# Load Shifting Demand Response in Energy Scheduling Based on Payment Cost Minimization Auction Mechanism

H.jafarirad[1], M. Rashidinejad[2*] and A. Abdollahi[3]

[1] Department of Electrical Engineering, Kerman Graduate University of Technology, Kerman, Iran. Email Address: hamed.jafarirad@gmail.com, Phone: 9809133472942.

[2*] Department of Electrical Engineering, Shahid Bahonar University of Kerman, Kerman, Iran. Email Address: mrashidi@uk.ac.ir, Phone: 9809133414044

[3]Department of Electrical Engineering, Shahid Bahonar University of Kerman, Kerman, Iran. Email Address: a.abdollahi@uk.ac.ir, Phone:9809131989225

## Abstract

Demand response (DR) is proven to be very efficacious for load mitigation especially in peak time period. On the other hand, DR benefits both consumers and system operators to reduce their payment and system operating cost, respectively. Offer cost minimization is currently used as the clearing mechanism with locational marginal pricing scheme to determine the consumers' payment. These clearing and pricing mechanisms are inconsistent as the system cost is being minimized, but the final payments are calculated based on marginal prices. Payment cost minimization (PCM) auction as a price-based clearing mechanism is envisaged to be an effective alternative to solve the issue. This paper shows how to include DR in PCM mechanism to further reduce the consumers' payment. It facilitates utilizing price responsive consumers for load shifting DR (LSDR) in a PCM auction. The optimization problem is modeled as a mixed-integer nonlinear bi-level programming. Duality theorem, Karush-Kuhn-Tucker conditions and integer algebra are used to convert such a problem to a single level mixed-integer linear programing problem. This problem is then solved by CPLEX solver in GAMS. The impacts of LSDR are studied by implementing the proposed formulation to solve the clearing problem in the case studies deriving promising numerical results.

**Keywords**: payment cost minimization, load shifting demand response, social welfare maximization, offer cost minimization, marginal pricing scheme, bi-level programing.

## 1. Introduction

According to US Federal Energy Regulatory Commission order no. 888 [1], the goal of deregulation was to encourage investments to provide cheaper electric power generation by competing power producers. Under deregulation, increasing electric demand can impose unlimited market power to a few large power plants typically having fossil fuel generators that may have a fast start and ramping time, resulting in relatively large market clearing prices (MCPs) both in energy as well as ancillary services [2]. Offer cost minimization (OCM) auction mechanism which is similar to the classical unit commitment (UC) in the case of inelastic demand, is being used in most electricity markets for market clearing, while marginal pricing schemes are then usually used to determine final prices [3]. This type of auctions, when the supply bids represent the real production costs may lead to a social welfare maximization as a factual goal. Because of producers strategic biding for making more benefits, such an assumption does not hold in reality. Moreover, this type of auctions might be inconsistent with marginal pricing schemes, since total payment cost is different from the minimizing the total offer costs. Therefore, consumer payments can be significantly higher than the minimized offer costs [4].

Consumer payment minimization has been proposed as a solution to the lack of incentives for suppliers to offer their actual costs. Such an auction mechanism directly minimizes the payment costs that can be considered as an instrument to protect consumers against exercising market power by suppliers via submitting the production bids higher than their actual costs [5]. From mathematical view point, the objective function of payment cost minimization (PCM) is more complicated than OCM. The reason lies in the existence of

MCPs in the payment terms of each consumer, which leads to a self-referring optimization problem. Non-linear terms in the objective function and constraints of PCM problem may increase its complexity. In [4] the authors propose an augmented Lagrange Relaxation (LR) employing surrogate optimization method to solve PCM problem, while the results are near optimal and some modifications should be done to guarantee the solution feasibility. References [6] and [7] also applied an almost the same optimization technique with some modifications to include the impact of transmission network, where in literature PCM is addressed both with and without transmission network constraints [8]. In the case of neglecting transmission constraints, marginal pricing scheme results in a uniform price as MCP [9], in which is compared with the results of OCM mechanism. In [10], the authors formulate the optimization problem as a general bi-level programming problem, where the resulting such bi-level programming formulation is transformed into an equivalent single level mixed integer linear programing by means of Karush-Kuhn-Tucker (KKT) optimality condition doing the conversion of some nonlinearity to the linear equivalent.

Considering of transmission network constraint makes the problem more complicated, where each buses of the network has its own locational marginal price (LMP) [11]. The authors propose a method for solving joint energy and reserve PCM auction incorporating network security constraints. In [12], LMP’s behavior under PCM and OCM mechanisms are compared showing that the sensitivity of LMPs under the PCM mechanism is lower comparing to the OCM mechanism. Some uncertainties such as load fluctuation and component availability are added to the main problem in [13], in which the proposed model resulted to a tri-level optimization problem that is solved after converting to an equivalent single level programming problem. In another study, a PCM unit commitment model is proposed to incorporate the uncertainty associated with wind generation [14], while the optimization problem is solved using GA, where the global optimality cannot be guaranteed.

It should be noted that none of the aforementioned studies did not consider the demand side participation in their models.

It should be noted that, in the absence of demand-side participation, the price spikes, supply shortages and market power may occur seriously. If the retail consumers purchase electricity based on time-invariant prices, they have no incentive to respond to the wholesale prices. As shown in Fig.1, when the supply is restricted due to some reasons, e.g., unexpected generation outage and/or transmission congestion, substantial reduction of price ($P-P'$) may take place even if a small fraction of the load ($Q-Q'$) responds to the price variations [15].

There are some studies investigating the benefits of demand response (DR). In [16], an economic model based on price elasticity and consumer benefit function is introduced for analyzing incentive-based DR programs on the load curve characteristics improvement. Time-based DR in [17] and generators rescheduling as a demand side bidder in [18] was introduced as proper tools for congestion management. Emergency DR program (EDRP) as an incentive-based DR was included in the unit commitment problem in [19]. Incorporating load shifting demand response (LSDR) in the security constraint unit commitment problem is proposed in [20], while in [21] and [22], stochastic models of DR for reserve scheduling has been discussed. A dynamic economic model of DR programs based on the concept of the flexible elasticity and the consumer benefit function is proposed in [23]. Proposing an effective mechanism for demand-side participation in electricity market is a step of utmost importance in market design, since some large consumers may have the storage facilities and the ability of direct participation in the wholesale market. In such a condition, they can produce and store electricity during low-price periods and may use it at high-price periods [24]. Price responsive loads were incorporated in PCM mechanism in the following studies. In [25] demand bids are considered in a two-layer structure where there is no solution methodology provided. Although, in [26] and [27], some simple nonstandard pricing schemes were applied, but such a simplified pricing scheme cannot be implemented in practice. It should be noted that in [28] the authors pointed out the main advantage of DR, i.e., the load shifting capability while it is not employed. In fact, the demand-side bids might be rejected if their values are lower than the MCPs.

Here in this paper, a particular type of DR, the so-called, LSDR is utilized. A partial LSDR will be implemented in a day-ahead wholesale PCM-based electricity market. Applying such a mechanism, the effects of the percentage of load shifting demand on the alleviated load profiles and MCP will be discussed, afterwards. Because of having the product of MCPs and consumption levels as two continuous decision variables, associated with integer decision variables, the optimization is nonconvex problem that can be treated a mixed integer non-linear (MINLP) problem. Similar to studies in [29], a bi-level programing framework is applied here to schedule the generating units where the price-responsive loads are determined in the upper level along with the unit commitment status, and generation/consumption levels are accounted in lower level. Bi-level programming is suitable to model such problems where one agent, the leader, optimizes its objective function (upper-level problem) incorporating a second agent, the follower, will react by optimizing its own objective function (lower-level problem). These models are relevant in those situations where the actions of the follower affect the decision making of the leader. This is the case in price-based market clearing: the selection of accepted bids and offers (upper-level problem) depends on MCPs (lower-level problem), which are in turn determined based on the set of accepted bids and offers. MCPs in different hours are computed as the shadow prices of power balance constraints. By applying a primal-dual transformation to a mixed integer nonlinear problem that is converted to a single level mixed integer nonlinear problem [30]. Bilinear terms of the single level mixed integer nonlinear problem regarding to product of energy prices and consumption levels are linearized by use of complementary slackness of KKT optimality condition at lower level. In fact, a mixed integer linear problem can be solved via shelf branch and bond method that guarantees the optimality [31]. Here, a pseudo novel approach for LSDR on a portion of forecasted load in PCM auction mechanism is proposed. Other than the load balance and capacity constraints, the intertemporal constraints of generation scheduling are also incorporated in the proposed methodology.

The remaining parts of the paper is organized as follows: Section 2 describes a load shifting model and its mathematical constraints. In Section 3, the formulation of scheduling problem with price sensitive demands is presented, while section 4 describes the proposed solution methodology. Simulation studies are carried out, while numerical results and discussions are presented in section 5. Finally concluding remarks and possible future works are provided in Section 6.

## 2. LSDR model

As it is mentioned in the introduction, the DR program employed in this paper as a part of total forecasted load ($P_{jt}^{d}$), is suitable for the industrial consumers with storage facilities. The other types of loads, e.g., residential loads, are considered as inelastic loads ($D_{jt}^{z}$), as shown by Eq. (1).

$$D_{jt}^{total} = D_{jt}^{z} + P_{jt}^{d} \tag{1}$$

The basic concept of the proposed modeling is that this type of consumers may produce and store electricity during the low-price periods in order to meet the demand at high-price periods. The load participation factor (LPF) is then defined as the ratio of price responsive demand to the total demand, Eq. (2).

$$\text{LPF}=\frac{P_t^d}{D_t^{total}} \tag{2}$$

As Fig. 2 shows, total load in the auction framework consists of two categories, the price taking demand and the price-responsive demand. Price taking demand will receive a specified volume ($D_t^z$) for all hours of the scheduling horizon.

The benefit of consuming demand by price taking bidders due to computational reason is taken as zero. The price-responsive bid allows consumers to submit their bids for the amount of their demand that are sensitive to electricity price. Therefore, similar to generators' offer blocks, consumers' multi-segment bids have two important characteristics, i.e., benefit of consuming demand and consumption limits. Equation (3) shows a gross surplus of price sensitive loads based on the accepted demand side bids ($P_{bjt}^d$) and regarding the marginal values that the consumers submit for these bids ($C_{bjt}^d$).

$$GS_t = \sum_j \sum_b C_{bjt}^d P_{bjt}^d \tag{3}$$

In Eq. (3), *b* is the index of a bidding block and *j* is the index of a demand-side bidder. Total consumption level for price responsive demand ($P_{jt}^d$) is provided in Eq. (4).

$$P_{jt}^d = \sum_{b \in \beta_j} P_{bjt}^d \tag{4}$$

Hourly consumption limit showed by Eq. (5) and daily energy requirement limit, showing by Eq. (6), are two constraints for the load shifting characteristics of price-sensitive demands.

$$V_{jt} P_{jt\min}^d \le P_{jt}^d \le V_{jt} P_{jt\max}^d \tag{5}$$

$$0 \le \sum_t P_{jt}^d \le E_j \tag{6}$$

$$0 \le P_{bjt}^{d} \le P_{bjt\max}^{d} \tag{7}$$

$P_{jt\min}^{d}$ and $P_{jt\max}^{d}$ are the minimum and maximum amounts of active power that can be consumed during scheduling period *t*. $V_{jt}$ is the acceptance status of demand *j* and $E_j$ is the maximum amount of energy that is required by bidder *j* over the optimization horizon.

## 3. PCM with LSDR problem formulation

In this section mathematical formulation of optimization problem is presented. As mentioned before, the problem is for energy scheduling of day-ahead pool-based electricity market considering LSDR based on PCM auction [32]. All intertemporal constraints of generation and marginal pricing scheme except transmission constraints are modeled in this section. A bi-level programing technique is modeled by the following mathematical statements [33].

Higher level problem:

$$\min \sum_{t \in T} (\lambda_t D_t + \sum_{i \in I} (SU_{it} + SD_{it} + V_{it} O_{it}^{NL})) \tag{8}$$

s.t:

$$SU_{it} \ge O_{it}^{su} (V_{it} - V_{it-1}) \qquad i \in I, t \in T \tag{9}$$

$$SD_{it} \ge O_{it}^{sd} (V_{it-1} - V_{it}) \qquad i \in I, t \in T \tag{10}$$

$$\begin{aligned}
&\sum_{t=1}^{L_i} (1 - V_{it}) = 0 && i \in I \\
&\sum_{q=t}^{t+UT_i-1} V_{iq} \ge UT_i (V_{it} - V_{it-1}) && i \in I, t = (L_i + 1)...(\mathrm{n}_T \text{-} UT_i + 1) \\
&\sum_{q=t}^{n_T} (V_{iq} - (V_{it} - V_{it-1})) \ge 0 && i \in I, t = (\mathrm{n}_T - UT_i + 2)...(\mathrm{n}_T) \\
&L_i = \min(n_T, (UT_i - UT_i^{0}) V_{i0})
\end{aligned} \tag{11}$$

$$\sum_{t=1}^{F_i}(V_{it})=0 \qquad i\in I$$

$$\sum_{q=t}^{t+DT_i-1}(1-V_{iq})\geq DT_i(V_{it-1}-V_{it}) \qquad i\in I, t=(F_i+1)...(\mathrm{n}_T\text{-}DT_i+1) \tag{12}$$

$$\sum_{q=t}^{n_T}(1-V_{iq}-(V_{it-1}-V_{it}))\geq 0 \qquad i\in I, t=(n_T-DT_i+2)...(n_T)$$

$$F_i=\min(n_T,(DT_i-DT_i{}^0)(1-V_{i0})$$

$$V_{it}\in(0,1) \qquad i\in I, t\in T \tag{13}$$

$$V_{jt}\in(0,1) \qquad j\in J, t\in T, \tag{14}$$

The on-off status of generation units ($V_{it}$) and on-off status of consumers offer acceptance ($V_{jt}$) are the binary variables of upper level. The start-up ($SU_{it}$) and shut-down ($SD_{it}$) costs of generation unit *i* is found according to offers submitted by this unit ($O_{it}^{sd}, O_{it}^{su}$) and units status following (9-10). The upper level problem minimizes consumer payment and comprises two terms. First term is consumers' energy payment, in which $\lambda_t$ and $D_t$ are continuous variables associated with hourly energy marginal price and consumption level. This term makes the problem complicated due to the bilinear product of two continuous variables. The second term is related to the start-up, shut-down and no-load costs of generation units. These costs are fully compensated in the objective function, while minimum up- and down-time constraints are provided in Eqs. (11) and (12), respectively. The integrality constraints of binary variables are provided in Eqs. (13)-(14).

Lower level problem:

$$\max(\sum_t(GS^t-OC^t)=(\sum_t\sum_j\sum_b C_{bjt}^d P_{bjt}^d-\sum_t\sum_i\sum_o C_{oit}^g P_{oit}^g)) \tag{15}$$

s.t:

$$\sum_{j\in J}D_{jt}^z+\sum_{j\in J}P_{jt}^d=\sum_{i\in I}P_{it}^g \qquad \mathrm{t}\in\mathrm{T} \qquad (\lambda_t) \tag{16}$$

$$V_{it}P_{it\min}^g\leq P_{it}^g\leq P_{it\max}^g\mathrm{V}_{it} \qquad \mathrm{t}\in\mathrm{T}, \mathrm{i}\in\mathrm{I} \qquad (\theta_{it}^{lo},\theta_{it}^{up}) \tag{17}$$

$$0\leq P_{oit}^g\leq P_{oit\max}^g \qquad \mathrm{t}\in\mathrm{T}, \mathrm{i}\in\mathrm{I}, \mathrm{o}\in\mathrm{O}_i \qquad (\beta_{oit}^{lo},\beta_{oit}^{up}) \tag{18}$$

$$P_{it}^{g} \le P_{it-1}^{g} + RU_i V_{it} - P_{it\max}^{g}(1-V_{it}) \qquad t \in T,\ i \in I \qquad (\xi_{it}) \tag{19}$$

$$P_{it}^{g} \ge P_{it-1}^{g} - RD_i V_{it-1} - P_{it\max}^{g}(1-V_{it-1}) \qquad t \in T,\ i \in I \qquad (\delta_{it}) \tag{20}$$

$$P_{it}^{g} \le P_{it\max}^{g} V_{it+1} + RD_i (V_{it} - V_{it+1}) \qquad t=1....n_T-1,\ i \in I \qquad (\varepsilon_{it}) \tag{21}$$

$$P_{it}^{g} = \sum_{o \in O_i} P_{oit}^{g} \qquad t \in T,\ i \in I \qquad (\gamma_{it}) \tag{22}$$

$$V_{jt} P_{jt\min}^{d} \le P_{jt}^{d} \le V_{jt} P_{jt\max}^{d} \qquad t \in T,\ j \in J \qquad (\rho_{jt}^{lo}, \rho_{jt}^{up}) \tag{23}$$

$$0 \le \sum_{t} P_{jt}^{d} \le E_j \qquad j \in J \qquad (\alpha_j^{lo}, \alpha_j^{up}) \tag{24}$$

$$P_{jt}^{d} = \sum_{b \in \beta_j} P_{bjt}^{d} \qquad t \in T,\ j \in J \qquad (\vartheta_{jt}) \tag{25}$$

$$0 \le P_{bjt}^{d} \le P_{bjt\max}^{d} \qquad t \in T,\ j \in J,\ b \in \beta_j \qquad (\mu_{bjt}^{lo}, \mu_{bjt}^{up}) \tag{26}$$

The lower level objective function, Eq. (15) is declared social welfare maximization (SWM) which is the difference between consumer surplus and generation cost according to offered bid blocks. This optimization is in fact a multi-period economic dispatch bearing in mind that the on/off variables $V_{it}$ and $V_{jt}$ are supplied by the upper level optimization. Power generations ($P_{it}^{g}$), power consumptions ($P_{jt}^{d}$), awarded levels of generation offer ($P_{oit}^{g}$) and demand bidding blocks ($P_{bjt}^{d}$) are continuous variables at this level. Generation load balance in each hour Eq. (16), the capacity limitations of generation units Eq. (17), generation limit in each offer block Eq. (18), ramp-up and start-up ramp rate Eq. (19), ramp-down Eq. (20) and shut-down ramp rate Eq. (21) form the list of generation-side constraints in this optimization. It is assumed that start-up and ramp-up rates are the same. The same assumption applies to ramp-down and shut-down ramp rate. Equations (22) -(26) describe the consumption constraints as mentioned before.

## 4. Solution methodology

In bi-level programing, any solution procedure attempting to find a global optimum must devise a system to enumerate the solution space. Such approach cannot be taken for large scale systems. Without such state enumeration, only the local optima can be guaranteed. In this paper, following Eq. [28], the proposed solution methodology is to convert the mixed integer nonlinear bi-level program with bilinear terms which was introduced in the previous Section to an equivalent single-level mixed integer linear problem. To this end, the duality theorem of linear programming, integer algebra and KKT optimality conditions are employed through the following two-step procedure.

*Step1: nonlinear single-level equivalent*

In the bi-level formulation of the original problem, the lower level problem is a linear programming problem because lower level problem, i.e, Eqs. (15)-(26), is parameterized in terms of the upper level binary variables. Therefore, it can be replaced by its equivalent KKT optimality conditions, where the Lagrangian function associated with the lower level is presented in Eq. (27).

$$
\begin{aligned}
&L(P_{it}^{g}, P_{oit}^{g}, P_{jt}^{d}, P_{bjt}^{d}, \lambda_t, \theta_{it}^{lo}, \theta_{it}^{up}, \beta_{oit}^{lo}, \beta_{oit}^{up}, \xi_{it}, \gamma_{it}, \delta_{it}, \varepsilon_{it}, \vartheta_{jt}, \rho_{jt}^{lo}, \rho_{jt}^{up}, \alpha_{j}^{lo}, \alpha_{j}^{up}, \mu_{bjt}^{lo}, \mu_{bjt}^{up}) = \\
&-\sum_t \sum_j \sum_b C_{bjt}^{d} P_{bjt}^{d} + \sum_t \sum_i \sum_o C_{oit}^{g} P_{oit}^{g} + \sum_t \lambda_t (\sum_j D_{jt}^{z} + \sum_j P_{jt}^{d} - \sum_i P_{it}^{g}) + \sum_t \sum_i (\theta_{it}^{lo}(-P_{it}^{g} + V_{it} P_{it\,\min}^{g}) + \\
&\theta_{it}^{up}(P_{it}^{g} - V_{it} P_{it\,\max}^{g})) - \sum_t \sum_i \gamma_{it}(P_{it}^{g} - \sum_o P_{oit}^{g}) + \sum_t \sum_i \sum_o (\beta_{oit}^{up}(P_{oit}^{g} - P_{oit\,\max}^{g}) - \beta_{oit}^{lo} P_{oit}^{g}) \\
&+\sum_t \sum_i \xi_{it}(P_{it}^{g} - (P_{it-1}^{g} + RU_i V_{it} - P_{it\,\max}^{g}(1 - V_{it}))) + \sum_t \sum_i \delta_{it}(-P_{it}^{g} + P_{it-1}^{g} - RD_i V_{it-1} - P_{it\,\max}^{g}(1 - V_{it-1})) \\
&+\sum_t \sum_i \varepsilon_{it}(P_{it}^{g} - (P_{it\,\max}^{g} V_{it+1} + RD_i (V_{it} - V_{it+1}))) + \sum_t \sum_j (\rho_{jt}^{lo}(-P_{jt}^{d} + V_{jt} P_{jt\,\min}^{d})) + \\
&\sum_t \sum_j \rho_{jt}^{up}(P_{jt}^{d} - V_{jt} P_{jt\,\max}^{d}) - \sum_j (\alpha_{lo}^{j} \sum_t P_{jt}^{d}) + \sum_j (\alpha_j^{up}(\sum_t P_{jt}^{d} - \mathrm{E}_j)) - \sum_t \sum_j (\vartheta_{jt}(P_{jt}^{d} - \sum_{b \in \beta_j} P_{bjt}^{d})) \\
&+\sum_t \sum_j \sum_b (\mu_{bjt}^{up}(P_{bjt}^{d} - P_{bjt\,\max}^{d}) - \mu_{bjt}^{lo} P_{bjt}^{d})
\end{aligned}
\tag{27}
$$

Primal feasibility constraints (16)-(26), dual feasibility constraints (28)-(38) and complementary slackness conditions and KKT optimality conditions replace the lower level.

$$\frac{\partial L}{\partial P_{it}^{g}} = 0 \rightarrow -\lambda_t - \theta_{it}^{lo} + \theta_{it}^{up} - \gamma_{it} + \xi_{it} - \xi_{it+1} - \delta_{it} + \delta_{it+1} = 0 \quad t = 1 \ldots \mathrm{n}_T - 1, i \in I \tag{28}$$

$$\frac{\partial L}{\partial P_{it}^{g}} = 0 \rightarrow -\lambda_T - \theta_{iT}^{lo} + \theta_{iT}^{up} - \gamma_{iT} + \xi_{iT} - \delta_{iT} = 0 \quad t = \mathrm{n}_T, i \in I \tag{29}$$

$$\frac{\partial L}{\partial P_{jt}^{d}} = 0 \rightarrow \lambda_t - \rho_{jt}^{lo} + \rho_{jt}^{up} - \alpha_j^{lo} + \alpha_j^{up} - \vartheta_{jt} = 0 \quad t \in T, j \in J \tag{30}$$

$$\frac{\partial L}{\partial P_{oit}^{g}} = 0 \rightarrow C_{oit}^{g} - \beta_{oit}^{lo} + \beta_{oit}^{up} + \gamma_{it} = 0 \quad t \in T, i \in I, o \in O_i \tag{31}$$

$$\frac{\partial L}{\partial P_{bjt}^{d}} = 0 \rightarrow -C_{bjt}^{d} + \vartheta_{jt} - \mu_{bjt}^{lo} + \mu_{bjt}^{up} = 0 \quad t \in T, j \in J, b \in \beta_j \tag{32}$$

$$\theta_{it}^{lo}, \theta_{it}^{up} \geq 0 \quad t \in T, i \in I \tag{33}$$

$$\beta_{oit}^{lo}, \beta_{oit}^{up} \geq 0 \quad t \in T, i \in I, o \in O_i \tag{34}$$

$$\delta_{it}, \xi_{it}, \varepsilon_{it} \geq 0 \quad t \in T, i \in I \tag{35}$$

$$\rho_{jt}^{lo}, \rho_{jt}^{up} \geq 0 \quad t \in T, j \in J \tag{36}$$

$$\alpha_{jt}^{lo}, \alpha_{jt}^{up} \geq 0 \quad t \in T, j \in J \tag{37}$$

$$\mu_{bjt}^{lo}, \mu_{bjt}^{up} \geq 0 \quad t \in T, j \in J, b \in \beta_j \tag{38}$$

Based on the findings of [34], the linearization of complementary slackness conditions adds some more binary variables that increase the computational time. These complementary slackness condition help to replace the nonlinear terms of the objective function applying strong duality condition in (39).

$$
\begin{aligned}
&-\sum_t\sum_j\sum_b C^d_{bjt}P^d_{bjt} + \sum_t\sum_i\sum_o C^g_{oit}P^g_{oit} = \sum_t \lambda_t(\sum_j D^z_{jt}) - \sum_t\sum_i(\theta^{lo}_{it}(V_{it}P^g_{it\min}) + \theta^{up}_{it}(-V_{it}P^g_{it\max}))\\
&-\sum_t\sum_i\sum_o(\beta^{up}_{oit}(P^g_{oit\max})) - \sum_{t\neq 1}\sum_i \xi_{it}((RU_iV_{it} - P^g_{it\max}(1-V_{it})))\\
&-\sum_{t\neq 1}\sum_i \delta_{it}(RD_iV_{it-1} + P^g_{it\max}(1-V_{it-1})) - \sum_t\sum_i \varepsilon_{it}((P^g_{it\max}V_{it+1} + RD_i(V_{it} - V_{it+1})))\\
&+\sum_t\sum_j(\rho^{lo}_{jt}(V_{jt}P^d_{jt\min})) - \sum_t\sum_j \rho^{up}_{jt}(V_{jt}P^d_{jt\max}) - \sum_j(\alpha^{up}_j(\mathrm{E}_j)) - \sum_t\sum_j\sum_b(\mu^{up}_{bjt}(P^d_{bjt\max}) - \mu^{lo}_{bjt}P^d_{bjt})\\
&+\sum_i \delta_{i1}(\mathrm{P}^g_{i0} - \mathrm{RD}_i\,\mathrm{V}_{i0} - P^g_{i1\max}(1-\mathrm{V}_{i0})) + \sum_i \xi_{i1}(\mathrm{P}^g_{i0} - \mathrm{RU}_i\,\mathrm{V}_{it} + P^g_{i1\max}(1-\mathrm{V}_{i1}))
\end{aligned}
\tag{39}
$$

Resulting single-level problem is still nonlinear due to the product terms of binary variables and continuous Lagrange multipliers associated with lower level problem in strong duality equation. These nonlinear terms are linearized following [35] in Eq. (40).

$$
\begin{aligned}
&-\sum_t\sum_j\sum_b C^d_{bjt}P^d_{bjt} + \sum_t\sum_i\sum_o C^g_{oit}P^g_{oit} = \sum_t \lambda_t(\sum_j D^z_{jt}) - \sum_t\sum_i(a_{it}P^g_{it\min} - b_{it}P^g_{it\max})\\
&-\sum_t\sum_i\sum_o(\beta^{up}_{oit}(P^g_{oit\max})) - \sum_{t\neq 1}\sum_i(c_{it}(RU_i - P^g_{it\max}) - \xi_{it}P^g_{it\max})\\
&-\sum_{t\neq 1}\sum_i(d_{it}(RD_i - P^g_{it\max}) + \delta_{it}P^g_{it\max}) - \sum_{t\neq T}\sum_i(f_{it}(P^g_{it\max} - RD_i) + e_{it}RD_i)\\
&+\sum_t\sum_j(g_{it}P^d_{jt\min}) - \sum_t\sum_j(h_{it}P^d_{jt\max}) - \sum_j(\alpha^{up}_j(\mathrm{E}_j)) - \sum_t\sum_j\sum_b(\mu^{up}_{bjt}(P^d_{bjt\max}) - \mu^{lo}_{bjt}P^d_{bjt})\\
&+\sum_i \delta_{i1}(\mathrm{P}^g_{i0} - \mathrm{RD}_i\,\mathrm{V}_{i0} - P^g_{i1\max}(1-\mathrm{V}_{i0})) + \sum_i(c_{i1}(-\mathrm{RU}_i + P^g_{i\max}) - \xi_{i1}(P^g_{i0} + P^g_{i\max}))
\end{aligned}
\tag{40}
$$

$$0 \le a_{it} \le \theta^{lo}_{it\max}\mathrm{V}_{it} \tag{41}$$

$$0 \le \theta^{lo}_{it} - a_{it} \le (1-V_{it})\theta^{lo}_{it\max} \tag{42}$$

$$0 \le b_{it} \le \theta^{up}_{it\max}\mathrm{V}_{it} \tag{43}$$

$$0 \le \theta^{lo}_{it} - b_{it} \le (1-V_{it})\theta^{up}_{it\max} \tag{44}$$

$$0 \le c_{it} \le \xi_{it\max}\mathrm{V}_{it} \tag{45}$$

$$0 \le \xi_{it} - c_{it} \le (1-V_{it})\xi_{it\max} \tag{46}$$

$$0 \le d_{it} \le \delta_{it\max} \mathrm{V}_{it} \tag{47}$$

$$0 \le \delta_{it} - d_{it} \le (1 - V_{it}) \delta_{it\max} \tag{48}$$

$$0 \le e_{it} \le \varepsilon_{it\max} \mathrm{V}_{it} \tag{49}$$

$$0 \le \varepsilon_{it} - e_{it} \le (1 - V_{it}) \varepsilon_{it\max} \tag{50}$$

$$0 \le f_{it} \le \varepsilon_{it\max} \mathrm{V}_{it+1} \qquad \mathrm{t} \notin \mathrm{n}_T \tag{51}$$

$$0 \le \varepsilon_{it} - f_{it} \le (1 - V_{it+1}) \varepsilon_{it\max} \quad \mathrm{t} \notin \mathrm{n}_T \tag{52}$$

$$0 \le g_{jt} \le \rho_{jt\max}^{lo} \mathrm{V}_{jt} \tag{53}$$

$$0 \le \rho_{jt}^{lo} - g_{jt} \le (1 - V_{jt}) \rho_{jt\max}^{lo} \tag{54}$$

$$0 \le h_{jt} \le \rho_{jt\max}^{up} \mathrm{V}_{jt} \tag{55}$$

$$0 \le \rho_{jt}^{up} - h_{jt} \le (1 - V_{jt}) \rho_{jt\max}^{up} \tag{56}$$

Equations (41)-(56) are equations of integer algebra technique that is used for linearization of binary and continuous variables product. Therefore, equations (16)-(26), (28)-(38) and (40)-(56) represent an equivalent mixed integer linear form for the lower level problem. The upper bounds of dual variables are also required in order to solve this optimization. Devising a method to properly determine these parameters is of premium importance through which, over estimation slows down the solution and under estimation may render the optimization infeasible. Therefore, we use the values of the corresponding Lagrange multipliers resulting from the optimal solution for the associated OCM problem.

*Step2: Single-level linear equivalent*

Nonlinearity of the equivalent formulation lies in the bilinear terms in the formulation of energy payment. Some methodology based on binary expansion approach [36] and schur's decomposition [37] were proposed for linearization of bilinear products, but such techniques are based on some approximation and necessitate inclusion of the additional binary variables. In this section, we apply the strong duality theory of linear programing and KKT optimality condition and integer algebra for linearization of such bilinear terms. Using equation (30), (57)-(58) are found.

$$\sum_{t\in T}\lambda_t D_t^{total} = \sum_{t\in T}\lambda_t \sum_{j\in J}(D_{jt}^{z} + P_{jt}^{d}) \tag{57}$$

$$\sum_{t}\sum_{j}\lambda_t P_{jt}^{d} = \sum_{t}\sum_{j}(\rho_{jt}^{lo} - \rho_{jt}^{up} + \alpha_{j}^{lo} - \alpha_{j}^{up} + \vartheta_{jt})P_{jt}^{d} \tag{58}$$

Using complementary slackness conditions associated with lower level constraints (23) and (24), (59)-(62) are derived.

$$\rho_{jt}^{lo}(P_{jt}^{d} - V_{jt}P_{jt\min}^{d}) = 0 \rightarrow \rho_{jt}^{lo}P_{jt}^{d} = \rho_{jt}^{lo}V_{jt}P_{jt\min}^{d} \tag{59}$$

$$\rho_{jt}^{up}(P_{jt}^{d} - V_{jt}P_{jt\max}^{d}) = 0 \rightarrow \rho_{jt}^{up}P_{jt}^{d} = \rho_{jt}^{up}V_{jt}P_{jt\max}^{d} \tag{60}$$

$$\alpha_{j}^{lo}\sum_{t}P_{jt}^{d} = 0 \tag{61}$$

$$\alpha_{j}^{up}(E_j - \sum_{t}P_{jt}^{d}) = 0 \rightarrow \alpha_{j}^{up}\sum_{t}P_{jt}^{d} = \alpha_{j}^{up}E_j \tag{62}$$

Based on (32) and (26), (63)-(65) will be derived.

$$\vartheta_{jt} = C_{bjt}^{d} + \mu_{bjt}^{lo} - \mu_{bjt}^{up} \tag{63}$$

$$\mu_{bjt}^{lo}P_{bjt}^{d} = 0 \tag{64}$$

$$\mu_{bjt}^{up}(P_{bjt}^{d}-P_{bjt\max}^{d})=0\rightarrow\mu_{bjt}^{up}P_{bjt}^{d}=\mu_{bjt}^{up}P_{bjt\max}^{d} \tag{65}$$

Using (58)-(65) the energy payment term in (57) can be express in (66).

$$\sum_{t\in T}\lambda_t\sum_{j\in J}(D_{jt}^{z}+P_{jt}^{d})=\sum_{t}\sum_{j}\lambda_t D_{jt}^{z}+\sum_{t}\sum_{j}(\rho_{jt}^{lo}V_{jt}P_{jt\min}^{d}-\rho_{jt}^{up}V_{jt}P_{jt\max}^{d}) \\ -\sum_{j}\alpha_{j}^{up}E_{j}-\sum_{t}\sum_{j}\sum_{b}\mu_{bjt}^{up}P_{bjt\max}^{d}+\sum_{t}\sum_{j}\sum_{b}C_{bjt}^{d}P_{bjt}^{d} \tag{66}$$

In (66), there are two nonlinear terms associated with the product terms of binary and continuous variables. These terms are linearized in (67)-(71) using integer algebra technique.

$$\sum_{t\in T}\lambda_t\sum_{j\in J}(D_{jt}^{z}+P_{jt}^{d})=\sum_{t}\sum_{j}\lambda_t D_{jt}^{z}+\sum_{t}\sum_{j}(k_{jt}P_{jt\min}^{d}-L_{jt}P_{jt\max}^{d}) \\ -\sum_{j}\alpha_{j}^{up}E_{j}-\sum_{t}\sum_{j}\sum_{b}\mu_{bjt}^{up}P_{bjt\max}^{d}+\sum_{t}\sum_{j}\sum_{b}C_{bjt}^{d}P_{bjt}^{d} \tag{67}$$

$$0\leq k_{jt}\leq\rho_{jt\max}^{lo}\mathrm{V}_{jt} \tag{68}$$

$$0\leq\rho_{jt}^{lo}-k_{jt}\leq(1-V_{jt})\rho_{jt\max}^{lo} \tag{69}$$

$$0\leq L_{jt}\leq\rho_{jt\max}^{up}\mathrm{V}_{jt} \tag{70}$$

$$0\leq\rho_{jt}^{up}-L_{jt}\leq(1-V_{jt})\rho_{jt\max}^{up} \tag{71}$$

Finally, the single level mixed integer linear equivalent of original bi-level nonlinear program is presented in Eq. (72).

$$\min(\sum_{t}\sum_{j}\lambda_t D_{jt}^{z}+\sum_{t}\sum_{j}(k_{jt}P_{jt\min}^{d}-L_{jt}P_{jt\max}^{d})-\sum_{j}\alpha_{j}^{up}E_{j} \\ -(\sum_{t}\sum_{j}\sum_{b}\mu_{bjt}^{up}P_{bjt\max}^{d}-C_{bjt}^{d}P_{bjt}^{d})+\sum_{t\in T}\sum_{i\in I}(SU_{it}+SD_{it}+V_{it}O_{it}^{NL})) \tag{72}$$

Subject to: (9)-14), (16)-(26), (28)-(38), (40)-(56) and (68)-(71)

Flowchart of all steps are depicted in Fig. 3.

## 5. Simulation studies and results analysis

### *5.1. RTS-Based Case*

In this section, the proposed market clearing mechanism is implemented to the 24-buses IEEE reliability test system (RTS) comprising 32 generating units in 24 hours load variations. The economic viability of demand shifting and its impact on market with PCM auction are evaluated. The results are also compared with those achieved by conventional SWM proposed in [24]. The effects of LSDR in comparison with price-volume biding DR model, are illustrated. Generating data and all intertemporal constraints are given in Table 1.

It is assumed that generating units submit four offer blocks associated with their incremental heat rates. The hourly total forecasted system demand is shown in Table 2, where the load profile corresponds to the Wednesday of week 35 [38].

The demand shifting part of total load, parameters of bidding behavior are described by the following equations:

$$E_j = \frac{LPF}{K} \sum_t D_t^{total} \tag{73}$$

$$P_{jt\max}^d = E_j \tag{74}$$

$$P_{jt\min}^d = 0 \tag{75}$$

Three bidding blocks are considered for each one of all *K* bidders between the average and highest quantities of generating units offer blocks as descending staircase form with negative slope. It is assumed that all generation offers and demand bids are time invariant. The simulation has been performed on a computer with 2.67GHZ core i5 processor with 4GB of RAM using CPLEX [37] in GAMS 25.1.3 [39],[40]. The results of the proposed auction mechanism are first compared with those form a conventional SWM mechanism for *LPF*=0, *LPF*=0.02 and *K*=10. Fig. 4 and Fig. 5 show that with PCM auction mechanism, the electricity price in some hours is less than the price found using SWM mechanism. This leads to reduction in consumer payment. Also, as can be seen, because of load shifting capability, some loads of peak hours are shifted to light load hours. This leads to lower electricity prices in these hours. To gain a better perspective of demand shifting effects from the economical point of view, the index of effective cost (EC) based on equation (76) is used. This index represents the average marginal cost of consumers.

$$EC = \frac{\sum_{t \in T} MCP_t \times (D_t^z + P_t^d)}{\sum_{t \in T} (D_t^z + P_t^d)} \tag{76}$$

Table 3 shows that, for equal consumption level, the total payment of consumers in PCM auction with DR and effective cost are 6.76% and 8.37% less than those obtained under SWM maximization mechanism, while the social welfare is just decreased by amount of 0.29% under PCM mechanism. As mentioned before, consumer's benefit of price taking part is considered zero. This leads to the negative social welfare quantities.

The economic viability of the proposed auction is next evaluated with various amounts of LPFs. As it can be seen in Fig. 6 and Fig. 7, increasing the load participation makes total load profile smoother. This, in turn, means some amount of load shifts from peak to light load periods and subsequently reduces the electrical energy price in peak load hours.

In [28], applying price-volume biding DR, some bids will be rejected and energy requirement will then remain unsatisfied. As it can be seen in Fig. 8 and Table 4, for *LPF*=0.1 in the proposed method, the total load is unchanged and equal to total forecasted load.

It should be mentioned that at higher levels of *LPF*, some demand shifting bids may be rejected. Nevertheless, the total unsatisfied demand with demand shifting biding mechanism is never more that the case with price-volume bids.

### *5.2. 118-Bus System*

The second case study is based on the IEEE 118-bus system [41], [42] and comprises 54 generating units and 91 consumers over a 24-hour time span. Generation and load data can be found in [43]. Similar to the RTS-based case, offers and bids are not modified throughout the scheduling horizon. three-block energy offers are obtained from the linearization of the quadratic production costs. It should be noted that generation data, offers and bids remain unchanged over the time span. For this case study, *LPF*=0.05 and *K*=91. Table5 provides the size of problem in terms of the numbers of constraints, binary variables, and real variables.

As can be seen in table 6, Using a stopping criterion of 0% optimality gap, the proposed approach required 22.32 s to attain the optimal solution for 118-bus case while SWM method needs 1.61 s. It is worth mentioning that the computing times required to attain such optimal solution are lower than that required for the RTS-based case. This is an indication of the case-dependent behavior of the branch-and-cut algorithm. Hourly market-clearing prices associated with and without LSDR are depicted in Fig. 9 and Fig.10. These Figures also show the hourly system demand. Note that market-clearing prices follow the shape of the demand curve based on Fig.9.

As mentioned before DR decreases consumer's payment and it can be seen in Table 7.

## 6. Conclusion

In this paper, a framework was presented to incorporate the LSDR as a part of load in day ahead pool-based electricity market based on consumers' payment minimization auction. The effects of such DR modeling on daily load profile, total consumption of consumers and energy prices based on PCM auction mechanism were analyzed. The resulting bi-level mixed integer nonlinear problem with bilinear terms was converted to a single level mixed integer linear form and was effectively solved with zero optimality gap in an acceptable time. Compared to the previous works, the proposed modeling approach reduces the risks of consumers going unbalanced after the closure of the gate and benefits all consumers even those that do not participate in the load shifting activities. The next step is to develop a mathematical formulation for the proposed approach considering network and revenue constraints. Further research will also be devoted to the analysis of joint energy and spinning reserve PCM markets as a crucial ancillary service.

## 7. References


1. U.S. Federal Energy Regulatory Commission. Order No. 888, Promoting Wholesale Competition through Open Access Non-Discriminatory Transmission Services by Public Utilities; FERC: Washington, DC, USA, 1996. Available online: http://www.ferc.gov/legal/maj-ord-reg/land-docs/rm95-8-00v.txt.
2. Durvasulu, V. and Hansen, T. "Benefits of a Demand Response Exchange Participating in Existing Bulk-Power Markets", Energies, special issue on Demand Response in Electricity Markets, v.11, 2018.
3. Hamian, M. Darvishan, A. Hosseinzadeh, M. Janghorban, M. Ghadimi, N. and Nouri, A. "A framework to expedite joint energy-reserve payment cost minimization using a custom-designed method based on Mixed Integer Genetic Algorithm". Eng. Appl. of AI 72: 203-212, 2018.
4. Luh, P. B, Blankson, W. E, Chen. Y. et al. "Payment cost minimization auction for deregulated electricity markets using surrogate optimization", IEEE Trans. Power Syst., vol. 21, no. 2, pp. 568–578, May 2006.
5. Fernández-Blanco, R. Arroyo, J. M. and Alguacil, N. "Network-constrained day-ahead auction for consumer payment minimization", IEEE Trans. Power Syst., vol. 29, no. 2, pp. 526–536, Mar. 2014.
6. Bragin, M. A. Han, X. Luh, P. B. Chen, Y. and Yan J. H. "Payment cost minimization using Lagrangian relaxation and modified surrogate optimization approach", in Proc. IEEE Power Energy Soc. Gen. Meet., Jul. 24–29, 2011.
7. Chang, T. S. "Comments on surrogate gradient algorithm for Lagrangian relaxation", J. Optim. Theory Appl., vol. 137, no. 3, pp. 691–697, Jun. 2008.
8. Vázquez, C. Rivier, M. and Pérez-Arriaga, I. J. "Production cost minimization versus consumer payment minimization in electricity pools", IEEE Trans. Power Syst., vol. 17, no. 1, pp. 119–127, Feb. 2002.
9. Mendes, D. P. "Resource scheduling and pricing in a centralised energy market," in Proc. 14th Power Syst. Comput. Conf., Seville, Spain, pp. 1–7, Jun. 2002.
10. Fernandez-Blanco, R. Arroyo, J. M. and Alguacil, N. "A unified bilevel programming framework for price-based market clearing under marginal pricing", IEEE Trans. Power Syst., vol. 27, no. 1, pp. 517, 525, Feb. 2012.
11. Nouri, A. Hosseini, S.H. "Payment minimisation auction with security constraints" IET Gener. Transm. Distrib, pp. 1370–1380., 2017.

12. Nouri, A. Hosseini, S.H. “Comparison of LMPs' sensitivity under payment cost minimization and offer cost minimization mechanisms,” IEEE Systems Journal, vol. 9, no. 4, pp. 1507-1518, Dec. 2015
13. Nouri, A. Hosseini, S.H. Keane, A. “Stochastic network constrained payment minimization in electricity markets”, IET Gener. Transm. Distrib, 13, (11), pp. 2268 – 2279, 2019.
14. Bizhaniaram, B. Nouri, A. "Stochastic Payment Cost Minimization in Energy Markets with High Penetration of Renewables," 2018 IEEE International Conference on Environment and Electrical Engineering and IEEE Industrial and Commercial Power Systems Europe (EEEIC / I&CPS Europe), Palermo, pp. 1-4, 2018.
15. Su, C.-L. “Optimal demand-side participation in day-ahead electricity markets”, Ph.D. dissertation, Univ. Manchester, Manchester, U.K., 2007.
16. Aalami, H. A. Moghaddam, M. P. and Yousefi, G. R., “Demand response modeling considering Interruptible/Curtailable loads and capacity market programs”, Appl. Energy, vol. 87, no. 1, pp. 243-250, Jan. 2010.
17. Nikzad, M. Mozafari, B. Bashirvand, M. Solaymani, S. Ranjbar AM. “Designing time-of-use program based on stochastic security constrained unit commitment considering reliability index”, Energy;41(1):541e8, 2012.
18. Esmaili, M. Amjady, N. and Shayanfar, H. A.“Stochastic congestion management in power markets using efficient scenario approaches” , Energy Convers. Manag., vol.51, no.11, pp. 2285–2293, 2010.
19. Rahmani-Andebili, M., Abdollahi A., and Moghaddam M. P., “An investigation of implementing emergency demand response programs (EDRP) in unit commitment problem” ,in Proc. IEEE PES Gen. Meeting, San Diego, CA, USA, pp. 1–7, Jul. 2011.
20. Zarei, E. Hemmatpour, M.H and Mohammadian, M. “The Effects of Demand Response on Security-Constrained Unit Commitment”, Scientia Iranica, article in press doi: 10.24200/sci.2017.4536 (2017).
21. Ghahary, K. Abdollahi, A. Rashidinejad, M. Alizadeh, MI. “Optimal reserve market clearing considering uncertain demand response using information gap decision theory”, Int J Electr Power Energy Syst;101:213–22,2018.
22. Parvania, M. and Fotuhi-Firuzabad, M. “Demand response scheduling by stochastic SCUC”, IEEE Trans. Smart Grid, 1(1):89–98, 2010.
23. Abdollahi, A. Pour-Moallem, N. & Abdollahi, A. “Dynamic Negawatt Demand Response Resource Modeling and Prioritizing in Power Markets”, Scientia Iranica, doi: 10.24200/sci.2017.4406 (2017).
24. Kirschen, Su CL, DS. “Quantifying the effect of demand response on electricity markets”, IEEE Transactions on Power Systems.;24(3):1199–1207, August 2009.
25. Hao, S. and Zhuang, F. “New models for integrated short-term forward electricity markets”, IEEE Trans. Power Syst., vol. 18, no. 2,pp. 478–485, May 2003.
26. Chen, Y, Luh, P. B, Yan. J. H, et al. “Payment minimization auction with demand bids and partial compensation of startup costs for deregulated electricity markets”, presented at the IEEE PES Gen. Meeting, San Francisco, CA, USA,Jun. 2005.
27. Luh, P. B, Chen, Y, Blankson, W. E, et al. “Payment cost minimization with demand bids and partial capacity cost compensations for day-ahead electricity auctions”, in Economic Market Design and Planning for Electric Power Systems,J. Momoh and L. Mili, Eds. Hoboken, NJ, USA: Wiley, pp. 71–85, 2010.
28. Fernández-Blanco, R. Arroyo, J. M. Alguacil, N. and Guan, X. “Incorporating price-responsive demand in energy scheduling based on consumer payment minimization”, IEEE Trans. Smart Grid, vol. 7, no. 2, pp. 817–826, Mar. 2016
29. Bard, J. F. “Practical Bilevel Optimization. Algorithms and Applications.Norwell”, MA, USA: Kluwer, 1998.
30. Arroyo, J. M. “Bilevel programming applied to power system vulnerability analysis under multiple contingencies”, IET Gener. Transmiss Distrib., vol. 4, no. 2, pp. 178–190, Feb. 2010.
31. Nemhauser G. L. and Wolsey L. A. “Integer and Combinatorial Optimization”, Hoboken, NJ, USA: Wiley, 1999.

32. Zhao, F. Luh, P. B. Yan, J. H. Stern, G. A. and Chang, S.-C. “Payment cost minimization auction for deregulated electricity markets with transmission capacity constraints”, IEEE Trans. Power Syst., vol. 23, no. 2, pp. 532–544, May 2008.
33. Fernández-Blanco, R. Arroyo, J. M. and Alguacil, N. “Networkconstrained day-ahead auction for consumer payment minimization”, IEEE Trans. Power Syst., vol. 29, no. 2, pp. 526–536, Mar. 2014.
34. Arroyo, J. M. “Bilevel programming applied to power system vulnerability analysis under multiple contingencies”, IET Gener. Transmiss Distrib., vol. 4, no. 2, pp. 178–190, Feb. 2010.
35. Floudas, C. A. “Nonlinear and Mixed-Integer Optimization: Fundamentals and Applications”. New York, NY, USA: Oxford Univ. Press, 1995.
36. Pereira, M. V. Granville, S. Fampa, M. H. C. Dix, R. and Barroso, L. A. “Strategic bidding under uncertainty: A binary expansion approach”, IEEE Trans. Power Syst., vol. 20, no. 1, pp. 180–188, Feb. 2005.
37. Horn, R. A. and Johnson, C. R. “Matrix Analysis”, 2nd ed. New York, NY, USA: Cambridge Univ. Press, 2012.
38. Grigg, C. “IEEE Reliability Test System”, IEEE Trans. Power App. Syst., vol. PAS-98, no. 6, pp. 2047–2054, Nov. 1979.
39. The IBM ILOG CPLEX Website. Available: http://www-01.ibm.com/software/commerce/optimization/cplex-optimizer
40. (2019). The GAMS Development Corporation Website. [Online].Available: http://www.gams.com
41. Power Systems Test Case Archive, Dept. Elect. Eng., Univ.Washington, Seattle, WA, USA, 2015. [Online]. Available:http://www.ee.washington.edu/research/pstca
42. (2015). IEEE 118-Bus System. [Online]. Available: http://motor.ece.iit.edu/data/Data_118_Bus.pdf

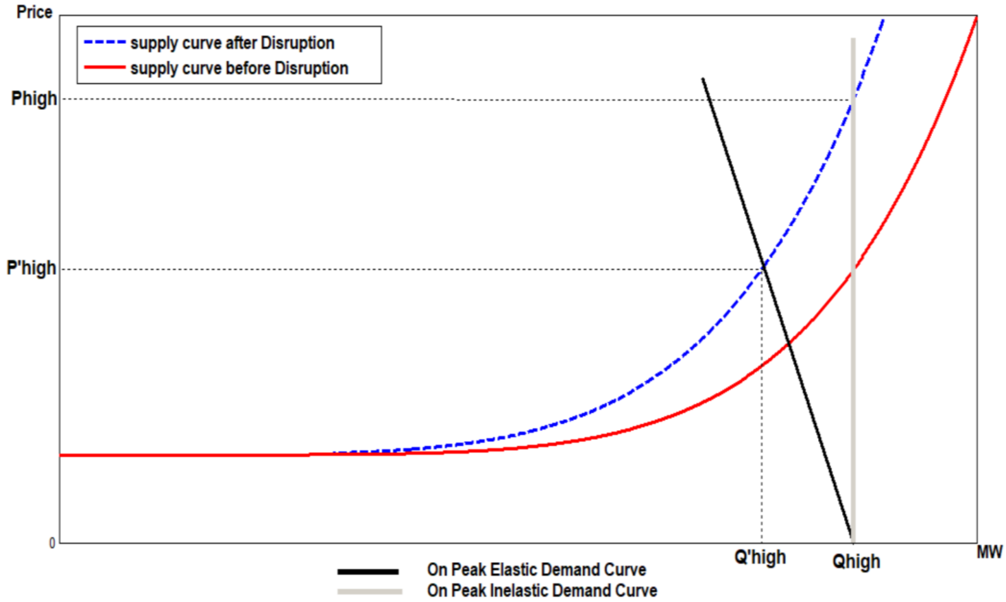


**Figure1.**

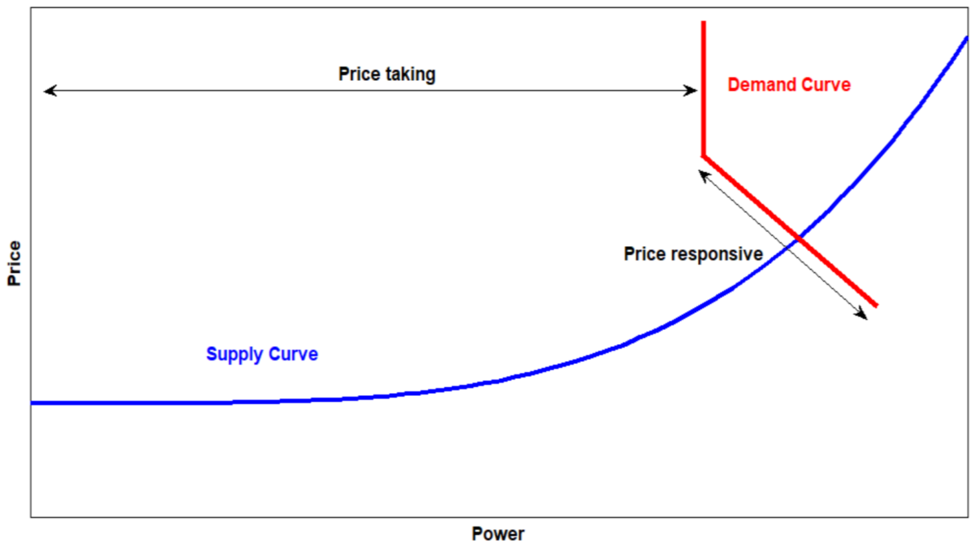


**Figure 2.**

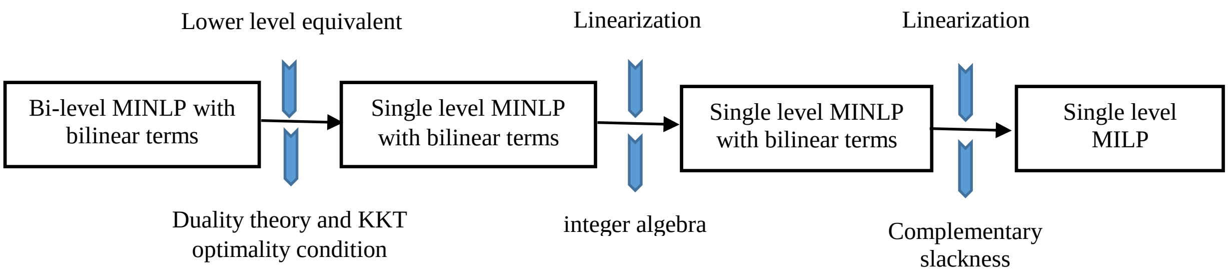


**Figure 3.**

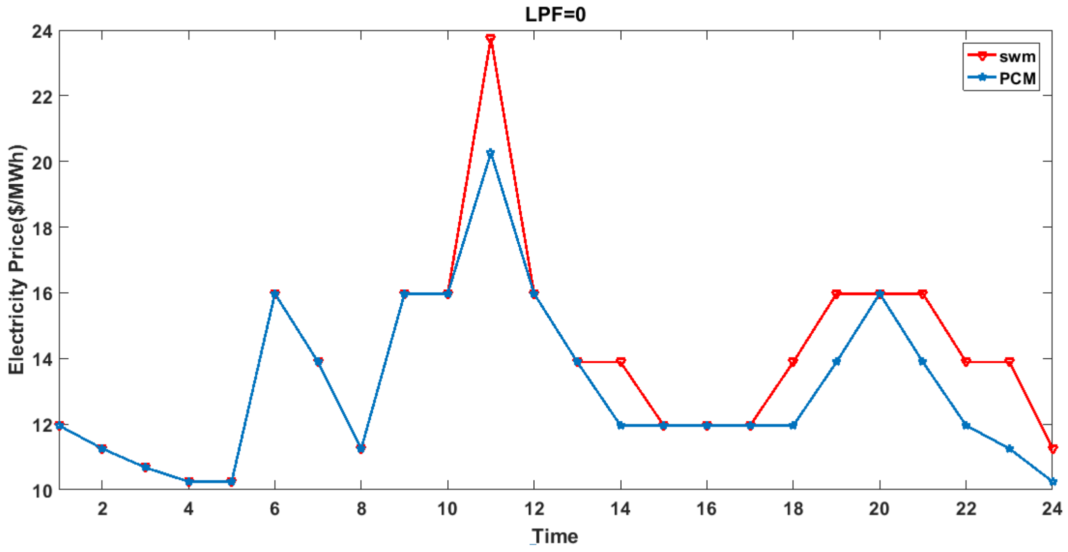


**Figure 4**.

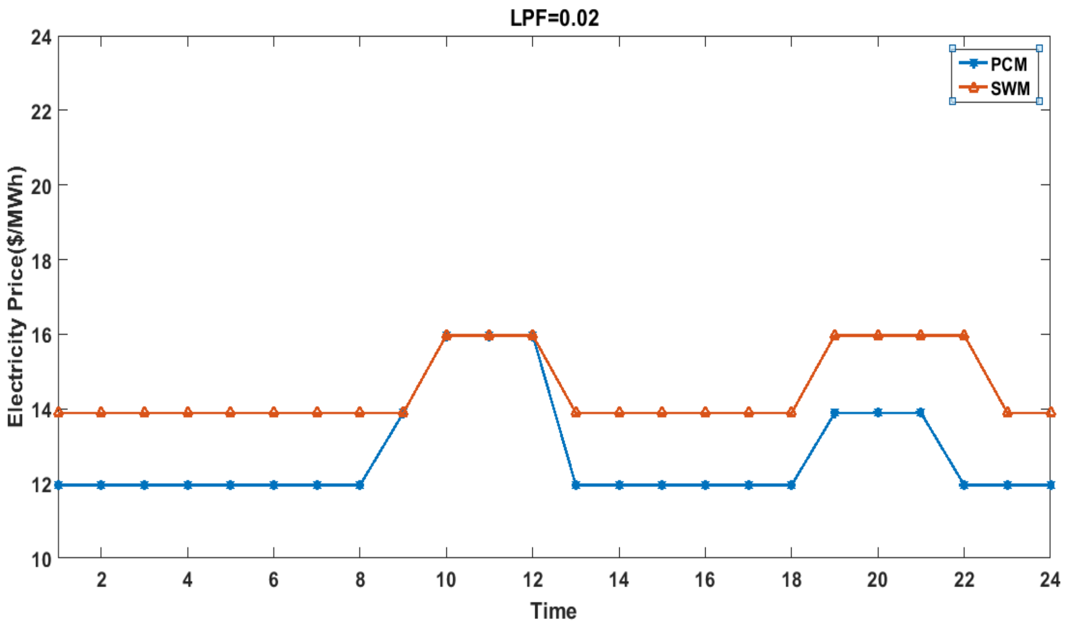


**Figure 5**.

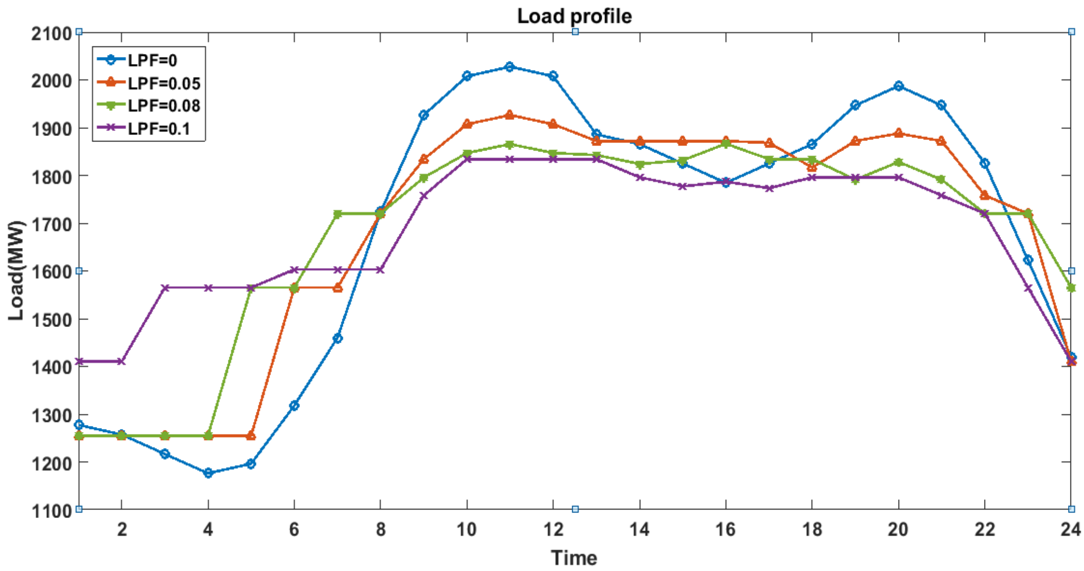


**Figure 6.**

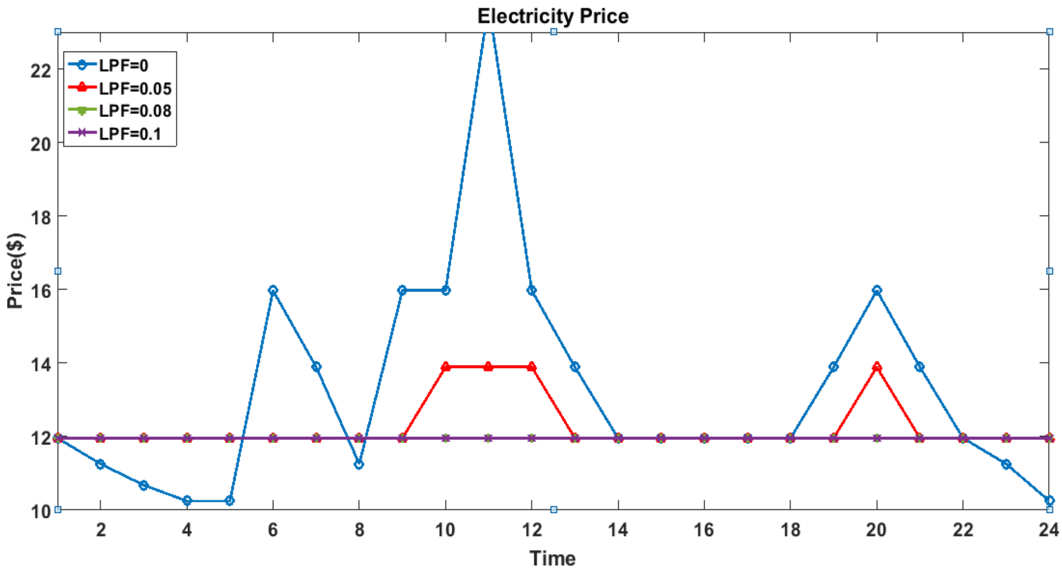


**Figure 7.**

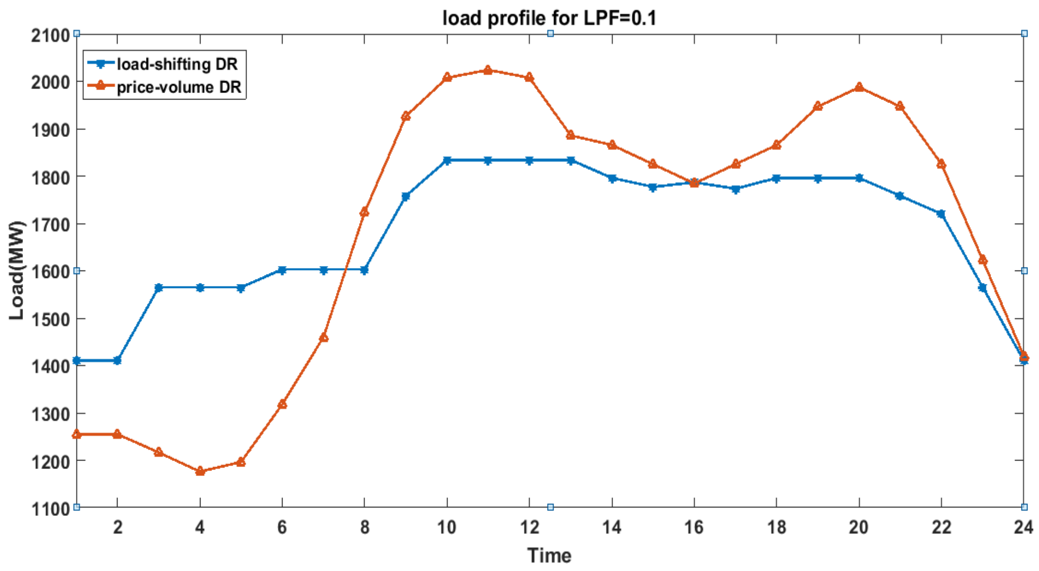


**Figure 8**.

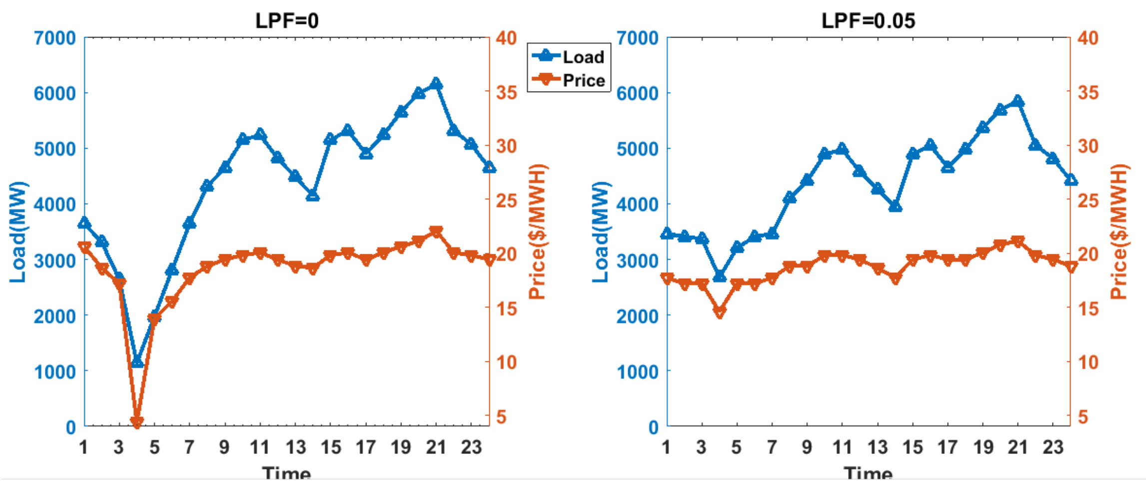


**Figure 9.**

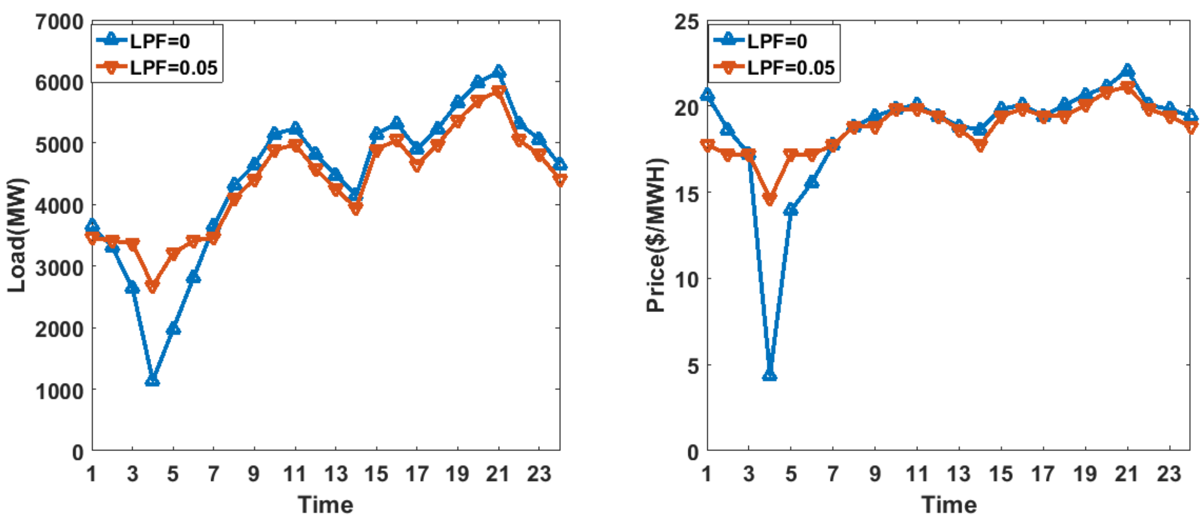


**Figure 10.**

**Table 1.**

| Unit group | Number Of units | $P^{g}_{it\,\min}$ (MW) | $P^{g}_{it\,\max}$ (MW) | $RU_i$ (MW/h) | $RD_i$ (MW/h) | $UT_i$ (h) | $DT_i$ (h) | $UT_i^{0}$ (h) | $DT_i^{0}$ (h) | $O^{su}_{it}$ ($) | $O^{sd}_{it}$ ($) | $O^{NL}_{it}$ ($) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| U12 | 5 | 2.4 | 12 | 12 | 12 | 4 | 2 | 0 | 2 | 87 | 50 | 56 |
| U20 | 4 | 15.8 | 20 | 20 | 20 | 1 | 1 | 0 | 1 | 15 | 10 | 467 |
| U50 | 6 | 0 | 50 | 50 | 50 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| U76 | 4 | 15.2 | 76 | 76 | 76 | 8 | 4 | 0 | 4 | 715 | 430 | 174 |
| U100 | 3 | 25 | 100 | 100 | 100 | 8 | 8 | 0 | 8 | 575 | 326 | 456 |
| U155 | 4 | 54.25 | 155 | 155 | 155 | 8 | 8 | 0 | 8 | 312 | 210 | 539 |
| U197 | 3 | 68.95 | 197 | 180 | 180 | 12 | 10 | 0 | 10 | 1019 | 600 | 1324 |
| U350 | 1 | 140 | 350 | 240 | 240 | 24 | 48 | 0 | 24 | 2298 | 950 | 1411 |
| U400 | 2 | 100 | 400 | 400 | 400 | 1 | 1 | 0 | 1 | 0 | 0 | 531 |

**Table 2.**

| period | Forecasted Load (MW) | period | Forecasted Load (MW) | period | Forecasted Load (MW) |
|---|---|---|---|---|---|
| 1 | 1277 | 9 | 1926 | 17 | 1824 |
| 2 | 1257 | 10 | 2007 | 18 | 1865 |
| 3 | 1216 | 11 | 2027 | 19 | 1946 |
| 4 | 1176 | 12 | 2007 | 20 | 1987 |
| 5 | 1196 | 13 | 1885 | 21 | 1946 |
| 6 | 1318 | 14 | 1865 | 22 | 1824 |
| 7 | 1459 | 15 | 1824 | 23 | 1622 |
| 8 | 1723 | 16 | 1784 | 24 | 1419 |

**Table 3.**

| | PCM with DR | SWM with DR | PCM without DR | SWM without DR |
|---|---|---|---|---|
| Payment ($) | 605453 | 649308 | 633130 | 655153 |
| Social welfare ($) | -311129 | -310228 | -346561 | -345028 |
| EC ($/MWh) | 12.91 | 14.09 | 13.53 | 14.14 |

**Table 4.**

| | Total daily load (MW) | Total Consumers payment($) |
|---|---|---|
| Load-shifting DR | 40392 | 566852 |
| Price-volume DR | 40263 | 642606 |

**Table 5.**

| | No of real variables | No of binary variables | No of constraints |
|---|---|---|---|
| 118-bus | 73107 | 3480 | 82814 |
| 24-bus RTS case | 26573 | 1008 | 31270 |

**Table 6.**

| | PCM with DR method (s) | SWM with DR method (s) |
|---|---|---|
| 118-bus | 22.32 | 1.61 |
| 24-bus RTS case | 46.64 | 6.18 |

**Table 7**.

| | PCM with DR | PCM without DR |
|---|---|---|
| EC ($) | 19.01 | 19.38 |

***Nomenclature***

PCM: payment cost minimization

OCM: offer cost minimization

MCP: market clearing price

LSDR: load shifting demand response

EDRP: emergency demand response program

LPF: load participation factor

SWM: social welfare maximization

*indices*

i generating unit

j demand

t time

b demand bid block

o generation offer block

sets

I generation unit indices

J consumer indices

T time period indices

$\beta_j$ demand bid block indeces of consumer j

$O_i$ generation offer block indeces of unit i

continous variables

$\lambda_t$ Dual variable of power balance equation

$P_{it}^{g}$ Power output of unit i in period t

$P_{jt}^{d}$ Power consumption of consumer j in period t

$P_{oit}^{g}$ Generation level awarded to unit i of block o

$P_{bjt}^{d}$ Consumption level awarded to consumer j of block b

$SU_{it}$ Payment for the start-up of unit i in period t

$SD_{it}$ Payment for the shut-down of unit i in period t

$\theta_{it}^{lo}$ Dual variable of minimum power generation of unit i constraint

$\theta_{it}^{up}$ Dual variable of maximum power generation of unit i constraint

$\beta_{oit}^{lo}$ Dual variable of minimum power generation of unit i of block o

$\beta_{oit}^{up}$ Dual variable of maximum power generation of unit i of block o constants

$P^{g}_{it\min}$ minimum generation power

$P^{g}_{it\max}$ maximum generation power

$P^{d}_{jt\min}$ minimum consumption power

$P^{d}_{jt\max}$ maximum consumption power

$P^{d}_{bjt\max}$ maximum consumption of bidded block

$P^{g}_{oit\max}$ maximum generation of offered block

$C^{d}_{bjt}$ price of bidded block b of consumer j

$C^{g}_{oit}$ price of offered block o of unit i

$RU_i$ Ramp-up rate of unit i

$RD_i$ Ramp-down rate of unit i

$UT_i$ up time of unit i

$DT_i$ down time of unit i

$UT_i^0$ up time of unit i at end of last period

$DT_i^0$ down time of unit i at end of last period

$O^{su}_{it}$ start up offer of unit i

$O^{sd}_{it}$ shut down offer of unit i

$O^{NL}_{it}$ no-load offer of unit i

$D^{total}_{t}$ total forcasted demand

$LPF$ load participation factor

$K$ number of consumers

$E_j$ total energy consumption of consumer j

$n_T$ Number of time periods

$n_i$ Number of generating units

$n_{\beta_j}$ Cardinality of $\beta_j$

$n_{o_i}$ Cardinality of $O_i$

$\theta^{lo}_{it\max}$ upper bound for $\theta^{lo}_{it}$

$\theta^{up}_{it\max}$ upper bound for $\theta^{up}_{it}$

$\xi_{it\max}$ upper bound for $\xi_{it}$

$\delta_{it\max}$ upper bound for $\delta_{it}$

$\varepsilon_{it\max}$ upper bound for $\varepsilon_{it}$

$\xi_{it}$ Dual variable of ramp-up and start-up ramp rate constraint of unit i

$\delta_{it}$ Dual variable of ramp-down rate constraint of unit i

$\varepsilon_{it}$ Dual variable of shut-down ramp rate constraint of unit i

$\gamma_{it}$ Dual variable associated with the definition of $P^{g}_{it}$

$\vartheta_{jt}$ Dual variable associated with the definition of $P^{d}_{jt}$

$\rho^{lo}_{jt}$ Dual variable of minimum power consumption of consumer j

$\rho^{up}_{jt}$ Dual variable of maximum power consumption of consumer j

$\mu^{lo}_{bjt}$ Dual variable of minimum power consumption of consumer j of block b

$\mu^{up}_{bjt}$ Dual variable of maximum power consumption of consumer j of block b

$\alpha^{lo}_{j}$ Dual variable of minimum daily energy requirment constraint of unit j

$\alpha^{up}_{j}$ Dual variable of maximum daily energy requirment constraint of unit j

$a_{it}$ Auxiliary variable equal to the product $\theta^{lo}_{it\max}V_{it}$

$b_{it}$ Auxiliary variable equal to the product $\theta^{up}_{it\max}V_{it}$

$c_{it}$ Auxiliary variable equal to the product $\xi_{it\max}V_{it}$

$d_{it}$ Auxiliary variable equal to the product $\delta_{it\max}V_{it}$

$e_{it}$ Auxiliary variable equal to the product $\varepsilon_{it\max}V_{it}$

$f_{it}$ Auxiliary variable equal to the product $\varepsilon_{it\max}V_{it+1}$

$g_{it}$ Auxiliary variable equal to the product $\rho^{lo}_{jt\max}V_{jt}$

$h_{it}$ Auxiliary variable equal to the product $\rho^{up}_{jt\max}V_{jt}$

$k_{jt}$ Auxiliary variable equal to the product $\rho^{lo}_{jt\max}V_{jt}$

$L_{jt}$ Auxiliary variable equal to the product $\rho^{up}_{jt\max}V_{jt}$

Binary variables

$V_{it}$ on-off statuse of unit i at time t

$V_{jt}$ on-off statuse of consumer j offer acceptance at time t

$\rho_{jt\max}^{lo}$ upper bound for $\rho_{jt}^{lo}$

$\rho_{jt\max}^{up}$ upper bound for $\rho_{jt}^{up}$

## Biographies

**Hamed jafarirad**: received his B.Sc. degree in electrical engineering from Shahid Bahonar University, Kerman, Iran, in 2009 and M.Sc. degree in electrical engineering from Power and Water University of Technology (PWUT), Tehran, Iran, in 2012. He is currently pursuing the Ph.D. degree in Kerman Graduate University of Technology (KGUT), Kerman, Iran. His research interests include energy management, power system optimization and power system operation.

**Masoud Rashidinejad**: received the B.Sc. degree in electrical engineering and the M.Sc. degree in systems engineering from the Isfahan University of Technology, Isfahan, Iran, and the Ph.D. degree in electrical engineering from Brunel University, London, U.K., in 2000. He is currently a Professor with the Department of Electrical Engineering, Shahid Bahonar University of Kerman, Kerman, Iran. His research interests include the areas of power system optimization, power system planning, electricity restructuring, and energy management in smart electricity grids.

**Amir Abdollahi**: received his B.Sc. degree in electrical engineering from Shahid Bahonar University, Kerman, Iran, in 2007 and M.Sc. degree in electrical engineering from Sharif University of Technology, Tehran, Iran, in 2009. He received his PhD in electrical engineering from Tarbiat Modarres University (TMU), Tehran, Iran, in 2012. He is currently an Associate Professor in the Department of the Engineering, Shahid Bahonar University of Kerman, Iran. His research interests include demand side management, optimization, planning and economics in smart grids.