Efficiency Measurement for Hierarchical Network Systems Using Network DEA and Intuitionistic Fuzzy ANP

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Abstract

According to the high importance of the university in the growth and development of the country, the efficiency of educational and research groups in universities is very important. The black box Data Envelopment Analysis (DEA) model is mathematical programming for measuring the relative efficiency of a set of Decision-making units (DMUs), without considering the operations of the component processes that may have misleading results. To overcome this defect, network models are recommended. This paper intends to propose a hybrid Intuitionistic Fuzzy Analytic Network Process (IFANP) and Network DEA (NDEA) technique to evaluate the efficiency of the faculty of basic sciences of Islamic Azad University. IFANP is used to evaluate the overall weights among all the criteria and sub-criteria and these weights are used in the NDEA model to measure the relative efficiency

The hypothetical example shows that the efficiency of all DMUs is equal to 1 by using the DEA, and there is no ranking between DMUs. The results of the IFANP-NDEA are more meaningful because of the full ranking of DMUs, considering component process operations. Also, it can prioritize efficient DMUs, provide the efficiencies of the DMU's functions which enables managers to identify areas of weakness.

Keywords: Efficiency, Ranking, Network Data Envelopment Analysis, Hierarchy Structure, Intuitionistic Fuzzy, Analytic Network process

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1. Introduction

Efficiency measurement refers to an important task in management, which not only indicates the past achievements of a unit but also represents the orientations of development in the future. The evaluation system is necessary for universities like every other organization, to be aware of the desirability of their activities, especially in complex and dynamic environments. Considering the role of education, research and generating science that are considered today for universities and higher education institutions, it is normal that, there is a need for designing a system for efficiency evaluation of these institutions in order to ensure that this task is realized and/or facilitated, and in this process, the weaknesses and strengths of their subsystems should be examined using specific criteria and scientific principles.

The Data Envelopment Analysis (DEA) model is considered as an effective method to measure the relative efficiency of Decision-Making Units (DMUs), which uses multiple inputs to generate multiple outputs. The system evaluation using conventional DEA models considers DMUs as black boxes regardless of their internal structure [1]. As a result, a system may be introduced as an efficient system, while all its process components may not be efficient. Significantly, there are cases where all process components of a DMU work worse than other DMUs, and at the same time, they have a better system performance.

For the first time, Färe & Grosskopf [2] presented an article entitled "Network Data Envelopment Analysis", which the importance of Network DEA (NDEA) was studied.

There are different NDEA models due to different structures of the systems. A hierarchical system can be divided into two groups of multicomponent or multi-function depending on whether each section has the same efficiency. In the multicomponent system, sections have the same function at all levels. In this type of system, there is no need to have the same number of sections in each level for different DMUs. Multicomponent hierarchy system is a network system with different levels of sections, while all sections use similar inputs to generate similar outputs. A multifunction system consists of several sections with a different function, where various inputs are used to generate different outputs.

Nowadays, we encounter several Multiple-criteria decision-making (MCDM) or MCDM analysis in daily life. The decision-making process faces many quantitative and qualitative criteria. The Analytic Hierarchy Process (AHP) has been introduced by Saaty and is considered to be one of the most comprehensive systems designed for MCDM. Also, the compatibility and incompatibility of the decision can be obtained by this method. It can be used when a decision-making practice faces with multi alternatives and several decision-making indicators. In 1996, Saaty [3] has provided a method for multi-criteria decision-making, called the Analytic Network Process (ANP).

The AHP method was developed and named the ANP method, which can consider correlations and feedback between effective elements in decision-making. In ANP, the decision-makers' opinions are expressed in the form of a certain number. But it may be impossible due to ambiguity and uncertainty in the evaluation because many of the criteria were inherently qualitative and subjective and it is impossible for the decision-maker to assign a certain number for their evaluation.

Some decision-making problems usually include imprecise, uncertain data that the decision-making process can cause more complex. To reflect the attribute - related information, most decision-makers tend to give linguistic variables or fuzzy variables instead of crisp values.

Fuzzy set theory was proposed by Zadeh [4,5] to reflect the uncertainties in human judgment. One of the applications of a fuzzy theory is fuzzy MCDM. For this purpose, in the most solving methods of AHP, fuzzy numbers have been used to do the paired comparison and the membership function is the basis of the determining of the weight of the criteria and sub-criteria. Huang [6] proposed a novel fuzzy ANP (FANP) model by solving a mathematical programming problem. Li et al. [7] used a generalized fuzzy number, which represents a different fuzzy number when the parameter changes.

In the decision-making process, the weight indicators or experts play an important role, as they have a direct effect on the accuracy of decision-making and ranking the results of the alternatives. The evaluation criteria usually include diverse opinions, so it is impossible to assume that each evaluation criteria is equally important [8]. The subjective methods determine the weights of indicators in terms of subjective preference or decision-makers. Delphi method [9], AHP [10] are a sample of subjective methods.

In objective methods such as DEA [11] and Entropy, the weight of indicators is achieved using objective decision matrix information or solving mathematical models. Subjective and objective methods have both advantages and disadvantages. For example, subjective methods can use the opinions of experts; yet objective methods are not dependent on human factors and ignore the opinion and experience of experts. For careful and scientific decisions, decision-makers need to provide qualitative or quantitative evaluations to determine the relative importance and importance of evaluation criteria.

Therefore, some integration methods have been proposed in many references [12-16]. Li et al. [17] proposed a dynamic fuzzy MCDM method. The method considers the integrated weight of the decision makers with subjective and objective preference.

The classical DEA method has considered organizations as a black box and limited their calculations to primary inputs and outputs and neglected internal processes. Cook et al. [18] during a study titled "Hierarchy and Groups in DEA" introduced a separate model to determine the efficiency of existing units at each level, and then, integrated the scores of two levels using three-step approaches. Sexton and Lewis [19] presented a two-stage DEA method to measure the efficiency of units that are produced in two stages. Mikhailov [20] presented a new approach to extract priorities from a paired comparison matrix based on the fuzzy alpha-cut analysis in a series of interval comparisons. Cook and Green [21] in another study entitled “Evaluating power plant efficiency: a hierarchical model” have introduced a hierarchical model that simultaneously calculates the efficiency of existing units at each level, with considering maximizing efficiency at a higher level. Kao [22] conducted a study entitled “Efficiency decomposition in network DEA: A relational model”. In this study, each network system is converted into a serial system using virtual processes, each step of which is a series system includes parallel structure. Chen et al. [23] introduced producing two-stage models of the DEA analysis by changing the technology and ranked the DMUs based on it. Castelli et al. [24] classified the main DEA models to evaluate the efficiency of DMUs when their internal structure is considered as a black box, but domestic processes have been considered. Kao and Hwang [25] used two serial and parallel structures of several models to evaluate the network DMUs that are defined based on the efficiency of the units. Therefore, they measured the efficiency of the two-step process for 24 insurance companies in Taiwan. Cook et al. [26] presented a multi-stage NDEA with a parallel process. Chen and Yan [27] proposed a DEA model to evaluate the supply chain performance in centralized, decentralized and hybrid modes. Despotis et al. [28] examined the NDEA by presenting a multi-objective linear programming model to the assessment of the academic research activity. Kashim et al. [29] presented a NDEA model with a parallel structure for effectiveness measurement of the university. The model included internal operations of educational and research functions in calculating the effectiveness of the academic system. Guo et al. [30] studied two-stage NDEA models with shared resources. Shafaghizadeh et al. [31] proposed combining two approaches Resilience and chance-constrained NDEA to measure performance decision-making and analyzing of a resilience supply chain using DEA in conditions of uncertainty

Mikhailov and Singh [32] presented a fuzzy extension of ANP that uses unknown human preferences as input data to the decision-making process, and a new fuzzy preference programming (FPP) method has been implemented. Mikhailov and Tsvetinov [33] introduced a new approach to cope with the uncertainty and ambiguity of the service evaluation process. Liu and Wang [34] presented new methods for solving multi-criteria decision-making problem in an intuitionistic fuzzy environment. Rouyendegh and Erol [35] introduced a DEA-FANP model to fully rank the departments of Amir Kabir University. Lin [36] used decision support tools using the Fuzzy DEA and ANP to select the personnel of the electrical and machinery company in Taiwan. Chen et al. [37] used the ANP-DEA model for optimization of decision-making on railway emergency plans. Özdemir [38] used a two-stage approach by combining ANP and DEA to evaluate the financial performance of banks. Zhang and Liu [39] proposed integrating ANP and game cross-efficiency DEA model to analyze the vulnerability of interdependent infrastructures. Ehsanifar [40] has performed a full ranking of DMUs using the DEA-ANP model. The proposed Fuzzy Priority method and uses the comparison ratios instead of exact numerical values and converts the initial fuzzy preference into nonlinear programming. Cui and Fang [41] used a hybrid DEA-ANP method permits assessing the relative complexity scores of engineering projects. Kumar et al. [42] used the Fuzzy AHP (FAHP) and DEA hybrid approach to achieve relative efficiency to identify inefficient service providers. Tavakoli et al. [43] introduced the ANP-DEA method for ranking organizational units as well as prioritizing the organization's human capital management. Abdullah & Najib [44] applied new IFAHP to establish a preference in the sustainable energy planning decision-making problem. Hu et al. [45] proposed the DEA/AHP hybrid approach, that pairwise comparison of AHP obtained with fuzzy DEA and used AHP to rank the units completely. Shariati et al. [46] proposed a new model based on the intuitionistic fuzzy sets (IFS) and ANP technique is proposed to evaluate the critical factors of the application of nanotechnology in the construction industry. Salehian et al. [47] proposed a novel hybrid algorithm based on FAHP and DEA to measure the efficiency of product transportation in road fleets of Iranian provinces. Mazumder et al. [48] developed a decision support framework by integrating the ANP and DEA approach in a manufacturing environment. Li et al. [49] investigated an intuitionistic fuzzy multiple attribute decision-making method based on weighted induced distance and its application to investment selection.

Kao [50] proposed the development of a NDEA model for hierarchical structure systems. Also, Kao [51] studied on NDEA and multi- stage serial processes and provided a full classification of studies in NDEA with respect to the type of network structure and the model used. The series and the parallel production processes are widely studied in literature. Lee and Worthington [52] presented a NDEA model for Australian universities operations in the area of research with regard to both quality and quantity. Koronakos et al. [53] evaluated the research performance of computer science in the UK using the NDEA approach. Koronakos

[54] described the underlying notions of NDEA methods and their advantages over the classical DEA ones. Also provided a full classification of the large volume of DEA literature in a unified manner. Chen et al. [55] presented several pitfalls in modeling NDEA starting from a simple two - stage network structure where only intermediate measures exist between the two stages and the first stage has inputs only and the second stage outputs only, and they discuss the difference between multiplier and envelopment two types of NDEA models and points out the functions of each of them.

This study was aimed to investigate the efficiency of the Islamic Azad University - faculty of sciences in two regions of education and research using the Intuitionistic Fuzzy ANP (IFANP) and NDEA methodology.

This paper is organized as follows: A preliminary introduction to the NDEA and IFANP is presented in section 2. This section introduces the basic definitions of intuitionistic fuzzy and triangular fuzzy numbers. In addition, an insight into some of the basic definitions in the IFSs is addressed. Section 3 explains the methodology and a numerical example is discussed in Section 4 and the conclusion is presented in Section 5.

2. Basic concepts

2.1. Intuitionistic Fuzzy Set

Atanassov [56,57] introduced Logic and IFSs in 1986. The author presented the IFS as a generalization of fuzzy sets that are known as by the membership function, the non-membership function and the hesitancy function. Intuitionistic fuzzy can be used, when there are doubts. Indeed, IFSs consider the degree of membership and non-membership proposes a suitable method for confronting the uncertain space governing the issues. uncertainty degree reflects the fact that decision-makers will not be able to select a certain membership degree always. However, this study is considered novel research because presents a new model using Intuitionistic fuzzy data.

Definition1: If X is a fixed infinite set, the IFS A in the reference set x is defined as:

$$A := \{< x, \mu_A(x), \upsilon_A(x) > x \in X\} \qquad (1)$$

Where, $\mu_A(x): \mathrm{x} \to [0,1]$ and $\upsilon_A(x): \mathrm{x} \to [0,1]$, determine the degree of membership and the degree of non-membership of the element $x \in X$. For each $x \in X$, for μ_A and υ_A we have:

$$0 \le \mu_A(x) + \upsilon_A(x) \le 1 \qquad (2)$$

Definition2: $\pi_{\bar{a}}(x)$ is the hesitancy intuitionistic fuzzy index of the element x in $\bar{a}$, if $\pi_{\bar{a}}(x) = 1 - \mu_{\bar{a}}(x) - \upsilon_{\bar{a}}(x)$.

Definition3: Triangular Intuitionistic fuzzy number (TIFN): $\tilde{a} = \langle (\underline{a}, a, \bar{a}); \omega_{\tilde{a}}, u_{\tilde{a}} \rangle$ is an IFS that is defined on the set of real numbers of R that has the membership function $\mu_{\tilde{a}}(x)$ and non-membership function $\upsilon_{\tilde{a}}(x)$ as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{(x-\underline{a})\omega_{\tilde{a}}}{(a-\underline{a})} & if \quad \underline{a} \le x < a, \\ \omega_{\tilde{a}} & if \quad x = a, \\ \dfrac{(\bar{a}-x)\omega_{\tilde{a}}}{(\bar{a}-a)} & if \quad a < x \le \bar{a}, \\ 0 & if \quad x < \underline{a} \ or \ x > \bar{a} \end{cases} \qquad (3) \qquad \upsilon_{\tilde{a}}(x) = \begin{cases} \dfrac{a - x + (x-\underline{a})u_{\tilde{a}}}{(a-\underline{a})} & if \quad \underline{a} \le x < a, \\ u_{\tilde{a}} & if \quad x = a, \\ \dfrac{[x - a + (\bar{a}-x)u_{\bar{a}}]}{(\bar{a}-a)} & if \quad a < x \le \bar{a}, \\ 1 & if \quad x < \underline{a} \ or \ x > \bar{a} \end{cases} \qquad (4)$$

As shown in Figure 1, the values $u_{\tilde{a}}, \omega_{\tilde{a}}$ representing the maximum degree of membership and the minimum degree of non-membership, respectively, as conditions $0 \le u_{\tilde{a}} + \omega_{\tilde{a}} \le 1, 0 \le u_{\tilde{a}} \le 1, 0 \le w_{\tilde{a}} \le 1$ are met.

2.2. Intuitionistic Fuzzy ANP

The ANP process has been widely used for decision-making in real problems, but despite its simplicity and efficiency, it has been criticized due to not considering the inaccuracy and uncertainty of perceptions of decision-makers and reflecting their views as a definite number. In the normal ANP, the decisions of the decision-makers are expressed in the form of a definite number, but this may not be possible due to the ambiguity and uncertainty in the assessment, because many criteria are intrinsically qualitative and subjective and assigning a definite number for their evaluation is impossible for the decision-maker. Therefore, decision-makers prefer to use fuzzy numbers for this purpose. Factors such as inadequate information and knowledge, complexity and intrinsic uncertainty in decision-making environments and the lack of appropriate criteria, make decision-makers more vulnerable to prioritization. Making comparisons based on fuzzy numbers is easier for the decision-maker because is more consistent with the uncertain nature of human judgments. For this reason, many researchers have tried to develop the ANP process to fuzzy space using the fundamental concepts of fuzzy sets theory and especially fuzzy numbers. In this attempt, triangular and trapezium fuzzy numbers have been more used, because the membership function of these continuous numbers is uniformly ascending or descending, therefore it can be understood by decision-makers easily, as well as performing math operations such as summation and multiplication, etc. will be easier. The purpose of the IFANP method is to maximize the membership function as well as minimize the non-membership function. For this reason, multi-objective modeling is used. The optimal solutions to this problem include the solutions that make maximize the degree of membership and minimize the degree of non-membership. According to this definition, the optimal solution will be obtained from the interfaces of the constraints and the objective function.
Mikhailov's fuzzy preferences linear programming can be used to solve the problems as follows. Consider a preference problem with n elements that paired comparison judgments are shown with normal fuzzy sets or fuzzy numbers. Mikhailov and Singh [32] proposed FPP to derive priority vectors from a set of interval comparisons.
Assume that the decision-maker provides a set of $F = \{\tilde{a}_{ij}\}$ with
$i = 1, ..., n-1; j = 2, ..., n; i < j; \mathrm{m} = \mathrm{n}(\mathrm{n}-1)$
fuzzy paired comparison in the form of triangular fuzzy numbers $\tilde{\mathrm{a}}_{ij} = (l_{ij}, \mathrm{m}_{ij}, u_{ij}), \delta_{ij}, \varepsilon_{ij}$. In other methods, $n(n-1)$ comparisons are needed to calculate the weight vector for the n factors, but in the Mikhailov method, the weight of the factors can be calculated with any number of comparisons $m = n(n-1)$.
Suppose that $\tilde{\mathrm{a}}_{ij} = (l_{ij}, \mathrm{m}_{ij}, u_{ij}), \delta_{ij}, \varepsilon_{ij}$, then the priority vector of $w = (w_1, ..., w_n)^T$ can be obtained based on the matrix of mental judgments and membership and non-membership functions, whose ratios are almost true in the initial judgments, $\tilde{\mathrm{a}}_{ij} = (l_{ij}, \mathrm{m}_{ij}, u_{ij})$. The set of n relative priorities should be normalized to sum of one, $\sum_{i=1}^{n} w_i = 1, w_i > 0, i = 1, ..., n$, so the number of independent local priorities
is (n − 1).

If the matrix of judgments is consistent, there will be several distinct weight vectors which are true in the following inequality:

$$l_{ij} \leq \frac{w_i}{w_j} \leq u_{ij}, i = 1,...,n-1; j = 2,...,n; i < j \quad (5)$$

But if the judgments are incompatible; there will not be any weight vector that was true simultaneously in the above inequality for all the elements of the matrix, therefore, it is reasonable that, rather than looking for achieving inequalities which are true for all conditions, we can determine the weights in such a way that "as far as possible" to be true in the inequalities. This means that a desirable solution is a solution which makes true the above inequality for almost all elements of the matrix, we have:

$$l_{ij} \tilde{\leq} \frac{w_i}{w_j} \tilde{\leq} u_{ij}, i = 1,...,n-1; j = 2,...,n; i < j \quad (6)$$

Where $\tilde{\leq}$ indicates "less than or equal".

In order to solve the inequality Equation 6, we introduce the following two simple inequality constraints:

$$\begin{aligned} & w_i - w_j u_{ij} \tilde{\leq} 0, \\ & -w_i + w_j l_{ij} \tilde{\leq} 0, \qquad\qquad (7) \\ & \mathrm{i} = 1,...,\mathrm{n}-1, \quad \mathrm{j} = 2,...,\mathrm{n}, \mathrm{i} < \mathrm{j}. \end{aligned}$$

The above set of m fuzzy constraints indicates in the form of a matrix as $R\omega \tilde{\leq} 0$, Where the matrix $R \in \Re^{m \times n}, m = n(n-1)$.

In order to determine the values of w_i, w_j, the feasible region of the interface of constraints is determined using the minimum or the maximum operator and the solution of the model can be achieved using minimum-maximum or maximum-minimum approach.

Consider a group with k decision-makers to evaluate n elements (clusters, criteria, sub-criteria, or alternatives). The membership function of the fuzzy linear constraint of the k[th] row of Rw which is defined as $R_k w \tilde{\leq} 0, k = 1,...,m$ can be expressed as follows:

$$\mu_k(R_k\omega) = \begin{cases} 1 & R_k\omega \leq 0, \\ 1 - \dfrac{R_k\omega}{d_k} & 0 \leq R_k\omega \leq d_k, \\ 0 & R_k\omega > d_k, \end{cases} \quad (8)$$

Where R_k denotes the k[th] row of R and w indicates priority vector, $w = (w_1,...,w_n)^T$.

Also d_k indicates the deviation from the correctness of definitive inequality of $R_k \leq 0$. In fact, it indicates the degree of deviation or tolerance of this equality. High membership function indicates decision-maker satisfaction by a particular weight vector and this satisfaction is, in fact, a symbol of the accuracy of the K[th]-constraint is in accordance with Equation 8. If a definite constraint $R_k \leq 0$ is severely violated, $\mu_k(R_k\omega)$ will be equal to zero. If it is met as much as possible, the relationship increases approximately linearly and approaches 1, and when the equation is fully met, it is larger than 1. The membership function is shown in Figure 2.

Now it is necessary to determine the common feasible region of these constraints first and then to identify their optimum point (maximum).
This problem can be solved using the Mikhailov FPP method as follows based on two assumptions [58]:
The first one requires the existence of a nonempty feasible region P on the (n-1) dimensional simplex Q^{n-1} plane.

$$Q^{n-1} = [(w_1, ..., w_n) \Big| w_i > 0, \sum_{i=1}^{n} w_i = 1] \qquad (9)$$

The second assumptions of the FPP method specifies a selection rule, which determines a priority vector with the highest degree of membership in the aggregated membership function. For the set of constraints of Equation 7, the membership function of the fuzzy feasible region, $\tilde{p}$ as follows:

$$\mu_{\tilde{P}}(w) = Min[\mu_1(\mathrm{R}_1\, w), ..., \mu_m(\mathrm{R}_m\, w) \; \big| w_1 + ... + w_n = 1] \qquad (10)$$

According to Equation 10, each of the membership functions $\mu_k(R_k \omega), \mathrm{k} = 1, ..., \mathrm{m}$ represents the distance, which can be optimally converted to Equation 11 by using infinity norm L_∞. More details are included in the appendix. The fuzzy feasible region is defined as an interface of all fuzzy constraints. If the initial judgment matrix is incompatible, in order to avoid an empty region (the state of the absence of a feasible region), the acceptable deviation value, d_k must be selected sufficiently large enough.
The feasible region indicates the overall consumer satisfaction with the definitive weight vector. Therefore, a priority vector must be defined at this stage that maximizes its degree of satisfaction.
$\mu_P(w)$ Is a convex set, and since a fuzzy feasible region $\tilde{P}$, and all fuzzy constraints are convex sets, there is always a point of w^* in the feasible region that maximizes the degree of membership in $\tilde{P}$.
the first step is to determine the priority vector with the highest degree of membership. It must be given a preference vector for membership functions to maximize the general degree of membership.
The FPP method assumes the priority vector with the highest membership degree. A priority vector must be defined for membership functions which maximize the total membership degree.
The maximum solution is a definite weight vector as w^*_{Max} that gives the maximum fuzzy feasible region:

$$\mu_{\tilde{P}}(w^*_{Max}) = \underset{w \in Q^{n-1}}{Max}\, Min[\mu_1(\mathrm{R}_1\, w), ..., \mu_m(\mathrm{R}_m\, w) \big| \, w_1 + ... + w_n = 1] \qquad (11)$$

The maximum-minimum fuzzy linear problem can be converted into a definite linear problem. Bellman & Zadeh (1970) proposed the basic decision-making processes in a fuzzy environment. They proposed maximal-minimum operator to find the maximum solution for decision-making problems with fuzzy goals and constraints and defined the variable α as follows:

$\alpha = Min[\mu_1(\mathrm{R}_1\, w),..., \mu_m(\mathrm{R}_m\, w)]$ (12)

$R_k w$, is Kth row of the fuzzy constraints set of membership functions. Therefore, the objective function to maximize membership functions is as follows:

$Max\ \alpha$ (13)

Here, the variable α indicates the degree of membership of a particular weight vector in the fuzzy feasible region of $\tilde{P}$. In order to solve the Equation 12, it is necessary α to be smaller than all degrees of membership within the function.

Using Equations 8 and 11, the following programming model that represents degree of membership of the fuzzy feasible region $\tilde{P}$, can be obtained:

$$\begin{aligned}&Max\ \alpha\\&s.t.\ d_k\alpha + R_k w \le d_k,\quad k = 1,...,m,\\&\sum_{i=1}^{n} w_i = 1, w_i > 0,\\&0 \le \alpha \le 1\end{aligned} \quad (14)$$

The objective function in terms of membership functions is defined as follows:

$$\begin{aligned}&Max\ \alpha\\&s.t.\ \alpha \le \mu_k(\mathrm{R}_k \mathrm{w}),\\&k = 1,...,m.\end{aligned} \quad (15)$$

The optimal solution for the linear problem is the (w^*, α^*) vector. The first component w^* is the relative vector, which represents the maximum degree of membership in the fuzzy feasible region, and the second component represents the maximum value of the degree of $\alpha^* = \mu_{\tilde{P}}(w^*)$.

Since the fuzzy feasible region $\tilde{P}$ and all fuzzy constraints are convex sets, there is always a point of w^* in the feasible region that minimizes the degree of membership in $\tilde{P}$.

The use of the minimum-maximum operator is the general method for finding solutions for decision problems with fuzzy goals and constraints.

At this stage, a priority vector for non-membership functions must be defined which minimizes total degree of non-membership. Therefore, it must find the vector w^*_{Min} in which the minimum the fuzzy feasible region is obtained, that is:

$$\mu_{\tilde{P}}(w^*_{Min}) = \underset{w \in Q^{n-1}}{Min}\ Max[\mu_1(\mathrm{R}_1\, w),..., \mu_m(\mathrm{R}_m\, w) \,|\, w_1 + ... + w_n = 1]$$

(16)

In order to obtain the minimizing solution, the variable β is defined as follows:

$\beta = \max\{\upsilon_1(R_1 w),..., \upsilon_m(R_m w)\}$ (17)

The non-membership function of the fuzzy linear constraint of the $R_k w \tilde{\le} 0, k = 1,...,m$ can be expressed as $\upsilon_k(R_k w)$.

Therefore, the objective function for non-membership functions is converted as follows:

$$\begin{aligned}&Min\beta\\&s.t.\ \beta \ge \upsilon_k(R_k w),\quad \mathrm{k} = 1,...,\mathrm{m}\end{aligned} \quad (18)$$

where υ_k the linear non-membership function characterizing the kth constraint $R_k w \tilde{\le} 0$.

According to the obtained equations 15 and 18 in the two previous steps, the multi-objective final model for both the membership and non-membership constraints will be written as follows:

$$\begin{aligned}
&Max\ \alpha \\
&Min\beta \\
&s.t.\ \alpha \le \mu_k(R_k w), \\
&\quad \beta \ge \upsilon_k(R_k w), \\
&\quad \sum_{i=1}^{n} w_i = 1, \\
&\quad \mathrm{k} = 1, \dots, \mathrm{m}.
\end{aligned} \tag{19}$$

For solving this model, for each α , once the minimum model $(f_0 : Min\ \alpha)$, and the maximum model, $(f_1 : Max\ \alpha)$ are calculated to obtain the maximum and minimum acceptable solution. Then, the minimum $(g_0 : Min\beta)$ and maximum $(g_1 : Max\beta)$ models are solved separately to minimize the maximum acceptable solution β as follows:

$$\begin{aligned}
&f_0 : Min\alpha \\
&s.t.\ \alpha \le \mu_k(R_k w), \\
&\quad \beta \ge \upsilon_k(R_k w), \\
&\quad \sum_{i=1}^{n} w_i = 1, \\
&\quad \mathrm{k} = 1, \dots, \mathrm{m}.
\end{aligned} \tag{20}$$

$$\begin{aligned}
&f_1 : Max\alpha \\
&s.t.\ \alpha \le \mu_k(R_k w), \\
&\quad \beta \ge \upsilon_k(R_k w), \\
&\quad \sum_{i=1}^{n} w_i = 1, \\
&\quad \mathrm{k} = 1, \dots, \mathrm{m}.
\end{aligned} \tag{21}$$

$$\begin{aligned}
&g_0 : Min\beta \\
&s.t.\ \alpha \le \mu_k(R_k w), \\
&\quad \beta \ge \upsilon_k(R_k w), \\
&\quad \sum_{i=1}^{n} w_i = 1, \\
&\quad \mathrm{k} = 1, \dots, \mathrm{m}.
\end{aligned} \tag{22}$$

$$\begin{aligned}
&g_1 : Max\beta \\
&s.t.\ \alpha \le \mu_k(R_k w), \\
&\quad \beta \ge \upsilon_k(R_k w), \\
&\quad \sum_{i=1}^{n} w_i = 1, \\
&\quad \mathrm{k} = 1, \dots, \mathrm{m}.
\end{aligned} \tag{23}$$

In the following, considering the solutions given for the above models, the membership functions of the two variables α, β are obtained as follows:

$$\mu_\alpha = \begin{cases} 0 & if \;\; \alpha \le f_{0(\alpha)} \\ \dfrac{\alpha - f_{0(\alpha)}}{f_{1(\alpha)} - f_{0(\alpha)}} & if \;\; f_{0(\alpha)} \le \alpha \le f_{1(\alpha)} \\ 1 & if \;\; \alpha \ge f_{1(\alpha)} \end{cases} \quad (24)$$

$$\mu_\beta = \begin{cases} 1 & if \;\; \beta \le g_0 \\ \dfrac{g_1 - \beta}{g_1 - g_0} & if \;\; g_0 \le \beta \le g_1 \\ 0 & if \;\; \beta \ge g_1 \end{cases} \quad (25)$$

According to the above membership functions, a maximal solution, a definite weight vector in the form of w^* which results in the maximum of the fuzzy acceptable region is defined as follows:

$$\mu_{\bar{P}}(w^*) = Max\left\{Min(\mu_\alpha, \mu_\beta\right\} \quad (26)$$

A model for the membership functions is defined using the maximum-minimum operator, where the variable θ is assumed as follows:

$$\theta = Min(\mu_\alpha, \mu_\beta) \quad (27)$$

Therefore, the objective function for the membership functions can be converted into the Equation 28:

$$\begin{aligned} & Max \;\; \theta \\ & s.t. \;\; \theta \le \mu_\alpha \\ & \qquad \theta \le \mu_\beta \end{aligned} \quad (28)$$

The final model of research will be as follows using the membership and non-membership functions:

$$\begin{aligned} & Max \;\; \theta \\ & s.t. \;\; \alpha - \theta(f_1 - f_0) \ge f_0 \\ & \qquad \beta + \theta(g_1 - g_0) \le g_1 \\ & \qquad \alpha w_j (m_{ij} - l_{ij}) - (w_i - l_{ij} w_j)\delta_{ij} \le 0; \\ & \qquad \alpha w_j (\mathrm{u}_{ij} - m_{ij}) - (\mathrm{u}_{ij}\, w_j - w_i)\delta_{ij} \le 0; \\ & \qquad m_{ij} w_j - w_i + (w_i - l_{ij} w_j)\varepsilon_{ij} - \beta w_j (m_{ij} - l_{ij}) \le 0; \\ & \qquad w_i - m_{ij} w_j + (\mathrm{u}_{ij}\, w_j - w_i)\varepsilon_{ij} - \beta w_j (\mathrm{u}_{ij} - m_{ij}) \le 0; \\ & \qquad \mathrm{w}_1 + \ldots + w_n = 1; \\ & \qquad \mathrm{w}_1, \ldots, w_n > 0; \\ & \qquad i = 1, \ldots, \mathrm{n}-1; \; \mathrm{j} = 2, \ldots, n; i < j. \end{aligned} \quad (29)$$

The equation 29 is a nonlinear model. Therefore, its solution is not possible using the Simplex method due to the nonlinearity of the model and it should be solved using appropriate methods and software such as lingo.

2.3. Network DEA

Although almost all organizations have a hierarchical structure, such systems have been considered less attention.

The organization usually has several units at the first level and several subunits in the second level. The subunits may be large, in this case, there are several sub-subunits with different functions at the third level, and this leveling can continue if necessary. At the highest level,

level 0, the inputs are distributed to the subset units at the first level. Similarly, each unit at the first level distributes the inputs to its subset units at the second level.
Suppose there is a system as Figure 3.

In the hierarchy system studied, each DMU has the same number of a unit at the first level with separate running functions. At the first level, if a unit has sub-units at a lower level, then the other DMUs must have the same number of sub-units with similar running functions so that the units of a DMU have a one-to-one correspondence with other DMUs.
The system is named according to this rule that the level 0 is known as the highest level, so that the subset of this level has four subunits of one, two, three, and four at the first level. Each sub-unit in the first level has two sub-units at the lower level. Subunit 1 has two sub-units (1,1), (1,2) and sub-unit 2 has two sub-units (2,1) and (2,2), sub-units 3 has two sub-units (3,1) and (3,2), the subset 4 has two sub-units (4,1) and (4,2). It should be noted that sub-units of one level don't have to have the same number of sub-subunit, even can have any sub-subunit.
Suppose that there are n DMUs with a similar structure to measure comparable efficiency and $X_{ij}^{(p)}$, $Y_{rj}^{(p)}$ are the i[th] input and the r[th] output of DMU_j from the unit p respectively. This system uses m inputs to generate s outputs. The inputs have two categories of shared and specific inputs and outputs are specific. In the hierarchical system method, the top management of the organization assigns inputs to the first level for the division into lower levels. Then, the inputs assigned to the first level divide into the second level. The input allocation process continues in this way up to the last level (the lowest level). The outputs divided at the lowest level are what is generated. Each unit has a specific input. It is assumed that the shared input is $x_i, i=1,...,n$; then, $\alpha_i x_i$ is distributed to unit 1, $\beta_i x_i$ to unit 2, $\gamma_i x_i$ to unit 3, $\delta_i x_i$ to unit 4 so that $\sum_{i=1}^{n}(\alpha_i+\beta_i+\gamma_i+\delta_i)=1$. In fact, the input entered unit 1 is equal to the sum of the specific input and the shared input. That is, the input of unit 1 is equal to $\alpha_i x_i + x^{(1)}$, thus the input of unit 2 is equal to $\alpha_i x_i + x^{(2)}$ and the input of unit 3 is equal to $\alpha_i x_i + x^{(3)}$ and the input of unit 4 is equal to $\alpha_i x_i + x^{(4)}$. Although each unit at different levels may not consume all m inputs and also does not generate the total s outputs. Each unit distributes its own inputs from its parent to its subunits in order to generate outputs. For example, unit (1) receives shared input $\alpha_i x_i$ from the parent unit (0) and also has its specific input as $x^{(1)}$.

The traditional black box DEA model is looking for coefficients u_r and v_i to generate the highest integrated share of outputs generated by the evaluating DMU to the total inputs used considering that the limitation of this ratio for each DMU should be less than or equal to one. In practice, the black box efficiency (E_{BB}) measurement model DMU_K with the assumption of a constant return to scale is as follows:

$$E_K^{BB} = \max \frac{\sum_{r=1}^{s} u_r (Y_{rk}^{(1,1)} + Y_{rk}^{(1,2)} + Y_{rk}^{(2,1)} + Y_{rk}^{(2,2)} + Y_{rk}^{(3,1)} + Y_{rk}^{(3,2)} + Y_{rk}^{(4,1)} + Y_{rk}^{(4,2)})}{\sum_{i=1}^{m} v_i X_{ik}} \tag{30}$$

$$\begin{aligned} s.t. \quad & \sum_{r=1}^{s} u_r (Y_{rk}^{(1,1)} + Y_{rk}^{(1,2)} + Y_{rk}^{(2,1)} + Y_{rk}^{(2,2)} + \\ & Y_{rk}^{(3,1)} + Y_{rk}^{(3,2)} + Y_{rk}^{(4,1)} + Y_{rk}^{(4,2)}) \\ & - \sum_{i=1}^{m} v_i X_{ij} \leq 0, \\ & j = 1, \ldots, n; \quad u_r, v_i \geq \varepsilon; \quad r = 1, \ldots, s; \quad i = 1, \ldots, m \end{aligned}$$

If the internal operations of the hierarchical system units are considered, then network efficiency (E^{NW}) can be measured. Since this system has four sub-units at different levels, there will be four constraints sets in the network model. By maximizing the efficiency of the DMU, which unit (0) at level (0), the objective function is as follows:

$$\begin{aligned} & E_k^{NW} = Max \frac{\sum_{r=1}^{s} u_r Y_{rk}}{\sum_{i=1}^{m} v_i X_{ik}} \\ & s.t. \\ & Unit(0): \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leq 0, \; j = 1, \ldots, n, \\ & Unit(1): \sum_{r=1}^{s} u_r Y_{rj}^{(1)} - \sum_{i=1}^{m} v_i (X_{ij}^{(1)} + \alpha_{ij} X_{ij}) \leq 0, \; j = 1, \ldots, n, \\ & Unit(2): \sum_{r=1}^{s} u_r Y_{rj}^{(2)} - \sum_{i=1}^{m} v_i (X_{ij}^{(2)} + \beta_{ij} X_{ij}) \leq 0, \; j = 1, \ldots, n, \\ & Unit(3): \sum_{r=1}^{s} u_r Y_{rj}^{(3)} - \sum_{i=1}^{m} v_i (X_{ij}^{(3)} + \gamma_{ij} X_{ij}) \leq 0, \; j = 1, \ldots, n, \\ & Unit(4): \sum_{r=1}^{s} u_r Y_{rj}^{(4)} - \sum_{i=1}^{m} v_i (X_{ij}^{(4)} + \delta_{ij} X_{ij}) \leq 0, \; j = 1, \ldots, n. \\ & u_r, v_i \geq \varepsilon, \quad r = 1, \ldots, \mathrm{s}, \; \mathrm{i} = 1, \ldots, \mathrm{m}. \end{aligned} \tag{31}$$

Such that

$$\begin{aligned} Y_{rj}^{(1)} &= Y_{rj}^{(1,1)} + Y_{rj}^{(1,2)} \\ Y_{rj}^{(2)} &= Y_{rj}^{(2,1)} + Y_{rj}^{(2,2)} \\ Y_{rj}^{(3)} &= Y_{rj}^{(3,1)} + Y_{rj}^{(3,2)} \\ Y_{rj}^{(4)} &= Y_{rj}^{(4,1)} + Y_{rj}^{(4,2)} \end{aligned} \tag{32}$$

The objective function is nonlinear, but can be converted into a linear form by assigning a value of 1 to a denominator, and can be eliminated from the numerator of the objective function.

$$
\begin{aligned}
& E_k^{NW} = Max \sum_{r=1}^{s} u_r Y_{rk} \\
& s.t. \quad \sum_{i=1}^{m} v_i X_{ik} = 1, \\
& Unit(0): \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0, \; j = 1,\ldots,n, \\
& Unit(1): \sum_{r=1}^{s} u_r Y_{rj}^{(1)} - \sum_{i=1}^{m} v_i (X_{ij}^{(1)} + \alpha_{ij} X_{ij}) \le 0, \; j = 1,\ldots,n, \\
& Unit(2): \sum_{r=1}^{s} u_r Y_{rj}^{(2)} - \sum_{i=1}^{m} v_i (X_{ij}^{(2)} + \beta_{ij} X_{ij}) \le 0, \; j = 1,\ldots,n, \\
& Unit(3): \sum_{r=1}^{s} u_r Y_{rj}^{(3)} - \sum_{i=1}^{m} v_i (X_{ij}^{(3)} + \gamma_{ij} X_{ij}) \le 0, \; j = 1,\ldots,n, \\
& Unit(4): \sum_{r=1}^{s} u_r Y_{rj}^{(4)} - \sum_{i=1}^{m} v_i (X_{ij}^{(4)} + \delta_{ij} X_{ij}) \le 0, \; j = 1,\ldots,n. \\
& u_r, v_i \ge \varepsilon, \quad r = 1,\ldots,\mathrm{s}, \; \mathrm{i} = 1,\ldots,\mathrm{m}.
\end{aligned} \tag{33}
$$

The network model (32) includes additional constraints. The sum of the constraints corresponding to units (1), (2), (3) and (4) is similar to the parent unit (0). Thus, the constraints corresponding to the unit (0) are additional and can be eliminated.

$$
\begin{aligned}
& E_k^{NW} = Max \sum_{r=1}^{s} u_r Y_{rk} \\
& s.t. \quad \sum_{i=1}^{m} v_i X_{ik} = 1, \\
& \quad Unit(1): \quad \sum_{r=1}^{s} u_r Y_{rj}^{(1)} - \sum_{i=1}^{m} v_i (X_{ij}^{(1)} + \alpha_{ij} X_{ij}) \le 0, \; j = 1,\ldots,n, \\
& \quad Unit(2): \quad \sum_{r=1}^{s} u_r Y_{rj}^{(2)} - \sum_{i=1}^{m} v_i (X_{ij}^{(2)} + \beta_{ij} X_{ij}) \le 0, \; j = 1,\ldots,n, \\
& \quad Unit(3): \quad \sum_{r=1}^{s} u_r Y_{rj}^{(3)} - \sum_{i=1}^{m} v_i (X_{ij}^{(3)} + \gamma_{ij} X_{ij}) \le 0, \; j = 1,\ldots,n, \\
& \quad Unit(4): \quad \sum_{r=1}^{s} u_r Y_{rj}^{(4)} - \sum_{i=1}^{m} v_i (X_{ij}^{(4)} + \delta_{ij} X_{ij}) \le 0, \; j = 1,\ldots,n. \\
& \quad u_r, v_i \ge \varepsilon, \quad r = 1,\ldots,\mathrm{s}, \; \mathrm{i} = 1,\ldots,\mathrm{m}.
\end{aligned} \tag{34}
$$

3. IFANP-NDEA Method

Managing many organizations has a hierarchical structure. DEA method has a general disadvantage due to its merely mathematical and therefore is not able to consider qualitative, inferential, and intuitive indicators. In the meantime, this disadvantage can be solved by some of the multi-criteria decision-making techniques, including ANP due to their specific characteristics. In practice, both DEA and ANP methods have been used. Both of them have restrictions, but their hybrid model has benefits of both methods and also remove restrictions.

The paper framework consists of four steps as Shown in Figure 4.

The present study has investigated the Faculty of Science of the Islamic Azad University. Education and research are the two main functions of a university. In many cases, the system consists of several parallel processes; for example, a university includes departments that use multiple inputs to generate multiple outputs. The Charnes, Cooper and Rhodes (CCR) model can be used to measure the efficiency of a university [11]. If the decision-maker is interested in the performance of a particular department, the model is applied for inputs and outputs of that department. Theoretically, if all the components of its processes are efficient, the system will be efficient. However, there is possible to design some examples that are inconsistent with this property due to system and process efficiency must be calculated independently. DEA investigates a system as a black box that ignores the internal relations of processes. The NDEA model is considered as one of the approaches to evaluating the system with several processes that consider the processes and the internal relations to calculate the efficiency. The main goal of the NDEA research is to open the black box of a system. It considers the elements of components when computing the efficiency. Figure 5 shows the structure of the Faculty of Basic Sciences with inputs and outputs.

The faculty of Basic Sciences includes mathematics, biology, chemistry, physics groups. The university can be studied from different aspects of research, educational, administrative, financial, student and cultural. Each group can be investigated in terms of education and research dimensions.

Each group has three specific inputs and three shared inputs. Specific inputs include specialized professors, bachelor students, and master and Ph.D. students. Shared inputs include general professors, staff and Area. The research outcomes of each group include books, research papers, ISI articles, research projects and outcomes of education include graduates. In the present study, there are 9 DMUs. Table 1 shows information about DMUs and in this Table.

Since the number of DMUs is less than three multiplied the sum of the inputs and outputs and in fact, the relation of $n \geq 3(m+s)$ is not met, for this reason, the weight restrictions are applied according to experts. The measurement model of network efficiency DMU_K is as follows with the assumption of a constant return to scale:

$$
\begin{aligned}
E_k^{NW} = Max \;\; & u_1 Y_{1k} + u_2 Y_{2k} + u_3 Y_{3k} + u_4 Y_{4k} + \\
& u_5 Y_{5k} + u_6 Y_{6k} + u_7 Y_{7k} + u_8 Y_{8k} + u_9 Y_{9k} + \\
& u_{10} Y_{10k} + u_{11} Y_{11k} + u_{12} Y_{12k} + u_{13} Y_{13k} + \\
& u_{14} Y_{14k} + u_{15} Y_{15k} + u_{16} Y_{16k} + u_{17} Y_{17k} + \\
& u_{18} Y_{18k} + u_{19} Y_{19k} + u_{20} Y_{20k} ; \\
\\
s.t. \quad & v_1 X_{1k} + v_2 X_{2k} + v_3 X_{3k} + v_4 X_{4k} + \\
& v_5 X_{5k} + v_6 X_{6k} + v_7 X_{7k} + v_8 X_{8k} + \\
& v_9 X_{9k} + v_{10} X_{10k} + v_{11} X_{11k} + v_{12} X_{12k} + \\
& v_{13} X_{13k} + v_{14} X_{14k} + v_{14} X_{14k} + v_{15} X_{15k} = 1 \\
& (u_1 Y_{1j} + u_2 Y_{2j} + u_3 Y_{3j} + u_4 Y_{4j}) + u_5 Y_{5j} - \\
& (v_1 X_{1j} + v_2 X_{2j} + v_3 X_{3j}) - \\
& (\alpha_1 v_{13} X_{13j} + \alpha_2 v_{14} X_{14j} + \alpha_3 v_{15} X_{15j}) \le 0, \\
& (u_6 Y_{6j} + u_7 Y_{7j} + u_8 Y_{8j} + u_9 Y_{9j}) + u_{10} Y_{10j} - \\
& (v_4 X_{4j} + v_6 X_{6j} + v_7 X_{7j}) - \\
& (\beta_1 v_{13} X_{13j} + \beta_2 v_{14} X_{14j} + \beta_3 v_{15} X_{15j}) \le 0, \\
& (u_{11} Y_{11j} + u_{12} Y_{12j} + u_{13} Y_{13j} + u_{14} Y_{14j}) + \\
& u_{15} Y_{15j} - (v_8 X_{8j} + v_9 X_{9j} + v_{10} X_{10j}) - \\
& (\gamma_1 v_{13} X_{13j} + \gamma_2 v_{14} X_{14j} + \gamma_3 v_{15} X_{15j}) \le 0, \\
& (u_{16} Y_{16j} + u_{17} Y_{17j} + u_{18} Y_{18j} + u_{19} Y_{19j}) + \\
& u_{20} Y_{20j} - (v_{10} X_{10j} + v_{11} X_{11j} + v_{12} X_{12j}) - \\
& (\delta_1 v_{13} X_{13j} + \delta_2 v_{14} X_{14j} + \delta_3 v_{15} X_{15j}) \le 0, \\
& \mathrm{u}_1, \ldots, u_{20} \ge 0; \quad v_1, \ldots, \mathrm{v}_{15} \ge 0;
\end{aligned}
\tag{35}
$$

As shown in the above model, $\alpha_i, \beta_i, \gamma_i, \delta_i, i = 1,2,3$ is the distributed input coefficient to each unit. The value of distributed input to each unit is obtained from the IFANP method.

3.1. Calculation of shared inputs' weights using IFANP

In this step, the paired comparisons matrix should be made. Using the Table 2, language variables are converted to triangular fuzzy numbers.

The model has three shared inputs named as x_{13}, x_{14}, x_{15}, it is necessary to calculate the coefficient of every shared input that distributed to each unit. The intuitionistic fuzzy paired comparisons have been used to calculate the distribution coefficient. A paired comparisons matrix has been made by a survey of 10 experts. Suppose X is a set of all the experts who participated in the survey.

$A(x) =$ The number of experts who agree; $D(x) =$ The number of experts who disagree;

$U(x) =$ The number of experts who are neither agrees nor disagree (no decision);

Suppose that n is the total number of experts who participate in the survey. We have:

$$\mu(x) = \frac{A(x)}{n}; \quad \nu(x) = \frac{D(x)}{n};$$
$$\pi(x) = 1 - \frac{A(x)}{n} - \frac{D(x)}{n} = \frac{n - A(x) - D(x)}{n}; \qquad (36)$$

Table 3 shows the results of the experts' survey for the paired comparisons of the groups for input x_{13}.

For example, for input x_{13}, in relation with the comparison between the group of biology with the mathematical group, 16 people were agreed, which are shown with the symbol $A(x)$ and two experts were not agreed, that represented by the symbol $D(x)$ and two experts did not participate in voting, that represented by the symbol $U(x)$.

The intuitionistic indicators of certainty and uncertainty and hesitancy are calculated according to the above formula. The results are shown in Table 4.

For example, a certain degree of 0.6 is calculated as follows:

$$\mu(x) = \frac{6}{10} = 0.6 \qquad (37)$$

The global weight that is distributed from each input to each group is as in Table 5.

According to the hierarchical structure, the sum of weights is equal to 1.

$$\alpha_1 + \alpha_2 + \alpha_3 + \beta_1 + \beta_2 + \beta_3 + \gamma_1 + \gamma_2 + \gamma_3 + \delta_1 + \delta_2 + \delta_3 = 1 \qquad (38)$$

The second stage is related to the use of the weights obtained from IFANP in the NDEA model. The IFANP-NDEA model is as follows:

$$
\begin{aligned}
E_k^{NW} = Max\ & u_1Y_{1k} + u_2Y_{2k} + u_3Y_{3k} + u_4Y_{4k} + u_5Y_{5k} + \\
& u_6Y_{6k} + u_7Y_{7k} + u_8Y_{8k} + u_9Y_{9k} + u_{10}Y_{10k} + u_{11}Y_{11k} + \\
& u_{12}Y_{12k} + u_{13}Y_{13k} + u_{14}Y_{14k} + u_{15}Y_{15k} + u_{16}Y_{16k} + u_{17}Y_{17k} + \\
& u_{18}Y_{18k} + u_{19}Y_{19k} + u_{20}Y_{20k} \\
s.t.\quad & v_1X_{1k} + v_2X_{2k} + v_3X_{3k} + v_4X_{4k} + v_5X_{5k} + v_6X_{6k} + \\
& v_7X_{7k} + v_8X_{8k} + v_9X_{9k} + v_{10}X_{10k} + v_{11}X_{11k} + \\
& v_{12}X_{12k} + v_{13}X_{13k} + v_{14}X_{14k} + v_{14}X_{14k} + v_{15}X_{15k} = 1 \\
& (u_1Y_{1j} + u_2Y_{2j} + u_3Y_{3j} + u_4Y_{4j}) + u_5Y_{5j} - (v_1X_{1j} + v_2X_{2j} + v_3X_{3j}) - \\
& (0.067*v_{13}X_{13j} + 0.119*v_{14}X_{14j} + 0.098*v_{15}X_{15j}) \le 0, \\
& (u_6Y_{6j} + u_7Y_{7j} + u_8Y_{8j} + u_9Y_{9j}) + u_{10}Y_{10j} - (v_4X_{4j} + v_6X_{6j} + v_7X_{7j}) - \\
& (0.067*v_{13}X_{13j} + 0.079*v_{14}X_{14j} + 0.080*v_{15}X_{15j}) \le 0, \\
& (u_{11}Y_{11j} + u_{12}Y_{12j} + u_{13}Y_{13j} + u_{14}Y_{14j}) + u_{15}Y_{15j} - (v_8X_{8j} + v_9X_{9j} + v_{10}X_{10j}) - \\
& (0.084*v_{13}X_{13j} + 0.079*v_{14}X_{14j} + 0.081*v_{15}X_{15j}) \le 0, \\
& (u_{16}Y_{16j} + u_{17}Y_{17j} + u_{18}Y_{18j} + u_{19}Y_{19j}) + u_{20}Y_{20j} - (v_{10}X_{10j} + v_{11}X_{11j} + v_{12}X_{12j}) - \\
& (0.066*v_{13}X_{13j} + 0.079*v_{14}X_{14j} + 0.098*v_{15}X_{15j}) \le 0, \\
& 5.7\times v_{13} \le v_{14};\quad 3.5\times v_{15} \le v_{13}; \\
& 5.7\times v_3 \le v_1;\quad 2.8\times v_2 \le v_3; \\
& 2.9\times v_6 \le v_4;\quad 2.5\times v_5 \le v_6; \\
& 5.8\times v_9 \le v_7;\quad 6.5\times v_8 \le v_9; \\
& 6.9\times v_{12} \le v_{10};\quad 4.5\times v_{11} \le v_{12}; \\
& 5.9\times u_1 \le u_3;\quad 5.7\times u_4 \le u_1;\quad 7.5\times u_2 \le u_4; \\
& 6.9\times u_6 \le u_8;\quad 5.6\times u_9 \le u_6;\quad 7.8\times u_7 \le u_9; \\
& 4.9\times u_{11} \le u_{13};\quad 3.9\times u_{14} \le u_{11};\quad 5.7\times u_{12} \le u_{14}; \\
& 4.8\times u_{16} \le u_{18};\quad 6.8\times u_{19} \le u_{16};\quad 5.7\times u_{17} \le u_{19}; \\
& \alpha_1, \alpha_2, \alpha_3, \alpha_4 \ge 0, \\
& \mathrm{u}_1, \dots, u_{20} \ge 0.00001; \\
& v_1, \dots, \mathrm{v}_{15} \ge 0.00001;
\end{aligned}
$$

(39)

4. Conclusion

The evaluation of the system with the traditional DEA model is like a black box which only focuses on the evaluation of the inputs and outputs, and the internal operations and interactions of processes are ignored. Therefore, a system may be introduced as an efficient system, but all of its process components may not be efficient. Significantly, there are cases where all the components of the process are worse than the other ones, while the performance of the system has a better system. In order to obtain meaningful efficiency, it is better to consider the internal structure of a system whenever the data is available. The approach that considers the operation component operations in DEA is called a NDEA. Systems can have different structures and there will be different NDEA models.

The IFANP method, a combination of IFS and ANP tools, is a powerful technique to obtain the relative importance of the evaluation criteria. The main objective of the present study is to investigate the efficiency of the Faculty of Basic Sciences of Islamic Azad University from two dimensions of education and research using the IFANP-NDEA method.

This paper employs the IFANP technique to obtain the relative weights of the evaluation criteria considered in the process of modeling the problem and

to rank the faculties based on their corresponding weights in NDEA model.

The Faculty of Science consists of mathematics, biology, chemistry and physics groups. Each group is investigated in terms of the educational and research aspects. Each group has three specific inputs and three shared inputs. Specialized inputs include specialized professors, bachelor students, and master and Ph.D. students and shared inputs include general professors, staff and area. The research outputs of each group include books, research papers, ISI articles,

research projects, and education outputs include graduates. In the present study, 9 DMUs are considered. Information about the 9 DMUs has been investigated. The results are as follows:
The results of the IFANP-NDEA model are as in Table 6.

As shown in the above table, in the second column the efficiency of the DMUs is calculated using the traditional black box DEA, the efficiency of all DMUs is equal to 1, and there is no ranking between DMUs. In fact, the DMUs are considered as a black box in calculating the efficiency using the traditional DEA method; that is, its internal structure is ignored, we will get the results shown in the second column of Table 6, in which there are twelve efficient universities and eight inefficient ones.

The efficiency is calculated using NDEA method by IFANP-NDEA model (E_{NW}) in the third column. In this calculation, the weights of shared inputs are obtained using IFANP. Then, these weights are used in the NDEA model. The DMU5 with the efficiency value of 0.66 has the lowest efficiency, and the DMU7 with the efficiency of 0.91 has the highest efficiency. For DMU5, the chemistry department with the efficiency of 0.55 has the lowest efficiency and the physics department with the efficiency of 0.78, has the highest efficiency among the groups. In addition, the internal efficiency of the groups mathematics$(E_{NW-Math})$, Biology $(E_{NW-Biology})$, chemistry$(E_{NW-Chemistry})$, and physics $(E_{NW-Physics})$ is also achieved in the fourth to seventh columns by using IFANP-NDEA model. The solution to this model is identical with considering or removing the zero unit.

This model allows managers to identify regions of weakness and improve overall efficiency by focusing on deficiencies. According to the results of the numerical example, the proposed model can achieve more significant results than DEA, because this model considers the operations of the internal processes and can also prioritize efficient units.

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Figures:

Figure 1. TIFN $\tilde{a} = \langle (\underline{a}, a, \overline{a}); \omega_{\tilde{a}}, u_{\tilde{a}} \rangle$

Figure 2. Membership Function

Figure 3. Hierarchical system

Figure 4. Methodology algorithm

Figure 5. DMU structure with inputs & outputs

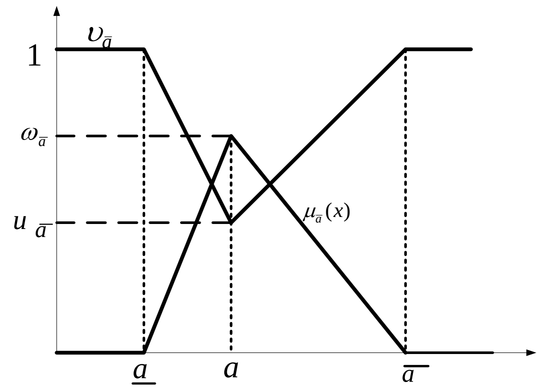

Figure 1. TIFN $\tilde{a} = \langle(\underline{a}, a, \bar{a}); \omega_{\tilde{a}}, u_{\tilde{a}}\rangle$

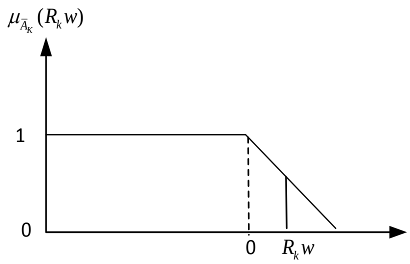

Figure 2. Membership Function

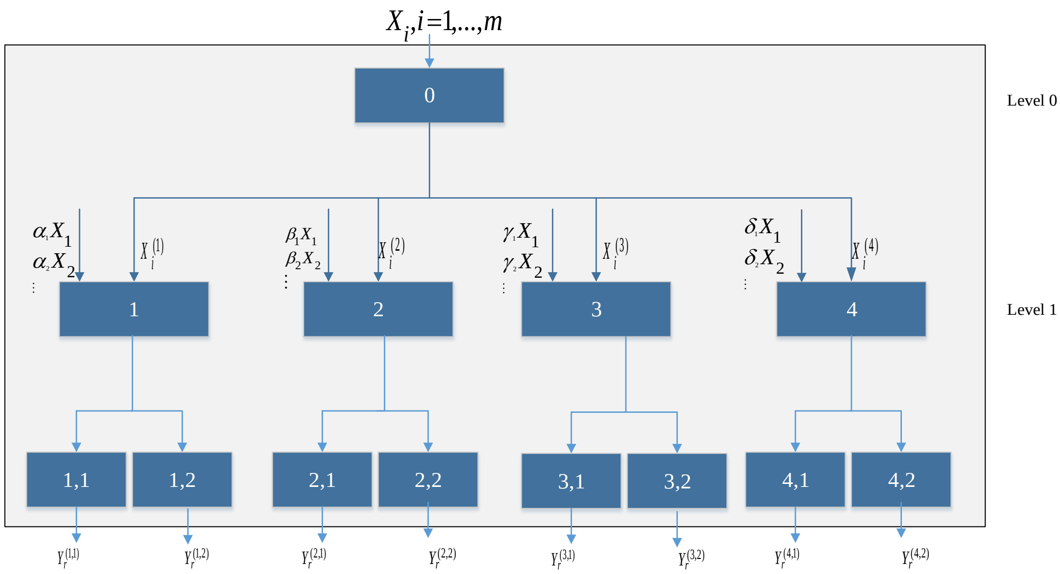

Figure 3. Hierarchical system

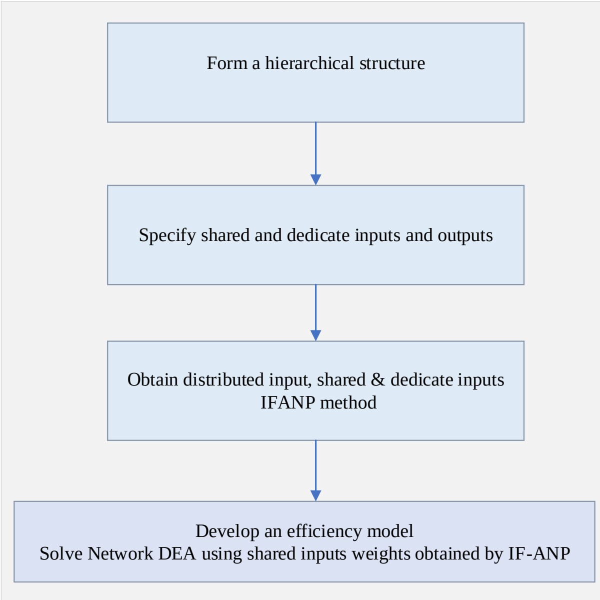

Figure 4. Methodology algorithm

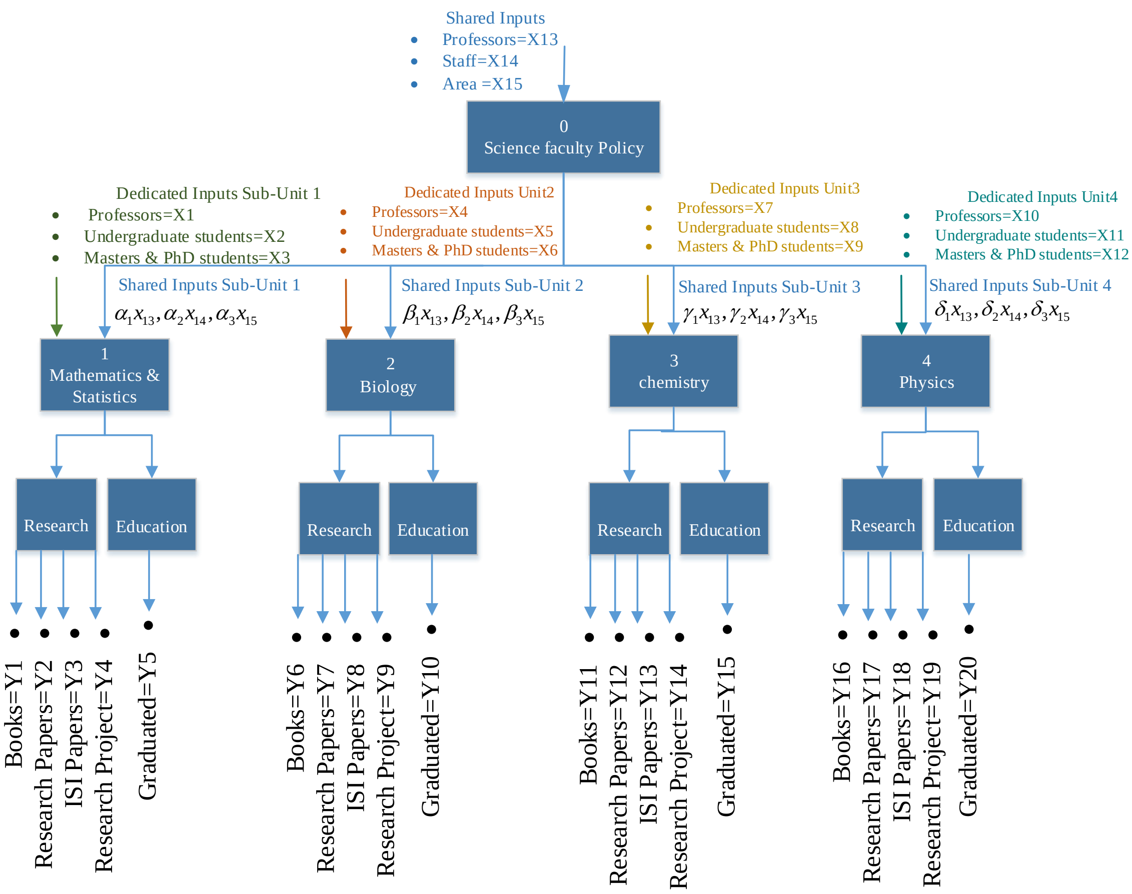

Figure 5. DMU structure with inputs & outputs

Tables:

Table 1. Information for DMUs

Table 2. Triangular Fuzzy Scale

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Table 1. Information for DMUs

2015-2016	*Inputs & Outputs*	*Mean*	*variance of entire population (Var.p)*	*variance of a sample set of data (Var.s)*	*standard deviation for the entire population (Std.p)*	*standard deviation for a sample set of data (Std.s)*
Mathematics and Statistics	X_1=Specialized Faculty members	22	41	46	6	7
	X_2=Undergraduate students	211	24081	27091	155	165
	X_3 =Masters & PhD students	261	8559	9629	93	98
	Y_1=Books	4	1	1	1	1
	Y_2=Research papers	26	76	85	9	9
	Y_3=International scientific indexing (ISI) Papers	10	27	30	5	5
	Y_4=Research projects	59	668	752	26	27
	Y_5=Graduated Students	171	2434	2738	49	52
Biology	X_4 =Specialized Faculty members	23	76	86	9	9
	X_5= Undergraduate students	524	49498	55685	222	236
	X_6= Masters & PhD students	354	9449	10630	97	103
	Y_6=Books	3	4	4	2	2
	Y_7=Research papers	44	406	457	20	21
	Y_8=ISI Papers	42	239	269	15	16
	Y_9=Research projects	47	488	550	22	23
	Y_{10}=Graduated Students	210	4508	5071	67	71
Chemistry	X_7 = Specialized Faculty members	28	178	201	13	14
	X_8 = Undergraduate students	451	41368	46539	203	216
	X_9= Masters & PhD students	167	4126	4641	64	68
	Y_{11}=Books	3	3	4	2	2
	Y_{12}=Research papers	15	22	25	5	5
	Y_{13}=ISI Papers	39	103	116	10	11
	Y_{14}=Research projects	48	722	812	27	28
	Y_{15}=Graduated Students	55	339	382	18	20
Physics	X_{10}= specialized Faculty members	59	224	252	15	16
	X_{11}=Undergraduate students	580	22938	25805	151	161
	X_{12} =Masters & PhD students	226	6223	7001	79	84
	Y_{16}=Books	2	1	2	1	1
	Y_{17}=Research papers	5	1	1	1	1
	Y_{18}=ISI Papers	49	220	248	15	16
	Y_{19}=Research projects	323	14200	15975	119	126
	Y_{20}=Graduated Students	365	11061	12443	105	112
Shared	X_{13}=General Teachers	28	176	198	13	14
	X_{14}=Staff	38	227	255	15	16
	X_{15}=Area	538	51407	57833	227	240

Table 2. Triangular Fuzzy Scale

Linguistic scale for importance	Triangular fuzzy scale	Triangular fuzzy reciprocal scale
Just Equal	(1, 1, 1)	(1, 1, 1)
Equally important (EI)	(1/2, 1, 3/2)	(2/3, 1, 2)
Weakly more important (WMI)	(1, 3/2, 2)	(1/2, 2/3, 1)
Strongly more important (SMI)	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
Very strongly more important (VSMI)	(2, 5/2, 3)	(1/3, 2/5, 1/2)
Absolutely more important (AMI)	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)

Table 3. Paired comparisons of the groups for input x_{13}

x_{13}	Mathematics			Biology			Chemistry			Physics		
	$A(x)$	$D(x)$	$U(x)$	$A(x)$	$D(x)$	$U(x)$	$A(x)$	$D(x)$	$U(x)$	$A(x)$	$D(x)$	$U(x)$
Mathematics				6	3	1	6	2	2	6	2	2
Biology	6	3	1				6	3	1	6	2	2
Chemistry	6	2	2	6	3	1				6	3	1
Physics	6	2	2	6	2	2	6	3	1			

Table 4. Certainty and uncertainty and hesitancy degree

x_{13}	Mathematics			Biology			Chemistry			physics		
	$\mu(x)$	$\nu(x)$	$\pi(x)$	$\mu(x)$	$\nu(x)$	$\pi(x)$	$\mu(x)$	$\nu(x)$	$\pi(x)$	$\mu(x)$	$\nu(x)$	$\pi(x)$
Mathematics				0.6	0.3	0.1	0.6	0.2	0.2	0.6	0.2	0.2
Biology	0.6	0.3	0.1				0.6	0.3	0.1	0.6	0.2	0.2
Chemistry	0.6	0.2	0.2	0.6	0.3	0.1				0.6	0.3	0.1
Physics	0.6	0.2	0.2	0.6	0.2	0.2	0.6	0.3	0.1			

Table 5. Global weights

Groups / Inputs	X_{13}	X_{14}	X_{15}
Mathematics	$\alpha_1 = 0.067$	$\alpha_2 = 0.119$	$\alpha_3 = 0.098$
Biology	$\beta_1 = 0.067$	$\beta_2 = 0.079$	$\beta_3 = 0.080$
Chemistry	$\gamma_1 = 0.084$	$\gamma_2 = 0.079$	$\gamma_3 = 0.081$
Physics	$\delta_1 = 0.066$	$\delta_2 = 0.079$	$\delta_3 = 0.098$

Table 6. Efficiency Results

DMU	Efficiency of whole system by Black box DEA (E_{BB})	Efficiency of whole system by IFANP-NDEA (E_{NW})	Efficiency of mathematics by IFANP-NDEA ($E_{NW-Math}$)	Efficiency of Biology by IFANP-NDEA ($E_{NW-Biology}$)	Efficiency of chemistry by IFANP-NDEA ($E_{NW-Chemistry}$)	Efficiency of physics by IFANP-NDEA ($E_{NW-Physics}$)
1	1	0.90	0.99	0.83	0.92	0.99
2	1	0.81	0.92	0.68	0.99	0.89
3	1	0.87	0.51	0.99	0.65	0.80
4	1	0.83	0.79	0.99	0.42	0.84
5	1	0.66	0.75	0.67	0.55	0.78
6	1	0.84	0.60	0.91	0.39	0.99
7	1	0.91	0.78	0.93	0.99	0.99
8	1	0.81	0.98	0.62	0.85	0.98
9	1	0.89	0.83	0.98	0.42	0.98

Appendix:

Consider model (10) again, we used the L_∞-norm as formula (40),

$$\mu_{\tilde{P}}(w) = \underset{k}{Min}\left\{\begin{array}{l} \left\| 1 - \frac{R_k w}{d_k} \right\|_\infty ; 0 \le \mathrm{R}_k\ w \le d_k, \\ 1; \mathrm{R}_k\ w \le 0, \quad 0; \mathrm{R}_k\ w \ge d_k, \\ | w_1 + ... + w_n = 1] \end{array}\right\} \qquad (40)$$

So we will have:

$$\mu_{\tilde{P}}(w^*_{\max}) = \underset{k}{MaxMin}\left\{\begin{array}{l} \left\| 1 - \frac{R_k w}{d_k} \right\|_\infty ; 0 \le \mathrm{R}_k\ w \le d_k, \\ 1; \mathrm{R}_k\ w \le 0, \quad 0; \mathrm{R}_k\ w \ge d_k, \\ | w_1 + ... + w_n = 1] \end{array}\right\} \qquad (41)$$

We named this as α as follows:

Finally, model (43) is formulated, that this model is the same as model (14), so we Converted the multi-objective function into a single objective by using L_∞-norm.

$$\begin{aligned} & Max\ \alpha \\ & s.t. \quad \alpha \le 1 - \frac{R_k w}{d_k}, k = 1, ..., m, \\ & \qquad \sum_{i=1}^{n} w_i = 1, w_i > 0, \\ & \qquad 0 \le \alpha \le 1. \end{aligned} \qquad (43)$$

$$\alpha = \underset{k}{MaxMin}\left\{\begin{array}{l} \left\| 1 - \frac{R_k w}{d_k} \right\|_\infty ; 0 \le \mathrm{R}_k\ w \le d_k, \\ 1; \mathrm{R}_k\ w \le 0, \quad 0; \mathrm{R}_k\ w \ge d_k, \\ | w_1 + ... + w_n = 1] \end{array}\right\} \qquad (42)$$

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