The effect of buckling and post-buckling behavior of laminated composite plates with rotationally restrained and Pasternak foundation on stacking sequence optimization

Sina Farahani\textsuperscript{a}, Mojtaba Fathi\textsuperscript{b*}, Ebrahim Nazarimofrad\textsuperscript{c}

\textsuperscript{a}Department of Civil Engineering, Razi University, Kermanshah, Iran, Email: farahani.sina@razi.ac.ir, ORCID: 0000-0001-9970-091X

\textsuperscript{b}Assistant Professor, Department of Civil Engineering, Razi University, Kermanshah, Iran, Corresponding author: Email: fathim@razi.ac.ir, ORCID: 0000-0003-3554-6831

\textsuperscript{c}Researcher, Department of Civil Engineering, Bu Ali Sina University, Hamedan, Iran, Email: enazarimofrad@yahoo.com, ORCID: 0000-0001-9961-0033

Abstract

This paper presents a stacking sequence optimization for maximizing the buckling load of rotationally restrained laminated composite rectangular plates with different boundary conditions resting on an elastic Pasternak foundation subjected to uniaxial and biaxial in-plane static loads. The Mindlin Plate Theory (MPT), which considers the first-order shear deformation effect, is used to extract the characteristic equations of the plates under in-plane loading, including plate-foundation interaction. The buckling problem of the laminated plates is analyzed by the Rayleigh–Ritz method. The aim of optimization is to maximize the buckling load and post-buckling load capacity by using the Genetic Algorithm (GA) method, and the design variable is the ply orientation. The results showed that the optimal orientation, $\theta$, of the laminated square plate under biaxial in-plane loading with various conditions is $45^\circ$ approximately. The existence of a foundation, clamped boundary conditions, and high aspect ratio lead to increase the optimal orientation.

**Keywords:** Stacking sequence optimization, Buckling and post-buckling behavior, Rotationally restrained laminated composite plates, Pasternak foundation, Mindlin plate theory.

1. Introduction

A large number of Polymer matrix laminated composite structures are widely utilized in structural, aeronautics, aerospace, marine engineering, resource tank and automotive industries for their low weight, high strength, high stiffness, and corrosion properties. For instance, the use of composite laminates in commercial aircraft structures has increased considerably over the past two decades, about 50\% of the Airbus A350 XWB structure is made of composite material [1].

Due to this revolutionary tendency in the industry, many researchers have worked to develop method for the composite laminate structures [2]. Composite structures are generally modeled as plate thus identifying their behaviors is important to understand the response of the structures under extreme loadings [3-5]. The variation of plies, thickness, orientation and stacking sequence
makes easy and possible tailoring to achieve the desired mechanical properties, such as the in-plane, flexural and buckling behaviors of composite laminates. Therefore, optimization of composite laminates has recently received more attention. [6-9].

There are several methods for obtaining the critical buckling load in isotropic and orthotropic plates. Shokrani et al [10] utilized two-variable refined plate theory to investigate buckling behavior of double-orthotropic nanoplates bonded in elastic media under biaxial and uniaxial loading. Shahraki et al [11] utilized Frobenius solution method to analyze the buckling in orthotropic rectangular plate under biaxial in-plane loading with non-uniform distribution. Lei et al. [12] studied buckling analysis of functionally graded carbon nanotube-reinforced composite plates by employing the first-order shear deformation theory to consider the impacts of rotary inertia and transverse shear deformation. Yu et al. [13] presented a new efficient and precise approach to simulate buckling problems of laminated composite plates with cutouts by combining the isogeometric analysis and the level set. Kheirikhah and Babaghasabha [14] explored buckling behavior of soft core corrugated sandwich plates under uniaxial load. The face sheets were considered as composite laminates and three-dimensional finite element method was utilized to analyze.

To describe the plate-foundation interactions, several types of elastic foundation models have been proposed that one of the simplest models is Winkler model [15]. The model consists of separated independent linear springs that are close to each other. This kind of foundation is a one-parameter model in which only the springs in the loaded area are affected. Farahani and Mohebkah [16] utilized Winkler’s model for developing their models to investigate the sensitivity of DDB designed frames to the effect of foundation flexibility. Pasternak [17] added a shear layer to Winkler’s model to improve the model. The layer establishes a shear interaction between the independent springs. The model is broadly utilized to illustrate the mechanical behavior of structure–foundation interactions. Nazarimofrad et al. [18,19] presented the buckling analysis of an orthotropic rectangular Mindlin plate resting on an elastic Pasternak foundation. They utilized Rayleigh–Ritz method to solve the governing equations based on the Mindlin–Reissner plate theory.

Gue et al. [20] proposed a deep collocation method based on a feedforward deep neural network (DNN) to solve the formed partial differential equations of buckling analysis plates. Samaniego et al. [21] developed an energy approach based on approximation DNN based method for the solution of the formed partial differential equations. The results of computational mechanics illustrated the capabilities of the proposed method. However due to variety coupled parameters of composite plates, the previous work by Shokrollahi and Shafagh [2] showed that using the Rayleigh–Ritz based method could be reliable and accurate method to develop analysis method of composite laminates. Akhavan [22] presented exact solutions for the buckling analysis of rectangular isotropic plates located on Pasternak foundation with various types of boundary conditions subjected to uniformly and linearly in-plane loadings. Golmakani and Rezatalab [23] investigated the buckling analysis of orthotropic graphene sheets embedded within elastic medium by Pasternak model under non-uniform biaxial loading. The first-order shear deformation theory and differential quadrature method is used to derive the nano-plate equilibrium equations and solve the governing equations for various boundary conditions, respectively.

In the optimization for maximum buckling load of laminated composite plates, the aim is often achieved by changing the stacking sequence, ply orientation and ply thickness as design variables. Many researches have been done on the problem of optimal design of composite laminates. Topal and Uzman [24] provided optimization of laminated composite plates with simple supports subject
to in-plane loads. The objective function was maximizing the buckling load capacity of composite plates by changing the design variable (ply orientation). The first-order shear deformation theory was used for the finite element analysis. Genetic algorithm (GA) is one of the first effective approaches well succeeded in the optimization of composite laminates. Manh and Lee [25] obtained maximum bending, buckling and post-buckling capacity of imperfect laminated composite plates by GA and NURBS-based finite element iso-geometric analysis. The fiber orientation was the design variable. Vu-Bac et al. [26] proposed a new method to develop material and geometric nonlinearities for inverse analysis. The NURBS-based finite element analysis was also utilized to capture the stable shape changes. Vosoughi et al. [27] maximized buckling load of thick laminated composite plate by the optimization of the stacking sequence of plate. They employed the finite element, GA and particle swarm optimization methods for it. Ehsani and Rezaeezazhand [28] employed GA to optimize the stacking sequence and pattern composition of the laminated grid plate. The objective function was the buckling load while the design variables were the pattern and orientation of the grid layer. Ritz method which is a classical laminated plate theory was considered to determine the buckling loads.

The success achieved with GA has led to several studies on the application of other approaches to the optimization of laminated composites. Jing [29], by using the permutation search algorithm and considering the ply orientation as a design variable, carried out optimization for obtaining the maximum of buckling load. de Almeida [30] maximized the buckling load of a symmetric laminated plate in a stacking sequence optimization by using the harmony search algorithm. They indicated that although harmony search algorithm is less reliable than a special version of GA, it is more effective than other metaheuristic methods. Kaveh et al. [31] presented the optimization of stacking sequence of laminated composites using biogeography-based optimization algorithm to maximize the buckling load of a symmetric composite laminated plates. de Almeida [32] proposed a new optimization method of composite structure based on harmony search algorithm. The results of this study showed that composite structures could be better optimized by the harmony search algorithm than other prior approaches could. To maximize load capacity and improve dynamic performance of laminated composite plates, Serhat and Basdogan [33] developed a multi-objective optimization method. Atri and Shojaee [34] developed method for buckling response of laminated composite plates using coupling of Truncated Hierarchical B-splines method. The results of numerical cases considering different fiber orientations of laminated plates and various geometrical shapes showed high accuracy for all test models. Recently, Nguyen et al. [35] proposed a new optimization method based on gradient-based interior point algorithm for increasing the biaxial bulking load capacity of laminated composite plates. Jing et al. [36] presented an enhanced permutation search (EPS) algorithm for simply supported orthotropic plates to maximize the buckling load. The obtained results of EPS algorithm showed that the computational cost decreases dramatically compared to traditional algorithms. In spite of several researches on optimization of laminated composite plates for obtaining the maximum of buckling load, to our knowledge, there is no research conducted on optimization of laminated composite plates considering the routinely restrained and elastic Pasternak foundation effects.

The main objective of this study is to use GA for the buckling optimization of a rotationally restrained composite plate resting on an elastic Pasternak foundation. To solve the problems in which the objective function is discontinuous, non-differentiable, stochastic, or highly nonlinear, GA can be utilized that is categorized as search heuristics. GA is a global search algorithm that uses a population of designs instead of single point in the design space; hence, the probability of finding the global optimum during the optimization process is high and local optimum points are
The optimization implementation is done using GA [38] and the Toolbox of MATLAB software [39]. The analysis procedure is based on the MPT considering the first-order shear deformation effects, including plate-foundation interaction. The buckling problem of laminated plate is analyzed by Rayleigh–Ritz method. The aim of the optimization is to maximize the buckling load capacity, while Ply orientation is taken as design variable.

2. Theoretical formulation
Consider a rectangular laminated composite plate with the constant thickness of \( h \) and the in-plane dimensions of \( a \) and \( b \) resting on an elastic Pasternak foundation, as illustrated in Figure 1. Two of boundary conditions are rotationally restrained and another two boundaries have simply supported conditions. By ignoring the axial in-plan deformations, the governing strain energy equation of the laminated composite plate resting on the Pasternak foundation can be written as below [40]:

\[
\Pi = \frac{1}{2} \int_0^a \int_0^b \left[ D_{11} \left( \frac{d\psi_x}{dx} \right)^2 + 2D_{12} \frac{d\psi_y}{dy} \frac{d\psi_x}{dx} + 2D_{16} \left( \frac{d\psi_x}{dy} + \frac{d\psi_y}{dx} \right) \right] dx dy
\]

\[
+ D_{22} \left( \frac{d\psi_y}{dy} \right)^2 + 2D_{26} \frac{d\psi_y}{dy} \left( \frac{d\psi_x}{dx} + \frac{d\psi_y}{dy} \right) + D_{66} \left( \frac{d\psi_x}{dy} + \frac{d\psi_y}{dx} \right)^2
\]

\[
+ KA_{44} \left( \frac{dw_0}{dy} + \psi_y \right)^2 + KA_{55} \left( \frac{dw_0}{dy} + \psi_y \right)^2
\]

\[
+ 2KA_{45} \left( \frac{dw_0}{dy} + \psi_y \right) \left( \frac{dw_0}{dy} + \psi_x \right) + k_w w_0^2 + K_s \left( \frac{dw_0}{dx} \right)^2 + \left( \frac{dw_0}{dy} \right)^2 \right] dx dy
\]

Where \( w_0 \), \( \psi_x \) and \( \psi_y \) are the transverse displacement, rotational displacement about \( x \) axe and rotational displacement about \( y \) axe, respectively. furthermore, \( k_w \) and \( k_s \) are lateral stiffness of resistant elements of foundation in vertical and shear directions, respectively. Indeed, these two parameters define the Pasternak foundation, as shown in Figure 1a. Parameter \( k \) is defined as shear correction coefficient which is considered as \( 5/6 \). Considering the classical plate theory for laminated composite structure, \( D \) is defined as the flexural stiffness matrix as follows:

\[
D = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}
\]

\[
D_{ij} = \sum_{k=1}^{N_k} \int_{z_k}^{z_{k+1}} \bar{Q}^{(k)}_{ij} z^2 dz \quad (i, j) = (1, 2, 6)
\]

\( A_{ij} \) is the extensional stiffness matrix as follows:

\[
A_{ij} = \sum_{k=1}^{N_k} \int_{z_k}^{z_{k+1}} \bar{Q}^{(k)}_{ij} dz \quad (i, j) = (4, 5)
\]
where, $Q^{(k)}_{ij}(1, 2, 4, 5, 6)$ are the transformed stiffness coefficients of the $k$th layer of the plate, and $z_k$ and $z_{k+1}$ are the coordinates of the lower and upper surfaces of the $k$th layer of the plate along $z$ direction, respectively.

The external potential energy $\Pi_e$ can be written as below in which in-plane applied loads are $N_{xx}^0$ and $N_{yy}^0$:

$$\Pi_e = -\frac{1}{2} \int_0^a \int_0^b \left( N_{xx}^0 \left( \frac{dw_0}{dx} \right)^2 + N_{yy}^0 \left( \frac{dw_0}{dy} \right)^2 \right) dx dy$$

(4)

Considering $k$, as the stiffness value could be stored in system as shown in Figure 1c, the restraining energy $\Pi_s$ can be calculated as follows:

$$\Pi_s = \frac{k}{2} \int_0^a \left[ \psi_y^2 (y = 0) + \psi_y^2 (y = b) \right] dx$$

(5)

Then total elastic potential $\Pi$ of the laminated composite system would be:

$$\Pi = \Pi_e + \Pi_s + \Pi_s$$

(6)

Using the appropriate shape function into each term of Eq. (6) (i.e. Eqs. (1), (4) and (5)), the eigenvalue problem of buckling could be solved in conjunction with the Rayleigh–Ritz method. All boundary conditions should be satisfied by shape functions. The shape functions for $w_0$, $\psi_x$, and $\psi_y$ are considered in the following equations:

$$w_0 = W w_1(x) w_2(y), \quad \psi_x = X \psi_{x1}(x) \psi_{x2}(y), \quad \psi_y = Y \psi_{y1}(x) \psi_{y2}(y)$$

(7)

where, $w_1(x)$, $w_2(y)$, $\psi_{x1}(x)$, $\psi_{x2}(y)$, $\psi_{y1}(x)$ and $\psi_{y2}(y)$ are the shape functions based on satisfaction of boundary conditions. The unknown parameters $W$, $X$, $Y$ will remain as uncalculated parameters due to buckling theory [40]. Using Eq. (7) into Eq. (6), the final form of elastic potential energy can be written as:
In Eq. (8), the shape functions \( w_1(x), w_2(y), \psi_{x_1}(x), \psi_{x_2}(y), \psi_{y_1}(x) \) and \( \psi_{y_2}(y) \) are subjected to integration with respect to \( x \) or \( y \). The integrals must be solvable because the shape functions are known. The following abbreviations are introduced to write Eq. (8) in simpler form.
Considering the above defined abbreviations, Eq. (8) can be rewritten as follows:

\[
\begin{align*}
\Pi &= \frac{1}{2} D_{11} X^2 I_{11} J_{11} + D_{12} X Y I_{22} J_{22} + D_{16} X^2 J_{33} + D_{16} X Y I_{44} J_{44} + \frac{1}{2} D_{22} Y^2 J_{55} + D_{26} X Y I_{66} J_{66} + D_{26} Y^2 I_{77} \\
&\quad + \frac{1}{2} D_{66} X^2 J_{88} + D_{66} X Y J_{99} + \frac{1}{2} D_{66} Y^2 J_{1010} + \frac{1}{2} K A_{44} W^2 I_{11} J_{11} + K A_{44} W Y I_{12} J_{12} + \frac{1}{2} K A_{44} Y^2 I_{13} J_{13} \\
&\quad + K A_{45} W^2 I_{14} J_{14} + K A_{45} W Y I_{15} J_{15} + K A_{45} Y X I_{16} J_{16} + K A_{45} X Y I_{17} J_{17} + \frac{1}{2} K A_{45} Y^2 I_{18} J_{18} \\
&\quad + K A_{55} W X I_{19} J_{19} + \frac{1}{2} K A_{55} X^2 J_{20} + \frac{1}{2} w W^2 I_{21} J_{21} + \frac{1}{2} k W^2 J_{22} + \frac{1}{2} k W^2 I_{23} J_{23} + \frac{1}{2} k W^2 I_{24} J_{24} \\
&\quad - \frac{1}{2} N_{33}^0 W^2 I_{25} J_{25} - \frac{1}{2} N_{33}^0 W^2 I_{26} J_{26} + \frac{1}{2} k Y^2 I_{55} (C_{11} + C_{22})
\end{align*}
\]
When the elastic potential energy $\Pi$ achieves a stationary value, buckling of laminate plates occurs. Indeed, the first variation $\delta \Pi$ vanishes: $\delta \Pi = 0$. As can be shown in Eq. (10), the only variable quantities in $\Pi$ are constants $W, X$ and $Y$, i.e. $\Pi = \Pi (X, Y, Z)$. Thus, the main buckling condition $\delta \Pi = 0$ decreases to the well-known Ritz equations as follows:

$$
\frac{d \Pi}{dW} = 0 \Rightarrow \frac{d \Pi}{dW} = \left( KA_{44} I_{11} J_{11} + 2KA_{45} I_{14} J_{14} + KA_{55} I_{18} J_{18} + k_w I_{11} J_{11} + k_s I_{11} J_{11} - N_0^0 I_{18} J_{18} - N_0^0 I_{11} J_{11} \right) W + \left( KA_{45} I_{16} J_{16} + KA_{55} I_{19} J_{19} \right) X + \left( KA_{44} I_{12} J_{12} + KA_{45} I_{15} J_{15} \right) Y = 0
$$

$$
\frac{d \Pi}{dX} = 0 \Rightarrow \frac{d \Pi}{dX} = \left( KA_{44} I_{16} J_{16} + KA_{55} I_{19} J_{19} \right) W + \left( D_{11} I_{11} J_{11} + 2D_{16} I_{3} J_{3} + D_{66} I_{9} J_{9} + KA_{55} I_{8} J_{19} \right) X + \left( 2D_{26} I_{7} J_{7} + D_{66} I_{10} J_{10} + KA_{44} I_{5} J_{10} \right) Y = 0
$$

$$
\frac{d \Pi}{dY} = 0 \Rightarrow \frac{d \Pi}{dY} = \left( KA_{44} I_{12} J_{12} + KA_{45} I_{15} J_{15} \right) W + \left( D_{12} I_{12} J_{12} + D_{66} I_{6} J_{6} + D_{66} I_{9} J_{9} + KA_{45} I_{6} J_{4} \right) X + \left( 2D_{26} I_{7} J_{7} + D_{66} I_{10} J_{10} + KA_{44} I_{5} J_{10} \right) Y = 0
$$

By defining the following abbreviations:

$$
\lambda_{11} = KA_{44} I_{11} J_{11} + 2KA_{45} I_{14} J_{14} + KA_{55} I_{18} J_{18} + k_w I_{11} J_{11} + k_s I_{11} J_{11} + k_s I_{11} J_{11}
$$

$$
\lambda_{12} = KA_{45} I_{16} J_{16} + KA_{55} I_{19} J_{19}
$$

$$
\lambda_{13} = KA_{44} I_{12} J_{12} + KA_{45} I_{15} J_{15}
$$

$$
\lambda_{22} = D_{11} I_{11} J_{11} + 2D_{16} I_{3} J_{3} + D_{66} I_{9} J_{9} + KA_{55} I_{8} J_{19}
$$

$$
\lambda_{23} = ( D_{12} I_{12} J_{12} + D_{66} I_{6} J_{6} + D_{66} I_{9} J_{9} + KA_{45} I_{6} J_{4} )
$$

$$
\lambda_{33} = ( D_{12} I_{12} J_{12} + 2D_{26} I_{7} J_{7} + D_{66} I_{10} J_{10} + KA_{44} I_{5} J_{10} + k_s I_{5} ( C_{1rot} + C_{2rot} ) )
$$

$$
\lambda_{31} = I_{18} J_{18} \alpha I_{11} J_{11}
$$

$$
N_{yy}^0 = \alpha N_{xx}^0
$$

The equation system (12) can be written in a matrix form as:

$$
\begin{bmatrix}
\lambda_{11} - N_{cr}^0 \bar{\lambda}_{11} & \lambda_{12} & \lambda_{13} \\
\lambda_{21} & \lambda_{22} & \lambda_{23} \\
\lambda_{31} & \lambda_{32} & \lambda_{33}
\end{bmatrix}
\begin{bmatrix}
W \\
X \\
Y
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
$$

(13)

The coefficient matrix determinant of the equation system (13) must be equal to zero due to avoid a non-trivial solution. Therefore, the closed-form of the buckling load $N_{cr}^0$ is obtained as follows:

$$
N_{cr}^0 = \frac{\lambda_{11} ( \lambda_{22} \lambda_{33} - \lambda_{23} \lambda_{32} ) - \lambda_{21} ( \lambda_{12} \lambda_{33} - \lambda_{13} \lambda_{32} ) + \lambda_{31} ( \lambda_{12} \lambda_{23} - \lambda_{13} \lambda_{22} )}{\bar{\lambda}_{11} ( \lambda_{22} \lambda_{33} - \lambda_{23} \lambda_{32} )}
$$

(14)
In addition, non-dimensional buckling load, non-dimensional Winkler foundation parameter and non-dimensional Pasternak foundation parameter present as below, respectively:

\[
\overline{N}^0_{cr} = N^0_{cr} \frac{b^2}{E_2 \cdot h^3}, \quad KW = k_w \frac{b^4}{E_2 \cdot h^3}, \quad KS = k_s \frac{b^2}{E_2 \cdot h^3}
\] (15)

Based on Kuehn et al. [40], the following shape functions can be utilized for the plate that has simple support at the two opposite edges and rotationally restrained conditions on the other two edges.

\[
w_1(x) = \sin \left( \frac{m \pi x}{a} \right)
\]

\[
w_2(y) = (1 - \beta) \sin \left( \frac{\pi y}{b} \right) + \beta \frac{1}{2} \left[ 1 - \cos \left( \frac{2 \pi y}{b} \right) \right]
\]

\[
\psi_{y1}(x) = \cos \left( \frac{m \pi x}{a} \right)
\]

\[
\psi_{y2}(y) = (1 - \beta) \sin \left( \frac{\pi y}{b} \right) + \beta \frac{1}{2} \left[ 1 - \cos \left( \frac{2 \pi y}{b} \right) \right]
\]

\[
\psi_{x1}(x) = \sin \left( \frac{m \pi x}{a} \right)
\]

\[
\psi_{x2}(y) = (1 - \beta) \cos \left( \frac{\pi y}{b} \right) + \beta \sin \left( \frac{2 \pi y}{b} \right)
\] (16)

In the Eq. (16), the factor \( \beta \) can be calculated by interpolating between the two models of a simply supported laminate plate (i.e. \( k_r = 0 \)) and a fully supported laminate plate (i.e. \( k_r \rightarrow \infty \)). These two models correspond to \( \beta = 0 \) and \( \beta = 1 \), respectively. Therefore, the factor \( \beta \) can be obtained as:

\[
\beta = \frac{k_r b}{2D_{22} \pi + k_r b}
\] (17)

3. Results and Discussion

This section is divided into two subsections. The first part deals with the validation of the formulas, and the second part is the desired optimization.

3.1. Modeling and validation

In first subsection, to validate presented method, the non-dimensional buckling load factors of the laminated plates under two type of loading (i.e. uniaxial and biaxial loading) are evaluated with previous works. The material properties of laminated plates are considered as follows:

\[
\frac{E_{1}}{E_{2}} = 40, \quad \frac{G_{12}}{E_{2}} = \frac{G_{13}}{E_{2}} = 0.6, \quad \frac{G_{23}}{E_{2}} = 0.5, \quad \nu_{12} = \nu_{13} = 0.25
\]

It is assumed that the ratio of the length of \( b \) to the thickness of the plate is \( b/h = 10 \). Table 1 shows the non-dimensional buckling load factors (uniaxial and biaxial) of a symmetrically laminated composite plate with 3-ply [0/90/0] on Winkler and Pasternak foundations for various values of aspect ratios for the present method, as compared with the works of Setoodeh [41] and Xiang [42]. The results showed a good correlation between the present method and the related
literature. The results of Table 1 also indicated that the developed method is reliable to utilize for calculating of non-dimensional buckling load factors.

As another validation by using reference [43], the optimal stacking sequence of several states was investigated with above mentioned material. It is assumed that the aspect ratios of the thickness of plate are \( b/h = 10 \) and \( b/h = 30 \). Table 2 shows the non-dimensional buckling load factors of a symmetrically laminated composite plate with 8-ply \([\theta/-\theta/\cdots]s\) without foundations for various values of aspect ratios. The results are compared with the present method that shows a good correlation between the present method and the mentioned reference. As another example to validate the formulas, an angle-ply composite plate made of T300/5208 graphite/epoxy layers is considered. The composite material’s properties are given as \( E_1 = 4 \times 10^7 \, MPa \), \( E_2 = 1 \times 10^6 \, MPa \), \( G_{12} = 6 \times 10^5 \, MPa \) and also \( \nu_{12} = 0.25 \). All optimization problems consider that the plies have the same thickness of 0.125 mm. For numerical example, we consider a rectangular composite plate with dimensions \( b = 500 \, mm \) and \( a = 500 \, mm \) and also \( a = 300 \, mm \). The plate has four layers with Stacking sequence \([30\text{-}30\text{-}30\text{-}30]\). The laminated composite plate has simple support and semi-clamped support \((k_r = 0 \text{ and } 1 \times 10^5)\). Table 3 and Figure 2 shows the results of buckling analysis. The results showed a good correlation between the outcome of the present method and that of ABAQUS software [44].

3.2. Optimization by Genetic Algorithm (GA)

By using the proposed method that is validated in previous subsection, this subsection estimates the optimum fiber orientation angles of laminated plate employing the GA approach. The aim is to maximize the buckling load capacity of laminated plates considering the orientation (\( \theta \)) as design variables. To capture the effect of boundary support of plates, three different boundary conditions are assumed by changing the boundary stiffness ratio (i.e. \( k_r \) as shown in Figure 1c) as: simple support, semi-clamped support and clamped support. 4-ply \([\theta/-\theta/\theta/\theta] \) symmetric laminates for several aspect ratios \((a/b), \alpha, K_W \text{ and } K_S\) are considered. The aim of the optimization is that by using GA, the buckling load’s capacity is maximized, while the design variable is the fiber orientation \( \theta \). Thus, it should find optimal orientation \( \theta \) for various models. Overlaying the plies as negative and positive is in accordance with Refs. [24, 45]. They concluded that this type of stacking sequence has the best performance. In the analysis, the elastic lamina’s properties are considered to be as below:

\[
\frac{E_1}{E_2} = 40, \quad \frac{G_{12}}{E_2} = \frac{G_{13}}{E_2} = 0.6, \quad \frac{G_{23}}{E_2} = 0.5, \quad \nu_{12} = \nu_{13} = 0.25
\]

The ratio of the length of \( b \) to the thickness of the plate is considered to be \( b/h = 10 \).

3.2.1. The plate with simple support

At first, a laminated composite plate with simple support \((k_r = 0)\) is considered. Table 4, Figure 3 and Figure 4 show the optimal orientation (\( \theta \)) of the plate for several aspect ratios \((a/b), \alpha, K_W \text{ and } K_S\). As it can be seen, if there is uniaxial in-plane loading \((\alpha = 0)\), the optimal orientation in all models would be below \(35^\circ\). It is obvious that the plies tend to locate in the loading direction. By adding Winkler foundation, the optimal orientation decreases. The optimal orientation has the greatest reduction when Pasternak foundation is added. For aspect ratios greater than 1, changing of the optimal orientation is negligible.
If there is biaxial in-plane loading ($\alpha=1$ and $\alpha=2$), the optimal orientation tends to increase. In the aspect ratios ($a/b$) equal 1 under biaxial in-plane loading, in the situations with and without presenting the foundation, the optimal orientation ($\Theta$) is $45^\circ$ [24,46,47]. The orientation increases for the aspect ratios greater than 1. However, in the aspect ratios between 2 and 8, the orientation does not change. In $\alpha=1$, the foundations cause to decrease the optimal orientation that is approximately $57^\circ$. But in $\alpha=2$, the foundations have low effect on the optimal orientation. According to Table 5, by increasing $\alpha$, the buckling load decreases. Also, by adding foundations (especially Pasternak foundation), the buckling load increases. Increasing the aspect ratio has low effect on the buckling load; however, increasing the aspect ratio causes to increase the half-wave of buckling. Assuming the same total thickness of the plate, the above results can be obtained for the plate with different layers (e.g. the plate with 8 layers).

### 3.2.2. The plate with semi-clamped support

Table 6 and Figure 5 illustrate a laminated composite plate with semi-clamped support ($k_r=1\times10^9$). As shown, if there is uniaxial in-plane loading ($\alpha=0$), the optimal orientation $\Theta$ in all models is below $30^\circ$. Therefore, all values are less than those of the plate with simple support. In addition, the change of optimal orientation versus the change of aspect ratio is low. If there is biaxial in-plane loading ($\alpha=1$ and $\alpha=2$), in the aspect ratio equal to one, with and without presenting the foundations, the optimal orientation $\Theta$ is $45^\circ$. However, in the aspect ratios greater than 1, under biaxial loading, the optimal orientation increases. The optimal orientation in the plate with semi-clamped support is a little less than that of the plate with simple support. According to Table 7, in all models, the buckling load in the plate with semi-clamped support is a little more than that of the plate with simple support.

### 3.2.3. The plate with clamped support

Table 8 and Figure 6 indicate a laminated composite plate with clamped support ($k_r=1\times10^{12}$). As shown, if there is uniaxial in-plane loading ($\alpha=0$), the optimal orientation $\Theta$ in all models would be below $30^\circ$. Therefore, the values are similar to those of the plate with semi-clamped support. In the plate under biaxial loading, the optimal orientation in all models is more than $45^\circ$. In the models that $\alpha=1$, the change of optimal orientation versus the change of aspect ratio is almost negligible. In various supporting conditions, when $\alpha=2$ and the aspect ratio is greater than 1, the optimal orientation is more than $60^\circ$. It is shown that the plies' orientation tends to locate in the y direction, because in the long plate, the effect of in-plane loading in the y direction is more. According to Table 9, in all models, the buckling load in the plate with clamped support is a little more than that of the plates with simple support and semi-clamped support.

As another example, a clamped supported laminated composite plate ($k_r=1\times10^{12}$) with different foundation stiffness values ($K_{W}=200$ and $K_{S}=20$) is considered. The results are presented in Figures 7-8 and Tables 10-11. As shown, if there is uniaxial in-plane loading ($\alpha=0$), the optimal orientation in all models is below $15^\circ$. Therefore, the values are very low than those of the clamped supported plate with lower foundation stiffness. In the plate under biaxial loading, the optimal orientation in all models is higher than $45^\circ$. In the models that $\alpha=1$, the change of optimal orientation versus various aspect ratios is almost negligible (i.e., it is similar to that of the clamped supported plate with lower foundation stiffness).
In various supporting conditions, when $\alpha = 2$ and the aspect ratio is greater than 1, the optimal orientation is more than $65^\circ$. According to Table 10, high foundation stiffness results in increase of the buckling load.

4. Numerical results of post-buckling

4.1. Comparison of the analytical solutions for the post-buckling analysis

In this subsection, the accuracy of the present analytical solutions for the load–deflection curves of clamped supported laminated composite plates with configurations (90/0/45/-45) and (67.5/-67.5) under shear loads so that all individual laminate layers have the same thickness of 0.3 mm and 0.6 mm, respectively. Also, it is assumed that the material properties for all lamina are the same. In the analysis, elastic lamina properties are assumed to be:

\[ E_{11} = 113 \text{GPa}, \ E_{22} = 9 \text{GPa}, \ G_{12} = 3.82 \text{GPa}, \ v_{12} = 0.302 \]

A finite element software is used to validate the analytical solution. The analytical and FEM curves have been illustrated in Figures 9 and 10. The results show that the analytical curves have a positive agreement with the FEM curves in the post-buckling process.

4.2. Optimization scheme

In the second subsection, the optimum stacking sequences of unsymmetrical rotationally-restrained laminated composite plates under shear loading are presented. In the optimization process, number of generations and number of population of run are considered as 100 and 200 for GA algorithm, respectively. Figure 11 shows the load-deflection curves of three kinds of laminates with different torsion stiffness ($\bar{K} = 0,10,100$) and different number of plies. In Table 12, the optimal stacking sequence of the plate is brought for three kinds of laminates with different torsion stiffness. It can be observed that the bearing capacity is higher than that of the laminate with configurations (90/0/45/-45). This increase is approximately 80% that shows by the optimal stacking sequence is very efficient. In addition, an increase at edge torsion stiffness results in increasing the bearing capacity of the laminates. On the other hand, it can be seen from the results that as the number of layer increases, the load-deflection curves become almost the same. These results can be referred to the design engineers of composite plate-like structures. In another example, width $b$ is considered 500 mm and the other properties of above example remain the same. Figure 12 shows the load-deflection curves of new laminates with different torsion stiffness ($\bar{K} = 0,10,100$) and different number of plies. In Table 13, the optimal stacking sequence of the plate is brought for three kinds of laminates with different torsion stiffness.

It can be observed that the increase of the width has no influence on the optimal stacking sequence. However, the increase of the width results in decreasing the bearing capacity of the plate. In addition, an increase at edge torsion stiffness leads to increase the bearing capacity of the laminates. On the other hand, it can be seen from the results that as the number of layer increases, the load-deflection curves become almost the same.

As the last example, the effects of the change in the modulus of elasticity $E_1$ on the bearing capacity of the laminates are examined. Three different modulus of elasticity $E_1$ are considered and the other properties of first example remain the same. Figure 13 shows the load-deflection curves of new laminates with different torsion stiffness ($\bar{K} = 0,10,100$) and different modulus of elasticity $E_1$. In Table 14, the optimal stacking sequence of the plate is brought for three kinds of
laminates with different torsion stiffness. It can be observed that the increase of the modulus of elasticity $E_1$ has not important influence on the optimal stacking sequence. However, the increase of the modulus of elasticity $E_1$ results in increasing the bearing capacity of the plate.

5. Conclusion
Identification of buckling characteristics of a laminated composite plates is a beneficial tool to capture and assess the performance of a composite plates under biaxial and uniaxial loading. By finding the updated buckling characteristics of a plates, then it can be possible to obtain the maximum buckling loading capacity of plates. To have better evaluate of this characteristics, the studied laminated plates needs to be analyzed precisely considering not only the composite plates characteristics but also the boundary condition characteristics. Thus, in current study, new modified algorithm based on GA was employed to maximize the buckling load considering optimization of the fiber orientations and different boundary supported conditions. The results of the present study showed that if there is a uniaxial in-plane loading ($\alpha=0$), the optimal orientation in all Models would be below $35^\circ$. It is obvious that under this condition the plies tend to locate in the loading direction. By adding the elastic foundation, especially Pasternak foundation, the optimal orientation decreases for all boundary conditions. For the aspect ratio greater than 1 under uniaxial in-plane loading, the change of optimal orientation is almost negligible.

For the plates under biaxial in-plane loading ($\alpha=1$ and $\alpha=2$) with the aspect ratios equal to 1 and with and without presenting the foundation, the optimal orientation tends to be almost $45^\circ$ for all boundary conditions. However, in the plates with clamped boundary conditions, under the above-mentioned situations, the optimal orientation tends to be $55^\circ$ approximately. Furthermore, it can be seen from the results, when the number of layers is increased in laminated plates, the slightly effect are acquired in term of capacity of buckling load. However, in post-buckling behavior, the plates with large number of layers has the highest convergence rate. Under biaxial in-plane loading with the aspect ratios greater than 1, and with and without presenting the foundation, the optimal orientation is greater than $45^\circ$; however, clamped support and Pasternak foundation cause to little decrease of the optimal orientation.

It can be observed that the bearing capacity of the laminate with the optimal stacking sequence is higher (approximately 80%) than that of the laminate with the normal stacking sequence. On the other hand, it can be seen from the results that as the number of layer increases, the load-deflection curves become almost the same. It can be observed that the increase of the width has not influence on the optimal stacking sequence. In addition, an increase at the edge torsion stiffness and the modulus of elasticity ($EI$) results in increasing the bearing capacity of the laminates.

References


Biographies

Sina Farahani was born in Hamedan, Iran, in 1990. He is now a Visiting Researcher at Division for Structures, Materials and Geotechnics of Aalborg University, Denmark. He was a Ph.D. candidate of structural engineering at the Razi University, Kermanshah, Iran. He received his MS degree in Structural Engineering from the Malayer University, Malayer, Iran. His research interests include performance- based seismic design of structures, direct displacement- based design of structures, nonlinear dynamics analysis, soil–structure interaction effect, optimization, seismic control of structures, and buckling of laminated composite structures.

Mojtaba Fathi was born in Kermanshah. He received his BS degree from Tehran University and his MS and PhD degrees from Tarbiat Modares University of Tehran in 1997 and 2004, respectively. At present, he works as an Assistant Professor at the Department of Civil Engineering in Razi University. His main research interests are: structural dynamics, Composite materials, Modern technology in structures, Semi-rigid connections, Steel structures and Concrete technology. He has authored more than 100 publications in journals and conferences. He has been a reviewer in some top international journals.

Ebrahim Nazarimofrad was born in Hamedan, Iran, in 1985. He received his MSc in Structural Engineering from Bu- Ali Sina University, Hamedan, in 2013. His main research interests are computer programming (C# and MATLAB), structural dynamics, earthquake engineering, passive–active control of vibration, structural health monitoring, thin- walled structures, laminated composite structures, buckling and instability, reliability of structures, shape memory alloys, finite element model, soil–structure interactions, and optimization. He has authored many publications including more than 10 journals and as well as 10 conference papers. He has been a reviewer in some top international journals as well as a structural designer in a few companies.
List of Figures captions

Figure 1. Rectangular laminated composite plate resting on an elastic Pasternak foundation
Figure 2. Deformation caused by the buckling of the composite plate
Figure 3. Optimal orientation $\theta$ of the plate with simple support on the elastic foundation
Figure 4. Non-dimensional buckling load factors of the plate with simple support on the elastic foundation
Figure 5. Optimal orientation $\theta$ of the plate with semi-clamped support on elastic foundation
Figure 6. Optimal orientation $\theta$ of the plate with clamped support on elastic foundation
Figure 7. Optimal orientation $\theta$ of the plate with clamped support on elastic foundation
Figure 8. Non-dimensional buckling load factors of the plate with simple support on elastic foundation
Figure 9. Laminated composite plates with configurations (90/0/45/-45)
Figure 10. Laminated composite plates with configurations (67.5/-67.5)
Figure 11. Load-deflection curves of laminates with different number of plies for Model 1
Figure 12. Load-deflection curves of laminates with different number of plies for Model 2
Figure 13. Load-deflection curves of laminates with different number of plies with different modulus of elasticity

List of Tables captions

Table 1. Uniaxial and biaxial non-dimensional buckling load factors of the laminated composite
Table 2. Non-dimensional buckling load factors of a symmetrically laminated composite
Table 3. The results of buckling analysis
Table 4. Optimal orientation $\theta$ of the plate with simple support on the elastic foundation
Table 5. Non-dimensional buckling load factors of the plate with simple support on the elastic foundation
Table 6. Optimal orientation $\theta$ of the plate with semi-clamped support on elastic foundation
Table 7. Non-dimensional buckling load factors of the plate with semi-clamped support on elastic foundation
Table 8. Optimal orientation $\theta$ of the plate with clamped support on elastic foundation
Table 9. Non-dimensional buckling load factors of the plate with clamped support on elastic foundation
Table 10. Optimal orientation $\theta$ of the plate with clamped support on elastic foundation
Table 11. Non-dimensional buckling load factors of the plate with clamped support on elastic foundation
Table 12: Optimum stacking sequence from GA algorithm for Model 1
Table 13: Optimum stacking sequence from GA algorithm for model 2
Table 14: Optimum stacking sequence from GA algorithm with different modulus of elasticity
Figure 1. Rectangular laminated composite plate resting on an elastic Pasternak foundation
Figure 2. Deformation caused by the buckling of the composite plate

Figure 3. Optimal orientation $\theta$ of the plate with simple support on the elastic foundation
Figure 4. Non-dimensional buckling load factors of the plate with simple support on the elastic foundation

Figure 5. Optimal orientation $\theta$ of the plate with semi-clamped support on elastic foundation
Figure 6. Optimal orientation $\theta$ of the plate with clamped support on elastic foundation

Figure 7. Optimal orientation $\theta$ of the plate with clamped support on elastic foundation

Figure 8. Non-dimensional buckling load factors of the plate with simple support on elastic foundation
Figure 9. Laminated composite plates with configurations (90/0/45/-45)

Figure 10. Laminated composite plates with configurations (67.5/-67.5)
Figure 11. Load-deflection curves of laminates with different number of plies for Model 1.
Figure 12. Load-deflection curves of laminates with different number of plies for Model 2
Figure 13. Load-deflection curves of laminates with different number of plies with different modulus of elasticity
Table 1. Uniaxial and biaxial non-dimensional buckling load factors of the laminated composite

<table>
<thead>
<tr>
<th>a/b</th>
<th>$K_w$</th>
<th>$K_S$</th>
<th>Uniaxial: $\alpha=0$</th>
<th></th>
<th></th>
<th></th>
<th>Biaxial: $\alpha=1$</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>22.234</td>
<td>22.315</td>
<td>22.312</td>
<td>9.942</td>
<td>10.202</td>
<td>10.520</td>
<td>22.235</td>
<td>22.447</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>0</td>
<td>32.235</td>
<td>32.447</td>
<td>32.439</td>
<td>11.923</td>
<td>12.228</td>
<td>12.235</td>
<td>49.226</td>
<td>50.751</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>10</td>
<td>49.226</td>
<td>50.751</td>
<td>50.743</td>
<td>21.866</td>
<td>22.228</td>
<td>22.237</td>
<td>49.039</td>
<td>49.266</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>16.424</td>
<td>16.434</td>
<td>16.426</td>
<td>3.269</td>
<td>3.286</td>
<td>3.295</td>
<td>32.354</td>
<td>32.447</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>0</td>
<td>32.354</td>
<td>32.447</td>
<td>32.426</td>
<td>9.345</td>
<td>9.590</td>
<td>9.621</td>
<td>49.039</td>
<td>49.266</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>10</td>
<td>49.039</td>
<td>49.266</td>
<td>49.251</td>
<td>19.140</td>
<td>19.590</td>
<td>19.622</td>
<td>49.039</td>
<td>49.266</td>
</tr>
</tbody>
</table>

Table 2. Non-dimensional buckling load factors of a symmetrically laminated composite

<table>
<thead>
<tr>
<th>a/h</th>
<th>a/b=1.0</th>
<th>a/b=1.5</th>
<th>a/b=2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>40</td>
<td>36.85</td>
<td>34.26</td>
</tr>
<tr>
<td>30</td>
<td>45</td>
<td>59.06</td>
<td>56.77</td>
</tr>
</tbody>
</table>

Table 3. The results of buckling analysis

<table>
<thead>
<tr>
<th>Example</th>
<th>a (mm)</th>
<th>b (mm)</th>
<th>$k_r$ (N.mm)</th>
<th>Programming Results</th>
<th>ABAQUS [44] Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>500</td>
<td>0</td>
<td>17.7</td>
<td>18.4</td>
</tr>
<tr>
<td>2</td>
<td>3000</td>
<td>500</td>
<td>0</td>
<td>3.50</td>
<td>3.69</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>500</td>
<td>$1 \times 10^9$</td>
<td>20.00</td>
<td>21.95</td>
</tr>
<tr>
<td>4</td>
<td>3000</td>
<td>500</td>
<td>$1 \times 10^9$</td>
<td>9.59</td>
<td>10.72</td>
</tr>
</tbody>
</table>
### Table 4. Optimal orientation $\theta$ of the plate with simple support on the elastic foundation

<table>
<thead>
<tr>
<th>Case</th>
<th>alpha</th>
<th>$K_w$</th>
<th>$K_x$</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>35.54</td>
<td>33.18</td>
<td>34.22</td>
<td>34.42</td>
</tr>
<tr>
<td>Model 2</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>22.94</td>
<td>30.41</td>
<td>28.79</td>
<td>28.30</td>
</tr>
<tr>
<td>Model 3</td>
<td>0</td>
<td>100</td>
<td>10</td>
<td>22.56</td>
<td>22.56</td>
<td>24.03</td>
<td>24.17</td>
</tr>
<tr>
<td>Model 4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>45.00</td>
<td>66.54</td>
<td>66.65</td>
<td>66.66</td>
</tr>
<tr>
<td>Model 5</td>
<td>1</td>
<td>100</td>
<td>0</td>
<td>45.00</td>
<td>56.99</td>
<td>57.45</td>
<td>57.50</td>
</tr>
<tr>
<td>Model 6</td>
<td>1</td>
<td>100</td>
<td>10</td>
<td>45.00</td>
<td>56.99</td>
<td>57.45</td>
<td>57.50</td>
</tr>
<tr>
<td>Model 7</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>45.00</td>
<td>75.06</td>
<td>74.71</td>
<td>74.77</td>
</tr>
<tr>
<td>Model 8</td>
<td>2</td>
<td>100</td>
<td>0</td>
<td>45.00</td>
<td>69.66</td>
<td>69.66</td>
<td>69.66</td>
</tr>
<tr>
<td>Model 9</td>
<td>2</td>
<td>100</td>
<td>10</td>
<td>45.00</td>
<td>74.99</td>
<td>74.64</td>
<td>74.70</td>
</tr>
</tbody>
</table>

### Table 5. Non-dimensional buckling load factors of the plate with simple support on the elastic foundation

<table>
<thead>
<tr>
<th>Case</th>
<th>alpha</th>
<th>$K_w$</th>
<th>$K_x$</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>37.51</td>
<td>36.75</td>
<td>36.56</td>
<td>36.53</td>
</tr>
<tr>
<td>Model 2</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>42.02</td>
<td>41.31</td>
<td>41.18</td>
<td>41.18</td>
</tr>
<tr>
<td>Model 3</td>
<td>0</td>
<td>100</td>
<td>10</td>
<td>54.52</td>
<td>54.52</td>
<td>54.50</td>
<td>54.47</td>
</tr>
<tr>
<td>Model 4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>19.45</td>
<td>16.04</td>
<td>15.98</td>
<td>15.97</td>
</tr>
<tr>
<td>Model 5</td>
<td>1</td>
<td>100</td>
<td>0</td>
<td>24.51</td>
<td>23.08</td>
<td>22.85</td>
<td>22.82</td>
</tr>
<tr>
<td>Model 6</td>
<td>1</td>
<td>100</td>
<td>10</td>
<td>34.51</td>
<td>33.08</td>
<td>32.85</td>
<td>32.82</td>
</tr>
<tr>
<td>Model 7</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>12.96</td>
<td>9.06</td>
<td>9.06</td>
<td>9.06</td>
</tr>
<tr>
<td>Model 8</td>
<td>2</td>
<td>100</td>
<td>0</td>
<td>16.34</td>
<td>13.50</td>
<td>13.50</td>
<td>13.50</td>
</tr>
<tr>
<td>Model 9</td>
<td>2</td>
<td>100</td>
<td>10</td>
<td>23.01</td>
<td>19.12</td>
<td>19.12</td>
<td>19.12</td>
</tr>
</tbody>
</table>

### Table 6. Optimal orientation $\theta$ of the plate with semi-clamped support on elastic foundation

<table>
<thead>
<tr>
<th>Case</th>
<th>alpha</th>
<th>$K_w$</th>
<th>$K_x$</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>28.49</td>
<td>29.04</td>
<td>30.83</td>
<td>29.82</td>
</tr>
<tr>
<td>Model 2</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>22.86</td>
<td>24.74</td>
<td>24.92</td>
<td>25.50</td>
</tr>
<tr>
<td>Model 3</td>
<td>0</td>
<td>100</td>
<td>10</td>
<td>22.29</td>
<td>21.80</td>
<td>20.70</td>
<td>21.26</td>
</tr>
<tr>
<td>Model 4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>44.54</td>
<td>60.03</td>
<td>60.15</td>
<td>60.17</td>
</tr>
<tr>
<td>Model 5</td>
<td>1</td>
<td>100</td>
<td>0</td>
<td>45.08</td>
<td>50.38</td>
<td>50.51</td>
<td>50.53</td>
</tr>
<tr>
<td>Model 6</td>
<td>1</td>
<td>100</td>
<td>10</td>
<td>45.08</td>
<td>50.38</td>
<td>50.51</td>
<td>50.53</td>
</tr>
<tr>
<td>Model 7</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>45.28</td>
<td>71.06</td>
<td>70.79</td>
<td>70.83</td>
</tr>
<tr>
<td>Model 8</td>
<td>2</td>
<td>100</td>
<td>0</td>
<td>45.93</td>
<td>65.40</td>
<td>65.42</td>
<td>65.42</td>
</tr>
<tr>
<td>Model 9</td>
<td>2</td>
<td>100</td>
<td>10</td>
<td>46.23</td>
<td>71.04</td>
<td>70.77</td>
<td>70.82</td>
</tr>
</tbody>
</table>
Table 7. Non-dimensional buckling load factors of the plate with semi-clamped support on elastic foundation

<table>
<thead>
<tr>
<th>Case</th>
<th>alpha</th>
<th>Aspect Ratio</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>40.66</td>
<td>39.45</td>
</tr>
<tr>
<td>Model 2</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>43.68</td>
<td>43.63</td>
</tr>
<tr>
<td>Model 3</td>
<td>0</td>
<td>100</td>
<td>10</td>
<td>0</td>
<td>56.68</td>
<td>56.68</td>
</tr>
<tr>
<td>Model 4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>21.21</td>
<td>19.43</td>
</tr>
<tr>
<td>Model 5</td>
<td>1</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>26.16</td>
<td>25.93</td>
</tr>
<tr>
<td>Model 6</td>
<td>1</td>
<td>100</td>
<td>10</td>
<td>0</td>
<td>36.16</td>
<td>35.93</td>
</tr>
<tr>
<td>Model 7</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>14.03</td>
<td>11.09</td>
</tr>
<tr>
<td>Model 8</td>
<td>2</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>17.31</td>
<td>15.45</td>
</tr>
<tr>
<td>Model 9</td>
<td>2</td>
<td>100</td>
<td>10</td>
<td>0</td>
<td>23.93</td>
<td>21.10</td>
</tr>
</tbody>
</table>

Table 8. Optimal orientation \( \theta \) of the plate with clamped support on elastic foundation

<table>
<thead>
<tr>
<th>Case</th>
<th>alpha</th>
<th>Aspect Ratio</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>26.75</td>
<td>29.32</td>
</tr>
<tr>
<td>Model 2</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>26.75</td>
<td>26.75</td>
</tr>
<tr>
<td>Model 3</td>
<td>0</td>
<td>100</td>
<td>10</td>
<td>0</td>
<td>20.23</td>
<td>21.40</td>
</tr>
<tr>
<td>Model 4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>53.75</td>
<td>52.60</td>
</tr>
<tr>
<td>Model 5</td>
<td>1</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>48.62</td>
<td>47.31</td>
</tr>
<tr>
<td>Model 6</td>
<td>1</td>
<td>100</td>
<td>10</td>
<td>0</td>
<td>48.62</td>
<td>47.31</td>
</tr>
<tr>
<td>Model 7</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>54.72</td>
<td>63.89</td>
</tr>
<tr>
<td>Model 8</td>
<td>2</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>54.73</td>
<td>60.10</td>
</tr>
<tr>
<td>Model 9</td>
<td>2</td>
<td>100</td>
<td>10</td>
<td>0</td>
<td>54.74</td>
<td>65.11</td>
</tr>
</tbody>
</table>

Table 9. Non-dimensional buckling load factors of the plate with clamped support on elastic foundation

<table>
<thead>
<tr>
<th>Case</th>
<th>alpha</th>
<th>Aspect Ratio</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>42.37</td>
<td>42.28</td>
</tr>
<tr>
<td>Model 2</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>44.90</td>
<td>44.90</td>
</tr>
<tr>
<td>Model 3</td>
<td>0</td>
<td>100</td>
<td>10</td>
<td>0</td>
<td>57.72</td>
<td>57.54</td>
</tr>
<tr>
<td>Model 4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>24.91</td>
<td>24.88</td>
</tr>
<tr>
<td>Model 5</td>
<td>1</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>28.98</td>
<td>28.85</td>
</tr>
<tr>
<td>Model 6</td>
<td>1</td>
<td>100</td>
<td>10</td>
<td>0</td>
<td>38.98</td>
<td>38.85</td>
</tr>
<tr>
<td>Model 7</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>15.86</td>
<td>15.52</td>
</tr>
<tr>
<td>Model 8</td>
<td>2</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>18.62</td>
<td>18.50</td>
</tr>
<tr>
<td>Model 9</td>
<td>2</td>
<td>100</td>
<td>10</td>
<td>0</td>
<td>24.99</td>
<td>24.57</td>
</tr>
</tbody>
</table>
Table 10. Optimal orientation $\theta$ of the plate with clamped support on elastic foundation

<table>
<thead>
<tr>
<th>Aspect Ratio</th>
<th>Case</th>
<th>alpha</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0</td>
<td>9.08</td>
<td>9.8</td>
<td>11.23</td>
<td>14.47</td>
<td></td>
</tr>
<tr>
<td>Model 2</td>
<td>1</td>
<td>43.74</td>
<td>42.23</td>
<td>42.48</td>
<td>42.55</td>
<td></td>
</tr>
<tr>
<td>Model 3</td>
<td>2</td>
<td>54.76</td>
<td>66.35</td>
<td>66.25</td>
<td>66.41</td>
<td></td>
</tr>
</tbody>
</table>

Table 11. Non-dimensional buckling load factors of the plate with clamped support on elastic foundation

<table>
<thead>
<tr>
<th>Aspect Ratio</th>
<th>Case</th>
<th>alpha</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0</td>
<td>69.82</td>
<td>69.84</td>
<td>69.91</td>
<td>69.98</td>
<td></td>
</tr>
<tr>
<td>Model 2</td>
<td>1</td>
<td>52.68</td>
<td>52.4</td>
<td>52.32</td>
<td>52.31</td>
<td></td>
</tr>
<tr>
<td>Model 3</td>
<td>2</td>
<td>34.12</td>
<td>33.61</td>
<td>33.59</td>
<td>33.59</td>
<td></td>
</tr>
</tbody>
</table>

Table 12: Optimum stacking sequence from GA algorithm for Model 1

<table>
<thead>
<tr>
<th>Number of plies</th>
<th>$\bar{K}$</th>
<th>Optimal stacking sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
<td>[-64/-63/-64/-65]</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>[-65/-61/-64/-65]</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>[-59/90/-51/-58]</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>[-65/-63/-61/-48/54/-67/-63/-63]</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>[-64/-66/-61/-59/-52/-65/-69/-69]</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>[-58/-60/75/76/-11/-47/-64/-58]</td>
</tr>
</tbody>
</table>

Table 13: Optimum stacking sequence from GA algorithm for Model 2

<table>
<thead>
<tr>
<th>Number of plies</th>
<th>$\bar{K}$</th>
<th>Optimal stacking sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
<td>[-64/-62/-64/-65]</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>[-65/-63/-63/-65]</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>[-59/81/-61/-57]</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>[-63/-63/-67/62/50/-68/-68/-63]</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>[-67/-62/-70/-59/-70/-60/-63/-65]</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>[-56/-66/-88/-86/-25/-57/-54/-61]</td>
</tr>
</tbody>
</table>

30
Table 14: Optimum stacking sequence from GA algorithm with different modulus of elasticity

<table>
<thead>
<tr>
<th>$\bar{K}$</th>
<th>$E_1$ (GPa)</th>
<th>Optimal stacking sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>113</td>
<td>[-65/-64/-63/-64]</td>
</tr>
<tr>
<td>0</td>
<td>225</td>
<td>[-67/-68/-66/-69]</td>
</tr>
<tr>
<td>0</td>
<td>360</td>
<td>[-70/-69/-68/-71]</td>
</tr>
<tr>
<td>10</td>
<td>113</td>
<td>[-65/-63/-66/-65]</td>
</tr>
<tr>
<td>10</td>
<td>225</td>
<td>[-61/-63/-88/-56]</td>
</tr>
<tr>
<td>10</td>
<td>360</td>
<td>[-64/78/61/-44]</td>
</tr>
<tr>
<td>100</td>
<td>113</td>
<td>[-60/-62/82/-56]</td>
</tr>
<tr>
<td>100</td>
<td>225</td>
<td>[-49/-69/-55/-53]</td>
</tr>
<tr>
<td>100</td>
<td>360</td>
<td>[-55/67/-32/-46]</td>
</tr>
</tbody>
</table>