Cooperative advertising with two local advertising options in a retailer duopoly

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This paper considers the issue of cooperative advertising with local advertising options in a channel with three players, including a manufacturer and two retailers. The current study, expands the cooperative advertising literature to a case where there exist two options for local advertising investment. Moreover, this paper compares two cases of presence and absence of cooperative advertising, which has almost been neglected in cooperative advertising literature. The purpose is to determine equilibrium strategy of retailers' advertising options, players' advertising expenditures and the manufacturer' participation rates on retailers' investment. The aforementioned problem is analyzed as a three-stage game, using backward induction. In the first and second stages, advertising investments of players are determined analytically. Then, in the third stage, the Nash equilibrium pair of advertising options can be found using numerical study. The problem is solved using illustrative examples in two cases of presence and absence of the cooperative advertising contract. Finally, the conditions for which offering the contract is win-win for all players, are identified. A Sensitivity analysis has been carried out to explain the efficacy of the model.

Keywords: cooperative advertising; game theory; advertising options; retail competition; equilibrium solution.

1. Introduction

Supply chain members may behave as a part of a unified system and coordinate with each other in order to improve the overall efficiency of supply chain, hence "coordination" comes into focus [1]. Supply chain coordination is a vehicle between channel members to redesign decision rights, workflow, and resources in order to get higher profit margins, improved customer service performance, and faster response time [2]. Retail fixed markup [3], Buyback [4], revenue sharing [5], and quantity discount [6] have been among the widely used coordination contracts in practice.

Vertical cooperative (co-op) advertising introduced by [7] is typically a cost-sharing mechanism and advertising coordination scheme that adopted by manufacturers to influence behavior of their retailers. Manufacturers advertise to promote the brand of their products; however, retailers locally advertise in order to achieve short-term sales [8]. In a cooperative advertising program, manufacturer's participation in paying a fraction of local advertising costs increases the retailer's incentive to invest more on advertising and as a result achieve further sales. This participation is often expressed as a percentage of the retailer's advertising costs [9]. Although Zhang et al. [10] showed bilateral participation in cooperative advertising achieve channel coordination, the traditional cooperative advertising may not lead to channel coordination. However, it may result in a better channel performance as well as the win-win or Pareto efficient outcome for supply chain members. A contract is Pareto-efficient if all players in the supply chain are not worse-off and at least one is strictly better-off with the existing contract comparing to any other contracts [11].

Manufacturers' participation in co-op advertising is nowhere near insignificant [12]. There are many reasons for such cooperation between manufacturers and the retailers. Hutchins [13] argues that manufacturers adopt cooperative advertising because the program leads to a quick sale. National advertising by the manufacturer creates a brand image and is more general than the retailers' advertising, while the retailer's local advertising mostly influences on the price and promotion of goods. Thus, national advertising mostly focuses on generalizing a product but does not necessarily lead to an actual demand [14]. Due to such supplementary effects, manufacturers are partly dependent on retailer's advertising so that their local advertising may look unsatisfactory by the manufacturer [15]. In such circumstances, the cooperative advertising program could act as an incentive policy to satisfactorily increase the level of local advertising as enough.

Cooperative advertising has been highly promoted in today's marketing practices. The budget allocated to local advertising of the GE Company has been three times more than its national advertising budget [16]. Moreover, costs associated with Intel's cooperative advertising have increased from $ 800 million in 1999 to $ 5.1 billion in 2001 [17]. Companies such as IBM [18] and Apple [19] also have benefited from this mechanism. Small, online co-op advertising particularly exist in automotive and durable goods [12]. The interested readers may refer to "*Co-op Advertising Programs Sourcebook*" (Available on www.co-opsourcebook.com) which lists thousands of available co-op programs in 52 product classifications. The categories range from agricultural products to toys. However, most companies arbitrarily set their participation rate without having an analysis; this emphasizes the need for analytic studies [20].

In this article, the issue of cooperative advertising with local advertising options in a channel with three players, including a manufacturer and two retailers is discussed. The market demand is influenced by advertising efforts of the players; and also, the retailers' market share depends on their selected local advertising options among which each retailer can only choose one for advertising. With the aim of profit maximization, each player determines the optimal level of advertising investment. In addition, the retailers in the downstream echelon of the supply chain also set their advertising options. Using game theory, the equilibrium solution of the problem is determined in three stages.

According to "*Co-op Advertising Programs Sourcebook*", Some $50 billion of media advertising (television, magazines, online, newspapers, radio, billboards, etc.) is financed through co-op programs. Most studies assume that there is only one advertising option available for retailers' advertising. However, different advertising options have a different impact on demand and market share. Clearly if the effect of advertising options is ignored in the problem, it results in sub-optimal solutions.

Most studies on cooperative advertising literature assume that players accept this contract in a non-cooperative or cooperative framework. In the cooperative case, the problem is often analyzed by bargaining games and the contract parameters set so that none of the players are worse-off comparing to the non-cooperative solution [21-24]; however, in most cases, some players may not be motivated enough to accept a non-cooperative contract. In other words, some players' profit in the absence of contract may be more than the case where there exists a contract. Therefore, in this article, absence and presence of the cooperative advertising contract are examined. Finally, the conditions for which cooperative advertising can benefit all players are identified.

This paper differs from previous works in several ways. First, this study attempts to examine the cooperative advertising problem with two advertising options. Each retailer decides on the optimal level of investment after selecting the appropriate option. Second, competition of the retailers in the channel is considered so that each competes for greater market share. The retailers' selected advertising options directly affect their market share, which is closer to the real-world assumptions. The third difference is that this study compares two cases of presence and absence of cooperative advertising, which has almost been neglected in cooperative advertising literature. Then, the necessary conditions for a win-win cooperative advertising contract are identified.

The reminder of the paper is organized as follows. In Section 2, we review the related literature. In Section 3, after introducing the symbols and market demand function; the profit functions of players are formulated. In Section 4, the defined problem is solved as a three-stage game for two cases of presence and absence of the cooperative advertising. In Section 5, the numerical results of two cases for the system variables and profits are discussed, and the necessary conditions for a win-win cooperative advertising contract are identified using sensitivity analysis. Finally, Section 6 contains conclusions and recommendations for future research.

2. Literature review

In recent years, extensive studies have been conducted on cooperative advertising. See, for example, Aust & Buscher [25] and Jørgensen & Zaccour [26], for the comprehensive review of the literature.

Most studies on cooperative advertising considered monopoly at upstream/downstream echelon so that only one supplier/manufacturer located at the upstream echelon and only one retailer/purchaser exists on the downstream echelon [22, 27]. However, a number of researchers developed models of duopoly [28-32] and oligopoly (Jørgensen & Zaccour [33]), in which rivals compete with each other to achieve greater benefits. In this paper, the main focus is on the models which consider a duopoly market.

The budgeting among various media alternatives is becoming an increasingly difficult marketing task. Since, each may have a different impact on market demand and market share. So, ignoring this issue may lead to sub-optimal solutions. Malthouse et al. [34] investigated a problem where an organization must choose among multiple media vehicles for a marketing campaign. They argued that different vehicles have different marginal costs per impression. Their model is solved to maximize profit. The different media such as television, radio, magazines, newspapers, billboards, cinema, etc. can differ widely in their short- and/or long-run effectiveness [35]. Naik & Peters [36] analyzed an advertising campaign for cars involving six media (television, magazines, newspapers, radio, internet banners and sponsored search). They found that radio advertising is the most cost effective, followed by newspapers, TV and magazines.

There are a few studies on cooperative advertising that investigate sufficient incentives of players to accept the contract. For example, Jorgensen et al. [37] examined dynamic advertising and promotional strategies in a retailer-manufacturer marketing channel. They compared stationary feedback equilibria for two non-cooperative games, including Nash Equilibrium without Promotional Support and Stackelberg Equilibrium with Promotional Support. Their result showed that a cooperative advertising support program is Pareto improving. Yang et al. [38], in a retailer-manufacturer channel, compared the models with/without the cooperative advertising contract. Their model does not include pricing decisions, but retailers' fairness concern effect is taken into account. They identified the necessary conditions for perfect channel coordination when the retailer has fairness concerns. Chen [39] and Giri & Bardhan [40] studied ordering and advertising decisions in a retailer-manufacturer channel and propose a profit-sharing contract. They also examined the presence and absence of the cooperative advertising contract and since the contract does not provide sufficient incentives for channel members, another mechanism called channel rebates is offered in order to get a win-win condition. Gou et al. [41] utilized differential game theory to evaluate three scenarios, including no cooperation, joint venture cooperation, and contractual alliance cooperation. They showed that joint venture cooperation and contractual alliance cooperation are more profitable than non-cooperation. Karray & Hassanzadeh Amin [42] investigated the effects of cooperative advertising in a channel with competing retailers. They found that coop advertising may not be profitable for the retailers or for the channel, especially when the market is characterized by low levels of price competition and high advertising competition between retailers. This study also compares the presence and absence of the cooperative advertising and identifies the necessary conditions for the contract to be win-win.

3. Description of the Model

In this study, we consider a two-level supply chain in which a manufacturer in the first level sells a product through two retailers in the second level. The market demand function is deterministic and affected by advertising expenditures of the members. The manufacturers nationally advertise to promote the brand name of his product, while the two retailers locally advertise in order to achieve short-term sales and to get a greater share of the market. Each retailer can choose between two available advertising options; each of these options has a different impact on market share.

The purpose of the model is to determine optimal decision variables for each player. This means that to maximize their profits, first the retailers select the appropriate advertising options, and make decisions about the optimal level of investment in the selected option. The profit function depends on the market share, profit margin, market demand and associated advertising costs.

Throughout the paper, we will use the following symbols:

Parameters

J	The set of local advertising options for each retailer: $J = \{1,2\}$
i,j	Retailers' local advertising options index
Ω	The set of retailers' pairs of local advertising options. $\Omega=\{(1,1),(1,2),(2,1),(2,2)\}$
(i,j)	The pair of local advertising options where the retailers 1 and 2 select i and j, respectively: $(i,j)\in\Omega$
k_m	The efficiency coefficient of national advertising in market demand
k_i	The efficiency coefficient of local advertising option i in demand
c_i	The cost coefficient of local advertising option i for each retailer
C	The cost coefficient of national advertising of manufacturer
ρ_m	Manufacturer's profit margin for each product unit
ρ_1,ρ_2	Retailers 1 and 2's profit margin for each product unit
η_{ij}	Retailer 1's market share for pair (i,j)
$\Pi_{m\|(i,j)}$	Manufacturer's profit for pair (i,j)
$\Pi_{1\|(i,j)}$	Retailers 1's profit for pair (i,j)
$\Pi_{2\|(i,j)}$	Retailers 2's profit for pair (i,j)
$D(A,a_i,b_j)$	Market demand function for pair (i,j)

Decision variables

A	Manufacture's national advertising level
$a_{i\|(i,j)}$	Retailer 1's local advertising level in advertising option i for pair (i,j)
$b_{j\|(i,j)}$	Retailer 2's local advertising level in advertising option j for pair (i,j)
t_1^{ij}	The participation rate of Manufacture in retailer 1's advertising for pair (i,j)
t_2^{ij}	The participation rate of Manufacture in retailer 2's advertising for pair (i,j)

The product flow as well as the financial flow in the channel can be as **Figure 1**. Our analysis in this paper takes some assumptions and definitions into account, which are described in the following:

Assumption 1. *The value of participation rates in the proposed cooperative advertising program varies between 0 and 1; thus negative participation rates are not reasonable.*

Assumption 2. *Retailers have credible information exchange regarding the profit functions and profit margins.*

"Please, insert Fig 1 here"

Definition 1. The η-specific game. *We assume that if retailers choose the same advertising options, either (1,1) or (2,2), regardless of value of η, the market is equally shared between them. However, if they choose different advertising options, either (1,2) or (2,1), the market share of retailer who chooses option 1, is η, and another retailer's is 1-η. We denote this game with $G(\eta)$ as illustrated in* **Table 1**.

"Please, insert Table 1 here"

Definition 2. Win-win strategy. *We say that the strategy s leads to win-win condition over the strategy s', if for all $k\in\{1,2,m\}$, it is the case that $\Pi_k^s \geq \Pi_k^{s'}$ and at least for one*

$k' \in \{1,2,m\}$, we have $\Pi_{k'}^{s} > \Pi_{k'}^{s'}$. Note that 1, 2 and m corresponds to the retailer 1, retailer 2 and manufacturer, respectively.

In this paper, we are going to specify win-win condition for cooperative advertising contract. So, the strategies s and s' correspond to the presence and absence of contract.

3.1 Market demand function

The market demand function is assumed to be affected by the retailers' local advertising and the manufacturer's national advertising. The price effect on the demand is ignored, and pricing decisions are not taken into account. This assumption is discussed in articles such as [8, 38, 43]. The market demand function for any pair of advertising options (i, j) is obtained from Equation (1).

$$D\left(a_i, b_j, A \mid (i,j) \in \Omega\right) = \mathrm{k}_i \sqrt{a_{i|(i,j)}} + \mathrm{k}_j \sqrt{b_{j|(i,j)}} + \mathrm{k}_m \sqrt{A} \tag{1}$$

The above demand function is used by [31] for retailers' duopoly. However, their model does not include advertising options. Likewise, in this study, the retailers' advertising efficiency is considered equal because they believe that this parameter mostly depends on the impressibility of the customers rather than the behavior of retailers.

3.2 Objective functions of players

As previously noted, each retailer faces two available advertising options. Each of them can select one. Profit of each retailer depends on his market share, which itself is dependent on his and the rival retailer's advertising options. Considering the defined market demand function and the retailers' market share, the profit functions can be formulated. Equations (2), (3) and (4) show the profit functions of the manufacturer, retailers 1 and 2, respectively.

$$\Pi_{m|(i,j)} = \rho_m \left(\mathrm{k}_i \sqrt{a_{i|(i,j)}} + \mathrm{k}_j \sqrt{b_{j|(i,j)}} + \mathrm{k}_m \sqrt{A}\right) - t_1^{ij} c_i a_{i|(i,j)} - t_2^{ij} c_j b_{j|(i,j)} - CA \tag{2}$$

$$\Pi_{1|(i,j)} = \rho_1 \eta_{ij} \left(\mathrm{k}_i \sqrt{a_{i|(i,j)}} + \mathrm{k}_j \sqrt{b_{j|(i,j)}} + \mathrm{k}_m \sqrt{A}\right) - (1 - t_1^{ij}) c_i a_{i|(i,j)} \tag{3}$$

$$\Pi_{2|(i,j)} = \rho_2 \left(1 - \eta_{ij}\right)\left(\mathrm{k}_i \sqrt{a_{i|(i,j)}} + \mathrm{k}_j \sqrt{b_{j|(i,j)}} + \mathrm{k}_m \sqrt{A}\right) - (1 - t_2^{ij}) c_j b_{j|(i,j)} \tag{4}$$

In equation (2) the first term (the product of profit margin and total demand) is the profit from the sale; the second and the third terms represent the participation in retailer 1's and 2's advertising cost, respectively; and the fourth term is the national advertising cost. In equations (3) and (4) also the first and the second terms represent the profit from the sale, and the retailers' advertising costs, respectively.

4. Solution approach

Game theory is the commonly used method in the literature to analyze the issue of cooperative advertising [26]. In this paper, the behavior of the players is studied by non-cooperative games, in which, each player makes decisions independently with the aim of optimizing his/her objective. When the cannel members have equal power, make their decisions independently and simultaneously, with the aim of optimizing their objectives. In this case, the Nash equilibrium provides the solution [44], in which, none of the players precede over the other in decision-making. However, in a leadership Stackelberg game, players are divided into leaders and followers. This game is based on sequential decision-making and usually begins when some players have a certain privilege that allows them to move first. In a channel, the more powerful party might be the leader. The game is played in two stages, in which the leader chooses his strategy first, and then the follower determines his strategy with regard to the leader's strategy [45]. Manufacturers have mostly been treated as the leaders and retailers as the followers of the Stackelberg game in the cooperative advertising literature [25].

4.1 The sequence of events and decision-making

The sequence of events has a remarkable effect on sequence of decisions and the type of game between players. The sequence of events in the problem is as follows. First, the manufacturer announces the level of national advertising investment as well as the participation rates to retailers. Thus $\{A, t_1^{ij}, t_2^{ij} \mid (i,j) \in \Omega\}$ is proposed to the retailers. Second, according to the manufacturer's announced values, the retailers determine their best solution *(a_i,b_j)* for each pair of advertising options $(i,j) \in \Omega$ and then, based on all pairs' outcome, choose the best *(i^*,j^*)*. It should be noted that the retailers' decision-making in the downstream echelon is performed in a bi-level form. Each retailer, firstly, chooses an advertising option, then determines the amount of investments in the selected option. The next two subsections present the solution to the presence and absence of cooperative advertising contract, respectively.

4.2 The manufacturer offers cooperative advertising

According to the previous section, backward induction is used to determine the equilibrium solution. The solution stages are as follows: *<u>The first stage</u>*: for an arbitrary amount of the national advertising, participation rates and any pair of advertising options *(i,j)*, the best response function of retailers are obtained. The retailers play a Nash game to determine local advertising expenditures for each pair *(i,j)*; *<u>The second stage</u>*: by substituting the retailers' best responses in the manufacture's objective function for any pair *(i,j)*, the optimal national advertising expenditure and participation rates are determined, as the manufacturer offers $\{A, t_1^{ij}, t_2^{ij} \mid (i,j) \in \Omega\}$ values knowing the retailer's best responses; *<u>The third stage</u>*: regarding to retailers' outcome for any pair $(i,j) \in \Omega$, they simultaneously decide on an equilibrium pair of advertising options. Each stage of the solution is described in the following.

- *The first stage*

At the first stage, it is assumed that retailers 1 and 2 have selected advertising options *i* and *j*, respectively, where $(i,j) \in \Omega$. It is also assumed that the manufacturer has announced the values of national advertising and participation rates as $\{A, t_1^{ij}, t_2^{ij} \mid (i,j) \in \Omega\}$. The goal is to find each retailer's best response according to these two assumptions. Therefore, the necessary conditions are obtained through derivation of Equations (3) and (4) with respect to variables $a_{i|(i,j)}$ and $b_{j|(i,j)}$, respectively as Equations (5) and (6):

$$\frac{\partial \Pi_{1|(i,j)}}{\partial a_{i|(i,j)}} = 0 \Rightarrow \rho_1 \eta_{ij} \left(\frac{\mathrm{k}_i}{2\sqrt{a_{i|(i,j)}}} \right) - (1 - t_1^{ij}) c_i = 0 \tag{5}$$

$$\frac{\partial \Pi_{2|(i,j)}}{\partial b_{j|(i,j)}} = 0 \Rightarrow \rho_2 \left(1 - \eta_{ij}\right) \left(\frac{\mathrm{k}_j}{2\sqrt{b_{j|(i,j)}}} \right) - (1 - t_2^{ij}) c_j = 0 \tag{6}$$

By solving Equations (5-6), **Proposition 1** is obtained:

Proposition 1. *In the presence of cooperative advertising, for any pair of advertising options* $(i,j)\in\Omega$ *and arbitrary values of national advertising and participation rates* $\left\{A, t_1^{ij}, t_2^{ij} \mid (i,j)\in\Omega\right\}$, *the retailers' best responses will be as follows:*

$$a_{i|(i,j)} = \left(\frac{\rho_1 \eta_{ij} \mathrm{k}_i}{2c_i(1-t_1^{ij})}\right)^2,$$

$$b_{j|(i,j)} = \left(\frac{\rho_2\left(1-\eta_{ij}\right)\mathrm{k}_j}{2c_j(1-t_2^{ij})}\right)^2$$

It should be noted that the objective functions of retailers 1 and 2 are concave with respect to variables $a_{i|(i,j)}$ and $b_{j|(i,j)}$, since:

$$\frac{\partial^2 \Pi_{1|(i,j)}}{\partial a_{i|(i,j)}^2} = -\frac{\rho_1 \eta_{ij} \mathrm{k}_i}{4\sqrt{a_{i|(i,j)}^3}} < 0 \tag{7}$$

$$\frac{\partial^2 \Pi_{2|(i,j)}}{\partial b_{j|(i,j)}^2} = -\frac{\rho_2\left(1-\eta_{ij}\right)\mathrm{k}_j}{4\sqrt{b_{j|(i,j)}^3}} < 0 \tag{8}$$

- *The second stage*

At this stage, the manufacturer must determine the optimal value of national advertising investment, and participation rates taking into account the retailers' best responses in **Proposition 1**. Thus the maximization problem of the manufacturer according to Equation (2) and **Proposition 1** will be as follows:

$$\begin{aligned}
&\max_{A_{ij}, t_1^{ij}, t_2^{ij}} \Pi_{m|(i,j)} = \rho_m\left(\mathrm{k}_i\sqrt{a_{i|(i,j)}} + \mathrm{k}_j\sqrt{b_{j|(i,j)}} + \mathrm{k}_m\sqrt{A}\right) \\
&-t_1^{ij} c_i a_{i|(i,j)} - t_2^{ij} c_j b_{j|(i,j)} - CA \\
&s.t. \\
&a_{i|(i,j)} = \left(\frac{\rho_1 \eta_{ij} \mathrm{k}_i}{2c_i(1-t_1^{ij})}\right)^2 \\
&b_{j|(i,j)} = \left(\frac{\rho_2\left(1-\eta_{ij}\right)\mathrm{k}_j}{2c_j(1-t_2^{ij})}\right)^2 \\
&t_1^{ij}, t_2^{ij} \geq 0
\end{aligned} \tag{9}$$

where the constraints are the best response of retailers that can be easily substituted in the objective function. The necessary conditions for optimality of the above function are obtained through its derivation to variables A, t_1^{ij} and t_2^{ij} as follows.

$$\frac{\partial \Pi_{m|(i,j)}}{\partial A} = 0 \Rightarrow \frac{\rho_m \mathrm{k}_m}{2\sqrt{A}} - C = 0 \tag{10}$$

$$\frac{\partial \Pi_{m|(i,j)}}{\partial t_1^{ij}} = 0 \Rightarrow \frac{\rho_m \mathrm{k}_i}{1-t_1^{ij}} \frac{\rho_1 \eta_{ij} \mathrm{k}_i}{2c_i(1-t_1^{ij})} - c_i\left(\frac{\rho_1 \eta_{ij} \mathrm{k}_i}{2c_i(1-t_1^{ij})}\right)^2\left(\frac{1+t_1^{ij}}{1-t_1^{ij}}\right) = 0 \tag{11}$$

$$\frac{\partial \Pi_{m|(i,j)}}{\partial t_2^{ij}} = 0 \Rightarrow \frac{\rho_m \mathrm{k}_j}{1-t_2^{ij}} \frac{\rho_2\left(1-\eta_{ij}\right)\mathrm{k}_j}{2c_j(1-t_2^{ij})} - c_j\left(\frac{\rho_2\left(1-\eta_{ij}\right)\mathrm{k}_j}{2c_j(1-t_2^{ij})}\right)^2\left(\frac{1+t_2^{ij}}{1-t_2^{ij}}\right) = 0 \tag{12}$$

By simultaneously solving Equations (10-12), the optimal values of variables are determined, and by substituting them into **Proposition 1**, the optimal local advertising investments are also determined. **Proposition 2** summarizes the results.

Proposition 2. *In the presence of cooperative advertising, for any pair of local advertising options* $(i, j) \in \Omega$*, the equilibrium solution is as follows:*

If $2\rho_m - \rho_1\eta_{ij} \geq 0$ *and* $2\rho_m - \rho_2\left(1-\eta_{ij}\right) \geq 0$*:*

$$t_1^{ij} = \frac{2\rho_m - \rho_1\eta_{ij}}{2\rho_m + \rho_1\eta_{ij}}$$

$$t_2^{ij} = \frac{2\rho_m - \rho_2\left(1-\eta_{ij}\right)}{2\rho_m + \rho_2\left(1-\eta_{ij}\right)}$$

$$A = \left(\frac{\rho_m \mathrm{k}_m}{2C}\right)^2$$

$$a_{i|(i,j)} = \left[\frac{\mathrm{k}_i}{4c_i}\left(2\rho_m + \rho_1\eta_{ij}\right)\right]^2$$

$$b_{j|(i,j)} = \left[\frac{\mathrm{k}_j}{4c_j}\left(2\rho_m + \rho_2\left(1-\eta_{ij}\right)\right)\right]^2$$

Otherwise there is no feasible cooperative advertising contract.

Proof: According to **Assumption 1**, participation rates are assumed to be non-negative, so **Proposition 2** is true until $2\rho_m - \rho_1\eta_{ij} \geq 0$ and $2\rho_m - \rho_2(1-\eta_{ij}) \geq 0$. This assumption is among the underlying assumptions, and its violation is beyond the scope of the research. The sufficient conditions of optimality are provided in **Appendix A**.

❖ *The third stage*

Each retailer chooses his advertising option with regard to his rival's advertising option based on his own and his rival's outcome (profit). Since both retailers have the same power in decision-making, they simultaneously and independently choose their own strategies. Therefore, the equilibrium pair of options could be determined by Nash equilibrium. Using **Proposition 2**, we can get each players' profit as summarized in **Result 1**.

Result 1. *In the presence of cooperative advertising, for any pair of local advertising options* $(i, j) \in \Omega$*, the optimal profits of the players are*

$$\Pi_{m|(i,j)} = \frac{\mathrm{k}_i^2}{16c_i}\left(2\rho_m + \rho_1\eta_{ij}\right)^2 + \frac{\mathrm{k}_j^2}{16c_j}\left(2\rho_m + \rho_2\left(1-\eta_{ij}\right)\right)^2 + \frac{\rho_m^2\mathrm{k}_m^2}{4C}$$

$$\Pi_{1|(i,j)} = \rho_1\eta_{ij}\left[\frac{\mathrm{k}_i^2}{8c_i}\left(2\rho_m + \rho_1\eta_{ij}\right) + \frac{\mathrm{k}_j^2}{4c_j}\left(2\rho_m + \rho_2\left(1-\eta_{ij}\right)\right) + \frac{\rho_m\mathrm{k}_m^2}{2C}\right]$$

$$\Pi_{2|(i,j)} = \rho_2\left(1-\eta_{ij}\right)\left[\frac{\mathrm{k}_i^2}{4c_i}\left(2\rho_m + \rho_1\eta_{ij}\right) + \frac{\mathrm{k}_j^2}{8c_j}\left(2\rho_m + \rho_2\left(1-\eta_{ij}\right)\right) + \frac{\rho_m\mathrm{k}_m^2}{2C}\right]$$

Referring to **Table 1**, which depicts the profit of the retailers for all available pairs of local advertising options, we can determine the game equilibrium. According to the theory of Nash equilibrium for a two-player game with *J* being the set of each player's strategies, the strategies (i^*,j^*) form a Nash equilibrium if and only if:

$$\Pi_{l|(i^*,j^*)} = \max_{i\in J} \Pi_{l|(i,j^*)} \quad \forall l = 1,2 \tag{13}$$

In other words, if retailer 1 selects a particular option (e.g. *i*), the retailer 2 chooses his best response which provides him the most profit. Similarly, the retailer 1 has the same behavior. To find the equilibrium solution (i^*,j^*), we should use **Result 1** in order to compute the game outcome and form the game as **Table 1**. This stage is not analytically possible, so, this is done by numerical examples in Section 5.

4.3 The manufacturer doesn't offer cooperative advertising

In this case, the manufacturer does not offer a cooperative advertising contract to the retailers, so each member incurs his or her own costs. The players' objective functions for this case are similar to Equations (2-4) as long as the participation rates set to be zero, i.e. $t_1^{ij} = t_2^{ij}=0$. Here, the manufacturer determines the value of national advertising investment, while the retailers make decisions to choose their advertising options and local advertising investment. To avoid redundancy and given that similar analysis needs to be performed for this case, only the main results to the problem are presented.

Proposition 3. *In the absence of cooperative advertising, for any pair of local advertising options* $(i,j)\in\Omega$*, the equilibrium solution is as follows:*

$$A = \left(\frac{\rho_m \mathrm{k}_m}{2C}\right)^2$$

$$a_{i|(i,j)} = \left(\frac{\rho_1 \eta_{ij} \mathrm{k}_i}{2c_i}\right)^2$$

$$b_{j|(i,j)} = \left(\frac{\rho_2 \left(1-\eta_{ij}\right) \mathrm{k}_j}{2c_j}\right)^2$$

It can be easily proved that the manufacturer's objective function is concave to *A*.

Result 2. *In the absence of cooperative advertising, for any pair of local advertising options* $(i,j)\in\Omega$*, the optimal profits of the players are*

$$\Pi_{m|(i,j)} = \rho_m \left(\frac{\rho_1 \eta_{ij} \mathrm{k}_i^2}{2c_i} + \frac{\rho_2\left(1-\eta_{ij}\right)\mathrm{k}_j^2}{2c_j} + \frac{\rho_m \mathrm{k}_m^2}{4C}\right)$$

$$\Pi_{1|(i,j)} = \rho_1 \eta_{ij} \left(\frac{\rho_1 \eta_{ij} \mathrm{k}_i^2}{4c_i} + \frac{\rho_2\left(1-\eta_{ij}\right)\mathrm{k}_j^2}{2c_j} + \frac{\rho_m \mathrm{k}_m^2}{2C}\right)$$

$$\Pi_{2|(i,j)} = \rho_2 \left(1-\eta_{ij}\right) \left(\frac{\rho_1 \eta_{ij} \mathrm{k}_i^2}{2c_i} + \frac{\rho_2\left(1-\eta_{ij}\right)\mathrm{k}_j^2}{4c_j} + \frac{\rho_m \mathrm{k}_m^2}{2C}\right)$$

Similarly, we can find the equilibrium solution *(i^*,j^*)* as described in the previous subsection. Note that, the solution procedure can be easily generalized to an oligopoly case and the case where there exist multiple advertising options to retailers.

5. Numerical study

In this section, two examples of market configuration are considered for the problem, which are presented in **Table 2**. There exist two local advertising options to retailers: television (T) and radio (R). The manufacturer also uses television advertising for national advertising. It should be noted that the television advertising has different cost and efficiency in the national and local advertising, since these parameters depend on advertising time and television channel. Given the value of η in each case, $G(\eta)$ is played at the retailers' level. Note that, according to Table 1, in a game $G(\eta)$, we have: $\eta_{11}=0.5$, $\eta_{12}=\eta$, $\eta_{21}=1-\eta$ *and* $\eta_{22}=0.5$.

"Please, insert Table 2 here"

5.1 Detailed Result of Example 1

The detailed results are summarized in **Table 3** and **Table 4**, respectively for the absence and the presence of cooperative advertising contract. It is obvious that the Nash equilibrium of each game is (T, T). Comparing the equilibrium of two cases, the profit of each retailer increased by 54% by offering cooperative advertising.

"Please, insert Table 3 here"
"Please, insert Table 4 here"

The manufacturer's profit corresponding the equilibrium solution to the absence and presence of cooperative advertising is 7750 and 11470, which increased by 48%. The values of variables are as follows:

- The national advertising (A) is 25 seconds in both cases.
- In equilibrium (T,T) in the absence and presence of cooperative advertising, the retailer 1's local advertising investment in television (a^{TT}), is 1.5 and 31.6 seconds, respectively.
- In equilibrium (T,T) in the absence and presence of cooperative advertising, the retailer 2's local advertising investment in television (b^{TT}), is 2.25 and 33 seconds, respectively.
- The manufacturer's participation rates on retailer 1's and retailer 2's advertising (t_1^{TT} and t_2^{TT}) is 77% and 74%, respectively.

5-2 Sensitivity analysis

In this section, the sensitivity of the solutions to the changes on value of parameters is checked and the changes on profits, percentage improvements with offering the contract as well as the changes on the values of variables are reported. **Table 5** shows the results of sensitivity analysis of market share η which changes from 0.23 to 0.5. The equilibrium and profits are computed for two cases of the absence and presence of contract. In the absence of contract, for $\eta \leq 0.44$, the equilibrium is (T,T) and the profits are not sensitive to changes. By increasing the market share parameter, the equilibrium is changed to (R,R) and profits are increased. The similar shift occurs in the presence of contract when the market share changes from 0.41 to 0.44. The percentage improvements of the profits with offering contract are also reported in the last three column of the table. The results show that offering the contract always improves the profit of members. **Table 6** also shows the values of variables in both cases of the absence and presence of the contract. National advertising in both cases is independent of the market share parameter and does not change with the changes on this parameter. Participation rates also do not change, because although these variables are dependent on the market share parameter, but given that the equilibrium is either (R,R) or (T,T), the market share of each retailer is 0.5 and these variables have not changed. Local advertising variables have also increased only when the equilibrium has changed.

"Please, insert Table 5 here"
"Please, insert Table 6 here"

Table 7 shows the results of sensitivity analysis of television advertising cost, c_2, which changes from 15 to 105. The equilibrium in both cases is (T,T). Since the equilibrium does not change with changing the c_2, considering the profit function of members, it is expected that with the increase of this parameter, the profits will be reduced. Results of the table confirm our expectation. The results show that offering the contract always improves the profit of members. Problem variables are also shown in **Table 8**. The national advertising and participation rates are constant for the same reasons as described for η. By increasing c_2, and given that both retailers use television advertising in equilibrium, local advertising variables have declined, which confirm our expectations.

"Please, insert Table 7 here"
"Please, insert Table 8 here"

Tables 9 and **10** show the results of sensitivity analysis of the efficiency of television advertising, k_2. The equilibrium in the absence of contract is always (T,T); however, it is changed in the presence of contract. When the equilibrium is (T,T), with the increase of k_2, the profits increased and the local advertising variables also increased, which is in line with the expectation. However, when the equilibrium is (R,R), the profits and local advertising variables do not experience any changes. The results show that offering the contract always improves the profit of members.

"Please, insert Table 9 here"
"Please, insert Table 10 here"

5-3 Sensitivity analysis of win-win condition

In the example 1, it was shown that offering the contract improves the profits of members. This result is still valid when only one parameter changes, and the rest stay constant. In this section, this topic is examined for different combinations of parameters. Because this analysis is not analytically possible, discrete values are assigned to the parameters. Then, the analysis is performed for simultaneously changing some combinations of parameters. We consider that the parameter values can change in the ranges given in **Table 11**.

Without loss of generality, it is assumed that the 2nd advertising option is more efficient and also costs more than the 1st one. So, we always have $c_2 \geq c_1$ and $k_2 \geq k_1$. This is true in the real world for television and radio advertising. So, c_2 starts from c_1 and k_2 starts from

k_1. Also, as described before for game $G(\eta)$, η is the market share of retailer 1 for outcome (R,T), considering the efficiency of advertising options, η cannot be greater than 0.5.

To analyze the sensitivity of the win-win condition, simultaneous change of two or three parameters are examined in each run. Then, it is examined whether offering the contract improves the profits of all members or not. For example, to examine the simultaneous changes of η and ρ_1, the problem is solved for all possible combinations between these two parameters. Since we have $\eta \in (0.01, 0.02, ..., 0.49)$ and $\rho_1 \in (5, 6, ..., 15)$, then we need to solve the problem 49×11=539 times. It should also be noted that, when simultaneously checking two or three parameters, the values of other parameters are set in accordance with their base value in Example 1.

"Please, insert Table 11 here"

The sensitivity analysis results are as follows:

- Offering the contract improves the profits of all members when any pair of parameters in Table 9 are simultaneously changed. Since, we have 10 parameters, there exist 45 possible pair of parameters.
- In order to evaluate changing three parameters simultaneously, we run the problem for some combination of parameters. The results show that offering the contract improves the profits of all members for the following combinations of the parameters:

$$\begin{Bmatrix} \{\eta,\rho_1,\rho_2\},\{\eta,\rho_1,\rho_m\},\{\eta,\rho_2,\rho_m\},\{\eta,c_1,c_2\},\{\eta,c_1,C\}, \\ \{\eta,c_2,C\},\{\eta,k_1,k_2\},\{\eta,k_1,k_m\},\{\eta,k_2,k_m\},\{\eta,k_2,c_2\} \end{Bmatrix}$$

For further analysis of the win-win condition, sensitivity analysis is performed on Example 2.

Figure 2-(a) shows the result according to changes on η and k_2. The determined area is outside of the win-win condition; that is where at least one member worse off by offering the contract. It is reasonable for the manufacturer to offer the contract only in the win-win condition. **Figure 2-(b)** shows the result for changing on η and c_2; and **Figure 2-(c)** shows the result for changing on c_2 and k_2. Furthermore, **Figure 2-(d)** shows the result according to changes on η, c_2 and k_2. The determined space between two surfaces is outside of the win-win condition.

"Please, insert Fig 2 here"

6. Conclusion

When advertising expenditures of supply chain players directly affect the market demand, cooperative advertising program might be an incentive mechanism by manufacturers to motivate retailers to invest more on local advertising. This article studies the cooperative advertising problem in a channel, including a manufacturer and two retailers. The current paper extended the previous works on three directions. First, cooperative advertising problem with two local advertising options is considered. Second, the retailers compete to get greater market share. Third, two cases of presence and absence of cooperative advertising are compared in order to identify win-win conditions. The aforementioned problem is analyzed as a non-cooperative three-stage game using backward induction. The problem is solved in two cases of presence and absence of the cooperative advertising contract using illustrative examples and sensitivity analysis. The results indicate that if certain conditions are met; proposing a cooperative advertising contract, is win-win for all players. There are some possible directions for further research. For example, some extension can be made by considering inventory decisions [46], and multiple performance measures of the supply chain [6] which apparently have almost been neglected in cooperative advertising literature so far. Moreover, some limitations of the current study can be relaxed. Firstly, since the cooperative advertising program does not always lead to win-win condition, another hybrid mechanism can be proposed. Secondly, another extension can be made to a situation without information exchange between retailers.

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Appendix A

To prove that the manufacturer's objective function is concave to variables A, t_1^{ij} and t_2^{ij}, just prove that the given Hessian matrix (A.1) is negative definite.

$$H=\begin{bmatrix} \frac{\partial^2\Pi_{m|(i,j)}}{\partial A^2} & \frac{\partial^2\Pi_{m|(i,j)}}{\partial A\partial t_1} & \frac{\partial^2\Pi_{m|(i,j)}}{\partial A\partial t_2} \\ \frac{\partial^2\Pi_{m|(i,j)}}{\partial t_1\partial A} & \frac{\partial^2\Pi_{m|(i,j)}}{\partial t_1^2} & \frac{\partial^2\Pi_{m|(i,j)}}{\partial t_1\partial t_2} \\ \frac{\partial^2\Pi_{m|(i,j)}}{\partial t_2\partial A} & \frac{\partial^2\Pi_{m|(i,j)}}{\partial t_2\partial t_1} & \frac{\partial^2\Pi_{m|(i,j)}}{\partial t_2^2} \end{bmatrix} \tag{A.1}$$

It should be noted that in the above relation, superscripts of variables t_1^{ij} and t_2^{ij} has been removed for convenience. Using the equations (10), (11) and (12) the second-order partial derivatives are calculated as follows:

$$\frac{\partial^2\Pi_{m|(i,j)}}{\partial A^2}=-\frac{\rho_m \mathrm{k}_m}{4\sqrt{A^3}} \tag{A.2}$$

$$\frac{\partial^2 \Pi_{m|(i,j)}}{\partial t_1^2} = \frac{\rho_1 \eta_{ij} \mathrm{k}_i^2}{c_i \left(1-t_1\right)^4} \times \left[\left(1-t_1\right) \rho_m - \left(\frac{2+t_1}{2} \right) \rho_1 \eta_{ij} \right] \tag{A.3}$$

$$\frac{\partial^2 \Pi_{m|(i,j)}}{\partial t_2^2} = \frac{\rho_2 \left(1-\eta_{ij}\right) \mathrm{k}_j^2}{c_j \left(1-t_2\right)^4} \times \left[\left(1-t_2\right) \rho_m - \left(\frac{2+t_2}{2} \right) \rho_2 \left(1-\eta_{ij}\right) \right] \tag{A.4}$$

$$\frac{\partial^2 \Pi_{m|(i,j)}}{\partial A \partial t_1} = 0 \tag{A.5}$$

$$\frac{\partial^2 \Pi_{m|(i,j)}}{\partial A \partial t_2} = 0 \tag{A.6}$$

$$\frac{\partial^2 \Pi_{m|(i,j)}}{\partial t_1 \partial t_2} = 0 \tag{A.7}$$

Due to the obtained equations (A.2-A.7), the only major diameter of the matrix is nonzero, and all other elements are zero. To prove the matrix as negative definite, each element of the diagonal should be proved to be negative.

Equation (A.2) is always negative and equation (A.3) is also negative under the following conditions:

$$t_1^{ij} > t_1'^{ij} = \frac{2\rho_m - 2\rho_1 \eta_{ij}}{2\rho_m + \rho_1 \eta_{ij}} \tag{A.8}$$

Similarly, equation (A.4) is negative under the following conditions:

$$t_2^{ij} > t_2'^{ij} = \frac{2\rho_m - 2\rho_2 \left(1-\eta_{ij}\right)}{2\rho_m + \rho_2 \left(1-\eta_{ij}\right)} \tag{A.9}$$

where values t'^{ij}_1 and t'^{ij}_2 are threshold values. The above conditions cannot be definitively met, yet clearly, the optimal values of the participation rates in **Proposition 2** satisfy the above conditions. Therefore, even though the concavity of the manufacturer's objective function could not be proved with certainty, as has been proved, the function is concave around the solution obtained in **Proposition 2**.

Table 1. Representation of $G(\eta)$, the η-specific game

	1	**2**
1	$\Pi_{1\|(1,1)}(0.5)$, $\Pi_{2\|(1,1)}(0.5)$	$\Pi_{1\|(1,2)}(\eta)$, $\Pi_{2\|(1,2)}(1-\eta)$
2	$\Pi_{1\|(2,1)}(1-\eta)$, $\Pi_{2\|(2,1)}(\eta)$	$\Pi_{1\|(2,2)}(0.5)$, $\Pi_{2\|(2,2)}(0.5)$

Table 2. Market configuration for Example 1 and 2

Parameter	**Example 1**	**Example 2**
ρ_m	15	20
ρ_1	10	10
ρ_2	12	12
k_m	100	100
k_1	50	40
k_2	50	50
C	250	200
c_1	20	30
c_2	100	100
η	0.4	0.4

Table 3. The equilibrium of *G(0.4)* in the absence of cooperative advertising

	R	**T**
R	(3633, 4280)	(2573, 4692)
T	(3993, 3067)	**(3031, 3600)**

Table 4. The equilibrium of *G(0.4)* in the presence of cooperative advertising

	R	**T**
R	(7067, 8440)	(4353, 8886)
T	(7447, 5214)	**(4641, 5550)**

Table 5. Changes on profits with regard of changing in η

Problem #	η	**Absence of cooperative advertising**				**Presence of cooperative advertising**				**% Improvement**		
		Eq.	Π_1	Π_2	Π_m	*Eq.*	Π_1	Π_2	Π_m	Π_1	Π_2	Π_m
1	0.23	(T,T)	3031	3600	7750	(T,T)	4641	5550	11470	53	54	48
2	0.26	(T,T)	3031	3600	7750	(T,T)	4641	5550	11470	53	54	48
3	0.29	(T,T)	3031	3600	7750	(T,T)	4641	5550	11470	53	54	48
4	0.32	(T,T)	3031	3600	7750	(T,T)	4641	5550	11470	53	54	48
5	0.35	(T,T)	3031	3600	7750	(T,T)	4641	5550	11470	53	54	48
6	0.38	(T,T)	3031	3600	7750	(T,T)	4641	5550	11470	53	54	48

7	0.41	(T,T)	3031	3600	7750	(T,T)	4641	5550	11470	53	54	48
8	0.44	(T,T)	3031	3600	7750	(R,R)	7067	8440	18803	133	134	143
9	0.47	(R,R)	3633	4280	10867	(R,R)	7067	8440	18803	95	97	73
10	0.5	(R,R)	3633	4280	10867	(R,R)	7067	8440	18803	95	97	73

Table 6. Changes on variables with regard of changing in η

Problem #	η	**Absence of cooperative advertising**			**Presence of cooperative advertising**				
		A	*a*	*b*	t_1	t_2	*A*	*a*	*b*
1	0.23	25	1.6	2.3	0.78	0.74	25	31.6	33.1
2	0.26	25	1.6	2.3	0.78	0.74	25	31.6	33.1
3	0.29	25	1.6	2.3	0.78	0.74	25	31.6	33.1
4	0.32	25	1.6	2.3	0.78	0.74	25	31.6	33.1
5	0.35	25	1.6	2.3	0.78	0.74	25	31.6	33.1
6	0.38	25	1.6	2.3	0.78	0.74	25	31.6	33.1
7	0.41	25	1.6	2.3	0.78	0.74	25	31.6	33.1
8	0.44	25	1.6	2.3	0.78	0.74	25	225	235.1
9	0.47	25	11.1	16	0.78	0.74	25	225	235.1
10	0.5	25	11.1	16	0.78	0.74	25	225	235.1

Table 7. Changes on profits with regard of changing in c_2

Problem #	c_2	**Absence of cooperative advertising**				**Presence of cooperative advertising**				**% Improvement**		
		Eq.	Π_1	Π_2	Π_m	Eq.	Π_1	Π_2	Π_m	Π_1	Π_2	Π_m
11	15	(T,T)	6042	7000	23333	(T,T)	16771	20000	48135	178	186	106
12	25	(T,T)	4625	5400	16000	(T,T)	11063	13200	30881	139	144	93
13	35	(T,T)	4018	4714	12857	(T,T)	8616	10286	23487	114	118	83
14	45	(T,T)	3681	4333	11111	(T,T)	7257	8667	19378	97	100	74
15	55	(T,T)	3466	4091	10000	(T,T)	6392	7636	16764	84	87	68
16	65	(T,T)	3317	3923	9231	(T,T)	5793	6923	14954	75	76	62
17	75	(T,T)	3208	3800	8667	(T,T)	5354	6400	13627	67	68	57
18	85	(T,T)	3125	3706	8235	(T,T)	5018	6000	12612	61	62	53
19	95	(T,T)	3059	3632	7895	(T,T)	4753	5684	11811	55	56	50
20	105	(T,T)	3006	3571	7619	(T,T)	4539	5429	11162	51	52	47

Table 8. Changes on variables with regard of changing in c_2

Problem #	c_2	**Absence of cooperative advertising**			**Presence of cooperative advertising**				
		A	*a*	*b*	t_1	t_2	*A*	*a*	*b*
11	15	25	69.4	100	0.78	0.74	25	1406.3	1469.4
12	25	25	25	36	0.78	0.74	25	506.3	529
13	35	25	12.8	18.4	0.78	0.74	25	258.3	269.9
14	45	25	7.7	11.1	0.78	0.74	25	156.3	163.3
15	55	25	5.2	7.4	0.78	0.74	25	104.6	109.3

16	65	25	3.7	5.3	0.78	0.74	25	74.9	78.3
17	75	25	2.8	4	0.78	0.74	25	56.3	58.8
18	85	25	2.2	3.1	0.78	0.74	25	43.8	45.8
19	95	25	1.7	2.5	0.78	0.74	25	35.1	36.6
20	105	25	1.4	2	0.78	0.74	25	28.7	30

Table 9. Changes on profits with regard of changing in k_2

Problem #	k_2	Absence of cooperative advertising				Presence of cooperative advertising				% Improvement		
		Eq.	Π_1	Π_2	Π_m	Eq.	Π_1	Π_2	Π_m	Π_1	Π_2	Π_m
21	25	(T,T)	2633	3150	5688	(R,R)	7067	8440	18803	168	168	231
22	30	(T,T)	2691	3216	5990	(R,R)	7067	8440	18803	163	162	214
23	35	(T,T)	2760	3294	6348	(R,R)	7067	8440	18803	156	156	196
24	40	(T,T)	2840	3384	6760	(R,T)	3929	8504	13681	38	151	102
25	45	(T,T)	2930	3486	7228	(T,T)	4234	5066	10241	45	45	42
26	50	(T,T)	3031	3600	7750	(T,T)	4641	5550	11470	53	54	48
27	55	(T,T)	3143	3726	8328	(T,T)	5090	6086	12829	62	63	54
28	60	(T,T)	3265	3864	8960	(T,T)	5583	6672	14317	71	73	60
29	65	(T,T)	3398	4014	9648	(T,T)	6118	7310	15935	80	82	65
30	70	(T,T)	3541	4176	10390	(T,T)	6696	7998	17682	89	92	70

Table 10. Changes on variables with regard of changing in k_2

Problem #	k_2	Absence of cooperative advertising			Presence of cooperative advertising				
		A	a	b	t_1	t_2	A	a	b
21	25	25	0.4	0.6	0.78	0.74	25	225	235.1
22	30	25	0.6	0.8	0.78	0.74	25	225	235.1
23	35	25	0.8	1.1	0.78	0.74	25	225	235.1
24	40	25	1	1.4	0.82	0.69	25	215.1	22.3
25	45	25	1.3	1.8	0.78	0.74	25	25.6	26.8
26	50	25	1.6	2.3	0.78	0.74	25	31.6	33.1
27	55	25	1.9	2.7	0.78	0.74	25	38.3	40
28	60	25	2.3	3.2	0.78	0.74	25	45.6	47.6
29	65	25	2.6	3.8	0.78	0.74	25	53.5	55.9
30	70	25	3.1	4.4	0.78	0.74	25	62	64.8

Table 11. Discrete values of parameters for analyzing win-win condition

Parameter	Discrete values
ρ_m	(15, 16, …, 25)
ρ_1	(5, 6, …, 15)
ρ_2	(7, 8, …, 17)
k_m	(75, 76, …, 125)
k_1	(30, 31, …, 50)
k_2	(k1, k1+1, …, 80)
C	(150, 151, …, 250)
c_1	(20, 21, …, 40)
c_2	(c1, c1+1, …, 150)
η	(0.01, 0.02, …, 0.49)

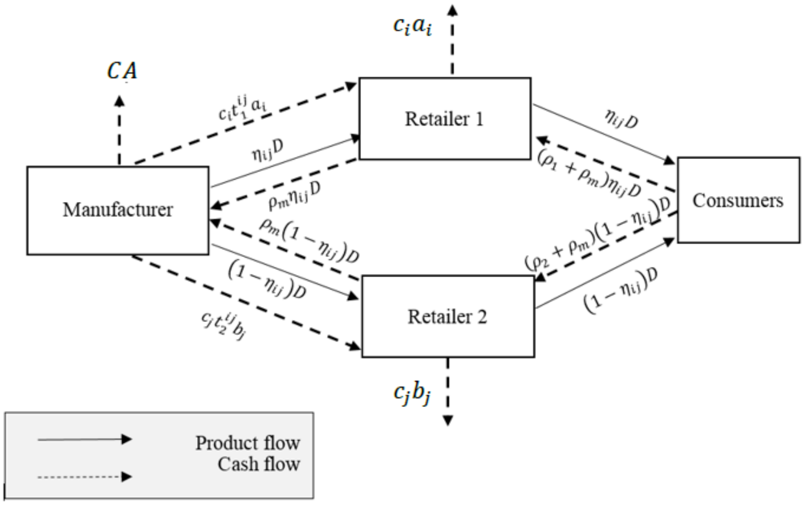

Figure 1. Product flows and financial flows among players when the retailers choose the pair *(i,j)*

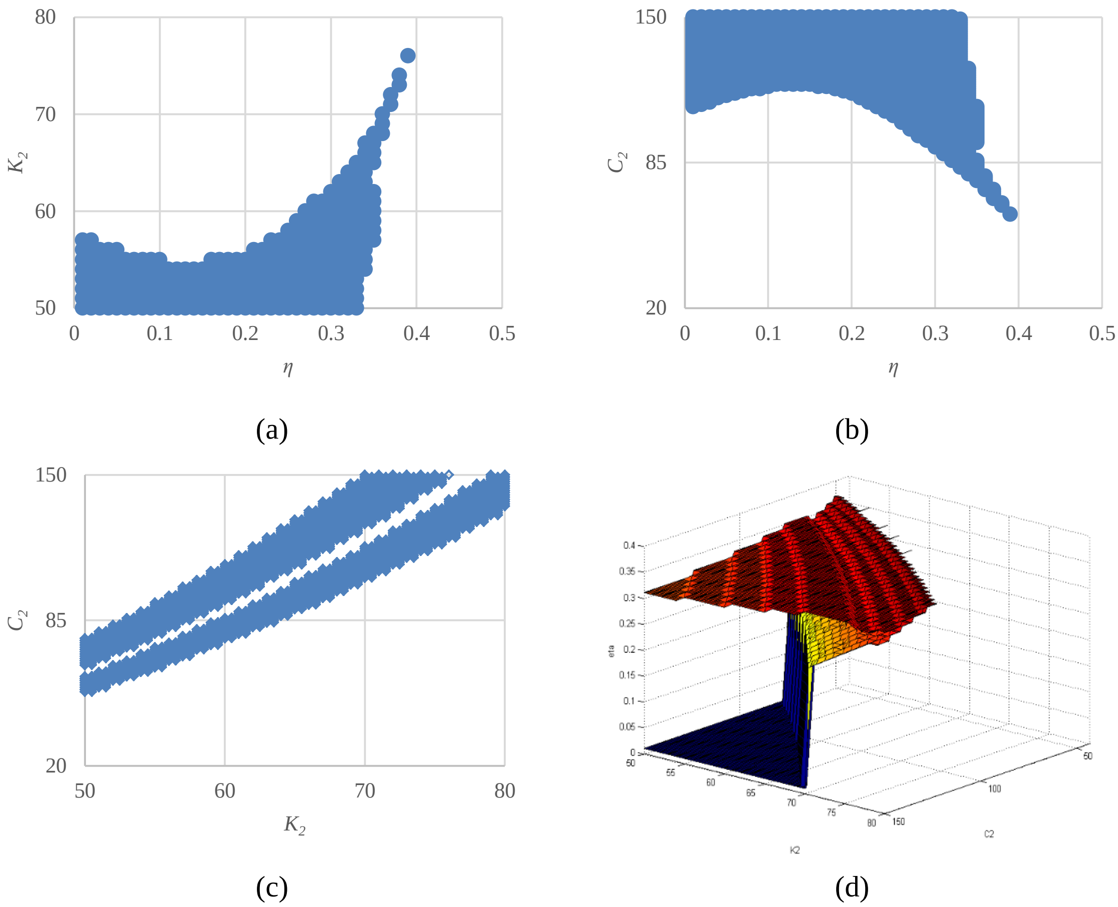

Figure 2. The area (space) outside the win-win condition according to changes on: a) η and k_2; b) η and c_2; c) c_2 and k_2; d) η, c_2 and k_2.