Bundle Pricing, Reservation and Refund Policies in a Two-Level Supply Chain

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Abstract

Refunding and bundling reservation are known as two popular methods to increase profit where in recent years have gained attention of researchers. One main application of refunding policy emerges for online product sale methods, where consumers can be refunded by returning goods which are not favorite according to their interest. Examining three scenarios including refunding, bundle reservation and refunding along with bundle reservation policies, we will investigate a model for each corresponding scenario. We try to compare two refund and bundle reservation pricing policies in a two-level supply chain including one manufacturer and one wholesaler, and we provide a combined model including two products. The demand is constant and also the population-related information about the division of the population into two types of consumers, strategic consumers (consumers who can predict the second stage discount) and myopic consumers (consumers who can not predict the second stage discount) are available. In addition, the percentage of consumers who refund the product due to regret, the inability to install the product or other reasons, is constant and is independent of the amount of refund. We show that the combined model is optimal and has a higher profit margin than any other policy alone.

Keywords: *Pricing; product refund policy; reserved product; bundling; return policy; inventory, supply chain*

1. Introduction and Literature Review

Granting the facility to the buyer based on the possibility of refund of the product and, consequently, the increase in demand, is one of the common methods to increase the profit of the products for manufacturers and suppliers of the goods. In this regard, pricing policies are based on predicting the increase in demand by studying the parameters of each population. In this field, the effect of the amount refund on the amount of sales is significant. The refund policy is applied in many cases. Online purchasing approach has specially developed during recent years. In online purchasing from a manufacturing site, the consumer has no opportunity to experience the product before making a decision to purchase. Therefore, in the refund model, this opportunity is given to the consumer in order to be able to refund the product if this product is not suitable. This assurance will increase demand. Regardless of the value of refund, some consumers tend to return products just because of some reasons like regret, incapability to install or utilize product or even as a habitual behavior.

Bundling pricing has been also known as one of the ways to increase profits. In this regard, the bundling product type and the number of each bundling, and studying the past information of the consumers and the population interest in this subject has been examined for this subject by the researchers. Bundling pricing includes two policies of pure bundling policy (merely offering in bundle and avoiding single-sail) and mixed bundling policy (using both bundle and single selling methods). Complementary pricing policy is established in case of proposing a bundle of products which are considered to be supplement to each other (complementary goods are those items consumed together and each items acts as a complement to other item to meet a specific need, for example candy and tea, brush and toothpaste).

One of the other ways to increase profit is pricing based on the product reserved for the consumer. This method is more used for multi-product manufacturers or suppliers. In this method, after purchasing the first product at full price, the product or other products are reserved for the consumer in the second stage by providing a deceptive discount, which, if desired, will be offered at a discount to the consumer in the second stage. This is to encourage consumers to purchase more. In this section of paper, with respect to two subjects of refunding and bundling, results of recent studies are mentioned respectively.

1.1. Pricing with the refund policy

Assarzadegan and Rasti-Barzoki (2019) [1] introduced CLSC (Closed Loop Supply Chain) including one producer and two sellers in which sold products can be returned in two groups of defective and non-defective items. In the first scenario the producer pays to the retailer 1, the price for defective product, and in the second one the producer pays the first retailer's price in order to motivate him to offer Money Back Guarantee for defective products. Toomas Hinnosaar and Keiichi Kawai (2018) [2] developed the sales mechanism according to the vendor's uncertainty about the feedback structure of the buyer. They showed that by offering a generous return policy, the vendor can significantly reduce this kind of uncertainty. Jong Yeob Kim (2015) [3] presented the pricing and optimal return rates for heterogeneous customers with the uncertainty of evaluation. He showed that when the return policy is active, the price of the product decreases. He examined the optimal price changes and an optimal return rate by a model that is dependent on the return policy of heterogeneous consumer. Heydaryan and Taleizadeh (2016a) [4] have examined the return pricing policy in a green supply chain. The supply chain produces two types of products, Green and non-green products. These products have similar performance, but they have a different effect on selling prices and environmental issues. The return policy assumed for each product specifies the customer's value. Different models of pricing strategy and return policy have been developed in both green and non-green modes. Taleizadeh and Heydarian (2017) [5] have developed an issue of pricing optimization with the return policy in a two-level supply chain consisting of a supplier and producer who produces both green and non-green products in both concentrated and decentralized modes. The features of both products are the same in use and performance, but they vary in price and environment aspects. The return policy has been considered for both products. The system's performance analysis has been conducted in both centralized and decentralized supply chains. Stackelberg game theory has been used in the centralized chain and Rubinstein bargaining theory has been used in a decentralized chain. The results show that centralized chains increase the profit and demand for both types of products. By increasing the amount of potential demand, the amount of return and the profit of green product are increased in centralized and non-centralized chains and the amount of non-green product return is decreased. Heydaryan and Taleizadeh (2016b) [6] developed the policies of pricing, return and supply chain coordination in a two-stage supply chain. Their model was based on a new return policy that depended on the amount of return and product sales price. They developed a combined optimization issue about pricing and return policies, which used the theory of co-operative and non-cooperative games. Giri et al. (2017) [7] developed a multi-producer supply chain model that producers sell a product through a public retailer channel. In their model, demand at the end of the retail sale depends on retail sale price and product quality. Each producer supplies its product distinctly from other producers. They analyzed the pricing and quality management strategies of producers and retailer in each scenario for centralized and decentralized systems. Noori-daryan and Taleizadeh (2015) [8] developed a low-volume model of production in a three-stage supply chain consisting of supplier, producer, and wholesaler under two scenarios. In the first scenario, they consider a return contract between the vendor and the supplier, as well as between the producer and the wholesaler, and the return policy between the producer and the wholesaler has not been considered in the second scenario. Li et al. (2012) [9] examined optimal policies and pricing in the management of the supply chain of fashion products (for example, a fashion clothing category) according to the product returns between supply chain partners (B2B). They examined the channel performance and optimal policies, two randomized models have been designed for a centralized channel and a decentralized channel for both new and out-of-season sales management. In a centralized channel, closed solutions were suggested for optimum ordering and pricing decision for new fashion products. Taleizadeh et al. (2017a) [10] examined analytically a pricing and solidarity issue in a two-level supply chain with a new upstream input with the return policy with the motive of the importance of the supply chain continuity strategy. They created various mathematical models for different unifying strategies and solved their optimization issues with a game theory approach. Batarfi et al. (2017) [11] considered a paper on a leading system and a reverse system consisting of a major manufacturer of equipment and a retailer. A return policy agreement was also considered where dissatisfied consumers may return items by buying a new item. Returned items are collected and only those that can be repaired are repaired through the contractor and then given to consumers at lower prices than new items. A return policy agreement that is also used for refurbished equipment. This paper has used a linear affiliate demand function where consumers are sensitive to the prices and return policy of the sold items (new and refurbished). Yan and Ke (2015) [12] in a study examined two dynamic pricing strategies, means, the matching of the final price and the delay in the matching of the final price. These strategies are used by vendors to consider consumer behavior in the market by considering several types of consumers. They analyzed the price of the season with low sales, purchasing balance, and the price of the regular sales season using equilibrium theory and induced backward method. Comparing these two strategies showed that both strategies would enable vendors to increase prices regularly during the sales season. Li et al. (2013) [13] developed a model that has had the source of the return of the sold product. This paper includes the following activities: The first distributor on his website describes product information such as appearance and application. Then, customers will decide on buying the products on the basis of product descriptions, feelings of need and price. A customer can not experience the product until he/she receives it. When the customer receives the product, he or she will make its final decision within a specified period to accept or return the product. Li et al. (2017) [14] examined the strategic effect of return policies in a two-channel supply chain, in which a manufacturer can supply the products directly to end-consumers as well as indirectly through independent retailers. The manufacturer decides whether to implement the return policy either directly or indirectly or on both channels. When return policy is provided, the cost of returned products must be covered by the relevant channel. They considered four possible strategies, including full refund only on the direct channel, full refund only on indirect channel, full refund on both channels, and non-refund on both channels. A joint optimization model of pricing strategies, was introduced by Taleizadeh et al. (2017c) [15]. Their model a suppy chain model considering effort decisions, quality levels, and return policies. They developed several structures for channel power including: retailer Stackelberg, Centralised, vertical Nash, third party Stackelberg, and manufacturer Stackelberg. Taleizadeh et al. (2018b) [16] developed a pricing strategies as well as the quality consideration and marketing cost of the producer, seller, and third party operating. Moshtagh and Taleizadeh (2016) [17] research, last few decades coincided with considerable attention to inventory management of manufactured, recovered and returned items in closed loop supply chain. Roy et al. (2016) [18] developed a supply chain including a manufacturer and a seller to find optimum order size, selling price, service level and promotional measures, under uncertain demand. Modak et al. (2016a) [19] stated the concept of recycling of used products to reduce pollution and increasing consumption of natural resources as a necessary means to maintain a sustainable life. Modak et al. (2017) [20] developed a two echelon closed-loop supply chain under recycling, quality consideration and pricing decisions. Modak et al. (2016b) [21] studied channel coordination and profit division of a three level chain producer–distributer– two retailers. One definite characteristic of problem was supplying lot-size of the product that contains a random portion of imperfect quality item by producer. Taleizadeh et al. (2018a) [22] stated in a paper that firms have recently moved to online purchasing or electronic shopping, but they pointed that there is no way for buyers to confirm the quality of target products, a matter that makes return policy as a significant signal for customers to trust the quality of goods since customers are able to return purchased goods and consequently be paid back in case of not being satisfied with quality. Moreover, return policy was considered as a function of refund and quality level. The Return quantity of the other product affects return quantity of product. Finally, authors by providing a numerical example attempted to show the profit optimization model. Xu et al. (2018) [23] believe that advantages of consumers return policy have vastly investigated in other researches of this area. Hence they, tried to investigate the potential disadvantages of return policy for retailer. Authors developed an analytic framework to examine economic impacts of consumers return among three groups including customers, retailers and supply chain. Modak et al. (2016c) [24] developed a coordination problem with pricing in a two-level supply chain invluding two retailers and a producer. Establishing manufacturer-Stackelberg game setting, corporate social responsibility was regarded for manufacturer and Cournot and Collusion games were to be played by retailers. This paper aimed to explore the impacts of social accountability on making best decisions as well as comparing optimal decisions by retailers. Roy et al. (2015) [25] developed a two-level supply chain including a producer and two retailers in which demand rate depends on selling price and customer arrivs randomaly. Considering competitors' strategies for retailers, authors analyzed a single-period newspaper to specify optimal order amount. Retailers' unsold products were repaid at lower prices compared to suggested prices from manufacturer. On the other hand, retailers encounter paucity since demand is inherently indefinite. Liu et al. (2020) [26] in their study focused on pricing of different items in chain including a manufacturer and a seller. A significant result of this study was that: "as manufacturers make a direct online selling channel to sell produced items, low-risk products must be considered as well as services which need the lowest level of experience.

1.2. Pricing with the bundling policy

yan and Bandyopadhyay (2011) [27] tried to provide a framework that can help companies to obtain optimal bundle product categories and pricing strategies that maximize their profits, this study examines a model for maximizing profits. The results indicated that there are optimal bundlings and price strategies; in particular, if a company uses a bundle strategy to sell its products, it should combine it with complementary products and offer a comparatively low price. The value of a bundle strategy with market size and price sensitivity is always increased. Managers can use the model framework and advice and relevant examples for planning their bundle strategies. Mesa Arango (2015) [28]'s bundling services and truck pricing are a key strategic decision for people working in this area. Benisch and Sandholm (2012) [29] we provide a framework to offer discounts for high-profit packed products sale based on previous consumer purchasing data. We create several search algorithms that identify the prices of maximum profits and product bundling discounts. We introduce a more efficient probable evaluation model than previous work considering complementary, replacement and covariance, and we present a combined search method to fit this model with previous purchase data. The new purchasing information that has been gathered is attached to the evaluation model and results in an online technique that continuously markets the prices and discounts of the products bundle. Musa (2017) [30] this paper aims at examining the effect of a bundling pricing strategy on consumer purchasing decision making in a home-based sharing online product in Makasar, Indonesia. This research has been conducted by distributing the questionnaire to 369 people and responding to them and random sampling method. We found using regression analysis that a bundling pricing strategy has a significant effect on purchasing decisions on a home-based sharing online product in Makasar. A bundling strategy with advertisements can help a company obtain a higher performance than a bundling strategy without advertising (Pan and Zhou, 2017) [31]. Beladev et al. (2015) [32] adviser systems will increase the sales of e-commerce by recommending products related to consumers. Advisers seek to run the company's internet marketing strategy to increase revenue. The production of product bundles is an example of a marketing strategy aims at meeting the needs and preferences of the consumer while at the same time increasing the range of consumer purchases and company income. Therefore, finding and recommending a personal and optimal bundle is very important. In this paper, a new model of packaging and bundling recommendations has been presented that integrates filtering techniques of personal demand functions and price modeling. This model provides the recommended list finding the pair of products that can be purchased by the user and the income resulted from the sale of the package and bundle. Chu et al. (2011) [33] Multi-product companies can set separate prices for all possible combinations of their products (MB Mix bundling). However, it is not applied for companies with more than a few products, because the price increases with the number of products exponentially. We have found that simple pricing strategies are often optimal. Specifically, we show that the BSP Bundle-Size Pricing, which depends on the size of the purchased bundles, is more beneficial than providing separate products, and tends to almost benefit from mix bundle. Jain and Oosterlee (2015) [34] this paper describes a practical simulation based algorithm, which we call it a Stochastic Grid Bundling Method (SGBM) for evaluating multidimensional Bermudan options (means, functional selection). This method of direct pricing choosing estimator produces the optimal policy of initial pricing as well as a lower bound value for price selection. The benefit of SGBM is that this method can be used for fast approximation (i.e. derivatives due to the base point price, such as delta, gamma, etc.) for the Bermudan lightweight options. The computational results for various multidimensional options of the Bermudan show the simplicity and efficiency of the proposed algorithm. Taleizadeh et al. (2017b) [35] the selling of related products is related to vendor's interactive sales, which is the main factor of revenue management and vendor's costs. Interactive selling is a phenomenon that when demand for products is interconnected, the demand for one of the dependent products automatically causes another demand. In these cases, different sales tactics, such as a Bundling, a bundle of tying products, a combined bundle, etc., are used to sell the items. Xiao and Shi (2016) [36], with their growing innovation and e-commerce development experience, manufacturers are more beginning to sell their products through dual channels: a retail channel and a direct channel. The channel's priority strategy affects supply uncertainty, price, and sales, and thus affects the profitability of the two channels. Considering these factors, the channel priority strategy has been examined in the presence of random operation in both centralized and decentralized environments in this paper. Yang et al. (2015) [37] examined the reservation pricing issue for a two-level fashion supply chain in which a downstream manufacturer with private information at his or her operating costs (low or high costs) reserves an essential component of the upstream supplier before doing the final order. They considered the time when demand forecast has been updated to some extent. They found that a new menu of reservation contracts, which includes a reserve cost with a reservation and final order, could encourage the manufacturer to accurately reflect the cost of his operations. Prasad et al. (2015) [38] examine that this research originates from the pricing of a product, is a two-way profit-making that in this method, the vendor offers two products at its initial price, and after purchasing the product by consumers, only consumers who have a bought one product and not both offer a discount for the second product. Taleizadeh et al. (2019) [39] have studied two types of supply chains: 1) centralized supply chain and 2) decentralized supply chain. In this study coordination of both mentioned supply chains was regarded as possible and considered a situation of supplier-retailer interaction. As a two-step decision making model, supplier initially decides to determine the required capacity for retailers according to prior knowledge and in the second step the supplier updates information about demand, retailing price, packing cost, and the quantity of ordered packages. In this paper, supply chain consists of a supplier and a retailer intending to trade two complementary products as a package of production. The package has a short term chapter along with random price depending on high level of uncertainty.

Being dependent on sail price and random noise in market, it was assumed that there was not a definite value for demand rate. In order to determine decisions about reserved production capacity, order quantity of bundled products and the bundle-selling price, profit maximization models have been expanded for two supply chains, the centralized and decentralized ones. The applicability of the developed models and solution method were illustrated with a numerical example. Finally, a brief literature is presented in Table 1.

Based on the research literature, none of the researchers have considered the combination of refund and reservation. Therefore, in this research, we develop the optimal pricing fewer than two policies of refund and bundle reservation in a two-level supply chain with two types of products, and the division of society into two types of strategic and myopic consumers with a given coefficient. The questions that may be posed in this research are as follows.

- What is the basis for the division of society into strategic and myopic consumers?

- What is an appropriate structured method that can be used to identify those types of consumers with good approximation?

- Does the developed model have the relative advantage and superior for online purchases?

The order of this paper is as follows. In Section 2, we define the problem and in Section 3, the analytical models have been presented in all three methods, and in Section 4, we will analyze the sensitivity and comparison of each method in the consumer society. At the end of Section 5, we have summarized and concluded.

2. Define the problem

In order to define the problem, for example, consider the two real-world sales of "E27 to E40 encoder conversion" and "60-watt incandescent light " at a website. Figure 1 shows schematic shape.

Insert Figure 1 about Here

Insert Table 1 about Here

Refund scenario: Selling any product online on the website at a price of 6.5 $, with the possibility of refund each product at most after one week and paying 5.5 $ to the consumer for each refundable product.

Reserve scenario: Sale of each product at a price of 6.5 $ and simultaneously at a price of 13 $ that can not be refunded, and only for consumers who buy a product at a price of 6.5 $ for the first stage, offer a second product at a price of 5 $ in the second stage without refund ability

Refund and Reservation scenario: The combination of the above conditions means the sale of each product at a price of 6.5 $ and simultaneously at a price of 13 $ with the possibility of refund each product after a maximum of one week and paying 5.5 $ to the consumer for each refundable product and only for consumers who bought a product in the first stage for 6.5 $ and did not refund it after a week later, offer to buy a second product at a price of 5 $ in the second stage without the possibility of refund.

Scenario 1: The refund scenario

The origin of this scenario is refund the sold product, in this scenario, sales managers must determine the product's refundable policy with the response of different consumers to the demand and the expectation of refund on the product. They must select the appropriate pricing method.

Scenario 2: The reservation scenario

This scenario has been caused by the pricing of the reserved product. The nature of this scenario is a kind of two-way profitability. In this way, the vendor proposes two products at its initial price, and after buying the first product by consumers, only consumers who buy one product, not both, have a reserved discount for the second product. This is what the vendor keeps discounting of the product in the reserved form. Therefore, the price can not be reduced for a part of the sales that was sold in the first stage of both products. In this way, if the consumer expects a reasonable expectation for the second step of discount and simply delays purchases until a discount is offered, the price will be reduced in the second phase. So the success of this scenario depends on the percentage of strategic consumers (consumers who can predict the second stage discount) and myopic consumers (consumers who can not predict a second stage discount). Therefore, collecting additional information about consumers can be the key to success of this method.

Scenario 3: The refund and reservation scenario

This scenario has been based on a combination of the two methods. In this method, in addition to the possibility of a product refund, for a consumer who has bought a product at a first stage at a full price and has not refunded it during a specified period, a second product with a discount and no refund is suggested. Figure 2 shows the combination of these two scenarios schematically.

Insert Figure 2 about Here

Now in this study, we seek to compare the above methods in different societies. In this method, consumer recognition in terms of the behavior to product refund and the possibility of predicting the second stage discount has particular importance. We seek to present a combined model for the third scenario and compare it with other scenarios.

2-1 Assumptions

The demand is certain and the supply chain is two-level and it is assumed that the manufacturer has two products available. The values of α, a, b, c, k, and w are normal values and range between 0 and 1. According the above parameters the R value is between 0 and 1 and always $R \leq D$. Effect of quality is not considered in the refund function. It is assumed that the price of both products is the same.

2.2 Parameters

D demand value

w the value of the sales price of product,

k the price of the refund from the sale

a the ratio of consumers who refund the product due to regret or the reason for the inability to install the product and other reasons. These people are fixed and not related to the amount of the refund.

b the sensitivity ratio, the amount of consumer refund to the amount of refund. It means that the more k is more R,

c the cost of product producing.

δ the price discount

δ_e the expected discount amount for the strategic consumers.

α the predictive factor of consumers, that is, if it is equal to 1, the whole society can not predict and if zero, the whole society can predict.

3. Modeling

We will continue modeling with each scenario. These models are such that the same parameters in all models have been represented by same symbol.

3.1. Refund scenario

Li et al. (2013) presented a model for a refundable product. They provided a model based on product quality and refund parameters. In this scenario, this model has been simplified.

The demand function is defined as follows:

$$D = f(w,k) = 1 - w + k \tag{1}$$

Which minus sign of w indicates a decrease in demand with increasing the price , and positive sign of k indicates an increase in demand by increasing the refund amount. In online purchasing, after receiving the product, the consumer decides to use or refund the product within the determined period. The refund function is as follows:

$$R = a + bk \tag{2}$$

Considering the above cases, the profit function will be as follows:

$$\pi = (w-c)D - kR = (w-c)(1-w+k) - k(a+bk) \tag{3}$$

By deriving the profit function compared to k and w, the following equations are obtained.

$$w = \frac{1+k+c}{2} \tag{4}$$

$$k = \frac{w - c - a}{2b} \tag{5}$$

The negative second derivation in both equations indicates that the obtained number is the maximum profit value (see Appendix B). According to the above assumptions, we must consider that there is always the restriction r and $c \le w \le 1$. Therefore, the following equations are obtained: (see Appendix A)

$$w^* = \frac{2bc + 2b - c - a}{4b - 1} \qquad b \neq 0.25 \tag{6}$$

$$k^* = \frac{1 - c - 2a}{4b - 1} \qquad b \neq 0.25 \tag{7}$$

$$\pi^* = \frac{4b^2c^2 + 4a^2b - 8b^2c - bc^2 + 4b^2 - a^2 + 4abc - 4ab + 2bc - ac + a - b}{(4b-1)^2} \qquad b \neq 0.25 \tag{8}$$

And if $b = 0.25$ Then $c = (1 - 2a)$.

3.2 Reserve scenario

This model includes both consumers, strategic and myopic consumers. The vendor will maximize his profit function by offering two products. The profit function is defined as follows:

$$\max\left[(w-c)+(w-\delta-c)\delta\right]2\alpha(1-w)+(1-\alpha)\left\{2(w-c)\left[w-w^2+\frac{(1-w)^2}{2}+\frac{\delta^2}{4}\right]+2(w-c-\delta)\left[\frac{(1+\delta-w)^2-0.5(2\delta-\delta_e)^2}{2}\right]\right\} \tag{9}$$

By deriving the above equation and putting $\delta = \delta_e$, we will have the following equations:

$$\delta = 0.5(w-c); \alpha = 1 \tag{10}$$

$$w = \frac{c+1}{2} + \frac{(1-c)^2}{18 - 2c + 2\sqrt{(9-c)^2 + 3(1-c)^2}} \tag{11}$$

Otherwise:

$$\delta = \frac{4w - 4 + \sqrt{16(1-w)^2 - 2(1-\alpha)^2(1-w)^2 + 4(w-c)(1-\alpha)(1-w)}}{1-\alpha} \tag{12}$$

3.3 Refund and reserve scenario.

We examine this scenario separately from the two types of consumers and then combine them.

3.3.1 Determine the demand for myopic consumers

Here, for myopic consumers, we present α coefficient, and the remaining $1-\alpha$ coefficient will be for strategic consumers. The demand in the first stage is equal to $2(1-w+k)$ in the situation where k is the amount of refund and $0 < k < 1$ at the price level w and in the second stage is equal to

$2(1-w+R)$ at the level Price $w-\delta$. We assume that the actual amount of the second stage discount is equal to δ. Then we will have:

$$\pi = \alpha\left\{\left[2(w-c)(1-w+k)+2(w-\delta-c)(1-w+k)\delta\right]-2k(a+kb)\right\} \quad (13)$$

3.3.2 Determine the demand for strategic consumers

Strategic consumers usually have a rational expectation for discount at the second-stage. In this case, we consider δ_e as the expected value of this type of consumers about the discount amount. Figure 3 shows the structure of the demand function for the first stage of sales. Strategic consumers do not buy the first two products in the first stage, compared to myopic consumers. The first stage demand is equal to $\left[(w-k)(1-w+k)+\frac{(1-w+k)^2}{2}+\frac{\delta^2}{4}\right]$ at the price of $w-c$, since demand in a non-refundable state is less than this value. In Figure 3, the hatchback section shows the demand for a situation in which the refund of the products is not allowed, but if you accept the assumption of refund, the area of the black part will be added to the demand. The demand is obtained in the second stage according to Figure 4. This demand is equal to $\frac{(1-w+\delta+k)^2-0.5(2\delta-\delta_e)^2}{2}$ provided at price $w-c-\delta$.

Insert Figure 3 about Here **Insert Figure 4 about Here**

3.3.3 Combination of profit from both types of consumers (strategic and myopic)

The ptofit function for combination of both types of customers is shown in Equation (14) as below.

$$\pi = \max\alpha\left\{\left[2(w-c)(1-w+k)+2(w-\delta-c)(1-w+k)\delta\right]-2k(a+kb)\right\} \quad (14)$$

$$+(1-\alpha)\left\{2(w-c)\left[(w-k)(1-w+k)+\frac{(1-w+k)^2}{2}+\frac{\delta^2}{4}\right]+2(w-\delta-c)\left[\frac{(1-w+\delta+k)^2-0.5(2\delta-\delta_e)^2}{2}\right]-2k(a+kb)\right\}$$

Assuming that a, and b, are the percentages of consumers who are accustomed to refunding, and a group of consumers who are encouraged to refund the items according to the refund amount is constant in both stags.

4. A numerical example, sensitivity analysis and managerial insights

Here, numerical examples are examined with hypothetical data to the parameters of each scenario in ascending order and the best conditions for the profit function. We begin examining the scenarios in order

4.1. Refund scenario

The data of this scenario is that the parameters a, b and c have been given from 0.05 to 0.85, with a range of 0.05, respectively. Based on the results of this scenario, the values of k, w and π

have been calculated according to Table 2. The diagram of the profit trend compared to the cost is shown in Figure 5. This diagram indicates that with the increase in the c, the profit product decreases exponentially. It seems logical that the profit margin increases if the product cost reduced. Figure 6 also shows changes in profit by changing the ratio of the sensitivity of the refund amount to the refund amount, that is, parameter b.

Insert Table 2 about Here Insert Figure 5 about Here

The data of this diagram can be seen in Table 3, which has been obtained by increasing the 0.05 from 0.04 to 0.84 of the above parameters. As it is clear, the best way to increase profits is that b is of about 50%. It means, if the population is such that 50% of the refund is proportional to the refund amount, we will get the maximum profit.

Insert Table 3 about Here Insert Figure 6 about Here

4.2. Reserve scenario

The data from this scenario is that the parameters a, b and c have been given from 0.05 to 0.95, with a range of 0.07, respectively. Based on the results of this scenario, the values k, w and π have been calculated according to Table 4. Figure 7 shows the amount of profit based on the population's predictive coefficient, that is, the parameter α. The diagram shows that if the population as a whole be myopic consumer, the profits will increase, and if 90% of the population be strategic consumer, the profit will be maximized.

Insert Table 4 about Here Insert Figure 7 about Here

Figure 8 shows the amount of profit compared to the cost changes, which indicates that the profit will increase as the cost drops. This is the same as in the previous scenario. The data has been shown in Table 5.

Insert Table 5 about Here Insert Figure 8 about Here

Figure 9 shows the amount of profit compared to the price of the product, specifies the best selling price at 50%. The data in this diagram have been presented in Table 6.

Insert Table 6 about Here Insert Figure 9 about Here

4.3. Refund and reserve scenario

The data for this scenario is that the parameters a, b and c have been set to be 0.05 to 0.95, respectively, with a value of 0.1. Based on the results of this scenario, the values k, w and π have been calculated according to Table 7. Figure 10 shows the profit diagram in terms of refund value.

Insert Table 7 about Here   Insert Figure 10 about Here

This diagram indicates that increasing refund will cause to increase the profit. Of course, the rate of these changes, as seen, will be slower in refund endpoints.

Diagram in Figure 11 shows the profit in terms of the discount amount given for the second product. The effect of this factor alone is that by decreasing the amount of discount then the profit will be increased, there is an inflection point in this diagram, indicating that with the constant amount of other parameters, the rate of profit is changed in the areas of about 70 %. The data in this diagram has been presented in Table 8.

Insert Table 8 about Here   Insert Figure 11 about Here

4.4. Comparison of methods

In Tables 9 and 10, the values of the maximum profit of each scenario have been compared to the different values of p and c. These values indicate that the refund and reserve scenario significantly maximizes the profit function compared to the other two scenarios.

Insert Table 9 and Table 10 about Here

Figure 12 examines the maximum profit for the various values of parameter c. As it is clear, the scenario of reserve and refund is better than other scenarios. Figure 13 shows the three scenarios for the different values of the price of the product or the parameter w, which it is clear that the third scenario is better than the other.

Insert Figure 12 about Here

Insert Figure 13 about Here

4.5 Managerial Insights

The topic of current research can be applied for a vast area of industries like auto parts manufacturing, food industry and clothing. For all mentioned industries, there is a need for companies to know: how to identify α (the predictive factor of consumers)? This method benefits what products? How to determine time period for secondary discount? Does this method work better for a specific time of selling? What are success factors in this method? How to determine b (the sensitivity ratio, the amount of consumer refund to the amount of refund)? We have achieved the following managerial insights as most significant:

1- As shown in Figure 6, the parameter α, which is the ratio of the strategic consumer, represents the division of population. Profit margins will be more, whatever the myopic consumers to be more.

Products that the consumer is forced to buy due to short of time, despite the discount prediction, and is considered to be the myopic consumers, such as seasonal products or university term books that should be purchased at the beginning of the semester, will increase profits. Some policies, such as changing in time discounts, can also lead to a misleading of strategic consumers and increase in the percentage of myopic consumers and, consequently, increase profits. Another factor that increases the strategic consumers is consumer loyalty. That is, if due to the quality, good services and the refund, some consumers to be changed from a simple consumer to a loyal consumer, they will increase the number of consumers in the population, in which case it will be necessary to examine what is the situation of the consumer's turning profit to a loyal consumer compared to reduce the profit due to the being strategic consumers? To determine the predictive factor of consumers, we can consider customers' data base and corresponding records of their history.

2-The method presented in this paper applies to companies whose product variety is high by classifying the products and providing a bundle with a policy of refund. However, it works better for complementary products. Also, the use of this method is in companies whose maturity of product is not long, online sales is a tool to help increase demand, and because of the policy of refund, increases sales.

3- Time period of secondary discount must be selected appropriately to encourage consumers to buy proposed products. This time interval must not be as long as making consumers feel there is no need to consume anymore and must not be as short as making them feel cheated by seller's trick.

4- It's worth mentioning that this method is also very useful in some companies that use the discount method and reduced price to make up the sales in special times and special occasions such as Christmas holidays or Norouz holidays.

5- One main factor to guarantee the accomplishment of this method is employing appropriate advertising and informing approaches for target population of consumers.

6- According to Figure 5, the parameter b, which indicates the sensitivity parameters of the population to the amount of the refund, comes to the highest amount of profit, when about 50% of the rational consumers relate the amount of the refund to the refund price. This means that the application of this method will be in the populations that will be introduced by product advertisements and the product refund policy will be communicated to consumers. Meanwhile, proper service of the products refund chain improves this parameter and approaches the appropriate level. Performing consumer loyalty activities also brings this index closer to the desired level. In order to estimate this parameter, statistical methods can be used.

5. Conclusion and future development

In this research, a model based on the refundable product was first presented. After obtaining the optimal answer to this model, another model based on the sale of the two products in the second order reserve was examined considering the division of the consumer's population into two subsets of the strategic and myopic consumers and its modeling was performed and the optimal values were determined. This model was kind of bundling. Then, we showed that a combined model can be achieved in studying the parameters of the population and determining the degree of predictability of the population towards the discount in multi-product companies and also the study of the parameters of population regarding the refund function, by combining the two methods of pricing based on the reserved product and the refund of the product. We showed that the combined model that its optimal answer was obtained would improve the profit function for each method alone. The combined scenario with numerical examples of each of the scenarios was examined to prove better, the results of scenarios were compared, which indicated the supremacy of the combined scenario. As it was observed, combined model acts far better than any model individually. Moreover, the introduced model is a practical model which can be utilized in various industries. There is no evidence for joint consideration of both areas among researches in literature of study.

Regarding the limitation of having two kind of products, for future developments, it is suggested that the above model was developed in the mode of product more diversity. Also it is possible to considering the quality in the model. Increasing the supply chain level can also be posed as another option in this regard. Considering different prices for bundle products instead of identical prices as it was assumed in current research, is another idea to develop this study. Another limitation of study was about deterministic values of demand which can be suggested as an approach for future studies. This paper can also be continued in an uncertainty space or composition with fuzzy theory. In addition, considering the point of working on a bi-level supply chain, three level supply chains might be of interest. In order to expand proposed model, Markowitz model can be used where demand variation is targeted to minimum through at least average profit limitation.

References

1. Assarzadegan, P., & Rasti-Barzoki, M. “A game theoretic approach for pricing under a return policy and a money back guarantee in a closed loop supply chain”, *International Journal of Production Economics* (2019).
2. Toomas Hinnosaar, Keiichi Kawai. “Robust Pricing with Refunds”, JEL: D82, C79, D42, pp. 1-22 (2018),
3. Jong Yeob Kim. “Optimal Pricing with Return Policy under Valuation Uncertainty”, *Creative Commons*, pp. 1-46, (2015),

4. Heydaryan, H., Taleizadeh, A. A. "Pricing strategy and return policy of one-echelon green supply chain under both green and hybrid productions", *Journal of Industrial and Systems Engineering*, 9, 17-29 (2016a),
5. Taleizadeh, A. A., & Heydarian, H. "Pricing, refund, and coordination optimization in a two stages supply chain of green and non-green products under hybrid production mode", *Journal of Remanufacturing,* 7(1), pp. 49-76 (2017),
6. Heydaryan, H., Taleizadeh, A. A. "Pricing strategy, return policy and coordination in a two-stage supply chain", *International Journal of Systems Science: Operations & Logistics*, 4(4), pp. 384-391 (2016b),
7. Giri, B. C., Roy, B., Maiti, T. "Multi-manufacturer pricing and quality management strategies in the presence of brand differentiation and return policy", *Computers & Industrial Engineering*, 105, pp.146-157, (2017),
8. Noori-daryan, M., & Taleizadeh, A. A. "Optimizing Pricing and Ordering Strategies in a Three-Level Supply Chain under Return Policy", *11th International Industrial Engineering Conference*, (2015),
9. Li, Y., Wei, C., Cai, X. "Optimal pricing and order policies with B2B product returns for fashion products", *International Journal of Production Economics,* 135(2), pp. 637-646 (2012),
10. Taleizadeh, A. A., Soleymanfar, V. R., & Choi, T. M. Optimal pricing and alliance strategy in a retailer-led supply chain with the return policy: a game-theoretic analysis. Information Sciences, 420, 466-489, (2017a).
11. Batarfi, R., Jaber, M. Y., & Aljazzar, S. M. "A profit maximization for a reverse logistics dual-channel supply chain with a return policy". *Computers & Industrial Engineering*, 106, 58-82 (2017).
12. Yan, B., & Ke, C. (2015). "Two strategies for dynamic perishable product pricing to consider in strategic consumer behaviour". International Journal of Production Research, 56(5), 1757-1772.
13. Li, Yongjian., Xu, Lei., Li, Dahui. "Examining relationships between the return policy, product quality, and pricing strategy in online direct selling", *International Journal of Production Economics,* 144(2), pp. 451-460, (2013).
14. Li, G., Li, L., Sethi, S. P., & Guan, X. "Return strategy and pricing in a dual-channel supply chain". *International Journal of Production Economics*, 215, 153-164 (2018).
15. Taleizadeh, A. A., Moshtagh, M. S., & Moon, I. "Optimal decisions of price, quality, effort level and return policy in a three-level closed-loop supply chain based on different game theory approaches". *European Journal of Industrial Engineering,* 11(4), 486-525 (2017c).
16. Taleizadeh, A. A., Moshtagh, M. S., & Moon, I. Pricing, "Product quality, and collection optimization in a decentralized closed-loop supply chain with different channel structures: Game theoretical approach". *Journal of Cleaner Production*, 189, 406-431 (2018b).
17. Moshtagh, M. S., & Taleizadeh, A. A. "Stochastic integrated manufacturing and remanufacturing model with shortage, rework and quality based return rate in a closed loop supply chain". *Journal of Cleaner Production,* 141, 1548-1573 (2016).
18. Roy, A., Sana, S. S., & Chaudhuri, K. "Joint decision on EOQ and pricing strategy of a dual channel of mixed retail and e-tail comprising of single manufacturer and retailer under stochastic demand". *Computers & Industrial Engineering,* 102, 423-434 (2016).

19. Modak, N. M., Panda, S., & Sana, S. S. “Two-echelon supply chain coordination among manufacturer and duopolies retailers with recycling facility”. The *International Journal of Advanced Manufacturing Technology*, 87(5-8), 1531-1546 (2016a).
20. Modak, N. M., Modak, N., Panda, S., & Sana, S. S. “Analyzing structure of two-echelon closed-loop supply chain for pricing, quality and recycling management”. *Journal of Cleaner Production*, 171, 512-528 (2017).
21. Modak, N. M., Panda, S., & Sana, S. S. “Three-echelon supply chain coordination considering duopolistic retailers with perfect quality products”. *International Journal of Production Economics*, 182, 564-578 (2016b).
22. Taleizadeh, A.A., Beydokhti, S.R., Cárdenas-Barrón, L.E. “Joint determination of the optimal selling price, refund policy and quality level for complementary products in an online purchasing”, *European Journal of Industrial Engineering*, 12(3), pp. 332-363 (2018a).
23. Xu, L., Li, Y., Govindan, K., & Yue, X. “Return policy and supply chain coordination with network-externality effect”. *International Journal of Production Research*, 56(10), 3714-3732 (2018).
24. Modak, N. M., Panda, S., & Sana, S. S. “Pricing policy and coordination for a two-layer supply chain of duopolistic retailers and socially responsible manufacturer”. *International Journal of Logistics Research and Applications*, 19(6), 487-508 (2016c).
25. Roy, A., Sana, S. S., & Chaudhuri, K. “Optimal Pricing of competing retailers under uncertain demand-a two layer, supply chain model”. *Annals of Operations Research*, 260(1-2), 481-500 (2015).
26. Liu, Y., Li, J., Ren, W., & Forrest, J. Y. L. “Differentiated products pricing with consumer network acceptance in a dual-channel supply chain”. *Electronic Commerce Research and Applications*, 39, 100915 (2020).
27. Yan, R., Bandyopadhyay, S. “The profit benefits of bundle pricing of complementary products”, *Journal of Retailing and Consumer Services*, 18(4), pp.355-361 (2011).
28. Mesa-Arango, R. (2015), “Algorithms for bundling and pricing trucking services: Deterministic and stochastic approaches”.
29. Benisch, M., Sandholm, T., “A framework for automated bundling and pricing using purchase data”, In International Conference on Auctions, Market Mechanisms and Their Applications (pp. 40-52). *Springer, Berlin, Heidelberg* (2012).
30. Musa, M. I. “Bundling Pricing Strategy on Purchasing Decision: A Case of Indihome Product”, *Management*, 7(3), pp. 126-130 (2017).
31. Pan, L., Zhou, S. “Optimal bundling and pricing decisions for complementary products in a two-layer supply chain”, *Journal of Systems Science and Systems Engineering*, 26(6), pp.732-752 (2017).
32. Beladev, M., Rokach, L., Shapira, B. “Recommender systems for product bundling”, *Knowledge-Based Systems*, 111, pp. 193-206 (2015).
33. Chu, C. S., Leslie, P., Sorensen, A. “Bundle-size pricing as an approximation to mixed bundling”, *American Economic Review*, 101(1), pp. 263-303 (2011).
34. Jain, S., Oosterlee, C. W. “The stochastic grid bundling method: Efficient pricing of Bermudan options and their Greeks”, *Applied Mathematics and Computation*, 269, pp. 412-431 (2015).

35. Taleizadeh, A. A., Babaei, M. S., Niaki, S. T. A., Noori-daryan, M. “Bundle pricing and inventory decisions on complementary products”, *Operational Research*, pp. 1-25 (2017b).
36. Xiao, T., & Shi, J. J. “Pricing and supply priority in a dual-channel supply chain”. *European journal of operational research*, 254(3), 813-823 (2016).
37. Yang, D., Xiao, T., Choi, T. M., & Cheng, T. C. E. “Optimal reservation pricing strategy for a fashion supply chain with forecast update and asymmetric cost information”. *International Journal of Production Research*, 56(5), 1960-1981 (2015).
38. Prasad, A., Venkatesh, R., Mahajan, V. “Product bundling or reserved product pricing? Price discrimination with myopic and strategic consumers”, *International Journal of Research in Marketing*, 32(1), pp. 1-8 (2015).
39. Taleizadeh, A.A., Cárdenas-Barrón, L.E., Sohani, R. “Coordinating the supplier-retailer supply chain under noise effect with bundling and inventory strategies”, *Journal of Industrial and Management Optimization*, 15(4), 1701-1727 (2019).

Appendix A

According to Eq. (3) we have:

$$\pi = (w-c)(1-w+k) - k(a+bk) = w - w^2 + kw - c + cw - ck - ak - bk^2$$

$$\frac{\partial \pi}{\partial w} = 0 \Rightarrow 1 - 2w + k + c = 0 \Rightarrow w = \frac{1+k+c}{2}$$

$$\frac{\partial \pi}{\partial k} = 0 \Rightarrow w - c - a - 2bk = 0 \Rightarrow k = \frac{w-c-a}{2b}$$

$$w = \frac{1+k+c}{2} \Rightarrow w = \frac{1+\frac{w-c-a}{2b}+c}{2} = \frac{2b+w-c-a+2bc}{4b} \Rightarrow 4bw - w = 2b - c - a + 2bc \Rightarrow w^* = \frac{2bc+2b-c-a}{4b-1}$$

$$r = \frac{w-c-a}{2b} = \frac{\frac{1+k+c}{2} - c - a}{2b} = \frac{1+k+c-2c-2a}{4b} = \frac{1+k-c-2a}{4b} \Rightarrow 4kb - k = 1 - c - 2a \Rightarrow k^* = \frac{1-c-2a}{4b-1}$$

$$\pi = w - w^2 + kw - c + cw - ck - ak - bk^2 \Rightarrow$$

$$\begin{aligned}\pi^* &= \tfrac{2bc+2b-c-a}{4b-1} - \left(\tfrac{2bc+2b-c-a}{4b-1}\right)^2 + \left(\tfrac{1-c-2a}{4b-1}\right)\left(\tfrac{2bc+2b-c-a}{4b-1}\right) - c \\ &\quad + c\left(\tfrac{2bc+2b-c-a}{4b-1}\right) - c\left(\tfrac{1-c-2a}{4b-1}\right) - a\left(\tfrac{1-c-2a}{4b-1}\right) - b\left(\tfrac{1-c-2a}{4b-1}\right)^2 \\ &= \frac{A}{(4b-1)^2}\end{aligned}$$

$$\begin{aligned}A &= (4b-1)(2bc+2b-c-a) - (2bc+2b-c-a)^2 + (1-c-2a)(2bc+2b-c-a) - c(4b-1)^2 \\ &\quad + (4bc-c)(2bc+2b-c-a) - (4bc-c)(1-c-2a) - (4ab-a)(1-c-2a) - b(1-c-2a)^2\end{aligned}$$

$$A = 8b^2c + 8b^2 - 4bc - 4ab - 2bc - 3b + c + a - 4b^2 - 8b^2c + 4bc^2 + 4bc + 4ab - 2ac + 2bc + 2b$$
$$- 2bc^2 - 2bc + ac - 4ab + 2ac - 2bc - 16b^2c - c + 8bc + 4b^2c^2 + 8b^2c - 4bc^2$$
$$- 2ac - 4bc + 2bc^2 + 4abc - a^2 - 4ab + 8a^2b - bc^2 - 4a^2b + 2bc + 4ab \Rightarrow$$

$$A = 4b^2c^2 + 4a^2b - 8b^2c - bc^2 + 4b^2 - a^2 + 4abc - 4ab + 2bc - ac + a - b$$

Appendix B

According to Appendix A we have:

$$\frac{\partial^2 \pi}{\partial w^2} = -2 < 0, \quad \frac{\partial^2 \pi}{\partial k^2} = -2b < 0 \quad (0 < b < 1)$$

Biographies

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Figure 1. Schematic shape of "60-watt incandescent light" and "E27 to E40 encoder conversion"

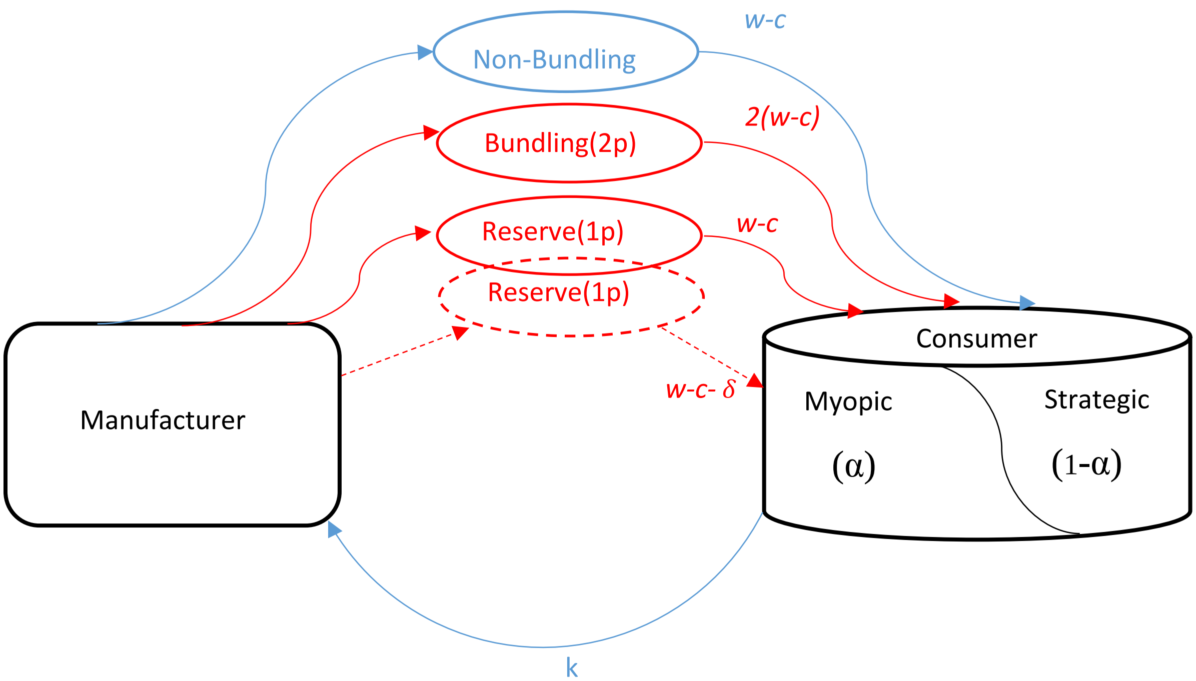


Figure 2. Combination refund and reserve scenario

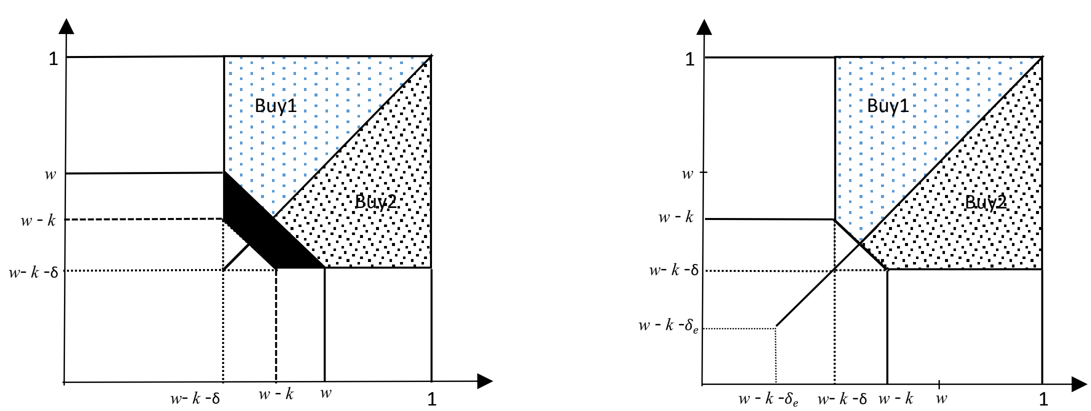


Figure 3. Demand diagram in the first stage

Figure 4. Demand diagram in the second stage

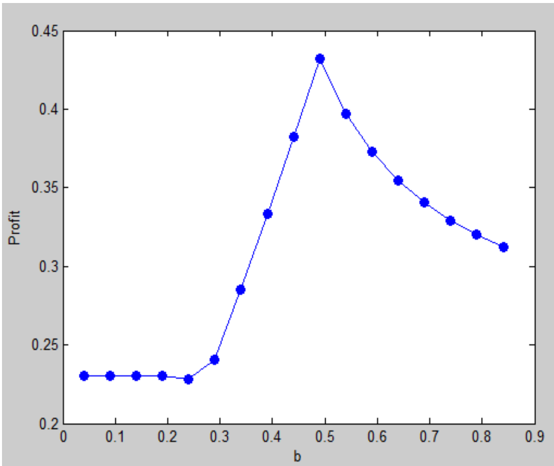


Figure 5. Profit changes diagram compared to b

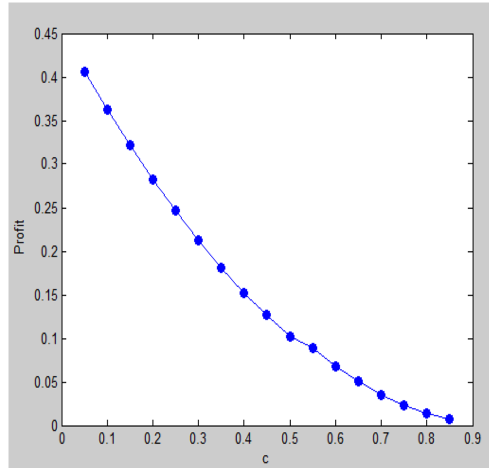


Figure 6. Profit changes diagram compared to c

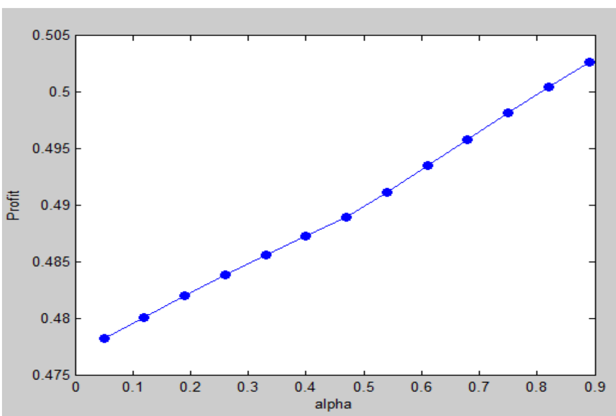


Figure 7. Diagram of profit value based on the coefficient of α

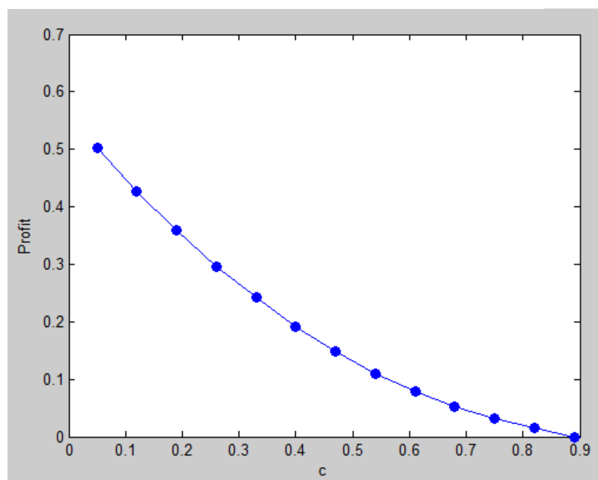
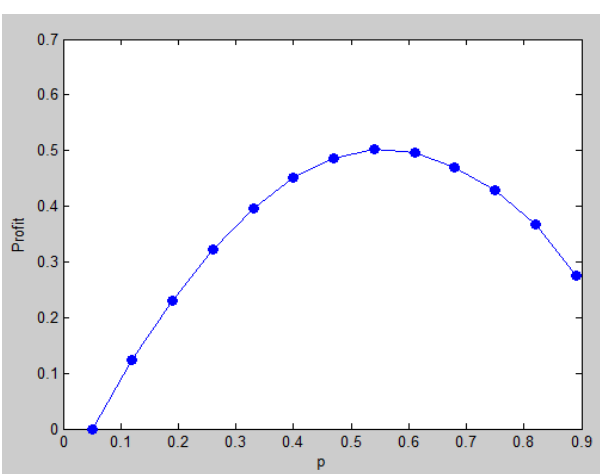


Figure 8. Profit value diagram according to the cost of the product



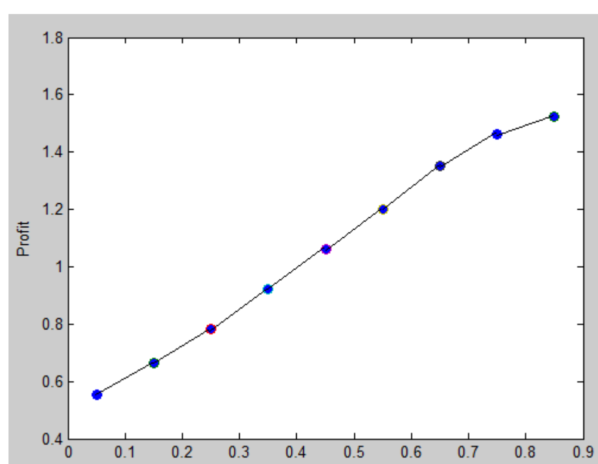


Figure 9. Diagram of profit according to product price

Figure 10. Profit diagram in terms of refund value

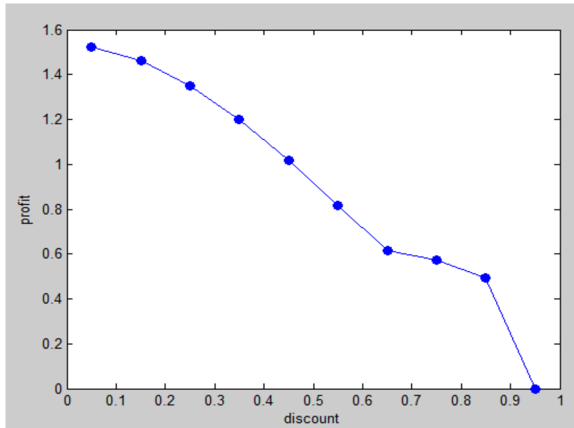


Figure 11. Diagram of profit in terms of a discount

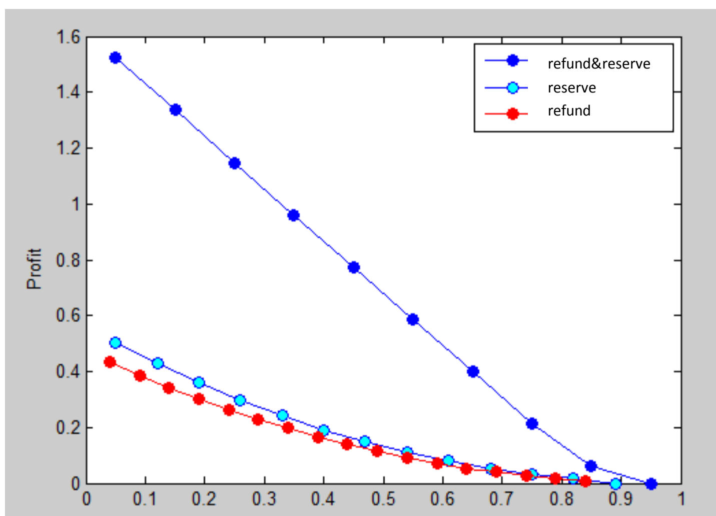


Figure 12. Review of three scenarios for different values of cost

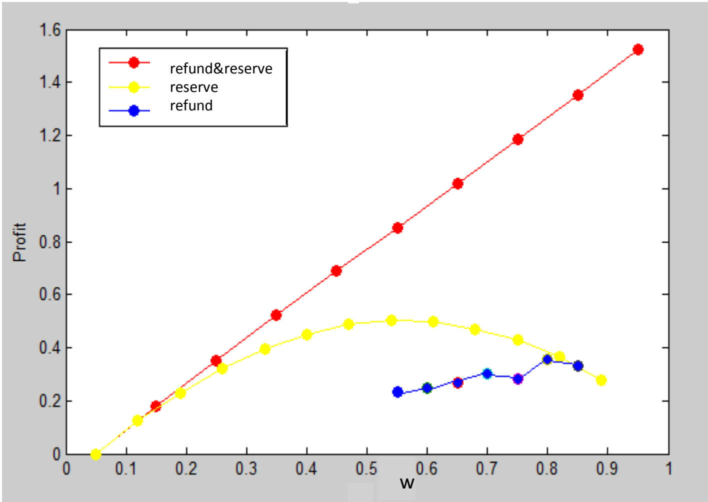


Figure 13. Review of three scenarios for different values of the product sales price

Table 1. Comparison for relevant literature

Research Work	Pricing	Return	Refund	Bundling	Reserve	Online Purchasing
Assarzadegan and Rasti-Barzoki (2019) [1]	✓	✓	✓			
Toomas Hinnosaar and Keiichi Kawai (2018) [2]	✓	✓	✓			
Jong Yeob Kim (2015) [3]	✓	✓	✓			
Heydaryan and Taleizadeh (2016a) [4]	✓	✓	✓			
Taleizadeh and Heydarian (2017) [5]	✓	✓	✓			
Heydaryan and Taleizadeh (2016b) [6]	✓	✓	✓			
Giri et al. (2017) [7]	✓	✓	✓			
Noori-daryan and Taleizadeh (2015) [8]	✓	✓				✓
Li et al. (2012) [9]	✓	✓				
Taleizadeh et al. (2017a) [10]	✓	✓	✓			
Batarfi et al. (2017) [11]	✓	✓	✓			✓
Yan and Ke (2015) [12]	✓		✓			
Li et al. (2013) [13]	✓	✓	✓			✓
Li et al. (2017) [14]	✓	✓	✓			✓
Taleizadeh et al. (2017c) [15]	✓	✓				
Taleizadeh et al. (2018b) [16]	✓	✓				✓
Moshtagh and Taleizadeh (2016) [17]	✓	✓				
Roy et al. (2016) [18]	✓	✓				✓
Modak et al. (2016a) [19]	✓	✓				
Modak et al. (2017) [20]	✓	✓				
Modak et al. (2016b) [21]	✓					
Taleizadeh et al. (2018a) [22]	✓	✓	✓	✓		✓
Xu et al. (2018) [23]	✓					✓
Modak et al. (2016c) [24]	✓					
Roy et al. (2015) [25]	✓	✓				
Liu et al. (2020) [26]	✓					✓
yan and Bandyopadhyay (2011) [27]	✓			✓	✓	
Mesa Arango (2015) [28]	✓	✓	✓	✓		
Benisch and Sandholm (2012) [29]	✓			✓		
Musa (2017) [30]	✓			✓		
Pan and Zhou (2017) [31]	✓			✓		
Beladev et al. (2015) [32]	✓			✓		
Chu et al. (2011) [33]	✓			✓		
Jain and Oosterlee (2015) [34]	✓			✓		
Taleizadeh et al. (2017b) [35]	✓			✓		
Xiao and Shi (2016) [36]	✓					✓
Yang et al. (2015) [37]	✓				✓	
Prasad et al. (2015) [38]	✓			✓	✓	
Taleizadeh et al. (2019) [39]	✓			✓		
This Paper	✓	✓	✓	✓	✓	✓

Table 2. Optimal profit changes respect to cost

c	a	b	k^*	w^*	π^*
0.05	0.05	0.5	0.85	0.95	0.4065
0.1	0.05	0.5	0.8	0.95	0.3627
0.15	0.05	0.5	0.75	0.95	0.3215
0.2	0.05	0.5	0.7	0.95	0.2827
0.25	0.05	0.5	0.65	0.95	0.2464
0.3	0.05	0.5	0.6	0.95	0.2127
0.35	0.05	0.5	0.55	0.95	0.1814
0.4	0.05	0.5	0.5	0.95	0.1526
0.45	0.05	0.5	0.45	0.95	0.1264
0.5	0.05	0.5	0.4	0.95	0.1026
0.55	0.05	0.45	0.44	0.99	0.089
0.6	0.05	0.45	0.38	0.99	0.0682
0.65	0.05	0.45	0.31	0.98	0.0502
0.7	0.05	0.45	0.25	0.98	0.0351
0.75	0.05	0.45	0.19	0.97	0.0227
0.8	0.05	0.4	0.17	0.98	0.0142
0.85	0.05	0.35	0.13	0.99	0.0072

Table 3. Optimal profit changes compared b

b	a	c	k^*	w^*	π^*
0.04	0.49	0.04	0.02	0.53	0.2303
0.09	0.49	0.04	0.03	0.54	0.2302
0.14	0.49	0.04	0.05	0.54	0.2302
0.19	0.49	0.04	0.08	0.56	0.23
0.24	0.49	0.04	0.5	0.77	0.2279
0.29	0.44	0.04	0.5	0.77	0.2404
0.34	0.34	0.04	0.78	0.91	0.2848
0.39	0.24	0.04	0.86	0.95	0.3333
0.44	0.14	0.04	0.89	0.97	0.3825
0.49	0.04	0.04	0.92	0.98	0.4321
0.54	0.04	0.04	0.76	0.9	0.3973
0.59	0.04	0.04	0.65	0.84	0.3728
0.64	0.04	0.04	0.56	0.8	0.3545
0.69	0.04	0.04	0.5	0.77	0.3404
0.74	0.04	0.04	0.45	0.74	0.3292
0.79	0.04	0.04	0.41	0.72	0.32
0.84	0.04	0.04	0.37	0.71	0.3124

Table 4. Optimal profit changes based on the α

α	c	w	δ	π^*
0.05	0.05	0.61	0.18	0.4783
0.12	0.05	0.61	0.18	0.4802
0.19	0.05	0.61	0.19	0.482
0.26	0.05	0.61	0.2	0.4839
0.33	0.05	0.61	0.21	0.4856
0.4	0.05	0.61	0.21	0.4873
0.47	0.05	0.61	0.22	0.489
0.54	0.05	0.54	0.19	0.4912
0.61	0.05	0.54	0.2	0.4935
0.68	0.05	0.54	0.2	0.4959
0.75	0.05	0.54	0.21	0.4982
0.82	0.05	0.54	0.22	0.5004
0.89	0.05	0.54	0.23	0.5027

Table 5. Optimal profit changes compared to changes in the cost of the product

c	α	w	δ	$\pi*$
0.05	0.89	0.54	0.23	0.5027
0.12	0.89	0.61	0.23	0.4272
0.19	0.89	0.61	0.2	0.36
0.26	0.89	0.68	0.2	0.2961
0.33	0.89	0.68	0.17	0.2425
0.4	0.89	0.68	0.13	0.1906
0.47	0.89	0.75	0.13	0.1493
0.54	0.89	0.75	0.1	0.11
0.61	0.89	0.82	0.1	0.0794
0.68	0.89	0.82	0.06	0.052
0.75	0.89	0.89	0.07	0.0318
0.82	0.89	0.89	0.03	0.0156

Table 6. Optimal profit changes compared to the price of product

k	α	c	δ	$\pi*$
0.05	0.95	0.05	0.25	0.553547
0.15	0.95	0.05	0.35	0.663228
0.25	0.95	0.05	0.35	0.781303
0.35	0.95	0.05	0.35	0.921966
0.45	0.95	0.05	0.35	1.061028
0.55	0.95	0.05	0.35	1.199691
0.65	0.95	0.05	0.25	1.351197
0.75	0.95	0.05	0.15	1.462378
0.85	0.95	0.05	0.05	1.522884

Table 7. Optimal profit change compared to refund value

w	α	c	δ	$\pi*$
0.12	0.89	0.05	0.01	0.1234
0.19	0.89	0.05	0.05	0.2305
0.26	0.89	0.05	0.08	0.3215
0.33	0.89	0.05	0.12	0.3954
0.4	0.89	0.05	0.16	0.4509
0.47	0.89	0.05	0.19	0.487
0.54	0.89	0.05	0.23	0.5027
0.61	0.89	0.05	0.27	0.4967
0.68	0.68	0.05	0.28	0.469
0.75	0.05	0.05	0.26	0.428
0.82	0.05	0.05	0.29	0.3675
0.89	0.05	0.05	0.3	0.2752

Table 8. Optimal profit changes compared to discount (some data)

k	α	c	δ	$\pi*$
0.05	0.95	0.05	0.25	0.553547
0.15	0.95	0.05	0.35	0.663228
0.25	0.95	0.05	0.35	0.781303
0.35	0.95	0.05	0.35	0.921966
0.45	0.95	0.05	0.35	1.061028
0.55	0.95	0.05	0.35	1.199691
0.65	0.95	0.05	0.25	1.351197
0.75	0.95	0.05	0.15	1.462378
0.85	0.95	0.05	0.05	1.522884

Table 9. Compare three scenarios for different values of the cost

C	Refund Scenario	Reserve Scenario	Refund &Reserve Scenario
0.1	0.405	0.448239	1.62
0.2	0.32	0.349969	1.44
0.3	0.326667	0.263343	1.26
0.4	0.24	0.192643	1.08
0.5	0.175	0.128881	0.9
0.6	0.106667	0.083746	0.72
0.7	0.135	0.042084	0.54
0.8	0.06	0.020468	0.36
0.9	0.015	2.66E-17	0.18

Table 10. Compare three scenarios for different amounts of the sales price

W	Refund Scenario	Reserve Scenario	Refund &Reserve Scenario
0.1		0.181368	1.8
0.2		0.330342	1.62
0.3		0.444987	1.44
0.4		0.522115	1.26
0.5	0.25	0.558534	1.08
0.6	0.28	0.551044	0.9
0.7	0.29	0.502694	0.72
0.8	0.34	0.424745	0.54
0.9	0.41	0.283746	0.36