The response of nano-ceramic doped fluids in heat convection models: A characteristics-based numerical approach

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Abstract

In this paper, forced, free, and mixed convections in incompressible flow were studied numerically. Nano-sized Al₂O₃, TiO₂, MgO, and ZnO ceramics with water were considered as nano-fluids. Simulations were carried out for cavity flow with different boundary conditions and aspect ratios, as well as flow over stationary and rotating cylinders. The mean Nusselt number ($\overline{Nu}$) and friction factor for cavity flow and $\overline{Nu}$ for flow over a cylinder were compared for different nano-fluids. A new code was developed in FORTRAN 95 for numerical simulations. A fifth-order Runge-Kutta method for time discretization and a characteristic-based scheme for convective terms were used in this code. The averaging scheme on the secondary cells is used to obtain viscous fluxes. Primary results are validated with other researcher’s outputs. Results showed that MgO-water and ZnO-water had maximum and minimum heat transfer rates, respectively. Moreover, maximum and minimum shear stresses were recorded for the Al₂O₃-water and TiO₂-water, respectively. Using nano-fluid increases the heat transfer rate between 15 and 37 percent depending on the Richardson number and selected nano-particles.

Keywords: Nano-fluid; Mixed convection; Cavity; Flow over a cylinder; Richardson number.
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1. Introduction

Optimization of thermal systems and using renewable energy sources are inevitable ways to reduce the danger of air pollution, global warming, environmental concerns as well as decreasing the related costs. Heat transfer enhancement in thermal systems is a basic part of system optimization. There are two major groups of heat transfer enhancement, active and passive methods [1]. In active methods, some types of external energies including electric field, magnetic field, and acoustic force are employed to perturb the fluid flow and increase the heat transfer rate, whereas, in passive methods the enhancement achieved by a change in the geometry of the system or manipulation of the working fluid [2]. Nano-fluids, as newborn working fluids, are categorized in the group of passive methods. The term, nano-fluids, refers to a stable dispersion of nano-sized particles in a base fluid. Solid materials, generally possess high thermal conductivities than conventional fluids, therefore the addition of solid particles improves their thermal properties [3]. The interesting properties of nano-fluids have motivated the researchers and engineers to investigate their application in the various fields of engineering including micro electro-mechanical systems (MEMS), fuel cells, heat treatment of metals [4]. It is still needed a wide range of experimental works to provide a reliable database to propose a comprehensive theory about the nano-fluids. Accurate experimental researches are costly and time-consuming. Moreover, working with nanoparticles has its dangers about the safety of the experimenter. On the other hand, extracting accurate data is difficult and sometimes impossible, for instance obtaining the temperature of each point in micro-channels. Uncertainties related to measuring instruments and sources of error including instrumental, environmental, procedural, and human are other drawbacks of using experimental methods. Numerical methods, however, are a powerful tool for analyzing almost all physical phenomena and can provide detailed information at each desired location.

Maïga et al. [5] aimed to study the effect of Al$_2$O$_3$ dispersion in two different media, water, and ethylene glycol. They investigated the forced convection of the mentioned nano-fluid numerically and reported that using alumina nanoparticles enhanced the heat transfer of both fluids. They found that the ethylene glycol–Al$_2$O$_3$ combination yielded a better heat transfer enhancement than water–Al$_2$O$_3$. However, wall shear stress was higher at the same time. Kefayati [6] studied natural convection flow with Cu-water nano-fluid by the Lattice-Boltzmann scheme in a cavity with different aspect ratios. The magnetic field was applied in the flow domain. The results showed that the influence of the nanoparticles increases for high
Hartmann numbers while the aspect ratio increases. Mahmoodi [7] studied the effect of particle type on natural convection of the cavity flow with different heaters in it. They employed the water as a base fluid and performed a series of numerical simulations using the SIMPLER algorithm. Their results demonstrated that the horizontally located heater had a higher Nu at small Ra numbers than the perpendicularly placed heater, but the location of the radiator did not affect Nu in high Ra numbers. Rahmati and Tahery [8] simulated the laminar natural convection in a cavity using TiO$_2$-water nano-fluid. They proposed some obstacles in the base geometry and used the Lattice-Boltzmann method to predict the velocity field and thermal behavior of the nano-fluid. They found that the $\overline{Nu}$ increased by an increase in Rayleigh number and the volume fraction of nanoparticles. They also reported that the obstacle dimension had a direct effect on thermal behavior of the cavity, so that the $\overline{Nu}$ improved for the obstacle dimensions up to 0.5 L, however the Nusselt number deteriorated as the obstacle dimensions increased to 0.7 L.

Akbarinia and Behzadmehr [9] studied numerically the mixed convection of Al$_2$O$_3$-water nano-fluid within a tube. Their results showed that the concentration of nanoparticles had no direct effect on the secondary flow, axial velocity, and friction coefficient. They claimed that the rising volume fraction of nanoparticles increased Nu and reduced friction factor (f). Heydari et al. [10] simulated the three-dimensional nano-fluid flow in a heat exchanger. They used SiO$_2$, CuO, Au, Cu, Fe, Al$_2$O$_3$, and Fe$_2$O$_3$ in ethylene glycol and water-based fluids. It was reported that the ethylene glycol-based nano-fluids had higher efficiencies. Uysal et al. [11] performed a three-dimensional simulation in a circular tube using ZnO-ethylene glycol nano-fluid. Their results showed a 10% enhancement in Nusselt number for 1% concentration. Sunil and Kumar [12] used the Lattice-Boltzmann scheme to simulate the Al$_2$O$_3$-water nano-fluid. The other outcome was that the increase in the heat transfer rate does not depend on Reynolds number variation. Kouz et al. [13] simulated two-dimensional in the cavity using Al$_2$O$_3$-Air nano-fluid. They used two fins in the hot wall to increase the effect of nano-fluid on the Nusselt number rising. Natural laminar convection flow was considered in this work and Knudsen number was between 0 and 0.1 and the Rayleigh number was between $10^5$ and $10^6$. They found that the Nu was a function of Rayleigh number and reverse of Knudsen number. They proposed a new relation for the Nusselt number as $Nu = 0.219Ra^{0.0829}Kn^{-0.511} \phi^{-0.104}$.

Zhang et al. [14] performed a numerical study to simulated nano-fluid flow. They used an adoptive grid method to ensure accuracy. Using nano-fluid is industrial to improve heat transfer rate has been increased in recent years. The numerical method costs less than experimental.
methods. Hence, numerical methods are excellent for the initial perception of nano-fluid flow behavior. As a result, different numerical studies had been done for nano-fluid flow [15-18]. The literature review shows that valuable numerical simulations have been performed to predict the behavior of the nano-fluids. However, the unknown nature of the involved mechanisms, is still a big challenge to completely cover the thermal behavior of these fluids, so that in some cases the numerical simulations are considerably far from the experimental data. Therefore it is needed to examine the different schemes to reduce the related numerical errors. The present work aims to solve the governing equations of the nano-fluids using the characteristic-based scheme proposed previously by our group [19-21]. To investigate the potential of the present scheme, four types of nanoparticles including Al₂O₃, TiO₂, MgO, and ZnO ceramics in base water are considered. Various types of convections are simulated numerically for nano-fluids and a pure fluid. The present scheme was examined on two different geometries, cavity flow and flow over a cylinder. The cavity flow was simulated at different aspect ratios (ARs), boundary conditions, and Richardson numbers. The flow over the cylinder was simulated in two cases of stationary and rotating cylinders.

2. The governing equations and the numerical procedure

Nano-fluids can be considered as single-phase fluids in numerical studies where their physical characteristics depend on the characteristics of the base fluid and the nanoparticles. The following are Navier-Stokes equations for a two-dimensional nano-fluid with heat transfer:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}), \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -g - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu (\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}), \\
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{1}{\rho C_p} (u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y}) + \frac{k}{\rho C_p} (\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) + \frac{\mu}{\rho C_p} \Phi, \\
\Phi &= \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial u}{\partial x} \right) \left( \frac{\partial v}{\partial x} \right) + \left( \frac{\partial v}{\partial y} \right)^2 
\end{align*}
\]

(1)

The artificial compressibility of Chorin [22] is applied and the gradient of pressure is added to the continuity equation. As a result, the pressure can be obtained by numerical schemes. Also, the nature of governing equations is changed and the characteristics-based scheme that
was proposed by ourselves can be used as a numerical scheme. This scheme solved the problems of the averaging scheme [19-21]. The instability of the averaging scheme was a serious problem that was solved by our proposed characteristics-based scheme.

$$\frac{1}{\varepsilon} \frac{\partial p}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

(2)

where $\varepsilon$ is the artificial compressibility factor [22]. The governing equations can be transformed into dimensionless forms and shown in the matrix shape as

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = \frac{\partial R}{\partial x} + \frac{\partial S}{\partial y} + H$$

$$Z = \begin{bmatrix} p \\ u \\ v \\ T \end{bmatrix}, \quad F = \begin{bmatrix} \beta u \\ p + u^2 \\ uv \\ (T - Ec p)u \end{bmatrix}, \quad G = \begin{bmatrix} \beta v \\ uv \\ p + v^2 \\ (T - Ec p)v \end{bmatrix}, \quad R = \frac{1}{Re} \begin{bmatrix} 0 \\ \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} \\ 1 \frac{\partial T}{Pr \partial x} \end{bmatrix},$$

$$S = \frac{1}{Re} \begin{bmatrix} 0 \\ \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial y} \\ 1 \frac{\partial T}{Pr \partial y} \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ 0 \\ \frac{Gr}{Re^2}T \\ \frac{Ec}{Re} \Phi \end{bmatrix}, \quad \Phi = \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 + 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2.$$

(3)

Dimensionless parameters are defined as:

$$Re = \frac{\rho_{ref} u_{ref} L_{ref}}{\mu}, \quad Pr = \frac{c_p \mu}{k}, \quad Ec = \frac{u_{ref}^2}{c_p (T_{ref 2} - T_{ref 1})},$$

$$Gr = \frac{\beta_{ex} g (T_{ref 2} - T_{ref 1}) L_{ref}^3}{\nu^2}, \quad Ri = \frac{Gr}{Re^2}.$$

(4)

The reference length for a cavity flow is the width of the cavity, and for the flow over a cylinder is the diameter of the cylinder. The reference velocity and density are the far-field velocity and density respectively. The Mach number of the flow is less than 0.3, hence, the
flow is considered incompressible flow, but the Boussinesq assumption is applied for density change in the y-direction because of the temperature gradient in the flow field. Reference temperatures are the temperature of warm and cold walls for cavity and are the far-field and wall of the cylinder for flow over the cylinder. The properties of water, nanoparticles, and nano-fluids are extracted from Ref. [23]. The following equations are used for nano-fluids in order to find the thermodynamically characteristics of the nano-fluid [24].

\[
\frac{\rho_{nf}}{\rho_{bf}} = 1 - C + C \frac{\rho_p}{\rho_{bf}}; \quad \frac{\mu_{nf}}{\mu_{bf}} = 1 + CN_v; \quad \frac{(\rho C_p)_{nf}}{(\rho C_p)_{bf}} = 1 - C + C \frac{(\rho C_p)_p}{(\rho C_p)_{bf}};
\]

\[
\frac{k_{nf}}{k_{bf}} = 1 + CN_c; \quad \frac{\beta_{nf}}{\beta_{bf}} = \frac{\rho_{nf}}{\rho_{bf}} \left[ (1 - C) + C \frac{(\rho \beta)_p}{(\rho \beta)_{bf}} \right].
\]

The governing equations are discretized by the FVM, where the fifth-order Rung-Kutta is used for time discretization as

\[
Z_{ijk}^{(q)} = Z_{ijk}^{(n)} - F(Z_{ijk}^{(q-1)}). \quad \alpha_q = \frac{1}{4}, \frac{1}{6}, \frac{3}{8}, \frac{1}{2}, 1, \quad q = 1, 2, ..., 5.
\]

The time step is considered $10^{-4}$ for a stable solution. The usual upwind scheme can be used for time discretization but the Rung-Kutta method is more stable [25, 26]. The convective fluxes are calculated by the characteristic-based scheme introduced by ourselves [20, 21]. The first-order derivatives (convective fluxes) in cell centers are changed to normal parameters on cell boundaries by Green’s theorem. A simple method to calculate the convective fluxes is the averaging method. In this method, the convective fluxes are obtained by a simple average of cell center data. But this method is not stable. So, it is replaced by a characteristics-based method that was introduced by the authors. The introduced scheme is stable. The pseudo wave propagation is considered in this method and convective fluxes are calculated by the pseudo waves. This scheme is explained with details in [20, 21]. The viscous fluxes (The second-order derivatives) in the cell center are replaced by the first-order derivatives on the cell boundaries. Obtained first-order derivatives are replaced by normal parameters on the boundaries of secondary cells by the aim of the Green theorem (equation 7).
\[
\frac{\partial \zeta}{\partial x} \bigg|_{AB} = \frac{1}{A} \int_{A} \frac{\partial \zeta}{\partial x} ds = \frac{1}{A} \int_{A} \zeta dy = \frac{1}{A} \sum_{k=1}^{4} \zeta \Delta y_k, \quad \zeta = u, v, T. \tag{7}
\]

where, \( f \) and \( \text{Nu} \) are calculated as:

\[
f = \frac{2 \mu_{gf} \partial u}{\text{Re} \mu_{gf} \partial y} = \frac{2 \mu_{gf} u_2 - u_1}{\text{Re} \mu_{gf} y_2 - y_1}, \quad \text{Nu} = \frac{2 k_{gf} \partial T}{\text{Re} k_{gf} \partial y} = \frac{2 k_{gf} T_2 - T_1}{\text{Re} k_{gf} y_2 - y_1}. \tag{8}
\]

Also, \( \bar{f} \) and \( \bar{\text{Nu}} \) for cavity flow are obtained from the equations below:

\[
\bar{f} = \int_{0}^{1} f dx = \frac{1}{N} \sum_{j=1}^{N} f_j; \quad \bar{\text{Nu}} = \int_{0}^{1} \text{Nu} dx = \frac{1}{N} \sum_{j=1}^{N} \text{Nu}_j; \tag{9}
\]

\( \bar{\text{Nu}} \) for flow over a cylinder is obtained from Eq. 6:

\[
\bar{\text{Nu}} = \int_{0}^{2\pi} \text{Nur} d\theta = \frac{1}{2\pi} \sum_{j=1}^{N} \text{Nu}_j \Delta \theta \tag{10}
\]

Boundary conditions for flow over the circular cylinder and cavity flow are shown in Fig. 1.

3. Results and discussion

3.1. Grid independence and convergence history

The quadrilateral grids were used in this work. Different grid sizes were selected and simulation was done to ensure grid independence. The grid independence check for flow over a cylinder and cavity flow is shown in Fig. 2., which compares horizontal velocity variations in the vertical centerline of the cavity for different grids. Nusselt number variations on cylinder wall for different grids are compared in terms of grid independence.

The error is calculated by:

\[
\text{Error} = \frac{\sum_{j=1}^{N} \sum_{i=1}^{M} (u_{i,j}^{k+1} - u_{i,j}^{k})}{NM} \tag{11}
\]

The convergence history was obtained and displayed in Fig. 3.
3.2. Validation

For validation, the results of the proposed numerical method for cavity flow are compared with that of Muthtamilselvan et al.[27] in Table 1. The applied boundary conditions are the same as the work by Muthtamilselvan et al.[27] at Ri=1. Table 1 shows that the applied novel numerical approach can predict the Nu number accurate enough for different nano-fluid concentrations. For more certainty, validation is done with the results of Ghia et al. [28]. For this purpose, the velocities in the x and y directions are obtained on the centerlines of the cavity and are compared with that of Ghia et al.[28] (Fig. 4.). Good agreement is observed, which proves that our new characteristics-based scheme can solve the governing equation numerically with acceptable accuracy.

3.3. Results of simulation of cavity flow

The Al$_2$O$_3$–water nano-fluid flow in a cavity was simulated by the proposed novel numerical approach, and the streamlines for different Ri numbers are shown in Fig. 5. The Al$_2$O$_3$ concentration in the fluid was assumed 5%. At high Ri numbers, the dominant heat transfer mechanism is natural convection, while for low Ri numbers the forced convection is the dominant one. At low Richardson numbers, two vortices are produced because of hydrodynamic boundary conditions. Upper and lower walls move right and force fluids in the boundary layer to move right. Therefore, two almost symmetric vortices are made in the upper and lower parts. With an increase in Ri number, the natural convection gets more important, however, it does not dominates and there are two vortices in medium Richardson numbers the same as small ones, but they are not symmetric anymore. At Ri=10 the natural convection influences the hydrodynamics of the flow and the number of vortices increases, and the formation of four vortices is shown in Fig. 5. Due to the effect of the moving upper and lower walls, there are two vortices suppressed to the right wall. Two other ones are the result of the buoyancy effect, which is stronger at the right side because of the higher temperature difference between lower and upper walls. The isotherms at different Ri numbers for 5% Al$_2$O$_3$-water nano-fluid is demonstrated in Fig. 6, which indicates that forced convection dominates in low Richardson numbers, and the effect of the thermal boundary layer is significant in these cases. The isotherms show that heat penetration is more from/to both upper and lower walls, and the value of isotherms are greater for Ri=0.1 in comparison with Ri=10. Another important parameter in cavity flow can be the AR. The novel developed code was also used to simulated the cavity flow with AR=2. In this part, the nano-fluid was assumed ZnO-water with C = 5%.
The streamlines are shown in Fig. 7 for different Ri numbers. Fig.7 shows that for Ri=0.1, two almost symmetric vortices are formed the same as Fig. 5 since the natural convection is negligible. At Ri=10 due to the effect of gravity and buoyancy effect, two more vortices are formed which is one more than AR=1. Due to the thermal boundary conditions at the upper and lower wall, the buoyancy effect is more powerful at the ride hand side of the cavity. Therefore, at the top right side of the cavity, the temperature of the upper wall is lower than the fluid and the buoyancy effect reinforced the formation of bigger vortices at this part. Nu for the upper and lower walls of the cavity are compared at different Ri numbers in Fig. 8.

MgO-water nano-fluid with C=5% is considered as the nano-fluid in this case. The effect of Ri is negligible on Nu of the upper wall since the upper wall is warm and the buoyancy can not influence the flow regime considerably. While, on the lower wall, the temperature gradient results in a higher buoyancy effect, and Nu increases when the Richardson number decreases. There are some extremums in the Nu-X chart since hydrodynamic and thermal boundary conditions are very complicated and periodic. Effect of different nano-fluids and Ri number on the average friction factor \(\overline{f}\) at the upper and lower wall is shown in Figs. 9 and 10, respectively. The Al\(_2\)O\(_3\)-water and the TiO\(_2\)-water nano-fluids have the maximum and minimum \(\overline{f}\) for upper and lower walls, respectively. The reason that the relative viscosity for The Al\(_2\)O\(_3\)-water and the TiO\(_2\)-water is maximum and minimum respectively. \(\overline{f}\) is minus for lower wall because it moves right. As a result, shear stress and \(\overline{f}\) are minus in the lower wall. It worth noting that the amount of average friction factor at the lower wall increases when the Richardson number decreases. While, at the upper wall, the increase of Ri numbers increases the average friction factor. It is the result of the temperature boundary condition on the lower and upper walls. As it was mentioned previously, the hot upper wall can not influence considerably the hydrodynamics of flow, while for the lower wall it is completely an effective parameter on the boundary layer formation. The buoyancy tends to separate the fluid from the lower wall, so at higher Ri numbers, when the natural convection dominates, the average friction factor decreases. Average Nu of different nano-fluids at the upper and lower wall versus Ri is demonstrated in Figs. 11 and 12, respectively. The MgO-water and the ZnO-water nano-fluids have maximum and minimum \(\overline{Nu}\) values for the upper and lower walls, respectively, which can be attributed to the higher thermal conductivity of these nano-fluids. \(\overline{Nu}\) decreases when the Richardson number increases in all cases, which was expected since natural convection shows lower heat transfer in comparison with forced convection. Since the influence of natural convection on the upper wall is not as much as the lower wall, for mixed
and forced convection (Ri=0.1 and 1), the Nu at upper is almost unchanged. The effect of AR on the average Nu and friction factor of the cavity flow is shown in Table 2. The nano-fluid was assumed MgO-water with C=5% and the Ri was 0.1. In this case, forced convection is dominant and the moving walls can not move the fluid at the center of the cavity easily when the AR increases. Therefore, both $\overline{Nu}$ and $\overline{f}$ decrease when the AR increases.

3.4. Results of simulation of flow over a cylinder

In this section, flow over stationary and rotating cylinders is simulated. The $\text{Al}_2\text{O}_3$, $\text{TiO}_2$, MgO, and ZnO are used as ceramic dopants in water. Streamlines and isotherms are shown flow over stationary and rotating cylinders in Figs. 13 and 14, respectively. MgO-water with C = 5% is considered as a nano-fluid in this case. Streamlines and isotherms are symmetric for the stationary cylinder. No vortex is observed for forced convection flow over the rotating cylinder. Angular velocity of cylinder forces fluid near the cylinder to rotate around the center of the cylinder. Therefore, the vortices disappear around the rotating cylinder compared to the stationary one. However, it should be noted that despite the stationary cylinder, the lift force acts on the rotating one due to the asymmetric pressure distribution. Streamlines and isotherms are compared for different kinds of convections for flow over the stationary cylinder in Figs. 15 and 16, respectively. ZnO-water nano-fluid with C = 5% is used in this simulation. Free convection destroys the vortices in medium and high Richardson numbers since free convection is dominant in such cases. Forced convection dominates in lower Richardson numbers, and forces the fluid downward. Fluid conveys heat from cylinders to the downward. As a result, the temperature of the downward fluid is higher in lower Ri. Shear stress and pressure coefficient are compared for different Ri values at flow over the stationary cylinder in Figs. 17 and 18, respectively. TiO$_2$-water with C = 5% is considered as a nano-fluid in this simulation. Shear stress and pressure coefficient are almost symmetric for lower Richardson numbers because free convection is negligible in this state. Shear stress is high in the angles between 0º and 90º. The pressure coefficient is usually high in the station point (0º). Nusselt numbers are compared for different Richardson numbers at flow over the rotating cylinder and are shown in Fig. 19. $\text{Al}_2\text{O}_3$-water with C = 5% is considered as the nano-fluid in this case. Combination of mixed convection and the effect of cylinder rotation results in complex Nusselt number variation. Nusselt number is maximum in the lower part of the cylinder in lower and medium Richardson numbers and in the upper part of the cylinder at higher Richardson numbers. The $\overline{Nu}$, calculated for four selected nano-fluids and pure water, is compared for different Richardson numbers in Fig. 20. MgO-water has maximum $\overline{Nu}$ between selected nano-
fluids in all Richardson numbers. Al₂O₃-water has the minimum $\overline{Nu}$ in low Richardson numbers and ZnO-water- has the minimum $\overline{Nu}$ in medium and high Richardson numbers.

4. Conclusions

The various models of heat convections were simulated in this work. The case studies were cavity flow and flow over stationary and rotating cylinders. Results showed that when Richardson number increases, the number of vortices increases in cavity flow, but vortices vanish for flow over a stationary cylinder. Also, when Richardson number increases, $\tilde{f}$ increases and $\overline{Nu}$ decreases in cavity flow. $\tilde{f}$ and $\overline{Nu}$ decrease when the aspect ratio of cavity increases. Comparisons between different nano-fluids demonstrated maximum and minimum $\overline{Nu}$ values for MgO-water and ZnO-water, respectively for flow over the rotating cylinder, when rotational speed increases, $\overline{Nu}$ and Nusselt number variations decrease on the cylinder wall.

References


Table 1. Comparison of average Nu number for cavity flow obtained by the present numerical approach and Muthamilselvan et al. [32].

Table 2. The $f$ and the $\bar{N}u$ for MgO-water nano-fluid at C = 5%, Re=20, and Ri = 0.1.

Fig. 1. The boundary conditions for cavity flow and flow over a circular cylinder.

Fig. 2. Grid independence for Al2O3-water nano-fluid at Ri = 0.2, Re=20, and C = 5%.

Fig. 3. Convergence history

Fig. 4. Velocities on the centerlines of the cavity (comparing our results and the results of Ghia et al. [28])

Fig. 5. Streamlines for different Richardson numbers for Al2O3-water nano-fluid at C=5%.

Fig. 6. Isotherms at different Richardson numbers for Al2O3-water nano-fluid at C=5%, Re=20

Fig. 7. Streamlines for free, mixed, and forced convections (ZnO-water nano-fluid at C=5%, Re=20, and AR=2).

Fig. 8. Nu at upper and lower walls for different Ri numbers (MgO-water, C=5%, Re=20).

Fig. 9. $f$ at the upper wall for different nano-fluids at Re=20, and C=5%

Fig. 10. $f$ at the lower wall for different nano-fluids at Re=20, and C=5%

Fig. 11. $\bar{N}u$ at the upper wall number for different nano-fluids at Re=20, and C=5%

Fig. 12. $\bar{N}u$ at the lower wall for different nano-fluids at Re=20, and C=5%

Fig. 13. Streamlines of flow over stationary and rotating cylinders (MgO-water nano-fluid, Ri = 0, C = 5%, and Re=20).

Fig. 14. Isotherms for flow over stationary and rotating cylinders (MgO-water nano-fluid, Ri = 0, C = 5%, and Re=20).

Fig. 15. Comparison of stream lines at different Richardson numbers in stationary cylinder (ZnO-water nano-fluid, Ri = 0, C = 5%)

Fig. 16. Comparison of isotherms at different Richardson numbers in the stationary cylinder (ZnO-water nano-fluid, Ri = 0, C = 5%).

Fig. 17. Comparison of shear stress for different Richardson numbers in the stationary cylinder (TiO2-water nano-fluid, C = 5%).

Fig. 18. Comparison of pressure coefficient for different Richardson numbers in the stationary cylinder (TiO2-water nano-fluid, C = 5%).
Fig. 19. Comparison of Nu for different Ri values in the rotating cylinder (Al₂O₃-water nano-fluid, C=5%, w = 10)

Fig. 20. $\overline{Nu}$ for the stationary cylinders for different Richardson numbers and nano-fluids at Re=20, C=5%.

Table 1.

<table>
<thead>
<tr>
<th>Nano particles concentration %</th>
<th>C=2%</th>
<th>C=4%</th>
<th>C=6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muthamilselvan et al.[27]</td>
<td>2.40</td>
<td>2.56</td>
<td>2.73</td>
</tr>
<tr>
<td>Present work</td>
<td>2.41</td>
<td>2.54</td>
<td>2.76</td>
</tr>
</tbody>
</table>

Table 2.

<table>
<thead>
<tr>
<th>W/L ratio</th>
<th>$\overline{Nu}$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upper wall</td>
<td>Lower wall</td>
</tr>
<tr>
<td>0.5</td>
<td>5.423</td>
<td>12.668</td>
</tr>
<tr>
<td>1.0</td>
<td>4.270</td>
<td>12.061</td>
</tr>
<tr>
<td>2.0</td>
<td>3.582</td>
<td>10.666</td>
</tr>
<tr>
<td>4.0</td>
<td>2.659</td>
<td>8.128</td>
</tr>
</tbody>
</table>
Fig. 1.
Stationary cylinder

Fig. 2.
Fig. 5.
Fig. 6.
Ri = 0.1  
Ri = 1.0  
Ri = 10  

Fig. 7.

Fig. 8.
Fig. 9.
Fig. 10.
Fig. 12.
Stationary cylinder

Rotating cylinder

Fig. 13.
Stationary cylinder

Rotating cylinder

Fig. 14. 
Fig. 15.
Fig. 17.
Fig. 19.
Biography of authors

1. Tohid Adibi currently is an assistant professor at the University of Bonab. He graduated in Mechanical Engineering in B.Sc., M.Sc., and Ph.D. from the Iran University Science and Technology, Sharif University of Technology, and the University of Tabriz, respectively. His research interests include computational fluid dynamics, HVAC, and heat transfer. He has been published several conference and journal papers in the field of CFD, combustion, HVAC and heat transfer.

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Fig. 20.
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