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Stochastic nonlinear ground response analysis: A case study site in Shiraz, Iran

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Borehole data;
Stochastic analysis;
Fundamental period.

Abstract. The present study aims to investigate the impact of the uncertainties of dynamic soil property on the ground response analysis in a case study. To this end, nonlinear time-domain ground response analysis and uncertainties of the soil parameters were coupled by a MATLAB code. To take full advantage of the real data, two investigation boreholes were drilled in the site. The analysis was deterministically conducted and then, extended to the stochastic context to consider the variability of the plastic index, shear wave velocity, and unit weight of the soil. Furthermore, the capability of the three different methods including modal analysis, approximate method, and nonlinear method to predict the stochastic fundamental period was investigated. The comparison revealed that the mean value for the approximate method provided closer predictions regarding the fundamental periods obtained through the nonlinear method. In the stochastic analysis, the maximum Coefficient Of Variation (COV) of the peak ground motion parameters including fundamental period, response spectrum, and amplification factor was calculated. The results demonstrated that the heterogeneity of soil parameters had a significant effect on the variation of the surface Peak Ground Displacement (PGD). Among the other stochastic responses, the fundamental period received the least effect from uncertainty of soil parameters.

1. Introduction

The study of earthquakes is essential to preventing the potential loss of life and property damages. Earthquakes are the undeniable proof of the activities of the dynamic forces deep inside the earth. Stored stress and strain energy inside the earth increase over time, and earthquakes are the release of this stored energy. Ground vibrations caused by earthquakes result from the upward transfer of stress waves from the bedrock to surface soft-soils. Earthquakes are considered one of the most important natural factors that destroys man-made structures. Therefore, nowadays, one of the most significant issues in seismic geotechnical engineering is the assessment of the ground response to seismic waves [1–3].

Earthquake waves are constantly changing due to movement away from the focal point and passing through the alluvial layers. Although earthquake waves pass through rock and soil layers, soil deposits have major effect on the surface ground shaking characteristics [4,5]. Factors related to the distance from the earthquake focal point are known as path effects, and those related to the characteristics of the soil layers above the bedrock are known as site effects. Ground response analysis is a tool for considering the local site effects used for estimating the ground surface movement. Ground response analyses are carried out to predict ground surface movements, provide a

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response spectrum, and measure seismic forces causing instability of ground and structures.

Geotechnical studies have shown that the stress-strain relationships of soils are nonlinear and hysteretic, especially for shear strains greater than 10^{-5} to 10^{-4} [6]. In addition, evaluation of soil deposit responses resulting from earthquake case histories over the recent decades has pointed to the nonlinearity of behavior of soils. The prominent role of soil nonlinearity in site response analysis has been proven, particularly after the occurrence of the 1990 Manjil-Rudbar ($M_w = 7.4$), 1994 Northridge ($M_w = 6.7$), 1995 Kobe ($M_w = 6.9$), and 2001 Gujarat earthquake ($M_w = 7.7$).

In recent years, numerous studies have been dedicated to modeling the nonlinear behavior of soils under earthquake loading. Schnabel et al. [7] attempted to develop a shear strain compatible frequency-domain method called equivalent linear method, which is capable of considering some nonlinear effects, such as increasing the damping and decreasing the shear modulus, upon increasing the effective shear strain. Although this approach remains more cost-effective than nonlinear time-domain analysis, its main limitation is the consideration of time-independent shear modulus and damping during ground shaking. Several researchers have extended this method to include the frequency and pressure-dependent soil properties [8,9]. Nevertheless, the equivalent linear method does not account for several characteristics of soil layers in case of strong earthquakes including real effects of higher modes, permanent ground deformations, and moving the predominant period to the long-period range upon increase in the input motion intensity. In this regard, the time-domain nonlinear method has drawn considerable attention.

The first presented models for estimating the nonlinear soil stress-strain relations were based on Masing rules [10]. In the following, with the development of laboratory equipment and experimental achievements in geotechnical engineering, several models have been proposed to elaborate the nonlinear soil behavior based on the skeleton curve equation concept [11,12]. For instance, Pyke [13] and Vucetic [14] generalized the basic Masing rule to the extended Masing criteria which defined the unloading-reloading behavior under general cyclic loading. As a result of this progress, a nonlinear solution of the shear wave propagation was provided by several researchers.

Lee and Finn [15] employed a hyperbolic model to develop a One-Dimensional (1-D) nonlinear site response analysis program. Matasovic and Vucetic [16] modified the Kondner and Zelasko hyperbolic model [17] and used it in the D-MOD program coupled with the extended Masing criteria. Hashash and Park [18] developed a new nonlinear 1-D site response analysis model (DEEPSOIL software) that considered the dependency of the confining pressure on soil properties. Lo Presti et al. [19] employed the constitutive model of Ramberg and Osgood [12] coupled with a modified Masing criterion in the ONDA computer program to conduct a nonlinear ground response analysis. Phillips and Hashash [20] proposed two new formulations of soil damping for both small and large strains. Of note, among the available codes, only the DEEPSOIL and ONDA accounted for soil strength. Markham et al. [21] utilized the DEEPSOIL software [22] to model the response of potentially liquefiable soils during strong shakings in a case study of the Christchurch area. Angina et al. [23] carried out a study to determine the free-field seismic response of the site of Pisa tower whose results demonstrated that the response spectra obtained from the nonlinear code ONDA [19] and the corresponding amplification factors were significantly lower than those computed using the equivalent linear codes.

To qualify the applicability of the currently employed ground response analysis methods, several studies have compared the results of nonlinear time-domain through equivalent frequency-domain methods [24–26]. The results of these investigations indicated that the nonlinear modeling of ground shaking was essential in case the Peak Ground Acceleration (PGA) of the input acceleration or the shear strains in the soil exceeded some critical levels, where the equivalent linear predictions became insufficient.

Advanced constitutive models can also be employed to simulate the seismic soil behavior. For instance, Constantopoulos et al. [27] used the Ramberg-Osgood soli model [12] to present a method for calculating the nonlinear seismic response of a profile. Joyner and Chen [28] employed the Iwan model to determine the ground response of horizontal soil layers during an earthquake [29]. Borja et al. [30] utilized a bounding surface plasticity model to simulate the cyclic soil behavior response at a site in Taiwan. Anthi et al. [31] used the 1-D version of the TA-GER sand model [32] for the non-linear ground response analysis of layered sites. Finally, results of the proposed method were compared with those form two STRATA [33] and NL-DYAS [34] programs.

Although effective stress methods using advanced constitutive models could provide better prediction than the total stress methods, they require a lot of field data acquisition and laboratory tests to calibrate their parameters. This amount of effort for achieving accurate models often limits their frequency of use. Therefore, the nonlinear total stress methods are often employed as an applicable method with relatively good accuracy, especially where the liquefaction is not the critical issue.

The seismic ground response analysis is associated with different types of uncertainty including the specification of the input rock motions, characterization of the shear-wave velocity profile, characterization of the nonlinear soil properties, and selection of the analysis method [35]. During the past few years, numerous studies have taken into account different types of uncertainty in ground response analysis. In this respect, Field and Jacob [36] performed Monte Carlo Simulations (MCS) of the linear elastic response of a site in California subjected to weak motions. Their results indicated that uncertainties in the shear-wave velocity profile and small strain damping ratio significantly affected the amplification predictions. Rahman and Yeh [37] attempted to assess the uncertainties of soil parameters in frequency-domain ground response analysis by coupling MCS with the Finite Element Method (FEM). Wang and Hao [38] considered the effect of soil properties and random variations on the estimated amplification factors in the frequency-domain using the Point Estimate Method (PEM) [39]. Nour et al. [40] simulated the uncertainties of soil layers in seismic response analysis by combining MCS and FEM. Andrade and Borja [41] employed SHAKE and SPECTRA to conduct a stochastic-deterministic site response analysis. Lopez-Caballero and Modaressi-Farahmand-Razavi [42] studied the seismic site response with stochastic soil parameters and input motions. Rota et al. [43] presented a method for estimating the stochastic site amplification of a case study in central Italy. Johari and Momeni [44] proposed the stochastic site amplification model using the non-recursive algorithm in the Hybrid Frequency Time Domain (HFTD) approach. Medel-Vera and Ji [45] validated their proposed stochastic ground motion model of Northwest Europe for simulating accelerograms. Berkane et al. [46] investigated the heterogeneity effects of soil properties due to their natural randomness on the spatial variation of the response spectra at different sites. Their results showed that the randomness of soil properties could significantly affect the amplitudes of the response spectra. Johari et al. [47] proposed a method for conducting system reliability analysis of the surface PGA with consideration of the cross-correlation of soil layers. To this end, the sequential compounding method was employed to reduce the computational costs of determining the PGA reliability index through the frequency-domain site response analysis. The results showed that the proposed method had reasonable accuracy for calculating the reliability indices of ground surface PGAs at the sites under study.

A common feature in the mentioned studies of the stochastic ground response analysis is the application of linear or equivalent linear methods, while application of nonlinear time-domain methods could better reveal the real soil behavior. The main objective of this paper was to provide a method for evaluating the effect of the uncertainty of soil parameters on the results of site response analysis. To this end, a MATLAB program was coded to combine the capability of fully nonlinear time-domain analyses with a random variable using MCS. To take full advantage of the nonlinear method, the Darendeli [48] model was used to predict the dynamic soil stiffness and minimum viscous damping at small strains. This model was coupled with the extended Masing rules [13,14] to better simulate the hysteresis soil behavior under irregular seismic loads. The accuracy of the proposed code results was deterministically verified by DEEPSOIL software. To evaluate the uncertainty of the soil parameters, a case study site with two boreholes data was selected and the mean and standard deviations of the stochastic variables including Plastic Index (PI), shear wave velocity (V_s), and unit weight (γ) were directly obtained from the measured data. The effects of the uncertainty of these soil parameters on the seismic responses of the case study site were assessed using stochastic nonlinear ground response analysis. The means and standard deviations of the ground motion parameters and amplification factor were determined by a MATLAB coded program. Moreover, the variation of the stochastic fundamental period was investigated through three different methods including modal analysis, approximate method, and nonlinear method.

2. Nonlinear site response analyses

In the geotechnical earthquake engineering field, ground response analysis is a powerful tool for evaluating the effects of the local site on the upward shear wave motions caused by an earthquake and it is used to measure the ground surface movement. The near-surface soil layers can filter the coming waves and alter their amplitude and frequency contents.

In the 1-D time-domain method, the site profile is assumed to be made of several homogeneous horizontal soil layers with an infinite extent, which can be idealized as Multi-Degree-Of-Freedom (MDOF) lumped mass system, as shown in Figure 1. In this figure, the mass (m_i) of the soil layers is lumped at the layer interfaces. The soil stiffness (k_i) is considered by a nonlinear spring which should be updated at each time step to capture soil nonlinearity. Moreover, to avoid unrealistic resonance at very small strains in nonlinear analysis, a viscous damping coefficient (c_i) is also applied to the model, here schematically illustrated as dashpots.

The dynamic equation of the MDOF system with the mass matrix $[M]$, viscous damping matrix $[C]$, and stiffness matrix $[K]$ subjected to the acceleration excitation $\ddot{u}_g$ at the base of the soil column can be written as follows:

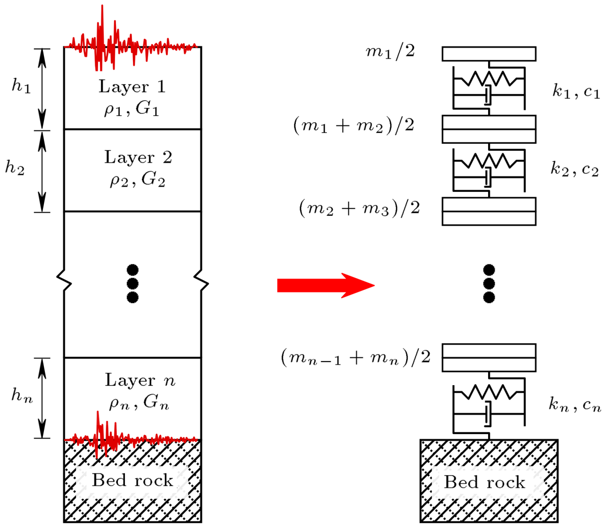

Figure 1. Schematic of 1D site response analysis of Multi-Degree-of-Freedom (MDOF) system with a rigid base.

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -M\{I\}\ddot{u}_g, \tag{1}$$

where $\{\ddot{u}\}$ is the vector of relative nodal accelerations, $\{\dot{u}\}$ the vector of relative nodal velocities, $\{u\}$ the vector of relative nodal displacements, and $\{I\}$ the unit vector.

In the time-domain method, the equation of motions is solved using a direct numerical integration scheme. Among the most significant time integration schemes are the central difference, Newmark β [49], and Wilson θ methods [50].

2.1. Nonlinear modified hyperbolic models

Several empirical models have been proposed in the literature to estimate the shear modulus reduction and damping curves. Among significant contributions, the models proposed by Darendeli [48], Menq [51], and Kishida et al. [52] were employed to simulate the dynamic soil parameters in the site response analyses when no dynamic tests were conducted. It is worth mentioning that the model proposed by Menq [51] is only calibrated for non-plastic granular soils and that proposed by Kishida et al. [52] is able to predict the dynamic properties of highly organic soils.

Generally, a nonlinear simple-shear model is established on two main principles:

(a) Backbone curve that includes a basic formula for determining the stress-strain path at the initial loading;

(b) A series of rules for determining the unloading and reloading behavior of soil.

The commonly used criteria are the so-called Masing rules. The basic assumption of Masing rules is that both the backbone curve and cyclic response are stable. However, in practice, the accumulation of cyclic loading effects may cause gradual changes in the backbone curve [53]. This issue makes the Masing rules only acceptable for regular cycle loads. Therefore, the extended Masing rules [13,14] were proposed to determine the stress-strain curve under irregular cyclic loading.

In this paper, the modified hyperbolic model proposed by Darendeli [48] was considered in modeling the backbone curve. Moreover, the extended Masing rules [13,14] were used for governing the unloading and reloading behavior of subsequent cycles.

2.2. Darendeli nonlinear model

Darendeli [48] proposed a new modified hyperbolic model by adding a curvature coefficient "a" to the basic model of Hardin and Drnevich [54] as follows:

$$\tau = F_{bb}(\gamma) = \frac{G_{\max}\gamma}{1 + (\gamma/\gamma_r)^a}, \tag{2}$$

where a is equal to 0.919 and γ_r is the shear strain when $G/G_{\max}$ equals 0.5, which can be obtained through the following equation:

$$\gamma_r = \left(0.0352 + 0.001 \times PI \times OCR^{0.3246}\right)\left(\sigma'_m\right)^{0.3483}, \tag{3}$$

where PI is the plasticity index, OCR the over-consolidation ratio, and σ'_m the mean effective confining pressure.

Based on Darendeli's assumption, the total damping is made of two main parts, namely small strain damping ($D_{\min}$) caused by internal friction and material viscosity and the damping corresponding to soil nonlinearity or hysteresis behavior (D_{Masing}) [48]:

$$D = F \times D_{\text{Masing}} + D_{\min}, \tag{4}$$

where D is the total damping and F is a reduction factor. Here, D_{Masing} is proportional to the ratio of the dissipated energy to the stored strain energy in one complete cycle of motion and $D_{\min}$ is calculated through the following equation [48]:

$$D_{\min}(\%) = (\sigma'_0)^{-0.2889} \left(0.8005 + 0.0129PI \times OCR^{-0.1069}\right) \left(1 + 0.2919\ln(f)\right), \tag{5}$$

where f is the loading frequency.

3. Computer program

In this study, a computer program capable of conducting total stress nonlinear time-domain ground response analysis was developed in MATLAB to take full advantage of its built-in math functions and matrix operations capability. The major abilities of the program are as follows:

- Conducting modal analysis to determine the natural periods and frequencies of soil profile corresponding to different modes;
- Considering soil strength and minimum viscous damping at small strain based on the empirical formulation of Darendeli [48];
- Applying the viscous damping matrix through Rayleigh damping formulation [55];
- Filtering and baseline correction of input motion, if needed;
- Simulating the fully nonlinear behavior of soil profile using the model proposed by Darendeli [48];
- Solving the equation of motion through the Newmark β method coupled by Newton-Raphson procedure with controllable accuracy and convergence;
- Determining the acceleration, velocity, and displacement time histories for each layer;
- Calculating the peak ground motion parameters, acceleration response spectra (S_a), and amplification factor for each layer.

4. Characteristics of the case study site

This study aimed to investigate the local site effects as well as uncertainties in the soil profile characteristics on the dynamic site response. In this respect, a case study site in Shiraz, Iran was taken into account. Located in the south-central Iran with more than 1.8 million residents, Shiraz is the fifth most populous city and the capital of Fars Province. In terms of seismicity, based on the Iranian Seismic Code [56], this city is located in an area with high seismic risk and is surrounded by a number of important faults. In order to investigate the soil properties of subsurface layers at the case study site, two boreholes (BH.1 and BH.2) with approximately 30m depth from the ground surface were drilled. For each borehole, several tests were carried out including down-hole, plasticity index, and unit weight. The results are summarized in Tables 1 and 2. As observed, the soil classification for the upper layers are mainly fine. Soil specimens obtained from layers 12 to 15 show that these layers are composed of granular soil with non-plastic fines. In these tables, *PI*, γ, and V_s are the mean of plasticity index, unit weight, and shear wave velocity of the soil layers, respectively. For seismic classification of the site, the average shear-wave velocity of the top 30.0 m layers, V_{S30}, is calculated for each borehole based on the Iranian Seismic Code [56], as shown in the following:

$$V_{S30} = \frac{\sum d_i}{\sum \left(\frac{d_i}{V_{si}}\right)}, \tag{6}$$

where d_i and V_{si} are the height and shear wave velocity of the ith layer, respectively. Through this formula, V_{S30} of BH.1 and BH.2 can be calculated as 352 and 349 (m/s), respectively. Based on the Iranian Seismic Code [56], this site is categorized in the "Type III" class (175 m/s $\leq V_s \leq$ 375 m/s).

In order to obtain a representative profile of the site, the mean and standard deviations of the data from the boreholes should be calculated. The mean and standard deviations of the selected stochastic parameters (V_s, γ, and *PI*) are given in Table 3.

Table 1. Geotechnical soil properties of borehole no. 1 (BH.1).

Layer no.	Classification	Depth (m)	*PI* (%)	γ (kN/m^3)	V_s (m/sec)
1	CL–ML	0–2	6.21	17.29	156.00
2	CL–ML	2–4	6.17	17.37	176.48
3	CL–ML	4–6	6.45	17.94	214.48
4	CL–ML	6–8	5.45	18.38	260.62
5	CL	8–10	7.19	18.70	301.39
6	CL–ML	10–12	5.08	18.78	289.30
7	SM	12–14	3.67	18.84	269.60
8	ML	14–16	4.36	19.07	305.55
9	CL–ML	16–18	4.58	19.37	368.26
10	SM	18–20	5.12	19.33	360.14
11	ML	20–22	2.45	20.28	468.59
12	SP	22–24	N.P.*	20.30	496.62
13	GP	24–26	N.P.*	20.62	518.82
14	GP	26–28	N.P.*	20.65	528.12
15	GP	28–30	N.P.*	20.85	568.74

*: Non-plastic granular soil.

Table 2. Geotechnical soil properties of borehole no. 1 (BH.2).

Layer no.	Classification	Depth (m)	PI (%)	γ (kN/m^3)	V_s (m/sec)
1	CL–ML	0–2	5.64	17.08	168.50
2	CL–ML	2–4	4.69	17.06	170.62
3	CL–ML	4–6	5.78	17.53	195.62
4	CL–ML	6–8	4.45	17.85	232.97
5	CL–ML	8–10	5.67	18.19	274.62
6	ML	10–12	3.92	18.10	257.62
7	SM	12–14	2.91	18.17	262.17
8	ML	14–16	3.18	18.60	323.26
9	ML	16–18	3.10	19.43	390.60
10	SM	18–20	3.60	19.71	401.71
11	SM	20–22	3.90	19.86	454.96
12	SP	22–24	N.P. *	19.99	461.73
13	GP	24–26	N.P. *	20.43	534.60
14	GP	26–28	N.P. *	20.43	507.82
15	GP	28–30	N.P. *	21.10	611.48

*: Non-plastic granular soil.

Table 3. The mean and standard deviations of the site borehole data.

Layer no.	Depth (m)	PI_{mean} (%)	γ_{mean} (kN/m^3)	$(Vs)_{\text{mean}}$ (m/sec)	σ_{PI}	σ_γ	σ_{vs}
1	0–2	5.93	17.18	162.25	0.40	0.15	8.84
2	2–4	5.43	17.21	173.55	1.05	0.22	4.14
3	4–6	6.12	17.74	205.05	0.47	0.29	13.34
4	6–8	4.95	18.11	246.79	0.71	0.38	19.55
5	8–10	6.43	18.45	288.00	1.07	0.36	18.93
6	10–12	4.5	18.44	273.46	0.82	0.49	22.41
7	12–14	3.29	18.50	265.89	0.54	0.47	5.25
8	14–16	3.77	18.84	314.40	0.83	0.33	12.52
9	16–18	3.84	19.40	379.43	1.05	0.04	15.79
10	18–20	4.36	19.52	380.93	1.07	0.27	29.40
11	20–22	3.18	20.07	461.78	1.03	0.29	9.64
12	22–24	0	20.15	479.18	0.00	0.22	24.67
13	24–26	0	20.53	526.71	0.00	0.14	11.16
14	26–28	0	20.54	517.97	0.00	0.16	14.35
15	28–30	0	20.97	590.11	0.00	0.18	30.22

5. Verification of the proposed method

To determine the accuracy of the proposed method in obtaining the response of soil layers, an example problem was simulated using both MATLAB coded program and DEEPSOIL software. DEEPSOIL is a 1-D site response analysis program with the capability of performing both nonlinear and equivalent linear analyses. In the present study, the nonlinear time-domain analysis option of DEEPSOIL is employed to validate the proposed method outputs. In order to ensure the mobilization of nonlinear soil behavior, it is necessary to consider a relatively strong earthquake. To this end, in this study, the 1999 Bala-Deh earthquake ($M_w = 6.1$) with the PGA of 0.43 g that occurred on May 6th at about 70 km away from Shiraz city was considered as the input motion at the bottom of the soil profile. Figures 2 and 3 show the recorded input motion accelerogram and the shear wave velocity profile, respectively. The mean values for soil parameters are given in Table 3, utilized in this analysis. Figures 4 and 5 show the estimated normalized modulus reduction and damping ratio curves for the site profile through the model proposed by Darendeli [48].

The predicted absolute acceleration time histories and a close-up view from 6 to 11 s time window as well as the S_a of the surface layer for the proposed method and the DEEPSOIL are compared in Figures 6 to 8, respectively. These graphs represent good agreement between the predicted responses of the proposed method and those of the DEEPSOIL.

Figure 9 demonstrates the predicted stress-strain loops through the proposed method and DEEPSOIL computer program. This figure indicates that the proposed method provides a good match with DEEPSOIL, and the slight mismatch between the results

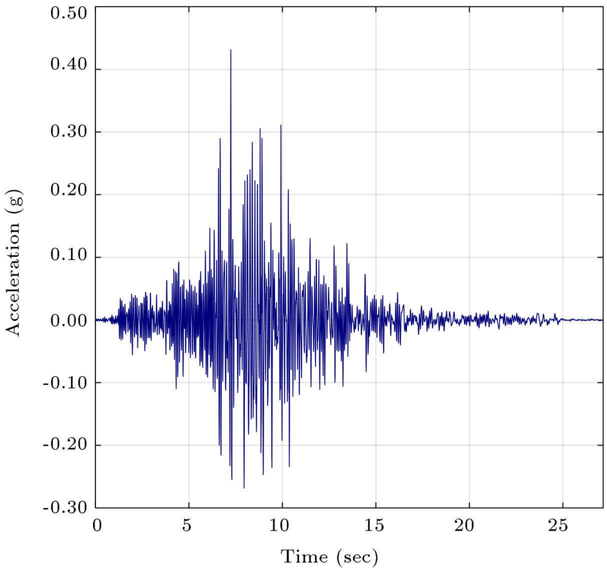

Figure 2. The 1999 Bala-Deh earthquake acceleration time history.

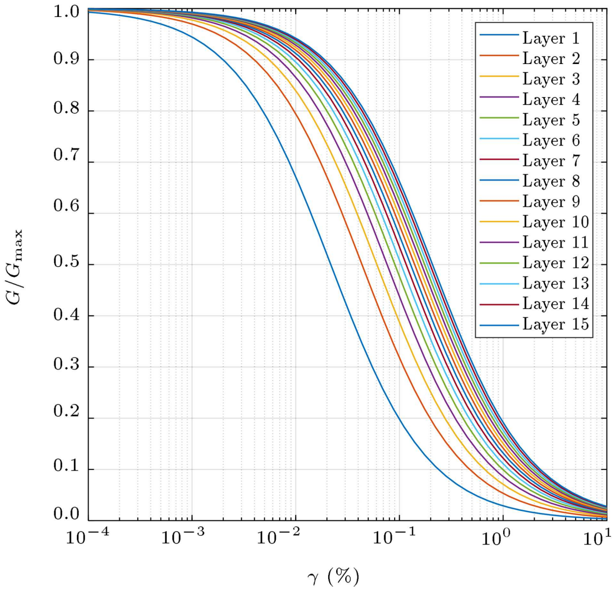

Figure 4. Predicted normalized modulus reduction curves for the soil layers.

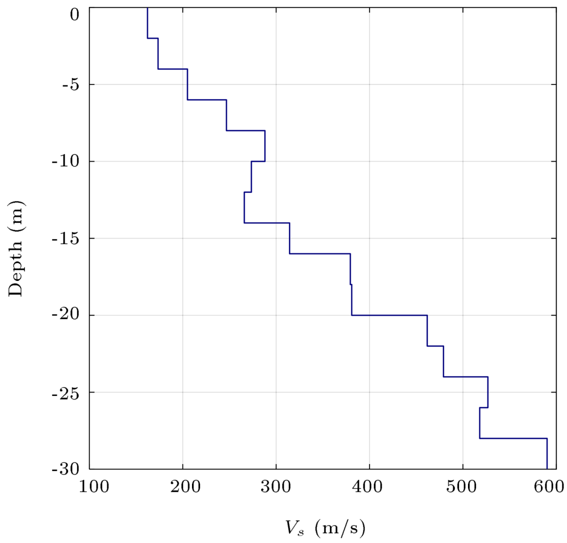

Figure 3. Variation in the mean value of V_s in the soil profile.

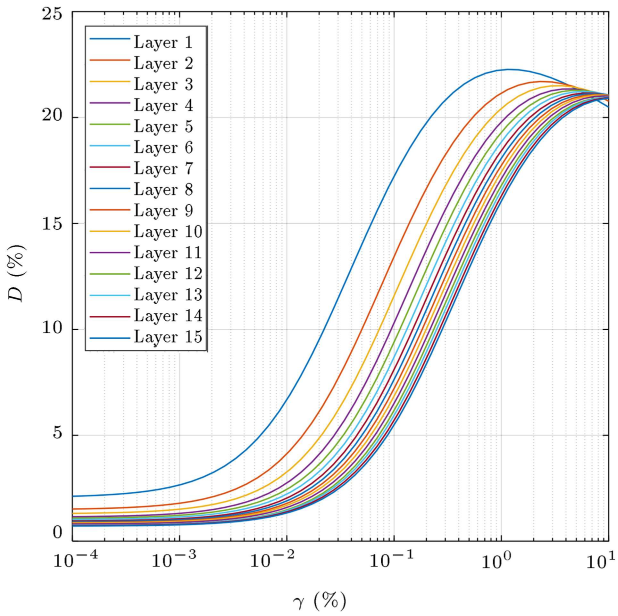

Figure 5. Predicted damping ratio curves for the soil layers.

from differences between the employed soil models in these two programs, i.e., the Darendeli [48] model in MATLAB code and the modified model of Konder and Zelasko [12] in the DEEPSOIL.

Table 4 shows the comparative results of the proposed method and those of the DEEPSIOL program including the peak of S_a, maximum and minimum values of acceleration, and normalized stress (τ/σ'_0). This table also highlights the acceptable agreement between the results obtained from these two methods.

Table 4. Comparison of the proposed method and DEEPSOIL results.

Method	Acceleration (g)		τ/σ'_0		Peak of S_a
	Min.	Max.	Min.	Max.	
Proposed method	–0.58	0.62	–1.61	1.32	3.89
DEEPSOIL	–0.57	0.62	–1.45	1.31	3.61

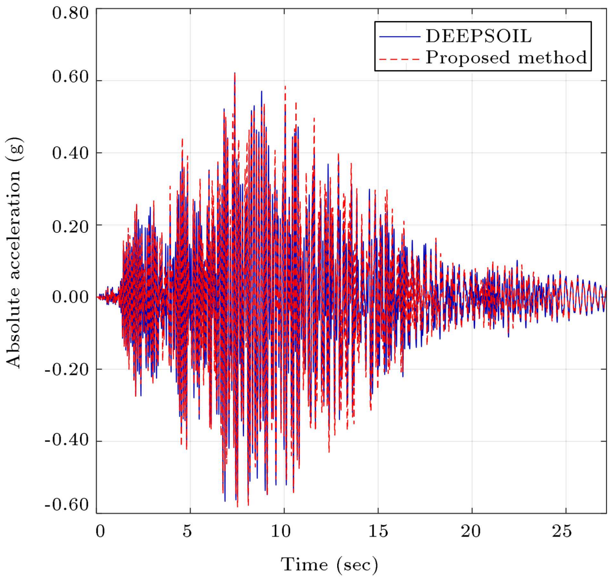

Figure 6. Absolute acceleration time histories on the ground surface through the proposed method and DEEPSOIL.

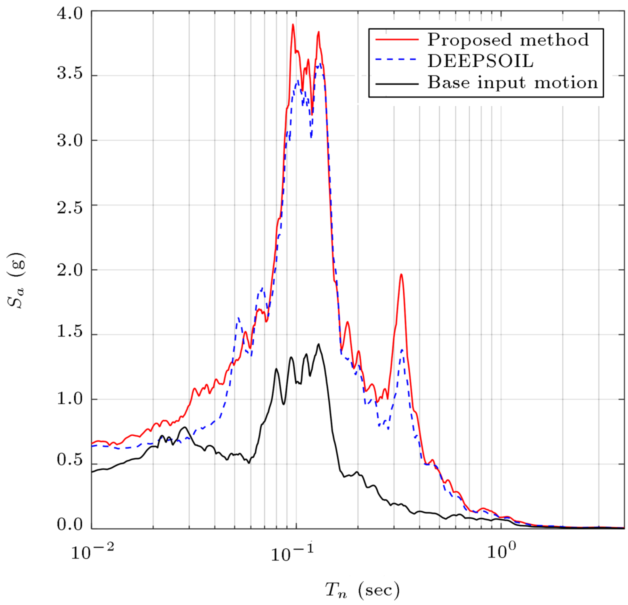

Figure 8. Ground surface S_a by the proposed method and DEEPSOIL.

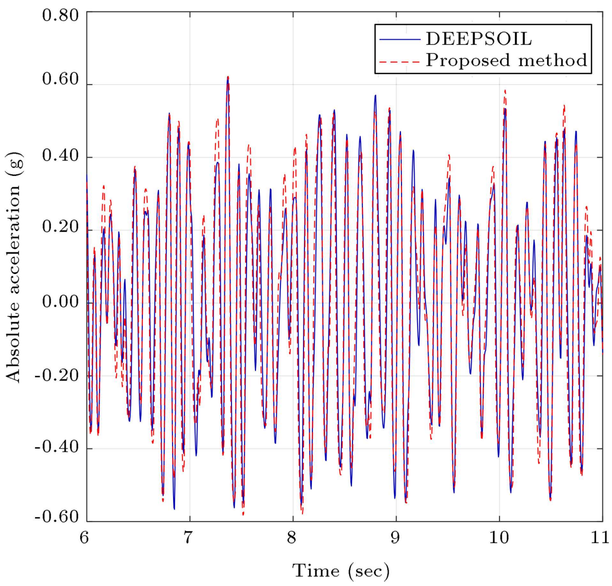

Figure 7. A close-up view of the absolute acceleration time histories for a time window from 6 s to 11 s.

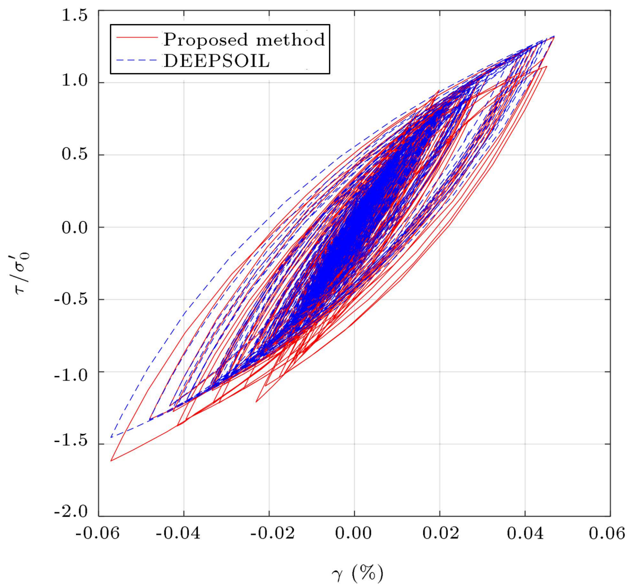

Figure 9. Stress-strain loops predicted by the proposed method and DEEPSOIL.

6. The stochastic ground response analysis procedure

Generally, subsurface soil properties can be obtained through field investigation at discrete survey points or logs. Therefore, In Situ tests can only provide good information about the sub-soil properties at the test locations, implying that the soil properties in engineering simulations are characterized by uncertain nature. This uncertain aspect of the soil parameters highlights the significance of considering the probabilistic nature of soil properties in seismic analyses. In other words, to capture the uncertain behavior of soil profiles, probabilistic methods must be employed so that the uncertainties of soil parameters can be taken into account [57,58]. In this respect, the proposed deterministic computer program was extended to ensure the generation of stochastic input parameters.

For a long time, probabilistic methods such as First Order Reliability Method (FORM), Second Order Reliability Method (SORM), MCS method, and PEM have been used to consider the uncertainties associated with stochastic analysis. Among these methods, only MCS and PEM can be applied to nonlinear problems. To be specific, MCS allows a full mapping of the uncertainty of the input into the corresponding uncertainty of outputs and it can provide a probability distribution for outputs. Therefore, in the present study, MCS was used as a basic method for investigating the effects of the uncertainty of the dynamic soil parameters on the corresponding uncertainty of the ground responses.

In order to carry out the stochastic ground response analysis at the first step, random variables and their characteristics including the mean and standard deviations as well as the statistical distribution type should be determined. According to these statistical properties, the selected stochastic soil parameters are generated to the required numbers. Then, the ground response analysis is carried out for each simulation. Finally, the statistical distributions of outputs including Peak Ground Displacement (PGD), PGA, fundamental period of the profile, acceleration response spectra, and amplification factor of the site are calculated.

As mentioned earlier, the main objective of this study was to investigate the soil heterogeneity effects on the nonlinear seismic responses through the stochastic analysis. In this study, the Darendeli model was employed to examine the nonlinear shear modulus and damping variation of soils under seismic loads. Based on this model, the shear modulus and damping are related to three independent soil parameters including PI, unit weight, and shear wave velocity. As a result, in this study, these three soil parameters (V_s, γ, and PI) were considered as independent stochastic parameters.

In this regard, the calculated mean and standard deviations (Table 3) were utilized to generate the stochastic parameters by the log-normal distribution. The loading and geometric parameters, such as input motion and thickness of layers, were regarded as deterministic parameters. Then, the stochastic computer program was employed to conduct stochastic ground response analysis.

The number of the required MCS depends on both the desired level of confidence in the solution and the number of variables [59]. Since the required trial numbers based on the proposed statistical equations in the literature had a high computational cost, a modest number of simulations were typically performed in the time-consuming dynamic simulations, especially for the nonlinear time-domain analyses [60,61].

In this study, a statistical analysis was carried out to determine the required number of stochastic simulations. The mean of PGA was considered as the target parameter for checking the stability of the stochastic analysis. The employed criterion for this stability analysis was based on a number of simulations where the average PGA variation for at least 50 consecutive numbers was less than 0.01 g. According to Figure 10, in case the number of simulations approaches 1,413 (red line), the mean PGA of the surface layer becomes stable; hence, 1,500 simulations

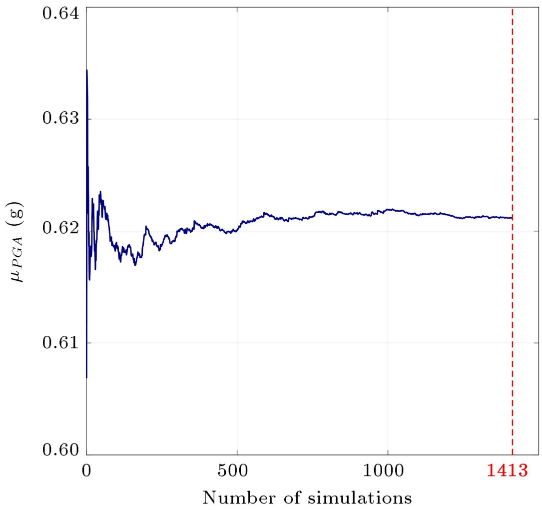

Figure 10. Number of simulations versus mean of Peak Ground Acceleration (PGA) of the surface layer.

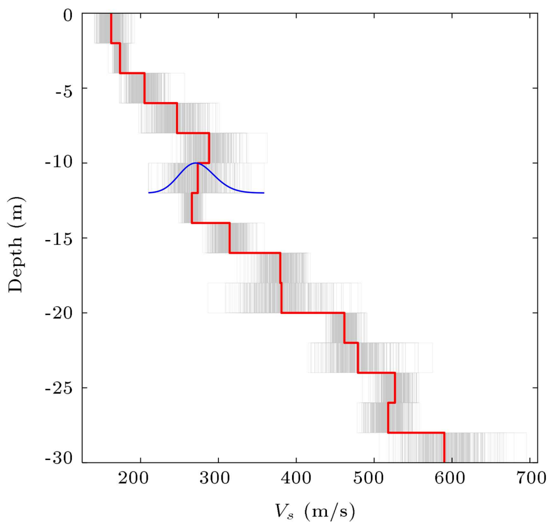

Figure 11. Graphical variability of V_s in the site profile.

are sufficient to conduct the stochastic analysis of the studied site. In this study, all of the following stochastic analyses are prepared using 1,500 simulations. The variability of V_s, γ, and PI of the soil layers is shown in Figures 11 to 13, respectively. In these figures, in other to demonstrate the log-normal distribution of the generated profiles, the Probability Density Function (PDF) of the corresponding parameters in an arbitrary layer (i.e., the 6th) is plotted. Furthermore, in these figures, red and gray lines correspond to the mean value of soil parameters and 1500 realizations of the random profile, respectively.

7. Stochastic results of the case study site

The stochastic results of seismic ground response analysis can be described in terms of PGD, PGA, surface

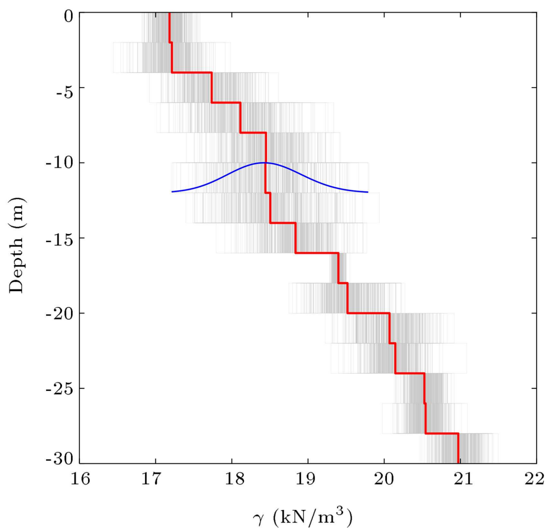

Figure 12. Graphical variability of γ in the site profile.

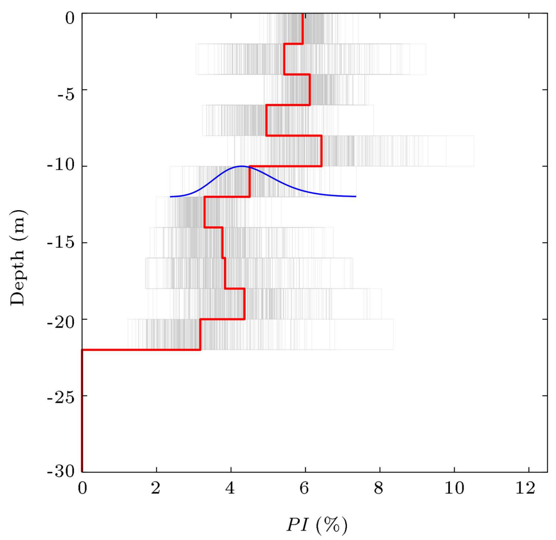

Figure 13. Graphical variability of *PI* in the site profile.

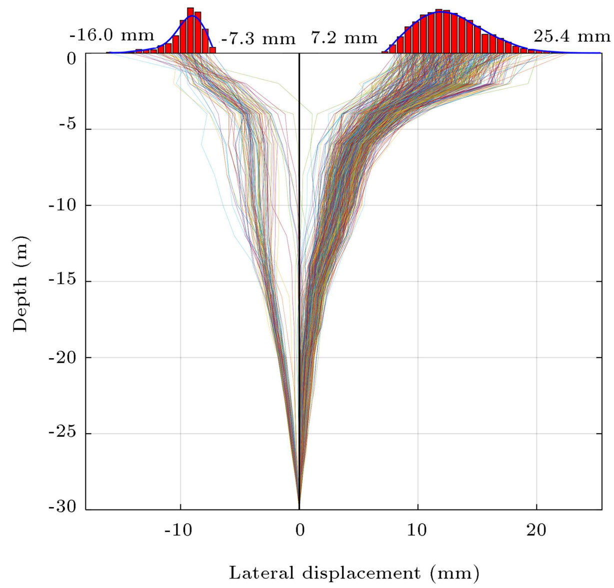

Figure 14. Variation of ground surface Peak Ground Displacement (PGD) and corresponding lateral displacement of the soil sub-layers.

Table 5. Stochastic parameters of the Peak Ground Displacement (PGD).

PGD	Min. (mm)	Max. (mm)	Mean (mm)	Std. (mm)
Right side	7.2	25.4	12.9	2.7
Left side	–7.3	–16.0	–9.6	1.3

acceleration response spectra, fundamental periods, and amplification factor. These stochastic results will be presented in the following subsections.

7.1. Peak Ground Displacement (PGD)

Earthquakes can cause some residual and permanent deformations, generating excess pressure and stress concentration in subsurface structures. PGD is a significant parameter in earthquake-resistant design, especially for buried structures such as pipelines and deep foundations. Figure 14 illustrates the PGD variations across the soil profile due to the selected stochastic soil parameters. A comparison of displacements on the bedrock and ground surface demonstrated the high potential of the profile of the site in magnifying the earthquake waves. As shown in this figure, PGD can occur on both left and right sides of the zero lateral displacement axis. For both sides, these graphs are plotted by obtaining the PGD of the ground surface with its time and then, capturing the displacements of the sub-layers at the corresponding time. Furthermore, the PGDs of both sides have a log-normal distribution with the stochastic parameters summarized in Table 5.

7.2. Peak Ground Acceleration (PGA)

PGA is a measure of the maximum amplitude of motion and is defined as the largest absolute value of acceleration time history. It is the most commonly used ground motion parameter in engineering applications, especially in building codes and earthquake-resistant design. Figure 15 shows the bar chart and the predicted PDF of PGA for the site with the mean and standard deviations of 0.62 g and 0.033 g, respectively.

In this figure, the bar chart of the PDF of the PGA is fitted to gain a smooth PDF of the PGA. As shown, two different types of distributions, i.e., log-normal and Burr [62], were used in this procedure. According to the bell-shaped PDF of PGA, both normal and lognormal distribution could be suitable for fitting; however, due to the observed skewness in the PDFs, they could provide better compliance with the derived values. The Burr [62] distribution is a flexible distribution family with the ability to be fitted to a wide range of distributions including gamma, lognormal, log-logistic, bell-shaped, and J-shaped beta distribution. In terms of the fitted lognormal and Burr [62], the distributions are shown in Figure 15 by blue and red color lines, respectively. It can be concluded that based on this figure, the Burr distribution can provide a better match with the predicted PGA data (correlation coefficient of 0.99 versus 0.93 for lognormal distribution).

7.3. The fundamental period of stochastic profiles

The fundamental period (the first mode of the natural period) of the local soil profile is an important parameter for the evaluation of the seismic site effects and design of buildings and infrastructures. The modal analysis and approximate methods are the most popular techniques for estimating the fundamental period which are based on calculating the lowest eigenvalue of the system and weighted average of the shear wave velocities of the soil layers, respectively.

The fundamental period is strongly influenced by the uncertainty associated with the site soil properties [63]. In this study, to evaluate the effects of the heterogeneity of the site soil profile, stochastic analysis of the fundamental period was carried out. To this end, the modal analysis as well as the approximate formulation, Eq. (7), were employed to calculate the fundamental period of each profile of the generated stochastic shear wave velocity [64]:

$$T = \frac{4H}{\overline{V}_s}, \tag{7}$$

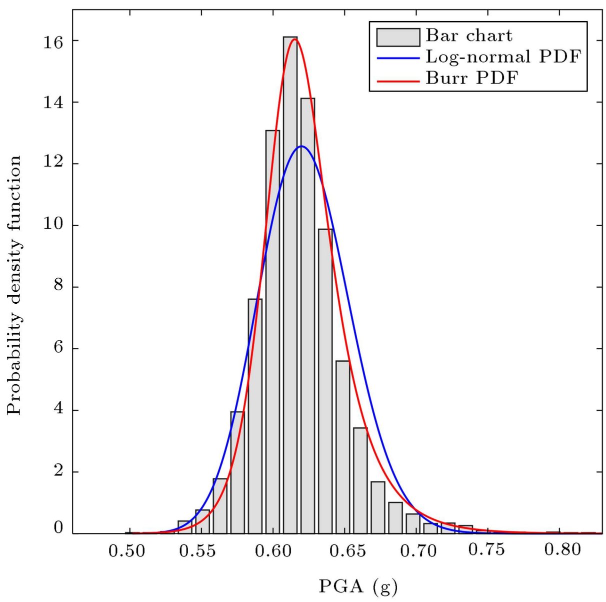

Figure 15. The predicted probability density function of Peak Ground Acceleration (PGA) on the site surface.

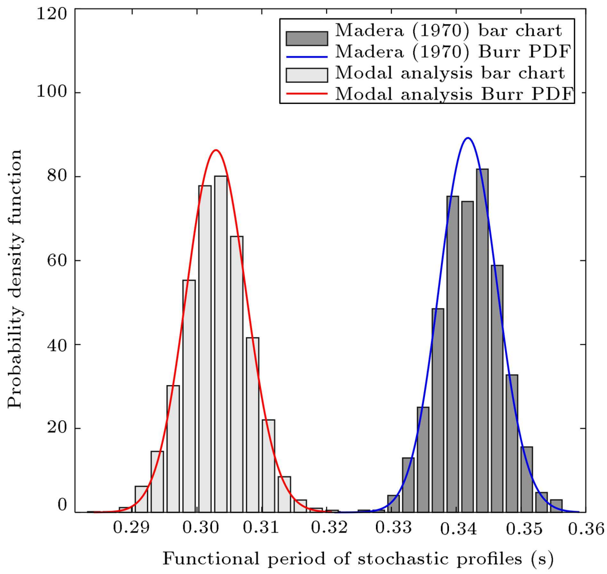

Figure 16. Variation of the fundamental period at the case study site.

Table 6. Stochastic parameters of the fundamental period.

Method	Min. (s)	Max. (s)	Mean (s)	Std. (s)
Modal analysis	0.28	0.32	0.3	0.0046
Madera (1970)	0.32	0.36	0.34	0.0045

where $\overline{V}_s$ is the average shear wave velocity of the profile obtained from Eq. (6).

Figure 16 depicts the variation of the fundamental period in the case study site as well as the bar chart and fitted PDF of the fundamental period. As observed in the considered site, the modal analysis could predict the fundamental period varying from 0.28 to 0.32 s. On the contrary, the prediction made by the empirical formula, Eq. (7), showed the fundamental period variations in the range of 0.32 to 0.36 s. The stochastic parameters of the fundamental period through the mentioned methods are listed in Table 6.

7.4. Acceleration response spectrum

The acceleration response spectrum (S_a) is a useful tool for designing structures under earthquake excitations. In this study, stochastic S_a was estimated through the proposed method. Figures 17 and 18 show the variations of nonlinear S_a obtained from the surface response acceleration for 5% structural damping. Figure 18 represents the calculated mean of S_a on the ground surface (red line), mean plus or minus one (gray), and two (green) standard deviations. Furthermore, S_a of the input accelerograms at the bedrock is presented by the dashed line.

The rule of thumb for estimating the natural

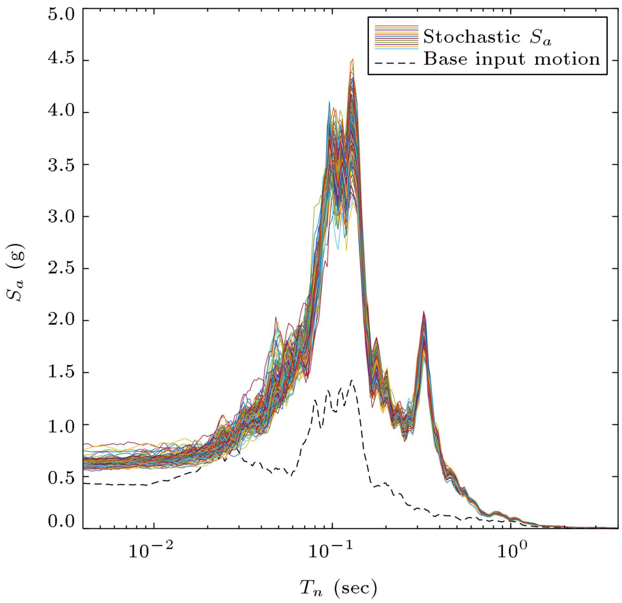

Figure 17. The stochastic S_a at the ground surface through the proposed method.

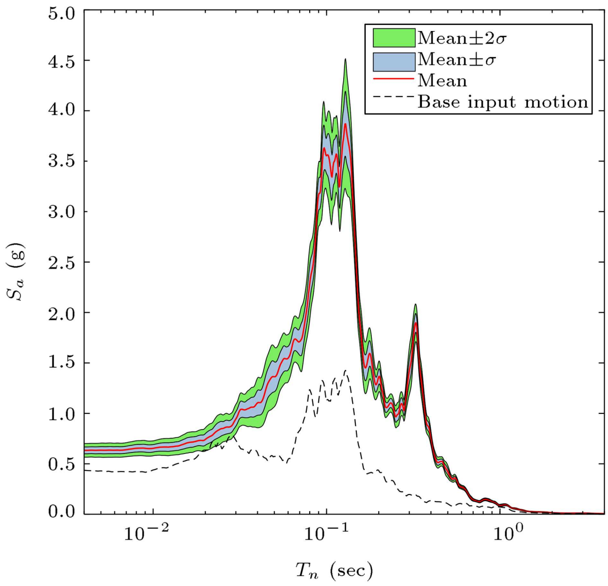

Figure 18. The mean S_a plus or minus one and two standard deviations on the ground surface.

period of buildings whose lateral-force resisting system provided by moment-resisting frames equals the number of stories divided into 10. Natural periods vary from about 0.1 to 0.5 s for one- to four-story buildings, respectively. Other factors such as the structural system, construction materials, and geometric proportions of the building also affect the period; however, height is the most important factor [65,66]. Based on Figures 17 and 18, it can be concluded that in evaluating the site through the considered earthquake, buildings with short and medium heights were more affected than high-rise buildings. A comparison between the input motion S_a and ground surface S_a revealed that both input motion S_a and ground surface S_a followed a similar trend with a peak at about 0.13 s, except that the ground surface S_a had another peak value at 0.33 s. Therefore, it can be concluded that the soil layers of the studied site magnified the input earthquake waves in this period. In other words, this period represents the fundamental period of the site.

7.5. Amplification factor

The amplification factor is specified as the ratio of displacement response spectra (with 5% damping) on the ground surface to the response spectra of the input motion on the bedrock. Figure 19 shows the variations of the amplification factor on the ground surface due to the uncertainties of the soil parameters. As observed, the peak of the amplification factor for the case study site occurred at 0.33 s, indicating that, unlike the deterministic method that could only provide one amplification factor value per each period, in the stochastic method, there is a range of amplification factors in each period such that in the fundamental period, the amplification factor for the considered site fluctuated in the range of 7.6 to 11.6. Figure 20 represents the predicted mean amplification factor (red line), mean plus or minus one (gray), and two (green) standard deviations on the ground surface.

The peak of the amplification factor diagram represents the fundamental period of the site. Considering these stochastic results reveals that the fundamental period of the case study site is located in the range of 0.32 to 0.34 s. Figure 21 shows the amplification factor variations of the site in the middle of this range (0.33 s) as well as the bar chart and fitted PDF of the amplification factor of this site. According to this

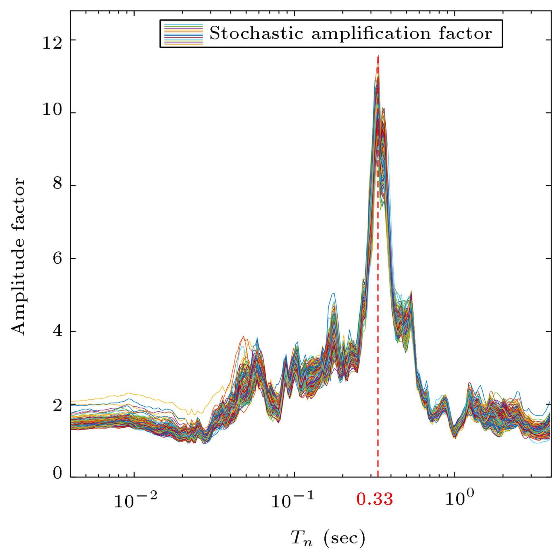

Figure 19. Variation of the amplification factor on the ground surface.

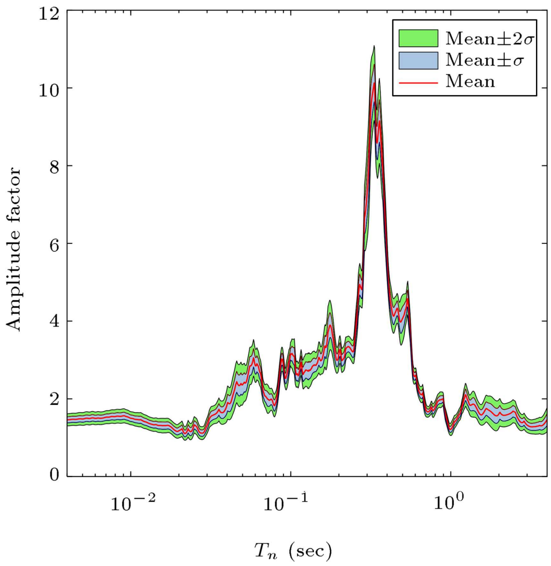

Figure 20. The mean amplification factor plus or minus one and two standard deviations on the ground surface.

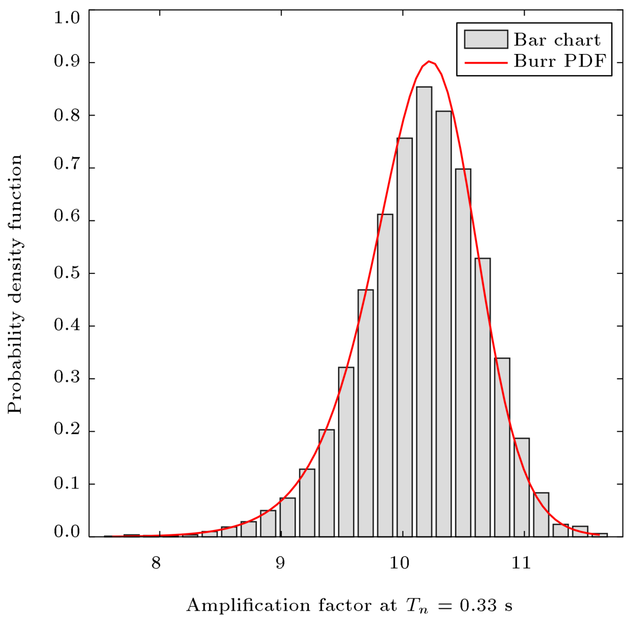

Figure 21. Variation of the amplification factor at the fundamental period of the case study site.

figure, the peak of the amplification factor had a right-skewed bell-shaped distribution between 7.6 and 11.6 with a mean of 10.13 and a standard deviation of 0.48.

8. Results and discussion

In the previous section, the PDFs and stochastic parameters of the seismic responses of the case study site were obtained. In this section, some of the presented stochastic results will be discussed and compared from a different point of view in two subsections.

Table 7. Stochastic parameters of the fundamental period.

Method	Min. (s)	Max. (s)	Mean (s)	Std. (s)
Modal analysis	0.28	0.32	0.30	0.0046
Madera (1970)	0.32	0.36	0.34	0.0045
Nonlinear method	0.32	0.34	0.33	0.0033

8.1. Performance of fundamental period estimation methods

In this section, the results of different methods for obtaining the fundamental periods are discussed. In this respect, Figures 16 and 19 illustrate a comparison of the predictions among different methods. Since the nonlinear ground response analysis considers the real behavior of soil layers at large strains, implementing the nonlinear amplification factors can lead to the most accurate estimation of the fundamental period.

Figure 16 compares two different methods for determining the stochastic fundamental period of the case study site. On the contrary, Figure 19 represents the variation of the amplification factors obtained by the nonlinear method on the ground surface, and the peak of these diagrams can also be considered as the fundamental period of the site.

Table 7 demonstrates the stochastic parameters of the fundamental period through the mentioned methods. According to the table, it can be concluded that in comparison to modal analysis, the approximate formulation [64] predicts closer minimum, maximum, and mean values in terms of the fundamental period obtained through the amplification factors of the site in the nonlinear method. This difference is caused by the linear assumptions of the modal analysis method. The predicted fundamental period through the nonlinear ground response analysis occurred in slightly longer periods, which was in agreement with the finding of the previous studies. This table also indicates that in the studied site, the results obtained from the nonlinear method exhibited lower dispersion than the other two methods. However, the predicted variation ranges of all these methods were close to each other. Therefore, in order to prevent the occurrence of resonance phenomenon, the natural period of the structures must be far away from this range of periods in this case study site.

8.2. Effects of soil heterogeneity on the seismic responses

In this section, a stochastic comparison is made between the seismic responses. To this end, the Coefficient Of Variation (COV) was employed as a measure of relative dispersion of events. The COV of an event is defined as the ratio of standard deviation to mean,

Table 8. Comparison of the stochastic results of the case study.

Stochastic response	Mean	Std.	Maximum COV (%)
Right side PGD	12.9 mm	2.7 mm	20.93
Left side PGD	9.6 mm	1.3 mm	13.54
PGA	0.62 g	0.033 g	5.32
Fundamental period (Modal analysis)	0.30 s	0.0046 s	1.53
Fundamental period (Modera 1970)	0.34 s	0.0045 s	1.32
Acceleration response spectra, S_a	1.36 g	0.16 g	11.76
Amplification factor, A_f	1.57	0.20	12.66

and it is commonly used for comparing different types of quantitative likelihood or probability distribution.

Table 8 represents the maximum COV of different stochastic results which facilitates their comparison. In other words, by considering the uncertainty of the soil in this site, these uncertainty values may be reflected in the dynamic responses. As indicated in this table, in this case study site, the heterogeneity of the soil parameters had a significant effect on the surface PGD variation. Among the other stochastic responses, the fundamental period was less affected by the uncertainty of soil parameters.

9. Conclusions

Ground response analysis is an attempt at determining the seismic site effects by solving the equations of motion. The nonlinear time-domain method using a stepwise integration procedure provides a more accurate framework for simulating the real behavior of soil deposits. However, the seismic ground response was mainly dependent on the dynamic behavior of the soil, which was always associated with a significant degree of heterogeneity.

In this paper, first, a MATLAB program was developed to conduct the nonlinear time-domain ground response analysis, which was verified by a commonly used ground response analysis program, DEEPSOIL. The results showed acceptable agreement between the results of the proposed method and DEEPSOIL outputs.

In the following, to study the effect of the uncertainties of the dynamic soil property on seismic responses, the deterministic nonlinear response analysis code was extended to perform stochastic soil parameters and MCS-based iterative calculations.

Then, a case study site in Shiraz, Iran with two boreholes was taken into account to investigate the effects of soil heterogeneity on the nonlinear seismic responses through the stochastic analysis. To this end, the mean and standard deviations of the selected stochastic soil variables (V_s, γ, and PI) were calculated using borehole data at every depth. The statistical analysis was carried out to determine the required number of stochastic simulations, the results of which revealed that in case the number of simulations approached 1,500, the mean Peak Ground Acceleration (PGA) of the surface layer was stable. The obtained uncertainties of the soil parameters were employed to generate random profiles through Monte Carlo Simulations (MCS). After conducting stochastic nonlinear ground response analysis, the stochastic seismic responses, amplification factors, and fundamental period of the site profile were obtained. Some conclusions are summarized below.

The comparison between the displacement on bedrock and that on the ground surface indicated the high potential of the site profile in magnifying the earthquake waves. Moreover, the Peak Ground Displacements (PGDs) of both sides had a log-normal distribution.

Based on the stochastic PGA assessment, it can be concluded that the surface PGA distribution had a mean and standard deviation of 0.62 g and 0.033 g, respectively. Based on the distributions fitted with the predicted surface PGAs, it can be concluded that the Burr distribution could provide a better match than the lognormal distribution.

As a result of stochastic analysis of acceleration response spectra, it was illustrated that in the assessed site under the considered earthquake, buildings with short and medium heights are more affected while high-rise buildings are not influenced significantly. The standard deviations of S_a for the range of periods lower than 0.4 s are high. It can be seen that the dispersion of S_a decreased with increasing period.

In another part of the paper, the effect of the soil heterogeneity on the amplification factor was investigated. It was observed that the fundamental period of this site, which is corresponding to the peak of the amplification factors, occurred in the range of periods between 0.32 and 0.34 s. It was concluded that in these periods, the site had a significant potential for amplifying the earthquake-induced ground motions.

According to the assessment of the stochastic fundamental periods of the case study site, it can be concluded that in comparison to modal analysis, the approximate formulation provides closer predictions regarding to the fundamental period obtained through the nonlinear site amplification factors. In contrast, due to its linear assumptions, the modal analysis method predicts lower values for the site fundamental period. However, the predictions by all these methods are close to each other. Therefore, the natural period of the structures at this case study site must be far away from the range of 0.28 s to 0.36 s to prevent the occurrence of resonance.

Comparison of the maximum Coefficient Of Variations (COVs) of different stochastic results depicted that at this case study site, the heterogeneity of the soil parameters had a significant effect on the variation of the surface PGD. Among the other stochastic responses, the fundamental period experienced the least effect from the uncertainty associated with the soil parameters.

Future directions in research should investigate the capability of employing interpolation method for a reasonable estimation of boreholes data. Although progress has been made in understanding the effect of soil parameters uncertainty in the nonlinear ground responses, a more comprehensive study should investigate the effect of input motion uncertainty on nonlinear seismic responses of the case study site.

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