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Global practical stabilization of discrete-time switched affine systems via switched Lyapunov functions and state-dependent switching functions

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Abstract. The present study addresses the problem of global practical stabilization of discrete-time switched affine systems using switched Lyapunov functions and aims to achieve less conservative stability conditions and less conservative size for the ultimate invariant set of attraction. This study also makes its main contribution by proposing a state-dependent switching controller synthesis that ensures the invariance and global attractive properties of a convergence set around a desired equilibrium point. This set is constructed through the intersection of a family of ellipsoids associated with each of switched quadratic Lyapunov functions. The global practical stability conditions are proposed as a set of Bilinear Matrix Inequalities (BMIs) for which an optimization problem is established to minimize the size of the ultimate invariant set of attraction. A DC-DC buck converter is considered to illustrate the effectiveness of the proposed stabilization and controller synthesis method.

1. Introduction

Switched systems are defined as a set of continuous dynamics featuring a controlled switching function that decides which continuous dynamic should be selected from this set at any time for the current continuous state evolution [1]. Not only do switched systems gain significance in the study of many real-world systems such as power systems and power electronics, automotive control, aircraft and air-traffic control as well as network control systems, but also they should be properly investigated for theoretical reasons while studying dynamical systems subjected to sudden parameter variations and assessing multi-controller switching techniques [2,3]. Although switched systems occupy a rich proportion of the literature with related several books [2–6] and survey papers [7–10] in this context, a majority of studies conducted in this domain considered a *common equilibrium* point for all isolated subsystems. However, one of the most significant subclasses of switched systems is the *switched affine systems* that are very common in practice, especially in power electronics [11–17].

In this specific subclass, the equilibrium point varies discontinuously during switching among subsystems; therefore, to achieve *asymptotic stability* at a desired equilibrium point, the switching frequency approaches infinity, which is not realizable in practice [18–20]. One of the well-known solutions to the problem of chattering phenomenon is to consider an upper bound on the frequency of switching functions through time discretization. This is a motivation for the analysis and synthesis of switched affine systems in the discrete-time domain. In addition, since the implementation of any controller nowadays is handled

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by computers and discrete-time samplers, study of the discrete-time systems and their computer-based control is of particular significance.

However, due to the frequency limitations for the switching signal, stability of a set can be achieved rather than a particular point [21,22]. As a result, many theoretical findings about the stability of switched systems with a common equilibrium point cannot be directly applied to the stability analysis of switched affine systems without any common equilibrium point. In this regard, the notion of *practical stability* has been proposed in the literature to analyze the stability of switched systems with no common equilibria [23–27].

There are two main types of Lyapunov functions used for stability analysis of switched systems, namely *common* and *multiple* Lyapunov functions. However, there are numerous examples of stable switched systems that do not admit any common Lyapunov function [4,7]. In this respect, to achieve less conservative stability conditions, the rich and more complex classes of multiple Lyapunov functions were utilized to stabilize the switched systems, among which max-type Lyapunov functions [28,29], min-type Lyapunov functions [19], and switched Lyapunov functions [30–34] can be named. In this study, the switched quadratic Lyapunov functions are employed to design switching rules for the global practical stabilization of discrete-time switched affine systems.

1.1. Practical stabilization of switched affine systems

The practical stabilization problem of switched systems without common equilibria or, in a particular case, switched affine systems has been investigated over the last years in the form of either a general or particular problem formulation, especially those problems related to the switching power converters [12,35,36]. There are very few studies on the global practical stabilization of discrete-time switched affine systems using multiple Lyapunov functions and state-dependent switching functions. Here, a brief review of the most relevant studies is presented.

While local practical stability as well as stabilization of continuous-time nonlinear time-variant switched systems through a single Lyapunov-like function were proposed in [26], global practical asymptotic stabilization of time-invariant switched nonlinear systems in continuous-time and discrete-time domains was investigated using a single quadratic Lyapunov function in [23,25], respectively. However, these studies did not propose any constructive and systematic way to compute the respective Lyapunov functions.

In [19], the global practical stability conditions were proposed as a set of Bilinear Matrix Inequalities (BMIs) for discrete-time switched affine systems using min-type multiple Lyapunov functions. In this study, a single ellipsoidal set containing an actual convergence set was employed as an invariant set of attraction, thus leading to more conservative results in size estimation of the ultimate convergence set. Moreover, the authors proposed a more complex methodology with two different theorems, each of which was associated with the attractive and invariant properties, to compute the final invariant set of attraction.

A set of BMI conditions was proposed in [34] for the global practical stabilization of continuous-time switched affine systems in the framework of sampled data systems using switched Lyapunov functions. However, only the attractiveness property of the convergence set can be guaranteed.

In [18,20,34], the global practical stability conditions were proposed for discrete-time and continuous-time switched affine systems, respectively, using a common quadratic Lyapunov function. These studies shared one limitation, that is, the invariant set of attraction must contain an equilibrium point that belongs to a predetermined set of attainable ones. This can be a barrier to the applicability of the proposed conditions because these equilibrium points are generated in a (Schur or Hurwitz) stable matrix calculated by the convex combination of each of affine subsystems. However, investigating the existence of a stable matrix as a convex combination of a family of matrices requires special algorithms that can be represented as an *NP-hard* problem [28,37–40]. This limitation may be relaxed by applying multiple Lyapunov functions and proposing less conservative stability conditions [20,34].

The problem of robust and global practical stabilization of the switched affine systems in the discrete-time domain was addressed in [41]. The proposed switching functions here were established based on the existence of a common Lyapunov function; however, no constructive method was proposed to calculate them. In [32,33], the local and global practical stabilization of continuous-time nonlinear switched systems was investigated, respectively, using time-dependent switching instead of its state-dependent counterpart as well as switched Lyapunov functions.

1.2. The contributions, objectives, and organization of this paper

To the best of our knowledge, according to the preceding literature review, it is for the first time that the switched quadratic Lyapunov function has been adopted in the context of global practical stabilization of discrete-time switched affine systems using state-dependent switching rules that guarantee invariant and attractive properties of the convergence set simultaneously. Use of multiple Lyapunov functions yields less conservative stability conditions. In addition, application of a family of the corresponding ellipsoids around the desired equilibrium point instead of a single one and guidance of the state trajectories to their intersection yield a less conservative size for the ultimate invariant set of attraction. In this regard, the theoretical foundations for practical stability of discrete-time switched systems without a common equilibrium point are presented through some basic definitions of different types of practical stability in Section 2 and Lemma 1 in Section 3. Next, a state-dependent switching rule (switching Algorithm 1) is proposed together with a set of BMI-based stability conditions in Theorems 1 and 2 through which the global practical stability of discrete-time switched affine systems is guaranteed. In Section 4, the optimization problems corresponding to the stability conditions of Theorems 1 and 2 are formulated to minimize the size of the invariant set of attraction. Finally, Section 5 discusses the applicability of the proposed stabilization method to a DC-DC buck converter as an illustrative example. Finally, concluding remarks are made in Section 6.

Notation: $\mathbb{R}$, $\mathbb{R}_{\geq 0}$, and $\mathbb{Z}_{\geq 0}$ are used to denote the set of real, nonnegative real, and nonnegative integer numbers, respectively. Moreover $\mathbb{R}^N$ and $\mathbb{R}^{m\times n}$ denote the real-valued N-dimensional column vectors and $m\times n$ matrices, respectively. Further, I_n, 1_n, and $0_{m\times n}$ denote the $n\times n$ identity matrix, $n\times n$ matrix with all elements of 1 and the $m\times n$ zero matrix, respectively. In addition $\forall$ and $\exists$ present the "all" and "there exists", respectively and $\Rightarrow$ is the logical implication. For matrix $M\in\mathbb{R}^{m\times n}$, M^T denotes its transpose and for a square matrix $M\in\mathbb{R}^{n\times n}$, M^{-1}, $tr(M)$, and $\lambda_i(M)$ are inverse, trace, and i^{th} eigenvalue of M, respectively. Moreover $\lambda_{\max}(M)$ and $\lambda_{\min}(M)$ are the largest and smallest eigenvalues of M, respectively. For $x\in\mathbb{R}^n$, $||x||$ denotes the Euclidean vector norm, i.e., $||x|| = (x^Tx)^{1/2}$. Further, $M\prec 0$ and $M\preceq 0$ indicate that M is a negative definite and negative semi-definite matrix, respectively. For the set $\mathcal{I}$, $|\mathcal{I}|$ shows the number of its elements (cardinality). In symmetric matrices, $*$ denotes each of their symmetric blocks. Consider the set $\mathbb{K}=\{1,\dots,N\}$ as a collection of N first positive integer numbers. Then, the convex combination of the matrices $\{M_1,\dots,M_N\}$ is denoted by $M_\lambda=\sum_{i\in\mathbb{K}}\lambda_i M_i$ with $\lambda\in\Lambda$ where $\Lambda := \{\lambda\in\mathbb{R}^N||\lambda_i\geq 0,\sum_{i\in\mathbb{K}}\lambda_i=1\}$ is the unitary simplex.

2. Problem statement

The discrete-time switched affine system is considered as follows:

$$x(k+1)=A_{\sigma(x(k),k)}x(k)+b_{\sigma(x(k),k)},\quad x(0)=x_0,\quad (1)$$

where $k\in\mathbb{Z}_{\geq 0}$ is the discrete-time instant, $x(k)\in\mathbb{R}^n$ is the state, and $\sigma(x(k),k){:}\mathbb{R}^n\times\mathbb{Z}_{\geq 0}\rightarrow\mathbb{K}$ is a switching function, which is continuous from right everywhere, that selects one of the N available subsystems $(A_i,b_i), i\in\mathbb{K}$ at any instant of time $k\in\mathbb{Z}_{\geq 0}$. In case the switching law does not depend on time, i.e., if for each $x_0\in\mathbb{R}^n$, $\sigma(x_0,k_1)=\sigma(x_0,k_2)$, $\forall\ k_1,k_2\in\mathbb{Z}_{\geq 0}$, the signal is said to be state-dependent. This is the class of switching signals to be discussed here. This study also intends to design a state-dependent switching function $\sigma(x(k))$ to apply the asymptotic convergence of state trajectories $x(k), k\in\mathbb{Z}_{\geq 0}$ to the *neighborhood* of a desired equilibrium point for all initial conditions $x_0\in\mathbb{R}^n$. In general, such an equilibrium point does not coincide with any other isolated subsystems, namely, $x_{ei}=(I_n-A_i)^{-1}b_i$. For a desired set point x_e, it is possible to reformulate the stabilization problem around the null equilibrium point by defining the error state vector $e(k)=x(k)-x_e, \forall k\in\mathbb{Z}_{\geq 0}$ that complies with the error dynamics as:

$$e(k+1)=A_{\sigma(e(k))}e(k)+l_{\sigma(e(k))},\quad e(0)=e_0,\quad (2)$$

with $\sigma(e(k))=\sigma(x(k)-x_e)$, $l_i=(A_i-I_n)x_e+b_i, \forall i\in\mathbb{K}$. In addition, this study attempts to design the the switching function $\sigma(e(k))$ using a switched quadratic Lyapunov function $v(e(k))=V_{\sigma(e(k))}=e(k)^TP_ie(k)$, where $P_i=P_i^T\succ 0$ such that the set $\mathcal{V}$ defined as:

$$\mathcal{V}=\bigcap_{i=1}^{N}\mathcal{E}_i,\ \mathcal{E}_i=\{e(k)\in\mathbb{R}^n||e(k)^TP_ie(k)\leq 1\},\quad (3)$$

is an invariant set of attraction for the switched affine system (2) according to the following definition.

Definition 1. *The bounded set $\mathcal{V}\subseteq D$ containing a ball $B_r=\{e\in\mathbb{R}^n|||e||\leq r\}$, $r>0$ is an* invariant set of attraction *in a given domain $D\subset\mathbb{R}^n$ for System (2) by the switching function $\sigma(e(k))$ if the following conditions are simultaneously satisfied:*

(a) $0_{n\times 1}\in\mathcal{V}$,

(b) If $e(k)\in\mathcal{V}$ then $e(k+1)=A_{\sigma(e(k))}e(k)+l_{\sigma(e(k))}\in\mathcal{V}$,

(c) *If $e(0)\in D-\mathcal{V}$, there is a $T=T(e(0))\geq 0$ such that $e(k)\in\mathcal{V}$, $\forall k\geq T$.*

Based on condition (b), the trajectories starting within $\mathcal{V}$ can, never escape from it; therefore, according to (a) and boundedness of $\mathcal{V}$, they will remain around the null set point. In case conditions (a) and (b) are satisfied, the set $\mathcal{V}$ will be *invariant* for System (2) under the switching function $\sigma(e(k))$. Furthermore, according to conditions (a) and (c), the trajectories starting outside $\mathcal{V}$ evolve in time towards the point $e(k)=0_{n\times 1}$, but they will never reach it. In this case, the set $\mathcal{V}$ is claimed to be *attractive* for System (2) under the switching function $\sigma(e(k))$.

Definition 1 falls into the category of practical stability, indicating that the trajectories either tend to the set $\mathcal{V}$ or remain inside it. Definitions 2, 3, and 4 clarify these notions.

Definition 2. *System (2) is* locally practically stable *with respect to an invariant set of attraction $\mathcal{V}$ in the domain D under the switching function $\sigma(e(k))$ if there exist sets $\mathcal{V}$ and D satisfying the conditions of Definition 1 and $\mathcal{V} = D$.*

Definition 3. *System (2) is practically stable in the large with respect to an invariant set of attraction $\mathcal{V}$ in the domain D under the switching function $\sigma(e(k))$ if there exist sets $\mathcal{V}$ and D satisfying conditions of Definition 1 and $\mathcal{V} \subset D$.*

Definition 4. *System (2) is* practically stable in the whole *or is* globally practically stable *if it is* practically stable in the large *and $D = \mathbb{R}^n$.*

Definition 5. *In Definitions 2–4, the set $D - \mathcal{V}$ is referred to as the* domain of attraction *of System (2) under the switching rule $\sigma(e(k))$.*

3. Stability analysis and controller synthesis

This section presents the main results obtained in this study. Lemma 1 determines under what conditions System (2) is *practically stable in the large* in the sense of Definition 3 using Lyapunov functions.

Lemma 1. *System (2) is practically stable in the large in a given domain $D \subset \mathbb{R}^n$ containing the origin in the sense of Definition 3 if there exist a bounded set $\mathcal{V} \subset D$ and a scalar function $v(e(k)) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ such that:*

(a) $0_{n\times 1} \in \mathcal{V}$;

(b) *If $e(k) \in \mathcal{V}$ then $e(k+1) = A_{\sigma(e(k))}e(k) + l_{\sigma(e(k))} \in \mathcal{V}$;*

(c) *If $e(k) \in D - \mathcal{V}$ then $v(e(k+1)) - v(e(k)) = \Delta v(e(k)) \leq -\phi(||e(k)||) < 0$ where $\phi(||e(k)||)$ is a nondecreasing scalar function such that $\phi(0) = 0$ and $\phi(||e(k)||) > 0$, $\forall e(k) \in D - \mathcal{V}$;*

(d) *$v(e(k)) \geq w(||e(k)||) > 0$ where $e(k) \in D - \mathcal{V}$ and $w(||e(k)||)$ is a continuous and nondecreasing scalar function such that $w(0) = 0$.*

Proof. Conditions (a) and (b) are the same as Conditions (a) and (b) of Definition 1, thus fulfilling the invariant property of the bounded set $\mathcal{V}$. To prove the attractiveness property of set $\mathcal{V}$ according to Condition (c) of Definition 1 it is necessary to show that to begin any initial state $e(0) \in D - \mathcal{V}$, there exists a finite time $T = T(e(0)) > 0$ such that for $k \geq T$ the state $e(k)$ eventually enters the set $\mathcal{V}$, i.e., $\exists T > 0$; hence, $e(k) \in \mathcal{V}$ for $k \geq T$. This can be justified within two stages in the following. According to Conditions (c) and (d), since the sequence $v(e(k))$ is decreasing and lower bounded, then based on Weierstrass theorem $\lim_{k\to\infty} v(e(k)) = h \geq 0$. Therefore, the following two cases are introduced.

Case 1. $h > 0$: Our proof in this case is made by contradiction. Assume that the state trajectory $e(k)$ never intersects with $\mathcal{V}$. Now $h > 0$, implies that there exists $\gamma > 0$ such that $||e(k)|| > \gamma, \forall k \in \mathbb{Z}_{\geq 0}$. Then, since $\phi(||e(k)||)$ is a nondecreasing function, we have $v(e(k+1)) - v(e(k)) \leq -\phi(\gamma) < 0$. In view of this, one can write:

$$v(e(k)) = v(e(0)) + \sum_{n=0}^{k-1}(v(e(n+1)) - v(e(n))) \leq v(e(0)) - k\phi(\gamma). \tag{4}$$

The right side of Eq. (4) will be eventually negative when k takes large values. This leads to contradiction against Condition (d) where it is assumed that $v(e(k))$ is positive definite on $D - \mathcal{V}$.

Case 2. $h = 0$: On the other hand, if $h = 0$, according to condition (d), one can write:

$$0 = \lim_{k\to\infty} v(e(k)) \geq \lim_{k\to\infty} w(||e(k)||) \geq 0. \tag{5}$$

Eq. (5) implies that $\lim_{k\to\infty} w(||e(k)||) = 0$. Since $w(||e(k)||)$ is a continuous, nondecreasing, and positive definite function, it can be concluded that $\lim_{k\to\infty} ||e(k)|| = 0$, indicating that $\forall\epsilon > 0$, $\exists T(\epsilon) > 0$ such that in case $k > T(\epsilon)$, $||e(k)|| < \epsilon$. Therefore, there exists a finite time $T = T(\epsilon) > 0$ such that in the case of $k \geq T$, the state $e(k)$ eventually enters the set $\mathcal{V}$. As a result, $\mathcal{V}$ is an invariant set of attraction according to Definition 1. In addition, according to Definition 3, System (2) is practically stable in the large in the domain D under switching function $\sigma(e(k))$. Hence the proof is completed.□

Lemma 1 does not determine how one can choose the bounded set $\mathcal{V}$. In this respect, one approach to selecting $\mathcal{V}$ among all possibilities is to choose the bounded level sets of the function $v(e(k))$ as follows:

$$\mathcal{V} = \{e(k) \in \mathbb{R}^n || v(e(k)) \leq r\}, \tag{6}$$

with $r > 0$. In this regard, an additional condition is required on the function $v(e(k))$ that can guarantee the boundedness of its level set $\mathcal{V}$ defined in Eq. (6). This condition is stated as a growth condition presented in Eq. (7):

$$\lim_{||e(k)||\to\infty} v(e(k)) \to \infty. \tag{7}$$

The function satisfying Relation (7) is *radially unbounded.* According to Condition (7), $\forall r > 0$, $\exists R > 0$ such that:

$$||e(k)|| > R \Rightarrow v(e(k)) > r. \tag{8}$$

A contrapositive statement of Relation (8) is:

$$v(e(k)) \leq r \Rightarrow ||e(k)|| \leq R, \tag{9}$$

indicating that $\mathcal{V} \subset B_R$ where B_R denotes the ball defined as $B_R = \{e(k) \in \mathbb{R}^n | ||e(k)|| \leq R\}$. As a result, the set $\mathcal{V}$ defined in Eq. (6) is bounded.

Remark 1. *Definitions 1–5, Lemma 1 and its proof can be applied to the nonlinear switched systems without a common equilibrium point where the functions $f_i(e(k))$, $i \in \mathbb{K}$ in $e(k+1) = f_i(e(k))$ do not necessarily need to be in the affine form, as in System(2). In this context, just replace condition (b) in Definition 1 and Lemma 1 using the following order: (b) if $e(k) \in \mathcal{V}$ then $e(k+1) = f_{\sigma(e(k))}(e(k)) \in \mathcal{V}$.*

Remark 2. *In Lemma 1, if $D = \mathbb{R}^n$, according to Definition 4, System (2) is globally practically stable.*

Note that the switched Lyapunov function $v(e(k)) = e(k)^T P_{\sigma(e(k))} e(k)$ with $P_i^T = P_i$, $i \in \mathbb{K}$ is a radially-unbounded positive definite function since $v(e(k)) \geq \min_{i\in\mathbb{K}} \lambda_{\min}(P_i)||e(k)||^2$. Moreover, $0_{n\times 1} \in \mathcal{V}$ where the set $\mathcal{V}$ is defined as shown in Eq. (3). It can be verified that the level set of the switched Lyapunov function $v(e(k)) = e(k)^T P_{\sigma_{(e(k))}} e(k)$ is the *union* of the ellipsoids $\mathcal{E}_i = \{e(k) \in \mathbb{R}^n | e(k)^T P_i e(k) \leq r\}$. By dividing both sides of $e(k)^T P_i e(k) \leq r$ into r and then substituting $P_i \leftarrow \frac{P_i}{r}$, the ellipsoids $\mathcal{E}_i$ can be represented as $\mathcal{E}_i = \{e(k) \in \mathbb{R}^n | e(k)^T P_i e(k) \leq 1\}$ with the same notation given in Eq. (3). In the context of practical stability of switched systems, since this study aims to obtain the invariant set of attraction with small size as much as possible [18,19,33,42], the *intersection* of the ellipsoids $\mathcal{E}_i$, instead of their union, is selected, as shown in Eq. (3). Note that since:

$$\bigcap_{i=1}^{N} \mathcal{E}_i \subseteq \bigcup_{i=1}^{N} \mathcal{E}_i, \tag{10}$$

the boundedness of the right side of Relation (10) guarantees that of its left side. Furthermore, it is obvious that $0_{n\times 1} \in \mathcal{V}$ where $\mathcal{V}$ is given in Eq. (3). Therefore, the set $\mathcal{V}$ defined in Eq. (3) is bounded and the switched Lyapunov function $v(e(k)) = e(k)^T P_{\sigma_{(e(k))}} e(k)$ satisfies Conditions (a) and (d) of Lemma 1. In the next subsection, a switching rule and some conditions on matrices $P_i = P_i^T \succ 0$ are proposed such that Conditions (b) and (c) of Lemma 1 are fulfilled for System (2) as well.

3.1. State-dependent switching rule synthesis

As discussed earlier, this study aims to design a switching rule $\sigma(e(k))$ for System (2) that drives the state trajectories towards the set $\mathcal{V}$ defined in Eq. (3) using a switched Lyapunov function $v(e(k)) = V_{\sigma(e(k))} = e(k)^T P_i e(k)$, $P_i = P_i^T \succ 0$. A min-type state feedback switching function is employed in this study shown in the following:

Algorithm 1. *Switching Law:*

1. `Set` $k = 0$.
2. `If the trajectory is outside the set` $\mathcal{V}$ `given in (3), i.e.,` $e(k) \notin \mathcal{V}$, `switch to the subsystem` $\sigma(e(k))$ `given by the following switching function:`

$$(i^*, j^*) = arg \min_{i,j\in\mathbb{K}} [(A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k)]$$

$$\sigma(e(k)) = j^* \tag{11}$$

3. `If the trajectory is inside the set` $\mathcal{V}$ `defined in Eq. (3), i.e.,` $e(k) \in \mathcal{V}$, `first, construct the set` $\mathcal{I} = \{j \in \mathbb{K} : (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) \leq 1, \forall i \in \mathbb{K}\}$. `Now:`
   - 3.1. `If` $|\mathcal{I}| = 1$, `switch to the subsystem` $\sigma(e(k)) = j$, `such that` $j \in \mathcal{I}$.
   - 3.2. `If` $|\mathcal{I}| > 1$, `compute` N `switching indices:`

$$\sigma_i = arg \min_{j\in\mathcal{I}} (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j), \quad \forall i \in \mathbb{K}.$$

   `If` $\sigma_1 = \sigma_2 = \ldots = \sigma_N$, `switch to subsystem` $\sigma(e(k)) = \sigma_j$, $j \in \mathbb{K}$. `Otherwise, choose` $\sigma(e(k)) = arg \min_{j\in\mathcal{I}} tr (P_j^{-1})$.
4. `Put` $k = k + 1$ `and go to item 2.`

Set $\mathcal{I}$ in the above switching algorithm contains all subsystem indices such that the state trajectory $e(k+1)$ will remain in the invariant set of attraction $\mathcal{V}$ by switching to them. In item (3.2), if there is a degree of freedom for the controller to choose suitable subsystems and keep the trajectory inside the set $\mathcal{V}$, the controller selects either the subsystem that governs the state trajectory as close as possible to the null equilibrium point or that which takes the state trajectory closer to the ellipsoid $\mathcal{E}_i = \{e(k) \in \mathbb{R}^n | e(k)^T P_i e(k) \leq 1\}$ with minimum size in terms of the sum of the squares of the ellipsoid semiaxes [42]. Lemmas 2 and 3 are frequently used in the next derivations.

Lemma 2. *For a set of functions $f_i : D \to \mathbb{R}$, $D \subseteq \mathbb{R}^n$, $i \in \mathbb{K}$ as $f_1(x), \ldots, f_l(x), \ldots, f_N(x)$, the following statements are equivalent.*

i) $\forall x \in D$, $\exists l \in \mathbb{K}$, such that $f_l(x) < 0$.

ii) $\forall x \in D$, $\exists(\lambda_1 \geq 0, \ldots, \lambda_l \geq 0, \ldots, \lambda_N \geq 0)$, such that $\sum_{i\in\mathbb{K}} \lambda_i f_i(x) < 0$, and $\sum_{i\in\mathbb{K}} \lambda_i > 0$.

Proof. *i)* $\Rightarrow$ *ii)* Assume that $\forall x \in D \subseteq \mathbb{R}^n$, at least one of the functions, say $f_l(x)$, $l \in \mathbb{K}$ satisfies the inequality $f_l(x) < 0$. Now, by choosing the set of parameters $\lambda_l > 0, \lambda_i = 0, i \neq l$, $i \in \mathbb{K}$, one can conclude $\sum_{i\in\mathbb{K}} \lambda_i f_i(x) = \lambda_l f_l(x) < 0$ and $\sum_{i\in\mathbb{K}} \lambda_i = \lambda_l > 0$.

ii) $\Rightarrow$ *i)* This can be shown by contradiction: Assume that $\exists x \in D \subseteq \mathbb{R}^n$ such that all $f_l(x)$, $l \in \mathbb{K}$ fulfill $f_l(x) \geq 0$. Through multiplication of these inequalities to $\lambda_l \geq 0$, but not all zero (since $\sum_{i\in\mathbb{K}} \lambda_i > 0$), and summation of all the terms one can reach $\sum_{l\in\mathbb{K}} \lambda_l f_l(x) \geq 0$. However, this contradicts the fact that $\forall x \in D \subseteq \mathbb{R}^n$ there exists a set of nonnegative parameters $\lambda_l \geq 0$, $l \in \mathbb{K}$ such that $\sum_{i\in\mathbb{K}} \lambda_i f_i(x) < 0$.□

Lemma 3 is a nonstrict form of Lemma 2.

Lemma 3. *For a set of functions* $f_i : D \to \mathbb{R}$, $D \subseteq \mathbb{R}^n$ $i \in \mathbb{K}$ *as* $f_1(x),\ldots,f_l(x),\ldots,f_N(x)$, *the following statements are equivalent:*

i) $\forall x \in D$, $\exists l \in \mathbb{K}$, *such that* $f_l(x) \geq 0$.

ii) $\forall x \in D$, $\exists(\lambda_1 \geq 0, \ldots, \lambda_l \geq 0, \ldots, \lambda_N \geq 0)$, *such that* $\sum_{i\in\mathbb{K}} \lambda_i f_i(x) \geq 0$, *and* $\sum_{i\in\mathbb{K}} \lambda_i > 0$.

Proof. The proof is similar to that of Lemma 2 and is omitted for the sake of brevity.

Theorem 1 provides sufficient conditions for which the switched affine system (2) under switching function in Algorithm 1 is globally practically stable according to Definition 1.

Theorem 1. *If there exist matrices* $P_i^T = P_i \succ 0$ *and nonnegative numbers* $\beta_{hi} \geq 0$, $\lambda_{hj} \geq 0$, $\lambda_j \geq 0$, $\tau_{1h} \geq 0$, $\tau_{2h} \geq 0$, $i, j, h \in \mathbb{K}$, *such that* $\sum_{i\in\mathbb{K}} \beta_{hi} > 0$, $\sum_{j\in\mathbb{K}} \lambda_{hj} > 0$, $\sum_{j\in\mathbb{K}} \lambda_j > 0$ *satisfying the system of inequalities:*

$$\begin{bmatrix} M_{1i} - \sum_{h\in\mathbb{K}} \tau_{1h} P_h & * \\ M_{2i} & M_{3i} + \sum_{h\in\mathbb{K}} \tau_{1h} \end{bmatrix} \preceq 0, \quad \forall i \in \mathbb{K}, \tag{12}$$

$$\begin{bmatrix} \tau_{2h} P_h + \tilde{M}_{1h} & * \\ \tilde{M}_{2h} & \tilde{M}_{3h} - \tau_{2h} \end{bmatrix} \prec 0, \qquad \forall h \in \mathbb{K}, \tag{13}$$

where:

$$M_{1i} = \sum_{j\in\mathbb{K}} \lambda_j A_j^T P_i A_j, \tag{14}$$

$$M_{2i} = \sum_{j\in\mathbb{K}} \lambda_j l_j^T P_i A_j, \tag{15}$$

$$M_{3i} = \sum_{j\in\mathbb{K}} \lambda_j (l_j^T P_i l_j - 1), \tag{16}$$

$$\tilde{M}_{1h} = \sum_{i\in\mathbb{K}} \sum_{j\in\mathbb{K}} \beta_{hi} \lambda_{hj} (A_j^T P_i A_j - P_j), \tag{17}$$

$$\tilde{M}_{2h} = \sum_{i\in\mathbb{K}} \sum_{j\in\mathbb{K}} \beta_{hi} \lambda_{hj} l_j^T P_i A_j, \tag{18}$$

$$\tilde{M}_{3h} = \sum_{i\in\mathbb{K}} \sum_{j\in\mathbb{K}} \beta_{hi} \lambda_{hj} l_j^T P_i l_j, \tag{19}$$

then, the switching strategy in Algorithm 1 ensures that System (2) is globally practically stable with respect to Definition 4, and that the set $\mathcal{V}$ *in Eq. (3) is an invariant set of attraction in the domain* $D = \mathbb{R}^n$ *with respect to Definition 1.*

Proof. As shown earlier, while matrix Inequalities (12) indicate the invariant property (Conditions (a) and (b) of Definition 1), matrix Inequalities (13) show the attractive property (conditions (a) and (c) of Definition 1). Pre-multiplying Inequality (12) by $[e(k)^T 1]$ and post-multiplying it by $[e(k)^T 1]^T$, one can reach:

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} M_{1i} - \sum_{h\in\mathbb{K}} \tau_{1h} P_h & * \\ M_{2i} & M_{3i} + \sum_{h\in\mathbb{K}} \tau_{1h} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq 0, \forall i \in \mathbb{K}. \tag{20}$$

Relation (20) can be rewritten as Relation (21).

$$-\tau_{11} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} P_1 & * \\ 0_{1\times n} & -1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \ldots$$
$$-\tau_{1h} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} P_h & * \\ 0_{1\times n} & -1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \ldots$$
$$-\tau_{1N} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} P_N & * \\ 0_{1\times n} & -1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}$$
$$+ \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} M_{1i} & * \\ M_{2i} & M_{3i} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq 0, \quad \forall i \in \mathbb{K}. \tag{21}$$

Using S-procedure [43,44], Relation (21) implies Relation (22):

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} P_1 & * \\ 0_{1\times n} & -1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq 0 \wedge, \ldots,$$

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} P_h & * \\ 0_{1\times n} & -1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq 0 \wedge, \ldots,$$

$$\wedge \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} P_N & * \\ 0_{1\times n} & -1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq 0 \Rightarrow$$

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} M_{1i} & * \\ M_{2i} & M_{3i} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq 0,$$

$$\forall i \in \mathbb{K}. \tag{22}$$

By substituting M_{1i}, M_{2i}, and M_{3i} from Eqs. (14)–(16) into Relation (22), one can write:

$$e(k)^T P_1 e(k) \leq 1 \wedge, \ldots, e(k)^T P_h e(k) \leq 1 \wedge, \ldots,$$

$$\wedge e(k)^T P_N e(k) \leq 1 \Rightarrow \sum_{j\in\mathbb{K}} \lambda_j (e(k)^T A_j^T P_i A_j e(k)$$

$$+ l_j^T P_i A_j e(k) + e(k)^T A_j^T P_j l_j + l_j^T P_i l_j - 1) \leq 0,$$

$$\forall i \in \mathbb{K}. \tag{23}$$

Since $\lambda_j \geq 0, j \in \mathbb{K}$ and $\sum_{j\in\mathbb{K}} \lambda_j > 0$, according to Lemma 3, Relation (23) is equivalent to:

$$e(k)^T P_1 e(k) \leq 1 \wedge, \ldots, e(k)^T P_h e(k) \leq 1 \wedge, \ldots,$$

$$\wedge e(k)^T P_N e(k) \leq 1 \Rightarrow \exists j \in \mathbb{K} \text{ such that } e(k)^T$$

$$A_j^T P_i A_j e(k) + l_j^T P_i A_j e(k) + e(k)^T A_j^T P_i l_j$$

$$+ l_j^T P_i l_j - 1 \leq 0, \quad \forall i \in \mathbb{K}. \tag{24}$$

After doing some algebra on the right side of Relation (24), one can reach:

$$e(k)^T P_1 e(k) \leq 1 \wedge, \ldots, e(k)^T P_h e(k) \leq 1 \wedge, \ldots,$$

$$\wedge e(k)^T P_N e(k) \leq 1 \Rightarrow \exists j \in \mathbb{K}, \text{ such that}$$

$$(A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) \leq 1, \qquad \forall i \in \mathbb{K}. \tag{25}$$

Based on Relation (25) and Item (3) of the switching rule in Algorithm 1, one can conclude that $|\mathcal{I}| \neq 0$. As a result, Relation (25) can be rewritten as follows:

$$e(k)^T P_1 e(k) \leq 1 \wedge, \ldots, e(k)^T P_h e(k) \leq 1 \wedge, \ldots,$$

$$\wedge e(k)^T P_N e(k) \leq 1 \Rightarrow \exists \sigma(e(k)) \in \mathbb{K} \text{ such that}$$

$$(A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T P_i (A_{\sigma(e(k))} e(k)$$

$$+ l_{\sigma(e(k))}) \leq 1, \quad \forall i \in \mathbb{K}. \tag{26}$$

Now, through the definition of set $\mathcal{V}$ in Eq. (3), Relation (26) can be rewritten as:

$$e(k) \in \mathcal{V} = \bigcap_{i=1}^{N} \mathcal{E}_i \Rightarrow e(k+1) \in \mathcal{V} = \bigcap_{i=1}^{N} \mathcal{E}_i. \tag{27}$$

Relation (27) implies that the set $\mathcal{V}$ in Eq. (3) is an *invariant* set for the switched affine system (2) under the switching law in Algorithm 1. In the sequel, an attempt is made to prove the attractiveness of the set $\mathcal{V}$ using matrix Inequalities (13). By Pre-multiplying matrix Inequalities (13) by $[e(k)^T 1]$ and post-multiplying them by $[e(k)^T 1]^T$, one can obtain:

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tau_{2h} P_h + \tilde{M}_{1h} & * \\ \tilde{M}_{2h} & \tilde{M}_{3h} - \tau_{2h} \end{bmatrix}$$

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix} < 0, \quad \forall h \in \mathbb{K}. \tag{28}$$

Relation (28) can be rewritten as Relation (29):

$$-\tau_{2h} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P_h & * \\ 0_{1\times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} + \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T$$

$$\begin{bmatrix} \tilde{M}_{1h} & * \\ \tilde{M}_{2h} & \tilde{M}_{3h} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} < 0, \quad \forall h \in \mathbb{K}. \tag{29}$$

Upon using S-procedure, from Relation (29), one can conclude in Relation (30) that $\forall e(k) \in \mathbb{R}^n$:

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P_h & * \\ 0_{1\times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} < 0 \Rightarrow$$

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_{1h} & * \\ \tilde{M}_{2h} & \tilde{M}_{3h} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} < 0,$$

$$\forall h \in \mathbb{K}. \tag{30}$$

By substituting $\tilde{M}_{1h}$, $\tilde{M}_{2h}$, and $\tilde{M}_{3h}$ from Eqs. (17)–(19) into Relation (30), and after doing some algebra, one can reach:

$$e(k)^T P_h e(k) > 1 \Rightarrow \sum_{i\in\mathbb{K}} \beta_{hi} \Bigg(\sum_{j\in\mathbb{K}} \lambda_{hj} [(A_j e(k) + l_j)^T$$

$$P_i (A_j e(k) + l_j) - e(k)^T P_j e(k)] \Bigg) < 0, \quad \forall h \in \mathbb{K}. \tag{31}$$

Since $\beta_{hi} \geq 0$, $i, h \in \mathbb{K}$, and $\sum_{i\in\mathbb{K}} \beta_{hi} > 0$ according to Lemma 2 and Relation (31) one can conclude that:

$$e(k)^T P_h e(k) > 1 \Rightarrow \exists i \in \mathbb{K}, \text{ such that } \sum_{j\in\mathbb{K}} \lambda_{hj}$$

$$[(A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k)]$$

$$< 0, \quad \forall h \in \mathbb{K}. \tag{32}$$

Again, according to Lemma 2, since $\lambda_{hj} \geq 0, j, h \in \mathbb{K}$, and $\sum_{j\in\mathbb{K}} \lambda_{hj} > 0$, Relation (32) implies that:

$$e(k)^T P_h e(k) > 1 \Rightarrow \exists i, j \in \mathbb{K}, \text{such that } (A_j e(k) + l_j)^T P_i(A_j e(k) + l_j) - e(k)^T P_j e(k) < 0, \quad \forall h \in \mathbb{K}. \tag{33}$$

Now, according to the definition of the set $\mathcal{V}$ in Eq. (3), we have:

$$e(k) \notin \mathcal{V} \Rightarrow \exists h \in \mathbb{K} \text{ such that } e(k)^T P_h e(k) > 1. \tag{34}$$

From Relations (33) and (34), one can infer:

$$e(k) \notin \mathcal{V} \Rightarrow \exists i, j \in \mathbb{K}, \text{ such that } (A_j e(k) + l_j)^T P_i(A_j e(k) + l_j) - e(k)^T P_j e(k) < 0. \tag{35}$$

Since $e(k) \notin \mathcal{V}$, according to Relation (5) and item (2) in the switching Algorithm 1, one can conclude that there exists $\sigma(e(k)), i, j \in \mathbb{K}$ satisfying the following expression:

$$(A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T P_i (A_{\sigma(e(k))} e + l_{\sigma(e(k))}) - e(k)^T P_{\sigma(e(k))} e(k) \leq (A_j e(k) + l_j)^T P_i(A_j e(k) + l_j) - e(k)^T P_j e(k). \tag{36}$$

Based on Relations (35) and (36), one can conclude that there exists $\sigma(e(k)), i \in \mathbb{K}$ such that:

$$e(k) \notin \mathcal{V} \Rightarrow \exists i, j, \sigma(e(k)) \in \mathbb{K}, \text{ such that } (A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T P_i (A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))}) - e(k)^T P_{\sigma(e(k))} e(k) \leq (A_j e(k) + l_j)^T P_i(A_j e(k) + l_j) - e(k)^T P_j e(k) < 0, \tag{37}$$

or equivalently:

$$e(k) \notin \mathcal{V} \Rightarrow \exists i, j \in \mathbb{K}, \text{ such that } v(e(k+1)) - v(e(k)) = \Delta v(e(k)) \leq (A_j e(k) + l_j)^T \ P_i(A_j e(k) + l_j) - e(k)^T P_j e(k) < 0. \tag{38}$$

Since $v(e(k)) \geq \min_{i \in \mathbb{K}} \lambda_{\min}(P_i) ||e(k)||^2 > 0$ when $e(k) \neq 0_{n\times 1}$, according to item (d) of Lemma 1, the continuous, nondecreasing, and positive definite scalar function $w(||e(k)||)$ can be taken as $w(||e(k)||) = \min_{i \in \mathbb{K}} \lambda_{\min}(P_i) ||e(k)||^2$. Moreover, according to item (c) of Lemma 1, we still need to find a nondecreasing and positive definite function $\phi(||e(k)||)$ such that $\Delta v(e(k)) \leq -\phi(||e(k)||) < 0$ when $e(k) \in D - \mathcal{V}$. In this regard, we define:

$$\phi_{i,j}(e(k)) = e(k)^T P_j e(k) - (A_j e(k) + l_j)^T P_i(A_j e(k) + l_j), (i, j) \in S, \tag{39}$$

$$S = \{(i, j) \in \mathbb{K} \times \mathbb{K} | \Delta v(e(k)) \leq -\phi_{i,j}(e(k)) < 0\}. \tag{40}$$

According to Relation (38), $|S| \geq 1$ and $\phi_{i,j}(e(k)) > 0$ when $e(k) \notin \mathcal{V}$. Now, we can define the function $\phi(s)$ as:

$$\phi(s) = \inf_{\substack{s \leq |e(k)| \\ s \geq 0 \\ e(k) \notin \mathcal{V}}} \min_{(i,j) \in S} \phi_{i,j}(e(k)). \tag{41}$$

The function $\phi(||e(k)||)$ is nondecreasing and positive definite when $e(k) \notin \mathcal{V}$. Moreover, according to Eq. (41) and Relation (38), one can write the following:

$$\Delta v(e(k)) \leq -\phi_{i,j}(e(k)) \leq -\phi(||e(k)||) < 0, \quad (i, j) \in S. \tag{42}$$

Relation (42) in conjunction with $0_{n\times 1} \in \mathcal{V}$ implies the attractive property of the set $\mathcal{V}$ according to items (a), (c) and (d) of Lemma 1. Therefore, according to Relations (27) and (42), all conditions of Lemma 1 are fulfilled. In the proof of Theorem 1, since no restriction is imposed on the selection of $e(k)$, i.e., $e(k) \in \mathbb{R}^n$, $D = \mathbb{R}^n$, according to Remark 2 the switched affine system (2) is globally practically stable under switching Algorithm 1 and the proof is finally concluded.

Remark 3. *The constraints in Relation (13) are highly non-convex due to the product of variables $\{\beta_{hi}, \lambda_{hi}, P_i\}$ in Eqs. (17)–(19). A more conservative, yet simpler, way to solve the conditions can be obtained by prefixing the variables β_{hi}. To this end, the matrix B whose elements are the parameters $\beta_{hi} \geq 0$, $(h, i) \in \mathbb{K} \times \mathbb{K}$ should be defined. Among all possibilities, one can choose matrix B as a simple form of $B = cI_n$, with $c > 0$. Another alternative is $B = c1_n$, with $c > 0$. Through this approach, all the conditions of Theorem 1 can be regarded as a set of BMIs without introducing the additional parameters β_{hi}.*

Theorem 2 presents another set of BMI conditions for the global practical stability of System (2) under switching Algorithm 1 which is more conservative than those proposed in Theorem 1, However, the new conditions in Theorem 2 belong intrinsically to the BMI form and do not require any prefixing stage of variables as in the case of Theorem 1 to come in the BMI form. Moreover, while the number of unknown variables in the conditions of Theorem 1 is $2N^2+3N+Nn(n+1)/2$, in the conditions of Theorem 2 this number is $2N^2 + 2N + Nn(n+1)/2$. Therefore, the number of unknown parameters in the BMI conditions of Theorem 2 is less than that in the proposed conditions in Theorem 1 by a quantity of N.

Theorem 2. *In case there are matrices $P_i^T = P_i \succ 0$ and nonnegative numbers $\lambda_{hj} \geq 0$, $\lambda_j \geq 0$, $\tau_{1h} \geq 0$,*

$\tau_{2hi} \geq 0$, $i, j, h \in \mathbb{K}$, such that $\sum_{j\in\mathbb{K}} \lambda_{hj} > 0$, $\sum_{j\in\mathbb{K}} \lambda_j > 0$, satisfying the system of inequalities:

$$\begin{bmatrix} M_{1i} - \sum_{h\in\mathbb{K}} \tau_{1h} P_h & * \\ M_{2i} & M_{3i} + \sum_{h\in\mathbb{K}} \tau_{1h} \end{bmatrix} \preceq 0,$$

$$\forall i \in \mathbb{K}, \tag{43}$$

$$\begin{bmatrix} \tau_{2hi} P_h + \tilde{M}_{1hi} & * \\ \tilde{M}_{2hi} & \tilde{M}_{3hi} - \tau_{2hi} \end{bmatrix} \prec 0, \quad \forall i, h \in \mathbb{K}, \tag{44}$$

where:

$$M_{1i} = \sum_{j\in\mathbb{K}} \lambda_j A_j^T P_i A_j, \tag{45}$$

$$M_{2i} = \sum_{j\in\mathbb{K}} \lambda_j l_j^T P_i A_j, \tag{46}$$

$$M_{3i} = \sum_{j\in\mathbb{K}} \lambda_j (l_j^T P_i l_j - 1), \tag{47}$$

$$\tilde{M}_{1hi} = \sum_{j\in\mathbb{K}} \lambda_{hj} (A_j^T P_i A_j - P_j), \tag{48}$$

$$\tilde{M}_{2hi} = \sum_{j\in\mathbb{K}} \lambda_{hj} l_j^T P_i A_j, \tag{49}$$

$$\tilde{M}_{3hi} = \sum_{j\in\mathbb{K}} \lambda_{hj} l_j^T P_i l_j, \tag{50}$$

then the switching strategy in Algorithm 1 ensures that System (2) will be globally practically stable based on Definition 4 and the set $\mathcal{V}$ in Eq. (3) is an invariant set of attraction in the domain $D = \mathbb{R}^n$ based on Definition 1.

Proof. Since the conditions in Relation (43) are the same as Conditions (12) in Theorem 1, the proof that the set $\mathcal{V}$ in Eq. (3) is invariant under the switching Algorithm 1 for System (2) is similar to the first part of the proof of Theorem 1 from Eqs. (20)–(27). In the sequel, the attractive property of the set $\mathcal{V}$ is proved by conditions in Relation (44). By pre-multiplying matrix Inequality (44) by $[e(k)^T 1]$ and post-multiplying it by $[e(k)^T 1]^T$, one can obtain:

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tau_{2hi} P_h + \tilde{M}_{1hi} & * \\ \tilde{M}_{2hi} & \tilde{M}_{3hi} - \tau_{2hi} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} < 0, \quad \forall i, h \in \mathbb{K}. \tag{51}$$

Relation (51) can be rewritten as:

$$-\tau_{2hi} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P_h & * \\ 0_{1\times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} + \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_{1hi} & * \\ \tilde{M}_{2hi} & \tilde{M}_{3hi} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} < 0, \quad \forall i, h \in \mathbb{K}. \tag{52}$$

Using S-procedure, from Relation (52), one can obtain Relation (53), hence $\forall e(k) \in \mathbb{R}^n$.

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P_h & * \\ 0_{1\times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} < 0 \Rightarrow \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_{1hi} & * \\ \tilde{M}_{2hi} & \tilde{M}_{3hi} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T < 0,$$

$$\forall i, h \in \mathbb{K}. \tag{53}$$

By substituting $\tilde{M}_{1hi}$, $\tilde{M}_{2hi}$, and $\tilde{M}_{3hi}$ from Eqs. (48)–(50) into Relation (53) and after doing some algebra, we obtain:

$$e(k)^T P_h e(k) > 1 \Rightarrow \sum_{j\in\mathbb{K}} \lambda_{hj} [(A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k)] < 0, \quad \forall i, h \in \mathbb{K}. \tag{54}$$

Since $\lambda_{hj} \geq 0, j, h \in \mathbb{K}$, and $\sum_{j\in\mathbb{K}} \lambda_{hj} > 0$, according to Lemma 2, Relation (54) implies Relation (55).

$$e(k)^T P_h e(k) > 1 \Rightarrow \exists j \in \mathbb{K}, \text{ such that } (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) < 0,$$

$$\forall i, h \in \mathbb{K}. \tag{55}$$

According to the definition of the set $\mathcal{V}$ in Eq. (3), we have:

$$e(k) \notin \mathcal{V} \Rightarrow \exists h \in \mathbb{K}, \text{ such that } (k)^T P_h e(k) > 1. \tag{56}$$

Now, through Relations (55) and (56), one can reach:

$$e(k) \notin \mathcal{V} \Rightarrow \exists j \in \mathbb{K}, \text{ such that } (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) < 0, \quad \forall i \in \mathbb{K}. \tag{57}$$

Since $e(k) \notin \mathcal{V}$, according to Relation (57) and Item (2) in the switching Algorithm 1, one can conclude that there is a $\sigma(e(k))$ satisfying the following expression:

$$(A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T P_i (A_{\sigma(e(k))} e + l_{\sigma(e(k))}) - e(k)^T P_{\sigma(e(k))} e(k) \leq (A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k), \quad \forall i \in \mathbb{K}. \tag{58}$$

Based on Relations (57) and (58), one can conclude:

$$e(k) \notin \mathcal{V} \Rightarrow \exists j \in \mathbb{K}, \text{such that } (A_{\sigma(e(k))}e(k)+l_{\sigma(e(k))})^T$$

$$P_i(A_{\sigma(e(k))}e(k) + l_{\sigma(e(k))}) - e(k)^T P_{\sigma(e(k))}e(k)$$

$$\leq (A_j e(k) + l_j)^T P_i(A_j e(k) + l_j) - e(k)^T P_j e(k)$$

$$< 0, \quad \forall i \in \mathbb{K}, \tag{59}$$

or equivalently:

$$e(k) \notin \mathcal{V} \Rightarrow \exists j \in \mathbb{K}, \text{ such that } v(e(k+1))v(e(k))$$

$$= \Delta v(e(k)) \leq (A_j e(k) + l_j)^T P_i(A_j e(k) + l_j)$$

$$-e(k)^T P_j e(k) < 0, \quad \forall i \in \mathbb{K}. \tag{60}$$

Similar to our argument in Theorem 1, it can be shown that there exist functions $w(||e(k)||)$ and $\phi(||e(k)||)$ that satisfy the respective properties in Lemma 1. This discussion is omitted here for the sake of brevity. Therefore, the attractiveness property of the set $\mathcal{V}$ is inferred according to items (a), (c), and (d) of Lemma 1. Therefore, all conditions for Lemma 1 are fulfilled. Similar to Theorem 1, since no restriction is imposed on the selection of $e(k)$, namely $e(k) \in \mathbb{R}^n$, we have $D = \mathbb{R}^n$, and according to Remark 2, the switched affine system (2) is globally practically stable under switching Algorithm 1, thus completing the proof.

Remark 4. *Theorem 1 is less conservative than Theorem 2. As observed in Relation (37) in Theorem 1, at any time, the decreasing condition of Lyapunov function, i.e.,* $\Delta v(e(k)) < 0$, *is imposed at least on one of the matrices* P_i, $i \in \mathbb{K}$, *while according to Relation (59) in Theorem 2, it is applied to all matrices* P_i, $i \in \mathbb{K}$.

4. Minimization of the invariant set of attraction

As already discussed in Section 3, the author intends to select the invariant set of attraction $\mathcal{V}$ defined in Eq. (3) to minimize its size based on some standard sense. As observed in Eq. (3), the formation of this set results from the intersection of N ellipsoids $\mathcal{E}_i = \{e \in \mathbb{R}^n | |e(k)^T P_i e(k) \leq 1\}$. Therefore, one approach to minimizing the size of the set $\mathcal{V}$ is to minimize the size of all ellipsoids $\mathcal{E}_i$, $i \in \mathbb{K}$. $\mathcal{E}_i$, and to do so, one can minimize the trace of the matrix P_i^{-1} that defines the sum of the squares of the ellipsoid $\mathcal{E}_i$ semiaxes [42]. To this end, this study employs the following optimization problem:

$$\inf_{P_i,\lambda_j,\lambda_{jh},\tau_{1h},\tau_{2h}} \sum_{i=1}^{i=N} tr(P_i^{-1}),$$

$$\text{subject to } [(12)-(13)] \text{ or } [(43)-(44)]$$

$$\lambda_j \geq 0, \quad \lambda_{jh} \geq 0, \quad \tau_{1ih} \geq 0, \quad \tau_{2ih} \geq 0. \tag{61}$$

Since both of the tool YALMIP/MATLAB [45] and the BMI solver PENBMI [46] are not able to handle the nonlinear terms $tr(P_i^{-1})$ in the objective function of (61), an upper bound $P_i^{-1} \preceq t_i I_n$ with $t_i > 0$, $P_i \succ 0$ $i \in \mathbb{K}$ is considered and rewritten as $I_n - t_i P_i \preceq 0$ with $t_i > 0$, $P_i \succ 0$, $i \in \mathbb{K}$. Accordingly, Relation (61) is replaced by the following optimization problem:

$$\inf_{P_i,\lambda_j,\lambda_{jh},\tau_{1h},\tau_{2h},t_i} \sum_{i=1}^{i=N} t_i,$$

$$\text{subject to } [(12)-(13)] \text{ or } [(43)-(44)]$$

$$\lambda_j \geq 0, \quad \lambda_{jh} \geq 0, \quad \tau_{1ih} \geq 0, \quad \tau_{2ih} \geq 0$$

$$I_n - t_i P_i \preceq 0, \quad t_i > 0, \quad P_i = P_i^T \succ 0. \tag{62}$$

In this study, the optimization problem in Relation (62) is solved using the BMI solver PENBMI [46] interfaced by YALMIP [45].

In the numerical simulations described in the next section, it was observed that in some cases, the optimization problem in Relation (62) was still too nonlinear and non-convex to obtain suitable and acceptable results from the PENBMI package. In this regard, additional constraints were added to the optimization problem in Relation (62) to put a limit on the search space and obtain desirable results from the solver, even at the expense of falling into conservatism. These additional constraints are suggested as follows:

$$\sum_{j\in\mathbb{K}} \lambda_{hj} = q_h, \quad \forall h \in \mathbb{K}, \tag{63}$$

$$\sum_{i\in\mathbb{K}} \lambda_j = q, \tag{64}$$

with $q_h, q > 0$. By taking the limited and reasonable values of the parameters q_h and q, one can control and limit the regions of non-negative variables λ_{hj} and λ_j in the conditions of Theorems 1 and 2. Of note, even after adding new constraints in Eqs. (63) and (64) to the conditions of Theorems 1 and 2, these theorems are still valid. The proof of this claim is derived from Lemmas 2 and 3 where the constraint $\sum_{i\in\mathbb{K}} \lambda_j > 0$ is implied by more restrictive constraints in Eqs. (63) and (64).

5. Application

A DC-DC buck converter is shown in Figure 1. The

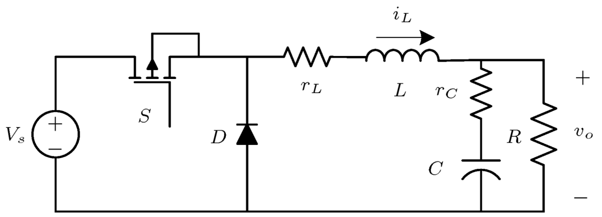

Figure 1. DC-DC buck converter.

state vector is defined as $x(t) = [i_L(t)\ v_o(t)]^T$, through which the continuous dynamics in Continuous Conduction Mode (CCM) associated with each mode are obtained as $\dot{x}(t) = A_{ci}x(t) + b_{ci}$, $i \in \{1, 2\}$ where:

$$A_{c1} = \begin{bmatrix} -\frac{r_L}{L} & -\frac{1}{L} \\ \frac{1}{C}\frac{R}{R+r_C}\left(1 - Cr_C\frac{r_L}{L}\right) & -\frac{1}{C}\frac{1}{R+r_C}\left(1 + Cr_C\frac{R}{L}\right) \end{bmatrix},$$

$$b_{c1} = \begin{bmatrix} \frac{V_s}{L} \\ V_s\frac{R}{R+r_C}\frac{r_C}{L} \end{bmatrix}, \tag{65}$$

$$A_{c2} = A_{c1}, \quad b_{c2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

In Eq. (65), $V_s = 50$ V is the input DC voltage, $R = 50\ \Omega$ the load resistance, $L = 2$ mH the inductor inductance, $r_L = 0.5\ \Omega$ the inductor resistance, $C = 100\ \mu$F the output capacitor capacitance, and $r_c = 0.1\ \Omega$ the equivalent series resistance of the capacitor [47]. In order to avoid ill-conditioned matrix inequalities and make the problem more amenable for the numerical purposes in the numerical simulations, the per-unit parameters [34,48] are employed. In this regard, the base values for the chosen per-unit system are $v_{base} = 50$ V, $i_{base} = 2.5$ A and $T_{base} = 10\ \mu$s. The sampling time is set to $T_s = 10\ \mu$s and consequently, the maximum value of the switching frequency is limited to $\frac{1}{2T_s} = 50$ kHz. The state-space matrices of the corresponding discrete-time system can be obtained as follows:

$$A_i = e^{A_{ci}T_s}, \quad b_i = \int_0^{T_s} e^{A_{ci}t}dt b_{ci}, \tag{66}$$

where $i \in \{1, 2\}$. Although the equilibrium point x_e can be arbitrarily chosen at the expense of obtaining the set $\mathcal{V}$ with a possibly greater size, in this study, the desired target was selected as the equilibrium point of the averaged system as $x_e = (I_n - A_{\lambda_{ref}})^{-1}b_{\lambda_{ref}} = [0.2970, 0.7426]^T$ corresponding to $\lambda_{ref} = 0.75$. Through Conditions (12) and (13) of Theorem 1, the solution of the optimization problem in Relation (62) corresponding to $B = 15\ 1_2$ can be obtained, considering the new constraint in Eq. (64), as $\lambda_1 + \lambda_2 = 6$ yields $t_1 = 0.0920$, $t_2 = 0.2066$, $\lambda_{11} = 1.0184E-4$, $\lambda_{12} = 0.0381E-4$, $\lambda_{21} = 1.4846E+7$, $\lambda_{22} = 2.0762E+7$, $\lambda_1 = 4.4980$, $\lambda_2 = 1.4993$, $\tau_{11} = 5.9862$, $\tau_{12} = -0.0826$, $\tau_{21} = -0.9141E-4$, $\tau_{22} = 1.5199E+7$ with the matrices P_1 and P_2 as:

$$P_1 = \begin{bmatrix} 10.8685 & -2.9654 \\ -2.9654 & 216.8589 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 4.9096 & -0.3452 \\ -0.3452 & 6.3089 \end{bmatrix}. \tag{67}$$

As can be observed, some of the variables have small deviations from the theoretical constraints; however, these results are still acceptable in practice, as confirmed by the numerical experiments. The invariant set of attraction as well as the state trajectories $x(k)$ corresponding to different initial conditions are presented in Figure 2. Though rare in practice, to show the wide range of the attractive region and validity of the obtained numerical solutions, the initial conditions are selected quite far from the equilibrium point x_e. Figure 3 illustrates an enlarged view of Figure 2 around

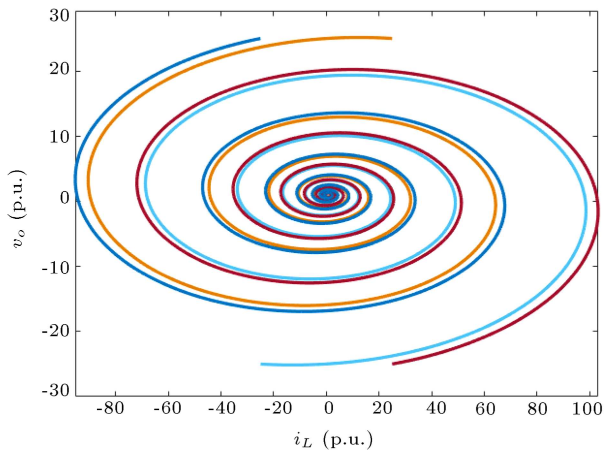

Figure 2. State trajectories corresponding to conditions of Theorem 1 and $B = 15\ 1_2$.

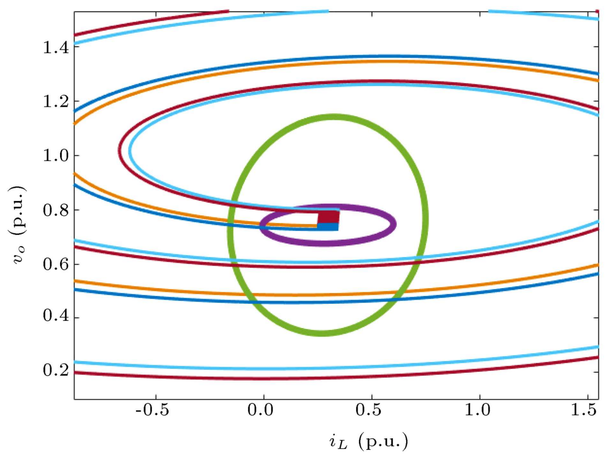

Figure 3. An enlarged view of Figure 2 around the attractive ellipsoids.

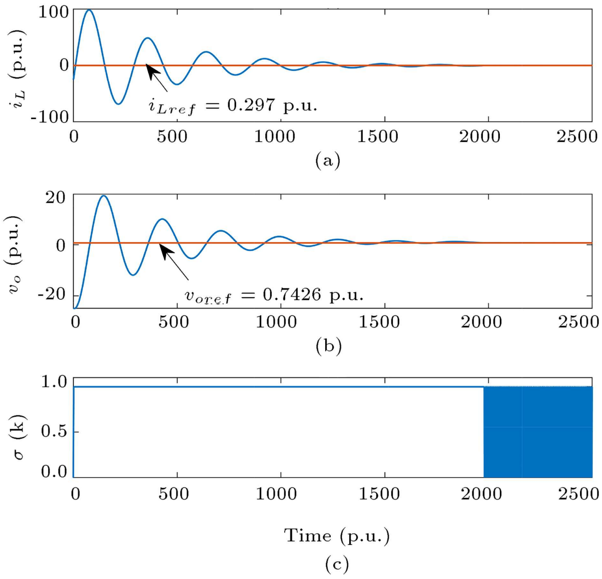

Figure 4. Time trajectories corresponding to the initial condition $i_L(0) = v_o(0) = -25 p.u.$: (a) Inductor current, (b) output voltage, and (c) switching function.

the attractive ellipsoids. Figure 4 shows the time trajectories of the inductor current, output voltage, and switching function corresponding to the initial condition $i_L(0) = v_o(0) = -25$ $p.u.$.

The optimization problem in Relation (62) corresponding to the proposed stability conditions in Theorem 2 and the additional constraints in Eqs. (63) and (64) as $\lambda_1+\lambda_2 = 1$, $\lambda_{11}+\lambda_{12} = 10$, and $\lambda_{21}+\lambda_{22} = 10$ yields $t_1 = t_2 = 0.3755$, $\lambda_{11} = \lambda_{21} = 7.5000$, $\lambda_{12} = \lambda_{22} = 2.4999$, $\lambda_1 = 0.7500$, $\lambda_2 = 0.2499$, $\tau_{11} = 0.2410$, $\tau_{12} = 0.7539$, $\tau_{211} = \tau_{212} = \tau_{221} = \tau_{222} = 0.0498$ with matrices P_1 and P_2 as:

$$P_1 = P_2 = \begin{bmatrix} 2.6627 & 0.0013 \\ 0.0013 & 53.4956 \end{bmatrix}. \qquad (68)$$

Compared to the results obtained by the conditions of Theorem 1, the stability conditions of Theorem 2 yield more conservative results regarding the smallest size of the ultimate invariant set of attraction. This can be realized by the comparison of the values t_1 and t_2 obtained corresponding to the conditions provided by these two theorems. Figures 5 and 6 illustrate the state trajectories of the converter under the stability conditions of Theorem 2.

Figure 7 illustrates the state trajectories starting from the null initial conditions and invariant set of attractions corresponding to the conditions of the Theorems 1 and 2. Although the size of the ultimate invariant set of attraction estimated by Theorem 1 was less conservative, the results obtained from the stability conditions of Theorem 2 were more desirable in terms of performance.

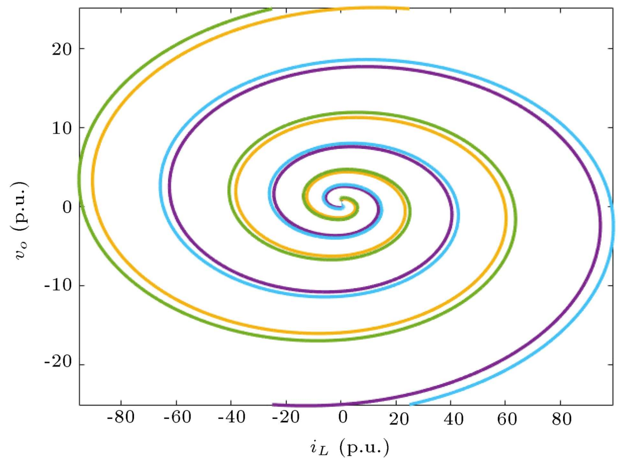

Figure 5. State trajectories corresponding to conditions of Theorem 2.

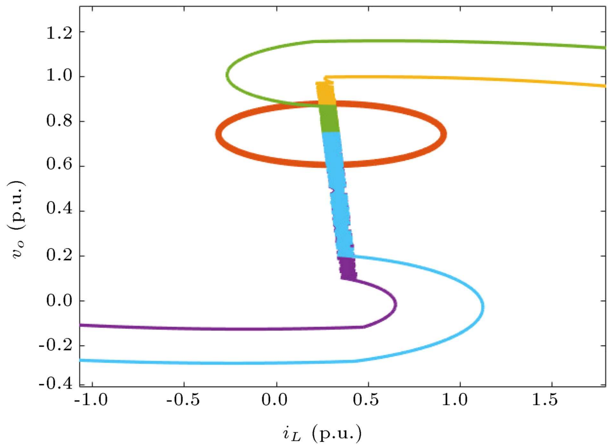

Figure 6. An enlarged view of Figure 5 around the attractive ellipsoids.

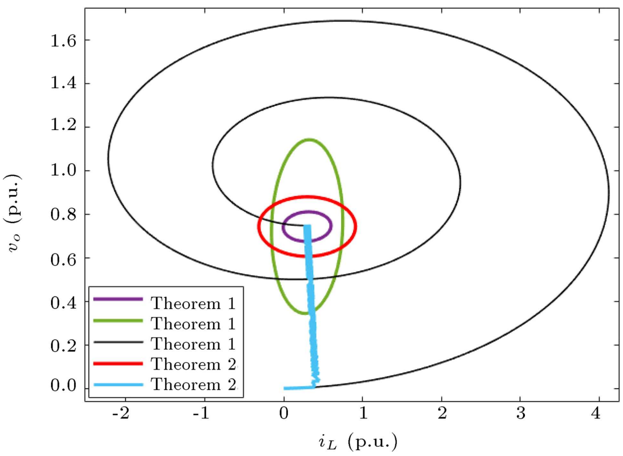

Figure 7. State trajectories corresponding to conditions of Theorems 1 and 2.

6. Conclusion

In this study, a switched Lyapunov function approach was proposed for the global practical stability analysis and controller synthesis of discrete-time switched affine systems. The proposed conditions were BMI-based conditions that could ensure both invariance and attractive properties of the ultimate convergence set simultaneously. Two theorems were suggested with two different sets of sufficient conditions. In the first theorem, the proposed conditions were not in the Bilinear Matrix Inequality (BMI) form; thus, prefixing of variables was recommended to convert them to the standard BMI form. The stability conditions in the second theorem were intrinsically in the BMI form, yet more conservative. The proposed design and stabilization method were successfully investigated on a DC-DC buck converter.

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References

1. *Handbook of Hybrid Systems Control*, J. Lunze and F. Lamnabhi-Lagarrigue, Eds., Cambridge University Press (2009).

2. Sun, Z. and Sam Ge, S., *Switched Linear Systems, Control and Design*, E.D. Sontag, M. Thoma, A. Isidori and J. H. van Schuppen, Eds., Springer-Verlag London (2005).

3. Zhang, L., Zhu, Y., Shi, P., et al., *Time-dependent Switched Discrete-time Linear Systems: Control and Filtering*, J. Kacprzyk, Ed., Springer International Publishing Switzerland (2016).

4. Liberzon, D., *Switching in Systems and Control*, T. Basar, Bikhauser Boston (2003).

5. Zhao, W., Kao, Y., Niu, B., et al., *Control Synthesis of Switched Systems*, Springer International Publishing Switzerland (2017).

6. Sun, Z. and Sam Ge, S., *Stability Theory of Switched Dynamical Systems*, A. Isidori, J.H. van Schuppen, E.D. Sontag, M. Thoma and M. Krstic, Eds., Springer-Verlag London (2011).

7. Liberzon, D. and Morse, A.S. "Basic problems in stability and design of switched systems", *IEEE Control Syst. Mag.*, **19**(5), pp. 59–70 (1999).

8. Hai Lin, H. and Antsaklis, P.J. "Stability and stabilizability of switched linear systems: a survey of recent results", *IEEE Trans. Autom. Control*, **54**(2), pp. 308–322 (2009).

9. Decarlo, R.A., Branicky, M.S., Pettersson, S., et al. "Perspectives and results on the stability and stabilizability of hybrid systems", *Proceedings of the IEEE*, **88**(7), pp. 1069–1082 (2000).

10. Shorten, R., Wirth, F., Mason, O., et al. "Stability criteria for switched and hybrid systems", *SIAM Review*, **49**(4), pp. 545–592 (2007).

11. Deaecto, G.S., Geromel, J.C., Garcia, F.S., et al. "Switched affine systems control design with application to DC-DC converters", *IET Control Theory A.*, **4**(7), pp. 1201–1210 (2010).

12. Baldi, S., Papachristodoulou, A., and Kosmatopoulos, E.B. "Adaptive pulse width modulation design for power converters based on affine switched systems", *Nonlinear Anal-Hybri.*, **30**, pp. 306–322 (2018).

13. Yoshimora, V.L., Assuncao, E., Pires da Silva, E.R., et al. "Observer-based control design for switched affine systems and applications to DCDC converters", *Journal of Control, Automation and Electrical Systems*, **24**(4), pp. 535–543 (2013).

14. Corona, D., Buisson, J., De Schutter, B., et al. "Stabilization of switched affine systems: An application to the buck-boost converter", *Proceedings of American Control Conf.*, New York, pp. 6037–6042 (2007).

15. Albea, C., Garcia, G., and Zaccarian, L. "Hybrid dynamic modeling and control of switched affine systems: application to DC-DC converters", *IEEE 54th Annual Conf. on Decision and Control*, Osaka, Japan, pp. 2264–2269 (2015).

16. Beneux, G., Riedinger, P., Daafouz, J., et al. "Adaptive stabilization of switched affine systems with unknown equilibrium points: application to power converters", *Automatica*, **99**, pp. 82–91 (2019).

17. Hejri, M., Giua, A., and Mokhtari, H. "On the complexity and dynamical properties of mixed logical dynamical systems via an automaton-based realization of discrete-time hybrid automaton", *Int. J. of Robust Nonlin.*, **28**(16), pp. 4713–4746 (2018).

18. Deaecto, G.S. and Geromel, J.C. "Stability analysis and control design of discrete-time switched affine systems", *IEEE Trans. Autom. Control*, **62**(8), pp. 4058–4065 (2017).

19. Egidio, L.N. and Deaecto, G.S. "Novel practical stability conditions for discrete-time switched affine systems", *IEEE Trans. Autom. Control*, **64**(11), pp. 4705–4710 (2019).

20. Albea Sanchez, C., Garcia, G., Sabrina, H., et al. "Practical stabilisation of switched affine systems with dwell-time guarantees", *IEEE Trans. Autom. Control*, **64**(11), pp. 4811–4817 (2019).

21. Zhai, G. and Michel, A.N. "On practical stability of switched systems", *Proceedings of the 41st IEEE Conf. on Decision and Control*, **3**, pp. 3488–3493 (2002). DOI: 10.1109/CDC.2002.1184415

22. Zhai, G. and Michel, A.N. "Generalized practical stability analysis of discontinuous dynamical systems", *42nd IEEE International Conf. on Decision and Con-*

trol (IEEE Cat. No.03CH37475), **2**, pp. 1663–1668 (2003). DOI: 10.1109/CDC.2003.1272851

23. Xu, X., Zhai, G., and He, S. "Some results on practical stabilizability of discrete-time switched affine systems", *Nonlinear Anal-Hybri.*, **4**(1), pp. 113–121 (2010).

24. Xu, X., Zhai, G., and He, S. "On practical stabilizability of discrete-time switched affine systems", *Joint 48th IEEE Conf. on Decision and Control*, Shanghai, China, pp. 2144–2149 (2009).

25. Xu, X., Zhai, G., and He, S. "On practical asymptotic stabilizability of switched affine systems", *Nonlinear Anal-Hybri.*, **2**(1), pp. 196–208 (2008).

26. Xu, X. and Zhai, G. "Practical stability and stabilization of hybrid and switched systems", *IEEE Trans. Autom. Control*, **50**(11), pp. 1897–1903 (2005).

27. Xu, X., Zhai, G., and He, S. "Stabilizability and practical stabilizability of continuous-time switched systems: a unified view", *Proceedings of the 2007 American Control Conf.*, New York City, USA, pp. 663–668 (2007).

28. Trofino, A., Assmann, D., Scharlau, C.C., et al. "Switching rule design for switched dynamic systems with affine vector fields", *IEEE Trans. Autom. Control*, **54**(9), pp. 2215–2222 (2009).

29. Scharlau, C.C., de Oliveria, M.C., Trofino, A., et al. "Switching rule design for affine switched systems using a max-type composition rule", *IEEE Trans. Autom. Control*, **68**, pp. 1–8 (2014).

30. Daafouz, J., Riedinger, P., and Iung, C. "Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach", *IEEE Trans. Autom. Control*, **47**(11), pp. 1883–1887 (2002).

31. Branicky, M.S. "Multiple Lyapunov functions and other analysis tools for switched and hybrid systems", *IEEE Trans. Autom. Control*, **43**(4), pp. 475–482 (1998).

32. Kuiava, R., Ramos, R.A., Pota, H.R., et al. "Practical stability of switched systems without a common equilibria and governed by a time-dependent switching signal", *Eur. J. Control*, **19**(3), pp. 206–213 (2013).

33. Perez, C., Azhmyakov, V., and Poznyak, A. "Practical stabilization of a class of switched systems: dwell-time approach", *IMA J. of Math. Control I.*, **32**(4), pp. 689–702 (2015).

34. Hetel, L. and Fridman, E. "Robust sampled-data control of switched affine systems", *IEEE Trans. Autom. Control*, **58**(11), pp. 2922–2928 (2013).

35. Senesky, M., Eirea, G., and Koo, T.J. "Hybrid Modeling and control of power electronics", *Hybrid Systems: Computations and Control, Lecture Notes in Computer Science*, Springer, pp. 450–465 (2003).

36. Albea Sanchez, C., Lopez Santos, O., Zambrano Prada, D.A., et al. "On the practical stability of hybrid control algorithm with minimum dwell time for a DC-AC converter", *IEEE Trans. Control Syst. Technol.*, **27**(6), pp. 2581–2588 (2019).

37. Blondel, V. and Tsitsiklis, J.N. "NP-Hardness of some linear control design problems", *SIAM J. Control and Optim.*, **35**(6), pp. 2118–2127 (1997).

38. Wicks, M., Peleties, P., and DeCarlo, R. "Switched controller synthesis for the quadratic stabilization of a pair of unstable linear systems", *Eur. J. Control*, **4**(2), pp. 140–147 (1998).

39. Trofino, A., Scharlau, C.C., and Coutinho, D.F. "Corrections to "Switching rule design for switched dynamic systems with affine vector fields"", *IEEE Trans. Autom. Control*, **57**(4), pp. 1080–1082 (2014).

40. Deaecto, G.S. "Dynamic output feedback H_∞ control of continuous-time switched affine systems", *Automatica*, **71**, pp. 44–49 (2016).

41. Hauroigne, P., Riedinger, P., and Iung, C. "Switched affine systems using sampled-data controllers: robust and guaranteed stabilization", *IEEE Trans. Autom. Control*, **56**(12), pp. 2929–2935 (2011).

42. Poznyak, A., Polyakov, A., and Azhmyakov, V., *Attractive Ellipsoids in Robust Control*, T. Basar, Ed., Birkhauser (2014).

43. Boyd, S., El Ghaoui, L., Feron, E., et al., *Linear Matrix Inequalities in Systems and Control Theory*, Society for Industrial and Applied Mathematics, SIAM (1994).

44. VanAntwerp, J.G. and Braatz, R.D. "A tutorial on linear and bilinear matrix inequalities", *J. Process Contr.*, **10**(4), pp. 365–385 (2000).

45. Lofberg, J. "YALMIP: A toolbox for modeling and optimization in MATLAB", *IEEE International Symposium on Computer Aided Control Systems Design*, Taipei, Taiwan, pp. 284–289 (2004).

46. Kocvara, M. and Stingl, M. "PENBMI Users Guide (Version 2.1)", www.penopt.com (2006).

47. Mariéthoz, S., Almér, S., Bâja, M. et al. "Comparison of hybrid control techniques for buck and boost DC-DC converters", *IEEE Trans. Control Syst. Technol.*, **18**(5), pp. 1126–1145 (2010).

48. Deaecto, G.S. and Santos, G.C. "State feedback H_∞ control design of continuous-time switched-affine systems", *IET Control Theory A.*, **9**(10), pp. 1511–1516 (2014).

Biography

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