The equilibrium of venture capital incentive contract: Optimization and Q-learning approaches

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Abstract

This paper presents an incentive contract model for allocating the income of venture projects. Venture Capital (VC), as one of the main sources of financing innovative projects, faces challenges like moral hazards, information asymmetry and interest conflicts (three agency problems). In addition to identifying the items that may affect the income of venture projects and the introduction of cost functions, we present an optimal incentive contract model from the perspective of both venture capitalists and entrepreneurs. In this model, a venture capitalist, as an active investor, provides managerial and training assistance to the entrepreneur. The results showed that the higher the initial ability of the entrepreneur, the less money the venture capitalist pays for training. Furthermore, the wealth that the contract parties can obtain if the venture contract is not accepted, is an influential factor in the contract payment function. This model has also been studied with bounded rationality hypothesis and has been implemented using the Q-learning algorithm. In addition, the results obtained from the Q-learning approach, are reasonably convergent with the Nash equilibrium.

Keywords: Venture capital, Agency problems, Active investor, Equilibrium values, Bounded rationality, Learning algorithms

1 Introduction

Entrepreneurial equity investments by venture capitalists, corporate venture capitalists, angel investors and more recently, crowdfunders and accelerators, represent the key sources of capital that may fuel innovation and development [1]. For a revolution of wealth creation and rapid economic, there is a need for risk finance and venture capital (VC) environment, which can leverage innovation, promote technology and harness knowledge based ideas [2]. In other words VC is an important intermediary, able to transform capital into innovations in a highly

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productive manner [3]. Among all kinds of entrepreneurial finance, Venture capitals, pay more attention to companies' revenue growth, business models, and current investors [4]. In the financial area, VC is regarded as the frontier of innovation, liberated from the regulation and precedents of traditional markets [5]. The VC market involves early-stage, and therefore, high-risk investments, and is made up of active investors [6].

In VC investment, capital and competence are transferred from the venture capitalist to the entrepreneurial firm in the value adding phase and the transformation of capital can be seen as the final ending of the investment decision phase [7]. Venture capitalists are mainly financial intermediaries, who not only provide financing, but also offer advice and monitoring, play a very active role in negotiating with suppliers, help recruit and compensate key individuals, and provide strategic advice and access to consultants and lawyers [8]. Therefore, the management team is an important factor for VCs [9].

VCs face four generic concerns in the investment process: 1) The entrepreneur will not work hard to maximize value after the investment is made. 2) The entrepreneur is more aware of his ability than the VC. 3) After the investment, there will be circumstances when the VC disagrees with the entrepreneur and the VC will want the right to make decisions. 4) The entrepreneur can “hold-up” the VC by threatening to leave the venture when the entrepreneur’s human capital is particularly valuable to the company [10]. VCs are in uncertain environments and fail to exploit even available information due to two obstacles: information asymmetry and agency problems [5]. The agency problem occurs when the goals of the principal (venture capitalist) and agent (entrepreneur) are different, and when it is difficult for the principal to know what the agent is actually doing [11]. Asymmetric information refers to a situation in which one party to a contract has access to relevant information that is not available to the other party to the contract. A venture capitalist needs to trade off benefits and costs when attempting to decrease agency problems in the investment process [12].

Kaplan and Stromberg reviewed the conflicts of interest between an entrepreneur and a venture capitalist, and identified a number of ways that the investor can mitigate these conflicts: First, the investor can engage in information collection before deciding whether to invest, in order to screen out the ex-ante unprofitable projects and bad entrepreneurs. Second, the investor can engage in information collection and monitoring once the project is under way. Third, financial contracts, i.e. the allocation of cash flow and control rights between the entrepreneur and investor can be designed to provide incentives for the entrepreneur to behave optimally [13]. Venture capital investments are made under conditions of potentially extreme information asymmetry and agency cost that motivates parties to design the financial contracts [14]. The contract is intended to create a mutual understanding between the parties and thus may be regarded as a basis for successful co-operation. This justifies the time spent on negotiation in the venture capital investment process [15].

Contracting and monitoring efforts can improve the portfolio performance by reducing possible losses caused by moral hazard and adverse selection [16]. Optimal contract arrangement can overcome the revenue conflict between venture capitalists and entrepreneurs and reach mutually motivating conditions [17].
Bitler et al. developed a principal-agent model in an entrepreneurial setting and tested the model’s predictions using unique data on entrepreneurial effort and wealth in privately held firms. They found that entrepreneurial ownership shares decrease with firm risk; the effort increases with ownership, and the effort increases firm performance [18]. Fluck et al. proposed a model to study the design of financial contracts between entrepreneurs and venture capitalists. They showed that staged financing alleviates the effort provision problem [19]. Zou and Zhou presented their optimal contract model based on the fact that both capitalists and entrepreneurs target at maximizing their own revenue. They also designed an optimal contract for short- and long-term venture capitals from perspectives of venture capitalists and entrepreneurs [17].

The advisory, networking and experience services provided by venture capitalists are complemented by the technological and innovative skills of entrepreneurs. These complementary skills create a partnership that has a significant impact on the company’s value [20]. Gonzalez-Uribe, referring to direct evidence based on patent citations, reported that venture capitalists diffuse knowledge about their existing patented innovations among the portfolio companies [21]. Helmers et al. suggested that information transmission through interlocking boards of directors has a significant positive effect on innovation [22]. A venture capitalist can transfer valuable knowledge to an entrepreneur and thus facilitate innovation. He/She can also communicate the entrepreneur's innovative knowledge to other portfolio companies. Actually, venture capitalists have a new role as knowledge intermediaries [23].

Common sense says that, for entering investment opportunities, people do not only consider economic factors; rather they also take into account psychological factors. In fact, individuals face bounded rationality in their decisions. Bounded rationality models try to answer the question: How do people with bounded time and knowledge make decisions? Rodriguez-Fernandez et al. pointed out that adequate analysis of past actions of market players can improve the decision-making process of negotiators and allow them to increase their outcomes. They proposed a new model based on an adaptation of the Q-learning reinforcement learning algorithm to estimate the expected prices that can be achieved in contracts under a special context. They showed that the learning method can identify the best scenario for each situation, since the behaviour of negotiators can change according to the negotiation environment [24].

According to the Vergara, Bonilla and Sepulveda article [20], the model of this paper considers issues such as the effect of potential market value on revenue function, the constraint of effort and capability level, and the exponential growth of these features. This model is designed to allocate proceeds from the innovative projects and tries to reduce the conflicts of interest between the contract parties by providing sufficient incentive for them and lead them to an optimal contract. Also venture capitalists have been looked at as active investors that play a key role in the success and thus increase the the project revenue by transferring experiences to entrepreneurs and training them. This model is designed from the perspectives of venture capitalists and entrepreneurs, in which the parameters of optimal contract arrangement are derived separately for each case. Furthermore, to achieve a unified strategy, equilibrium solutions have been extracted from optimal solutions.
Following this introductory section, Section 2 describes the proposed mathematical investment model from the perspective of venture capitalists and entrepreneurs, and the optimal solutions for each party are presented. In the remainder of this section, the equilibrium answers are calculated to reach an agreement between the parties. Finally, the problem is examined from the perspective of bounded rationality with the Q-learning method and the results are reported. Section 3 presents the most relevant conclusions of this work.

2 Model

In venture capital sphere, the entrepreneur’s effort and ability are the determinants of income function [17]. In addition, we believe that the potential market value can affect the income function of the venture capital and the concerning effort and ability, that it is not possible to increase the level of effort and ability indefinitely. Also in the early stages, the growth of effort and ability occurs at a higher rate; however, from one level onwards, we have to pay a lot to make a small change in the level of ability or effort. Therefore, some nonlinear functions with exponential form are considered for the entrepreneur’s effort and ability. We develop the ability function based on the desired properties in equation (3). Accordingly, we defined the venture capital income function (venture capitalist’s observable income) as follows:

\[
\pi(s, \theta, V) = f_s(x) f_\theta(T) V + \varepsilon
\]

Where, \( f_s(x) \) and \( f_\theta(T) \) represent the entrepreneur’s effort function and ability function respectively. Also V represents the potential market value of the company, which depends on the environment and market conditions of the innovative project. Finally, \( \varepsilon \) as the random error of venture capital income function, follows the normal distribution: \( \varepsilon \sim N(0, \sigma^2) \).

The realization of the entrepreneur’s innovative ideas requires his/her effort. In the effort function, the entrepreneur’s effort level is denoted by \( x \), which takes value in range \( [0, \infty) \). The effort function is regarded as a positive but descending function; i.e. \( f_s(0) = 0 \), \( f_s' > 0 \), \( f_s'' < 0 \).

The effort function is defined as follows:

\[
f_s(x) = 1 - e^{-\varnothing x}.
\]

Where, \( \varnothing \) is a constant coefficient and indicates the degree of convexity of \( f_s(x) \). In the discussed function, the greater the entrepreneur’s effort level, the closer the effort function is to 1, and for infinite effort level, it takes 1; \( 0 \leq f_s(x) \leq 1 \) [19].

To obtain the ability function, it is necessary to note that the venture capitalists actively invest their funds in existing plans; this means that in addition to investing, by holding training courses and providing management assistance, they increase the knowledge of entrepreneurs and thus improve their ability level. One of the advantages of venture capital companies over other methods of financing is the management and training assistance provided by venture capitalists.
Hence, entrepreneur’s ability function varies with the level of cost (T) paid by the venture capitalist to enhance the entrepreneur’s ability. So, it can be said that:

\[ f_\theta(T) = \ln(e + T) \theta_\theta, \quad T \geq 0, \quad 0 < \theta_\theta \leq 1 \]  

Equation (3)

Where, \( \theta_\theta \) is the ability level of entrepreneur.

In the defined function, it is obvious that if the investor (venture capitalist) does not invest in entrepreneur training, the initial ability level of the entrepreneur will not change, but if venture capitalist invests in entrepreneur training (T), the ability level will be greater than 1; on the other hand, learning can make up for the lack of effort, i.e. the ultimate value of the learning function can be greater than 1: \( \theta_\theta \leq f_\theta(T) \).

Knowing the effort and ability functions, the revenue function of the plan is defined as follows:

\[ \pi(s, \theta, V) = (1-e^{-\alpha s})(\ln(T + e) \theta_\theta)V + \epsilon \]  

Equation (4)

This function acts in such a way that if an infinite level is considered for entrepreneur’s effort and no money is allocated by the venture capitalist, both parties will reach the maximum value of the company. However, it is obviously costly and sometimes impossible to make an infinite effort. In such a situation the investor attempts to improve the entrepreneur’s ability level by spending money on training in order to achieve maximum predicted value for the company.

To ensure the effort level of entrepreneur and to maximize the venture project, both parties will need to work out an incentive contract. In this contract, the incentive payment function to the entrepreneur is defined as follows:

\[ S(\theta, \pi) = \alpha(\theta) + \beta(\theta)\pi \]  

Equation (5)

Here, \( \alpha(\theta) \) is the fixed component and \( \beta(\theta)\pi \) is the variable component of the payment [17].

By approving the contract, the venture capital parties undergo some costs. The cost to the entrepreneur depends on his effort and ability. The venture capitalist pays for managing, monitoring and training. Accordingly, the cost functions of venture capitalist and entrepreneur are defined as follows:

\[ c(\theta, x) = \frac{b e^{\mu x}}{f_\theta(T)} = \frac{b e^{\mu x}}{\ln(e + T) \theta_\theta} \]  

Equation (6)

Venture capitalist: \( c_i(r, x, T) = \frac{e^{\mu x}}{r} + T \)  

Equation (7)

Where, \( e^{\mu x} \) indicates the effort cost with \( x \) representing the effort level of entrepreneur, and \( \mu \) is a constant coefficient certified based on each person’s characteristics.

Subsequently, the monetary income of the contract parties is described as:

Monetary income of the venture capitalist:

\[ m_i = (1 - \beta(\theta))(1-e^{-\alpha s})(\ln(T + e) \theta_\theta)V + \epsilon - \alpha(\theta) - c_i(e, r) \]  

Equation (8)
Monetary income of entrepreneur:

\[ m_u = \alpha(\theta) + \beta(\theta) \left( (1 - e^{-\theta_0}) \left( \ln (T + e) \theta_0 + \varepsilon \right) - c(\theta, e) \right) \]  

Equation (9)

In this case, the certainty equivalent wealth for the venture capitalist and entrepreneur will be as follows, respectively:

Venture capitalist:

\[ E [m_u] - \sigma^2 \rho_u \frac{(1 - \beta(\theta))^2}{2} = - \]  

Equation (10)

\[ \alpha(\theta) + (1 - \beta(\theta)) \left( (1 - e^{-\theta_0}) \left( \ln (T + e) \theta_0 \right) V \right) - c_1(e, r) - \sigma^2 \rho_v \frac{(1 - \beta(\theta))^2}{2} \]

Entrepreneur:

\[ E [m_u] - \sigma^2 \rho_v \frac{\beta(\theta)^2}{2} = \alpha(\theta) + \beta(\theta) \left( (1 - e^{-\theta_0}) \left( \ln (T + e) \theta_0 \right) V \right) - \]  

Equation (11)

\[ c(\theta, e) - \sigma^2 \rho_u \frac{\beta(\theta)^2}{2} \]

Where, \( \rho_v \) is the risk aversion coefficient of venture capitalist and \( \sigma^2 \rho_v \frac{(1 - \beta(\theta))^2}{2} \) is the cost of his/her risk, and \( \rho_u \) is the risk aversion coefficient of entrepreneur and \( \sigma^2 \rho_u \frac{\beta(\theta)^2}{2} \) the cost of his/her risk [17].

2.1 Presenting the model

In this section, we will present a mathematical model to obtain the optimum contract for the venture capitalist and entrepreneur:

2.1.1 Venture capitalist

From the venture capitalist perspective, the optimum investment model is presented as follows:

\[
\max_{\alpha, \beta, \theta} - \alpha(\theta) + (1 - \beta(\theta)) \left( (1 - e^{-\theta_0}) \left( \ln (e + T) \theta_0 \right) V \right) - \left( \frac{e^{\mu_x}}{r} + T \right) 
\]

Equation (12)

\[
- \sigma^2 \rho_v \frac{(1 - \beta(\theta))^2}{2}
\]

s.t
\[
\alpha(\theta) + \beta(\theta) \cdot \left[ (1-e^{-\theta}) \cdot (\ln(e+T) \cdot \theta_0) \right] - \frac{be^{\mu x}}{\ln(e+T) \cdot \theta_0}
\]
Equation (13)

\[
-\sigma^2 \rho_u \frac{\beta(\theta)^2}{2} \geq m
\]

\[
\text{IC} : \max_x \alpha(\theta) + \beta(\theta) \cdot \left[ (1-e^{-\theta}) \cdot (\ln(e+T) \cdot \theta_0) \right] - \frac{be^{\mu x}}{\ln(e+T) \cdot \theta_0}
\]
Equation (14)

Where, \( m \) is the wealth that the entrepreneur can obtain if the venture contract is not accepted.

### 2.1.2 Entrepreneur

The optimum investment model, from entrepreneur’s perspective is as below:

\[
\max_{\alpha, \beta, x} \alpha + \beta \cdot \left[ (1-e^{-\theta}) \cdot (\ln(e+T) \cdot \theta_0) \right] - \frac{be^{\mu x}}{\ln(e+T) \cdot \theta_0} - \sigma^2 \rho_u \frac{\beta^2}{2}
\]
Equation (15)

s.t

\[
-\alpha(\theta) + (1-\beta(\theta)) \cdot \left[ (1-e^{-\theta}) \cdot (\ln(e+T) \cdot \theta_0) \right] - \left( \frac{e^{\mu x}}{r} + T \right)
\]
Equation (16)

\[
-\sigma^2 \rho_u \frac{(1-\beta(\theta))^2}{2} \geq q
\]

\[
\max_x -\alpha(\theta) + (1-\beta(\theta)) \cdot \left[ (1-e^{-\theta}) \cdot (\ln(e+T) \cdot \theta_0) \right] - \left( \frac{e^{\mu x}}{r} + T \right)
\]
Equation (17)

Where, \( q \) is the wealth that the investor can obtain if the venture contract is not accepted.

### 2.2 Optimal solution to the model

Based on the model presented in Section 2.1, this issue falls into the constrained multi-objective optimization problems category. To solve this model, we use the Karush, Kuhn, Tucker (K.K.T) method. To implement the conditions of the K.K.T, the model must be written as a maximization problem and the constraints must be \( (\leq 0) \). The conditions of the K.K.T are:

1- \( \nabla f(x) = \sum_{i=1}^{m} \lambda_i \nabla g_i(x) \)
2. \( \lambda_i g_i(x) = 0 \)

3. \( g_i(x) \leq 0 \)

4. \( \lambda_i \geq 0 \)

In the following, we will solve the model by applying these conditions:

### 2.2.1 Venture capitalist

To use the K.K.T, we first go to solve the IC constraint in section 2.1.1, that is a maximization problem without constraint:

\[
IC : \max_x \alpha(\theta) + \beta(\theta) \left[ (1-e^{-\omega x})(\ln(e+T),\theta_0) \right] - \frac{be^{\mu x}}{\ln(e+T),\theta_0} - \sigma^2 \rho_a \frac{\beta(\theta)^2}{2}
\]

To solve IC, we derive the function in terms of \( x \):

\[
\frac{\partial IC}{\partial x} = \beta (\ln(e+T),\theta_0) \cdot \partial V e^{-\omega x} - \frac{b \mu e^{\mu x}}{\ln(e+T),\theta_0}
\]

Equation (18)

So:

\[
x = \frac{-\ln\left( \frac{b \mu}{\beta \cdot \partial V \cdot (\ln(e+T),\theta_0)^2} \right)}{\partial + \mu}
\]

Equation (19)

By solving IC and obtaining the optimal value of \( x \), we remove the IC constraint from the model and then rewrite the model to the standard form:

\[
\max_{\alpha, \beta, x} f : -\alpha(\theta) + (1 - \beta(\theta)) \left[ (1-e^{-\omega x})(\ln(e+T),\theta_0) \right] - \left( \frac{e^{\mu x}}{r+T} \right)
\]

\[-\sigma^2 \rho_a \frac{(1-\beta(\theta))^2}{2}
\]

s.t.

\[
g_1 : m - \alpha(\theta) - \beta(\theta) \left[ (1-e^{-\omega x})(\ln(e+T),\theta_0) \right] + \frac{b e^{\mu x}}{ln(e+T),\theta_0} + \sigma^2 \rho_a \frac{\beta(\theta)^2}{2}
\]

\[\leq 0\]
According to the first condition of K.K.T:

\[ \nabla f(x) = \sum_{i=1}^{n} \lambda_i \nabla g_i(x) \]

\[
\begin{bmatrix}
\frac{\partial f}{\partial \alpha} & \frac{\partial f}{\partial \beta}
\end{bmatrix} = \lambda_i \begin{bmatrix}
\frac{\partial g_1}{\partial \alpha} & \frac{\partial g_1}{\partial \beta}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{\partial f}{\partial \alpha} \\
\frac{\partial f}{\partial \beta}
\end{bmatrix} = \begin{bmatrix}
-1 \\
(1-\beta)\rho_v \sigma^2 - (1-e^{-\varphi x}) \left( \ln(e + T \cdot \theta_0) \right) V
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{\partial g_1}{\partial \alpha} \\
\frac{\partial g_1}{\partial \beta}
\end{bmatrix} = \begin{bmatrix}
-1 \\
\beta \rho_a \sigma^2 - (1-e^{-\varphi x}) \left( \ln(e + T \cdot \theta_0) \right) V
\end{bmatrix}
\]

Therefore, applying the first condition of K.K.T leads to the following equations:

\[
\begin{cases}
-1 = \lambda_1 (-1) \\
(1-\beta)\rho_v \sigma^2 - (1-e^{-\varphi x}) V \left( \ln(e + T \cdot \theta_0) \right) = \beta \rho_a \sigma^2 - (1-e^{-\varphi x}) V \ln (e + T \cdot \theta_0)
\end{cases}
\]

As a result:

\[
\begin{align*}
\lambda_1 &= 1 \\
\beta^* (\theta) &= \frac{\rho_v}{\rho_v + \rho_a}
\end{align*}
\]

Equation (21)

By placing \( \beta^* (\theta) \) in (19), \( x^* \) is obtained:

\[
x^* = -\ln \left( \frac{b \mu (\rho_v + \rho_a)}{\rho_v \vartheta V \left( \ln (e + T \cdot \theta_0) \right)^2} \right) \]

Equation (22)

In the following, According to the second condition of K.K.T:

\[ \lambda_i g_i(x) = 0 \]
\[
\lambda_i \left( m - \alpha(\theta) - \beta(\theta) \left[ (1 - e^{\alpha x}) \left( \ln (e + T) \theta_0 \right) V \right] + \frac{b e^{\alpha x}}{\ln (e + T) \theta_0} + \sigma^2 \rho_x \beta \left( \theta \right)^2 \right) = 0
\]

So:
\[
\alpha^* = m + \frac{b e^{\alpha x^*}}{\ln (e + T) \theta_0} + \rho_x \beta^2 \sigma^2 - \beta \left( 1 - e^{-\alpha x^*} \right) V \left( \ln (e + T) \theta_0 \right)
\]

Equation (23)

The third and fourth conditions of K.K.T based on \( g_i(x) \leq 0 \) and \( \lambda_i \geq 0 \) are also established.

In summary we have come up the following optimal solutions for venture capitalist in table 1.

2.2.2 Entrepreneur

As described in the previous section, the model in 2.1.2 has also been solved from the entrepreneur's point of view and the final optimal values are listed in table 2.

2.3 Equilibrium in the optimal solutions of venture capital parties

By comparing the optimal results obtained for the contract parties of venture capital, we realize that the optimum values of the effort level (x) and the constant component of the contract (\( \alpha \)) are different for the parties. In this situation, it can be said that the parties will agree if their optimum answers are close.

Here, the equilibrium values of the parties are obtained; as indicated in Tables 1 and 2, how much does the venture capitalist pay (T) for training the entrepreneur and increasing his/her ability to equalize the effort level of the contracting parties? Also, in case of non-participation in the plan (m+q), how much should the total amount of income of the parties be for the constant component of the contract to make an equilibrium between the parties?

2.3.1 Parametric equilibrium values

To achieve the parametric equilibrium solution, the optimal values of the effort level, from both parties’ perspective as well as the optimal values of the constant component of the contract have to be equalized. To equalize the optimal effort level of the parties, we have to first determine how much the capitalist should pay to train the entrepreneur in order to balance the optimal efforts level of the parties. So:

\[
x_v^* = x_u^*
\]
By solving the above equation, we will have:

\[ \ln(e + T) = \frac{b(1 - \beta_r)}{\theta_0 \beta} \]

Then:

\[ T = e^{\frac{b(1 - \beta_r)}{\theta_0 \beta}} - e \]

Equation (27)

In the next step, the optimal answers of the constant component of the parties \((\alpha)\) have to be equalized. Doing so, if not participating in the plan, indicates how much the total income of the parties should be to accept the contract. Therefore:

\[ \alpha^* = \alpha_a^* \]

\[ m + \frac{b \mu}{\beta^* \mathcal{O} \mathcal{V} \cdot \left( \ln(e + T), \theta_0 \right)^2} = -\ln \left[ \frac{b \mu}{\beta^* \mathcal{O} \mathcal{V} \cdot \left( \ln(e + T), \theta_0 \right)^2} \right] \]

\[ = -\ln \left[ \frac{\mu}{(1 - \beta^*_e) \mathcal{O} \mathcal{V} \cdot \left( \ln(e + T), \theta_0 \right)} \right] \]

Then:

\[ m + q = (1 - \beta^*_e) \cdot \left( 1 - e^{\frac{\mu}{\beta^* \mathcal{O} \mathcal{V} \cdot \left( \ln(e + T), \theta_0 \right)^2}} \right) \cdot \left( \ln(e + T), \theta_0 \right) \cdot V - T \cdot \frac{e^{\frac{\mu}{\beta^* \mathcal{O} \mathcal{V} \cdot \left( \ln(e + T), \theta_0 \right)^2}}}{\theta_0} - \frac{(1 - \beta^*_e)^2 \cdot \mathcal{V} \cdot \mathcal{O} \cdot \mathcal{V} \cdot \left( \ln(e + T), \theta_0 \right)^2}{2} - q \]

Equation (28)

2.3.2 Stating numeric values

For funding through the venture capital, the entrepreneur must present his/her innovative plan to the venture capitalist in order to be assessed and examined. The capitalist’s assessment showed
that the entrepreneur’s innovative plan, if successful, will create a value of 15 units ($V=15$) for the venture company. Moreover, the proposed plan of the entrepreneur has been estimated 0.4 at the total investment required in the plan ($r = 0.4$). The initial ability level of the entrepreneur is placed in the closed interval, $[0,1]$ (in this study ($\theta_0 = 1$); this number will be changeable through holding training courses by the capitalist). The standard deviation, as a measurement of risk aversion, equals to 1 ($\sigma = 1$). In all studies, the entrepreneur is considered to be more risk averse than the venture capitalist; so, in this study, it is assumed that entrepreneur’s and venture capitalist’s risk aversion coefficients are 3 and 1, respectively ($\rho_u = 3, \rho_v = 1$). In the functions considered for effort and cost, there are constant coefficients that are used to determine the degree of convexity and gradient of the functions. In this numerical study, according to the personal characteristics of the entrepreneur and capitalist, the following values are estimated for these coefficients:

$$\mu = 0.2, \varnothing = 0.2, b = 1$$

### 2.3.3 Solving a numerical example and finding the equilibrium values

To find the optimal values, the aforementioned numbers are put into the optimal numbers as below:

$$\beta_v^* = \beta_u^* = \frac{1}{4} = 0.25$$

Now, we are looking for an optimal value for $T$ to create equilibrium at the effort level of the parties. By inserting the above values in Eq. (27), we will have:

$$T = e^{1.075 \times 0.4} - e^{1.025} - e$$

$$T = e^{1.2} - e = 0.6018$$

Finally, by knowing the optimal value for $T$, the optimal and equilibrium values of the effort level, from the perspective of the contract parties, will be as follows:

$$x_v^* = x_u^* = 4.216$$

By specifying the equilibrium values of $\beta$ and $x$, and also finding the desired cost of holding the training courses ($T$), we can create equilibrium in the values of the constant component of the contract ($\alpha$).

By inserting the numbers in Eq. (28), the following result will be obtained:

$$m + q = 1.5313$$
Therefore, if the total saved capital of the capitalist and the entrepreneur (if not participating in the plan) is 1.5313 units, they will reach equilibrium in the constant component of the contract.

Ultimately, with regard to the assumptions of the problem and the values defined for the variables, the optimal values from the parties’ perspective can be summarized as in Table 3.

It is notable that the optimal values given in Table 3 have been calculated assuming $T = 0.6018$.

As shown, $\alpha$, income and profit are sensitive to $m$ and $q$. The effect of $m$ and $q$ values on $\alpha$ and profit is illustrated in Figs. 1 and 2, respectively.

### 2.4 Bounded rationality Vs. Game theory

Here, the venture capitalist and the entrepreneur, as parties to the contract, are considered as parties to the game and are faced with numerous strategic decisions to cooperate with each other. Given the nature of the model and the variables used in it, the values of $\alpha$ and $x$ are assumed to be the entrepreneur’s strategies, and the values of $\beta$ and $T$ as the investor’s strategies. So, we create five-by-five matrices of possible strategies of each party involved in the investment and calculate their earnings. The logic behind this is that if the investor proposes a low $\beta$, he/she will allocate a higher cost for training the entrepreneur since the proposed $\beta$ is proportional to the ability of the entrepreneur, and choosing a low $\beta$ indicates that the investor believes that the entrepreneur’s ability is low and is forced to provide more educational services. But for the entrepreneur, there are two views:

1. Entrepreneur increases his/her effort with the increase of the proposed $\alpha$ (Table 4),
2. Entrepreneur decreases his/her effort with the increase of the proposed $\alpha$ (Table 5).

In this matrix, the entrepreneur’s strategies are shown in the first row, and the venture capitalist’s strategies in the first column. The values written in the matrix represent the players' income where the upper value is related to the venture capitalist and the lower value belongs to the entrepreneur. In each of the matrix cells, the numbers on the left correspond to the individuals’ profit and the number on the right (the larger number) corresponds to their income, regardless of the costs.

Considering the items such as the cost of risk taking in the cost section and consequently, the non-real and tangible costs, individuals' incomes are also calculated along with their benefits in order to clarify the results.

To meet the conditions of bounded rationality in the model, we assume that in the initial stage, the contract parties with bounded knowledge and incomplete recognition of each other will make one of the strategic decisions. Over time with additional information and a greater understanding of the environment, the parties will update their strategies; thus, learning will happen during the implementation of the contract and will affect the decisions of individuals. Therefore, learning algorithms are used in the model. In this study, we apply Q-learning algorithm, as one of the reinforcement learning algorithms.
In this learning algorithm, the player number \( i \) performs the activity \( a_t^i \) in the moment \( t \) and receives the reward \( R_t^i \left( a_t^i, a_t^{-i} \right) \) based on the activity \( a_t^{-i} \), which is played by the rival. Then, according to the following recursive formula, Function \( Q \) is updated:

\[
Q_{t+1}^i(a_t^i) = \begin{cases} 
(1-\alpha_t)Q_t^i(a_t^i) + \alpha_t[I(R_t^i + \gamma \max_{a_t^{i'}} Q_t^i(a_t^{i'}));ifa_t^i = a_t^i] & \text{if } a_t^i = a_t^i \\
Q_t^i(a_t^i) & \text{otherwise}
\end{cases}
\]

Equation (29)

Where, \( \gamma \) is the reducing factor and \( \alpha_t = C_{t-1}^{-i}(a_t^i)^{-\delta} \) is the learning rate variable. Then the next activity, \( a_{t+1}^i \) is randomly chosen from the probabilities determined by means of the following exponential decision making formula:

\[
\pi_t^i(a_t^i) = \frac{e^{\lambda Q_t^i(a_t^i)}}{\sum_{a_t'}e^{\lambda Q_t^i(a_t')}}
\]

Equation (30)

The results of this algorithm depend on many factors including the parameters used in the model; parameters such as \( \gamma \), \( \delta \) and \( \lambda \) have a great impact on the results. Other factors are the frequency of learning the model, and the model run time. Therefore, many studies have been carried out to find the appropriate values for these parameters. In this study, we applied these values: \( \gamma = 0.96 \), \( \delta = 0.63 \), and \( \lambda = 6 \) [25]. One of the most important and influential factors is the condition of stopping the algorithm (the degree of probability convergence of the chosen strategy), as well as the random choices in the algorithm. In this study, more than 0.999 probabilities are considered simultaneously for the players. On the other hand, the random choices of the algorithm cause the software output results not to be the same. To solve this problem, the algorithm is repeated so that learning takes place. By doing this, the probability of choosing each matrix cell and each strategy by the parties is determined. The results of the two games previously presented are given in table 6 and table 7. In the given matrix, the upper number in each cell is related to the number of repetitions and the lower number represents the corresponding probability.

On the other hand, the created matrices can be examined from the perspective of game theory. This game has two players who decide at the same time. Here, each player must maximize their expected income based on their perceptions of the other player. With these interpretations, we calculate the Nash equilibrium in the generated game matrices. The most important feature of Nash Equilibrium is that the outcome is not the greatest for the players. In this balance, the assumption of the rationality of the players is a necessary condition and the correctness of each player's belief in choosing the opponent is a sufficient condition. The distinct cell in tables 6 and 7 shows Nash equilibrium.

Therefore, the probability of choosing each strategy for the investment parties based on the first view and second view of entrepreneur is as shown in table 8 and table 9.
The basis of the calculations is the profit earned so that the results can be compared with the results of the mathematical solution.

3 Conclusion

In this paper, we focused on designing a model for venture capital contracts to show that, despite the different expectations of the contract parties, there are equilibrium points that convince the parties to sign the contract.

As shown above, the optimal values of the models from the perspective of each party are different. Therefore, it is impossible to achieve a single strategy and sign a contract in the real world. To find the equilibrium values and thus sign the contract, we need to equalize the optimal values of the effort level \((x)\) and the fixed component of the incentive payment function \((\alpha)\) from the perspective of each party. Then the values of \(T\) (the cost of entrepreneur training) and \(m+q\) (the total wealth that the parties to the contract can obtain if the contract is not accepted) are determined.

We expected that the higher the initial ability of the entrepreneur, the less money the venture capitalist will pay for his training. Also the higher the value of \(q\), the lower the value for \(\alpha\) (fixed component of the entrepreneur’s payment function) will be suggested by the venture capitalist, and the higher the value of \(m\), the more value the entrepreneur will call for \(\alpha\); the results confirm these expectations.

As we know, both entrepreneur and venture capitalist seek to optimize the model from their own prospective. This study shows that, if the values of \(T\) (the cost of entrepreneur training) and \(m+q\) (total wealth that the parties to the contract can obtain if the contract is not accepted) are close to the equilibrium points obtained in the models, the optimal value of decision variables in the models of each party will be equal. In addition, by moving away from the equilibrium values, the probability of concluding a contract will decrease because of the conflict of interest.

In the learning algorithms section, the computational results of the Q-learning algorithm indicated that, learning has been completely done for the investor because he/she only chooses the strategies which generate the most income in the first and second rounds, but learning has not been well done for the entrepreneur, and so he/she selects all of his/her strategies with different possibilities (higher than 10%). In the second scenario, the investor also learns very well, and is more likely to choose profitable strategies. Finally, learning happens for the entrepreneur, and he/she selects the profitable strategies with higher probabilities.

According to table 6, in the first scenario, the first matrix cell is introduced as the Nash equilibrium. Examining table 6, which shows the results of 100 iterations of the Q-learning algorithm, we find that the parties most likely choose the Nash equilibrium point. That is, Q-learning outcomes converge with Nash equilibrium results. Also, according to table 7, in the second scenario, the cell1*5 is selected as the Nash equilibrium. This cell was most likely selected by the parties compared to other cells. Therefore, the results of the Q-learning are reasonably convergent with Nash equilibrium.
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Figures and tables
Table 1 Optimum values of decision-making variables from the perspective of venture capitalist
\[ \beta_v^* (\theta) = \frac{\rho_v}{\rho_v + \rho_u}. \]

\[ x_v^* = \frac{-\ln \left[ \frac{b \mu}{\beta_v^* D V \cdot (\ln (e + T) \cdot \theta_0)^2} \right]}{\varnothing + \mu}. \]

\[ \alpha_u^* = m + \frac{b e^{\mu x_u^*} + \rho_u \beta_v^* \rho_v \sigma^2}{2} - \beta_v^* \cdot (1 - e^{-\varnothing x_u^*}) \cdot V \cdot (\ln (e + T) \cdot \theta_0) \]

Where, \( \beta_v^* \), \( x_v^* \) and \( \alpha_u^* \) are optimum values from the venture capitalist’s perspective.

**Table 2** Optimum values of decision-making variables from the perspective of entrepreneur

\[ \beta_u^* = \frac{\rho_v}{\rho_v + \rho_u} \quad \text{Equation (24)} \]

\[ x_u^* = \frac{-\ln \left[ \frac{\mu}{(1 - \beta_v^*) D r \cdot (\ln (e + T) \cdot \theta_0) V} \right]}{\varnothing + \mu} \quad \text{Equation (25)} \]

\[ \alpha_u^* = (1 - \beta_u^*) \cdot (1 - e^{-\varnothing x_u^*}) \cdot (\ln (e + T) \cdot \theta_0) V - T - \frac{e^{\mu x_u^*}}{r} - \frac{(1 - \beta_u^*)^2 \cdot \rho_v \sigma^2}{2} - q \quad \text{Equation (26)} \]

Where, Where, \( \beta_u^* \), \( x_u^* \) and \( \alpha_u^* \) are optimum values from the entrepreneur’s perspective.

**Table 3** Optimal values of numerical example from the perspective of the parties

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( x ) ( (T = 0.6018) )</th>
<th>( \alpha )</th>
<th>Income</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Venture capitalist</td>
<td>0.25</td>
<td>4.216</td>
<td>m - 0.5333</td>
<td>7.6905</td>
</tr>
<tr>
<td>Entrepreneur</td>
<td>0.25</td>
<td>4.216</td>
<td>0.9980 - q</td>
<td>3.5615 - q</td>
</tr>
</tbody>
</table>
Fig. 1 α sensitivity analysis relative to m and q

Fig. 2 Profit sensitivity analysis relative to m and q
### Table 4 Strategies matrix of entrepreneur and investor based on different strategic decisions (the first view of entrepreneur)

<table>
<thead>
<tr>
<th>Investor</th>
<th>( \alpha = 0.1 )</th>
<th>( \alpha = 0.2 )</th>
<th>( \alpha = 0.3 )</th>
<th>( \alpha = 0.4 )</th>
<th>( \alpha = 0.46 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta = 0.1 )</td>
<td>( \alpha = 0.1 )</td>
<td>( x = 4.2 )</td>
<td>( x = 4.7 )</td>
<td>( x = 5.2 )</td>
<td>( x = 5.7 )</td>
</tr>
<tr>
<td>( T = 0.85 )</td>
<td>2.6134, 9.7593</td>
<td>2.6099, 10.4648</td>
<td>2.4752, 11.1032</td>
<td>2.2089, 11.6808</td>
<td>1.9854, 12.0007</td>
</tr>
<tr>
<td>( \beta = 0.17 )</td>
<td>1.7823, 8.7377</td>
<td>1.7049, 9.3693</td>
<td>1.5034, 9.9409</td>
<td>1.1767, 10.4581</td>
<td>0.9197, 10.7444</td>
</tr>
<tr>
<td>( T = 0.72 )</td>
<td>-0.0293, 1.8896</td>
<td>0.0028, 2.1190</td>
<td>0.0018, 2.3361</td>
<td>-0.0332, 2.5420</td>
<td>-0.0711, 2.6607</td>
</tr>
<tr>
<td>( \beta = 0.25 )</td>
<td>0.8962, 7.6684</td>
<td>0.7415, 8.2227</td>
<td>0.47, 8.7243</td>
<td>0.08, 9.1782</td>
<td>-0.212, 9.4295</td>
</tr>
<tr>
<td>( T = 0.6 )</td>
<td>0.6312, 2.6561</td>
<td>0.7129, 2.9409</td>
<td>0.7556, 3.2081</td>
<td>0.7588, 3.4594</td>
<td>0.7414, 3.6032</td>
</tr>
<tr>
<td>( \beta = 0.32 )</td>
<td>0.137, 6.7392</td>
<td>-0.0848, 7.2263</td>
<td>-0.4171, 7.6672</td>
<td>-0.8621, 8.066</td>
<td>-1.1848, 8.2869</td>
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<tr>
<td>( T = 0.48 )</td>
<td>1.1254, 3.2714</td>
<td>1.2451, 3.6066</td>
<td>1.321, 3.9081</td>
<td>1.3528, 4.1958</td>
<td>1.3504, 4.3597</td>
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<td>( \beta = 0.4 )</td>
<td>-0.6868, 5.7341</td>
<td>-0.9814, 6.1486</td>
<td>-1.3794, 6.5237</td>
<td>-1.8839, 6.8631</td>
<td>-2.2393, 7.0510</td>
</tr>
<tr>
<td>( T = 0.35 )</td>
<td>1.6166, 3.9227</td>
<td>1.7756, 4.2991</td>
<td>1.8856, 4.6491</td>
<td>1.9464, 4.9754</td>
<td>1.9592, 5.1607</td>
</tr>
</tbody>
</table>

Source: Research calculations

### Table 5 Strategies matrix of entrepreneur and investor based on different strategic decisions (the second view of entrepreneur)

<table>
<thead>
<tr>
<th>Investor</th>
<th>( \alpha = 0.1 )</th>
<th>( \alpha = 0.2 )</th>
<th>( \alpha = 0.3 )</th>
<th>( \alpha = 0.4 )</th>
<th>( \alpha = 0.46 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta = 0.1 )</td>
<td>( \alpha = 0.1 )</td>
<td>( x = 4.2 )</td>
<td>( x = 3.7 )</td>
<td>( x = 2.7 )</td>
<td>( x = 2.4 )</td>
</tr>
<tr>
<td>( T = 0.85 )</td>
<td>2.6134, 9.7593</td>
<td>2.2848, 8.9796</td>
<td>1.8217, 8.1179</td>
<td>1.2205, 7.1655</td>
<td>0.7915, 6.5467</td>
</tr>
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<td>( \beta = 0.17 )</td>
<td>1.7823, 8.7377</td>
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<td>1.1624, 7.2681</td>
<td>0.6609, 6.4154</td>
<td>0.2967, 5.8614</td>
</tr>
<tr>
<td>( T = 0.72 )</td>
<td>-0.0293, 1.8896</td>
<td>0.1062, 1.8467</td>
<td>0.2096, 1.7886</td>
<td>0.2811, 1.714</td>
<td>0.3086, 1.6605</td>
</tr>
<tr>
<td>( \beta = 0.25 )</td>
<td>0.8962, 7.6684</td>
<td>0.7346, 7.0557</td>
<td>0.4562, 6.3786</td>
<td>0.059, 5.6303</td>
<td>-0.2374, 5.1441</td>
</tr>
<tr>
<td>( T = 0.6 )</td>
<td>0.6312, 2.6561</td>
<td>0.7107, 2.5519</td>
<td>0.7513, 2.4262</td>
<td>0.7524, 2.2768</td>
<td>0.7336, 2.1747</td>
</tr>
<tr>
<td>( \beta = 0.32 )</td>
<td>0.137, 6.7392</td>
<td>0.0497, 6.2007</td>
<td>-0.1467, 5.6057</td>
<td>-0.4532, 4.948</td>
<td>-0.6907, 4.5207</td>
</tr>
<tr>
<td>( T = 0.48 )</td>
<td>1.1254, 3.2714</td>
<td>1.1616, 3.118</td>
<td>1.1531, 2.9380</td>
<td>-0.1098, 2.7285</td>
<td>1.0438, 2.5874</td>
</tr>
<tr>
<td>( \beta = 0.4 )</td>
<td>-0.6868, 5.7341</td>
<td>-0.6939, 5.276</td>
<td>-0.8016, 4.7696</td>
<td>-1.0099, 4.2101</td>
<td>-1.1837, 3.8465</td>
</tr>
<tr>
<td>( T = 0.35 )</td>
<td>1.6166, 3.9227</td>
<td>1.6078, 3.7173</td>
<td>1.5482, 3.4798</td>
<td>1.4361, 3.2067</td>
<td>1.3428, 3.0243</td>
</tr>
</tbody>
</table>

Source: Research calculations
Table 6 The number of repetitions and the probability of choosing each matrix cell (the first view of entrepreneur)

<table>
<thead>
<tr>
<th>Investor</th>
<th>$\alpha = 0.1$</th>
<th>$\alpha = 0.2$</th>
<th>$\alpha = 0.3$</th>
<th>$\alpha = 0.4$</th>
<th>$\alpha = 0.46$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0.1$</td>
<td>30</td>
<td>13</td>
<td>8</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>$T = 0.85$</td>
<td>0.3</td>
<td>0.13</td>
<td>0.08</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>$\beta = 0.17$</td>
<td>11</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>$T = 0.72$</td>
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<td>0.07</td>
<td>0.07</td>
<td>0.06</td>
<td>0.04</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
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<td>$T = 0.6$</td>
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<td>0</td>
</tr>
<tr>
<td>$\beta = 0.32$</td>
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<td>0</td>
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<td>$\beta = 0.4$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 7 The number of repetitions and the probability of choosing each matrix cell (the second view of entrepreneur)

<table>
<thead>
<tr>
<th>Investor</th>
<th>$\alpha = 0.1$</th>
<th>$\alpha = 0.2$</th>
<th>$\alpha = 0.3$</th>
<th>$\alpha = 0.4$</th>
<th>$\alpha = 0.46$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0.1$</td>
<td>5</td>
<td>7</td>
<td>15</td>
<td>19</td>
<td>14</td>
</tr>
<tr>
<td>$T = 0.85$</td>
<td>0.05</td>
<td>0.07</td>
<td>0.15</td>
<td>0.19</td>
<td>0.14</td>
</tr>
<tr>
<td>$\beta = 0.17$</td>
<td>5</td>
<td>6</td>
<td>15</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>$T = 0.72$</td>
<td>0.05</td>
<td>0.06</td>
<td>0.15</td>
<td>0.1</td>
<td>0.03</td>
</tr>
<tr>
<td>$\beta = 0.25$</td>
<td>1</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>$T = 0.6$</td>
<td>0.01</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\beta = 0.32$</td>
<td>0</td>
<td>0</td>
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<td>$T = 0.48$</td>
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<tr>
<td>$\beta = 0.4$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$T = 0.35$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 8 The probability of choosing each party’s strategic decisions based on the first view of entrepreneur

<table>
<thead>
<tr>
<th>Investor</th>
<th>$\beta = 0.1,T = 0.85$</th>
<th>$\beta = 0.17,T = 0.72$</th>
<th>$\beta = 0.25,T = 0.6$</th>
<th>$\beta = 0.32,T = 0.48$</th>
<th>$\beta = 0.4,T = 0.35$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.1,x = 4.2$</td>
<td>0.65</td>
<td>0.35</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha = 0.2,x = 3.7$</td>
<td>0.44</td>
<td>0.2</td>
<td>0.15</td>
<td>0.14</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 9 The probability of choosing each party’s strategic decisions based on the second view of entrepreneur

<table>
<thead>
<tr>
<th>Investor</th>
<th>$\beta = 0.1,T = 0.85$</th>
<th>$\beta = 0.17,T = 0.72$</th>
<th>$\beta = 0.25,T = 0.6$</th>
<th>$\beta = 0.32,T = 0.48$</th>
<th>$\beta = 0.4,T = 0.35$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.1,x = 4.2$</td>
<td>0.60</td>
<td>0.39</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha = 0.2,x = 4.7$</td>
<td>0.11</td>
<td>0.13</td>
<td>0.3</td>
<td>0.29</td>
<td>0.17</td>
</tr>
</tbody>
</table>
Biographies

**Seyed Hossein Jafarpour Rezaei** is an MS graduate student in School of finance sciences at the Kharazmi University, Tehran, Iran. His research interests include private equity, venture capital, and contracting.

**Mohammad Ali Rastegar** is an Associate Professor of Industrial Engineering at Tarbiat Modares University, Tehran, Iran. His research interests include financial and banking risks modelling, Algorithmic trading, Asset-Debt Management, Agent-based simulation, and Investment management. He has published papers in journals such as Handbook of power systems II, Solar Energy, International Journal of Finance & Managerial Accounting, and etc.