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Home healthcare routing and scheduling problem under uncertainty considering patients' preferences and service desirability

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Abstract. Home Health Care (HHC) is the task of preparing medical and paramedical services for patients at their place of residence. In the HHC industry, it is imperative for decision-makers to appoint nurses to service patients and plan visiting patterns to confront with conflicting objectives and boost service quality. This study provides significant insights into Home Health Care Routing and Scheduling Problem (HHCRSP) by pursuing three patient-oriented objectives. The proposed model accounts for real-life constraints such as emergency patients, nurses' proficiency, and patients' preferences. Owing to the multi-objective nature of the model, both Augmented Epsilon Constraint (AEC) and Fuzzy Goal Programming (FGP) approaches are employed to accomplish the mentioned objectives. Further, getting as close as possible to the real-world problems, some parameters are considered uncertain. In this regard, a robust approach along with three dissimilar uncertainty sets is used to control uncertainty. The numerical results indicate that regardless of the type of the uncertainty set, increasing the control parameters would make the objective values farther than ideal, and the comparison made among the sets confirmed the stringency of the Box space. A unique indicator was also presented to validate the viability of the robust approaches according to which the features of all sets were almost the same in terms of equal optimality and feasibility criteria.

1. Introduction

Home Health Care (HHC) is an alternative to conventional clinical treatments that includes offering medical, paramedical, and social services to patients at their own home. The HHC industry has flourished significantly in the last decade due to several reasons, namely population aging, social changes, increase in the number of patients that suffer from chronic diseases, development of modern technologies, congestion of hospitals, and governmental pressures [1]. It is expected that the demand rate for the HHC services will be twice until 2030, mainly because the HHC services are becoming more accessible and prevailing than ever owing to their desirability and ability to alleviate the patients' stress [2]. In addition, the care and treatment HHC services can provide are different from the formal services in hospitals. For instance, the main difference between the HHC and hospital services is the

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location, i.e., home, a place where patients can stay with their family in a personalized environment with particular values, preferences, culture, and habits [3,4]. In the HHC companies, decisions made in the operations management are systematically arranged into four decision levels according to the time horizon [5] including the strategic (1–5 years), tactical (6–12 months), operational (weeks-months), and detailed operational levels (hours-days). The current study primarily focused on the detailed operational level problem which includes daily routing and scheduling of nurses. The essential resources for preparing home care services in advance are nurses, transportation vehicles, administrative buildings, medical equipment and instruments, and administrative staff. On the bulk scale, low payments and high operational costs are considered as the greatest challenges in the HHC industry, resulting in near-zero profit margin for HHC companies and even negative in rural areas [6,7].

In this context, HHC studies have explored some criteria represented as the constraints and objective functions in mathematical programming. The fundamental problem is concerned with employing nurses to provide their patients with healthcare and devise their daily visiting routes to achieve certain objectives. In real-world applications, a number of constraints may complicate the matter by showing the distinct requirements of a particular HHC company such as preferences of customers, continuity of care, break times, and interdependent services [1,8]. In general, HHC optimization problems are made up of three sub-problems:

(i)   Assignment of nurses and patients;

(ii) Scheduling appointments;

(iii) Daily routing for nurses.

In some research studies, only one of these problems was taken into account, while some others considered all three sub-problems. Those studies that deal with all three sub-problems follow two different approaches. The first approach is based on decomposition methodology. In this approach, the principal problem is divided into two main problems namely assignment problem and routing and scheduling problem. First, the assignment problem is solved and then, the schedules and routes are settled. The second approach is integrated solving all the three problems. Figure 1 shows a network including major topics in the field of Home Health Care Routing and Scheduling Problem (HHCRSP) which are based on the frequency of keywords selected by the authors in this field [9]. The keywords included in the figure are Time Windows (TWs), continuity of care, synchronization, and a variety of other solutions to these issues. Figure 2 shows that an increasing number of publications have been conducted on HHC since 2010, confirming that HHC is a promising and growing sector in the near future. Figure 3 shows the share of each of the top five countries in the total number of studies published by 2020. Surprisingly, according to the chart, the United States with a 41% share is the largest and most influential player in the field of HHC. Table 1 lists the main acronyms used throughout this paper.

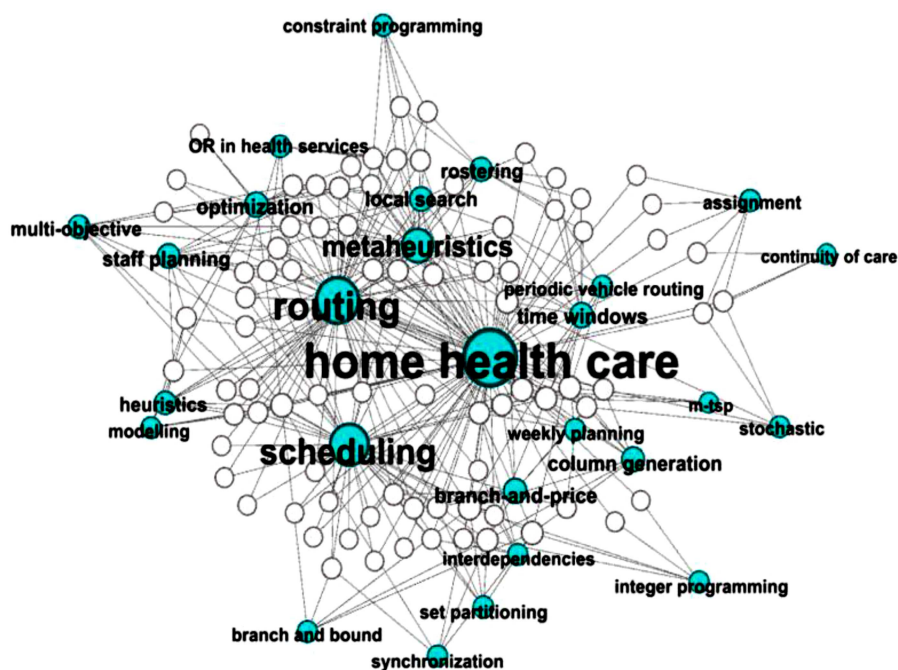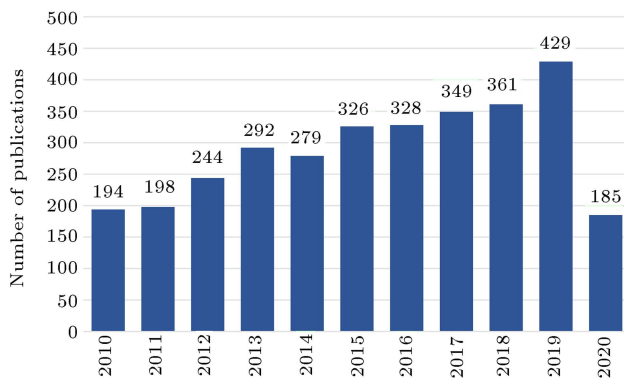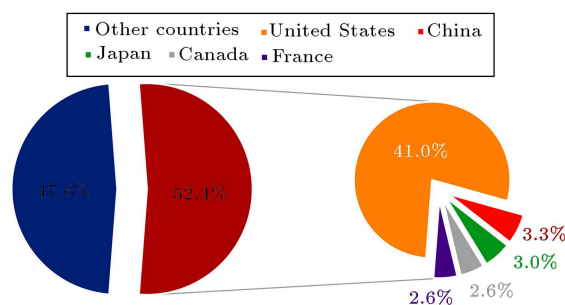In this research, a routing problem was integrated

Figure 1. Visualization of selected keywords in the field of HHCRSP [9].

Figure 2. The histogram of publications dealing with HHC in journals and conference proceedings by year.

Figure 3. Five leading countries in the field of HHC based on the number of studies published so far.

Table 1. List of acronyms.

Symbols	Description
EC	Epsilon Constraint
AEC	Augmented Epsilon Constraint
ELL	Ellipsoidal
GP	Goal Programming
FGP	Fuzzy Goal Programming
HHCRSP	Home Health Care Routing & Scheduling Problem
HHC	Home Health Care
HCC	Home Health Center
PH	Polyhedral
VRPTW	Vehicle Routing Problem with Time Windows

with the scheduling problem. In fact, the routing section itself can be Vehicle Routing Problem with the Time Window (VRPTW). The HHC problem is a collection of nodes and vectors. The nodes represent patients randomly distributed at the level of a geographic region, and the vectors connect these nodes to each other representing the communication paths through which nurses can visit patients. What makes the problem more challenging is that nurses' traveling times and patients' visit times related to their health conditions are usually not deterministic. Therefore, while planning for the HHC operation, such a source of uncertainty should be taken into consideration.

The contributions of this paper can be elaborated at three different levels. First, the present study considered uncertain traveling and service times in the HHCRSP. In this regard, three unique Robust Optimization Approaches (ROAs) were employed to control uncertainties. Given that in such problems with distinct patients, each with unique characteristics, three patient-oriented objectives would gain significance; hence, a multi-objective optimization approach was used to consider different managerial perspectives on the problem ahead. Moreover, Emergency Patients (EP) and preferred visit times were considered to get the model closer to the real-world problems. Due to their health conditions, the EP should be visited in the predetermined TWs as much as possible with the least earliness or tardiness in their visits.

The rest of the paper is organized as follows: Section 2 presents a review of the related HHCRSPs from the academic literature. Section 3 formally defines a tri-objective mathematical programming model for the problem. Section 4 uses Augmented Epsilon Constraint (AEC) and Fuzzy Goal Programming (FGP) methods in order to solve the multi-objective model proposed in the previous section and examine the trade-off between objectives. This section also elaborates ROA with three different uncertainty sets including Box, Polyhedral (PH), and Ellipsoidal (ELL) sets. Section 5 presents a computational experiment that highlights the findings based on the proposed approach. Section 6 presents a sensitivity analysis of the number of nurses on the objectives. Finally, Section 7 concludes the paper and points out future research directions.

2. Related literature

HHC planners may face tough and challenging optimization problems at distinct decision levels such as nurse assignment, shift scheduling, and routing decisions. In most cases, a set of different nurses should be assigned to heterogeneous patients who are spread over a specific area. In this regard, they are required to consider various constraints like Skill Matching (SM), patients' preferences, and real-world complications of HHC operations such as continuity of care.

The literature review given in this paper focuses on how to study the three core aspects that characterize recent works on the HHCRSP including:

- Objectives and performance measures;
- Decisions and constraints;
- Solution methodologies.

Table 2 presents an initial classification of the objectives and performance measures considered in the

Table 2. Common objectives and performance measures in the HHC area.

Abbreviation	Description
UN	Uncovered visits
BP	Balance between Patients
TD	Travel Distance
OT	Over Time
TC	Travel Cost
PP	Patient Preference

Table 4. Frequent decisions and constraints.

Abbreviation	Description
EP	Emergency Patients
SM	Skill Matching
TW	Time Window
U	Uncertainty
CC	Continuity of Care
B	Breaks
S	Synchronization
WT	Working Time regulation

HHC area. Each category contains an extensive class of possible objective functions or performance measures. Table 3 presents a summary of the objectives treated in the HHC literature.

According to this table, Traveling Costs (TC), Patients' Preferences (PP), and Over Times (OT) are the most common objectives in the HHC literature. Table 4 lists the most frequent decisions and constraints taken into account when planning HHC services. Table 5 identifies which of these constraints were handled in the reviewed studies.

As observed in Table 5, the relevant studies done to date have concentrated mostly on TW and SM rather than on other common constraints to ensure high service level. In addition, some service characteristics in this table namely EP and Uncertainty (U), which are prevailing in real-world problems, have

received scant attention in the literature. Hence, this paper attempts to fill a gap in the literature by considering EPs and U in detail. Considering hard TWs, i.e., no flexibility outside the specified time window, is a vital assumption, as numerous operations in HHC are time sensitive. Some examples are the provision of medication or insulin injection which must be completed in a specific time frame. Moreover, in order to respect PP, soft TW were taken into account in a range of articles [10–13]. If a certain task is performed outside the soft TW, a penalty is added to the objective function. Working time regulations, guaranteeing that nurses can only be scheduled for a certain amount of time, are included in most of the reviewed papers. The maximum working time during a day usually varies from 5 to 10 hours. In this context,

Table 3. Performance measures and objectives found in the HHC literature.

References	UN	PP	TC	OT	TD	BP
Rasmussen et al. [8]	✓	—	✓	—	—	—
Bertels and Fahle [16]	—	✓	✓	—	—	—
Braekers et al. [23]	—	✓	✓	✓	—	—
Fernandez et al. [7]	—	—	—	—	✓	—
Hiermann et al. [10]	—	✓	—	✓	—	—
Mankowska et al. [1]	—	—	—	—	✓	—
Nickel et al. [24]	—	—	—	✓	✓	—
Bard et al. [19]	—	—	✓	✓	—	—
Bachouch et al. [27]	—	—	—	—	✓	—
Shao et al. [28]	—	—	✓	✓	—	—
Trautsmawieser et al. [14]	—	✓	—	✓	—	—
Mısır et al. [21]	—	✓	—	✓	—	—
Riazi et al. [20]	—	—	—	—	✓	—
Shi et al. [22]	—	—	✓	—	—	—
Liu et al. [26]	—	—	✓	—	—	—
This paper	✓	✓	—	—	—	✓

Table 5. Decisions and constraints handled in the HHC literature.

Reference	EP	SK	U	TW	B	S	WT	CC
Rasmussen et al. [8]	—	✓	—	✓	—	✓	—	—
Bertels & Fahle [16]	—	✓	—	✓	✓	—	✓	—
Braekers et al. [23]	—	✓	—	✓	—	—	✓	—
Fernandez et al. [7]	—	—	✓	—	—	—	—	—
Hiermann et al. [10]	—	✓	—	✓	—	—	—	—
Mankowska et al. [1]	—	✓	—	✓	—	✓	—	—
Nickel et al. [24]	—	✓	—	✓	≚	—	✓	—
Bard et al. [19]	—	✓	—	✓	✓	—	✓	—
Bachouch et al. [27]	—	✓	—	✓	✓	—	—	✓
Shao et al. [28]	—	✓	—	✓	✓	—	—	—
Trautsmawieser et al. [14]	—	✓	—	✓	—	—	—	—
Mısır et al. [21]	—	✓	—	✓	—	—	✓	—
Riazi et al. [20]	—	✓	—	✓	—	—	✓	—
Shi et al. [22]	—	✓	✓	✓	—	—	—	—
Liu et al. [26]	—	—	—	✓	—	✓	—	—
This paper	✓	✓	✓	✓	—	—	✓	—

some authors [11,14,15] preferred working times and penalized violations in the objective function to respect nurses' preferences. Mandatory breaks, like lunch times, are less frequently taken into account. Some papers such as [16,17] considered a predefined compulsory break node that ought to be visited by every single nurse. A more recent paper [18] considered if and at what time a break should occur by setting a maximum cumulative working time without a break. Bard et al. [19] demonstrated that the presence of different components such as overtime and need for scheduling lunch breaks would cause different theoretical and computational difficulties. To find solutions, they developed a branch-and-price-and-cut algorithm procedure and a novel rolling horizon algorithm that could incrementally construct weekly schedules by modifying the linear cost functions. The proposed algorithms proved to be capable of finding near-optimal solutions to small instances within 50 minutes. Additionally, Mankowska et al. [1] and Rasmussen et al. [8] claimed that 10% to 30% of all services were either concurrent or should be performed in a certain order to help heavy patients or prepare injections before or after meals.

Many developed solution methods for HHCSRP were developed based on the VRP techniques in many variants. Riazi et al. [20] integrated the gossip algorithm with a local solver based on Column Generation (CG), which made it a constructive algorithm for larger instances. They claimed that in the extensive numer-

ical examples, gossip-CG outperformed pure CG in minimizing the total distance traveled by all caregivers. Mısır et al. [21] evaluated the performance of heuristics while solving the problems via routing and rostering characteristics including HHCRSP and personnel rostering. They employed a novel hyper-heuristic as an analytical tool to investigate the behavior of the heuristics and determine the requirements for solving the problems. The experimental results revealed that different low-level heuristics performed better in solving problems, particularly with a vehicle routing component. Shi et al. [22] studied the HHCRSP assuming that the nurses' travel times and service times for patients were uncertain. They employed the Gurobi solver, simulated annealing, Tabu search, and variable neighborhood search to solve the problem. They concluded that in case of uncertainties in carrying out a given schedule to visit patients, the solutions obtained by the stochastic model and the Robust model demonstrated the advantage of the Robust model.

While most of previous studies had dealt with single-objective optimization models, Braekers et al. [23] developed a bi-objective HHC routing and scheduling model for the first time, considering the total travelling distance and patient inconvenience as two conflicting objectives functions. Nickel et al. [24] attempted to solve the routing and scheduling problem based on Master Schedule Problem (MSP) and Operational Planning Problem (OPP). They also developed

heuristics formulated based on constraint programming. In order to obtain a valid planning, Decerle et al. [25] modeled a problem considering soft TW and synchronization constraints as well as working time balancing. Using an MACO algorithm, they illustrate that a balanced working time among nurses is vital to gaining fairness and satisfaction in a home-based hospitalization structure. In another research, Hiermann et al. proposed a multi-modal HHC routing and scheduling supported by Austrian HHC providers [10]. They used a two-stage solution approach based on constraint programming and metaheuristic algorithms to solve the problem. Liu et al. [26] investigated a special variant of the VRP considering the TWs and synchronized visits. Given that the synchronization constraints can complicate the problem, Adaptive Large Neighborhood Search (ALNS) heuristics was proposed to solve this problem. They succeeded in obtaining the highest quality solutions in a shorter computation time than that in the previous methods. Bachouch et al. [27] discussed the problem of assigning patients to different care workers using a routing problem with some specific constraints. In this regard, they proposed an integer linear program to decide (1) which human resource should be used and (2) when the service should be executed during the planning horizon to satisfy the care plan for each patient served by the HHC providers. Shao et al. [28] presented the first algorithm to support weekly planning at the healthcare agencies with contracts. In order to better match patients' demands with therapist skills while minimizing the treatment, travel, and administrative costs, they modeled the problem as a mixed-integer program while developing a two-phase greedy randomized adaptive search procedure. At Phase I, daily routes are constructed for the therapists and at Phase II, a high-level neighborhood search is executed to converge on a local optimum. With the help of real-data provided by a U.S. rehabilitation agency and random instances, they could illustrate the effectiveness of the procedure.

The current research aimed to develop a tri-objective Mixed Integer Linear Programming (MILP) model to deal with the HHC problem, considering both EPs and preferred visit times. To this end, it employed the FGP and AEC approaches to solve the proposed model. Finally, it adopted several robust methods to address the uncertainty inherent in the travel time and duration of each patient's visit.

To the best of the authors' knowledge, as detailed in the literature, the EP in the case of the HHCRSP has not been considered yet. Although extensive research has been carried out on HHCRSP, no single study that had already examined the uncertainty conditions and patients' preferences simultaneously was found. In the next section, a set of explanations and details about the developed problem is given.

3. The proposed HHCRSP model

3.1. Problem definition

Consider an HCC that plans to use its limited nursing teams to service a specific number of emergency and non-emergency patients at different locations each day. The optimal TW for each patient's visit is pre-determined. In the planning period (daily), each nurse is available for a specific period who visits a certain number of patients. First, the patients should be assigned to available nurses. Then, nurse routing is done for servicing patients, and the time schedule for each patient's visit is determined by the nurses. For example, as outlined in Figure 4, 10 patients are scheduled to be serviced by the HCC
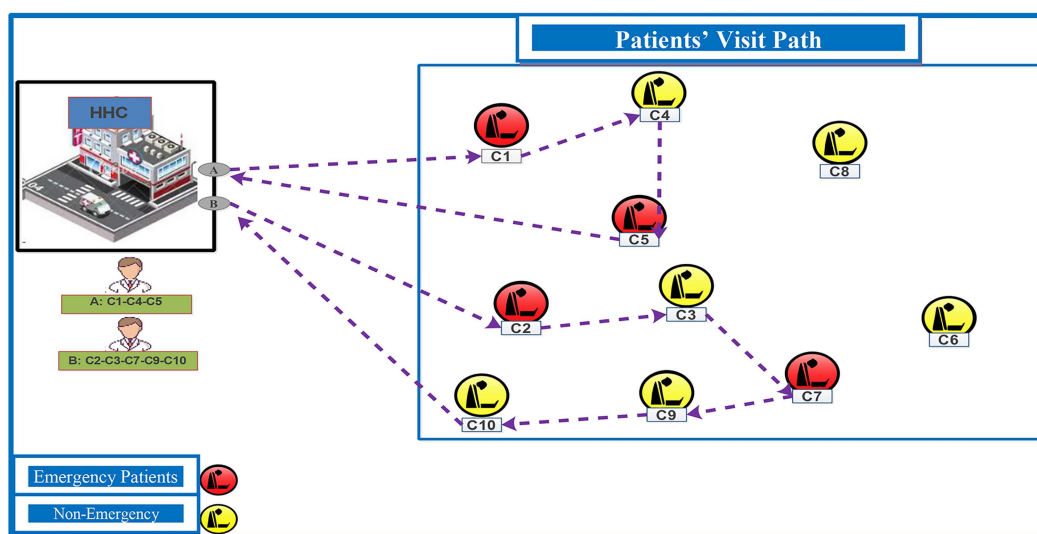

Figure 4. Schematic flow of patients' visits in the HHCRSP.

(two nurses). The assignment of nurses A and B to the patients set $C = \{C1, C2, \cdots, C10\}$ has been defined in advance. Therefore, patients $\{C1, C4, C5\}$ and $\{C2, C3, C7, C9, C10\}$ should be visited by nurse A and nurse B, respectively. The remaining patients will receive care in the next planning period. The patient's visit path is considered to be $C2 - C3 - C7 - C9 - C10$ for nurse B and $C1$–$C4$–$C5$ for nurse A. In this research, every nurse cannot visit any patient. Nurses in the HHC actually provide supportive care for patients who are sick, recovering, or disabled. They encounter patients with unique medical and nursing needs that must be fulfilled in order for these patients to be able to continue living at their own homes. Hence, there is a broad range of nurses' daily activities, e.g., giving treatment, cleaning, providing required drugs, conducting checkups, and so on. Moreover, nurses in the HHC are accountable to many patients; hence, they must perform their duties in the way required and contribute to good and safe care. It is precisely for this reason that it is assumed that nurses have restricted medical and nursing competencies indicating their skills to visit certain patients. In other words, nurses have a higher concentration on their patients' care and treatment.

Assume that the number of patients is equal to C. Consider the network N consisting of $C + 2$ nodes; the nodes $\{0\}$ and $\{C + 2\}$ are the same as HCC. At first, N nurses are placed in the HCC (i.e., node 0). After assigning nurses to patients, the path specific to each nurse is determined and ultimately, they return to the HCC (that is, the $N + 1$ node, which is possibly the same node as 0). Based on each nurse's path, the travelling time between the two nodes i and j of the network and duration of each patient's visit, patients' visit schedule (of course, if he/she is visited) is determined. The parameters of the travel time on the arc $(i - j)$ and stop time duration at each node i of the network N are among the parameters that should be considered in the HHCRSP to control the decision risk. According to the aforementioned explanations, the proposed optimization model pursues the following three objectives:

- Minimizing the number of patients who have not been visited;

- Minimizing the total (or maximum) deviation (earliness/tardiness) from the optimal TW for patients' visits;

- Maximizing patients who are visited in their optimal TW.

To accomplish the three above-mentioned objectives, the following decisions should be optimally made:

- How to allocate nurses to patients;

- How to prepare the patients' visit schedule (nurses' arrival time);

- How to determine the nurses' route.

3.2. Problem formulation

The most influential assumptions that are defined in modeling and solving the HHCRSP are as follows:

- The location of patients is given;

- The number of nurses is given;

- At moment 0, all nurses are present at the HCC (moment 0 can be taken, for example, as 7 am);

- The nurse's working time is determined;

- Based on the medical diagnosis and preferences of each patient, the optimal time interval for each patient's visit is predetermined;

- EP must be visited;

- Each patient is visited by only one nurse;

- The skills of nurses are restricted and every nurse cannot visit any patient;

- Travelling time between patients' locations and visit time durations is uncertain.

Since "travelling time between patients' locations" and "visit time durations" are uncertain, a nominal/average value with a perturbation value for these parameters is considered. For example, the nominal travel time between patients 4 and 7 is considered as 30 minutes with five-minute perturbation. Therefore, an interval between 25–35 minutes is considered as the travel time between patients 4 and 7. Prior to proposing a multi-objective optimization model, the nomenclature should be explained. The sets, parameters, and decision variables of the proposed optimization model are presented in Tables 6–8, respectively.

The objective functions and constraints of the proposed model are as follows:

Table 6. Sets of the proposed optimization model.

Sets	Definition
$C = \{C1, C2, \cdots, C10\}$	Set of patients
$D \subseteq C$	Set of emergency patients
$H = \{0, \lvert C \rvert + 1\}$	$0, \lvert C \rvert + 1$ are artificial nodes representing the origin and destination of the route (HCC)
$N = C \cup H$	Set of all nodes
$K = \{1, 2, \cdots, k, \cdots, \lvert K \rvert\}$	Set of nurses

Table 7. Parameters of the proposed optimization model.

Parameters	Definition
b_{kc}	If nurse k is qualified to visit patient c, it is equal to 1; otherwise, 0
$[\alpha_c - \beta_c]$	Optimal time window to service patient c
dm_c	Maximum permissible deviation from the optimal time window to visit patient c
vm_k	The maximum number of patients who can be visited by nurse k
am_k	Maximum working time duration of nurse k
$\widetilde{time_{i-j}}$	Travelling time between node i and node j (uncertain)
$\widetilde{vtime_c}$	Service duration of patient c (uncertain) $\widetilde{vtime_0} = \widetilde{vtime_{|C|+1}} = 0$
λ_c	The patient c's sensitivity coefficient compared to that of other patients ($\lambda_c > 1 \ \forall c \in D$ and $\lambda_c = 1 \ \forall c \notin D$)

Table 8. Decision variables of the proposed optimization model.

Decision variables	Definition
y_{kc}	Equals 1 if nurse k visits patient c; otherwise, 0.
g_c	Equals 1 if patient c is visited in the optimal time window; otherwise, 0.
x_{kij}	Equals 1 if nurse k travels arc $i - j$; otherwise, 0.
t_{kc}	Service start time of nurse k at patient c ($t_{k0} \geq 0$ when nurse k starts at HCC and $t_{k|C|+1} > 0$ when nurse k returns to HCC at the end of the working day)
e_c	Early arrival to visit patient c according to the optimal time window
l_c	Late arrival to visit patient c according to the optimal time window

$$\text{Minimize } F_1 = \sum_{c \in C} \left(1 - \sum_{k \in K} y_{kc} \right), \qquad (1)$$

$$\text{Minimize } F_2 = \max_{c \in C} (\lambda_c (e_c + l_c)), \qquad (2)$$

$$\text{Maximize } F_3 = \sum_{c \in C} g_c. \qquad (3)$$

Objective Function (1) minimizes the number of uncovered patients during the planning horizon (i.e., on a daily basis). Note that if a patient is visited, $\sum_{k \in K} y_{kc} = 1$ and $1 - \sum_{k \in K} y_{kc} = 0$; therefore, this patient is not considered an uncovered one. Objective Function (2) minimizes the maximum deviation from the optimal time window of the patients' services. This function aims to confine tardiness or earliness of services to patients. Note that $e_c.l_c = 0$ is permanent (i.e., merely earliness or tardiness takes place or both variables are equal to 0 in value). This linearization of the Objective Function (2) is performed by Eq. (4):

$$\text{Minimize } F_2 = u,$$

$$u \geq \lambda_c (e_c + l_c); \qquad \forall \ c \in C. \qquad (4)$$

Objective Function (3) maximizes the number of patients visited in their optimal TW.

s.t.:

$$\sum_{k \in K} y_{kc} \leq 1; \qquad \forall \ c \in C, \qquad (5)$$

$$\sum_{k \in K} y_{kc} = 1; \qquad \forall \ c \in D \subseteq C, \qquad (6)$$

$$\sum_{c \in C} y_{kc} \leq vm_k; \qquad \forall \ k \in K, \qquad (7)$$

$$y_{kc} \leq b_{kc}; \qquad \forall \ k \in K, \quad c \in C, \quad (8)$$

$$g_c \leq \sum_{k \in K} y_{kc}; \qquad \forall \ c \in C, \qquad (9)$$

$$\sum_{j \in N} x_{k0j} = 1; \qquad \forall \ k \in K, \qquad (10)$$

$$\sum_{i \in N} x_{ki(|C|+1)} = 1; \qquad \forall \ k \in K, \qquad (11)$$

$$\sum_{i \in N} x_{kic} = \sum_{i \in N} x_{kci}; \qquad \forall \ k \in K, \quad c \in C, \ (12)$$

$$\sum_{i \in N} x_{kic} = y_{kc}; \qquad \forall \ k \in K, \quad c \in C, \ (13)$$

$$t_{kj} \geq t_{ki} + \widetilde{\text{time}}_{i-j} + \widetilde{v\text{time}}_i + (x_{kij} - 1)M;$$
$$\forall \ k \in K, i, \quad j \in N, \qquad\qquad (14)$$

$$(\alpha_c - dm_c)y_{kc} \leq t_{kc} \leq (\beta_c + dm_c)y_{kc};$$
$$\forall \ k \in K, \quad c \in C, \qquad\qquad (15)$$

$$t_{k|C|+1} - t_{k0} \leq am_k; \qquad \forall \ k \in K, \qquad (16)$$

$$e_c \geq \alpha_c - t_{kc} - M(1 - y_{kc}); \qquad \forall \ k \in K, \quad c \in C, \ (17)$$

$$l_c \geq t_{kc} - \beta_c; \qquad \forall \ k \in K, \quad c \in C, \ (18)$$

$$g_c = \begin{cases} 1 & \text{if} \quad l_c + e_c = 0 \\ 0 & \text{if} \quad l_c + e_c > 0 \end{cases} \qquad \forall \ c \in C, \qquad (19)$$

$$\begin{cases} y_{kc} \in \{0,1\} \\ g_c \in \{0,1\} \\ x_{kij} \in \{0,1\} \\ t_{kc} \geq 0 \\ t_{k0} \geq 0 \\ t_{k|c|+1} \geq 0 \\ e_c \geq 0 \\ l_c \geq 0 \end{cases} \qquad\qquad (20)$$

Constraint (5) ensures that each patient is either visited by one nurse or uncovered (maximum of one nurse is assigned to each patient). Constraint (6) indicates that every single EP must be visited (a nurse will surely visit any EP). Constraint (7) restricts the number of patients assigned to each nurse. Constraint (8) guarantees that nurse k can visit patient c on condition that the nurse is sufficiently qualified for service patient c. In other words, if $b_{kc} = 0$, then $y_{kc} = 0$. Constraint (9) ensures that one nurse visits any patient who is supposed to be visited in the optimal TW ($\sum_{k \in K} y_{kc} = 1$). Constraints (10) and (11) ensure that the route of each nurse starts and ends in the HCC. Constraint (12) represents inflow-outflow conditions and ensures that nurse k, who visits patient i, should leave this patient after the service. Constraint (13) ensures that if nurse k is assigned to patient c, then patient c must be located on the path of nurse k. Constraint (14) calculates the start times of the service operations considering the traveling times and service durations. The constraint shows that the start times of services along a nurse's route are

explicitly increasing. In doing so, they evade cycles in the routes as well because a return to an already visited patient would violate the start time of the prior visit. Note that in this case, M is a large number. Obviously, if the arc $(i - j)$ is not traversed, this constraint will be deactivated. Given this constraint, the classic constraint of sub-tour elimination, which is as Constraint (21), will be redundant [29]:

$$w_{ki} - w_{kj} + |C| \cdot x_{kij} \leq |C| - 1;$$
$$\forall \ i, j \in C; \quad k \in K. \qquad\qquad (21)$$

A hard time window (optimal time window with the maximum deviation allowed $[\alpha_c - dm_c, \beta_c + dm_c]$) is considered by every nurse for each patient's visit through Constraint (15). Note that if nurse k is not assigned to patient c, then $y_{kc} = 0$ and $t_{kc} = 0$, and this constraint becomes deactivated. Nurses are only allowed to work within a given time in a day, which is handled by Constraint (16). It should be noted that the variable $t_{k|c|+1}$ represents the arrival back of the nurse k to the HCC, and if he/she leaves the HCC at moment 0 ($t_{k0} = 0$), then $t_{k|c|+1} - t_{k0}$ is equal to his/her working time. Constraint (17) computes the early arrival to visit patient c compared to the optimal TW. As mentioned above, M is a large number (as large as α_c), and it should be noted that if nurse k is not allocated to patient c ($y_{kc} = 0$), this inequality is deactivated. Similarly, Constraint (18) shows the nurse's late arrival to visit patient c compared to the optimal TW. Constraint (19) determines whether or not a patient is visited in his/her optimal TW. If no early arrival or late arrival take place ($l_c + e_c = 0$), then, $g_c = 1$; otherwise, $g_c = 0$. This equality can be linearized as follows:

$$g_c \leq 1 - \frac{(l_c + e_c)}{M}; \qquad \forall \ c \in C, \qquad (22)$$

$$g_c \geq 1 - (l_c + e_c)M; \qquad \forall \ c \in C. \qquad (23)$$

Constraint (20) defines the feasible domains of decision variables.

4. Solution approach

The general form of a mathematical model with n objective functions can be stated as follows:

$$\min(F_1(x), F_2(x), \cdots, F_n(x)); \qquad x \in X. \qquad (24)$$

An ideal solution for the problem formulated in Constraint (24) optimizes all the objective functions simultaneously while satisfying all constraints. However, most real-world problems involve conflicting objectives; hence, any feasible solution cannot optimize all the

objective functions at the same time. Accordingly, decision-makers seek a preferable and effective solution. The solution methods used to cope with the proposed multi-objective optimization model are as follows.

4.1. FGP method

FGP is a method for solving multi-objective optimization problems and it was first proposed by Rubin and Narasimhan [30]. This method is an extension of Goal Programming (GP) method that considers a goal for each objective and attempts to get as much closer as possible to these goals. FGP considers a goal for each objective using fuzzy numbers (fuzzy membership function) where the value of each goal varies from 0 to 1. Then, the weighted sum of goals is expressed as the objective function of the problem. Assume that the single-objective version of the problem formulated in Constraint (24) has been already solved. Based on the results of individual optimization, a payoff table can be obtained according to which $m_i = f_i^*$ and M_i are considered as the optimal values of the objective i and upper bound for this objective, respectively. Accordingly, in the FGP method, each objective i is expressed based on the following goal:

$$G_i(x) = \mu(x)_{f_i} = G(f_i(x))$$

$$= \begin{cases} 0; & f_i(x) \geq M_i \\ \frac{M_i - f_i(x)}{M_i - m_i}; & x \in X \\ 1; & f_i(x) \leq m_i \end{cases} \qquad (25)$$

Therefore, the Multi-objective Optimization Model (MODM) problem modeled in Constraint (24) can be rewritten and solved using the FGP method (on the basis of the fuzzy membership function in Eq. (25)):

$$\max \sum_i w_i G_i(x),$$

$$G_i(x) = \frac{M_i - f_i(x)}{M_i - m_i}; \qquad i = 1, 2, \cdots, n; \quad x \in X, \qquad (26)$$

where $f_i(x) \to m_i$ and $G_i(x) \to 1$. Of note, in the proposed FGP method, w_i represents the relative importance of objective i, which is assumed to be determined by the decision-maker(s).

4.2. AEC method

Consider the general expression of an MODM problem based on the optimization model (24). Suppose that the objective k is to be minimized provided that other objectives are limited to the predetermined upper bounds. In this case, based on the EC method, the following single-objective optimization model is obtained:

$$\min f_k(x),$$

$$f_i(x) \leq e_i; \qquad i = 2, 3, \cdots, n, \ i \neq k; \quad x \in X. \qquad (27)$$

In Constraint (27), different solutions can be achieved by changing the values of e_i that may not be effective (weakly efficient). However, the problem can be solved in a more effective manner by modifying the model in Constraint (27) based on the AEC method [31]. In this method, it is required that the payoff matrix as well as the minimum (m_i) and maximum (M_i) values for each objective be computed. Then, the range of changes in each e_i is defined as $e_i \in [m_i, M_i]$, and the value of $R_i = M_i - m_i$ is defined as a domain to normalize the objectives. If $e_i < m_i$, the problem becomes infeasible; if $e_i \geq M_i$, inefficient or repetitive solutions can be obtained. In the AEC method, the following model replaces the model in the EC method (Relation (27)):

$$\min f_k(x) - \sum_{i=1, i \neq k}^n \phi_i s_i,$$

$$f_i(x) + s_i = e_i;$$

$$i = 1, 2, 3, \cdots, n, \quad i \neq k; \quad x \in X; \quad s_i \geq 0. \qquad (28)$$

An effective solution is obtained in the AEC method by assigning distinct values to $e_i \in [m_i, M_i]$. Ultimately, the values of objectives with regard to different values of e_i generate the Pareto frontier.

4.3. Robust Optimization Approach (ROA)

Generally, the robust solution in an optimization problem under uncertainty conditions is a solution that acts justifiably with respect to uncertain data in most cases with less deviance than the optimal value of the objective function. Consider the general form of a linear optimization problem as Constraint (29):

$$\min \quad c^T x,$$

s.t.:

$$Ax \leq b. \qquad (29)$$

The above-mentioned optimization model will be an uncertain one if at least one of the components of the c and b vectors or the matrix A is uncertain. Here, $\xi = [c, A, b]$ is defined as parameters of Model (29). Consider the set U as all possible states for ξ (parameters of the problem). Clearly, if $|U| = 1$, the above-mentioned model is a deterministic optimization one. However, in most real-world problems, we have $|U| > 1$; hence, an uncertain optimization problem is obtained. Therefore, robust optimization methods are used in the state of $|U| > 1$. One of the robust optimization methods is the interval method of Ben-Tal et al. [32], which is an extension of the Soyster's strict method [33].

Accordingly, the interval method considers a closed bounded interval for the model parameters. In this case, the robust solution in most possible cases is made feasible for the corresponding intervals. In order to clarify this method, consider the following uncertain optimization problem:

$$\min\ z = \sum_j \widetilde{a_{0j}} x_j + \widetilde{b_0},$$

s.t.:

$$\sum_j \widetilde{a_{ij}} x_j \le \widetilde{b_i} \qquad \forall\ i = 1, 2, \cdots, m, \tag{30}$$

where for each $i = 0, 1, 2, \cdots, m$: $\widetilde{a_{ij}} \in [a_{ij}^L, a_{ij}^U]$ and $\widetilde{b_i} \in [b_i^L, b_i^U]$. Now, we define:

$$\widetilde{a_{ij}} = a_{ij} + \xi_{ij} \cdot \widehat{a_{ij}}, \tag{31}$$

$$\widetilde{b_i} = b_i + \xi_i \cdot \widehat{b_i}, \tag{32}$$

where a_{ij} and b_i are the nominal values for the parameters, and $\widehat{a_{ij}}$ and $\widehat{b_i}$ are the maximum perturbations of each parameter from its nominal value. The relationship between the nominal values for parameters and their associated maximum perturbations is defined through Eqs. (33) and (34).

$$a_{ij} = \frac{a_{ij}^L + a_{ij}^U}{2}, \qquad \widehat{a_{ij}} = a_{ij} - a_{ij}^L, \tag{33}$$

$$b_i = \frac{b_i^L + b_i^U}{2}, \qquad \widehat{b_i} = b_i - b_i^L. \tag{34}$$

According to the above-mentioned definitions, it can be shown that the variations of each parameter/data of Model (31) at its corresponding interval are equivalent to the variation in its corresponding ξ value at the $[-1, 1]$ interval. Note that if a parameter of the problem is deterministic, the upper and lower bounds of its corresponding uncertain interval are equal and also equal to the nominal value. Followed by normalizing the uncertainty interval of each parameter, Model (30) can be rewritten as Model (35), where all values of ξ_{ij}, ξ_i are at the $[-1, 1]$ interval.

$$\min\ z = \sum_j a_{0j} x_j + b_0 + \sum_i \xi_j \widehat{a_{0j}} x_j + \xi_0 \cdot \widehat{b_0},$$

s.t.:

$$\sum_j a_{ij} x_j + \sum_j \xi_{ij} \widehat{a_{ij}} x_j \le b_i + \xi_i \cdot \widehat{b_i},$$

$$\forall\ i = 1, 2, \cdots, m. \tag{35}$$

Followed by normalization in the interval robust method, Model (36) is considered as the robust counterpart of Model (30), where Θ_i is the control param-

eter of the uncertainty level in each constraint.

$$\min\ z = \max_{\xi_j, \xi_0} \Bigg(\sum_j a_{0j} x_j + b_0 + \sum_j \xi_j \widehat{a_{0j}} x_j + \xi_0 \cdot \widehat{b_0} \Bigg),$$

s.t.:

$$\sum_j a_{ij} x_j + \Theta_i \cdot \max_{\xi_{ij}, \xi_i} \Bigg(\sum_j \xi_{ij} \widehat{a_{ij}} x_j - \xi_i \cdot \widehat{b_i} \Bigg) \le b_i$$

$$\forall\ i = 1, 2, \cdots, m. \tag{36}$$

In the following, three prominent uncertainty spaces are described. These uncertainty spaces will be further used for controlling the uncertain parameters of the HHCRSP.

4.3.1. Box uncertainty space
In case ξ's vary independently at the $[-1, 1]$ interval in the objective function and each constraint of Model (36), the uncertainty space is called box uncertainty space. According to Li et al. [34], Model (37) is the robust counterpart of the model based on the box uncertainty space.

$$\min\ z,$$

s.t.:

$$z \ge \sum_j a_{0j} x_j + b_0 + \psi_0 \Bigg(\sum_j \widehat{a_{0j}} |x_j| + \widehat{b_0} \Bigg),$$

$$\sum_j a_{ij} x_j + \psi_i \sum_j \widehat{a_{ij}} |x_j| \le b_i - \psi_i \cdot \widehat{b_i}$$

$$\forall\ i = 1, 2, \cdots, m, \tag{37}$$

where the parameter ψ shows the risk aversion level or uncertainty coverage in the parameters of each constraint. Note that if $\psi = 0$, it can be implied that uncertain parameters are replaced with nominal values. In addition, if $\psi = 1$, the uncertainty coverage is considered as full, and the robust optimal solution obtained from Model (36) will be feasible in all possible cases for uncertain parameters.

4.3.2. ELL uncertainty space
Suppose that in Model (36), ξ_{ij}'s are constrained based on the ellipsoidal relationship (38).

$$\|\xi_{ij}\|_2 = \sqrt{\sum_j \xi_{ij}^2} \le \Omega_i^2$$

$$\forall\ i = 0, 1, 2, \cdots, m; \quad \xi_{ij} \in [-1, 1]. \tag{38}$$

The uncertainty space is called the ELL uncertainty space. According to Li et al. [34], the robust counterpart of the model based on the ELL uncertainty space is equivalent to Model (39):

$$\min \ z,$$

$$\text{s.t.:},$$

$$z \geq \sum_j a_{0j} x_j + b_0 + \Omega_0 \left(\sqrt{\sum_j \left(\widehat{a_{0j}} x_j \right)^2 + \widehat{b_0}^2} \right),$$

$$\sum_j a_{ij} x_j + \Omega_i \sqrt{\sum_j \left(\widehat{a_{ij}} x_j \right)^2} \leq b_i - \Omega_i \cdot \widehat{b_i}^2$$

$$\forall \ i = 1, 2, \cdots, m, \tag{39}$$

where the parameter Ω shows the risk aversion level or uncertainty coverage in each constraint. Note that if $\Omega = 0$, similar to the Box uncertainty space, uncertain parameters are replaced with the nominal values. In addition, if $\Omega = 1$, uncertainty coverage is fully applied in the defined ELL set. Moreover, if the control parameters ψ and Ω are equal in both Box and ELL uncertainty spaces, the Box uncertainty space is always greater than the ELL uncertainty space and the Box will be more risk averse.

4.3.3. PH uncertainty space
Suppose that in Model (36), ξ_{ij}'s are constrained based on the polyhedral relationship (Eq. (40)):

$$\| \xi_{ij} \|_1 = \sum_j |\xi_{ij}| \leq \Gamma_i$$

$$\forall \ i = 0, 1, 2, \cdots, m; \quad \xi_{ij} \in [-1, 1]. \tag{40}$$

According to [34], the robust counterpart of the model based on the PH uncertainty space is equivalent to Model (41):

$$\min \ z,$$

$$\text{s.t.:}$$

$$z \geq \sum_j a_{0j} x_j + \Gamma_0 \cdot t_0,$$

$$t_0 \geq \widehat{a_{0j}} |x_j| \quad \text{and} \quad t_0 \geq \widehat{b_0} \qquad \forall \ j = 1, 2, \cdots, n,$$

$$\sum_j a_{ij} x_j + \Gamma_i \cdot t_i \leq b_i,$$

$$t_i \geq \widehat{a_{ij}} |x_j| \quad \text{and} \quad t_i \geq \widehat{b_i} \qquad \forall \ j = 1, 2, \cdots, n, \tag{41}$$

where the parameter Γ indicates the risk aversion level or uncertainty coverage in the parameters of each constraint. In this case, if $\Gamma = 0$, uncertain parameters

are replaced by nominal values. In addition, if $\Gamma = 1$, the whole uncertainty of the defined PH set will be covered. For equal control parameters ($\psi = \Omega = \Gamma$), the Box state covers most uncertainties and the ELL state covers a space less than Box and more than PH. Figure 5 clearly illustrates the uncertainty spaces in the presence of two uncertain parameters for cases of the Box, ELL, and PH uncertainty spaces.

In the HHCRSP model presented in this study, the parameters of travel time between two nodes ($\widetilde{time_{i-j}}$) and the visit time of each patient ($\widetilde{vtime_i}$) were considered uncertain in the form of intervals. For each uncertain parameter, an uncertainty interval was defined as $\widetilde{data} \in [data^L, data^U]$ based on which the nominal value $data = \frac{data^L + data^U}{2}$ and deviation value $\widehat{data} = data^U - data$ can be obtained. Once the nominal values and deviation of each uncertain parameter are calculated, the robust counterparts can be determined based on each of the Box, ELL, and PH methods. The robust counterparts of Relation (14), containing two uncertain parameters of ($\widetilde{time_{i-j}}$) and ($\widetilde{vtime_i}$), are represented by using robust methods of Box, ELL, and PH through Relations (42), (43), and (44), respectively:

$$\begin{cases} (Box) \\ t_{kj} \geq t_{ki} + time_{i \to j} + vtime_i \\ \qquad + \psi_{ij} \left(\widetilde{time_{i \to j}} + \widetilde{vtime_i} \right) + (x_{kij} - 1)M; \\ \forall \ k \in K, i, \quad j \in N \end{cases} \tag{42}$$

$$\begin{cases} (Ell) \\ t_{kj} \geq t_{ki} + time_{i \to j} + vtime_i \\ \qquad + \Omega_{ij} \sqrt{ \left(\widetilde{time_{i \to j}^2} + \widetilde{vtime_i}^2 \right)} + (x_{kij} - 1)M; \\ \forall \ k \in K, \quad i, j \in N \end{cases} \tag{43}$$
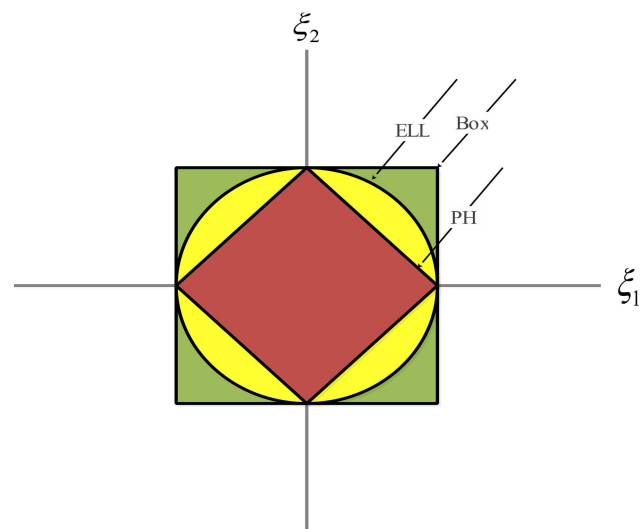
Figure 5. The uncertainty space in Box, ELL, and PH states.

$$\begin{cases} (PH) \\ t_{kj} \geq t_{ki} + time_{i \to j} + vtime_i \\ \qquad + \Gamma_{ij} \cdot t_{ij} + (x_{kij} - 1)M; \quad \forall\, k \in K,\ i,\ j \in N, \\ t_{ij} \geq \widehat{time_{i \to j}} \qquad\qquad \forall\, i, j \in N, \qquad (44) \\ t_{ij} \geq \widehat{vtime_i} \qquad\qquad\quad \forall\, i, j \in N \end{cases}$$

where $\widetilde{time_{i-j}} \in [time_{i \to j}^L - time_{i \to j}^U]$ and $\widetilde{vtime_i} \in [vtime_i^L - time_i^U]$ are taken into consideration, and their nominal values and deviations are defined as follows:

- $time_{i \to j} = \frac{(time_{i \to j}^L + time_{i \to j}^U)}{2}$,
- $\widehat{time_{i \to j}} = time_{i \to j}^U - time_{i \to j}$,
- $vtime_i = \frac{(vtime_i^L + vtime_i^U)}{2}$,
- $\widehat{vtime_i} = vtime_i^U - vtime_i$.

5. Computational results

To solve the developed Mixed Integer Linear Programming (MILP) model, the CPLEX Solver in the GAMS software version 24.7.1 was used on a personal computer with the important features of the "Central Processing Unit (CPU): Core™ i5 2.5 GHz" and "Random Access Memory (RAM): 6.0 GB". Since no benchmark data was available for the problem, the numerical example was randomly generated, in which one HCC with a specified location planned to schedule five nursing teams to serve 25 nursing patients daily and determine their travel routes. Among all 25 patients at this care center, there are seven EP that should be necessarily visited. It is desirable that nursing teams visit other patients (18 non-emergency patients), as well. The general information about the HHCRSP test problem is given in Table 9.

In Table 10, a two-dimensional coordinate is defined for each patient that shows his/her location. Based on the defined coordinates, their Euclidean distance is obtained to determine the nominal value of the time parameter for the displacement between every two nodes of the network. It is assumed that the time is proportional to the distance between the two

Table 9. The general information about the HHCRSP test problem.

Patients	Optimal time window $[\alpha_c - \beta_c]$	Maximum permissible deviation from time window	Visit duration (in distance) $\widetilde{vtime_c} \in [vtime_c^L - time_c^U]$	Patient's sensitivity coefficient λ_c	Location	
					X	Y
1	[20−40]	10	[5−10]	1	15	7
2	[15−50]	10	[5−10]	1	11	4
3	[30−40]	15	[10−20]	1.3	8.5	1
4	[60−70]	10	[10−15]	1	9.5	16
5	[40−60]	10	[5−10]	1	2	2.5
6	[30−50]	10	[5−10]	1	17	13
7	[45−72]	10	[10−20]	1.5	4	15
8	[33−63]	10	[10−20]	1.2	15	15
9	[55−72]	10	[10−20]	1.6	13	9
10	[43−74]	15	[10−20]	1.1	7	6.5
11	[35−80]	10	[5−10]	1	18	4.5
12	[52−92]	20	[10−15]	1	19	18
13	[74−93]	15	[5−10]	1	18.5	9.5
14	[61−85]	10	[5−10]	1	16	1
15	[70−95]	10	[10−20]	1.4	1	10
16	[68−93]	15	[5−10]	1	1.5	19
17	[82−100]	10	[10−20]	1.2	14.5	13
18	[95−104]	10	[5−10]	1	8	18
19	[83−110]	15	[5−10]	1	5	10
20	[90−100]	10	[5−10]	1	13	18.5
21	[85−120]	15	[5−10]	1	2	7.5
22	[75−118]	10	[10−15]	1	11.5	13.5
23	[92−110]	10	[10−15]	1	8	12
24	[80−115]	20	[5−10]	1	1	13
25	[90−105]	10	[5−10]	1	5.5	4
HCC	Starting from 0	—	—	—	10	10

Table 10. Information on patient in the numerical study of the HHCRSP.

Planning period	No. non-emergency patients	No. emergency patients	Total patients	No. nurses	No. HCC
1	18	7	25	5	1

Table 11. Information on nurses in the numerical study of HHCRSP.

	Nurse 1	Nurse 2	Nurse 3	Nurse 4	Nurse 5
vm_k	6	6	6	6	6
am_k	100	100	100	100	100

nodes. For the perturbation component in the value of this uncertain parameter, a random number between 10% and 40% is generated. This coefficient for the parameter $\widehat{time}_{i \to j}$ is multiplied by the nominal value.

Other patient information (emergency vs non-emergency, sensitivity coefficient, optimal time window of visit, etc.) is shown in Table 10. In addition, Tables 11 and 12 provide information about nurses (nurses' skills, availability of each nurse, and the number of patients that each nurse can visit per day, etc.).

Table 12. Information on nurses' skills in the numerical study of HHCRSP.

b_{kc}	Nurse 1	Nurse 2	Nurse 3	Nurse 4	Nurse 5
1	1	1	1	0	1
2	0	1	1	1	1
3	1	1	1	1	1
4	1	1	1	0	1
5	0	1	0	0	1
6	1	1	1	0	1
7	0	1	1	1	1
8	1	1	1	0	1
9	1	0	1	0	1
10	1	0	0	1	1
11	0	1	1	1	0
12	1	0	0	1	1
13	1	1	0	1	1
14	0	1	1	1	0
15	1	1	1	1	1
16	0	1	1	0	0
17	1	1	1	1	1
18	0	1	1	0	1
19	1	1	1	1	1
20	1	0	0	1	1
21	1	0	0	1	1
22	0	1	1	1	0
23	1	0	1	1	1
24	1	1	1	1	1
25	1	1	1	1	1

5.1. Solving the model using FGP and robust optimization based on the Box uncertainty set

The numerical example is solved using the FGP method and robust optimization in a way that uncertain parameters are controlled by a Box uncertainty space with the control parameter $\psi = 0.8$. Tables 13 and 14 show the optimal values of objectives as well as the payoff matrix of objectives, respectively, obtained from solving the model through FGP and robust optimization based on the Box uncertainty space. A pairwise comparison between the trends of objectives in Table 14 confirms that there is a contradiction between the objectives of the proposed optimization model. Given the maximum and minimum values of each objective function in the payoff matrix, the $G1$, $G2$, and $G3$ goals, defined for all the three objectives of the model, are as follows:

$$(M_1 = 11 \ \& \ m_1 = 0) \to G_1 = \frac{11 - F_1}{11},$$

$$(M_2 = 9.64 \ \& \ m_2 = 0) \to G_2 = \frac{9.64 - F_2}{9.64},$$

$$(M_3 = 14 \ \& \ m_3 = 3) \to G_3 = \frac{F_3 - 3}{11}.$$

Assume that the weights of all model objectives are the same, i.e., $w_1 = w_2 = w_3 = 1$. The objective function

Table 13. Optimal values of the objective functions (FGP and robust optimization based on the Box uncertainty set).

F_1	F_2	F_3
0	0	14

Table 14. Payoff matrix (FGP and robust optimization based on the Box uncertainty set).

Payoff (i, j)	F_1	F_2	F_3
F_1	0	11	11
F_2	9.64	0	0
F_3	3	14	14

of the FGP method, which should be maximized, is calculated as follows:

$$Z_{FGP} = w_1 G_1 + w_2 G_2 + w_3 G_3$$

$$= \frac{11 - F_1}{11} + \frac{9.64 - F_2}{9.64} + \frac{F_3 - 3}{11}.$$

Accordingly, the optimal solution presented in Table 15 is obtained using the FGP method. Table 16 shows the state of each patient's visit as well as the nurse assigned to any patient according to the FGP method. Figure 6 shows the route of each nurse for visiting the patients based on the FGP method.

Table 15. Optimal solution of the numerical example based on the FGP method.

F_1	F_2	F_3
5	5.32	8

5.2. Solving the model using AEC and robust optimization based on the Box uncertainty set

In order to use the AEC method, the payoff matrix must first be obtained. The payoff matrix of the FGP method can also be used for the AEC method. In this regard, the payoff matrix presented in Table 14 should be employed. Accordingly, the minimum, maximum, and range of variations to each objective are as follows:

$$(M_1 = 11 \ \& \ m_1 = 0) \rightarrow R_1 = 11,$$

$$(M_2 = 9.64 \ \& \ m_2 = 0) \rightarrow R_2 = 9.64,$$

$$(M_3 = 14 \ \& \ m_3 = 3) \rightarrow R_3 = 11.$$

In the proposed AEC method, the third objective is set as the main objective function and the first and second objectives are bounded using the upper limit of ε_1 and

Table 16. Patients' visits in the optimal solution of the FGP method (E: Emergency, NE: Not-Emergency).

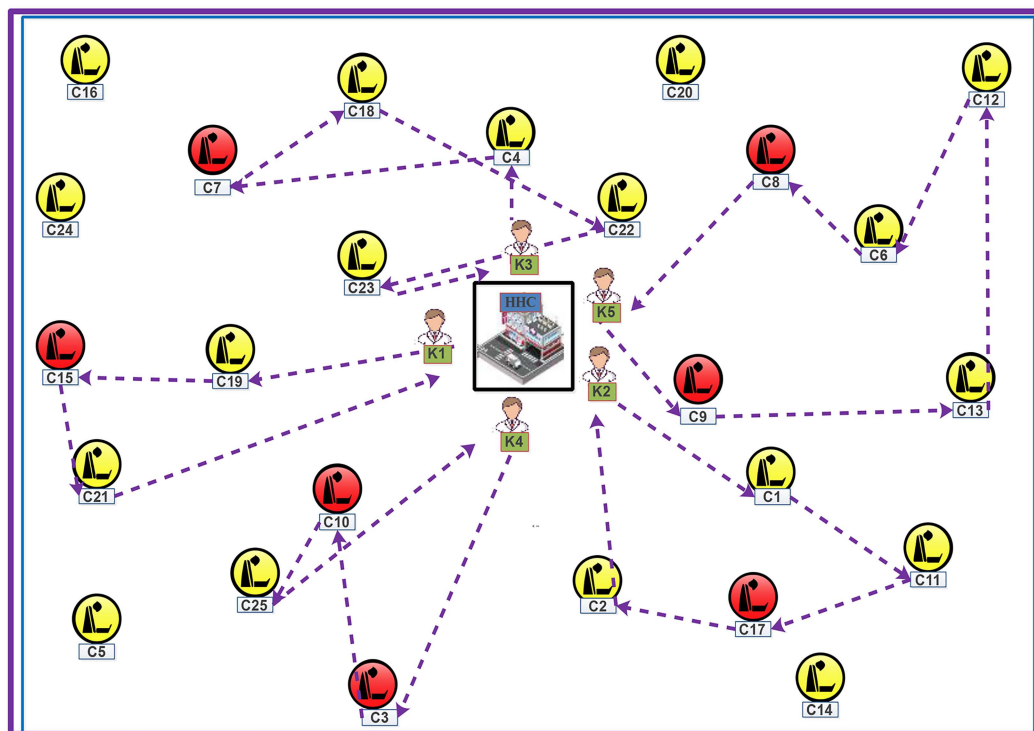|	Patient status	Visited (1) Not visited (0)	Visiting in optimal time window	The assigned nurse
1	NE	1	1	$K2$
2	NE	1	0	$K2$
3	E	1	1	$K4$
4	NE	1	0	$K3$
5	NE	0	0	—
6	NE	1	0	$K5$
7	E	1	1	$K3$
8	E	1	1	$K5$
9	E	1	0	$K5$
10	E	1	1	$K4$
11	NE	1	0	$K2$
12	NE	1	0	$K5$
13	NE	1	0	$K5$
14	NE	0	0	—
15	E	1	1	$K1$
16	NE	0	0	—
17	E	1	1	$K2$
18	NE	1	0	$K3$
19	NE	1	0	$K1$
20	NE	0	0	—
21	NE	1	0	$K1$
22	NE	1	0	$K3$
23	NE	1	1	$K3$
24	NE	0	0	—
25	NE	1	0	$K4$

Figure 6. Nurses' paths for patient visits based on the FGP method in the numerical study of HHCRSP.

Table 17. Pareto solutions obtained from the AEC method.

Pareto solution	F_3	F_2	F_1	ε_1
1	3	9.64	0	0
2	5	8.42	1	1
3	5	7.23	2	2
4	6	7.01	3	3
5	7	6.93	4	4
6	8	5.32	5	5
7	9	4.76	6	6
8	10	4.12	7	7
9	11	3.67	8	8
10	12	2.11	9	9
11	13	1.43	10	10
12	14	0	11	11

Figure 7. The 3D diagram of the Pareto front obtained from the AEC method.

ε_2. As a result, we will have $\varepsilon_1 \in [m_1 = 0, M_1 = 11]$, $\varepsilon_2 \in [m_2 = 0, M_2 = 9.64]$, and $\varphi_2 = \frac{R_3}{R_2} = \frac{11}{9.64} = 1.14$. Table 17 shows the Pareto solutions obtained from the AEC method. Figure 7 demonstrates the 3D diagram of the Pareto front obtained from the AEC method. The balance between the first and second objectives and the balance between the first and objectives based on the AEC method are given in Figures 8 and 9, respectively. Again, Figures 8 and 9 confirm the
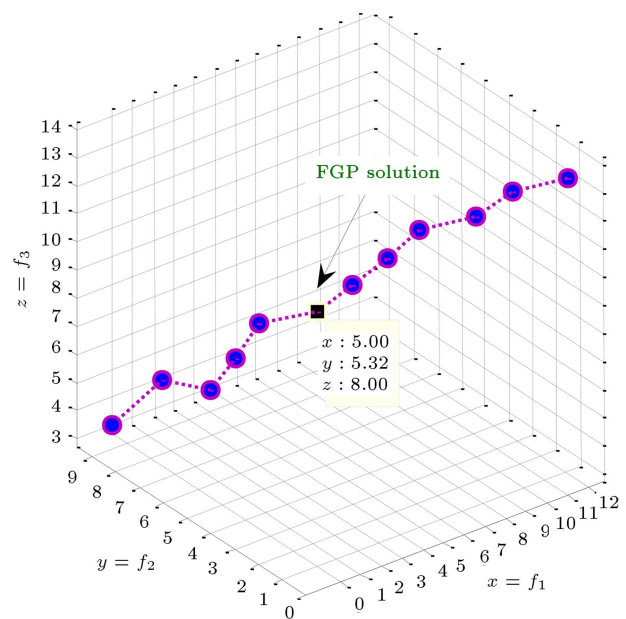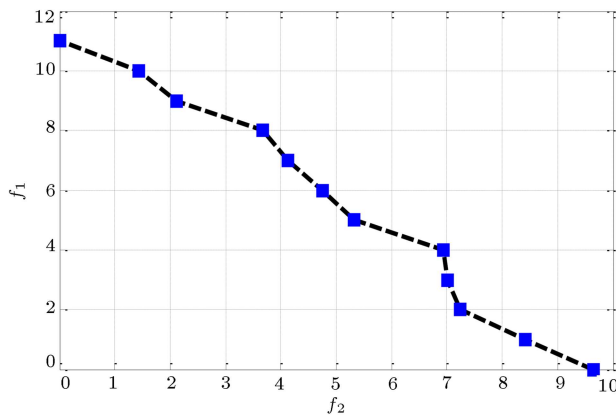
contradictions between the objectives of the proposed model.

As observed in the Pareto front diagram of the AEC method, one of the Pareto solutions obtained from the AEC method is equal to a solution previously obtained through the FGP method. In the AEC method, unlike the FGP method, a set of Pareto solutions can be obtained. The decision-maker selects

Figure 8. The balance of the first and second objectives based on the AEC method.
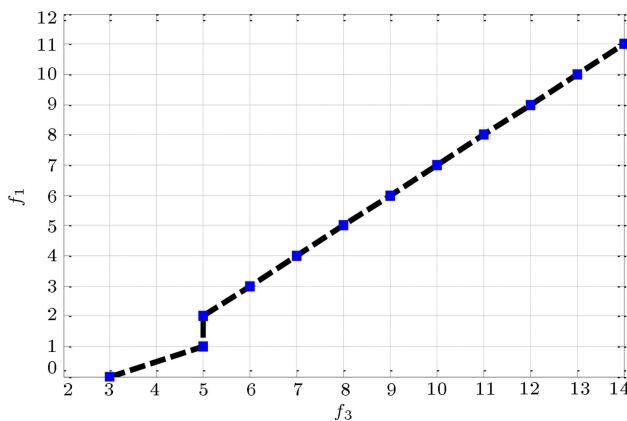
Figure 9. The balance of the first and third objectives based on AEC method.

one of the Pareto solutions for implementation. An important question may arise here: Which solution should be selected from the set of Pareto solutions of AEC method? To answer this question, Mean Ideal Distance (MID) criterion is of great help. Given the

optimal solutions reported in Table 14, ($F_1^* = 0, F_2^* = 0, F_3^* = 14$), it is considered as the ideal solution.

According to the payoff matrix and due to the contradiction between the objective functions, there is surely no feasible solution that optimizes the first, second, and third objective functions simultaneously. Although finding an ideal solution in most real-world problems is difficult, a feasible solution can be chosen that minimizes the maximum deviation of objectives from the ideal solution. If we assume that $F = (F_1, F_2, F_3)$ is an efficient solution to the Pareto solution set, its ID criterion is calculated as follows:

$$ID(F, Ideal) = \max \left\{ w_1 \left| F_1 - F_1^* \right|, w_2 \left| F_2 - F_2^* \right|, \right.$$

$$\left. w_3 \left| F_3 - F_3^* \right| \right\}.$$

Given the equal weights of all objective functions, the ID for the above-mentioned numerical example is expressed as follows:

$$ID(F, Ideal) = \max \{ F_1, F_2, 14 - F_3 \}.$$

Table 18 shows the values of the ID criterion for all Pareto solutions obtained from the AEC method. According to this criterion, Pareto solution no. 6 ($F_1 = 5, F_2 = 5.32, F_3 = 8$) and Pareto solution no. 7 ($F_1 = 6, F_2 = 4.76, F_3 = 9$) with $ID = 6$ have the lowest ID, hence selected. It should be noted that the Pareto solution No. 6 is the same as one solution already obtained from the FGP method. If we are to choose only one solution among the Pareto solutions, the solution no. 7 is more favorable since it has the least total weighted deviation from the ideal values (considering equal weights).

5.3. Solving the model using FGP and robust optimization based on ELL and PH uncertainty sets

In this section, the solution of the proposed model

Table 18. values of the ID criterion for all Pareto solutions obtained from the AEC method.

Pareto solution	Value of ID criterion	Deviation of the 3rd objective from the optimal solution	Deviation of the 2nd objective from the optimal solution	Deviation of the 1st objective from the optimal solution
1	11	11	9.64	0
2	9	9	8.42	1
3	9	9	7.23	2
4	8	8	7.01	3
5	7	7	6.93	4
6	**6**	**6**	**5.32**	**5**
7	**6**	**5**	**4.76**	**6**
8	7	4	4.12	7
9	8	3	3.67	8
10	9	2	2.11	9
11	10	1	1.43	10
12	11	0	0	11

Table 19. Changing the objective values by changing the value for the control parameter ψ (Box uncertainty set).

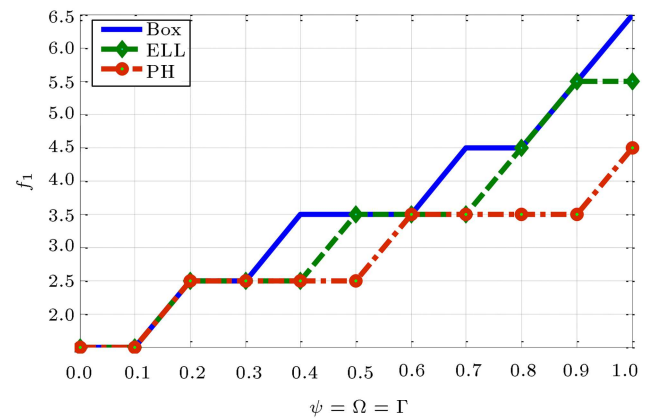| ψ	F_1	F_2	F_3
0	2	4.69	10
0.1	2	4.23	9
0.2	3	4.63	9
0.3	3	4.87	9
0.4	4	4.93	8
0.5	4	5.32	8
0.6	4	5.32	8
0.7	5	4.68	8
0.8	5	5.32	8
0.9	6	5.41	7
1	7	6.12	7

Table 20. Changing the objective values by changing the value for the control parameter Ω (ELL uncertainty set).

Ω	F_1	F_2	F_3
0	2	4.69	10
0.1	2	4.02	10
0.2	3	4.82	10
0.3	3	4.11	9
0.4	3	5.97	9
0.5	4	4.02	9
0.6	4	4.98	9
0.7	4	4.81	8
0.8	5	4.23	8
0.9	6	5.12	8
1	6	5.78	8

Table 21. Changing the objective values by changing the value for the control parameter Γ (PH uncertainty set).

Γ	F_1	F_2	F_3
0	2	4.69	10
0.1	2	3.89	10
0.2	3	4.04	10
0.3	3	4.02	9
0.4	3	5.00	9
0.5	3	4.21	9
0.6	4	4.52	9
0.7	4	5.03	9
0.8	4	4.71	9
0.9	4	5.41	9
1	5	5.23	8

Figure 10. The effect of different robust approaches on the first objective value using FGP method.
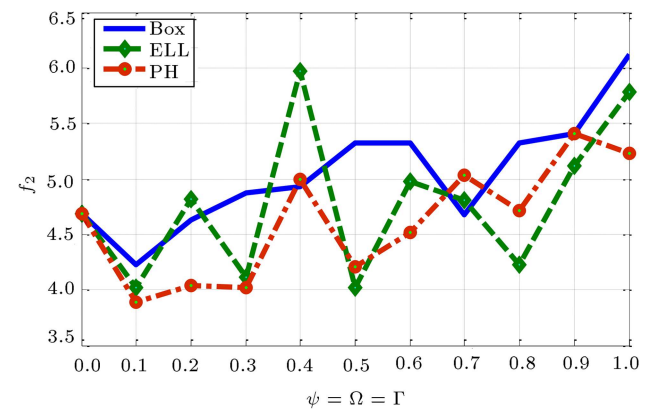
Figure 11. The effect of different robust approaches on the second objective value using FGP method.
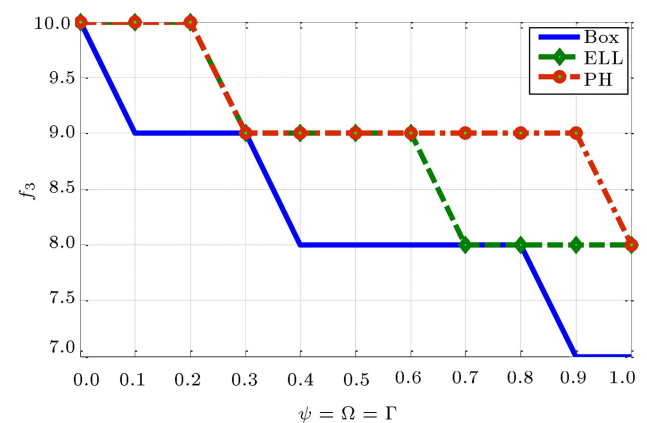
Figure 12. The effect of different robust approaches on the third objective value using FGP method.

obtained from the FGP is analyzed by changing the uncertainty space from the Box to the ELL and PH ones as well as the values of the control parameters ψ, Ω, and Γ. As shown in Tables 19, 20, and 21, upon increasing the values of the control parameters, the objective values will be more distant than the ideal state despite more coverage of the uncertainty space. In addition, for spaces $\psi = \Omega = \Gamma$, the solution

always encounters more strictness in the Box space. Consequently, the objective values of the Box space usually overcome those of the ELL and PH spaces. Figures 10, 11, and 12 confirm this statement. In the state of $\psi = \Omega = \Gamma = 0$, which is called the nominal state, the results of all the three robust approaches are the same.

5.4. Validation of the proposed robust optimization models

First, consider the general form of the following uncertain optimization problem.

$$\min \ Z = \widetilde{C} \cdot x,$$

s.t.:

$$\widetilde{A}x \geq \widetilde{b}.$$

The solution $x^*(M)$ is obtained by solving the above-mentioned optimization model. Assume that (C_d, A_d, b_d) are the deterministic values of the parameters of the above-mentioned problem and Z_d^* is the optimal value of the objective function in the deterministic state. In this case, $|Z_d^* - C_d \cdot x^*(M)|$ takes a value close to zero. In case the deterministic parameters are not available, the problem should be simulated. In any simulation run, the problem is first converted into a deterministic state and then, solved. Suppose that the deterministic optimization problem is simulated N times as follows:

$$\{\min \ Z_{d_i} = C_{d_i} \cdot x | A_{d_i}x \geq b_{d_i}; \quad i = 1, 2, \cdots, N\},$$

where $(C_{d_i}, A_{d_i}, b_{d_i})$ are the certain values of the parameters in the simulation run. Here, J_F is defined as a subset of these problems for which $x^*(M)$ is feasible. In addition, J_{UF} shows a subset of these problems where $x^*(M)$ violates at least one of the constraints $(|J_F| + |J_{UF}| = N)$. In addition, UF_i is defined as the set of violated constraints of the problem i in J_{UF}. The sensitivity coefficient for each percent of deviation from the optimality is represented by c^O and the sensitivity coefficient for each percent of violation for a constraint by c^F. Therefore, an indicator fundamentally based on feasibility and optimality concepts is proposed to validate the solution approach, which is defined as follows [35,36]:

$$Criteria = \frac{1}{N} \left(c^O \sum_{i \in J_F} \frac{|Z_{d_i}^* - C_{d_i} \cdot x^*(M)|}{|Z_{d_i}^*|} \right.$$
$$\left. + c^F \sum_{i \in J_{UF}} \sum_{k \in UF_i} \frac{|A_{d_i}^k \cdot x^*(M) - b_{d_i}^k|}{|b_{d_i}^k|} \right).$$

Obviously, the smaller the criterion value, the more valuable the solution approach to solving the problem under uncertainty. As mentioned earlier, in the mathematical model of the HHCRSP, such parameters as travel time, displacement of nurses, and duration of visit for each patient are uncertain, each considered as an interval. According to the above-mentioned validation procedure, the numerical experiment is simulated 10 times by random initialization of uncertain parameters, and the outputs of the ROAs based on the Box, ELL, and PH uncertainty sets are compared.

Table 22 shows the different values of the proposed indicator for all expressed approaches when the sensitivity coefficients of optimality (c^O) and feasibility (c^F) take the values of 0 and 1, respectively. All control parameters of the Box, ELL, and PH approaches are considered as 0.8. According to the obtained results, if we consider only the optimality criterion ($c^O = 1$), the nominal approach will outperform other approaches. The PH approach, which is less strict than both Box and ELL, covers smaller uncertainty space, thus yielding better results regarding this criterion. However, if we consider only the feasibility criterion ($c^F = 1$), the Box approach that covers more uncertainty space will yield the best result regarding this criterion. Finally, if we consider both the optimality and feasibility criteria ($c^F = c^O = 1$), the obtained results from all three approaches will be almost the same. In this case, the performance of the robust approach based on ELL uncertainty set is a slightly better than those of other approaches. Table 23 summarizes the output of the validation procedure.

Tables 24 and 25 compare the nominal, Box, ELL, and PH approaches in terms of feasibility and optimality criteria, respectively. Although the nominal approach exhibits a relatively good performance in terms of optimality criterion, it will not be still acceptable given that the feasibility criterion has been

Table 23. Preferred approach based on the proposed indicator.

$c^O = 1,\ c^F = 0$	Nominal
$c^O = 0,\ c^F = 1$	Box
$c^O = 1,\ c^F = 1$	Box, PH, and ELL are almost the same

Table 22. Values of the proposed indicator for the given approaches in case the c^O and c^F take the values of 0 and 1, respectively.

	Nominal	Box $(\psi = 0.8)$	ELL $(\Omega = 0.8)$	PH $(\Gamma = 0.8)$
$c^O = 1,\ c^F = 0$	0.09	0.18	0.14	0.11
$c^O = 0,\ c^F = 1$	0.26	0.05	0.08	0.13
$c^O = 1,\ c^F = 1$	0.35	0.23	0.22	0.24

Table 24. A comparison between the nominal, Box, ELL, and PH robust approaches in terms of feasibility criterion.

Test problem	Box (%)	ELL (%)	PH (%)	Nominal (%)
1	0	0	0	0
2	0	0	0	13
3	0	0	0	0
4	0	4	7	10
5	9	0	0	0
6	9	17	25	34
7	0	0	0	0
8	0	0	12	18
9	0	0	0	0
10	13	21	29	41

Table 25. A comparison between the nominal, Box, ELL, and PH robust approaches in terms of optimality criterion.

Test problem	Box (%)	ELL (%)	PH (%)	Nominal (%)
1	22	19	15	12
2	7	5	4	NA
3	19	17	12	11
4	3	NA*	NA	NA
5	7	6	4	1
6	NA	NA	NA	NA
7	8	6	3	0
8	6	3	NA	NA
9	17	15	12	8
10	NA	NA	NA	NA

*Note: The problem was infeasible.

considered; instead, the Box, ELL, and PH approaches exhibit better performance.

6. Sensitivity analysis

According to Table 16 and the results obtained from the FGP based on the Box robustization method with the control parameter $\psi = 0.8$, the optimal objective values were calculated as ($F_1 = 5, F_2 = 5.32, F_3 = 8$). In case the value of the first objective function in the payoff matrix in Table 15 is optimal (i.e., all patients are visited or the number of patients not visited is 0), the values of the second and third objective functions are not very favorable. In this case, objective values are ($F_1 = 0, F_2 = 9.64, F_3 = 3$).

Obviously, the number of nurses in the HCC and their skills in visiting patients has a significant impact on the three objectives defined in the HHCRSP. In the specified numerical study, five nurses are held accountable to visit 25 patients. To analyze the sensitivity of

Table 26. Impact of changing available budget (number of nurses) on the second and third objectives (FGP and robust optimization based on the Box uncertainty set).

Added budget ($)	No. of nurses added	F_1	F_2	F_3
30	1	0	5.01	11
60	2	0	4.12	15
90	3	0	3.97	17
120	4	0	4.09	19
150	5	0	5.12	22
180	6	0	0	25
210	7	0	0	25
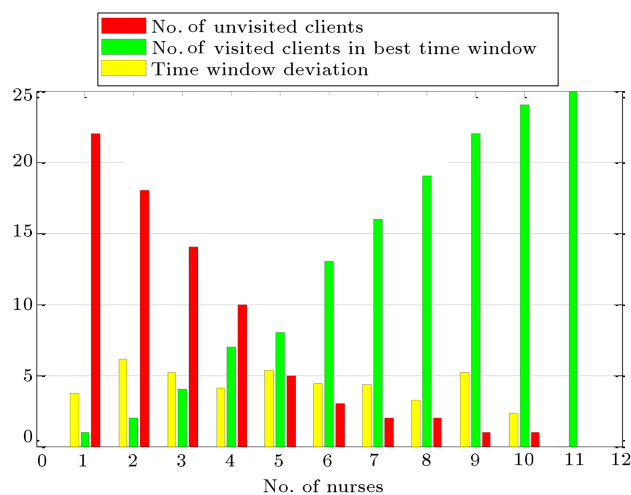
Figure 13. Sensitivity analysis of objective values relative to the number of nurses.

the objectives to the number of nurses, assume that the costs of adding each nurse and total available budget to hire new nurses are equal to C and B units (B is a multiple of C), respectively. As a result, the number of new nurses to be hired is obtained by $V = B/C$. In this numerical example, the daily cost of adding each nurse is considered to be $30. Therefore, a minimum additional budget of $180 is needed to hire six other nurses; hence, all patients will be visited in a favorable TW with no tardiness/earliness (see Table 26). In the latter case (spending $180 to hire six new nurses), the objective values are ($F_1 = 0, F_2 = 0, F_3 = 25$). Table 27 shows the trend of changing the objective values in response to variations in the available budgets which are proportional to hiring a number of new nurses. In other words, this table shows the degree of improvement in objectives with any increase in the budget (or number of nurses). Figure 13 shows a clear trend of changing all three objectives of HHCRSP in response to changing the number of nurses (1-12).

Table 27. Variations in the values of the objective functions at each budget level (FGP and robust optimization based on the Box uncertainty set).

Added budget ($)	Added no. of nurses	F_1	F_2	F_3	Absolute reduction of F_1	Absolute reduction of F_2	Absolute increase of F_3
0 (The current answer)	0	5	5.32	8	—	—	—
30	1	3	4.39	13	2	0.93	5
60	2	2	4.37	16	3	0.95	8
90	3	2	3.23	19	3	1.91	11
120	4	1	5.21	22	4	0.12	14
150	5	1	2.27	24	4	2.95	16
180	6	0	0	25	0	5.32	17

7. Conclusion and future research

In this paper, an Mixed Integer Linear Programming (MILP) model was proposed to deal with the Home Health Care Routing and Scheduling Problem (HHCRSP) considering three patient-oriented objectives. The first objective was to minimize the number of patients who have not been visited yet. The second objective was to minimize the maximum deviation from the optimal time window. Finally, the third objective was to maximize the number of patients visited in their optimal time window. To cope with the HHCRSP, first, nurses were assigned to patients and followed by allocation, each nurse's route in each work shift was determined. Finally, the starting time of visiting each patient was determined according to their optimal time window.

Considering the feedbacks from decision-makers in the Home Health Care (HHC) companies, nurses encounter distinct uncertainties namely road and weather conditions, driving skills, diagnosing time, etc. when carrying out a predefined schedule to visit patients. As nurses were authorized to adjust their plan in case of unpredictability, the final planning might not be optimal for strategic decision-making. Given this, two key elements namely "the time spent for relocation of nurses between patients" and "the duration of visit for each patient" were considered uncertain. The current study formulated an HHCRSP from the robust optimization perspective and particularly, controlled the non-deterministic variables based on the three prominent uncertainty spaces called Box, ELL, and PH uncertainty sets.

In order to solve the proposed multi-objective optimization model, both Fuzzy Goal Programming (FGP) and Augmented Epsilon Constraint (AEC) methods based on the Box uncertainty space were employed. Unlike the FGP method, AEC method yielded a set of Pareto solutions. MID criterion was indicative of the fact that two Pareto solutions with identical ID values were characterized by the lowest ID and one of them was the same as one solution already obtained from the FGP method. Further, the comparative numerical results revealed that regardless of the type of the uncertainty set, increasing the value of each control parameter for spaces $\psi = \Omega = \Gamma$ made the objective values farther than the ideal ones. In other words, in the state of $\psi = \Omega = \Gamma$, the solution always was bound to higher strictness in the Box space, and the objective values for the Box space usually conquered those for the ELL and PH spaces. In the nominal state when $\psi = \Omega = \Gamma = 0$, all the three robust approaches did not differ from each other in terms of results. In addition, an indicator basically based on feasibility and optimality concepts (c^F, c^O) was presented to validate the robust approaches. In case only the optimality criterion was considered $(c^O = 1)$, the nominal approach would outperform others. On the contrary, if only the feasibility criterion was considered $(c^F = 1)$, the Box approach that could cover a larger uncertainty space would yield the best result in terms of this criterion. Finally, if both the optimality and feasibility criteria were considered $(c^O = c^F = 1)$, the results from all the three approaches were almost the same. Finally, the sensitivity analysis of the number of available nurses revealed that hiring at least six other nurses guaranteed visiting all patients in a favorable time window with no tardiness/earliness.

There are also numerous other interesting extensions to this publication worthy of further consideration. Therefore, we plan to extend our work by adding other real-life practical constraints in the model such as the continuity of care by considering a longer planning horizon. For future research, the following areas seem to be attractive:

- Considering the possibility of rescheduling nurses' programs in response to the unpredicted absence of nurses, cancelation by patients, etc.;

- Considering sustainability issues, e.g., green transportation;

- Considering the continuity of care as a constraint or objective in the proposed optimization model;

- Considering some issues related to the convenience and comfort of nurses. For instance, nurses choose their own working hours and determine which patients they prefer to visit.

References

1. Mankowska, D.S., Meisel, F., and Bierwirth, C. "The home health care routing and scheduling problem with interdependent services", *Health Care Manag. Sci.*, **17**(1), pp. 15–30 (Mar. 2014). DOI: 10.1007/s10729-013-9243-1

2. Super, N. "Who will be there to give care? The growing gap between caregiver supply and demand", Natl. Heal. Policy Forum Georg. Washingt. Univ. Washington, DC (2002).

3. Lindahl, B., Lidén, E., and Lindblad, B.M. "A meta-synthesis describing the relationships between patients, informal caregivers and health professionals in home-care settings", *J. Clin. Nurs.*, **20**(3–4), pp. 454–463 (2011). DOI: https://doi.org/10.1111/j.1365-2702.2009.03008.x

4. Hariz, H.A., Dönmez, C.Ç., and Sennaroglu, B. "Siting of a central healthcare waste incinerator using GIS-based multi-criteria decision analysis", *J. Clean. Prod.*, **166**, pp. 1031–1042 (2017). DOI: https://doi.org/10.1016/j.jclepro.2017.08.091

5. Matta, A., Chahed, S., Sahin, E., et al. "Modelling home care organisations from an operations management perspective", *Flex. Serv. Manuf. J.*, **26**, pp. 295–319 (Aug. 2012). DOI : 10.1007/s10696-012-9157-0

6. NAHC, Home Care Profit Margins Update. Washington, DC: National Association for Home Care & Hospice (2006).

7. Fernandez, A., Gregory, G., Hindle, A., et al. "A model for community nursing in a rural county", *Oper. Res. Q.*, **25**(2), pp. 231–239 (Jun. 1974). DOI: 10.1057/jors.1974.40

8. Rasmussen, M.S., Justesen, T., Dohn, A., et al. "The home care crew scheduling problem: Preference-based visit clustering and temporal dependencies", *Eur. J. Oper. Res.*, **219**(3), pp. 598–610 (Jun. 2012). DOI: 10.1016/j.ejor.2011.10.048

9. Fikar, C. and Hirsch, P. "Home health care routing and scheduling: A review", *Comput. Oper. Res.*, **77**, pp. 86–95 (2017). DOI: 10.1016/j.cor.2016.07.019

10. Hiermann, G., Prandtstetter, M., Rendl, A., et al. "Metaheuristics for solving a multimodal home-healthcare scheduling problem", *Cent. Eur. J. Oper. Res.*, **23**(1), pp. 89–113 (Mar. 2015). DOI: 10.1007/s10100-013-0305-8

11. Allaoua, H., Borne, S., Létocart, L., et al. "A matheuristic approach for solving a home health care problem", *Electron. Notes Discret. Math.*, **41**, pp. 471–478 (Jun. 2013). DOI: 10.1016/j.endm.2013.05.127

12. Dohn, A., Kolind, E., and Clausen, J. "The manpower allocation problem with time windows and job-teaming constraints: A branch-and-price approach", *Comput. Oper. Res.*, **36**(4), pp. 1145–1157 (Apr. 2009). DOI: 10.1016/j.cor.2007.12.011

13. Mutingi, M. and Mbohwa, C. "Multi-objective home-care worker scheduling: A fuzzy simulated evolution algorithm approach", *IIE Trans. Healthc. Syst. Eng.*, **4**(4), pp. 209–216 (Oct. 2014). DOI: 10.1080/19488300.2014.966213

14. Trautsamwieser, A., Gronalt, M., and Hirsch, P. "Securing home health care in times of natural disasters", *OR Spectr.*, **33**(3), pp. 787–813 (Jul. 2011). DOI: 10.1007/s00291-011-0253-4

15. Trautsamwieser, A. and Hirsch, P. "Optimization of daily scheduling for home health care services", *J. Appl. Oper. Res.*, **3**(3), pp. 124–136 (2011).

16. Bertels, S. and Fahle, T. "A hybrid setup for a hybrid scenario: Combining heuristics for the home health care problem", *Comput. Oper. Res.*, **33**(10), pp. 2866–2890 (Oct. 2006). DOI: 10.1016/j.cor.2005.01.015

17. Eveborn, P., Flisberg, P., and Rönnqvist, M. "Laps Care-an operational system for staff planning of home care", In *European Journal of Operational Research*, **171**(3), pp. 962–976 (Jun. 2006). DOI: 10.1016/j.ejor.2005.01.011

18. Fikar, C. and Hirsch, P. "A matheuristic for routing real-world home service transport systems facilitating walking", *J. Clean. Prod.*, **105**, pp. 300–310 (Oct. 2015). DOI: 10.1016/j.jclepro.2014.07.013

19. Bard, J.F., Shao, Y., Qi, X., et al. "The traveling therapist scheduling problem", *IIE Trans. Institute Ind. Eng.*, **46**(7), pp. 683–706 (Jul. 2014). DOI: 10.1080/0740817X.2013.851434

20. Riazi, S., Wigstrom, O., Bengtsson, K., et al. "A column generation-based gossip algorithm for home healthcare routing and scheduling problems", *IEEE Trans. Autom. Sci. Eng.*, **16**(1), pp. 127—137 (Jan. 2019). DOI: 10.1109/TASE.2018.2874392

21. Mısır, M., Smet, P., and Vanden Berghe, G. "An analysis of generalised heuristics for vehicle routing and personnel rostering problems", *J. Oper. Res. Soc.*, **66**(5), pp. 858–870 (May 2015). DOI: 10.1057/jors.2014.11

22. Shi, Y., Boudouh, T., and Grunder, O. "A robust optimization for a home health care routing and scheduling problem with consideration of uncertain travel and service times", *Transp. Res. Part E Logist. Transp. Rev.*, **128**, pp. 52–95 (Aug. 2019). DOI: 10.1016/j.tre.2019.05.015

23. Braekers, K., Hartl, R.F., Parragh, S.N., et al. "A bi-objective home care scheduling problem: Analyzing the trade-off between costs and client inconvenience", *Eur. J. Oper. Res.*, **248**(2), pp. 428–443 (2016). DOI: 10.1016/j.ejor.2015.07.028

24. Nickel, S., Schröder, M., and Steeg, J. "Mid-term and short-term planning support for home health care services", *Eur. J. Oper. Res.*, **219**(3), pp. 574–587 (Jun. 2012). DOI: 10.1016/j.ejor.2011.10.042

25. Decerle, J., Grunder, O., Hajjam El Hassani, A., et al. "A hybrid memetic-ant colony optimization algorithm for the home health care problem with time window, synchronization and working time balancing", *Swarm Evol. Comput.*, **46**, pp. 171–183 (May 2019). DOI: 10.1016/j.swevo.2019.02.009

26. Liu, R., Tao, Y., and Xie, X. "An adaptive large neighborhood search heuristic for the vehicle routing problem with time windows and synchronized visits", *Comput. Oper. Res.*, **101**, pp. 250–262 (Jan. 2019). DOI: 10.1016/j.cor.2018.08.002

27. Ben Bachouch, R., Guinet, A., and Hajri-Gabouj, S. "A decision-making tool for home health care nurses' planning", *Supply Chain Forum An Int. J.*, **12**(1), pp. 14–20 (Jan. 2011). DOI: 10.1080/16258312.2011.11517250

28. Shao, Y., Bard, J.F., and Jarrah, A.I "The therapist routing and scheduling problem", *IIE Trans.*, **44**(10), pp. 868–893 (Oct. 2012). DOI: 10.1080/0740817X.2012.665202

29. Kallehauge, B., Larsen, J., Madsen, O.B.G., et al., *Vehicle Routing Problem with Time Windows*, In Column Generation, New York: Springer-Verlag, pp. 67–98 (2005).

30. Rubin, P.A. and Narasimhan, R. "Fuzzy goal programming with nested priorities", *Fuzzy Sets Syst.*, **14**(2), pp. 115–129 (Nov. 1984). DOI: 10.1016/0165-0114(84)90095-2

31. Mavrotas, G. "Effective implementation of the ε-constraint method in multi-objective mathematical programming problems", *Appl. Math. Comput.*, **213**(2), pp. 455–465 (Jul. 2009). DOI: 10.1016/j.amc.2009.03.037

32. Ben-Tal, A., El Ghaoui, L., and Nemirovskiĭ, A.S., *Robust Optimization*, Princeton University Press (2009).

33. Soyster, A.L. "Technical Note-convex programming with set-inclusive constraints and applications to inexact linear programming", *Oper. Res.*, **21**(5), pp. 1154–1157 (Oct. 1973). DOI: 10.1287/opre.21.5.1154

34. Li, Z., Ding, R., and Floudas, C.A. "A comparative theoretical and computational study on robust counterpart optimization: I. robust linear optimization and robust mixed integer linear optimization", *Ind. Eng. Chem. Res.*, **50**(18), pp. 10567–10603 (Sep. 2011). DOI: 10.1021/ie200150p

35. Bertsimas, D. and Sim, M. "The Price of robustness", *Operations Research*, **52**(1), pp. 35–53 (2004). DOI: 10.1287/opre.1030.0065

36. Ben-Tal, A., EI Ghaoui, L., and Nemirovski, A., *Robust Optimization*, Princeton University Press (2009).

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