Multi-Layer Advanced Fibre Hybridisation (Glass-Carbon-Kevlar) and Variable Stiffness Influence on Composite Structure Responses (Stress and Deformation): An FE Approach

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Abstract

The influence of one/more numbers advanced layer fibre hybridization effect on the composite structural stiffness (deformation resistance capability) and the stress behaviour under the variable loading (uniformly distributed and sinusoidal) is analyzed numerically through an isoparametric finite element approach. The hybrid composite panel model is derived in the framework of the higher-order kinematic model to satisfy the inter-laminar stress continuity via the strains. The necessary structural equilibrium equations under the influence of variable mechanical loadings are derived through the variational principle to compute the panel’s central point deflections, as well as the stress values. The varied structural stiffness and their corresponding deflection parameters due to the hybridization of different advanced fibres (Carbon/Glass/Kevlar) are obtained through an in-house MATLAB code by incorporating the necessary elastic constant through the constitutive relationship. Firstly, the steadiness of the numerical solution is confirmed and extended further to verify the necessary solution correctness by solving a different number of examples similar to the published results. Finally, the influences of variable structural parameters relevant to the geometry (thickness ratio, aspect ratio), boundary conditions and the order of hybridizing layers (Glass-Carbon-Kevlar) on the bending strength have been highlighted by solving a series of examples and explained in details.

Keywords: Flexural behaviour, hybrid composite laminates, HSDT, geometrical parameters, FEM, MATLAB.

1. Introduction

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The traditional materials have been successfully substituted by the hybrid composite materials in several lightweight and high strength applications. On the basis of reinforcement of two or more fibers in a single polymeric matrix, a new advanced type of material configuration renders great diversity in material properties, called the hybrid composites. Thus, there is a need to investigate the mechanical properties like hardness, tensile strength, compression strength, impact strength and flexural strength of this advanced hybrid material. Among these properties, the flexural strength is most commonly determined through a three-point bend test, in which the specimen having some span length deflects under the loading rate until fracture. Due to the anisotropic material property of the specimen, the failure in bending may be caused by tensile, compressive, shear or a combination of all these stresses. The hybrid composites are being investigated throughout the globe because they have enhanced properties as compared to their parent composites which are relatively cost effective. Various researchers have investigated the flexural properties of various types of hybrid materials via experimental, numerical and analytical approaches. The mechanical/thermo-mechanical behaviour such as tensile, compressive, flexural, inter-laminar and impact strength of woven fabric glass-kevlar/epoxy composites has been studied [1,2] under different types of loading such as uniformly distributed load and sinusoidal load. The aforementioned properties of the plain kevlar fabric have also been investigated and compared with glass-kevlar hybrid composite [3] to reveal that the hybrid composites show better strength than that of single fiber reinforced composite material. The hybridization effect of glass fiber on sisal fiber polypropylene composites has been studied [4] and it has been observed that the tensile, flexural and impact strength is enhanced without any effect on tensile and flexural moduli. Also, from the study on flexural behaviour of glass-hemp-glass hybrid composite it has been divulged that the hybrid composites have more economical, ecological, recycling value and specific fatigue strength [5]. Further, the flexural modulus of the carbon/basalt hybrid composite was found to depend upon their compositions according to the rule of mixture [6,7]. The investigation on tensile and compressive in-plane mechanical properties under quasi-static loading of different combination of glass-carbon hybrid composites showed higher tensile strength and ultimate tensile strain with the higher percentage of glass fiber [8]. The flexural strength and modulus of carbon-basalt hybrid composite is strongly dependent on the different sequences of fiber reinforcement [9]. Moreover, the flexural progressive failure modes of carbon and glass fiber inter-layer and intra-layer hybrid composites have also been investigated [10]. The experimental analysis on the flexural behaviour of two different sandwich hybrid composites- aramid fiber in the skin and carbon or glass in core; and aramid in core and carbon or glass in skin- has been carried out.
and the bending stress behaviour has been observed to be higher at the core-skin interface than at the core surface [11]. Dong and his colleagues [12,13] reported the flexural properties by taking different grades of glass and carbon fiber reinforced epoxy hybrid composites and suggest that flexure modulus decreases with increase the percentage of glass fiber [14] and flexural strength decreases when carbon/epoxy laminate are replaced by glass/epoxy laminas partially [15]. Naidu et al. [16] reported that the addition of up to 2% graphitic carbon nitride (g-C₃N₄) filler leads to an increase in bending strength of hybrid glass fiber composite. Doddi et al. [17] investigated the influence of fiber angle on the dynamic properties of pineapple leaf fiber hybridized with basalt reinforced epoxy composite. James D et al. [18] observed the effect of bagasse/sisal fiber layup scheme on the mechanical properties of hybrid composites.

So far as the numerical prediction of responses is concerned, several displacement kinematic models [19–24] have been utilized for the analysis of flexural and vibration analysis of composite structures such as laminated composite [25,26] as well as functionally graded structures [27–30], viscoelastic micro-composite beam [31] and sandwich doubly curved nano-composite panel [32]. Reddy et al. [33] employed higher-order shear deformation theory (HSDT) for determining the through-thickness variation of the transverse shear strains, influence of support conditions, loading type etc. in the simply supported cylindrical and spherical orthotropic laminated shells. Xiao et al. [34] analysed static infinitesimal deformations of thick laminated composite elastic plates using a higher-order shear and normal deformable plate theory (HOSNDPT) via a meshless method under different support conditions. The influence of delamination on the flexural responses has also been investigated in detail by numerous researchers [35,36]. The commercial FE tools such as ANSYS have been utilized by various researchers to study the free vibration responses of laminated composite intact [37] as well as damaged beams [38] and hybrid laminates [39]. The FEA in conjunction with the classical lamination theory (CLT) has also been employed to study the effect of stacking sequence on the flexural properties of glass-carbon fiber [40]. Moreover, an optimal design for the flexural behaviour of glass and carbon fiber reinforced polymer hybrid composites has been reported [41].

Subsequently, several studies on bending and stress behaviour of carbon nanotube-reinforced hybrid composite plates have been reported [42–47] The bending and free vibration of thin-to-moderately thick hybrid composite plates reinforced by single-walled carbon nanotubes was analysed [48]. Also, the influence of matrix cracks on bending and vibration behaviours of hybrid laminated plates made of carbon nanotube reinforced functionally graded (CNTR-FG)
layers and conventional graphite fiber reinforced composite layers has been investigated by Lei et al. [49].

This concise review of the literature portrays that numerous studies have been performed to analyse the structural performance of the laminated composite structures both numerically and experimentally. It is safe to infer that the majority of the work has been dedicated to the characterization and evaluation of the influence of hybridisation on the mechanical properties experimentally. Also, a variety of methods (analytical-numerical) have been employed for model verification and performance assessment of newly proposed solution methodologies. In fact, the deflections and stresses in the laminated hybrid composite panel due to varying load, structural parameters, support conditions and hybrid type in the framework of more accurate higher-order displacement model such as HSDT have been reported only in a few literatures. Moreover, the works analysing the flexural responses of curved hybrid composite panels are scarce. Therefore, in order to bridge this knowledge gap, the flexural behaviour of hybrid (Glass-Carbon-Kevlar) laminated composite curved shell panels subjected to varying loading conditions has been analysed. A finite element model has been implemented in the framework of HSDT. A nine-noded Lagrangian isoparametric element is used for discretization purpose. The curvature effect is incorporated to have cylindrical, spherical, elliptical, hyperboloid and flat shell panel geometries. The present model’s accuracy is ascertained by reproducing the benchmark results reported in the published literature using the present scheme. Subsequently, the present model is extended to solve several parametric examples to comprehend the flexural and stress characteristics of the laminated hybrid composites curved shell panels in the light of varying material and geometrical parameters.

2. Mathematical Formulation

The geometry of the curved laminated composite shell panel is depicted in Fig. 1. The length, width and the thickness of the panel are considered to be ‘a’, ‘b’ and ‘h’, respectively. The panel has curvature in longitudinal and transverse direction the radius of which is given by $R_x$ and $R_y$, respectively. Depending on the relation between $R_x$ and $R_y$ five geometrical shapes are realized namely, flat, cylindrical, spherical, elliptical and hyperboloid. Fig. 2 portrays the geometrical shapes and the corresponding relations between $R_x$ and $R_y$. 
Now, the displacement model for the deflection \((u_l, v_m, w_n)\) of any point within the panel in terms of the deflection of any point lying on the midplane is represented as:

\[
\begin{align*}
\{u_l\} & = \{u_l\} \{z^3\} \\
\{v_m\} & = \{v_m\} \{z^3\} \\
\{w_n\} & = \{w_n\} \{z^3\} \\
& \quad i = 0, 1, 2, 3
\end{align*}
\]

(1)

Here, the terms \([u_{l0}, u_{l1}, u_{l2}, u_{l3}]\), \([v_{m0}, v_{m1}, v_{m2}, v_{m3}]\), \([w_{n0}]\) represent the midplane deflections and are obtained from the Taylor series expansion of \(u_l, v_m, w_n\), respectively, and \(\{z^3\} = [1 \quad z \quad z^2 \quad z^3]^T\). The terms \(u_{l0}, v_{m0}\) and \(w_{n0}\) corresponding to \(i = 0\) are the midplane displacements in \(x, y\) and \(z\) directions, respectively. The terms \(u_{l1}\) and \(v_{m1}\) corresponding to \(i = 1\) represent the rotation of the transverse normal to the midplane about \(y\) and \(x\)-axes, respectively. The terms corresponding to \(i = 2\) and \(3\) are the same as the coefficients of \(z^2\) and \(z^3\), respectively.

The stress-strain relation for any lamina having the fibers oriented at an angle \(\theta_{\text{ply}}\) in reference to the principal material axes is expressed as:

\[
\{\sigma\} = \{Q\} \{\varepsilon\}
\]

(2)

where, \(\{\sigma\}, \{Q\}\) and \(\{\varepsilon\}\) are the stress tensor, reduced stiffness matrix, and strain tensors, respectively.

The strain vector of the laminated shell panel can be expressed as [50]:

\[
\{\varepsilon\} = \begin{bmatrix}
\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{zz} & \varepsilon_{xz} & \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{yz}
\end{bmatrix}^T
\]

(3)

The terms corresponding to \(i = 0\) are the midplane displacements in \(x, y, z\) directions, respectively. The terms corresponding to \(i = 1\) represent the rotation of the transverse normal to the midplane about \(y\) and \(x\)-axes, respectively. The terms corresponding to \(i = 2\) and \(3\) are the same as the coefficients of \(z^2\) and \(z^3\), respectively.

The stress vector is formulated as:

\[
\{\sigma\} = [D] \{\varepsilon\}
\]

(4)

The elements of the stiffness matrix \([D]\) are defined as:

\[
[D] = \sum_{k=1}^{n} \int_{z_k-1}^{z_k} \left( \{Q\}_{ij} \{1, z, z^2, \ldots, z^6\} \right) dz
\]

(5)
The numerical solution for the flexure of the curved hybrid laminated composite shell panels is obtained via FEM. The panel model is discretized using a nine-noded isoparametric Lagrangian element (for element details refer to [51]). Each node of the element has nine degrees of freedom In agreement with the displacement model given by Eq. (1). The deflection (\( \delta \)) of a point lying on the mid-surface can be interpolated using the nodal shape function (\( N_i \)) in terms of the nodal deflection \( (\delta)_i \) of the \( i^{th} \) node as:

\[
\delta = \sum_{i=1}^{n} N_i(x, y) \delta_i \tag{6}
\]

The strain vector represented by Eq. (3) is split into the product of the strain interpolation matrix \([Z_e]\) (interpolates strain over thickness) and the mid-plane strains \(\{\varepsilon\}\) as:

\[
\{\varepsilon\} = [Z_e] \{\varepsilon\} \tag{7}
\]

The mid-plane strain vector \(\{\varepsilon\}\) can be decomposed as the product mid-plane deflection \(\delta\) and the matrix of partially differentiated shape functions \([D_e]\) as:

\[
\{\varepsilon\} = [D_e] \{\delta\} \tag{8}
\]

Eq. (1) is now represented in discrete form as:

\[
\{\delta\} = \begin{bmatrix} u_i \\ v_m \\ w_n \end{bmatrix} = [Z_d] \{\delta\} \tag{9}
\]

Here, the matrix \([Z_d]\) interpolates deflection over the thickness coordinate.

The strain energy (U) of the hybrid laminated composite panel is given as [50]:

\[
U = \frac{1}{2} \int \{\epsilon\}^T \{\sigma\} dV = \frac{1}{2} \iint \int_{-h/2}^{+h/2} \{\epsilon\}^T \{\sigma\} dz dy 
\]

\[
= \frac{1}{2} \iint \iint (\{\delta\}^T [\Lambda] [D_e]^T \{\varepsilon\}) dx dy = \frac{1}{2} \iint \iint (\{\delta\}^T [D_e]^T [\Lambda] [D_e] \{\delta\}) dx dy \tag{10}
\]

Eq. (10) is further be rewritten by substituting into it Eq. (5) and Eq. (8) as:

\[
U = \frac{1}{2} \iint \left( \{\delta\}^T [\Lambda] [D_e]^T \{\varepsilon\} \right) dx dy = \frac{1}{2} \iint \left( \{\delta\}^T [D_e]^T [\Lambda] [D_e] \{\delta\} \right) dx dy \tag{11}
\]

where, \([\Lambda] = \int_{-h/2}^{+h/2} \left[ Z_e \right]^T \left[ \bar{Q} \right] Z_e dz\).
The total work done by an externally applied load, $F$, is given by:

$$ W = \int \{ \delta \}^T \{ F \} dA \quad (12) $$

The stiffness of the panel can be expressed in matrix $[K]$ form as [50]:

$$ [K] = \int \left( \sum_{\text{sym}} \right) \int [D_{ij}] \{ \Lambda \} [D_{ij}] d\zeta dA \quad (13) $$

The necessary mathematical equation for bending analysis is given by the relation,

$$ \partial \Pi = \partial (U - W) = 0 \quad (14) $$

where, $\partial$ is the variational symbol and $\Pi$ is the total energy functional.

Now, the final form of equation of equilibrium for the transversely deflected composite shell panel is given by the following equations.

$$ [K] \{ \delta \} = \{ F \} \quad (15) $$

where, $[K]$ and $\{ F \}$ are the global system stiffness matrix and load vectors, respectively. The detailed expression for obtaining bending deflection and bending stress can be referred from Hirwani et al. [36].

### 3. Numerical results and discussion

The HSDT based numerical scheme is now implemented to analyse the flexural responses of curved laminated composite shell panels. An FE code has been developed in MATLAB for the present analysis. Firstly, the convergence of the flexural responses (central deflection, normal and transverse shear stress) predicted by the present scheme is ensured. Also, the accuracy of the predicted responses is justified by comparing the present results with the standard results available in the published literature. Finally, the effect of material property and geometrical parameters on the nondimensional central deflection and stress responses of the hybrid laminated composite panels are computed and discussed in detail. Table 1 lists the material properties [50] (utilized in the present analysis) of hybrid composite flat panels fabricated by usual hand lay-up technique at NIT Rourkela and tested at CIPET, LARPM, Bhubaneswar.

The curved laminated composite shell panels have been considered to be subjected to the following types of mechanical loading (pictorially depicted in Fig. 3):
- Uniformly distributed load (UDL) is given by: \( p(x,y) = q_0 \)

- Sinusoidally distributed load (SDL) is given by: \( q = q_0 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right) \).

The maximum central deflection \( w_c \) of the panels is nondimensionalized as \( \bar{w} = \frac{w_c}{h} \), where, \( h \) is the thickness of panel.

The nondimensional form of normal stresses and shear stresses are given as:

\[
\bar{\sigma}_x = \sigma_x \left( \frac{h^2}{qa^2} \right), \quad \bar{\sigma}_y = \sigma_y \left( \frac{h^2}{qa^2} \right) \quad \text{and} \quad \bar{\tau}_{xz} = \tau_{xz} \left( \frac{h^2}{qa^2} \right)
\]

The coordinates for the maximum value of stresses for the present evaluation are as follows:

In-plane normal stress \( (\sigma_x) = (a/2, b/2, \pm h/2) \).

In-plane normal stress \( (\sigma_y) = (a/2, b/2, \pm h/2) \).

In-plane shear stress \( (\tau_{xz}) = (0, b/2, \pm h/2) \).

The different edge conditions are employed to avoid the rigid body motion for the computation of the numerical responses, details can be referred from Hirwani et al. [36].

### 3.1 Convergence and validation study

#### 3.1.1 Convergence and validation of nondimensional central deflection

Firstly, the convergence of the present responses is confirmed by computing the nondimensional central deflection \( \bar{w} = \frac{w_c}{h} \) for increasing mesh density. An example of a simply supported four-layered cross-ply laminated composite symmetric \([0^0/90^0]_s\) spherical shell panel subjected UDL and SDL taken from Reddy and Liu [33] is solved for the comparison purpose. Table 2 illustrates the present values alongside the reference values and it is inferred that the present responses converge well the mesh refinement. On this basis, a \((6 \times 6)\) mesh is adopted for computational purpose in the following analyses. Moreover, it is worthy to note that present values corresponding to a mesh of \((6 \times 6)\) are in fine agreement with the values reported by reference [33] emphasizing the efficacy of the proposed scheme in predicting the flexural responses.

Additionally, in order to strengthen the confidence in the correctness of the present responses, additional examples of simply supported composite spherical shell panels with different lay-up sequences such as \([0^0/90^0], [0^0/90^0/0^0]\) and \([0^0/90^0\), and varying curvature ratio are subjected to UDL and SDL are solved. Tables 3 and 4 present a comparison of present nondimensional...
central deflection with the values obtained by Reddy and Liu [33] using FSDT and HSDT. The results clearly indicate a close conformance between the present and the reference values thereby establishing the validity of the present approach.

3.1.2 Validation of bending stress

Similarly, the present bending stress has been validated by solving an example of a simply supported three-layered \([0^\circ/90^\circ/0^\circ]\) square plate are computed subjected to sinusoidally distributed load (SDL) as considered by Kant and Swaminathan [52]. The nondimensional normal stresses \((\sigma_x\) and \(\sigma_y\)) are presented in the Fig. 4 (a) and (b), respectively. It is clear that the stress responses obtained using the present model are close to the reference thus justifying the efficacy of the present model in obtaining the bending stress responses.

3.2 Numerical examples

The nondimensional central deflection, normal and transverse shear stresses developed in curved hybrid composite shell panels are now analysed via the present FE implementation of higher-order displacement kinematics. The structural parameters specifically, the thickness ratio \((a/h)\), aspect ratio \((a/b)\), support conditions, geometry, curvature ratio \((R/a)\) and hybrid types are varied in order to study their impact on the central deflections and stresses of the hybrid curved composite shell panels. The material properties (for four-layered antisymmetric \((0^\circ/90^\circ/0^\circ/90^\circ)\) cross ply) utilized for computation in the present analysis are listed in Table 1. The support conditions utilized in the present analysis have been adapted from the reference [36].

3.2.1 Effect of thickness ratio \((a/h)\) on flexural flat plate

The effect of thickness ratios \((a/h)\), i.e. width to thickness \((a/h = 5, 10, 20, 40, 60, 80, 100)\) on the nondimensional central deflections on the bending responses of laminated hybrid composite plate (4C) subjected to UDL and SDL are computed via the present formulation and illustrated in Fig. 5 (a) and (b), respectively. It is obvious that the nondimensional central deflections increases with the increase of thickness ratio as well as the load. It is due to the fact that by decreasing the thickness while keeping the length constant the structure became thin leading to decrease in its stiffness as the stiffness is directly proportional to the thickness. But, for all edges clamped condition, the nondimensional central deflection are more under UDL
than SDL as shown in Fig. 5 (b). The reverse happened for the simply supported case in which the nondimensional central deflections are more under SDL than UDL, shown in Fig. 5 (a).

Similarly, the thickness ratio is now varied as $a/h = 5, 10, 20, 50, 100$ and it’s influence on the nondimensional stresses $\bar{\sigma}_x, \bar{\sigma}_y$ and $\bar{\tau}_{xz}$ developed in the simply supported hybrid composite (4C) square plate subjected to sinusoidal load (SDL) of 0.20 MPa magnitude, is investigated and presented in Fig. 6 (a), (b) and (c) respectively. It is observed from the Fig. 6 (a) and (b) are, nondimensional normal stresses $(\bar{\sigma}_x, \bar{\sigma}_y)$ increase with increase of nondimensional thickness and also found that normal stresses follow approximately similar trend for all the values of thickness ratios (i.e., 5, 10, 20, 50, 100). However, the shear stress decreases as the thickness ratio increases and the same is evident from Fig. 6(c).

### 3.2.2 Effect of aspect ratio $(a/b)$ on the flexural behaviour

The aspect ratio is now varied as $a/b = 0.5, 0.8, 1.1, 1.4, 1.7, 2.0$ and its influence on the nondimensional central deflection of the hybrid composite plate 3G-1C under two types of loading (UDL and SDL) is studied using the proposed model and presented in Fig. 7 (a) and (b), respectively. From the Fig. 7, it is observed that the nondimensional central deflection decreases with increase in the aspect ratio. This is due to the fact that as the aspect ratio increases stiffness of plate decreases. Also, it is evident from the figure that the central deflection is more when all the edges are simply supported (SSSS) than all edges are clamped (CCCC). The deflection has higher value in case of UDL than SDL for both the SSSS and CCCC type cases.

Also, the nondimensional stresses $\bar{\sigma}_x, \bar{\sigma}_y$ and $\bar{\tau}_{xz}$ for the clamped hybrid laminated composite (3G-1C) square plate under sinusoidal load (SDL) of 0.15 MPa magnitude are computed using the present model for varying thickness ratios $(a/b = 0.5, 0.8, 1.1, 1.4, 1.7, 2.0)$ and presented in Fig. 8 (a), (b) and (c), respectively.

It is observed from the Fig. 8 (a) and (b) that the nondimensional normal stresses $(\bar{\sigma}_x, \bar{\sigma}_y)$ increase with the increasing nondimensional thickness. Also, the magnitude of normal stresses decreases with an increase in aspect ratio because of decrease in stiffness of the plate. The variation of normal stresses from top to bottom is more in case of lower aspect ratios. The nondimensional shear stress increases with the increase in aspect ratio, and the same is shown in the Fig. 8 (c).
3.2.3 Effect of geometry on the flexural behaviour

Fig. 9 (a) and (b) shows the nondimensional central deflections with varying geometry (plate, cylindrical, spherical, hyperboloid and elliptical) and load intensity \( (q_0 = 0.10, 0.15, 0.20, 0.25, 0.30 \text{ and } 0.35 \text{ MPa}) \) for a simply supported (SSSS) and fully clamped (CCCC) 2G-1C-1K hybrid laminated composite square shell panel subjected to both UDL and SDL. It has been observed that deflection is not much influenced by the change in geometry with variation of load. The deflection increases as the load increase. The deflection is higher in the case of simply supported panels as compared to all edges clamped condition. Also, the deflection is higher for SDL condition as compared to that under UDL condition.

A similar trend is observed in the variation of normal and shear stresses. Both the normal stress and shear stress values are approximately same for all types of geometry at a specified load as shown in the Fig. 10 (a), (b) and (c).

3.2.4 Effect of support condition on the flexural behaviour

The nondimensional central deflection with varying support condition (SSSS, CCCC, SCSC, CFFF, HHHH and SFSF) and load intensity \( (q_0 = 0.10, 0.15, 0.20, 0.25, 0.30 \text{ and } 0.35 \text{ MPa}) \) for a cylindrical and spherical 4K hybrid composite panel under both UDL and SDL condition are shown in the Fig. 11 (a) and (b), respectively. The deflection increases as the load increases. The deflection is high in case of cantilever type of support (CFFF) than all types of support followed by SFSF, HHHH, SCSC, CCCC and SSSS. Also, from the Fig. 11, it is seen that the deflection is higher under the uniform distributed load (UDL) for corresponding support condition as compared to the sinusoidal load (SDL).

3.2.5 Effect of hybrid types on the flexural behaviour

Fig. 12 (a) shows the nondimensional central deflections with varying hybrid types and load intensity for a square plate under simply support (SSSS) condition and Fig. 12 (b) shows for hyperboloid shell panel under all edges clamped condition (CCCC). The deflections have been carried out under UDL as well as SDL. The deflection increases with the increasing load \( (q_0) \) for both geometries and the same is evident from the Figs. 12 (a) and (b). The value of deflection is the highest in case of 4K hybrid composite panel followed by 3G1C, 2G1C1K, 2G2C and 4C, respectively, in the decreasing order. Also, it has been observed that the deflections in corresponding hybrid schemes are higher in case of uniformly distributed load (UDL) than under the sinusoidal load (SDL).
Interestingly, the variation in normal and shear stresses is insignificant across the hybrid types having cylindrical shell panel geometry with all edges clamped (CCCC) and subjected to UDL of magnitude $q_0 = 0.20$ MPa and the same can be observed from Figs 13 (a), (b) and (c), respectively. As seen in the previous example, the displacement decreases with the increase in the percentage of carbon fibre whereas the strength in bending seems to remain unaffected by the addition of carbon layers.

3.2.6 Effect of curvature ratio ($R/a$) on the flexural behaviour

Finally, Fig. 14 (a) and (b) shows the nondimensional central deflections with varying curvature ratio ($R/a = 5, 10, 20, 50, 100, 200$) for a simply supported (SSSS) and fully clamped (CCCC) spherical shell panel under both the UDL and SDL. The deflections increase with the increasing of curvature ratio for all loads. It is because of the fact that the curvature decreases with increasing curvature ratio leading to reduction in the stiffness of the panels. Moreover, in alignment with the observations made in the example for influence of support conditions, the deflection has higher value in case of all edge simply supported (SSSS) than all edges clamped (CCCC) condition.

4. Conclusion

The composite panel stiffness variation due to the hybridization of different kinds of advanced fibre is examined in this article first-time by utilizing a higher-order kinematic theory. In this regard, the structural strength is verified by evaluating the flexural deflection and stress values due to the inclusion of different advanced fibres (Carbon/Glass/Kevlar). The responses are predicted using the higher-order FEM by solving structural equilibrium equation obtained through the variational principle. To obtain the responses a computer code is prepared by using the formulation in a MATLAB environment. Based on the current numerical analysis by varying the input parameters, a few selective insights are discussed in the following lines.

- Firstly, the proposed numerical model validity is established as per the present convergence as well as the comparison study.
- The deflections are showing an increasing trend when the thickness ratio and the aspect ratio values increase.
- This is because the structure becomes thin when the thickness ratio increases, however slender due to the increase in aspect ratio regardless of the hybridization.
Further, the responses follow a reverse line, i.e. decreasing line when the panel curvature increases and the shallowness increases, which, in turn, reduces the stretching energy in comparison to the bending strength.

The maximum and minimum value of the deflection data can be observed for the simply-supported and clamped type due to the enhancement of the total structural stiffness.

The types of loading have a significant role in the structural deflections due to the total area of application, i.e. higher under UDL whereas lower for the SDL.

Moreover, the change of panel geometries has insignificant influences on the deflection parameter.

The deflection due to the hybridization is showing maximum i.e. flexural structure for 4K type of composite panels and followed by 3G1C, 2G1C1K, 2G2C and 4C, respectively.

The nondimensional form of the normal \((\sigma_x, \sigma_y)\) and shear stress \((\tau_{xz})\) values are following the similar line, as same as the deflection while the thickness and the aspect ratios increase.

Additionally, the nondimensional normal and the shear stress values are invariant for all types hybridization whereas constant for every kind of geometry at a specific loading intensity.

It can be noted that the normal and the shear stress values are higher for the hinged and cantilever types of end support, respectively.

**Nomenclature**

- \(a, b\) and \(h\): Length, width and thickness of the shell panel
- \(a/b\): Aspect ratio
- \(a/h\): Thickness ratio
- \(\delta\): The variational symbol
- \{\(\delta\)\}: Global displacement field vector
- \{\(\bar{\delta}\)\}: The global displacement vector
- \{\(\delta_i\)\}: Displacement vector for \(i^{th}\) node
- \{\(\varepsilon\)\}: Strain vectors
- \{\(\bar{\varepsilon}\)\}: Mid-plane strain vectors
- \{\(\sigma\)\}: The stress vector
- \(\sigma_x\) and \(\sigma_y\): In-plane normal stresses along \(x\) and \(y\)-directions
- \(\tau_{xz}\): Transverse shear stress
- \([D]\): The elements of the stiffness matrix
- \([D_l]\): Matrix of partial derivatives of shape function.
- \(E_1, E_2\) and \(E_3\): Young’s modulus in the principal material direction
\( G_{12} \) \hspace{1cm} In-plane shear modulus
\( G_{13} \) and \( G_{23} \) \hspace{1cm} Out-of-plane shear moduli
\( [K] \) \hspace{1cm} The global system stiffness matrix
\( [F] \) \hspace{1cm} The load vectors
\( N \) \hspace{1cm} Shape function (interpolation function)
\( N_i \) \hspace{1cm} Nodal interpolating functions at \( i^{th} \) node
\( \Pi \) \hspace{1cm} The total energy functional.
\( p, q \) \hspace{1cm} Load function
\( q_0 \) \hspace{1cm} Load factor
\( [\tilde{Q}] \) \hspace{1cm} Reduced stiffness matrix
\( R_x, R_y \) \hspace{1cm} Radius of curvature in longitudinal and transverse directions, respectively
\( R/a \) \hspace{1cm} Curvature ratio
\( u_x, v_m \) and \( w_n \) \hspace{1cm} Displacements of any point along the \( x, y \) and \( z \) principal material coordinate axes, respectively
\( u_{0x}, v_{m0} \) and \( w_{n0} \) \hspace{1cm} Corresponding displacements of a point on the mid-plane
\( u_{11} \) and \( v_{m1} \) \hspace{1cm} Rotations of normal to the about their reference i.e., \( y \) and \( x \)-axes at the mid-surface
\( u_{12}, v_{m2}, u_{13} \) and \( v_{m3} \) \hspace{1cm} Higher order terms assumed from Taylor series expansion
\( U \) \hspace{1cm} The strain energy
\( W \) \hspace{1cm} The total work done
\( w_c \) \hspace{1cm} Central deflection
\( \bar{w} \) \hspace{1cm} Nondimensional central deflection
\( [Z_d] \) and \( [Z_e] \) \hspace{1cm} The thickness coordinate matrices
\( \nu \) \hspace{1cm} Poisson's ratio
\( \rho \) \hspace{1cm} Density of material, kg/m³
\( x, y \) and \( z \) \hspace{1cm} Principal material coordinate axes

Compliance with Ethical Standards:

Funding: This research has not received any funding.

Conflict of Interest: The authors of the article declare that they have no conflict of interest.

References


### Tables:

**Table 1.** Material properties of hybrid composite laminates

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>4C</th>
<th>3G1C</th>
<th>2G2C</th>
<th>2G1C1K</th>
<th>4K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus, $E_1=E_2=E_3$ (GPa)</td>
<td>13.549</td>
<td>10.710</td>
<td>13.289</td>
<td>11.082</td>
<td>8.491</td>
</tr>
<tr>
<td>Shear modulus, $G_{12}=G_{13}$ (GPa)</td>
<td>5.376</td>
<td>4.284</td>
<td>5.316</td>
<td>4.433</td>
<td>3.396</td>
</tr>
<tr>
<td>Shear modulus, $G_{23}$ (GPa)</td>
<td>0.5 $G_{12}$</td>
<td>0.5 $G_{12}$</td>
<td>0.5 $G_{12}$</td>
<td>0.5 $G_{12}$</td>
<td>0.5 $G_{12}$</td>
</tr>
<tr>
<td>Poisson’s ratio ($\nu_{12}=\nu_{23}=\nu_{13}$)</td>
<td>0.26</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Density ($\rho$), kg/m$^3$</td>
<td>1394</td>
<td>1579</td>
<td>1524</td>
<td>1500</td>
<td>1000</td>
</tr>
</tbody>
</table>
Table 2. Convergence and validation of nondimensional central deflection ($\bar{w} = w_c/h$) of cross-ply laminated spherical shells under UDL and SDL ($E_1 = 25E_2$, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$, $\nu = 0.25$, $a/b = 1$, $R/a = 100$, $a/h = 10$, $0^\circ/90^\circ/0^\circ$, $q_0=100$)

<table>
<thead>
<tr>
<th>Mesh size</th>
<th>Nondimensional central deflection ($a/h = 10$)</th>
<th>Nondimensional central deflection ($a/h = 100$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UDL</td>
<td>SDL</td>
</tr>
<tr>
<td>2 × 2</td>
<td>14.3592</td>
<td>8.5065</td>
</tr>
<tr>
<td>4 × 4</td>
<td>11.8426</td>
<td>7.7272</td>
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<tr>
<td>5 × 5</td>
<td>11.1217</td>
<td>7.2658</td>
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<tr>
<td>6 × 6</td>
<td>11.1213</td>
<td>7.2530</td>
</tr>
<tr>
<td>7 × 7</td>
<td>11.1526</td>
<td>7.2811</td>
</tr>
<tr>
<td>8 × 8</td>
<td>11.1407</td>
<td>7.2683</td>
</tr>
<tr>
<td>9 × 9</td>
<td>11.1544</td>
<td>7.2821</td>
</tr>
<tr>
<td><strong>Reddy and Liu [33]</strong></td>
<td><strong>10.8980</strong></td>
<td><strong>7.1240</strong></td>
</tr>
</tbody>
</table>

Table 3. Validation of nondimensional central deflections ($\bar{w} = w_c/h$) of cross-ply laminated spherical shells under UDL and SDL ($a/h=10$) ($E_1 = 25E_2$, $G_{12}=G_{13}=0.5E_2$, $G_{23} = 0.2E_2$, $\nu = 0.25$, $a/b = 1$, $q_0 = 100$)

<table>
<thead>
<tr>
<th>Curvature ratio ($R/a$)</th>
<th>Theory</th>
<th>Nondimensional central deflection ($\bar{w} = w_c/h$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>UDL, $a/h=10$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0^\circ/90^\circ$</td>
</tr>
<tr>
<td></td>
<td>Present numerical (HSDT)</td>
<td>17.966</td>
</tr>
<tr>
<td></td>
<td>HSDT [33]</td>
<td>18.744</td>
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</tbody>
</table>
Table 4. Nondimensional central deflections \((\bar{w} = w_c/h)\) of cross-ply laminated spherical shells under UDL and SDL \((a/h=100)\) \((E_1=25E_2, G_{12}=G_{13}=0.5E_2, G_{23}=0.2E_2, v = 0.25, a/b = 1, q_0 = 100, SSSS)\)

<table>
<thead>
<tr>
<th>Curvature ratio ((R/a))</th>
<th>Theory</th>
<th>UDL, (a/h=100)</th>
<th>SDL, (a/h=100)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nondimensional central deflection ((\bar{w} = w_c/h))</td>
<td>Lamination scheme</td>
<td>Lamination scheme</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0^\circ/90^\circ) &amp; (0^\circ/90^\circ/0^\circ) &amp; ([0^\circ/90^\circ]_x) &amp; (0^\circ/90^\circ) &amp; (0^\circ/90^\circ/0^\circ) &amp; ([0^\circ/90^\circ]_x)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>FSDT [33]</td>
<td>1.753</td>
<td>1.511</td>
</tr>
<tr>
<td></td>
<td>HSDT [33]</td>
<td>1.751</td>
<td>1.509</td>
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<tr>
<td></td>
<td>Present numerical (HSDT)</td>
<td>1.771</td>
<td>1.523</td>
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<tr>
<td>10</td>
<td>FSDT [33]</td>
<td>5.542</td>
<td>3.644</td>
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<tr>
<td></td>
<td>HSDT [33]</td>
<td>5.538</td>
<td>3.642</td>
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<tr>
<td></td>
<td>Present numerical (HSDT)</td>
<td>5.574</td>
<td>3.684</td>
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<tr>
<td>--------</td>
<td>---------------------------</td>
<td>--------</td>
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<tr>
<td><strong>20</strong></td>
<td>FSDT [33]</td>
<td>11.273</td>
<td>5.547</td>
</tr>
<tr>
<td></td>
<td>HSDT [33]</td>
<td>11.268</td>
<td>5.550</td>
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<tr>
<td></td>
<td>Present numerical (HSDT)</td>
<td>11.302</td>
<td>5.618</td>
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<tr>
<td></td>
<td>Present numerical (HSDT)</td>
<td>15.734</td>
<td>6.571</td>
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</table>

Figures:
Fig. 1 Curved composite shell panel geometry and stacking sequence

Fig. 2 Curved shell panel shapes
Fig. 3 Types of mechanical loading (a) UDL, (b) SDL.
Plate, $[0^0/90^0/0^0]$, $a/b = 1$, SDL

(a)

Nondimensional normal stress ($\bar{\sigma}_x$)

(b)

Nondimensional normal stress ($\bar{\sigma}_y$)

(E_1/E_2 = 25, E_2 = E_3 = 7 GPa, G_{12} = G_{13} = 0.5 E_2, G_{23} = 0.2 E_2, \nu_{12} = \nu_{23} = \nu_{13} = 0.25$ [52])

Fig. 4 Validation of nondimensional in-plane normal stress (a) $\bar{\sigma}_x$, (b) $\bar{\sigma}_y$ through the nondimensional thickness ($z/h$) of simply supported square plate of three layers ($0^0/90^0/0^0$) under sinusoidally distributed transverse load.
Fig. 5 Nondimensional central deflection of the flat shell panel for different thickness ratio under UDL and SDL with two different support conditions (a) SSSS, (b) CCCC.
Hybrid type: 4C, Plate, $a/b = 1$, SSSS

$q_0 = 0.20$ MPa, SDL

(a)

(b)
Fig. 6 Variation of in-plane nondimensional stresses through the thickness of four-layered 4C hybrid simply supported square plate under sinusoidal transverse load (a) (\(\bar{\sigma}_x\)), (b) (\(\bar{\sigma}_y\)), and (c) (\(\bar{\tau}_{xz}\)).
Fig. 7 Nondimensional central deflection of the flat shell panel for different aspect ratio under UDL and SDL with two different support conditions (a) SSSS, (b) CCCC.
Hybrid type: 3G1C, Plate, \(a/h = 50\), CCCC
\(q_0 = 0.15\) MPa, SDL

(a)

(b)
Fig. 8 Variation of in-plane nondimensional stresses through the thickness of four-layered 3G-1C hybrid clamped square plate under sinusoidal transverse load (a) \( \sigma_x \), (b) \( \sigma_y \), and (c) \( \tau_{xz} \).
Fig. 9 Nondimensional central deflection for different geometry under UDL and SDL with different support conditions (a) SSSS, (b) CCCC.
Nondimensional thickness \( \left( \frac{z}{h} \right) \)

Nondimensional normal stress \( \left( \bar{\sigma}_x \right) \)

(a)

(b)

Hybrid type: 2G1C1K, CCCC, \( q_0 = 0.20 \) MPa, UDL

\( a/b = 1 \), \( a/h = 50 \), \( R/a = 50 \),

- Plate
- Cylindrical
- Spherical
- Hyperboloid
- Elliptical

Hybrid type: 2G1C1K, CCCC, \( q_0 = 0.20 \) MPa, UDL

\( a/b = 1 \), \( a/h = 50 \), \( R/a = 50 \),

- Plate
- Cylindrical
- Spherical
- Hyperboloid
- Elliptical
Fig. 10 Variation of in-plane nondimensional stresses through the thickness of 4-layer 2G1C1K hybrid clamped square plate under uniform distributed load (a) Normal stress ($\sigma_x$), (b) Normal stress ($\sigma_y$), and (c) shear stress ($\tau_{xz}$).
Fig. 11 Nondimensional central deflection for different support condition under UDL and SDL (a) Cylindrical, (b) Spherical.
Fig. 12 Nondimensional central deflection for hybrid types under UDL and SDL

(a) Plate, SSSS (b) Hyperboloid, CCCC.
Cylindrical, CCCC, \( q_0 = 0.20 \) MPa, UDL
\[ \frac{a}{b} = 1, \frac{a}{h} = 50, \frac{R}{a} = 50 \]

(a)

UDL, CCCC, \( q_0 = 0.20 \) MPa, Cylindrical
\[ \frac{a}{b} = 1, \frac{a}{h} = 50, \frac{R}{a} = 50 \]

(b)
Nondimensional shear stress ($\bar{\tau}_{xz}$)

Cylindrical, CCCC. $q_0 = 0.20$ MPa, UDL

$alb = 1, alh = 50, R/a = 50$

Nondimensional thickness ($z/h$)

**Fig. 13** Variation of in-plane nondimensional stresses through the thickness of clamped cylindrical shell panel of different hybrid schemes under uniform distributed load (a) ($\bar{\sigma}_x$), (b) ($\bar{\sigma}_y$), and (c) ($\bar{\tau}_{xz}$).
Fig. 14 Nondimensional central deflection of the spherical panel for different curvature ratio \( R/a \) under UDL and SDL with two different support conditions (a) SSSS, (b) CCCC.
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