented by an N-component vector. Harmony

Memory (HM) matrix is built using several solutions or harmonies.

$$\left[\begin{array}{cccc|c} \mathbf{x_1^1} & \mathbf{x_2^1} & \cdots & \mathbf{x_N^1} & \mathbf{f(x^1)} \\ \mathbf{x_1^2} & \mathbf{x_2^2} & \cdots & \mathbf{x_N^2} & \mathbf{f(x^1)} \\ \cdots & \cdots & \cdots & \cdots & \vdots \\ \mathbf{x_1^1} & \mathbf{x_2^1} & \cdots & \mathbf{x_N^{\mathit{HMS}}} & \mathbf{f}\left(\mathbf{x}^{\mathit{HMS}}\right) \end{array}\right]. \qquad (32)$$

Step 2: Create the first generation (initial values) for algorithm
In this step, harmony vectors are randomly created and stored in HM by as many as first-generation HMSs. Then, the objective function is calculated and stored for each of these vectors and stored in the matrix given in Figure 1.

Step 3: Create a new harmony
Each harmony vector includes the problem variables. To create value for the ith variable, first, produce a random number between zero and one. Then, compare this number with parameter HMCR. If it was is than its value, another value is chosen for ith variable from the memory matrix and ith column. Otherwise, a random value is chosen from the search space for ith variable with the probability as (1-HMCR). If a value is chosen from the memory matrix, we can set it to probability PAR.

Step 4: Update HM
The value of newly generated harmony is compared with that of the worst harmony available in the matrix memory. If it is higher than that of the worst harmony available in the matrix memory, the old harmony is replaced by a new one.

Step 5: Check the stopping condition
Steps 3 and 4 continue until a certain number of iterations (MaxIt), and the best solution is reported as the solution to the problem [49]. Despite the suitability of the harmony search algorithm, this algorithm quickly converges in some cases. In addition, this algorithm characterized by greedy nature seeks to improve the worst solution available in the HM, which reduces the variety of solutions on HM and leads to trapping by local optimum [50]. Given the problems listed for harmony search algorithm, efficiency improvements for the changes required to develop these algorithms are presented: Among the modifications of the harmony search algorithm is the improvement of the memory harmony. To this end, a cumulative idea was used for producing HM. The cumulative idea was first proposed in 2008 by Degertekin. In this idea, instead of all memory members randomly, random solutions are produced that are twice the HMS and places as many as HMSs of the best solutions in HM [51].

Among other improvements to the fixed algorithm is not to consider the choice probability parameter from HM HMCR so that the value of this parameter over the consequent iterations of algorithm changes linearly. This change occurs as follows [50]:

$$HMCR(t) = HMCR_I + \frac{(HMCR_F - HMCR_I)(t-1)}{MaxIt - 1}$$
$$t \in \{1, \cdots, MaxIt\}, \qquad (33)$$

where $HMCR_I$ represents the $HMCR$ value at the first iteration, and $HMCR_F$ the value at the last iteration. The harmony search algorithm's pseudo-code is shown as follows:

Input: initialize parameters and harmony memory
Repeat
$t = 1$
While (not_termination)
{
for ($I = 1$ to n)
  if ($rand1 \leq HMCR(t)$)
    $x(i)$ will be randomly chosen from harmony memory
   if ($rand2 \leq PAR$) pitch adjustment
    $x(i) = x(i \pm 1)$
   end if
  else
    $x(i)$ =random selection
end if
end for
$t = t + 1$
evaluate the fitness of each vector
update harmony memory
update $HMCR(t)$
}

5.2. Solution string

The model presented in this study is a location-allocation model. Followed by examination, it has been found that once the type of established facility is determined in each location, the solution time is dramatically reduced, and the model is solved easily and quickly using the exact model. Therefore, to solve

0	1	3	3	2	0	0	2	1	3	1	1	0	1	0	0	0	1	1	1	0	0	1	0	1	1	1	0	1	0
First part										Second part																			

Figure 1. Example of a solution string for the problem with $k = 3$ and 10 candidate points.

the model, the integration of the exact method with meta-heuristic method is used. Based on the proposed meta-heuristic algorithm at each iteration, the type of treatment centers located in any place is added to the model as an input parameter given to the model. Then, the model is solved using CPLEX solver, thus returning the resulting solution. To display the solution, the two-part string is used in which the first part has a length of n which is used for the establishment of permanent facilities, while the second part has multiple subparts equal to those of scenarios minus 1 each of length n, thus showing how to establish temporary facilities at any candidate point in any scenario. It is worth noting that in Scenario 1, no temporary center can be established. Numbers $0 - k$ can be set in each cell of the corresponding string in the first string. Number 0 implies that no facilities were built at the corresponding candidate point. Numbers $1 - k$ represent the facility establishment types $1 - k$ in the relevant places. Figure 1 presents an example with 10 candidate points and three kinds of permanent facilities based on the three scenarios.

According to Figure 1, no facilities were established at candidate points 1, 6, and 7. Facilities of types 1,2, and 3 were established at Points 2 and 9; Points 5 and 8; and Points 3, 4, and 10, respectively. The second part of the string, as shown in Figure 1, includes the subparts of 10 in length, each of which is related to one scenario except for the first scenario. In each cell of the corresponding subparts, Numbers 0 and 1 can be put. Number 0 means that at the desired candidate point, no temporary facility is built while Number 1 is indicative of the establishment of a temporary facility at the candidate point. As observed in Figure 2, no temporary facility is established in the first scenario under normal conditions. In this regard, Scenario 1 is not included in the solution string. In Scenario 2, in the above example, at each of points 1, 2, 4, 8, 9 and 10, a temporary facility is established. In Scenario 3, at Points 3, 5, 6, 7, and 9, a temporary facility is established. It should be noted that the presented string is interpreted in such a manner that if a permanent facility exists in node i and if this facility does not fail in one scenario, no temporary facility will be established in this place without paying attention to the part related to the corresponding scenario.

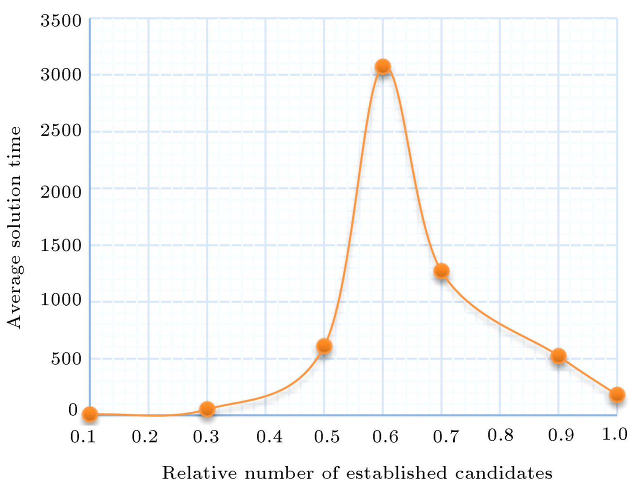

Figure 2. The average solution time according to the number of established facilities compared to all candidate points.

5.3. Tabu search algorithm

Tabu search algorithm is a meta-heuristic optimization algorithm that was first introduced in 1986 by Glover [52]. This method is widely used as a method of optimization. This technique is a holistic approach to conducting research for achieving good results in a complex solution space. To obtain the optimum results in an optimization problem, the tabu search algorithm begins to move from the initial solution and the list of prohibited actions is given. This list includes previous changes in the solution string, thus making it possible for those previous changes to remain unchanged at least in the next several moves. Then, the algorithm selects the best neighbor solution from the current neighbors. If the solution is not on the tabu list, the algorithm moves to the solution neighbors; otherwise, the algorithm checks a measure called the aspiration criterion. According to the aspiration criterion, in case the neighbor solution is superior to the best obtained neighbor result, the algorithm will move to this neighborhood, even if the solution is on the tabu list. Once the algorithm moves to the neighboring result, the tabu list is updated, meaning that the previous change that led to the current result is given in the tabu list to prevent the algorithm from returning and creating a cycle. In fact, a tabu list is a tool in the tabu search algorithm that prevents the algorithm from being trapped in the local optima. Followed by placing the previous move in the tabu list, some movements previously located in the list are removed from the list. The time the parameters are located on the list is determined by a parameter called tabu list length. Moving from the present solution to the neighbor solution continues until the stopping condition occurs. Different stopping conditions can be considered for the algorithm. Table 2 lists the parameters used in this section.

5.3.1. Proposed hybrid tabu search algorithm

To find an initial solution to the proposed hybrid tabu search algorithm, a simplified mathematical model was used. In this model, only the demand level of each service anywhere in the scenarios, cost of facilities established at different levels as well as shortage cost in each scenario regardless of the cost allocated to the patients, and number and location of the permanent

Table 2. The tabu search algorithm parameters.

Parameter	Definition
Sol	Current answer
S'	The answer resulting from neighborhood
Sol_{best}	The best found answer
$StCon$	The number of iterations that if the neighborhood does not reach the best answer, the algorithm goes on to the next neighborhood
$MaxIt$	Maximum repetition of algorithm
$StCon_{it}$	The maximum number of repetitions that if the best solution does not change, the algorithm stops
del	Counter of the number of not reaching improvement (in the neighborhood)
$No\ Im\ p$	Counter of the number of times that the best answer is not improved (total algorithm)
it	Counter of algorithm iterations

as well as temporary facilities are determined in each scenario. In the proposed tabu search algorithm, a variable neighborhood search algorithm is used. Its main idea is to change the neighborhood using local search. A variable neighborhood search algorithm was first proposed in 1997 by Mladenović and Hansen [53]. Simplicity of implementation and quality of the obtained results from Variable Neighborhood Search (VNS) quickly made this method an appropriate candidate for solving the optimization problems. Assume that A_i for $l = \{1, 2, \cdots, l_{\max}\}$ is a predetermined neighborhood structure, and $A_l(x)$ is the set of neighborhoods of x under the structure of A_l. Algorithm VNS has two main phases of shaking and local search. At the shaking phase, the algorithm moves to a neighborhood solution (S') from the current solution using the ith neighborhood structure. Besides, the local search phase is searched on S' using neighborhood search methods to reach optimized S'. In case the obtained local optima in the moving or non-moving part are better than the current solution (Sol), it will be replaced by it. Otherwise, the next neighborhood structure A_{l+1} is used to continue searching. This search continues until $l < l_{\max}$. The pseudo-code of the variable neighborhood algorithm is expressed as follows [54]:

Input: a set of neighborhood structures A_l, $l = 1, 2, \cdots, l_{\max}$
Sol=generate initial solution();
Repeat
$l = 1$;
While $(l \leq l_{\max})$
{
$s' = shaking(Sol, Al)$
$s'^{*} = Localsearch(s')$
if $f(s'^{*}) \leq f(Sol)$
$Sol \leftarrow s'^{*}$
$l = 1$;
else
$l = l + 1$;
}
Until stopping condition is met;
Output: The best solution;

Initial solution generation phase. According to the variable neighborhood search algorithm, the initial solution should be local optimal [54]. Accordingly, at each iteration, the latest result obtained from the tabu search algorithm up to this iteration is allocated to this algorithm.

Neighborhood creation phase (shaking phase). The purpose of this phase is to make a sudden change in solution [54]. Each of the methods for the creation of the neighborhood is called a k-neighborhood method. In the proposed algorithm, eight neighborhoods (operations) were considered based on the studies targeting the impact on the performance of the algorithm. These eight neighborhoods are listed in Table 3.

Local search phase. In the case of the improvement of the initial results from the tabu search algorithm at the shaking phase, they will be given to the tabu search algorithm to apply the local search using alternative techniques. At this phase, the algorithm at each iteration creates some neighborhoods with an interchange heuristic introduced by Narula and Ogbu [21], which is actually the developed form of the heuristic method presented by Teitz and Bart [55]. Through this technique at each iteration, a t-type facility ($t = 1, 2, 3, \cdots$) in position I may be replaced by zero in position j (zero value means that there is no facility in the corresponding point) or t-type facility in position I is likely to be replaced by a facility with other types in position j. Followed by the production of each neighborhood, if the obtained result outweighs the current solution, the resulting solution is replaced by the current solution and the neighborhood that

Table 3. The neighborhoods in tabu search algorithm.

	How to create a neighborhood
1	Delete a facility of high level and add a facility of moderate level
2	Delete a high-level facility and add two low-level facility in the case of having enough budget
3	Delete a facility of high level and add a low-level facility and a high-level facility in the case of having enough budget
4	Delete a high-level facility and add a low-level facility
5	Delete a moderate-level facility and add a low-level facility
6	Delete two high-level facilities and add three low-level facilities in the case of having enough budget
7	Delete two high-level facilities and add three moderate-level facilities in the case of having enough budget
8	Delete a low-level facility and add a moderate-level facility in the case of having enough budget

causes improvement is transferred to the tabu list, hence updated list. Then, the counter of the number of times of not improved solution becomes zero in the neighborhood (del); otherwise, one unit is added to the counter.

If the counter reaches a particular value given as the algorithm input (*StCon*), the production of this neighborhood stops, and the obtained result is considered as the local optimum solution. Finally, the obtained result returns to the VNS algorithm, and this round trip continues until the algorithm stops. The stopping condition in the proposed tabu search algorithm is in two forms, and each condition that occurred first will stop the algorithm. In one condition, there is no improvement in the solution with a particular number of consequent iterations ($NoImp > StConu$), while in another one, the maximum number of iterations occurs ($it > MaxIt$).

5.4. Setting parameters using Taguchi method

The parameters of the meta-heuristic algorithms affect their performance. A suitable combination of these factors can greatly improve the performance of the algorithms. There are some ways to design tests. One of the first ways presented in this area is the factorial method in which the number of tests is obtained from $N = L^m$.

A major drawback of this approach, however, is that in the case of the multiplicity of variables, too many tests are needed, hence not desirable in terms of time and cost. Taguchi method is a widely used method for setting parameters [56]. In this regard, the current paper employed Taguchi method to set the parameters. In the tabu search algorithm, four variables as the controllable factors are determined that include the total number of iterations of the algorithm in which no improvement occurred ($StCon_{it}$), number of not improved solutions in local searches (*StCon*), total number of iterations of the algorithm (*MaxIt*), and tabu list length (D).

Three levels and L_9 Taguchi design were used for Taguchi testing. Based on the results of the experiments obtained from Taguchi test, the values for each of the desired above-mentioned parameters were equal to 100, 10, 30 and finally, the tabu list length was equal to the square root of the number of neighborhoods divided by two. In the harmony search algorithm, five variables were determined as the controllable factors, including HMS, rate of choice from harmony memory in the first iteration ($HMCR_I$), rate of choice from Harmony Memory in the last iteration ($HMCR_F$), pace adjustment rate (PAR), and number of algorithm iterations (*MaxIt*). According to the results obtained from the Taguchi test, the considered values for the above parameters are 1000, 0.2, 0.5, 0.9, and 300, respectively.

5.5. Lower bound of Lagrangian relaxation

To compare the performance of algorithms in the case of a large-sized problem, Lagrangian relaxation method was used to produce a lower bound, more explanation of which is given in this section.

5.5.1. Lagrangian relaxation method

Lagrangian relaxation method is one of the most effective and efficient ways in a discrete optimization problem that can be used to produce lower bounds in minimization problems [57,58]. In a given problem, the choice of constraint or constraints is important, and those constraints should be relaxed that have much impact on the complexity of the problem. In a minimization problem, a more optimal value of Lagrangian dual function means stronger relaxation (release) [59]. Consider the following problem:

$$Z = \min \ cx, \tag{34}$$

$$Ax \geq b, \tag{35}$$

$$Dx \geq e, \tag{36}$$

$$x \geq 0. \tag{37}$$

Using the Lagrangian relaxation method, the problem is converted into the following form:

$$Z = \min\ cx + \lambda(b - Ax), \tag{38}$$

$$Dx \geq e, \tag{39}$$

$$x \geq 0. \tag{40}$$

Upon adding a constraint to the objective function, a negative value is added to the objective function ($\lambda \geq 0$) and consequently, the solution to the second question as a lower bound on the main problem can be raised. However, with the removal of Constraint (35), the problem solution does not get worse. As a result, the solution to the second question without Constraint (35) is a lower bound. In a Lagrangian maximization problem, as it finds a lower bound, we look for the highest objective function value (the greatest lower bound).

5.5.2. Applying Lagrangian relaxation method to the proposed model

In the Lagrangian problem, selection of the constraint added to the objective function is of high importance. In fact, a constraint should be selected to have both a great impact on the reduction of solution time (complexity) of the problem and elimination of the constraint. OF note, adding it to objective function will lead to the creation of good lower bounds (in minimization problem) or good upper bounds in the maximization problem. Followed by further investigations and selection of different constraints for the Lagrangian relaxation of Constraints (20), (21), and (23), Section 4 presents the model and the objective function of the Lagrangian relaxation method takes the following shape:

$$\begin{aligned} LR\text{: Minimize } Z + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=c}^{K}\sum_{c=1}^{C}\sum_{s=1}^{S}\lambda_{ijkcs}(x_{ijkcs} \\ -M \times y_{jk} \times dam_{js} + M \times temp_{js}) \\ + \sum_{h=1}^{n}\sum_{k=c}^{K}\sum_{c=1}^{C}\sum_{s=1}^{S}\beta_{hkcs}\left(\sum_{j=1}^{n} r_{jhkcs}\right. \\ \left. - M \times y_{hk} \times dam_{hs}\right). \end{aligned} \tag{41}$$

Other constraints of the Lagrangian optimization approach are the same as the robust model presented in Section 4. The Lagrangian optimization approach that determines the appropriate values for the Lagrangian coefficients is also important. A general method for determining the optimum values used for the Lagrangian coefficients is the sub-gradient optimization algorithm [21,60]. To find the optimal values for the Lagrangian coefficients, the sub-gradient optimization algorithm was employed in this study [61]. For the stopping condition in this algorithm, the maximum condition in this algorithm, the maximum number of iterations and step size were obtained as 100 and less than or equal to 0.01, respectively. The initial values of Lagrangian coefficients were considered as zero. The step size was initially measured as 2, and if no improvement is achieved at the lower bound after 10 iterations, this value is divided by two. The pseudo code for the sub-gradient algorithm is expressed as follows [60]:

1. **Initialization**
   - 1.1. Find a feasible solution Zh using the tabu search algorithm described in Subsection 5.3;
   - 1.2. Initialize Lagrangian multipliers;
   - 1.3. Initialize Lb ($:= -\infty$; lower bound for LR);
   - 1.4. Initialize parameters and counters.
2. **Do While** stop conditions not met:
   - 2.1. Solve Lagrangian problem LR obtaining ZLR;
   - 2.2. Update best lower bound so far, $Lb := \max[Lb; ZLR]$;
   - 2.3. Compute sub-gradient vectors;
   - 2.4. Update step size;
   - 2.5. Update Lagrangian multipliers;
   - 2.6. Check whether stop condition is met;
3. **END.**

To calculate the step size at each iteration of the sub-gradient algorithm, a possible solution is required. The possible required solution is obtained using a hybrid tabu search algorithm. This algorithm produces a feasible solution with a significant performance which remains unchanged in the entire algorithm.

6. Computational results

6.1. Analyzing difficulty of problem and production of test problems

To produce the sample problems, the difficulty of the problem and its dependence on the parameters were examined. To this end, 12 problems with eight candidate points, 12 problems with ten candidate points, and 12 problems with 12 candidate points with different budgets were randomly produced in a two-dimensional space $[10 * 10]$. Then, the patients' demands and shortage penalty were set, demanding that the candidate facility be established if sufficient budget in all parts is ensured. To compare the solution time, the demand value in each of the 12 problems with similar candidate points was considered fixed. The maximum total time for the problems was considered 9000 seconds, and the problems were solved using CPLEX. Figure 2 shows the average time needed to solve the problems with different levels of budget based on the number of established candidates, compared to all candidate points.

According to Figure 2, when the number of established candidates is small compared to the total candidate points (in low budgets), the complexity and solution time of the problems would decrease. On the contraty, upon increasing the budget and number of facilities to be established, the complexity and solution time of the problems would increase. This trend continues until significant budgets are appropriated and as in most of the candidate points, facilities are established. Then, the complexity and solution time of the problems decrease. It seems that, according to Figure 2, the complexity is mostly dependent on the budget and demand, especially when the need for establishment of facilities varies between 55% and 75% of the candidate points. To produce sample problems, this result was taken into consideration and the random samples were produced in both difficult and easy samples. To produce sample problems in small dimensions, the coordinates of the points were randomly determined in a two-dimensional space $[10 * 10]$ and in the problems with large dimensions, the coordinates of each point were randomly determined in a two-dimensional space $[50 * 50]$. In addition, three service centers and three levels were considered in the sample problems. To create scenarios in the sample problems, four different events were taken into account. In the first case, it is assumed that no disaster occurs and normalcy prevails. In all the next three other cases, a point was randomly generated in the solution space and was considered as the center of the disaster. In this case, a destruction radius was taken into consideration. Table 4 shows the destruction radius to handle large and small problems for different aspects. In the sample problems, it is assumed that all facilities within the destruction radius will fail with a probability of 80%.

To randomly calculate the demand rate for each of the initial points, the population residing at each of the demand points were generated in the range of 30–200. The demand rate at each of the points was considered proportional to the resident population (equal to 30% of the population). To calculate the demand rate in the disaster, Index ifk_k was defined which is the incidence rate in the kth disaster-inflicted region. The index value for the points that are not distant from the incidence center is equal to 80%, and that for the points that are far from the disaster center is twice the radius. The index value decreases linearly by about 80% to 10%. In addition, the value of Index ifk_k was considered as 10% in normalcy (when no disaster occurred).

Table 4. Failure radius intended to problems in different dimensions.

The radius of failure in the small-scale problems	The radius of failure in the macro-scale issues
2	4
4	6
6	10

In this research, the rate of demand from services was obtained at different levels based on the parameters estimated by Oppong based on the data collected from medical centers located in Suhum, Ghana. This demanded service is considered as follows [62]:

$$u_c = (u_1, u_2, u_3) = (0.609, 0.203, 0.188). \tag{42}$$

It should be noted that to create the sample problems, 20% of the injured are assumed to be directly transferred to the Level-1 service, 70% of them are referred to the Level-2 service and 10% of them are referred to Level-3 service. The transfer rate of the injured from the Level-2 service to the Level-3 service is considered as 25%. In addition, every temporary facility established in critical situations is assumed to receive equipment from permanent centers and medical staff (Level 1 service) up to 20% of its maximum capacity. Finally, regarding the issue of disaster, it is assumed that any temporary established facility can receive up to 20% of the capacity of the permanent centers including equipment and medical personnel (service level).

6.2. Computational results

This subsection analyzes the results of the proposed algorithms for both small- and large-sized problems. The results of the exact method, lower bound, and two meta-heuristics in small-sized problems are shown in Tables 5 and 6. In the small problems, the gap is obtained by the deviation percentage of the objective function resulting from the proposed algorithms and objective function obtained from the exact method. This value is calculated via Eq. (43):

$$GAP = \frac{Z - Z_{best}}{Z} \times 100. \tag{43}$$

In this equation, Z_{best} is the solution obtained from CPLEX, and Z the solution obtained from the algorithm. In the following tables, *CP*, *LR*, *HS*, and *C-TS* stand for the CPLEX, Lagrangian lower bound, harmony search algorithm, and hybrid tabu search algorithm methods, respectively.

As shown in Tables 5 and 6, the average error of the solution methods is not much different. The average errors of the hybrid tabu search algorithm in both small simple and difficult problems are equal to 0.06% and 0.27%. The average gaps of the harmony search algorithm in these two problems were measured as 4.79% and 4.26%, respectively. In both problems, Lagrangian method offered lower bounds with the

Table 5. Results of small size simple problems solving (CP: CPLEX, LR: Lagrangian lower bound, HS: Harmony Search algorithm, C-TS: Hybrid Tabu Search algorithm).

		CP		LR		HS			C-TS		
Problem	**Cu**	***Z***	***T***	***Z***	***GAP* (%)**	***Z***	***T***	***GAP* (%)**	***Z***	***T***	***GAP* (%)**
P1e	7	992.31	1.3	586.71	28.78	1003.82	201.82	1.15	992.31	68.05	0
P2e	7	743.28	8.6	554.63	17.31	783.86	202.33	5.17	743.28	60.97	0
P3e	8	1093.01	8.22	688.04	26.98	1140.54	203.55	4.16	1093.01	89.51	0
P4e	8	926.32	12.07	672.56	16.59	970.53	228.45	4.55	926.32	88.62	0
P5e	9	1256.83	20.52	785.06	25.60	1318.32	234.10	4.66	1257.08	134.92	0.02
P6e	9	1022.77	24.38	757.29	19.11	1064.93	258.97	3.95	1022.77	117.92	0
P7e	10	1595.56	50.96	978.30	19.88	1682.58	258.02	5.17	1595.56	188.95	0
P8e	10	1275.32	31.84	928.82	17.76	1334.50	296.66	4.43	1275.32	225.67	0
P9e	11	1873.78	58.64	1134.56	17.56	1949.70	295.94	3.89	1873.78	195.62	0
P10e	11	1458.15	255.11	1064.98	18.04	1527.61	339.43	4.54	1461.83	145.99	0.25
P11e	12	2134.72	533.96	1280.02	17.08	2226.37	383.08	4.11	2134.72	189.76	0
P12e	12	2005.25	2483.81	1244.66	16.98	2116.76	385.52	5.26	2005.25	168.06	0
P13e	12	1655.25	3068.68	1195.11	18.73	1731.33	384.21	4.39	1664.90	220.11	0.57
Average		**1387.12**	**504.46**	**1115.44**	**20.03**	**1450.06**	**282.46**	**4.26**	**1388.16**	**145.70**	**0.06**

Table 6. Results of small size difficult problems solving (CP: CPLEX, LR: Lagrangian lower bound, HS: Harmony Search algorithm, C-TS: Hybrid Tabu Search algorithm).

		CP		LR		HS			C-TS		
Problem	**Cu**	***Z***	***T***	***Z***	***GAP* (%)**	***Z***	***T***	***GAP* (%)**	***Z***	***T***	***GAP* (%)**
P1d	7	806.96	8.16	634.63	21.35	837.32	416.43	3.62	806.96	126.61	0
P2d	7	777.64	8.64	554.63	28.67	826.38	420.90	5.89	790.94	122.29	1.68
P3d	7	757.88	9.07	554.63	26.81	789.89	474.14	4.05	757.88	137.28	0
P4d	8	985.98	17.43	772.56	21.64	1029.52	477.68	4.23	995.89	131.17	0.99
P5d	8	962.68	27.68	762.56	20.78	995.84	518.84	3.32	962.68	159.66	0
P6d	9	1149.84	28.48	867.29	24.57	1205.02	523.59	4.58	1149.84	162.37	0
P7d	9	1094.93	74.90	867.29	20.79	1118.15	565.61	2.07	1098.16	141.11	0.29
P8d	10	1408.03	162.36	1068.82	24.09	1502.83	571.05	6.30	1408.03	173.59	0
P9d	10	1372.26	714.83	1068.82	22.11	1480.59	642.66	7.31	1380.51	184.22	0.59
P10d	11	1592.65	1121.13	1214.98	23.73	1679.54	645.51	5.17	1592.65	202.74	0
P11d	12	1975.62	1610.63	1444.66	26.87	2051.32	715.06	3.69	1975.62	228.36	0
P12d	12	1908.49	2765.41	1411.26	26.05	2035.22	716.75	6.22	1908.49	253.35	0
P13d	12	1822.95	6180.75	1385.11	24.01	1936.83	726.69	5.87	1822.95	264.01	0
Average		**1278.14**	**979.19**	**969.78**	**23.96**	**1345.26**	**570.38**	**4.79**	**1280.81**	**175.90**	**0.27**

gaps of 23.96% and 20.3%. According to the results of the hybrid tabu search algorithm and harmony search algorithm as an upper bound as well as the result of the relaxation method as a lower bound, the optimum solution was obtained with the average range of 20%. The solution time for the difficult problems significantly increased, compared to the corresponding time durations in the simple problems. In simple problems, on average, the required times to solve these problems in CPLEX, harmony search algorithm, and hybrid tabu search algorithm were about 504 s, 282 s, and 145 s, respectively. However, in difficult problems, on average, the solution times were about 979 s, 570 s, and 176 s. Figure 3 lists the solution times of the algorithms for small simple problems.

6.3. Results of large-scale computation

Tables 7 and 8 presents the results obtained from the proposed algorithms in large-sized problems as well as the small simple and difficult problems.

Table 7. Results of large size simple problems solving with proposed algorithms (LR: Lagrangian lower bound, HS: Harmony Search algorithm, C-TS: Hybrid Tabu Search algorithm).

		LR	HS			C-TS		
Problem	**Cu**	***Z***	***Z***	***T***	***GAP* (%)**	***Z***	***T***	***GAP* (%)**
P1be	15	1603.46	2032.12	918.33	18.97	1991.03	702.29	19.47
P2be	20	1859.01	2451.01	1367.34	21.17	2416.19	1035.14	23.07
P3be	25	1776.51	2392.10	1511.55	26.57	2373.56	2254.35	25.14
P4be	30	2229.43	3147.89	1979.45	30.95	3093.25	3728.03	27.93
P5be	35	3032.74	4405.79	2266.44	30.10	4309.81	4946.91	29.65
P6be	40	5368.19	7534.28	2761.48	29.12	7519.56	6728.64	28.63
P7be	50	6459.15	10432.62	3205.86	31.59	9244.90	7360.12	30.17
P8be	60	6549.17	10963.7	4166.78	40.26	10019.43	8133.42	34.64
Average		**1919.81**	**5419.93**	**2272.15**	**28.59**	**5120.96**	**4361.11**	**27.34**

Table 8. Results of large size difficult problems solving with proposed algorithms (LR: Lagrangian lower bound, HS: Harmony Search algorithm, C-TS: Hybrid Tabu Search algorithm).

		LR	HS			C-TS		
Problem	**Cu**	***Z***	***Z***	***T***	***GAP* (%)**	***Z***	***T***	***GAP* (%)**
P1bd	15	1437.17	1750.35	954.27	17.89	1733.35	763.28	17.08
P2bd	20	1519.58	1998.99	1466.95	23.98	1974.08	1235.611	23.02
P3bd	25	1841.93	2396.04	1662.69	23.12	2381.65	2961.56	22.66
P4bd	30	2303.24	3148.40	2162.66	26.84	3087.02	4110.11	25.38
P5bd	35	2426.39	3403.72	2947.30	28.71	3343.71	5324.23	27.43
P6bd	45	4888.73	6905.43	3140.64	29.20	6884.12	7028.03	28.98
P7bd	50	4729.15	7079.68	3608.92	33.20	6863.74	7899.14	31.09
P8bd	60	4937.89	7081.47	4351.46	30.27	6981.09	10000	29.26
Average		**3010.51**	**4220.51**	**2536.86**	**26.65**	**4156.09**	**4915.24**	**25.61**

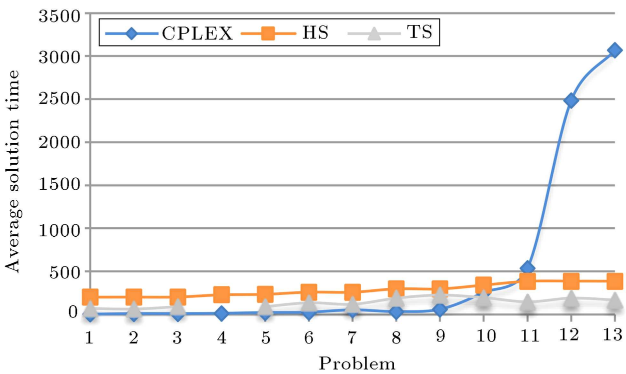

Figure 3. Comparing the solution time of CPLEX and HS and TS algorithms for solving small simple problems.

As indicated in Tables 7 and 8, the average gaps of the hybrid tabu search algorithm and harmony algorithm related to the lower bound were measured as about 26% and 27%. Given that the Lagrangian relaxation method for small-sized problems showed a 22% gap compared to the optimum solution and based on the assumption that the quality of Lagrangian relaxation method in large-sized problem did not decrease, as an optimistic assumption, we can conclude that the average error values of the hybrid tabu search algorithm and harmony algorithm were at most about 4% and 5%, respectively, in the large-sized problems. According to Tables 6 and 7, the average solution time in the simple problems was more efficient than that in the difficult problems for both meta-heuristic algorithms. In both simple and difficult problems, the solution times in the hybrid tabu search algorithm were, on average, about 4361 s and 4915 s, respectively. In addition, the same values in harmony search algorithm were 2536.86 s and 2272.15 s, respectively. It seems that a shorter solution time in the harmony search algorithm than the hybrid tabu search algorithm on a large scale results from the rapid convergence of this algorithm.

6.4. Impact of robust solution on results

To evaluate the performance of the robust model, an example was analyzed concerning different scenarios.

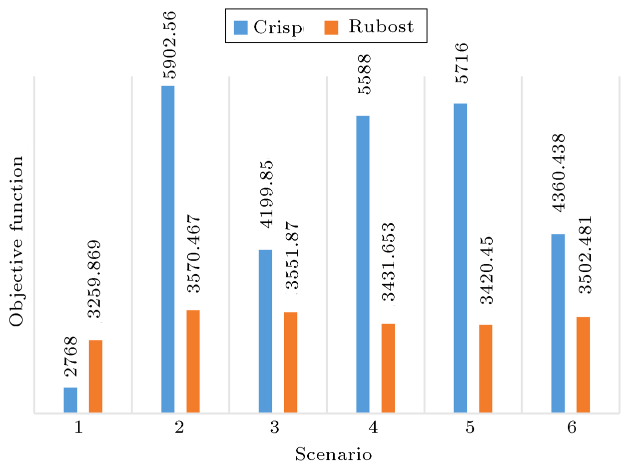

Figure 4. The objective function value obtained from robust model and crisp model for all the scenarios.

This example contained six scenarios. Initially, a crisp model under normal conditions (the first scenario) and a robust model were implemented. Then, the decisions of the first stage resulting from the two models were considered that included the type and location for the establishment of permanent health facilities to solve a crisp model with data from each of the six scenarios. Figure 4 shows the obtained results.

As shown in Figure 4, the objective function value obtained from the crisp model under normal conditions in different scenarios underwent many changes. It can be concluded that the crisp model outperformed the robust model only in normal scenarios, while the opposite result was obtained in case of disaster (Scenarios 2–6). In this example, the average values of the objective function in different scenarios for the robust and crisp models were 4755.808 and 3456.132, respectively. The standard deviations of the objective function in the robust and crisp models were 103.905 and 1106.52, respectively.

7. Conclusion

In this paper, a mathematical robust model was proposed for the construction of permanent hierarchical healthcare centers and temporary relief centers in both normal and critical situations. To solve the problem, two meta-heuristic methods, harmony search, and hybrid tabu search and variable search algorithm were employed. In addition, a lower bound-based Lagrangian relaxation method was presented to obtain a lower bound for the proposed problem. To check the quality of the proposed algorithms, the impact of parameters on the difficulty degree of the problem was evaluated. Based on the amount of available budget and ratio of facilities to be established in all the candidate points, the sample problems were divided into two categories of difficult and simple. Then, some problems were generated in each category in large and small dimensions. In the category of small and simple problems, the quality of solutions from the meta-heuristic algorithms was appropriate. Here, the average errors, compared to those of the exact solution of the hybrid tabu search and harmony algorithms, were measured as 4.48% and 0.18%, respectively. The distance of the lower bound obtained from the Lagrangian relaxation algorithm was about 22%, compared to the optimum solution. In large-scale problems, on the contrary, the results of the algorithms were compared to the lower bound obtained from the Lagrangian method. The mean deviations of the solution of the hybrid tabu search algorithm and harmony algorithm compared to the proposed lower bound by the Lagrangian relaxation method were, on average, equal to 27% and 26%, respectively. According to the 22% error associated with the lower bound resulting from Lagrangian relaxation algorithm in small sizes, compared to the optimum solution, and assuming that the quality of Lagrangian relaxation algorithm in great problems is not the worst, it can be concluded that the average error of hybrid tabu search and harmony algorithm on large scales is at most about 5% and 6%, respectively. Finally, reduction in the costs of using a robust model in disaster in a numerical example was investigated. The suggestion made for future studies is to consider such criteria as patient's waiting time for service and high equality in offering healthcare services.

References

1. Nikbakhsh, E. and Zanjirani Farahani, R. "Humanitarian logistics planning in disaster relief operations", *Logistics Operations and Management: Concepts and Models*, pp. 22291–332 (2011).

2. Jang, H.-C., Lien, Y.-N., and Tsai, T.-C. "Rescue information system for earthquake disasters based on MANET emergency communication platform", *International Conference on Wireless Communications and Mobile Computing: Connecting the World Wirelessly*, pp. 623–627, Germany (2009).

3. Akkihal, Anup Roop, T.-C. "Inventory pre-positioning for humanitarian operations", PhD Diss., Massachusetts Institute of Technology (2006).

4. Eaton, D.J., Daskin, M.S., Simmons, D.G., et al. "Determining emergency medical service vehicle deployment in Austin, Texas", *Interfaces*, **15**, pp. 96–108 (1985).

5. Jacobs, D.A., Silan, M.N., and Clemson, B.A. "An analysis of alternative locations and service areas of American Red Cross blood facilities", *Interfaces*, **26**, pp. 40–50 (1996).

6. McAleer, W.E. and Naqvi, I.A. "The relocation of ambulance stations: A successful case study", *Euro-*

pean Journal of Operational Research, **75**, pp. 582–588 (1994).

7. Osinuga, I.A., Oyebade, V.O., and Otun, W.O. "Integrated technique for the optimization of healthcare facility problem", *International Journal of Applied Operational Research*, **10**(1), pp. 11–20 (2020).

8. Cardoso, T., Oliveira, M.D., Barbosa-Póvoa, A., et al. "Moving towards an equitable long-term care network: A multi-objective and multi-period planning approach", *Omega*, **58**, pp. 69–85 (2016).

9. Dogan, K., Karatas, M., and Yakici, E. "A model for locating preventive health care facilities", *Central European Journal of Operations Research*, **28**, pp. 1091–1121 (2020).

10. Daskin, M.S., Hesse, S.M., and Revelle, C.S. "α-reliable p-minimax regret: a new model for strategic facility location modeling", *Location Science*, **5**, pp. 227–246 (1997).

11. Carson, Y.M. and Batta, R. "Locating an ambulance on the Amherst campus of the State University of New York at Buffalo", *Interfaces*, **20**, pp. 43–49 (1990).

12. Revelle, C., Schweitzer, J., and Snyder, S. "The maximal conditional covering problem", *Location Science*, **5**, pp. 77–91 (1997).

13. Snyder, L.V. "Supply chain robustness and reliability: Models and algorithms", Doctoral Dissertation, Northwestern University (2003).

14. Berman, O., Krass, D., and Menezes, M.B.C. "Facility reliability issues in network p-median problems: strategic centralization and co-location effects", *Operations Research*, **55**, pp. 332–350 (2007).

15. Landa-Torres, I., Manjarres, D., Salcedo-Sanz, S., et al. "A multi-objective grouping Harmony Search algorithm for the optimal distribution of 24-hour medical emergency units", *Expert Systems with Applications*, **40**, pp. 2343–2349 (2013).

16. Hoseinpour, P. and Ahmadi-Javid, A. "A profit-maximization location-capacity model for designing a service system with risk of service interruptions", *Transportation Research Part E: Logistics and Transportation Review*, **96**, pp. 113–134 (2016).

17. Shishebori, D. and Yousefi Babadi, A. "Robust and reliable medical services network design under uncertain environment and system disruptions", *Transportation Review*, **77**, pp. 268–288 (2015).

18. Şahin, G. and Süral, H. "A review of hierarchical facility location models", *Computers & Operations Research*, **34**, pp. 2310–2331 (2007).

19. Narula, S.C. "Hierarchical location-allocation problems: a classification scheme", *European Journal of Operational Research*, **15**, pp. 93–99 (1984).

20. Calvo, A.B. and Marks, D.H. "Location of health care facilities: An analytical approach", *Socio-Economic Planning Sciences*, **7**, pp. 407–422 (1973).

21. Narula, S.C. and Ogbu, U.I. "An hierarchal location-allocation problem.", *OMEGA, The International Journal of Management Science*, **7**, pp. 137–143 (1979).

22. Parr, J.B. "Health care facility planning: Some developmental considerations", *Socio-Economic Planning Sciences*, **14**, pp. 121–127 (1980).

23. Tien, J.M., El-Tell, K., and Simons, G.R. "Improved formulations to the hierarchical health facility location-allocation problem", *Systems, Man and Cybernetics, IEEE Transactions on*, pp. 1128–1132 (1983).

24. Gerrard, R.A. and Church, R.L. "A generalized approach to modeling the hierarchical maximal covering location problem with referral", *Papers in Regional Science*, **73**, pp. 425–453 (1994).

25. Boffey, B., Yates, D., and Galvao, R.D. "An algorithm to locate perinatal facilities in the municipality of Rio de Janeiro", *Journal of the Operational Research Society*, **54**, pp. 21–31 (2003).

26. Galvão, R.D., Acosta Espejo, L.G., Boffey, B., et al. "Load balancing and capacity constraints in a hierarchical location model", *European Journal of Operational Research*, **172**, pp. 631–646 (2006).

27. Yasenovskiy, V. and Hodgson, J. "Hierarchical location-allocation with spatial choice interaction modeling", *Annals of the Association of American Geographers*, **97**, pp. 496–511 (2007).

28. Hodgson, M.J. and Jacobsen, S.K. "A hierarchical location-allocation model with travel based on expected referral distances", *Annals of Operations Research*, **167**, pp. 271–286 (2009).

29. Pouraliakbarimamaghani, M., Mohammadi, M., and Mirzazadeh, A. "A queuing location-allocation model for a capacitated health care system", *Scientia Iranica*, **24**(2), pp. 751–764 (2017).

30. Aslanzadeh, M., Rostami, E.A., and Kardar, L. "Logistics management and SCM in disasters", *Supply Chain and Logistics in National, International and Governmental Environment*, Springer, pp. 221–252 (2009).

31. Toregas, C., Swain, R., ReVelle, C., and Bergman, L. "The location of emergency service facilities", *Operations Research*, **19**, pp. 1363–1373 (1971).

32. Chang, M.-S., Tseng, Y.-L., and Chen, J.-W. "A scenario planning approach for the flood emergency logistics preparation problem under uncertainty", *Transportation Research Part E: Logistics and Transportation Review*, **43**, pp. 737–754 (2007).

33. Najafi, M., Eshghi, K., and Dullaert, W. "A multi-objective robust optimization model for logistics planning in the earthquake response phase", *Transportation Research Part E: Logistics and Transportation Review*, **49**, pp. 217–249 (2013).

34. Bozorgi-Amiri, A., Jabalameli, M.S., and Al-e-Hashem, S.M.J.M. "A multi-objective robust stochastic programming model for disaster relief logistics under uncertainty", *OR spectrum*, **35**, pp. 905–933 (2013).

35. Shen, Z.J.M., Zhan, R.L., and Zhang, J. "The reliable facility location problem: Formulations, heuristics, and approximate algorithms", *INFORMS Journal on Computing*, **23**, pp. 470–482 (2011).

36. Shavandi, H. and Mahlooji, H. "Fuzzy hierarchical queueing models for the location set covering problem in congested systems", *Scientia Iranica*, **15**(3), pp. 378–388 (2008).

37. Motallebi Nasrabadi, A., Najafi, M., and Zolfagharinia, H. "Considering short-term and long-term uncertainties in location and capacity planning of public healthcare facilities", *European Journal of Operational Research*, **281**, pp. 152–173 (2020).

38. Mestre, A.M., Oliveira, M.D., and Barbosa-Póvoa, A.P. "Location-allocation approaches for hospital network planning under uncertainty", *European Journal of Operational Research*, **240**(3), pp. 791–806 (2015).

39. Ghezavati, V., Soltanzadeh, F., and Hafezalkotob, A. "Optimization of reliability for a hierarchical facility location problem under disaster relief situations by a chance-constrained programming and robust optimization", *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, **229**(6), pp. 542–555 (2015).

40. Canós, M.J., Ivorra, C., and Liern, V. "An exact algorithm for the fuzzy p-median problem", *European Journal of Operational Research*, **116**, pp. 80–86 (1999).

41. Darzentas, J. "A discrete location model with fuzzy accessibility measures", *Fuzzy Sets and Systems*, **23**, pp. 149–154 (1987).

42. Rao, J.R. and Saraswati, K. "Facility location problem on a network under multiple criteria–fuzzy set theoretic approach", *International Journal of Systems Science*, **19**(12), pp. 2555–2559 (1988).

43. Zhou, J. and Liu, B. "Modeling capacitated location—allocation problem with fuzzy demands", *Computers & Industrial Engineering*, **53**, pp. 454–468 (2007).

44. Mulvey, J.M., Vanderbei, R.J., and Zenios, S.A. "Robust optimization of large-scale systems", *Operations Research*, **43**, pp. 264–281 (1995).

45. Yu, C.-S. and Li, H.-L. "A robust optimization model for stochastic logistic problems", *International Journal of Production Economics*, **64**, pp. 385–397 (2000).

46. Megiddo, N. and Supowit, K.J. "On the complexity of some common geometric location problems", *SIAM J. Comput.*, **13**, pp. 182–196 (1984).

47. Geem, Z.W., Kim, J.H., and Loganathan, G.V. "A new heuristic optimization algorithm: harmony search", *Simulation*, **76**, pp. 60–68 (2001).

48. Geem, Z.W. "School bus routing using harmony search", *Genetic and Evolutionary Computation Conference*, Washington DC, USA, pp. 15–37 (2005).

49. Lee, K.S., Geem, Z.W., Lee, S.H., et al. "The harmony search heuristic algorithm for discrete structural optimization", *Engineering Optimization*, **37**(7), pp. 663–684 (2005).

50. Mahdavi, M., Fesanghary, M., and Damangir, E. "An improved harmony search algorithm for solving optimization problems", *Applied Mathematics and Computation*, **188**, pp. 1567–1579 (2007).

51. Degertekin, S.O. "Optimum design of steel frames using harmony search algorithm", *Structural and Multidisciplinary Optimization*, **36**, pp. 393–401 (2008).

52. Glover, F. "Future paths for integer programming and links to artificial intelligence", *Computers & Operations Research*, **13**, pp. 533–549 (1986).

53. Mladenović, N. and Hansen, P. "Variable neighborhood search", *Computers & Operations Research*, **24**, pp. 1097–1100 (1997).

54. Fleszar, K., Osman, I.H., and Hindi, K.S. "A variable neighbourhood search algorithm for the open vehicle routing problem", *European Journal of Operational Research*, **195**, pp. 803–809 (2009).

55. Teitz, M.B. and Bart, P. "Heuristic methods for estimating the generalized vertex median of a weighted graph", *Operations Research*, **16**, pp. 955–961 (1968).

56. Taguchi, G., Chowdhury, S., and Wu, Y., *Taguchi's Quality Engineering Handbook*, Wiley (2005).

57. Fisher, M.L. "An applications oriented guide to Lagrangian relaxation", *Interfaces*, **15**, pp. 10–21 (1985).

58. Geoffrion, A.M., *Lagrangean Relaxation for Integer Programming*, Springer (1974).

59. Lemaréchal, C. and Renaud, A. "A geometric study of duality gaps, with applications", *Mathematical Programming*, **90**, pp. 399–427 (2001).

60. Ignacio, A.A.V. and Galvão, R.D. "Lower and upper bounds for a two-level hierarchical location problem in computer networks", *Computers & Operations Research*, **35**, pp. 1982–1998 (2008).

61. Fisher, M.L. "The Lagrangian relaxation method for solving integer programming problems", *Management Science*, **50**, pp. 1861–1871 (2004).

62. Oppong, J.R. "Location-allocation models for primary health care in Suhum District, Ghana", PhD Diss, University of Alberta (1992).

Biographies

Mahdi Alinaghian is an Associate Professor at Isfahan University of Technology. He received PhD in Industrial Engineering, Iran University of Technology. His research interests include discrete event simulation, uncertain programming, supply chain management, and meta-heuristic algorithms. His published research articles appear in Expert Syst Appl, Nspec, J Intell Manuf, Ijamt, etc.

Seyed Reza Hejazi is a Full Professor at Isfahan University of Technology. He received PhD in Industrial

Engineering, Tarbiat Modares University. His research interests include fuzzy systems, operations research, multiple criteria decision making, supply chain management, etc. His published research articles appear in International Journal of Production Research, Fuzzy sets and systems, Computers & Operations Research, etc.

Nooshin Bajoul earned a MSc degree in Industrial Engineering from Isfahan University of Technology. Her research interests include discrete event simulation, uncertain programming, supply chain management, and meta heuristic algorithms.

Kosar Sadeghi Velni earned a BSc in Industrial Engineering from Tehran University and a Master's degree from Isfahan University of Technology. She received PhD in Industrial Engineering from Tehran University. Her research interests include supply chain management, meta heuristic algorithms, etc.