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Importance of activation energy and heat source on nanoliquid flow with gyrotactic microorganisms

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Abstract. This article addresses salient features of gyrotactic microorganisms and activation energy in the flow of nanofluid by rotating disks. An ESHS process is implemented to examine the thermal transport characteristics. Additionally, the nanoparticle mass flux condition is considered and the solutions are numerically computed. Impacts of various physical variables appearing in the solutions of non-linear systems are carefully analyzed. The current work identifies that temperature distribution of nanoliquid enhances via thermophoresis and Brownian diffusion variables. Moreover, activation energy and temperature difference parameters diminish nanoparticle concentration. Comparative study is provided in order to validate the outcomes.

1. Introduction

Analysis of nanofluids has received special attention in recent research. Such interest is stimulated by the fact that nanotechnology is now considered a noteworthy factor that influences the modern insurgency of the present century. In the previous couple of years, numerous scientists have been concentrating on demonstrating thermal conductivity and looking at the changed viscosities of nano-liquids. There is no doubt that heat transfer liquids like water and minerals, ethylene glycol and oil play important roles in many industrial sites, including chemical production, air conditioning, power generation, microelectronics, transportation, food, pharmaceuticals, etc. It is also recognized fact that most heat transfer liquids, for example, motor oil, ethylene glycol and water, have constrained/poor capabilities in terms of thermal features which thus create limitations in thermal procedures. On the other hand, metals are considered to have a thermal performance three times higher than these liquids. Various studies have been undertaken for improvement of the thermal conductivity of such conventional heat transfer liquids. A creative technique for raising the heat exchange of base liquids is to drop small particles into it [1]. Commonly, the nanoparticles are compacts of oxides (titania, alumina and carbides, copper oxide,) and metals including (gold and copper). Carbon nanotubes and diamond are also used in nano-liquids. The base liquids include ethylene glycol, oil, water, biofluids, some lubricants and polymer solutions. The scope of nanoliquid can be noted in many biomedical and engineering systems, such as computers, nuclear reactors, car engines, cancer therapy, solar energy, radiators, safer surgeries, safety issues emerging in nuclear reactors and X-rays. Moreover, nanoliquids can be used as a cooling agent in micro machines in micro reactors,

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cars, airplanes, CPU etc. The most important characteristics of such liquids are better spreading, sufficient viscosity, stability, dispersion and wetting on solid surfaces [2]. The addition of nanoparticles may result in a coefficient of heat transportation diminution but literature reveals the improvement in heat transport. Numerous mechanisms for illustration of thermal diffusion, micro-convection, thermophoretics, Brownian movement, interface of nanoparticles, and particle to particle coupling have been elaborated to illustrate the improvement regarding heat transport. Some remarkable endeavors in this area can be seen by refs. [3–35].

A reaction that releases energy initially needs some quantity of energy to achieve its energy releasing levels. This initial input energy, which precedes the reaction, is referred to as activation energy. Activation energy was introduced by Svante Arrhenius. Such energy has a noteworthy association in regard to assessment of a reaction [36]. Geothermal reservoirs, thermal oil recovery, chemical engineering, nuclear reactor chilling etc. are some examples of combined chemical reaction and activation energy. Initially, Bestman [37] addressed the simultaneous aspect of activation energy, along with chemical reaction, for convective mass transport analysis. Awad et al. [38] addressed unsteady stretched flow in a rotating liquid with activation energy and chemical reactions. Features of activation energy and chemical reactions in unsteady convective flow bounded by a radiated surface are scrutinized by Makinde et al. [39], who discussed the features of activation energy in radiated flow. Mixed convection radiated flow accounting for activation energy is studied by Maleque [40]. The flow of Maxwell materials via a surface subject to activation energy is explored by Mustafa et al. [41]. Abbas et al. [42] studied the impact of activation energy in chemically reacted Casson liquid flow. Recently, Zeeshan et al. [43] and Archana and Mahanthesh [44] numerically explored the behavior of activation energy in flow by considering Casson liquid and viscous models.

The objective of this paper is to analyze bioconvection flow comprised of both microorganisms and nanoparticles. A rotating disk with power-law stretching is used. A Buongiorno model has been implemented and zero mass flux condition is imposed. Further combined aspects of exponential heat source, binary chemical reactions and activation energy are addressed. The governing expressions are reduced to a set of nonlinear boundary layer problems. The numerical study is performed using the NDSolve technique [45,46] and the behavior of active variables are sketched via graphs. Tables are presented to reflect the computational analysis of skin frictions and Nusselt number. Moreover, a comparative table is also shown to justify the current outcomes with previous analyses. Major outcomes are listed in the conclusions.

2. Mathematical development

The flow, temperature, microorganism and concentration distributions are governed by the continuity expression, momentum expression, energy and mass transport expressions with exponential space dependent internal heat source and activation energy (see Figure 1). The following assumptions are made for the governing problem.

(i) Viscous nanomaterials are used;

(ii) Flow is steady, incompressible and two dimensional;

(iii) The nanoparticles have uniform size and shape;

(iv) There is relative movement between nanoparticles and regular liquid;

(v) Nanoparticles have no impact on the velocity of microorganisms and swimming direction;

(vi) The suspended nanoparticles are stable and do not accumulate in the liquid;

(vii) Thermophoretic and Brownian diffusions are present.

The problem statement using the above assumptions leads to the following equations [47–50]:

$$\frac{\partial(ru)}{\partial r}+\frac{\partial(rw)}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=\nu\frac{\partial^2 u}{\partial z^2}, \tag{2}$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\nu\frac{\partial^2 v}{\partial z^2}, \tag{3}$$

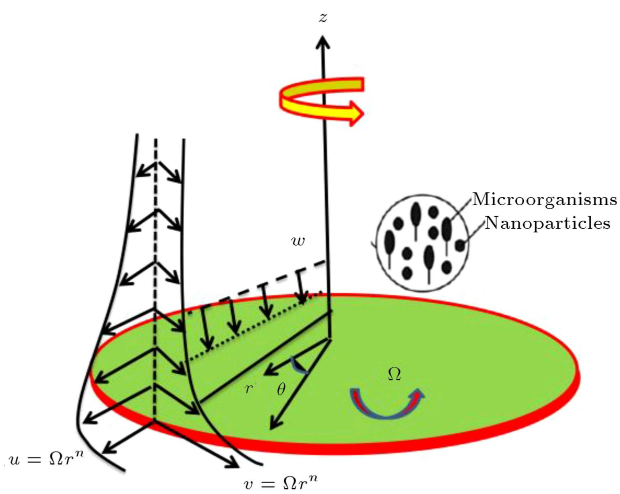

Figure 1. Physical sketch.

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha_f\frac{\partial^2 T}{\partial z^2}+\tau\left(D_B\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2\right)$$
$$+\frac{Q_0(T-T_\infty)}{(\rho c_p)_f}e^{-zn_1\sqrt{\frac{\Omega}{\nu}}r^{\frac{n-1}{2}}}, \tag{4}$$

$$u\frac{\partial C}{\partial r}+w\frac{\partial C}{\partial z}=D_B\left(\frac{\partial^2 C}{\partial z^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial z^2}\right)$$
$$-k_r^2(C-C_\infty)\left(\frac{T}{T_\infty}\right)^m\exp\left(\frac{-E_a}{\kappa T}\right), \tag{5}$$

$$u\frac{\partial N}{\partial r}+w\frac{\partial N}{\partial z}=-\frac{bW_c}{\Delta C}\left[N\frac{\partial^2 C}{\partial z^2}+\frac{\partial C}{\partial z}\frac{\partial N}{\partial z}\right]$$
$$+D_n\frac{\partial^2 N}{\partial z^2}, \tag{6}$$

$$u=aru_w(r)=ar^n,\qquad v=\Omega ru_w(r)=\Omega r^n,$$

$$w=0,\qquad T=T_w,\qquad D_B\frac{\partial C}{\partial z}+\frac{D_T}{T_\infty}\frac{\partial T}{\partial z}=0,$$

$$N=N_w\quad\text{at}\quad z=0, \tag{7}$$

$$u\to 0,\qquad v\to 0,\qquad T\to T_\infty,\qquad C\to C_\infty,$$

$$N\to N_\infty\quad\text{as}\quad z\to\infty. \tag{8}$$

Here, (u,v,w) are the velocity components in (r,φ,z); ν the kinematic viscosity, $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ the heat capacity ratio; n_1 the exponential index; α_f the thermal diffusivity of nanoliquid; ρ the liquid density; Q_0 heat generation/absorption variable; E_a the non-dimensional activation energy; D_B the diffusion coefficient; D_T the coefficient of thermophoretic diffusion; b the chemotaxis constant; (T,T_w,T_∞) fluid, surface and ambient temperatures, fluid, surface and ambient concentrations are (C,C_w,C_∞); the concentrations of microorganisms are (N,N_w,N_∞); D_n the diffusivity of microorganisms; and W_c the maximum cell swimming speed.

Note that energy expression is modeled via exponential heat source for the distribution of internal temperature [51,52]. Further, in Eq. (5) $\kappa=8.61\times 10$ (eV/K) as the Boltzmann constant, k_r^2 designates the reaction rate, and m $(-1<m>1)$ the fitted rate constant. Consider [16,47]:

$$u=\Omega r^n f'(\xi),\qquad v=\Omega r^n g(\xi),$$

$$w=-\sqrt{\Omega\nu}r^{\frac{n-1}{2}}\left(\frac{3+n}{2}f(\xi)+\frac{n-1}{2}\xi f'(\xi)\right),$$

$$\xi=\sqrt{\frac{\Omega}{\nu}}r^{\frac{n-1}{2}}z,\qquad \theta(\xi)=\frac{T-T_\infty}{T_w-T_\infty},$$

$$\phi(\xi)=\frac{C-C_\infty}{C_\infty},\qquad S(\xi)=\frac{N}{N_w}. \tag{9}$$

It is assumed that the longitudinal pressure gradient is zero. On using the above transformations, expression (1) is trivially verified and Eqs. (2)–(8) give:

$$f'''-nf'^2+\left(\frac{n+3}{2}\right)ff''+g^2=0, \tag{10}$$

$$g''-(n+1)f'g+\left(\frac{n+3}{2}\right)fg'=0, \tag{11}$$

$$\theta''+\Pr\left(\frac{n+3}{2}\right)f\theta'+N_b\theta'\phi'+\Pr Q\exp(-n_1\xi)$$
$$+N_t\theta'^2=0, \tag{12}$$

$$\phi''+\left(\frac{n+3}{2}\right)\mathrm{Le}f\phi'+\frac{N_t}{N_b}\theta''$$
$$-\mathrm{Le}\alpha_1(1+\delta\theta)^m\phi\exp\left(\frac{-E}{1+\delta\theta}\right)=0, \tag{13}$$

$$S''+\left(\frac{n+3}{2}\right)S_cfS'-\mathrm{Pe}\phi''S-\mathrm{Pe}\phi'S=0, \tag{14}$$

$$f'(0)=\alpha,\qquad f(0)=0,\qquad g(0)=1,$$

$$\theta(0)=1,\qquad \phi'(0)+\frac{N_t}{N_b}\theta'(0)=0,\qquad S(0)=1,$$

$$f'(\infty)\to 0,\qquad g(\infty)\to 0,\qquad \theta(\infty)\to 0,$$

$$\phi(\infty)\to 0,\qquad S(\infty)\to 0, \tag{15}$$

where prime designates differentiation via ξ, N_t the thermophoresis parameter, Pe the bioconvection Peclet number, N_b the Brownian motion variable, Le denotes Lewis number, Pr the Prandtl number, Q the (ESHS) variable, Sc the Schmidt number, δ the temperature difference variable, α_1 stands for non-dimensional reaction rate, E the dimensionless activation energy and α the ratio of stretching rate to angular velocity. These quantities are defined by:

$$N_t = \frac{(\rho c)_p}{(\rho c)_f}\frac{D_T(T_w - T_\infty)}{T_\infty \nu}, \qquad N_b = \frac{(\rho c)_p}{(\rho c)_f}\frac{D_B C_\infty}{\nu}$$

$$Q = \frac{Q_0}{\Omega(\rho c_p)_f}, \qquad \alpha = \frac{a}{\Omega}, \qquad \delta = \frac{(T_w - T_\infty)}{T_\infty},$$

$$\mathrm{Le} = \frac{\nu}{D_B}, \qquad \mathrm{Pr} = \frac{\nu}{\alpha_f}, \qquad S_c = \frac{\nu}{D_n},$$

$$\alpha_1 = k_r^2/\Omega, \qquad E = \frac{E_a}{\kappa T_\infty}, \qquad \mathrm{Pe} = \frac{bWc}{D_n}. \tag{16}$$

The typical quantities of key interest are the skin frictions in radial and azimuthal directions (C_{f_r}, C_{f_ϕ}), temperature gradient (Nu_r) and gradient of motile microorganisms (Nn_r).

$$C_{f_r} = \frac{\mu\left(\frac{\partial u}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \phi}\right)}{\rho(r^n\Omega)^2}, \qquad C_{f_\phi} = \frac{\mu\left(\frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \phi}\right)}{\rho(r^n\Omega)^2}, \tag{17}$$

$$\mathrm{Nu}_r = \frac{-kr\left(\frac{\partial T}{\partial z}\right)_{z=0}}{k(T_w - T_\infty)}, \tag{18}$$

$$Nn_r = \frac{-D_n r\left(\frac{\partial N}{\partial z}\right)_{z=0}}{D_n N_w}. \tag{19}$$

In dimensionless form, one has:

$$\sqrt{\mathrm{Re}_r}C_{f_r} = f''(0), \qquad \sqrt{\mathrm{Re}_r}C_{f_\phi} = g'(0), \tag{20}$$

$$(\mathrm{Re}_r)^{-0.5}\,\mathrm{Nu}_r = -\theta'(0), \tag{21}$$

$$(\mathrm{Re}_r)^{-0.5}\,\mathrm{Nn}_r = -S'(0), \tag{22}$$

where $\mathrm{Re}_r = u_w r/\nu$ depicts local Reynolds number.

3. Computational scheme

The governing problem consists of a non-linear system. Therefore, it is not possible to find exact solutions. However, an approximate solution can be computed through various techniques such as analytical and numerical etc. Here, in the problem considered, the NDSolve based Shooting scheme is employed. This technique is a numerical solver of differential systems. With the aid of NDSolve one can tackle different ODEs systems as well as special PDEs systems. General ODEs systems possess a number of equations n (i.e., $q_1, q_2, q_3 \cdots q_n$), independent variable x, number of dependent variables n (i.e., $v_1, v_2, v_3 \cdots v_n$), and boundary conditions according to the order of the PDEs system. Using the NDSolve technique, this system can be computed as follows:

$$\mathit{NDSolve}[\{q_1, q_2, q_3 \cdots q_n, \text{boundary conditions}\},$$

$$\{v_1, v_2, v_3 \cdots v_n\}, \{x, x_{\min}, x_{\max}\}.$$

This technique attains exceptional accuracy and is stable unconditionally. Furthermore, it provides the best outcomes in minimum CPU time and avoids lengthy expressions.

4. Discussion

The current intention is to predict the characteristics of sundry variables on velocity components (radial $f'(\xi)$ and tangential $g(\xi)$), temperature $\theta(\xi)$, concentration $\phi(\xi)$, and nanoparticle motile density $S(\xi)$. The values used for involved variables are: $n = 0.5$, $\mathrm{Pr} = 0.7 = \mathrm{Pe} = S_c$, $\delta = 0.3 = N_b$, $N_t = 0.1$, $\mathrm{Le} = 1.0 = \alpha_1$, $m = 0.5$, $Q = 0.2 = \alpha = n_1$, and $E = 3.0$. Figure 2 presents the characteristics of α on $f'(\xi)$. In fact, larger values of α result in an enhancement of stretching rate near the disk and so $f'(\xi)$ increase. Figures 3 and 4 are interpreted to inspect the physical impacts of embedding variables α and n on tangential velocity $g(\xi)$. Here, radial velocity enhances for higher α whereas a reverse trend is observed for power law index n (see Figure 4). Variations of $\theta(\xi)$ for multiple values of N_t are pointed out in Figure 5. Obviously, larger N_t yield higher $\theta(\xi)$ and corresponding thermal layer thickness. Physically higher N_t show stronger thermophoretic force on nanoparticles. Therefore, a large number of nanoparticles is shifted towards ambi-

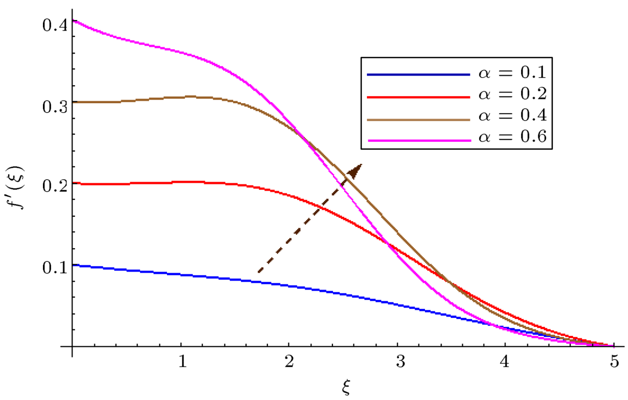

Figure 2. Behavior of f' via α.

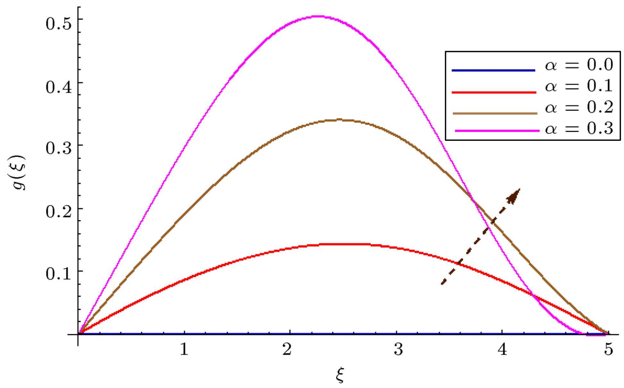

Figure 3. Behavior of g via α.

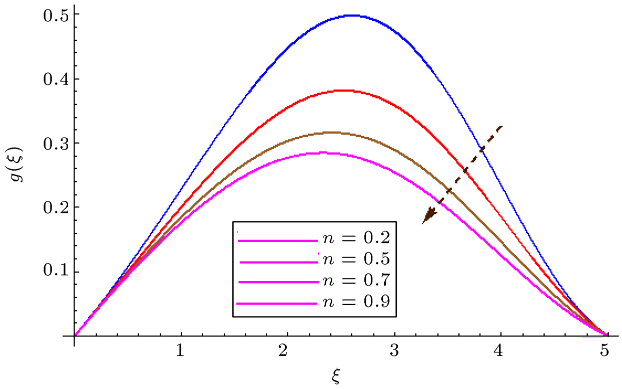

Figure 4. Behavior of g via n.

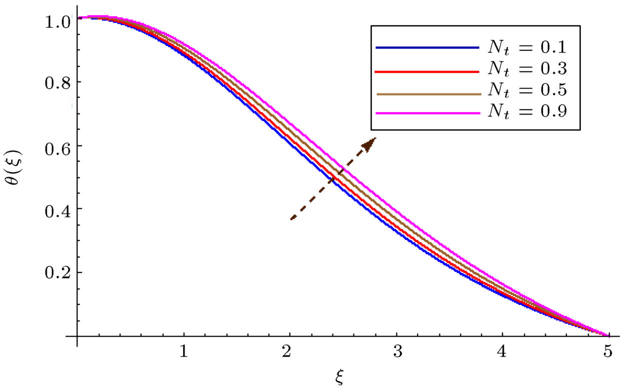

Figure 5. Behavior of θ via N_t.

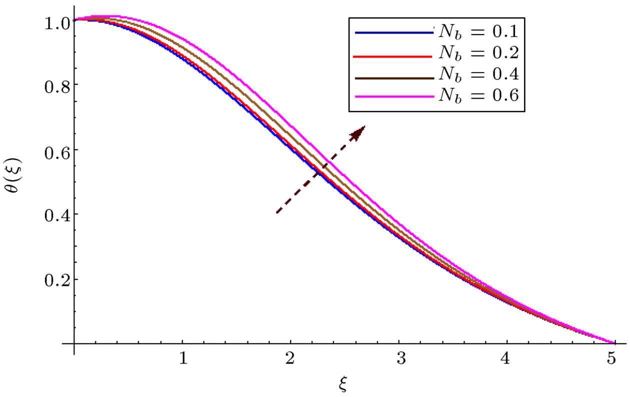

Figure 6. Behavior of θ via N_b.

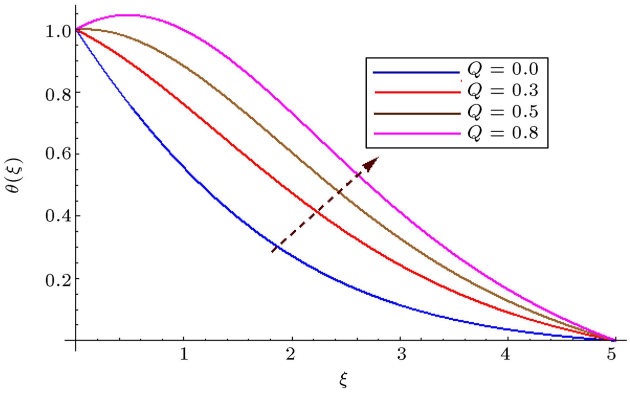

Figure 7. Impact of θ via Q.

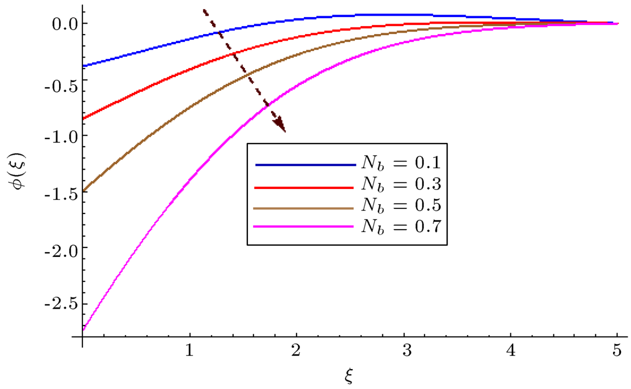

Figure 8. Behavior of ϕ through N_b.

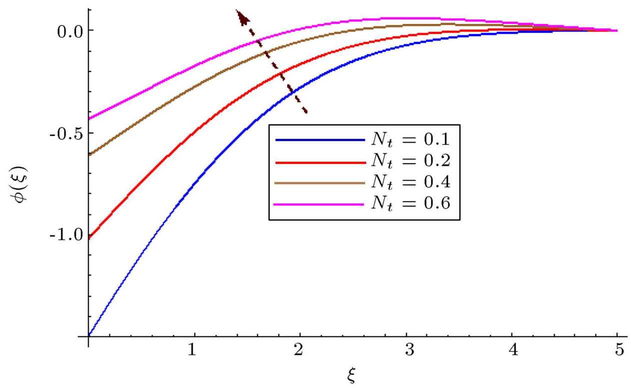

Figure 9. Behavior of ϕ through N_t.

ent liquid, which enhances the thermal field. Features of N_b on $\theta(\xi)$ are depicted in Figure 6. Higher N_b augments the material particles random motion, due to which, more heat is produced. That is why the $\theta(\xi)$ increases. Characteristic of ESHS parameter Q on $\theta(\xi)$ is illustrated in Figure 7. Here, the thermal field improves for larger Q. In view of physics, an increment in Q supplies heat in the fluid and the thermal field rises. Figure 8 shows the changes in N_b for $\phi(\xi)$. It has been pointed out that by enhancing N_b, an enhancement occurs in the thermal field. In fact, higher N_t improves the nanoparticle movement rates, which have distinct velocities because of Brownian diffusion. Figure 9 illustrates effect of thermophoresis parameter N_t on $\phi(\xi)$. The plot indicates that increasing estimations of N_t augment concentration. In a physical sense, larger N_t capitulate an increment in thermophoretic force and this frequently shifts nanoparticles from a higher to lower concentration region and so $\phi(\xi)$ boosts. Variations of $\phi(\xi)$ via E are plotted in Figure 10. Here, $\phi(\xi)$ enhances for larger activation energy parameter E. Physically an increment in E decays the exponential factor, which ultimately generates the chemical reaction due to which $\phi(\xi)$ enhances. Enhancement

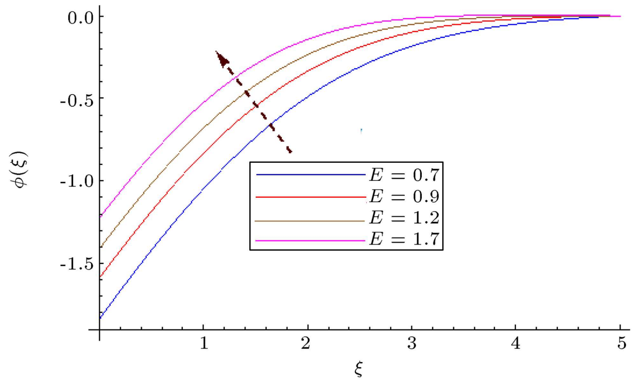

Figure 10. Behavior of ϕ through E.

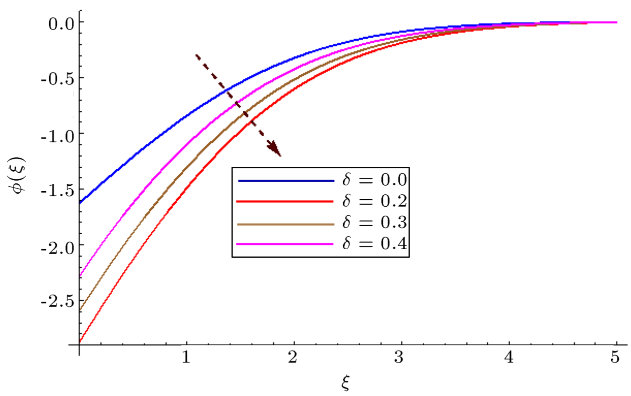

Figure 11. Behavior of ϕ through δ.

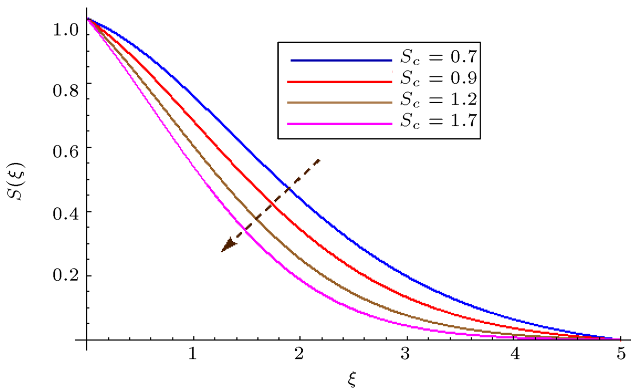

Figure 12. Effect of S through S_c.

in δ leads to a decay in concentration distribution. This behavior is depicted in Figure 11. $S(\xi)$ depicts decaying behavior against S_c (see Figure 12). The feature of Pe on $S(\xi)$ is exposed in Figure 13. It is found that $S(\xi)$ increases when Pe is enhanced. It is because larger Pe causes decay in the diffusivity of microorganisms and thus the motile density of liquid reduces. Aspects of diverse embedding variables on skin frictions, local Nusselt number and motile density number of microorganisms are reported in Tables 1 and 2. From Table 1, it is noted that skin friction along radial direction ($(\mathrm{Re}_r)^{0.5}C_{fr}$) enhances for larger

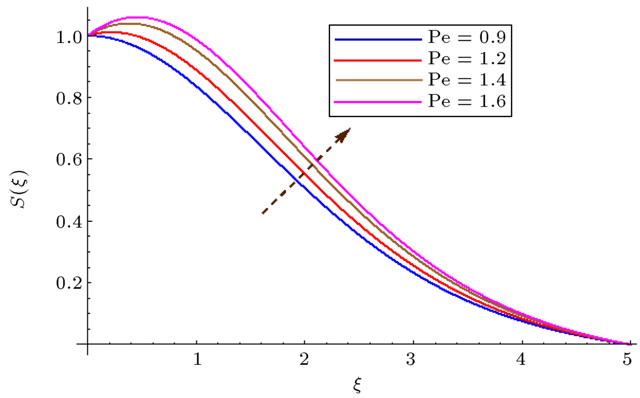

Figure 13. Effect of S through Pe.

Table 1. Numerical estimations of skin frictions for distinct values of n and α when $\mathrm{Pr} = 0.7 = \mathrm{Pe} = S_c$, $\delta = 0.3 = N_b$, $N_t = 0.1$, $\mathrm{Le} = 1.0 = \alpha_1$, $m = 0.5$, $Q = 0.2 = n_1$, and $E = 3.0$.

n	α	$(\mathrm{Re}_r)^{0.5}C_{fr}$	$(\mathrm{Re}_r)^{0.5}C_{f\varphi}$
0.5	0.2	0.0054969	0.1799612
0.9		0.0408849	0.0202541
1.0		0.0559298	0.0408849
0.5	0.2	0.0054969	0.2000000
	0.5	0.1580423	0.2707312
	1.0	0.6800625	0.6800621

Table 2. Numerical data of Nusselt number and density of the motile microorganisms for n, N_b, N_t and E when $\mathrm{Pr} = 0.7 = \mathrm{Pe} = S_c$, $\delta = 0.3$, $\mathrm{Le} = 1.0 = \alpha_1$, $Q = 0.2 = \alpha = n_1$ and $m = 0.5$.

n	N_b	N_t	E	$-\theta'(0)$	$-S'(0)$
0.2	0.3	0.1	3.0	0.152402	0.215945
0.5				0.027784	0.050162
1.0				0.047204	0.030000
0.5	0.1	0.1	3.0	0.024494	0.050838
	0.3			0.027784	0.050162
	0.8			0.090303	0.043765
0.5	0.1	0.1	3.0	0.027784	0.050162
		0.4		0.036857	0.054499
		0.6		0.050806	0.060307
0.5	0.3	0.1	1.0	0.027686	0.050096
			2.0	0.027706	0.050139
0.2	0.3	0.1	3.0	0.027784	0.050162

estimations of power law index n, whereas skin friction along the azimuthal direction ($(\mathrm{Re}_r)^{0.5}C_{f\phi}$) has a reverse behavior. From Table 2, it is pointed out that local Nusselt number (Nu_r) is enhanced for N_b, N_t and E. This table also provides the information that rising values of N_b and E decay $S'(0)$. Table 3 shows a comparison of the presented investigation with Chen

Table 3. Comparative data of $f''(0)$ and $g'(0)$ for various values of n and α when $Q = \alpha_1 = n_1 = E = \delta = 0$.

n	α	$g'(0)$		$f''(0)$	
		Chen et al. [47]	**Present outcomes**	**Chen et al. [47]**	**Present outcomes**
1	0.0	-0.6149	-0.61451	0.5080	0.50913
	1.0	-1.4870	-1.4863	-0.9486	-0.94820
	1.5	-1.7996	-1.7992	-1.9695	-1.9696
2	0.0	-0.6677	-0.6673	0.4639	0.4637
	1.0	-1.7418	-1.7417	-1.2878	-1.2875
	1.5	-2.4327	-2.4325	-2.5585	-2.5580

et al. [47] in a limiting sense for Q, α_1, n_1, E and δ. Here, a good match is noted.

5. Concluding remarks

The influence of activation energy and a space dependent internal heat source in the 3D flow of nanomaterials containing gyrotactic microorganisms is examined. Comparison is provided. The main points of analysis are mentioned below:

- Both radial and tangential velocities decay from an increase in power-law index n, whereas a contrary trend is seen for α;
- The temperature of nanoliquid increases for larger N_b and N_t;
- Features of δ and α_1 concentration are similar;
- Concentration of nanoliquid decays for higher E and δ;
- The behavior of Pe and S_c on motile density are quite opposite;
- Skin frictions are augmented for higher n;
- Heat and mass transmission rates are uplifted by N_t, N_b and E;
- Absolute estimations of motile microorganism transmission rate is diminished via N_b and E.

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