Modeling for radiated Marangoni convection flow of magneto-nanoliquid subject to Activation energy and chemical reaction

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Abstract: Simultaneous impacts of non-linear radiation and magnetohydrodynamics in Marangoni convection nanoliquid are addressed Novel aspects of activation energy and space dependent heat source are addressed. Nanoliquid attributes Brownain movement and thermophoresis diffusion. NDSolve base shooting technique is employed for the numerical simulation. Aspects of various embedded variables are focused on velocity, heat and mass transport distributions via graphical interpretations. Moreover temperature gradient at the surface is estimated and analyzed. Our study identified that exponential based space heat source (ESHS) parameter significantly enhanced the thermal field. Activation energy and temperature difference parameters decrease the nanoparticles concentration. Moreover temperature gradient enhances for higher Marangoni ratio parameter, Hartmann number, dimensionless activation energy and thermophoresis parameter.

Keywords: Marangoni convection; ESHS; Activation energy; Thermal radiation; Nanomaterials.

1. Introduction

Nanoliquid at present is busy topic of researchers. Choi [1] proposed novel idea of suspension of nanoparticles (dimensions less the $100nm$) in the traditional liquids. He extracted that such kind of suspension of solid particles in conventional liquids leads to thermal conductivity enhancement. There are several kinds of base liquids namely bio-liquids, lubricants, oils, water, polymer solutions, ethylene glycol etc. Such liquids have useful applications in transformer cooling, electronic cooling and heat exchanger [1-3]. Thus for the intensification of thermal potential, fine metalic particles (TiO_2, Cu, Ag, Al_2O_3, Fe and their oxides) are dispersed homogeneously in the operating liquids. Now sizeable information about nanoliquids with diverse features is available. Few representative attempts about nano-materials can be observed via [4-18]. On the other hand biodegradability, long blood retention time and low toxicity of magnetic nanoparticles become a principal material in biomedical uses. Distinct characteristics like coercivity, high magnetic susceptibility, large surface to volume ratio, superparamagnetism and low Curie temperature have improved the attention of scientists in magneto-nanoliquids. Nanoparticles composed of magnetic iron oxide are widespread in nature, chemically and physically stable, inexpensive to produce, biocompatible and environmentally safe [19]. The magnetite, hematite and maghemite are the typical examples of these materials. Numerous engineering applications regarding magnetics materials are like elimination of tumors, asthma treatment, magnetic hyperthermia, drug release, thermoblation, magnetic resonance imaging (MRI), targeted drug delivery, synergistic effects and biosensing (biomolecules, cells). Generally

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magnetized nanoparticles exhibit random motion inside the base liquid would be converted into uniform motion by employing an external magnetic field [20]. The coated magnetized nanoparticles dispersed in drug of anti-cancer and are injected into patient's body. The movement of drug to target region is restrained via magnetic field [21]. Some analysis in this regard may be noticed in the literature [22-25].

The layers which may arise along the liquid-liquid or liquid-gas interacts are referred as Marangoni layers. Marangoni flow can be both temperature and concentration gradients which is created due to surface tension gradient. Analysis of Marangoni convection remains an area of high curiosity for the engineers and scientists. Because of its appearance in several practical applications namely aerospace, crystal growth, materials science, welding, spreading of thin films, semiconductor processing, nuclear reactors etc. Firstly Napolitano [26,27] provided the basic work in this direction. After that numerous researchers scrutinized comprehensively the Marangoni flow with various aspects [28-32]. Xu and Chen [33] explored heat transport features of Marangoni flow in a copper-water nanoliquid. Sheikholeslami and Chamkha [34] examined the aspects of magnetohydrodynamic (MHD) on nanoliquid by considering the Marangoni convection. Thermal Marangoni convective flow of nanoliquid by rotating disk with irregular heat and solar radiation is studied by Mahanthesh et al. [35].

The energy escalates from a heated region to its absorption region in every directions like electromagnetic waves is referred as thermal radiation. It is created by the thermal tumult of composite atoms of body. Light-bulb and fire, heat from the microwave and sun radiation are typical examples of thermal radiation. Infrared regime of electromagnetic spectrum incorporates the radiation for most objects on this earth. Due to this reason the inspection of radiation is important in conversation of thermal frameworks. It possess remarkable role in different high temperature procedures. This concept is also widespread in gas turbines, furnaces, engine cooling, aircraft and boilers etc. Further technological uses of it can be consulted in the solar technology, nuclear power plants, technology related to power, combustion chambers and chemical processes. It has been remarked that aspects of thermal radiation become significant when difference between ambient and surface temperature is large [36-40].

Our objective here is to disclose the features of solutal-thermo Marangoni convection in flow of nanomaterials. Formulation is based upon conservation laws. Attributes of thermal radiation, Brownian motion and thermophoretic diffusion are accounted. In addition we intended to examine the impacts of activation energy. Transformations are used to convert flow expressions into ordinary one. NDSolve based Shooting technique is implemented for the solutions. Plots are declared to insight the behaviors physical variables. Nusselt number is estimated

2. Problem formulation

Radiative Marangoni convective flow of magneto-nanoliquid is addressed. Magnetic field of strength B_0 is implemented in $y-$direction (see Fig. 1). Varying temperature with power law is assumed at the surface. Heat and mass transport is subject to irregular heat source, thermal radiation and activation energy. Aspects of thermophoretic diffusion and Brownian motion are considered. No relative movement exists between base liquid and nanoparticles. Moreover the nanoparticles flux condition is intended. Modeled equations are [28-31, 41-45]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_1 B_0^2}{\rho_f}u, \tag{2}$$

$$u\frac{\partial T}{\partial y}+v\frac{\partial T}{\partial y}=\alpha_f\frac{\partial^2 T}{\partial y^2}+\tau\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_0}\left(\frac{\partial T}{\partial y}\right)^2\right)+\frac{Q_0(T-T_\infty)}{(\rho c_p)_f}e^{-yn\sqrt{\frac{a}{\nu}}}-\frac{1}{(\rho c_p)_f}\frac{\partial q_r}{\partial y}, \tag{3}$$

$$u\frac{\partial C}{\partial y}+v\frac{\partial C}{\partial z}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_0}\frac{\partial^2 T}{\partial y^2}-k_r^2(C-C_\infty)\left(\frac{T}{T_0}\right)^m\exp\left(\frac{-E_a}{\kappa T}\right), \tag{4}$$

with [46, 47]:

$$\left.\begin{aligned}\mu\left.\tfrac{\partial u}{\partial y}\right|_{y=0}=-\left.\tfrac{\partial\sigma}{\partial x}\right|_{y=0}=\sigma_0\left(\gamma_C\left.\tfrac{\partial C}{\partial x}\right|_{y=0}+\gamma_T\left.\tfrac{\partial T}{\partial x}\right|_{y=0}\right),\ v(x,0)=0,\\ T_w(x,0)=T_\infty+T_0X^2,\ D_B\left.\tfrac{\partial C}{\partial y}\right|_{y=0}+\tfrac{D_T}{T_0}\left.\tfrac{\partial T}{\partial x}\right|_{y=0}=0,\end{aligned}\right\} \tag{5}$$

$$u(x,\infty)\to 0,\ T(x,\infty)\to T_\infty,\ C(x,\infty)\to C_\infty. \tag{6}$$

Here velocity components parallel to (x,y) are signified by (u,v), ν the kinematic viscosity, n the exponential index, α_f the nanomaterials thermal diffusivity, ρ_f the liquid density, Q_0 heat generation/absorption variable, σ_1 the electrical conductivity, $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ the heat capacity ratio， E_a the non-dimensional activation energy, D_B the diffusion coefficient, D_T the coefficient of thermophoretic diffusion, $(T\ ,C)$ and $(T_\infty\ ,C_\infty)$ the respective ambient and nanoparticles temperature and concentration. Expression for surface tension (σ) is

$$\left.\begin{aligned}\sigma=\sigma_0[1-\gamma_C(C-C_\infty)-\gamma_T(T-T_\infty)],\\ \gamma_C=-\tfrac{1}{\sigma_0}\left.\tfrac{\partial\sigma}{\partial C}\right|_C,\ \gamma_T=-\tfrac{1}{\sigma_0}\left.\tfrac{\partial\sigma}{\partial T}\right|_T,\end{aligned}\right\} \tag{7}$$

where γ_T and γ_C designate the coefficients of temperature and concentration surface tension and σ_0 is a positive constant. Further in Eq. (4) $\kappa=8.61\times10\ eV/K$ as the Boltzmann constant, m $(-1<m>1)$ the fitted rate constant and k_r^2 the rate of reaction.

Radiative heat flux q_r is

$$q_r=-\frac{4\sigma^{**}}{3m^{**}}\frac{\partial(T^4)}{\partial y}=-\frac{16\sigma^{**}T_\infty^3}{3m^{**}}\frac{\partial^2 T}{\partial y^2}, \tag{8}$$

where m^{**} the coefficient of mean absorption and σ^{**} indicates the Stefan-Boltzman. On using Eq. (8), Eq. (3) yield

$$\left.\begin{aligned}u\tfrac{\partial T}{\partial y}+v\tfrac{\partial T}{\partial y}=\alpha_f\tfrac{\partial^2 T}{\partial y^2}+\tau\left(D_B\tfrac{\partial C}{\partial y}\tfrac{\partial T}{\partial y}+\tfrac{D_T}{T_0}\left(\tfrac{\partial T}{\partial y}\right)^2\right)\\ +\tfrac{Q_0(T-T_\infty)}{(\rho c)_f}e^{-yn_1\sqrt{\frac{a}{\nu}}}+\tfrac{1}{(\rho c)_f}\tfrac{16\sigma^{**}T_\infty^3}{3m^{**}}\tfrac{\partial^2 T}{\partial y^2}.\end{aligned}\right\} \tag{9}$$

Setting [47]:

$$\left.\begin{aligned}u=\tfrac{\partial\psi}{\partial y},\ v=-\tfrac{\partial\psi}{\partial x},\ \psi=\nu Xf(\eta),\ \eta=y/L,\ X=x/L,\\ T=T_\infty+T_0X^2\theta(\eta),\ C=C_\infty+C_0X^2\phi(\eta),\end{aligned}\right\} \tag{10}$$

expression (1) is verified trivially while other equations and boundary conditions are

$$f'''-f'^2+ff''-Haf'=0, \tag{11}$$

$$\left(1+Rd\right)\theta''+\Pr(f\theta'-2f'\theta)+N_b\theta'\phi'+N_t\theta'^2+\Pr Q\exp(-n\eta)=0, \tag{12}$$

$$\phi'' + Sc(f\phi' - 2f'\phi) + \frac{N_t}{N_b}\theta'' - Sc\alpha_1(1+\delta\theta)^m \phi \exp\left(\frac{-E}{1+\delta\phi}\right) = 0, \tag{13}$$

$$\left.\begin{array}{c} f''(0) = -2(1+r),\ f(0) = 0,\ \theta(0) = 1,\ \phi(0) + \frac{N_t}{N_b}\theta'(0) = 0, \\ f'(\infty) \to 0,\ \theta(\infty) \to 0,\ \phi(\infty) \to 0, \end{array}\right\} \tag{14}$$

where Ha denotes Hartmann number, N_b the Brownain motion parameter, $\Pr$ the Prandtl number, n the exponential index, Q the ESHS variable, N_t the thermophoresis parameter, Sc the Schmidt number, α_1 the reaction rate, L the reference length, Rd the radiation variable, δ the temperature difference parameter, r the Marangoni ratio parameter, E the dimensionless activation energy Ma the solutal and Ma_T thermal Marangoni numbers. These variables are quantified by

$$\left.\begin{array}{c} r = \frac{Ma_C}{Ma_T} = \frac{C_0\gamma_C}{T_0\gamma_T},\ Ma_C = \frac{\sigma_0 C_0 \gamma_C L}{\alpha\mu},\ Ma_T = \frac{\sigma_0 T_0 \gamma_T L}{\alpha\mu},\ N_t = \frac{(\rho c)_p}{(\rho c)_f}\frac{D_T T_0 X^2}{\nu}, \\ N_b = \frac{(\rho c)_p}{(\rho c)_f}\frac{D_B C_0 X^2}{\nu},\ Q = \frac{Q_0 L^2}{\nu(\rho c_p)_f},\ L = -\frac{\mu\nu}{\sigma_0 T_0 \gamma_T},\ \Pr = \frac{\nu}{\alpha_f},\ Ha = B_0 L\sqrt{\sigma_1/\mu}, \\ Sc = \frac{\nu}{D_B},\ \alpha_1 = L^2 k_r^2/\nu,\ E = \frac{E_a}{\kappa T_\infty},\ \delta = \frac{T_0 X^2}{T_\infty},\ Rd = \frac{16\sigma^{**}T_\infty^3}{3km^{**}}. \end{array}\right\} \tag{15}$$

2.1. Physical quantity

Local gradient of temperature (Nu_x) is defined as

$$Nu_x = \frac{q_w x}{k_f(T - T_\infty)}, \tag{16}$$

where

$$q_w = -\left(k_f \frac{\partial T}{\partial y} - \frac{16\sigma^{**}T_\infty^3}{3m^{**}}\frac{\partial T}{\partial y}\right)_{y=0}. \tag{17}$$

In dimensionless form we have

$$Nu_x = -\frac{x}{L}(1+Rd)\theta'(0). \tag{18}$$

2.2. Computational method

The solutions of Eqs. $(12-15)$ are computed numerically by adopting NDSolve based Shooting technique. A well known computer software Mathematica is utilized for the simulation of non-linear problems.

3. Discussion

Using the numerical technique highlighted in aformention section, interpretations have been carried out for several values of embedded variables like Hartmann number Ha, Brownain motion parameter N_b , Prandtl number $\Pr$, ESHS variable Q , Schmidt number Sc, thermophoresis variable N_t, reaction rate α_1, radiation parameter Rd, temperature difference parameter δ , Marangoni ratio parameter r and dimensionless activation energy E on non-dimensional velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$. The involved variables have fixed values $n = 0.2$, $\Pr = 0.7 = Sc,$ $\delta = 0.3 = N_b,$ $N_t = 0.1,$

$\alpha_1 = Rd = 0.2 = Q = Ha$, $m = 1.2 = r$ and $E = 3.0$. In Fig. 2 the impact of Ha on $f'(\eta)$ is designed. Here we noticed that velocity decays for higher Ha. For Lorentz forces increment, a back flow appears and velocity $f'(\eta)$ reduces. Thickness of momentum layer is also reduced. The effect of r on velocity $f'(\eta)$ is reported in Fig. 3. Clearly higher estimations of r correspond to more velocity. Influence of Ha on temperature $\theta(\eta)$ is elucidated in Fig. 4. It is found that larger Ha augment the nanoliquid $\theta(\eta)$. Physically strength in magnetic field corresponds to enhance Lorentz force. This force has a property to endure the liquid movement and thus rise $\theta(\eta)$. Moreover the situation of hydrodynamic is recovered for $Ha = 0$. Fig. 5 is interpreted to see changes in $\theta(\eta)$ via Q. It is concluded that higher estimations of Q significantly augment the temperature. In fact more heat is produced because of heat generation procedure within the considered liquid. Interestingly thermal field is enhanced significantly for very small estimations i.e (0.0 to 0.6) in the process of (ESHS) parameter. Importance of thermophoresis parameter N_t on $\theta(\eta)$ is captured in Fig. 6. Here thermal field is improved for larger N_t. Physically N_t assists the thermal diffusion. It means that large number of nanoparticles are shifted towards ambient liquid and thus uplift the thermal field and its layer thickness. Fig. 7 pointed out the behavior of N_b on $\theta(\eta)$. Here temperature is decaying function of N_b. Fig. 8 displayed the consequences of $\Pr$ on $\theta(\eta)$. As expected an increment in $\Pr$ declines the thermal diffusion and it increases the thermal capacity of liquid. As $\Pr$ has inverse link with thermal diffusion. Therefore strength in $\Pr$ declined the thermal diffusion which consequently drops the temperature. Fig. 9 disclose feature of r on temperature $\theta(\eta)$. Here $\theta(\eta)$ is reduced for higher estimations of r. An enhancement in Rd leads to augment the temperature distribution. This result is depicted in Fig. 10. Since kinetic energy progressively enhances due to an increment in Rd which makes the thermal layer thicker. Salient characteristics of N_t on concentration field $\phi(\eta)$ are reported in Fig. 11. Phenomenon of thermophoresis corresponds to disseminate the nanoparticles towards ambient liquid from hot region (as from heated surface more resistance is offered to the nanoparticles). Consequently the thermophoretic force allows nanoparticles to transport heat from the surface to the moving liquids and so $\phi(\eta)$ enhances. Feature of N_b on concentration distribution is declared in Fig. 12. Here $\phi(\eta)$ decaying function of N_b is enhanced. Physically Brownian motion occurs due to the correlation of nanoparticles with base liquid in a nanoliquid system. Brownian motion is influenced for higher values of N_b which ultimately drops the conduction distribution. Nanoparticles concentration designates decaying feature for higher activation energy variable (see Fig. 13). Fig. 14 is design to visualize the features of Sc on concentration $\phi(\eta)$ It is found that $\phi(\eta)$ is an increasing function of Sc. Impact of α_1 on $\phi(\eta)$ is interpreted in Fig. 15. Clearly strength in α_1 declined $\phi(\eta)$ Aspect of diverse embedding variables on local temperature gradient (Nu_x) is displayed in Table 1. It is noticed that Nusselt number is enhanced via Ha, r, N_t and E.

Table 1: Numerical estimations of temperature gradient (Nu_x) for E, Q, Ha, N_b, N_t and

Rd when $n = 0.2$, $\Pr = 0.7 = Sc$, $\delta = 0.3 = 0.1$, $\alpha_1 = 0.2$, and $m = 1.2$.

4. Concluding remarks

Magnetohydrodynamic Marangoni convective flow in presence of activation energy, ESHS and thermal radiation is inspected. Further zero mass flux condition is encountered. Outcomes of present analysis are summarized as follows:

- Velocity for Marangoni ratio and Hartmann number are opposite.
- Strength in Marangoni ratio on temperature and velocity has reverse trend.
- Temperature is increased with higher Rd and Q .
- Features of N_b and N_t on thermal field are quite opposite.
- Temperature gradient is enhanced via Marangoni ratio and activation energy parameters but reverse trend is observed for higher heat source parameter.
- Concentration enhances for larger Sc and α_1.

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Table and Figures Captions

Table 1: Numerical estimations of temperature gradient (Nu_x) for $E,\ Q,\ Ha,\ N_b,\ N_t$ and Rd when $n=0.2$, $\Pr=0.7=Sc,\ \delta=0.3=0.1,\ \alpha_1=0.2$ and $m=1.2$.

Fig. 1. Flow Physical schematic diagram.

Fig. 2. Behavior of $f'(\eta)$ via Ha.

Fig. 3. Feature of $f'(\eta)$ via r.

Fig. 4. Behavior of $\theta(\eta)$ via Ha.

Fig. 5. Behavior of $\theta(\eta)$ via Q.

Fig. 6. Behavior of $\theta(\eta)$ via N_t.

Fig. 7. Behavior of $\theta(\eta)$ via N_b

Fig. 8. Behavior of $\theta(\eta)$ via $\Pr$.

Fig. 9. Behavior of $\theta(\eta)$ via r.

Fig. 10. Behavior of $\theta(\eta)$ via Rd.

Fig. 11. Behavior of $\phi(\eta)$ via N_t.

Fig. 12. Behavior of $\phi(\eta)$ via N_b.

Fig. 13. Behavior of $\phi(\eta)$ via E.

Fig. 14. Behavior of $\phi(\eta)$ via Sc.
Fig. 15. Behavior of $\phi(\eta)$ via α_1.

Parameters (fixed values)	Parameters		$-\theta'(0)$
$n=0.2, \Pr=0.7=Sc, \delta=0.3=N_b, m=0.2,$ $N_t=0.1, \ \alpha_1=Rd=Ha=Q=0.2, E=3.0,$	r	0.0	1.00756
		1.0	1.33393
		2.0	1.56104
$n=0.2, \Pr=0.7=Sc, \delta=0.3=N_b, m=0.2,$ $N_t=0.1, \ \alpha_1=Rd=Ha=Q=0.2, E=3.0, r=1$	Ha	0.2	0.81202
		0.5	1.27351
		1.0	1.17967
$n=0.2, \Pr=0.7=Sc, \delta=0.3, m=0.2,$ $N_t=0.1, \ \alpha_1=Rd=Ha=Q=0.2, E=3.0, r=1$	N_b	0.1	1.27379
		0.2	1.33393
		0.3	1.45732
$n=0.2, \Pr=0.7=Sc, \delta=0.3=N_b, m=0.2,$ $\alpha_1=Rd=Ha=Q=0.2, E=3.0, r=1$	N_t	0.1	1.33393
		0.3	0.66894
		0.5	0.64616
$n=0.2, \Pr=0.7=Sc, \delta=0.3=N_b, m=0.2,$ $N_t=0.1, \ \alpha_1=Ha=Q=0.2, E=3.0, r=1$	Rd	0.0	1.57952
		0.5	1.09983
		0.9	0.90810
$n=0.2, \Pr=0.7=Sc, \delta=0.3=N_b, m=0.2,$ $N_t=0.1, \ \alpha_1=Rd=Ha=0.2, E=3.0, r=1$	Q	0.0	1.49673
		0.4	1.17325
		0.8	0.85797
$n=0.2, \Pr=0.7=Sc, \delta=0.3=N_b, m=0.2,$ $N_t=0.1, \ \alpha_1=Rd=Ha=Q=0.2, \ r=1$	E	0.0	1.33335
		1.0	1.33370
		3.0	1.33393

Figures are provided in separate file (both PDF and TIF format).