DEA altruism and exclusiveness cross-efficiency evaluation models

Lei Li, Wei Dai, Junfei Chu, Xiaohong Liu, Yuhong Wang

Abstract

As an effective method for evaluating efficiency, the cross-efficiency evaluation method has been widely used to assess the performance of decision-making units (DMUs). However, the non-uniqueness of optimal weights problem has reduced the effectiveness of this method. To address this problem, scholars proposed to use secondary goals and presented many secondary goal models. In this paper, two new secondary goal models are presented, in order to further extend the above-mentioned existing secondary goal models. Specifically, when the evaluated DMU is cooperative with other DMUs, our altruism cross-efficiency model is proposed, as this model will maximize the peer-efficiency scores of other DMUs. When the evaluated DMU is competing with other DMUs, our exclusiveness cross-efficiency model is presented. This model is designed to minimize the peer-efficiency scores of other DMUs. Compared to other cross-efficiency approaches, our model ensures that all the peer-efficiencies generated by each DMU are maximized (minimized) under the prior principle that the minimum among which is maximized (minimized). More importantly, our proposed al-
Algorithm guarantees the uniqueness of the optimal weights and the set of cross efficiency scores. Finally, two numerical examples are used to verify the effectiveness of our proposed models.

**Keywords:** Data envelopment analysis (DEA), cross-efficiency evaluation, altruism model, exclusiveness model, efficiency

1. **Introduction**

As an effective method for efficiency evaluations, data envelopment analysis (DEA) has been widely used to assess the performance and relative efficiency of decision making units (DMUs). The method was firstly introduced by Charnes et al. [1], who proposed the original DEA model (CCR model). Since then, the DEA method has been further extended by numerous scholars and researchers. Compared to other methods, DEA has many obvious advantages. On the one hand, DEA can be used to evaluate the efficiency of DMUs that have multiple inputs and multiple outputs [2]. On the other hand, providing any subjective information for decision makers before evaluation is unnecessary when DEA is used [3]. Therefore, DEA results are extremely objective and unbiased. However, the traditional DEA models also have a number of disadvantages. For example, DMUs are evaluated from the perspective of self-evaluation. In addition, the evaluated DMU will select its most preferable weights, in order to maximize its own efficiency score. Therefore, more than one DMU will be evaluated as DEA-efficient. These DEA-efficient DMUs cannot be further discriminated against by the DEA CCR model [4].

To overcome the shortcomings mentioned above, Sexton et al. [5] proposed the cross-efficiency evaluation method, as shown in Section 2. The core idea of the method is to obtain the peer-efficiency scores of other DMUs, by using the optimal weights of the evaluated DMU. Then, the average of n efficiency scores (including n-1 peer-efficiency scores and one self-efficiency score), is used as the final efficiency evaluation index. This method brings at least three benefits. Firstly, a unique ranking order of all DMUs can be obtained through the above
method \cite{5}. Secondly, this method can decrease the emergence of unrealistic weights \cite{6}. Finally, the cross-efficiency evaluation method can distinguish between good performers and bad performers \cite{7}. Thus far, the method has been widely applied in a variety of areas and studies \cite{8, 9, 10, 11, 12, 13}.

However, some defects still exist in the cross-efficiency evaluation method. For example, because of the non-unique optimal weights calculated by the CCR model, the peer-efficiency scores of other DMUs may also be non-unique. To address this problem, we propose the use of DEA models with secondary goals. Among all secondary goal models, the benevolent and aggressive models proposed by Sexton et al.\cite{5} and Doyle and Green \cite{14} are the most classical. These models will be introduced in Section 2. To date, and on the basis of the traditional benevolent and aggressive models, many scholars and researchers have presented a significant number of new benevolent and aggressive models, which can be utilized in different application scenarios. For instance, Liang et al. \cite{15} proposed three secondary goal models with different secondary goals (including minimum total deviation). Due to the unrealistic target efficiency of the models presented by Liang et al. \cite{15}, those models were further extended by Wang and Chin \cite{16}, whose models changed the target efficiency of each DMU from 1 to its CCR efficiency. In addition, Lim \cite{17} proposed two DEA models with new secondary goals, in order to maximize (or minimize) the peer-efficiency score of the DMU with the worst (or best) performance, respectively. Subsequently, Wu et al. \cite{18} pointed out that the target efficiency (CCR efficiency) of the Wang and Chin \cite{19} models was still unreachable for DMUs. Therefore, Wu presented a new method, designed to obtain the maximum and minimum peer-efficiency scores of every other DMU. They proposed other secondary goal models that could be used to calculate the peer-efficiency scores of other DMUs, taking into consideration the DMUs willingness to be close to their maximum peer-efficiency scores and as far as possible from their minimum peer-efficiency scores.

In addition, Wang and Chin \cite{16} pointed out that the evaluated DMU should ignore the influence of the other DMUs. The evaluated DMU should select its own most favorable weights. Therefore, these scholars proposed a neutral DEA
model, which can effectively avoid the appearance of zero output weights. However, because Wang and Chins model only considered the constraints of output weights, a new neutral DEA model was presented for cross-efficiency evaluation. This new model can avoid the simultaneous emergence of zero weights of outputs and inputs. What’s more, by incorporating an ideal DMU and an anti-ideal DMU into the neutral DEA model, Wang, Chin, and Luo [16] provided a new perspective in the study of neutral DEA models. They proposed a neutral DEA model in which the distance between the evaluated DMU and the ideal DMU, and the distance between the evaluated DMU and the anti-ideal DMU, are taken into account. In addition, other neutral DEA models have been proposed that are applicable for reducing the difference in weighted inputs and outputs. For instance, a weight-balanced DEA model was proposed by Wu et al. [20] to reduce the difference in weighted inputs and weighted outputs. In that model, each weighted input (and weighted output) is considered as an independent individual. In order to reduce the number of zero weights, Sun et al. [21] presented a weight-optimized DEA model. In addition, Sun et al. [21] model could reduce differences among weighted inputs and outputs.

Apart from the DEA models mentioned above, other DEA models applicable from different perspectives also exist. For example, Wu et al. [22] maintained that DMUs may be more concerned about their ranking order than about their efficiency scores in some cases, such as the selection of projects and preference voting. Therefore, these scholars proposed a rank priority model, as a means to optimize the ranking order of DMUs. In addition, Wu et al. [18] noticed that another shortcoming of cross-efficiency evaluation has traditionally been neglected. Namely, the obtained cross-efficiency scores may not be Pareto optimal. Therefore, many DMUs are unwilling to accept the average cross-efficiency scores as the final efficiency evaluation measure. To address this problem, they proposed a cross-efficiency evaluation method based on Pareto improvement. Song and Liu [23] pointed out that the traditional average cross-efficiency evaluation method fails to reflect the real performance of all DMUs. They presented a variation coefficient method as a means to aggregate the cross-efficiency scores. In addition,
Ruiz et al. [24] proposed a fuzzy approach as a means to rank DMUs which have imprecise data. To measure the efficiency scores of DMUs, Chen [25] presented an approach on the basis of lexicographical evaluations of DMUs. Chen’s approach can effectively avoid overestimating the efficiency scores of DMUs. In order to consider the decision makers’ risk preference attitude, Liu et al. [26] proposed a prospect cross-efficiency approach based on prospect theory. Authors concluded that other models are the special cases of the prospect cross-efficiency approach. Kao and Liu [27] integrated cross-evaluation method into the two-stage DEA for assessing the cross efficiencies of two basic network systems. Authors concluded that the proposed method not only improves the discriminating ability of the network system ranking, but also identifies the relationship between system and departmental efficiency.

In many practical applications, competition exists between DMUs. The traditional cross-efficiency evaluation method has difficulty dealing with this kind of situation. To solve the competition issue, scholars focused their attention on the study of the game cross-efficiency model. In the study of Liang, Wu, Cook, and Zhu [28], every DMU is deemed to be a player in a non-cooperative game. Each DMU is given a corresponding algorithm, which is proved to be convergent to the game cross-efficiency score. On the basis of the DEA game cross-efficiency model, Wu et al. [22] proposed a new method of ranking the candidates of a preferential election. In addition, by extending the model of Liang et al. [28], Wu et al. [29] presented a modified DEA game cross-efficiency model. Wu used the model to evaluate the performances of different countries in the 2004 Olympic Games. Roboredo et al. [13] used the DEA game cross-efficiency model to calculate the efficiency scores of Brazilian football teams. In Roboredos model, which each team was considered to be a DMU. Based on the Liang et al. [28] s algorithm, Sun et al. [30] proposed a DEA non-cooperation game model to allocate the emission permits. Wu et al. [18] introduce the concept of the satisfaction degree into cross-efficiency evaluation approach. Then the proposed method was applied to evaluate all the information technology to select the best one for the enterprise. Liu et al. [31] presented an equitable
cross-efficiency model by integrating undesirable outputs. Then, the proposed method was used to analyze the ecological efficiency of coal-fired power plants. Chen et al. [32] uses a game cross efficiency model to assess and analyze the China’s provincial power efficiency. By analyzing the efficiency result from the temporal and spatial perspective, authors suggested differential energy policies for Chinas provinces.

From the above literature review, we can conclude that the research into cross-efficiency evaluation has made great progress in recent years. However, most of the existing secondary goal models mainly focus on maximizing (or minimizing) the average peer-efficiency scores of all other DMUs, thus failing to maximize (or minimize) the peer-efficiency levels of each of the other DMUs. In addition, most of these models still cannot guarantee the uniqueness of optimal weights. In order to solve these problems and further extend the existing secondary goal models, this paper proposes two different models. These models are based on different application scenarios, i.e., the altruism and exclusiveness cross-efficiency models. When the evaluated DMU is cooperative with the other DMUs, the evaluated DMUs secondary altruism goal is to maximize the peer-efficiency score of each of the other DMUs. When the evaluated DMU is competing with other DMUs, the secondary exclusiveness goal is to minimize the peer-efficiency score of each of the other DMUs. To solve the proposed models linearly, corresponding algorithms are also presented.

The rest of this paper is organized as follows. The CCR model, classical cross-efficiency evaluation method and traditional benevolent and aggressive models are briefly introduced in Section 2. An altruism cross-efficiency model with the corresponding algorithm and an exclusiveness cross-efficiency model with the corresponding algorithm are presented in Section 3. Two illustrative examples are given in Section 4. Finally, our concluding remarks are presented in Section 5.
2. CCR model and DEA cross-efficiency evaluation

2.1. Notation summary

To facilitate model formulation, the notations used in this study are summarized in Table 1.

![Insert Table 1 here]

2.2. CCR model and cross-efficiency evaluation

Assume that there are \( n \) DMUs to be evaluated. Each DMU consumes \( m \) inputs to produce \( s \) outputs. For DMU \( j \) \((j = 1, 2, \ldots, n)\), the \( i \)th input is denoted as \( x_{ij} \); the \( r \)th output is denoted as \( y_{rj} \). The self-efficiency score of the evaluated DMU, which is denoted as \( E_{dd} \), can be obtained by the following linear programming model (1). This model was originally proposed by Charnes, Cooper, and Rhodes (CCR) [1], and is called the CCR model.

\[
\begin{align*}
\max \quad & E_{dd} = \sum_{r=1}^{s} u_{rd} y_{rd} \\
s.t. \quad & \sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} w_{id} x_{ij} \leq 0, \forall j \\
& \sum_{i=1}^{m} w_{id} x_{id} = 1 \\
& u_{rd}, w_{id} \geq 0, \forall i, r
\end{align*}
\]  

(1)

In model (1), \( w_{id}(i = 1, 2, \ldots, m) \) and \( u_{rd}(r = 1, 2, \ldots, s) \) are the weight of the \( i \)th input and the weight of the \( r \)th output of DMU \( d \), respectively. A set of optimal weights \((w_{id}^{*}, u_{rd}^{*}, \forall j)\) and the optimal self-efficiency score \( E_{dd}^{*} \) can be obtained by solving model (1). If \( E_{dd} = 1 \), then DMU \( d \) is called DEA-efficient. Otherwise, DMU \( d \) is DEA-inefficient. Utilizing the optimal weights of DMU \( d \), the peer-efficiency scores of DMU \( j \) \((j = 1, 2, \ldots, n)\) evaluated by DMU \( d \) could be calculated by the following formula (2).

\[
E_{dj} = \frac{\sum_{r=1}^{s} u_{rd}^{*} y_{rj}}{\sum_{i=1}^{m} w_{id}^{*} x_{ij}}, \forall j
\]  

(2)

For DMU \( j \) \((j = 1, 2, \ldots, n)\), the average of all its efficiency scores, denoted as \( E_{j} \), can be calculated as the following equation (3) for final efficiency evaluation measure.
\[ E_j = \frac{1}{n} \sum_{d=1}^{n} E_{dj}, \forall j \] (3)

The non-uniqueness of optimal weights in model (1) will most likely lead to different cross-efficiency scores in equation (2). To address this problem, secondary goals are incorporated into the cross-efficiency evaluation method. Of all secondary goal models, the most commonly utilized are the traditional benevolent and aggressive models proposed by Sexton et al. [5] and Doyle and Green [14]. These are shown below as model (4) and model (5), respectively.

\[
\begin{align*}
\text{max} & \quad \sum_{r=1}^{s} u_{rd} \left( \sum_{j=1, j \neq d}^{n} y_{rj} \right) \\
\text{s.t.} & \quad \sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} w_{id} x_{ij} \leq 0, \forall j \\
& \quad E_{dd} \cdot \sum_{i=1}^{m} w_{id} x_{id} - \sum_{r=1}^{s} u_{rd} y_{rd} = 0 \\
& \quad \sum_{i=1}^{m} w_{id} \left( \sum_{j=1, j \neq d}^{n} x_{ij} \right) = 1 \\
& \quad u_{rd}, w_{id} \geq 0 \forall r, i
\end{align*}
\] (4)

\[
\begin{align*}
\text{min} & \quad \sum_{r=1}^{s} u_{rd} \left( \sum_{j=1, j \neq d}^{n} y_{rj} \right) \\
\text{s.t.} & \quad \sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} w_{id} x_{ij} \leq 0, \forall j \\
& \quad E_{dd} \cdot \sum_{i=1}^{m} w_{id} x_{id} - \sum_{r=1}^{s} u_{rd} y_{rd} = 0 \\
& \quad \sum_{i=1}^{m} w_{id} \left( \sum_{j=1, j \neq d}^{n} x_{ij} \right) = 1 \\
& \quad u_{rd}, w_{id} \geq 0 \forall r, i
\end{align*}
\] (5)

The core idea of the traditional benevolent (aggressive) model is to identify a set of the optimal weighs of DMU_d. This model can thus maximize (or minimize) the average peer-efficiency scores of other DMUs as much as possible, while retaining the optimal self-efficiency score for each DMU. Although these two models can reduce the non-uniqueness of optimal weights to a certain extent, some drawbacks still exist in each of the two models. For example, only maximizing or minimizing the average peer-efficiency scores of other DMUs is likely to lead to the emergence of extreme results. Specifically, some cross-efficiency scores may be very large, while others may be very small, as seen in models (4) and (5).
3. The altruism and exclusiveness cross-efficiency models

This section will present two improved cross-efficiency models, namely, altruism and exclusiveness models. The altruism model first identifies the DMU (called DMU1) with the minimal peer-efficiency, and then maximize the peer-efficiency of the DMU1. Then, the altruism model continues to identify the DMU (called DMU2) with the minimal peer-efficiency from the remaining DMUs, and maximize the peer-efficiency of the DMU2. Repeat this step until all DMUs peer-efficiencies are obtained. However, the exclusiveness model obtains the peer-efficiencies of DMUs from an opposite angle. Specifically, the exclusiveness model first identify the DMU (called DMU1) with the maximal peer-efficiency, and then minimize the peer-efficiency of the DMU1. Then, the exclusiveness model continues to identify the DMU (called DMU2) with the maximal peer-efficiency from the remaining DMUs, and minimize the peer-efficiency of the DMU2. Repeat this step until all DMUs peer-efficiencies are obtained.

3.1. The altruism cross-efficiency model

If the evaluated DMU is cooperative with other DMUs, the evaluated DMUs weights should not only ensure the optimal self-efficiency score, but the peer-efficiency scores of other DMUs will also be maximized. We present the altruism cross-efficiency model (6) to deal with this situation.

\[
\begin{align*}
\max_{w,u} & \quad \min_{1 \leq j \leq n, j \neq d} E_{dj} = \frac{\sum_{r=1}^{s} u_{rd} y_{rj}}{\sum_{i=1}^{m} w_{id} x_{ij}} \\
\text{s.t.} & \quad \sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} w_{id} x_{ij} \leq 0, \forall j \\
& \quad \sum_{r=1}^{s} u_{rd} y_{rd} = E_{dd} \\
& \quad \sum_{i=1}^{m} w_{id} x_{id} = 1 \\
& \quad u_{rd}, w_{id} \geq 0, \forall i, r
\end{align*}
\]

(6)

The maximum peer-efficiency score of DMU \( j \) \((j = 1, 2, \ldots, n)\) as evaluated by DMU \( d \) can be obtained through model (6), while DMU \( d \) is retained as the optimal self-efficiency score \( (E_{dd}) \). In fact, model (6) is a multi-objective linear programming model. The first objective of model (6) is to identify the DMU
with the most minimal peer-efficiency score of all the DMUs. The second objective is to maximize the minimal peer-efficiency score through the weights of DMU\(_d\).

The corresponding steps are designed to find the final solution of model (6).

**Step 1:** let \( l = 1 \), model (6) can be converted into model (7).

\[
\begin{align*}
\max_{w,u} & \quad \beta \\
\text{s.t.} & \quad \sum_{r=1}^{s} u_{rd}y_{rj} - \sum_{i=1}^{m} w_{id}x_{ij} \leq 0, \forall j \\
& \quad \sum_{r=1}^{s} u_{rd}y_{rd} = E_{dd} \\
& \quad \sum_{i=1}^{m} w_{id}x_{id} = 1 \\
& \quad \sum_{r=1}^{s} u_{rd}y_{rj} - \beta \cdot \sum_{i=1}^{m} w_{id}x_{ij} \leq 0, \forall j, j \neq d \\
& \quad u_{rd}, w_{id} \geq 0 \forall i, r
\end{align*}
\]

(7)

Assume that the optimal solution of model (7) is \((\beta^*_1, u^*_{1d}, w^*_{1d}, \forall r, i)\). Let \( E^*_{d1} = \frac{\sum_{r=1}^{s} u^*_{rd}y_{rj}}{\sum_{i=1}^{m} w^*_{id}x_{ij}}, \forall j \), then \( J = j | j = 1, 2, ..., n, j \neq d \) can be divided into two mutually incompatible subsets as follows.

\[
J_1 = j | E^*_{d1} = \beta^*_1, \forall j \in J \\
J_2 = j | E^*_{d1} > \beta^*_1, \forall j \in J
\]

Through model (7), the peer-efficiency score of DMUs in is determined to be \( \beta^*_1 \). Assume that the number of DMUs in \( J_1 \) is \( n_1 \). If \( n_1 = m + s - 2 \), then the algorithm terminates, and \((\beta^*_1, u^*_{rd}, w^*_{rd}, \forall r, i)\) is the unique optimal solution of model (7) (The reason is explained by Theorem 2.). If \( n_1 = m + s - 2 \), then go to Step 2.

**Step 2:** \( l = 2 \), calculating the following model (8).

\[
\begin{align*}
\max_{w,u} & \quad \beta \\
\text{s.t.} & \quad \sum_{r=1}^{s} u_{rd}y_{rj} - \sum_{i=1}^{m} w_{id}x_{ij} \leq 0, \forall j \\
& \quad \sum_{r=1}^{s} u_{rd}y_{rd} = E_{dd} \\
& \quad \sum_{i=1}^{m} w_{id}x_{id} = 1 \\
& \quad \sum_{r=1}^{s} u_{rd}y_{rj} - \beta^*_1 \cdot \sum_{i=1}^{m} w_{id}x_{ij} = 0, \forall j \in J_1 \\
& \quad \sum_{r=1}^{s} u_{rd}y_{rj} - \beta \cdot \sum_{i=1}^{m} w_{id}x_{ij} \leq 0, \forall j \in J_2 \\
& \quad u_{rd}, w_{id} \geq 0 \forall i, r
\end{align*}
\]

(8)
Insuring that the self-efficiency score of DMU$_d$ is $E_{dd}$ and the peer-efficiency score of the DMUs in $J_1$ is $\beta^*_1$, the optimal solution of model (8) can be obtained, denoted as $(\beta^*_2, u^2_{rd}, w^2_{id}, \forall r, i)$. Let $E^2_{d_j} = \sum_{i=1}^m \frac{u^2_{rd}y_{ij}}{w^2_{id}x_{ij}}, j \in J_2$ ; $J_2$ is then divided into two mutually incompatible subsets as follows.

$$J_3 = j | E^2_{d_j} = \beta^*_2, \forall j \in J_2$$

$$J_4 = j | E^2_{d_j} > \beta^*_2, \forall j \in J_2$$

Using model (8), the peer-efficiency score of the DMUs in $J_3$ is determined as $\beta^*_2$. Assume that the number of DMUs in $J_3$ is $n_2$. If $n_1 + n_2 = m + s - 2$, then the algorithm terminates, and $(\beta^*_2, u^2_{rd}, w^2_{id}, \forall r, i)$ is the unique optimal solution of model (8) (The reason is explained by Theorem 2). If $n_1 + n_2 < m + s - 2$, then go to Step 3.

**Step 3:** $l = l + 1$, Solving the following model (9).

$$\max_{w, u, \beta} \beta$$

s.t.

$$\sum_{r=1}^s u_{rd}y_{rj} - \sum_{i=1}^m w_{id}x_{ij} \leq 0, \forall j$$

$$\sum_{r=1}^s u_{rd}y_{rd} = E_{dd}$$

$$\sum_{i=1}^m w_{id}x_{id} = 1$$

$$\sum_{r=1}^s u_{rd}y_{rj} - \beta^*_1 \cdot \sum_{i=1}^m w_{id}x_{ij} = 0, \forall j \in J_1$$

$$\sum_{r=1}^s u_{rd}y_{rj} - \beta^*_2 \cdot \sum_{i=1}^m w_{id}x_{ij} = 0, \forall j \in J_3$$

$$\sum_{r=1}^s u_{rd}y_{rj} - \beta \sum_{i=1}^m w_{id}x_{ij} \leq 0, \forall j \in J_{2l-3}$$

$$u_{rd}, w_{id} \geq 0, \forall i, r$$

Similarly, ensure that the self-efficiency score of DMU$_d$ is $E_{dd}$, and the peer-efficiency scores of the DMUs in $J_1, J_3, ..., J_{2L-3}$ are $\beta^*_1, ..., \beta^*_{l-1}$, respectively. We can assume that the optimal solution of model (9) is $(\beta^*_1, u^1_{rd}, w^1_{id}, \forall r, i)$. Let $E^1_{d_j} = \sum_{i=1}^m \frac{u^1_{rd}y_{ij}}{w^1_{id}x_{ij}}, j \in J_{2l-2}$, $J_{2l-2}$ is then divided into two subsets as follows.

$$J_{2l-1} = j | E^1_{d_j} = \beta^*_j, \forall j \in J_{2l-2}$$

$$J_{2l} = j | E^1_{d_j} > \beta^*_j, \forall j \in J_{2l-2}$$

Using model (9), the peer-efficiency score of the DMUs in $J_{2l-1}$ is determined as $\beta^*_j$. Assume that the number of DMUs in $J_{2l-1}$ is $n_l$. If $\sum_{i=1}^m = m + s - 2$, then
of model (7). Also, the optimal value of model (8) is larger than the optimal value of model (7). Theorem 1. The optimal weights of model (8) are also the optimal weights of model (7). Also, the optimal value of model (8) is larger than the optimal value of model (7).

Proof: The optimal solution of model (7) is \((\beta^*_1, u^1_{rd}, w^1_{id}, \forall r, i)\) and we have
\[
\sum_{r=1}^s u^1_{rd}y_{rj} = \beta^*_1 (j \in J_1) \quad \text{and} \quad \sum_{r=1}^s u^1_{id}x_{ij} > \beta^*_1 (j \in J_2).
\]
The optimal solution of model (8) is \((\beta^*_2, u^2_{rd}, w^2_{id}, \forall i, r)\), and we have
\[
\sum_{r=1}^s u^2_{rd}y_{rj} = \beta^*_2 (j \in J_1) \quad \text{and} \quad \sum_{r=1}^s u^2_{id}x_{ij} > \beta^*_2 (j \in J_2).
\]
Obviously, \(\beta^*_2 \geq \beta^*_1\). Assuming that \(\beta^*_1 \geq \beta^*_2\), we have \(J_3 \subseteq J_1\), which conflicts with the fact that \(J_3 \subseteq J_2\) and \(J_2 \cap J_1 = \emptyset\). Thus, we have \(\beta^*_2 > \beta^*_1\). Through the optimal solution \((\beta^*_2, u^2_{rd}, w^2_{id}, \forall i, r)\) obtained by model (8), the optimal value of model (7) is \(\beta^*_1\). Therefore, the optimal solutions of model (7) is the unique optimal solution of model (8), and the optimal solutions of model (8) is the optimal solution of model (7). Q.E.D.

Through Theorem 1, we know that the optimal weights of model (7) while \(l = k\) are also the optimal weights of model (7), and \(\beta^*_k > \beta^*_k > \ldots > \beta^*_1\).

Theorem 2. If \(n_1 = m + s - 2\), then \((\beta^*_1, u^1_{rd}, w^1_{id}, \forall i, r, j)\) is the unique optimal solution of model (7).

Proof: For model (7), if \(n_1 = m + s - 2\), we know that represents \(\sum_{r=1}^s u_{rd}y_{rj} - \beta^*_1 \cdot \sum_{i=1}^m w_{id}x_{ij} = 0, \forall j \in J_1\) equations. Note that model (7) contains two equations, specifically \(\sum_{i=1}^m w_{id}x_{id} = 1\) and \(\sum_{r=1}^s u_{rd}y_{rd} = E_{dd}\). The number of variables \((u_{rd}, w_{id}, \forall r, i)\) is \(m + s\), and the vectors \((x_{ij}, y_{rj}, \forall i, r, j)\) are mutually linearly independent. Because the number of variables and equations are both
m + s, the optimal solution of model (7) can be uniquely determined and denoted as \((\beta^*_1, u_{rd}^1, w_{id}^1, \forall r, i, d)\), and the algorithm terminates. If \(n_1 < m + s - 2\), then go to Step 2. Q.E.D.

Through Theorem 2, we know that if \(\sum_{t=1}^I n_t = m + s - 2\), then \((\beta^*_1, u_{rd}^1, w_{id}^1, \forall r, i)\) is the unique optimal solution of model (9), and the algorithm terminates.

### 3.2. The exclusiveness cross-efficiency model

On the contrary, if the evaluated DMU is competing with other DMUs, the evaluated DMUs secondary goal should minimize the peer-efficiency scores of all other DMUs as much as possible. Therefore, the exclusiveness cross-efficiency model is presented to cope with this situation. This model is shown as following model (10).

\[
\min_{w, u} \max_{1 \leq j \leq n, j \neq d} \quad E_{dj} = \frac{\sum_{r=1}^s u_{rd} y_{rj}}{\sum_{i=1}^m w_{id} x_{ij}} \\
\text{s.t.} \quad \sum_{r=1}^s u_{rd} y_{rj} - \sum_{i=1}^m w_{id} x_{ij} \leq 0, \forall j \quad (10) \\
\sum_{r=1}^s u_{rd} y_{rd} = E_{dd} \\
\sum_{i=1}^m w_{id} x_{id} = 1 \\
u_{rd}, w_{id} \geq 0, \forall i, r
\]

The minimum peer-efficiency score of DMU \(j \ (j = 1, 2, \ldots, n)\) as evaluated by DMU\(_d\) can be obtained through model (10), while DMU\(_d\) is retained as the optimal self-efficiency score. Model (10) is also a multi-objective programming model. The models first objective is to identify the DMU with the maximal peer-efficiency score of all the DMUs. The second objective is to minimize the maximal peer-efficiency score through the weights of DMU\(_d\).

The corresponding steps are designed to find the solution of model (10).
Step 1: let \( l = 1 \), translating model (10) into the following model (11):

\[
\begin{align*}
\min_{w,u} \quad & \alpha \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} w_{id} x_{ij} \leq 0, \forall j \\
& \sum_{r=1}^{s} u_{rd} y_{rd} = E_{dd} \\
& \sum_{i=1}^{m} w_{id} x_{id} = 1 \\
& \sum_{r=1}^{s} u_{rd} y_{rj} - \alpha \cdot \sum_{i=1}^{m} w_{id} x_{ij} \leq 0, \forall j, j \neq d \\
& u_{rd}, w_{id} \geq 0 \forall i, r 
\end{align*}
\]

(11)

Suppose that the optimal solution of model (11) is \((\alpha^*_1, u^*_1 rd, w^*_1 id, \forall r, i)\). Let \( E^*_1 dj = \sum_{r=1}^{s} u^*_1 rd y_{rj}, \forall j, j \neq d, J = j | j = 1, 2, \ldots, n, j \neq d \) is then divided into two incompatible subsets.

\[
\begin{align*}
J_1 &= j | E^*_1 dj = \alpha^*_1, \forall j \in J \\
J_2 &= j | E^*_1 dj > \alpha^*_1, \forall j \in J 
\end{align*}
\]

Using model (11), the peer-efficiency score of DMUs in \( J_1 \) is determined as \( \alpha^*_1 \). Assume that the number of DMUs in \( J_1 \) is \( n_1 \). If \( n_1 = m + s - 2 \), then the algorithm terminates, and \((\alpha^*_1, u^*_1 rd, w^*_1 id, \forall r, i)\) is the unique optimal solution of model (11) (The reason is explained by Theorem 4.). If \( n_1 = m + s - 2 \), then go to Step 2.

Step 2: \( l = 2 \), solving the following model (12):

\[
\begin{align*}
\min_{w,u} \quad & \alpha \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} w_{id} x_{ij} \leq 0, \forall j \\
& \sum_{r=1}^{s} u_{rd} y_{rd} = E_{dd} \\
& \sum_{i=1}^{m} w_{id} x_{id} = 1 \\
& \sum_{r=1}^{s} u_{rd} y_{rj} - \alpha^*_1 \cdot \sum_{i=1}^{m} w_{id} x_{ij} = 0, \forall j \in J_1 \\
& \sum_{r=1}^{s} u_{rd} y_{rj} - \alpha^*_1 \cdot \sum_{i=1}^{m} w_{id} x_{ij} \leq 0, \forall j \in J_2 \\
& u_{rd}, w_{id} \geq 0 \forall i, r 
\end{align*}
\]

(12)

Ensure that the self-efficiency score of DMU \( d \) is \( E_{dd} \) and the peer-efficiency score of the DMUs in \( J_1 \) is \( \alpha^*_1 \). Suppose that the optimal solution of model (12) is \((\alpha^*_2, u^*_2 rd, w^*_2 id, \forall r, i)\). Let \( E^*_2 dj = \sum_{r=1}^{s} u^*_2 rd y_{rj}, j \in J_2 \); \( J_2 \) is then divided into two subsets as follows.
Using model (12), the peer-efficiency score of the DMUs in $J_3$ is determined as $\alpha_2^*$. Assume that the number of DMUs in $J_3$ is $n_2$. If $n_1 + n_2 = m + s - 2$, then the algorithm terminates, and $(\alpha_2^*, u_{r, i}^t, w_{id}^t, \forall r, i)$ is the unique optimal solution of model (12) (The reason is explained by Theorem 4). If $n_1 + n_2 < m + s - 2$, then go to Step 3.

**Step 3:** $l = l + 1$, Solving the following general model (13).

$$
\max_{w, u} \alpha
$$

s.t. \hspace{1cm}

\begin{align*}
\sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} w_{id} x_{ij} & \leq 0, \forall j \\
\sum_{r=1}^{s} u_{rd} y_{rd} & = E_{dd} \\
\sum_{i=1}^{m} w_{id} x_{id} & = 1 \\
\sum_{r=1}^{s} u_{rd} y_{rj} - \alpha_1^* \cdot \sum_{i=1}^{m} w_{id} x_{ij} & = 0, \forall j \in J_1 \\
\sum_{r=1}^{s} u_{rd} y_{rj} - \alpha_2^* \cdot \sum_{i=1}^{m} w_{id} x_{ij} & = 0, \forall j \in J_3 \\
\vdots & \\
\sum_{r=1}^{s} u_{rd} y_{rj} - \alpha_{l-1}^* \cdot \sum_{i=1}^{m} w_{id} x_{ij} & = 0, \forall j \in J_{2l-3} \\
\sum_{r=1}^{s} u_{rd} y_{rj} - \alpha \cdot \sum_{i=1}^{m} w_{id} x_{ij} & \leq 0, \forall j \in J_{2l-2} \\
u_{rd}, w_{id} & \geq 0 \forall i, r
\end{align*}

Ensure that the self-efficiency score of DMU$_d$ is $E_{dd}$, and the peer-efficiency scores of the DMUs in $J_1, J_3, ..., J_{2L-3}$ are $\alpha_1^*, ..., \alpha_{l-1}^*$, respectively. Suppose the optimal solution of model (13) is $(\alpha_1^*, u_{r, id}^t, w_{id}^t, \forall r, i)$. Let $E_{d,j}^t = \frac{\sum_{r=1}^{s} u_{r, id}^t y_{rj} \sum_{i=1}^{m} w_{id}^t x_{ij}}{\sum_{i=1}^{m} w_{id}^t x_{ij}}$, $j \in J_{2l-2}$, $J_{2l-2}$ is then divided into two subsets as follows.

$$
J_{2l-1} = \{ j \} | E_{d,j}^t = \alpha_1^*, \forall j \in J_{2l-2} \\
J_{2l} = \{ j \} | E_{d,j}^t > \alpha_1^*, \forall j \in J_{2l-2}
$$

Using model (13), the peer-efficiency score of the DMUs in $J_{2l-1}$ is determined as $\alpha_1^*$. Assume that the number of DMUs in $J_{2l-1}$ is $n_l$. If $\sum_{l=1}^{l} = m + s - 2$, then the algorithm terminates, and $(\alpha_1^*, u_{r, id}^t, w_{id}^t, \forall r, i)$ is the unique optimal solution of model (13) (The reason is explained by Theorem 4). If $\sum_{l=1}^{l} < m+s-2$, then go to Step 3.

Assume that the above steps are repeated $w$ times ($w \leq n - 1$). The peer-
efficiency scores of DMU \(_j\), \(\forall j, j \neq d\), denoted as \(E_{dj}^*\), can be obtained as follows.

\[
E_{dj}^* = \alpha_{1j}^*, j \in J_1 \\
E_{dj}^* = \alpha_{2j}^*, j \in J_2 \\
\vdots \\
E_{dj}^* = \alpha_{k_j}^*, j \in J_{2k-1}
\]

**Theorem 3.** The optimal weights of model (12) are also the optimal weights of model (11), and the optimal value of model (12) is smaller than the optimal value of model (11).

**Proof:** The proof of this theorem is similar to that of Theorem 1. We omit it here.

Through Theorem 3, we know that the optimal weights of model (13) while \(l = 2\) are also the optimal weights of model (11), and \(\alpha_{aw}^* < \alpha_{2-1}^* < \ldots < \alpha_{1}^*\).

**Theorem 4.** If \(n_1 = m + s - 2\), then \((\alpha_{11}^*, u_{1r}^{sl}, u_{id}^{sl}, \forall i, r)\) is the unique optimal solution of model (11).

**Proof:** The proof of this theorem is similar to Theorem 2. We omit it here. Through Theorem 4, we know that if \(\sum_{t=1}^{l} n_t = m + s - 2\), then \((\alpha_{11}^*, u_{1r}^{sl}, u_{id}^{sl}, \forall i, r)\) is the unique optimal solution of model (11), and the algorithm terminates.

### 3.3. The solution method

It can be easily seen that the altruism cross-efficiency model and exclusiveness cross-efficiency model have some common properties which allow them to be solved using similar method. Here, we give an algorithm to solve the proposed altruism cross-efficiency model. Based on the algorithm, the exclusiveness cross-efficiency model can also be solved. First we use the following Figure 1 to show the workflow of the algorithm.

[Insert Figure 1 here]

This method notes that \(\beta\) is a parameter, and models (7), (8) and (9) should be solved as parametric linear programming models.
Step 1: Solving model (7). In model (7), the initial value of $\beta$ is set as $\min E_{dj}, \forall j$. Then, every time $\beta$ increases by a small positive number, for example, $\delta = 0.0001$, the increased times are denoted as $I$. Thus, $\beta = \min E_{dj}, \forall j + I \delta$, and $I = 0, 1, 2, \ldots$. Assume that when $I = I^*_1$, model (7) has no feasible solutions. The optimal value of model (7) can be obtained, that is $\beta^*_1 = \min E_{dj}, \forall j + (I^*_1 - 1) \delta$. If $n_1 = m + S - 2$, then the algorithm terminates, and $(\beta^*_1, u^*_{rd}, w^*_{ld}, \forall i, r)$ is the unique optimal solution of model (7). If $n_1 < m + S - 2$, then go to Step 2.

Step 2: Solving model (8). Since the optimal value of model (8) is larger than the optimal value of model (7), the initial value of $\beta$ in model (8) is set as $\min E_{dj}, \forall j + I^*_1 \delta$. Therefore, $\beta = \min E_{dj}, \forall j + I \delta$, and $I = I^*_1, I^*_1 + 1, I^*_1 + 2, \ldots$. Assume that when $I = I^*_2$, model (8) has no feasible solutions. The optimal value of model (8) can be obtained, that is $\beta^*_2 = \min E_{dj}, \forall j + (I^*_2 - 1) \delta$. If $n_1 + n_1 = m + S - 2$, then the algorithm terminates, and $(\beta^*_2, u^*_{rd}, w^*_{ld}, \forall i, r)$ is the unique optimal solution of model (8). If $n_1 + n_1 < m + S - 2$, go step 3.

Step 3: Solving model (9). Generally, in model (9), the initial value of $\beta$ is set as $\min E_{dj}, \forall j + I^*_{l-1} \delta$. Thus $\beta = \min E_{dj}, \forall j + I \delta$, and $I = I^*_{l-1}, I^*_{l-1} + 1, I^*_{l-1} + 2, \ldots$. Assume that when $I = I^*_l$, model (9) has no feasible solutions. The optimal value of model (9) can be obtained, that is, $\beta^*_l = \min E_{dj}, \forall j + (I^*_l - 1) \delta$. If $\sum_{t=1}^l n_t < m + S - 2$, repeat step 3 again, until we have $\sum_{t=1}^l n_t = m + S - 2$.

4. Numerical example

In this section, two numerical examples are used to verify the effectiveness and feasibility of the proposed models.

4.1. A simple example of 5 DMUs

Firstly, as shown in Table 2, there are five DMUs with three inputs (X1, X2, X3) and two outputs (Y1, Y2) to be evaluated.

| Insert Table 2 here |
| Insert Table 3 here |
The five DMUs are evaluated by CCR model (model (1)), the traditional cross-efficiency model (model (2)), the traditional benevolent model (model (4)) and the proposed altruism model. Table 3 shows the DMUs average efficiency scores and ranking orders, as obtained from each of the different models. The cross-efficiency scores of each DMU from models (2), (4) and (6) are given in Tables 4, 5 and 6, respectively. As shown in the second column of Table 3, four DMUs are evaluated as being efficient. This clearly indicates that DMUs cannot be effectively distinguished by the CCR model. In order to overcome this defect, the cross-efficiency evaluation model is used, including for models (2), (4) and the proposed altruism model. We found that the diagonal elements of Tables 4, 5 and 6 are identical, and these are the self-efficiency scores of the DMUs, as obtained from the CCR model. In addition, the peer-efficiency scores from model (2) are generally smaller than those from model (4) and model (6). For instance, the peer-efficiency scores of DMU1, as evaluated by DMU5 in models (2), (4) and the proposed altruism model are 0.595, 0.918, and 0.936, respectively. No large differences exist between the peer-efficiency scores of model (4) and model (6). This is because the main function of both model (4) and the altruism model is to maximize the other DMUs peer-efficiency scores, while simultaneously ensuring the self-efficiency score of the evaluated DMU is optimal. However, the main differences between model (4) and the altruism model lies in the following two aspects. On the one hand, model (4) has the objective of maximizing the average efficiency of other \( n - 1 \) DMUs, while the altruism model is supposed to max-min the efficiency of each of the other \( n - 1 \) DMUs. On the other hand, the altruism model could obtain a unique set of cross-efficiency scores, while model (4) fails to do that.
4.2. Application to flexible manufacturing systems

The numerical example from Shang and Sueyoshi [8] is then used to illustrate the proposed models. As shown in Table 7, there are 12 flexible manufacturing systems (FMSs) to be evaluated. Each FMS contains two inputs and four outputs.

- Input 1: the annual operating and depreciation cost ($100,000).
- Input 2: the floor space requirements of each specific system (1000 ft$^2$).
- Output 1: improvements in qualitative benefits (%).
- Output 2: work in process reduced (10).
- Output 3: average reduction in number of tardy jobs (%).
- Output 4: average increase in yield (%).

The 12 DMUs are evaluated by the CCR model (model (1)), the traditional cross-efficiency model (model (2)), the traditional benevolent model (model (4)), the traditional aggressive model (model (5)), the proposed altruism model and the proposed exclusiveness model. The corresponding results are reported in Table 8. Using model (1), the DMUs CCR efficiency scores and corresponding ranking order are obtained and are shown in the second and third columns of Table 8. Note that there are seven efficient DMUs, which cannot be further distinguished by model (1).

From Table 8, we found that the average cross-efficiency scores calculated by model (4) and the altruism model are larger than the corresponding average cross-efficiency scores calculated by model (5) and the exclusiveness model. This is because the intended objective of model (4) and the altruism model is to maximize other DMUs peer-efficiency scores. Conversely, the key objective of both model (5) and the exclusiveness model is to minimize the peer-efficiency scores of other DMUs.
The eleventh and thirteenth columns of Table 8 are the ranking orders of DMUs as obtained from the altruism model and exclusiveness model, respectively. We can see that these two ranking orders are totally different. For example, DMU7 is ranked in first place by the altruism model, whereas the same DMU is ranked in third place by the exclusiveness model. This indicates that different strategies will lead to different ranking results. Therefore, the decision makers should choose the appropriate model, according to the actual application.

Tables 9, 10, and 11 list the cross-efficiency scores obtained from model (1), the altruism model and the exclusiveness model, respectively. The diagonal elements in Tables 9, 10 and 11 are the self-efficiency scores of the DMUs, as obtained from model (1). By comparing those cross-efficiency scores, we make several findings. Firstly, we note that the peer-efficiency scores from the altruism model (or the exclusiveness model) are larger (or smaller) than the corresponding peer-efficiency scores from model (2). For example, the peer-efficiency scores of DMU1, as evaluated by DMU5 in model (2), the altruism model and the exclusiveness model are 0.879, 0.958, and 0.747, respectively. This indicates that the proposed altruism model (or exclusiveness model) has a good ability to maximize (or minimize) the peer-efficiency scores of other DMUs. Therefore, these models can be seen as an extension of traditional cross-efficiency evaluation methods. Secondly, the standard deviations of the DMUs cross-efficiency scores are reported in the last row. Note that the fluctuation of cross-efficiency scores in the altruism model is the smallest. This indicates that the altruism model can be used to reduce the differences in the cross-efficiency scores of DMUs. Finally, from Table 8, we observe that DMU9 is the worst of all the DMUs. In fact, DMU9 is always ranked in last place. An additional
finding is that most of the peer-efficiency scores evaluated by DMU9 are small. This indicates that if a DMUs peer-efficiency scores (when evaluated by other DMUs) are not large, small peer-efficiency scores will be given to other DMUs.

5. Conclusions

Cross-efficiency evaluation is an effective method for assessing efficiency. The major drawback of the cross-efficiency evaluation method is the appearance of non-unique optimal weights. To overcome this deficiency, a secondary goal is incorporated into cross-efficiency evaluation. Thus far, scholars have presented numerous secondary goal models, with multiple secondary goals. In this paper, two new secondary goal models are presented to further extend the existing secondary goal models. Specifically, when the evaluated DMU is cooperative with other DMUs, the altruism cross-efficiency model is proposed as a means to iteratively maximize the peer-efficiency score of each DMU. When the evaluated DMU is competing with other DMUs, the exclusiveness cross-efficiency model is presented as a means to minimize the peer-efficiency score of each of the other DMUs. The decision-makers can choose their preferred models, depending on the different application scenarios and requirements. In this paper, two numerical examples were used to illustrate the proposed models. Our results show that the proposed altruism model (exclusiveness model) has a good ability to maximize (or minimize) the peer-efficiency scores of other DMUs.

Our method brings at least three advantages to current cross-efficiency evaluation methods. Firstly, the competition and cooperation game theory are integrated into a DEA approach, in order to propose two cross-efficiency evaluation models. Compared to cross-efficiency evaluation methods, our proposed models aim to iteratively maximize (or minimize) each of the other DMUs efficiency ratings, as opposed to affecting the average overall efficiency of other DMUs. Secondly, although the models presented in this paper are nonlinear, an algorithm is proposed for solving the models. We have given two examples, in order to verify the effectiveness of the proposed algorithm. Finally, in order
to solve the non-uniqueness of cross-efficiency scores, we provide and prove the hypothesis of the unique solution of cross efficiency. This proof provides an even greater reason for all DMUs to accept the evaluation results.

The work of this paper can be extended in the future. For example, in some real-world applications, the input and output data are stochastic. In this case, our models are not applicable. Some further extensions might consider and address this problem, based on stochastic or fuzzy cross-efficiency evaluation methodologies.

Acknowledgement

This research is supported by Research Project of Philosophy and Social Sciences in Universities of Jiangsu (2016SJD630186) and Industry-University Collaborative Education Project of China’s Ministry of Education (No. 201801069100).
References


Appendix

Appendix A: Tables

Table 1: The notation

<table>
<thead>
<tr>
<th>Notations</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{ij}$</td>
<td>$i^{th}$ input consumed by DMU $j$</td>
</tr>
<tr>
<td>$y_{rj}$</td>
<td>$r^{th}$ output produced by DMU $j$</td>
</tr>
<tr>
<td>$\omega_{id}$</td>
<td>Weight of the $i^{th}$ input of DMU $d$</td>
</tr>
<tr>
<td>$\mu_{rd}$</td>
<td>Weight of the $r^{th}$ output of DMU $d$</td>
</tr>
<tr>
<td>$E_{dd}$</td>
<td>DMU $d$'s Self-evaluated efficiency</td>
</tr>
<tr>
<td>$E_{j}$</td>
<td>DMU $j$'s Cross-efficiency score</td>
</tr>
<tr>
<td>$E_{dj}$</td>
<td>Peer-efficiency score of DMU $j$ evaluated by DMU $d$</td>
</tr>
<tr>
<td>$e_{j}$</td>
<td>Maximum peer-efficiency score of DMU $j$ evaluated by DMU $d$</td>
</tr>
</tbody>
</table>

Table 2: The simple numerical example

<table>
<thead>
<tr>
<th>DMUs</th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>Y1</th>
<th>Y2</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU1</td>
<td>7</td>
<td>1</td>
<td>2</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>DMU2</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>DMU3</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>DMU4</td>
<td>5</td>
<td>6</td>
<td>1</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>DMU5</td>
<td>6</td>
<td>7</td>
<td>1</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 3: Efficiency scores and rankings of different models

<table>
<thead>
<tr>
<th>DMU</th>
<th>CCR model Rank</th>
<th>Classical model Rank</th>
<th>Benevolent model Rank</th>
<th>Altruism model Rank</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000 1</td>
<td>0.856 1</td>
<td>0.927 3</td>
<td>0.930 3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1.000 1</td>
<td>0.846 2</td>
<td>0.987 1</td>
<td>0.986 1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1.000 1</td>
<td>0.588 4</td>
<td>0.893 4</td>
<td>0.877 4</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>0.800 5</td>
<td>0.416 5</td>
<td>0.742 5</td>
<td>0.750 5</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>1.000 1</td>
<td>0.652 3</td>
<td>0.965 2</td>
<td>0.964 2</td>
<td>2</td>
</tr>
</tbody>
</table>

27
Table 4: Arbitrary cross-efficiency scores

<table>
<thead>
<tr>
<th>Rating DMU&lt;sub&gt;d&lt;/sub&gt;</th>
<th>Rated DMU&lt;sub&gt;j&lt;/sub&gt;</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.443</td>
<td>0.249</td>
<td>0.103</td>
<td>0.152</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.894</td>
<td>1.000</td>
<td>0.513</td>
<td>0.295</td>
<td>0.542</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.932</td>
<td>0.965</td>
<td>1.000</td>
<td>0.224</td>
<td>0.566</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.857</td>
<td>0.933</td>
<td>0.833</td>
<td>0.800</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.595</td>
<td>0.889</td>
<td>0.343</td>
<td>0.658</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Cross-efficiency scores of model (4)

<table>
<thead>
<tr>
<th>Rating DMU&lt;sub&gt;d&lt;/sub&gt;</th>
<th>Rated DMU&lt;sub&gt;j&lt;/sub&gt;</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
<td>0.804</td>
<td>0.684</td>
<td>0.862</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.941</td>
<td>1.000</td>
<td>0.865</td>
<td>0.798</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.918</td>
<td>1.000</td>
<td>1.000</td>
<td>0.686</td>
<td>0.964</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.857</td>
<td>0.933</td>
<td>0.833</td>
<td>0.800</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.918</td>
<td>1.000</td>
<td>0.964</td>
<td>0.743</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Cross-efficiency scores of the proposed altruism model

<table>
<thead>
<tr>
<th>Rating DMU&lt;sub&gt;d&lt;/sub&gt;</th>
<th>Rated DMU&lt;sub&gt;j&lt;/sub&gt;</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
<td>0.803</td>
<td>0.682</td>
<td>0.861</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.939</td>
<td>1.000</td>
<td>0.873</td>
<td>0.793</td>
<td>0.999</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.918</td>
<td>1.000</td>
<td>1.000</td>
<td>0.682</td>
<td>0.961</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.857</td>
<td>0.933</td>
<td>0.833</td>
<td>0.800</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.936</td>
<td>0.998</td>
<td>0.874</td>
<td>0.792</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>
Table 7: Data of 12 flexible manufacturing systems

<table>
<thead>
<tr>
<th>DMUs</th>
<th>Input 1</th>
<th>Input 2</th>
<th>Output 1</th>
<th>Output 2</th>
<th>Output 3</th>
<th>Output 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.02</td>
<td>5</td>
<td>42</td>
<td>45.3</td>
<td>14.2</td>
<td>30.1</td>
</tr>
<tr>
<td>2</td>
<td>16.46</td>
<td>4.5</td>
<td>39</td>
<td>40.1</td>
<td>13</td>
<td>29.8</td>
</tr>
<tr>
<td>3</td>
<td>11.76</td>
<td>6</td>
<td>26</td>
<td>39.6</td>
<td>13.8</td>
<td>24.5</td>
</tr>
<tr>
<td>4</td>
<td>10.52</td>
<td>4</td>
<td>22</td>
<td>36</td>
<td>11.3</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>9.5</td>
<td>3.8</td>
<td>21</td>
<td>34.2</td>
<td>12</td>
<td>20.4</td>
</tr>
<tr>
<td>6</td>
<td>4.79</td>
<td>5.4</td>
<td>10</td>
<td>20.1</td>
<td>5</td>
<td>16.5</td>
</tr>
<tr>
<td>7</td>
<td>6.21</td>
<td>6.2</td>
<td>14</td>
<td>26.5</td>
<td>7</td>
<td>19.7</td>
</tr>
<tr>
<td>8</td>
<td>11.12</td>
<td>6</td>
<td>25</td>
<td>35.9</td>
<td>9</td>
<td>24.7</td>
</tr>
<tr>
<td>9</td>
<td>3.67</td>
<td>8</td>
<td>4</td>
<td>17.4</td>
<td>0.1</td>
<td>18.1</td>
</tr>
<tr>
<td>10</td>
<td>8.93</td>
<td>7</td>
<td>16</td>
<td>34.3</td>
<td>6.5</td>
<td>20.6</td>
</tr>
<tr>
<td>11</td>
<td>17.74</td>
<td>7.1</td>
<td>43</td>
<td>45.6</td>
<td>14</td>
<td>31.1</td>
</tr>
<tr>
<td>12</td>
<td>14.85</td>
<td>6.2</td>
<td>27</td>
<td>38.7</td>
<td>13.8</td>
<td>25.4</td>
</tr>
</tbody>
</table>

Table 8: Efficiency scores and rankings of different models

<table>
<thead>
<tr>
<th>DMU</th>
<th>CCR model Rank</th>
<th>Classical model Rank</th>
<th>Benevolent model Rank</th>
<th>Aggressive model Rank</th>
<th>Altruism model Rank</th>
<th>Exclusiveness model Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>1</td>
<td>0.870</td>
<td>3</td>
<td>0.947</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>1.000</td>
<td>1</td>
<td>0.857</td>
<td>4</td>
<td>0.930</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>0.982</td>
<td>9</td>
<td>0.819</td>
<td>5</td>
<td>0.919</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>1.000</td>
<td>1</td>
<td>0.890</td>
<td>2</td>
<td>0.982</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>1.000</td>
<td>1</td>
<td>0.911</td>
<td>1</td>
<td>0.972</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>1.000</td>
<td>1</td>
<td>0.762</td>
<td>9</td>
<td>0.956</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>1.000</td>
<td>1</td>
<td>0.797</td>
<td>6</td>
<td>0.988</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0.961</td>
<td>10</td>
<td>0.764</td>
<td>8</td>
<td>0.931</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>1.000</td>
<td>1</td>
<td>0.560</td>
<td>12</td>
<td>0.749</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>0.954</td>
<td>11</td>
<td>0.653</td>
<td>11</td>
<td>0.815</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>0.983</td>
<td>8</td>
<td>0.776</td>
<td>7</td>
<td>0.908</td>
<td>9</td>
</tr>
<tr>
<td>12</td>
<td>0.801</td>
<td>12</td>
<td>0.701</td>
<td>10</td>
<td>0.773</td>
<td>11</td>
</tr>
</tbody>
</table>

29
Table 9: The arbitrary cross-efficiency scores

<table>
<thead>
<tr>
<th>Rating</th>
<th>DMU</th>
<th>Rated DMU</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.999</td>
<td>0.692</td>
<td>0.833</td>
<td>0.854</td>
<td>0.371</td>
<td>0.434</td>
<td>0.635</td>
<td>0.142</td>
<td>0.426</td>
<td>0.787</td>
<td>0.650</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.977</td>
<td>1.000</td>
<td>0.618</td>
<td>0.784</td>
<td>0.782</td>
<td>0.322</td>
<td>0.373</td>
<td>0.572</td>
<td>0.130</td>
<td>0.361</td>
<td>0.736</td>
<td>0.600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.000</td>
<td>0.959</td>
<td>0.982</td>
<td>0.927</td>
<td>1.000</td>
<td>0.932</td>
<td>1.000</td>
<td>0.927</td>
<td>0.422</td>
<td>0.760</td>
<td>0.977</td>
<td>0.801</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.955</td>
<td>0.981</td>
<td>0.784</td>
<td>1.000</td>
<td>0.990</td>
<td>0.501</td>
<td>0.550</td>
<td>0.676</td>
<td>0.257</td>
<td>0.490</td>
<td>0.746</td>
<td>0.736</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.879</td>
<td>0.879</td>
<td>0.775</td>
<td>0.925</td>
<td>1.000</td>
<td>0.398</td>
<td>0.463</td>
<td>0.572</td>
<td>0.095</td>
<td>0.401</td>
<td>0.671</td>
<td>0.719</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.707</td>
<td>0.699</td>
<td>0.837</td>
<td>0.882</td>
<td>0.889</td>
<td>1.000</td>
<td>0.991</td>
<td>0.767</td>
<td>0.891</td>
<td>0.731</td>
<td>0.679</td>
<td>0.685</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.683</td>
<td>0.661</td>
<td>0.871</td>
<td>0.884</td>
<td>0.932</td>
<td>0.990</td>
<td>1.000</td>
<td>0.747</td>
<td>0.779</td>
<td>0.745</td>
<td>0.654</td>
<td>0.700</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1.000</td>
<td>0.977</td>
<td>0.949</td>
<td>1.000</td>
<td>1.000</td>
<td>0.962</td>
<td>1.000</td>
<td>0.961</td>
<td>0.753</td>
<td>0.833</td>
<td>0.951</td>
<td>0.794</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.470</td>
<td>0.472</td>
<td>0.538</td>
<td>0.592</td>
<td>0.557</td>
<td>0.793</td>
<td>0.751</td>
<td>0.554</td>
<td>1.000</td>
<td>0.558</td>
<td>0.462</td>
<td>0.441</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.764</td>
<td>0.704</td>
<td>0.904</td>
<td>0.956</td>
<td>1.000</td>
<td>0.951</td>
<td>1.000</td>
<td>0.860</td>
<td>0.849</td>
<td>0.954</td>
<td>0.714</td>
<td>0.720</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1.000</td>
<td>0.966</td>
<td>0.924</td>
<td>0.896</td>
<td>0.927</td>
<td>0.954</td>
<td>1.000</td>
<td>0.945</td>
<td>0.672</td>
<td>0.783</td>
<td>0.983</td>
<td>0.759</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1.000</td>
<td>0.984</td>
<td>0.953</td>
<td>1.000</td>
<td>1.000</td>
<td>0.967</td>
<td>1.000</td>
<td>0.950</td>
<td>0.724</td>
<td>0.795</td>
<td>0.953</td>
<td>0.801</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard deviation
0.167 0.169 0.136 0.111 0.127 0.265 0.253 0.154 0.316 0.188 0.154 0.167

Table 10: Cross-efficiency scores of the altruism model

<table>
<thead>
<tr>
<th>Rating</th>
<th>DMU</th>
<th>Rated DMU</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.984</td>
<td>0.938</td>
<td>1.000</td>
<td>0.986</td>
<td>0.971</td>
<td>1.000</td>
<td>0.960</td>
<td>0.779</td>
<td>0.816</td>
<td>0.951</td>
<td>0.791</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.000</td>
<td>1.000</td>
<td>0.889</td>
<td>1.000</td>
<td>0.963</td>
<td>0.850</td>
<td>0.881</td>
<td>0.911</td>
<td>0.640</td>
<td>0.734</td>
<td>0.923</td>
<td>0.771</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.000</td>
<td>0.959</td>
<td>0.982</td>
<td>0.926</td>
<td>1.000</td>
<td>0.930</td>
<td>0.999</td>
<td>0.926</td>
<td>0.416</td>
<td>0.759</td>
<td>0.977</td>
<td>0.801</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.977</td>
<td>0.966</td>
<td>0.926</td>
<td>1.000</td>
<td>0.978</td>
<td>0.977</td>
<td>1.000</td>
<td>0.948</td>
<td>0.810</td>
<td>0.816</td>
<td>0.925</td>
<td>0.780</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.958</td>
<td>0.930</td>
<td>0.938</td>
<td>1.000</td>
<td>1.000</td>
<td>0.964</td>
<td>1.000</td>
<td>0.943</td>
<td>0.781</td>
<td>0.851</td>
<td>0.904</td>
<td>0.781</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.915</td>
<td>0.920</td>
<td>0.887</td>
<td>1.000</td>
<td>0.945</td>
<td>1.000</td>
<td>1.000</td>
<td>0.919</td>
<td>0.910</td>
<td>0.802</td>
<td>0.865</td>
<td>0.756</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.971</td>
<td>0.957</td>
<td>0.927</td>
<td>1.000</td>
<td>0.981</td>
<td>0.976</td>
<td>1.000</td>
<td>0.947</td>
<td>0.811</td>
<td>0.822</td>
<td>0.921</td>
<td>0.781</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1.000</td>
<td>0.977</td>
<td>0.949</td>
<td>1.000</td>
<td>1.000</td>
<td>0.962</td>
<td>1.000</td>
<td>0.961</td>
<td>0.753</td>
<td>0.833</td>
<td>0.951</td>
<td>0.794</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.780</td>
<td>0.808</td>
<td>0.810</td>
<td>1.000</td>
<td>0.888</td>
<td>1.000</td>
<td>0.973</td>
<td>0.850</td>
<td>1.000</td>
<td>0.780</td>
<td>0.725</td>
<td>0.700</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.764</td>
<td>0.704</td>
<td>0.904</td>
<td>0.956</td>
<td>1.000</td>
<td>0.951</td>
<td>1.000</td>
<td>0.860</td>
<td>0.849</td>
<td>0.953</td>
<td>0.714</td>
<td>0.720</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1.000</td>
<td>0.966</td>
<td>0.924</td>
<td>0.896</td>
<td>0.927</td>
<td>0.954</td>
<td>1.000</td>
<td>0.945</td>
<td>0.672</td>
<td>0.783</td>
<td>0.983</td>
<td>0.759</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1.000</td>
<td>0.984</td>
<td>0.953</td>
<td>1.000</td>
<td>1.000</td>
<td>0.967</td>
<td>1.000</td>
<td>0.950</td>
<td>0.723</td>
<td>0.795</td>
<td>0.953</td>
<td>0.801</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard deviation
0.086 0.087 0.044 0.036 0.036 0.039 0.034 0.037 0.146 0.055 0.090 0.032
### Table 11: Cross-efficiency scores of the exclusiveness model

<table>
<thead>
<tr>
<th>Rating</th>
<th>DMU&lt;sub&gt;i&lt;/sub&gt;</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.959</td>
<td>0.900</td>
<td>0.890</td>
<td>0.946</td>
<td>0.773</td>
<td>0.849</td>
<td>0.879</td>
<td>0.417</td>
<td>0.731</td>
<td>0.954</td>
<td>0.746</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.917</td>
<td>1.000</td>
<td>0.602</td>
<td>0.909</td>
<td>0.789</td>
<td>0.429</td>
<td>0.451</td>
<td>0.603</td>
<td>0.304</td>
<td>0.421</td>
<td>0.666</td>
<td>0.604</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.000</td>
<td>0.959</td>
<td>0.982</td>
<td>0.926</td>
<td>1.000</td>
<td>0.930</td>
<td>0.999</td>
<td>0.926</td>
<td>0.416</td>
<td>0.759</td>
<td>0.977</td>
<td>0.801</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.818</td>
<td>0.874</td>
<td>0.775</td>
<td>1.000</td>
<td>0.889</td>
<td>0.820</td>
<td>0.817</td>
<td>0.803</td>
<td>0.725</td>
<td>0.685</td>
<td>0.723</td>
<td>0.694</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.747</td>
<td>0.725</td>
<td>0.830</td>
<td>0.869</td>
<td>1.000</td>
<td>0.462</td>
<td>0.541</td>
<td>0.556</td>
<td>0.007</td>
<td>0.407</td>
<td>0.625</td>
<td>0.722</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.608</td>
<td>0.602</td>
<td>0.776</td>
<td>0.802</td>
<td>0.817</td>
<td>1.000</td>
<td>0.973</td>
<td>0.688</td>
<td>0.968</td>
<td>0.679</td>
<td>0.591</td>
<td>0.626</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.890</td>
<td>0.842</td>
<td>0.904</td>
<td>0.877</td>
<td>0.929</td>
<td>0.949</td>
<td>1.000</td>
<td>0.889</td>
<td>0.725</td>
<td>0.833</td>
<td>0.869</td>
<td>0.727</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1.000</td>
<td>0.977</td>
<td>0.949</td>
<td>1.000</td>
<td>1.000</td>
<td>0.962</td>
<td>1.000</td>
<td>0.961</td>
<td>0.753</td>
<td>0.833</td>
<td>0.951</td>
<td>0.794</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.359</td>
<td>0.367</td>
<td>0.423</td>
<td>0.482</td>
<td>0.436</td>
<td>0.699</td>
<td>0.643</td>
<td>0.451</td>
<td>1.000</td>
<td>0.468</td>
<td>0.356</td>
<td>0.347</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.764</td>
<td>0.704</td>
<td>0.904</td>
<td>0.956</td>
<td>1.000</td>
<td>0.951</td>
<td>1.000</td>
<td>0.860</td>
<td>0.849</td>
<td>0.953</td>
<td>0.714</td>
<td>0.720</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1.000</td>
<td>0.966</td>
<td>0.924</td>
<td>0.896</td>
<td>0.927</td>
<td>0.954</td>
<td>1.000</td>
<td>0.945</td>
<td>0.672</td>
<td>0.783</td>
<td>0.983</td>
<td>0.759</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1.000</td>
<td>0.984</td>
<td>0.953</td>
<td>1.000</td>
<td>1.000</td>
<td>0.967</td>
<td>1.000</td>
<td>0.950</td>
<td>0.723</td>
<td>0.795</td>
<td>0.953</td>
<td>0.801</td>
<td></td>
</tr>
</tbody>
</table>

|       | Standard deviation | 0.199 | 0.196 | 0.165 | 0.140 | 0.162 | 0.199 | 0.202 | 0.175 | 0.290 | 0.175 | 0.199 | 0.126 |

### Appendix B: Figures

![Figure 1: Workflow of the algorithm](image)

Figure 1: Workflow of the algorithm
Biographies

**Lei Li** is an assistant professor of School of Business, Applied Technology College of Soochow University. Her study mainly focuses on areas including performance evaluation and empirical study. He has published papers in journals such as Expert Systems and Entropy.

**Wei Dai** is a postgraduate student at School of Business, Soochow University, China. She obtained a Bachelor Degree from Shanghai Polytechnic University in 2016. Her research interests focus on operations research and decision-making models.

**Junfei Chu** received his doctoral degree in University of Science and Technology of China and CentraleSuplec. He is now an assistant professor at School of Business, Central South University. His research interests contain data envelopment analysis, supply chain management, and dynamic pricing. He has published papers in journals such as European Journal of Operational Research, International Journal of Production Research, Journal of Operational Research Society, and Annals of Operations Research.

**Xiaohong Liu** is a doctoral student at University of Science and Technology of China. Her research interests contain data envelopment analysis and environment and energy efficiency analysis. She has published articles in journals including Journal of Cleaner Production, Annals of Operations Research, and Energy systems.