



Research Note

Efficiency measurement for hierarchical network systems using network DEA and intuitionistic fuzzy ANP

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Abstract. Regarding the high importance of university in the growth and development of a country, the efficiency of educational and research groups in universities is a vital consideration. The black box Data Envelopment Analysis (DEA) model is mathematical programming for measuring the relative efficiency of a set of Decision-Making Units (DMUs) without considering the operations of the component processes that may have misleading results. To overcome this problem, network models are recommended. This paper intends to propose a hybrid Intuitionistic Fuzzy Analytic Network Process (IFANP) and Network DEA (NDEA) technique to evaluate the efficiency of the Faculty of Basic Sciences of Islamic Azad University. IFANP was used to evaluate the overall weights among all the criteria and sub-criteria and the weights were in turn used in the NDEA model to measure the relative efficiency. A hypothetical example showed that the efficiency of all DMUs was equal to 1 by using the DEA and there was no ranking among the DMUs. The results of the IFANP-NDEA could be more meaningful with full ranking of the DMUs considering the component process operations. Finally, the model could prioritize the efficient DMUs and determine the efficiencies of the DMUs' functions. This model enables managers to identify the areas of weakness in the subject under their study.

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1. Introduction

Efficiency measurement is an important task in management, which not only indicates the past achieve-

ments of a unit, but also represents the orientations of development in the future. On the other hand, having an efficient evaluation system is necessary for universities, like every other organization, to be aware of the desirability of their activities, especially in complex and dynamic environments. Considering the role of education and research in the production of science, which are today the responsibility of universities and higher education institutions, normally there is an essential need for designing systems of efficiency evaluation for such institutions in order to ensure that this task is realized and/or facilitated. In this regard,

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identifying the pros and cons the subsystems of these institutions should be examined using specific criteria and scientific principles.

The Data Envelopment Analysis (DEA) model is considered as an effective method to measure the relative efficiency of Decision-Making Units (DMUs), which uses multiple inputs to generate multiple outputs. System evaluation through the conventional DEA models considers DMUs as black boxes regardless of their internal structure [1]. As a result, a system may be introduced as an efficient system while all its process components are not efficient. Significantly, there are cases in which all process components of a DMU work worse than other DMUs and, at the same time, they have a better system performance.

For the first time, Färe and Grosskopf [2] presented an article that studied the importance of Network DEA (NDEA). There are different NDEA models with regard to various structures of the systems. A hierarchical system can be divided into two groups of multicomponent or multi-function depending on whether each section has the same efficiency. In the multicomponent system, sections have the same function at all levels. In this type of system, there is no need to have the same number of sections in each level for different DMUs. Multicomponent hierarchy system is a network system with different levels of sections, while all sections use similar inputs to generate similar outputs. A multifunction system consists of several sections with different functions, where various inputs are used to generate different outputs.

Nowadays, we encounter several types of Multi-Criteria Decision-Making (MCDM) analysis in daily life. The decision-making process faces many quantitative and qualitative criteria. The Analytic Hierarchy Process (AHP) has been introduced by Saaty and is considered as one of the most comprehensive systems designed for MCDM. Compatibility or incompatibility of a decision can also be evaluated by this method. This method can be employed when decision-making faces multiple alternatives and several decision-making indicators. In 1996, Saaty [3] provided a method for MCDM, called the Analytic Network Process (ANP).

The ANP method, which has been developed from the AHP, can consider correlations and feedback between effective elements in decision-making. In ANP, the decision-makers' opinions are expressed in the form of certain numbers. However, this is not always possible due to ambiguity and uncertainty in evaluation considering the fact that many of the criteria are inherently qualitative and subjective; hence, it becomes impossible for a decision maker to assign a certain number to such criteria.

Some decision-making problems usually include imprecise, uncertain data that make the decision making process more complex. To reflect the attribute-

related information, most decision-makers tend to give linguistic variables or fuzzy variables instead of crisp values.

Fuzzy set theory was proposed by Zadeh [4,5] to reflect the uncertainties in human judgment. One of the applications of the fuzzy theory is fuzzy MCDM. For this purpose, in most of the solution methods of AHP, fuzzy numbers are used to conduct pairwise comparisons, where membership functions are employed as the basis for determining the weights of the criteria and sub-criteria. Huang [6] proposed a novel Fuzzy ANP (FANP) model by solving a mathematical programming problem. Li et al. [7] used a generalized fuzzy number, which represented a different fuzzy number when the parameter changed.

In the decision-making process, the weight indicators or experts play an important role, as they have a direct effect on the accuracy of decision-making and ranking of the results for alternatives. The evaluation criteria usually include diverse opinions, so it is impossible to assume that each evaluation criterion is equally important [8]. The subjective methods determine the weights of indicators in terms of the subjective preference of decision-makers. The Delphi method [9] and AHP [10] are samples of subjective methods.

In objective methods such as DEA [11] and entropy, the weights of indicators are achieved using objective decision matrix information or solving mathematical models. Subjective and objective methods have both advantages and disadvantages. For example, subjective methods can use the opinions of experts, yet objective methods are not dependent on human factors and ignore the opinion and experience of experts. For careful and scientific decisions, decision-makers need to provide qualitative or quantitative evaluations to determine the relative importance of evaluation criteria.

In this regard, some integration methods have been proposed in many references [12–16]. For instance, Li et al. [17] proposed a dynamic fuzzy MCDM method, which considered the integrated weight of decision makers with subjective and objective preferences.

The classical DEA method considers organizations as a black box, limits their calculations to primary inputs and outputs, and neglects internal processes. Cook et al. [18] in a study introduced a separate model to determine the efficiency of existing units at each level and then, integrated the scores of two levels using three-step approaches. Sexton and Lewis [19] presented a two-stage DEA method to measure the efficiency of units that were produced in two stages. Mikhailov [20] presented a new approach to extracting priorities from a pairwise comparison matrix based on the fuzzy alpha-cut analysis in a series of interval comparisons. Cook and Green [21] in another study

introduced a hierarchical model that simultaneously calculated the efficiency of existing units at each level in order to maximize efficiency at a higher level. Kao [22] conducted a study in which each network system was converted into a serial system using virtual processes and each step comprised in a series of systems with parallel structure. Chen et al. [23] developed two-stage models of the DEA analysis by changing the technology and ranked the DMUs. Castelli et al. [24] classified the main DEA models to evaluate the efficiency of DMUs wherein the internal structures were considered as black boxes, but domestic processes were incorporated. Kao and Hwang [25] used two serial and parallel structures of several models to evaluate the network DMUs that were defined based on the efficiency of the units. They measured the efficiency of the two-step process for 24 insurance companies in Taiwan. Cook et al. [26] presented a multi-stage NDEA with a parallel process. Chen and Yan [27] proposed a DEA model to evaluate the supply chain performance in centralized, decentralized, and hybrid modes. Despotis et al. [28] examined the NDEA by presenting a multi-objective linear programming model into the assessment of the academic research activity. Kashim et al. [29] presented an NDEA model with a parallel structure for effectiveness measurement of universities. The model included internal operations of educational and research functions in calculating the effectiveness of an academic system. Guo et al. [30] studied two-stage NDEA models with shared resources. Shafaghizadeh et al. [31] proposed combining the two approaches of resilience and chance-constrained NDEA to measure the performance of decision-making and analyze the resilience of the supply chain using DEA in conditions of uncertainty.

Mikhailov and Singh [32] presented a fuzzy extension of ANP that used unknown human preferences as input data to the decision-making process and they implemented a new Fuzzy Preference Programming (FPP) method. Mikhailov and Tsvetinov [33] introduced a new approach to coping with the uncertainty and ambiguity of the service evaluation process. Liu and Wang [34] presented new methods for solving MCDM problems in an intuitionistic fuzzy environment. Rouyendegh and Erol [35] introduced a DEA-FANP model to fully rank the departments of Amir Kabir University. Lin [36] developed decision support tools by using the Fuzzy DEA and ANP in order to select the personnel of an electrical and machinery company in Taiwan. Chen et al. [37] used the ANP-DEA model for optimization of decision-making on railway emergency plans. Özdemir [38] used a two-stage approach by combining ANP and DEA to evaluate the financial performance of banks. Zhang and Liu [39] proposed integrating ANP and game cross-efficiency DEA model to analyze the vulnerability of

interdependent infrastructures. Ehsanifar [40] performed a full ranking of DMUs using the DEA-ANP model. In this study, a fuzzy priority method was proposed, the comparison ratios were employed instead of exact numerical values, and the initial fuzzy preference was converted into a nonlinear programming. Cui and Fang [41] used a hybrid DEA-ANP method that allowed assessing the relative complexity scores of engineering projects. Kumar et al. [42] used the Fuzzy AHP (FAHP) and DEA hybrid approach in order to achieve relative efficiency in identifying inefficient service providers. Tavakoli et al. [43] introduced the ANP-DEA method for ranking organizational units as well as prioritizing an organization's human capital management. Abdullah and Najib [44] adopted a new IFAHP to establish a preference in the sustainable energy planning decision-making problem. Hu et al. [45] proposed the DEA/AHP hybrid approach by conducting a pairwise comparison of AHP with fuzzy DEA and used AHP to completely rank the units. Shariati et al. [46] proposed a new model based on the Intuitionistic Fuzzy Sets (IFSs) and ANP technique to evaluate the critical factors of the application of nanotechnology to the construction industry. Salehian et al. [47] proposed a novel hybrid algorithm based on FAHP and DEA for measuring the efficiency of product transportation in road fleets of the Iranian provinces. Mazumder et al. [48] developed a decision support framework by integrating the ANP and DEA approach in a manufacturing environment. Li et al. [49] investigated an intuitionistic fuzzy multiple-attribute decision-making method based on weighted induced distance and applied it to an investment selection problem.

Kao [50] proposed the development of an NDEA model for hierarchical structure systems. Furthermore, Kao [51] studied the NDEA and multi-stage serial processes and provided a full classification of studies on NDEA with respect to the type of network structure and the model used. The series and the parallel production processes have been widely studied in literature. Lee and Worthington [52] presented an NDEA model for the research operations of the Australian universities with regard to both quality and quantity. Koronakos et al. [53] evaluated the research performance of computer science in the UK through the NDEA approach. Koronakos [54] provided descriptions of the notions underlying the NDEA methods and discussed their advantages over the classical DEA methodologies. Koronakos also provided a full classification of a large volume of literature on DEA in a unified manner. Chen et al. [55] identified several pitfalls in the modeling of NDEA, originating from a simple two-stage network structure wherein only intermediate measures existed between two stages, with the first stage having only input and the second stage

only output. They discussed the difference between two types of NDEA models, namely multiplier and envelopment, and pointed out the functions of each.

This study is aimed to investigate the efficiency of the Faculty of Basic Sciences of the Islamic Azad University, in two areas of education and research, using the Intuitionistic Fuzzy ANP (IFANP) and NDEA methodologies.

This paper is organized as follows: a preliminary introduction to the NDEA and IFANP is presented in Section 2. This section provides basic definitions for intuitionistic fuzzy and triangular fuzzy numbers. In addition, an insight into the basics of the IFSs is provided. Section 3 explains the methodology and a numerical example is discussed in Section 4. Finally, Section 5 concludes the research.

2. Basic concepts

2.1. Intuitionistic fuzzy set

Atanassov [56,57] introduced the logics of the IFSs in 1986. The author presented the IFS as a generalization of the fuzzy sets, known by the membership function, the non-membership function, and the hesitancy function. Intuitionistic fuzzy can be used when there are doubts. Indeed, IFSs consider degrees for membership and non-membership functions, offering a suitable method to tackle the uncertainty that may govern an issue in hand. The degree of uncertainty reflects the fact that decision-makers will not be able to select a certain membership degree in all cases. However, the novelty of the present paper lies in putting forward a new model using the intuitionistic fuzzy data.

Definition 1: If X is a fixed infinite set, the IFS A in the reference set x is defined as:

$$A := \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}, \tag{1}$$

where, $\mu_A(x) : x \rightarrow [0, 1]$ and $\nu_A(x) : x \rightarrow [0, 1]$ determine the degrees of membership and non-membership of the element $x \in X$, respectively. For each $x \in X$, for μ_A and ν_A , we have:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1. \tag{2}$$

Definition 2: $\pi_{\tilde{a}}(x)$ is the hesitancy intuitionistic fuzzy index of the element x in \tilde{a} , if $\pi_{\tilde{a}}(x) = 1 - \mu_{\tilde{a}}(x) - \nu_{\tilde{a}}(x)$.

Definition 3: Triangular Intuitionistic Fuzzy Number (TIFN) $\tilde{a} = \langle (\underline{a}, a, \bar{a}); \omega_{\tilde{a}}, u_{\tilde{a}} \rangle$ is an IFS that is defined on the set of real numbers of R with the membership function $\mu_{\tilde{a}}(x)$ and non-membership function $\nu_{\tilde{a}}(x)$ as follows:

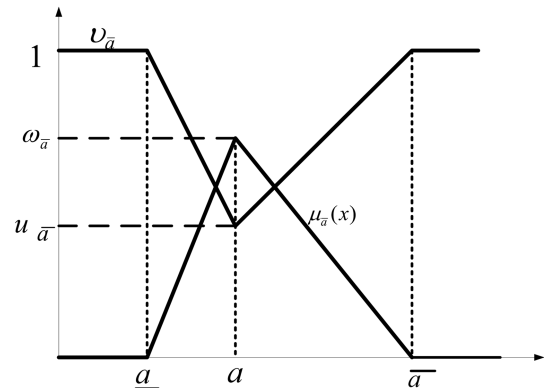


Figure 1. Triangular Intuitionistic Fuzzy Number (TIFN), $TIFN_{\tilde{a}=\langle (\underline{a}, a, \bar{a}); \omega_{\tilde{a}}, u_{\tilde{a}} \rangle}$.

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{(x-\underline{a})\omega_{\tilde{a}}}{(a-\underline{a})} & \text{if } \underline{a} \leq x < a, \\ \omega_{\tilde{a}} & \text{if } x = a, \\ \frac{(\bar{a}-x)\omega_{\tilde{a}}}{(\bar{a}-a)} & \text{if } a < x \leq \bar{a}, \\ 0 & \text{if } x < \underline{a} \text{ or } x > \bar{a} \end{cases} \tag{3}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} \frac{a-x+(x-\underline{a})u_{\tilde{a}}}{(a-\underline{a})} & \text{if } \underline{a} \leq x < a, \\ u_{\tilde{a}} & \text{if } x = a, \\ \frac{[x-a+(\bar{a}-x)u_{\tilde{a}}]}{(\bar{a}-a)} & \text{if } a < x \leq \bar{a}, \\ 1 & \text{if } x < \underline{a} \text{ or } x > \bar{a} \end{cases} \tag{4}$$

As shown in Figure 1, the variables $u_{\tilde{a}}$ and $\omega_{\tilde{a}}$ representing the maximum degree of membership and the minimum degree of non-membership, respectively, meet the condition $0 \leq u_{\tilde{a}} + \omega_{\tilde{a}} \leq 1$, and $0 \leq u_{\tilde{a}} \leq 1$, and $0 \leq \omega_{\tilde{a}} \leq 1$.

2.2. Intuitionistic fuzzy ANP

The ANP process has been widely used for decision-making in dealing with real problems. However, despite its simplicity and efficiency, the process has been criticized for coming short in incorporating the inaccuracy and uncertainty of the perceptions of decision-makers. In the normal ANP, the decisions of the decision-makers are expressed in the form of definite numbers, while this may not be appropriate due to the ambiguity and uncertainty of assessment. Of note, many criteria are intrinsically qualitative and subjective, hence assigning definite numbers to them in the evaluations is impossible for the decision-maker. Therefore, decision-makers prefer to use fuzzy numbers for this purpose. Factors such as inadequate information and knowledge, complexity and intrinsic uncertainty of decision-making environments, and the lack of appropriate criteria make decision-makers more vulnerable in prioritization. Making comparisons based on fuzzy numbers is easier for the decision-makers, because it is more consistent with the uncertain nature of human judgments. For this reason, many researchers have tried to develop the

ANP process to the fuzzy space using the fundamental concepts of fuzzy sets theory and, especially, fuzzy numbers. In regard to this, triangular and trapezium fuzzy numbers have more frequently been employed, because the membership functions of such continuous numbers are uniformly ascending or descending. As a result, they are more easily understandable and math operations such as summation and multiplication on them are done in a simpler way. The purpose of the IFANP method is to maximize the membership functions as well as to minimize the non-membership functions, for which multi-objective modeling is used. The optimal solutions to this problem are those that maximize the degree of membership and minimize the degree of non-membership. According to this definition, the optimal solution will be obtained from the interfaces of the constraints and the objective function.

Mikhailov’s fuzzy preferences linear programming can be used to solve such problems as follows.

Consider a preference problem with n elements wherein paired comparison judgments are shown with normal fuzzy sets or fuzzy numbers. Mikhailov and Singh [32] proposed FPP to derive priority vectors from a set of interval comparisons.

Assume that the decision-maker provides a set $F = \{\tilde{a}_{ij}\}$ with the fuzzy paired comparisons of:

$$i = 1, \dots, n - 1; \quad j = 2, \dots, n; \quad i < j;$$

$$m = n(n - 1),$$

in the form of triangular fuzzy numbers $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij}), \delta_{ij}, \varepsilon_{ij}$. In other methods, $n(n - 1)$ comparisons are needed to calculate the weight vector for n factors, but in the Mikhailov method, the weight of the factors can be calculated with any number of comparisons $m = n(n - 1)$.

Suppose that $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij}), \delta_{ij}, \varepsilon_{ij}$, then the priority vector of $w = (w_1, \dots, w_n)^T$ can be obtained based on the matrix of mental judgments and membership and non-membership functions, whose ratio is almost precise in the initial judgments, $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$. The set of n relative priorities should be normalized to the sum of one, $\sum_{i=1}^n w_i = 1, w_i > 0, i = 1, \dots, n$. Therefore, the number of independent local priorities is $(n - 1)$.

If the matrix of judgments is consistent, there will be several distinct weight vectors, which are true in the following inequality:

$$l_{ij} \leq \frac{w_i}{w_j} \leq u_{ij}, \quad i = 1, \dots, n - 1; \\ j = 2, \dots, n; \quad i < j. \tag{5}$$

However, if the judgments are incompatible, there will not be any weight vector that simultaneously holds in

the above inequality for all the elements of the matrix. Therefore, it is reasonable that, rather than looking for inequalities which hold for all conditions, we can determine the weights in such a way that they hold “as far as possible” in the inequalities. This means that a desirable solution is a solution which makes the above inequality holds for almost all elements of the matrix. We have:

$$l_{ij} \lesssim \frac{w_i}{w_j} \lesssim u_{ij}, \quad i = 1, \dots, n - 1; \quad j = 2, \dots, n; \\ i < j, \tag{6}$$

where \lesssim indicates “less than or equal to.”

In order to solve Ineq. (6), we introduce the following two simple inequality constraints:

$$w_i - w_j u_{ij} \lesssim 0, \\ -w_i + w_j l_{ij} \lesssim 0, \\ i = 1, \dots, n - 1, \quad j = 2, \dots, n, \quad i < j. \tag{7}$$

The above set of m fuzzy constraints is indicated in the form of a matrix as $R\omega \lesssim 0$, where $R \in \mathfrak{R}^{m \times n}$, $m = n(n - 1)$.

In order to determine the values of w_i and w_j , the feasible region for the interface of constraints is determined using the minimum or the maximum operator and the solution for the model can be achieved through the minimum-maximum or maximum-minimum approach.

Consider a group with k decision-makers to evaluate n elements (clusters, criteria, sub-criteria, or alternatives). The membership function of the fuzzy linear constraint of the k th row of Rw , which is defined as $R_k w \lesssim 0, k = 1, \dots, m$, can be expressed as follows:

$$\mu_k(R_k \omega) = \begin{cases} 1 & R_k \omega \leq 0, \\ 1 - \frac{R_k \omega}{d_k} & 0 \leq R_k \omega \leq d_k, \\ 0 & R_k \omega > d_k, \end{cases} \tag{8}$$

where R_k denotes the k th row of R and w indicates the priority vector, $w = (w_1, \dots, w_n)^T$.

Also, d_k indicates the deviation from correctness of the definitive inequality $R_k \leq 0$. In fact, it shows the degree of deviation or tolerance of the inequality. High membership function indicates decision-maker satisfaction if a particular weight vector and this satisfaction is, in fact, indicative of the accuracy of the K th constraint in accordance with Eq. (8). If a definite constraint $R_k \leq 0$ is severely violated, $\mu_k(R_k \omega)$ will be equal to zero. If it is as met as possible, the relationship rises in an approximately linear manner and approaches 1. Finally, when the equation is fully met, it is larger than 1. The membership function is shown in Figure 2.

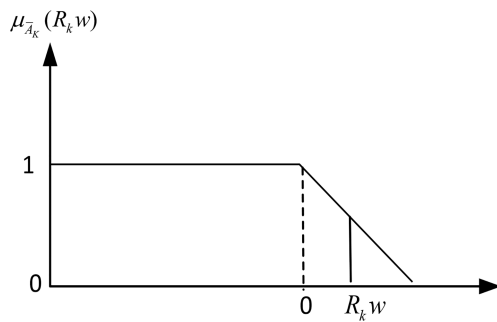


Figure 2. Membership function.

Now, it is necessary first to determine the common feasible region for the constraints and then to identify their optimum points (maximum).

This problem can be solved through the Mikhailov FPP method based on two assumptions as follows [58].

The first assumption requires the existence of a nonempty feasible region P on the $(n - 1)$ -dimensional simplex Q^{n-1} plane.

$$Q^{n-1} = \left[(w_1, \dots, w_n) \mid w_i > 0, \sum_{i=1}^n w_i = 1 \right]. \quad (9)$$

The second assumptions of the FPP method specifies a selection rule, which determines a priority vector with the highest degree of membership in the aggregated membership function.

For the set of constraints in Eq. (7), the membership function of the fuzzy feasible region \tilde{P} is as follows:

$$\mu_{\tilde{P}}(w) = \min[\mu_1(R_1 w), \dots, \mu_m(R_m w) \mid w_1 + \dots + w_n = 1]. \quad (10)$$

According to Eq. (10), each of the membership functions $\mu_k(R_k w)$, $k = 1, \dots, m$ represents the distance, which can be optimally converted to Eq. (11) by using the infinity norm L_∞ . More details are included in the appendix. The fuzzy feasible region is defined as an interface of all fuzzy constraints. If the initial judgment matrix is incompatible, in order to avoid an empty region (the state of the absence of a feasible region), the acceptable deviation value d_k must be selected sufficiently large.

The feasible region indicates the overall consumer satisfaction of the definitive weight vector. Therefore, a priority vector must be defined at this stage that maximizes the degree of satisfaction.

$\mu_{\tilde{P}}(w)$ is a convex set and since the fuzzy feasible region \tilde{P} and all the fuzzy constraints are convex sets, there is always a point w^* in the feasible region that maximizes the degree of membership in \tilde{P} .

The first step is to determine the priority vector with the highest degree of membership. A preference vector must be given for the membership functions to maximize the general degree of membership.

The FPP method assumes the priority vector with the highest membership degree. A priority vector must be defined for membership functions in a way that it maximizes the total membership degree.

The maximum solution is a definite weight vector w_{\max}^* that gives the maximum fuzzy feasible region:

$$\mu_{\tilde{P}}(w_{\max}^*) = \max_{w \in Q^{n-1}} \min[\mu_1(R_1 w), \dots, \mu_m(R_m w) \mid w_1 + \dots + w_n = 1]. \quad (11)$$

The maximum-minimum fuzzy linear problem can be converted into a definite linear problem. Bellman and Zadeh (1970) proposed the basic decision-making processes in a fuzzy environment. They proposed the maximal-minimal operator to find the maximum solution for decision-making problems with fuzzy goals and constraints, and defined the variable α as follows:

$$\alpha = \min[\mu_1(R_1 w), \dots, \mu_m(R_m w)]. \quad (12)$$

$R_k w$ is the K th row of the fuzzy constraints set of membership functions. Therefore, the objective function to maximize membership functions is as follows:

$$\max \alpha. \quad (13)$$

Here, the variable α indicates the degree of membership of a particular weight vector in the fuzzy feasible region of \tilde{P} . In order to solve Eq. (12), it is necessary for α to be smaller than all degrees of membership within the function.

Using Eqs. (8) and (11), the following programming model that represents the degree of membership of the fuzzy feasible region \tilde{P} can be obtained:

$$\begin{aligned} \max \alpha, \\ \text{s.t. } d_k \alpha + R_k w \leq d_k, \quad k = 1, \dots, m, \\ \sum_{i=1}^n w_i = 1, \quad w_i > 0, \\ 0 \leq \alpha \leq 1. \end{aligned} \quad (14)$$

The objective function in terms of membership functions is defined as follows:

$$\begin{aligned} \max \alpha, \\ \text{s.t. } \alpha \leq \mu_k(R_k w), \quad k = 1, \dots, m. \end{aligned} \quad (15)$$

The optimal solution for the linear problem is the (w^*, α^*) vector. The first component w^* is the relative vector, which represents the maximum degree of membership in the fuzzy feasible region, and the second component represents the maximum value of the degree of $\alpha^* = \mu_{\tilde{P}}(w^*)$.

Since the fuzzy feasible region \tilde{P} and all the fuzzy constraints are convex sets, there is always a point w^* in the feasible region that minimizes the degree of membership in \tilde{P} .

The use of the minimum-maximum operator is a general method for finding solutions to decision problems with fuzzy goals and constraints.

At this stage, a priority vector for non-membership functions must be defined, which minimizes the total degree of non-membership. Therefore, the vector w_{\min}^* should be identified by which the minimum fuzzy feasible region is obtained, that is:

$$\mu_{\tilde{P}}(w_{\min}^*) = \min_{w \in Q^{n-1}} \max[\mu_1(R_1 w), \dots, \mu_m(R_m w)]$$

$$[w_1 + \dots + w_n = 1]. \tag{16}$$

In order to obtain the minimizing solution, the variable β is defined as follows:

$$\beta = \max\{v_1(R_1 w), \dots, v_m(R_m w)\}. \tag{17}$$

The non-membership function of the fuzzy linear constraint of $R_k w \leq 0, k = 1, \dots, m$ can be expressed as $v_k(R_k w)$.

Therefore, the objective function for non-membership functions is converted into:

$$\min \beta,$$

$$\text{s.t. } \beta \geq v_k(R_k w), \quad k = 1, \dots, m, \tag{18}$$

where v_k is the linear non-membership function characterizing the k th constraint $R_k w \leq 0$.

According to the obtained Eqs. (15) and (18) in the two previous steps, the final multi-objective model for both the membership and non-membership constraints will be written as follows:

$$\max \alpha, \quad \min \beta,$$

$$\text{s.t. } \alpha \leq \mu_k(R_k w), \quad \beta \geq v_k(R_k w),$$

$$\sum_{i=1}^n w_i = 1, \quad k = 1, \dots, m. \tag{19}$$

For solving this model, for each α , the minimum model ($f_0 : \min \alpha$) and the maximum model ($f_1 : \max \alpha$) are calculated to obtain the maximum and minimum acceptable solutions. Then, the minimum ($g_0 : \min \beta$) and maximum ($g_1 : \max \beta$) models are solved separately to minimize the maximum acceptable solution β as follows:

$$f_0 : \min \alpha,$$

$$\text{s.t. } \alpha \leq \mu_k(R_k w), \quad \beta \geq v_k(R_k w),$$

$$\sum_{i=1}^n w_i = 1, \quad k = 1, \dots, m. \tag{20}$$

$$f_1 : \max \alpha,$$

$$\text{s.t. } \alpha \leq \mu_k(R_k w), \quad \beta \geq v_k(R_k w),$$

$$\sum_{i=1}^n w_i = 1, \quad k = 1, \dots, m, \tag{21}$$

$$g_0 : \min \beta,$$

$$\text{s.t. } \alpha \leq \mu_k(R_k w), \quad \beta \geq v_k(R_k w),$$

$$\sum_{i=1}^n w_i = 1, \quad k = 1, \dots, m, \tag{22}$$

$$g_1 : \max \beta,$$

$$\text{s.t. } \alpha \leq \mu_k(R_k w), \quad \beta \geq v_k(R_k w),$$

$$\sum_{i=1}^n w_i = 1, \quad k = 1, \dots, m. \tag{23}$$

In the following, considering the solutions given for the above models, the membership functions of the two variables α and β are obtained as follows:

$$\mu_\alpha = \begin{cases} 0 & \text{if } \alpha \leq f_0(\alpha) \\ \frac{\alpha - f_0(\alpha)}{f_1(\alpha) - f_0(\alpha)} & \text{if } f_0(\alpha) \leq \alpha \leq f_1(\alpha) \\ 1 & \text{if } \alpha \geq f_1(\alpha) \end{cases} \tag{24}$$

$$\mu_\beta = \begin{cases} 1 & \text{if } \beta \leq g_0 \\ \frac{g_1 - \beta}{g_1 - g_0} & \text{if } g_0 \leq \beta \leq g_1 \\ 0 & \text{if } \beta \geq g_1 \end{cases} \tag{25}$$

According to the above membership functions, a maximal solution in terms of a definite weight vector w^* , which leads to the maximum fuzzy acceptable region, is defined as follows:

$$\mu_{\tilde{P}}(w^*) = \max \{\min(\mu_\alpha, \mu_\beta)\}. \tag{26}$$

A model for the membership functions is defined using the maximum-minimum operator, wherein the variable θ is assumed as follows:

$$\theta = \min(\mu_\alpha, \mu_\beta). \tag{27}$$

Therefore, the objective function for the membership functions can be converted into Eq. (28):

$$\max \theta,$$

$$\text{s.t. } \theta \leq \mu_\alpha \quad \theta \leq \mu_\beta. \tag{28}$$

The final model using the membership and non-membership functions will be as follows:

$$\max \theta,$$

$$\text{s.t.}$$

$$\alpha - \theta(f_1 - f_0) \geq f_0,$$

$$\beta + \theta(g_1 - g_0) \leq g_1,$$

$$\alpha w_j(m_{ij} - l_{ij}) - (w_i - l_{ij} w_j) \delta_{ij} \leq 0;$$

$$\begin{aligned} \alpha w_j(u_{ij} - m_{ij}) - (u_{ij}w_j - w_i)\delta_{ij} &\leq 0; \\ m_{ij}w_j - w_i + (w_i - l_{ij}w_j)\varepsilon_{ij} - \beta w_j(m_{ij} - l_{ij}) &\leq 0; \\ w_i - m_{ij}w_j + (u_{ij}w_j - w_i)\varepsilon_{ij} - \beta w_j(u_{ij} - m_{ij}) &\leq 0; \\ w_1 + \dots + w_n &= 1; \\ w_1, \dots, w_n &> 0; \\ i &= 1, \dots, n - 1; \quad j = 2, \dots, n; \quad i < j. \end{aligned} \tag{29}$$

Eq. (29) is a nonlinear model. Therefore, achieving a solution to the equation is not possible using the Simplex method and it should be solved by employing the appropriate methods and software such as lingo.

2.3. Network DEA

Although almost all organizations have a hierarchical structure, due attention has been paid to such systems. An organization usually has several units at the first level and several subunits at the second level. In case the subunits are large, there may also be several sub-subunits with different functions at the third level and this leveling can continue if necessary. At the highest level, that is, level 0, the input is distributed to the subset units at the first level. Similarly, each unit at the first level distributes the input to its subset units at the second level. Suppose there is a system as in Figure 3.

In the hierarchy system studied, each DMU has the same number of units at the first level with separate running functions. At the first level, if a unit has subunits at a lower level, then the other DMUs must have the same number of subunits with similar running functions so that a DMU has a one-to-one correspondence in units with other DMUs.

The system is named according to this rule. Level 0 is known as the highest one, which has four subunits of one, two, three, and four at the first level. Each subunit in the first level has two subunits at the lower level. Subunit 1 has two subunits (1,1) and (1,2); subunit 2 has two subunits (2,1) and (2,2); subunit 3 has two subunits (3,1) and (3,2); and subunit 4 has two subunits (4,1) and (4,2). It should be noted that subunits of one level do not have to have the same number of sub-subunit and they can have any sub-subunits.

Suppose that there are n DMUs with a similar structure to measure comparable efficiency. $X_{ij}^{(p)}$ and $Y_{rj}^{(p)}$ are the i th input and the r th output of DMU_j for unit p , respectively. This system uses m inputs to generate s outputs. The input is divided into two categories of shared and specific, while the output is specific. In the hierarchical system method, the top management of the organization assigns input to the first level for the division into lower levels. Then, the input assigned to the first level is divided at the second level. The input allocation process continues in this way up to the last level (the lowest level). The output divided at the lowest level is what has been generated. Each unit has a specific input. It is assumed that the shared input is x_i , $i = 1, \dots, n$; then, $\alpha_i x_i$ is distributed to unit 1, $\beta_i x_i$ to unit 2, $\gamma_i x_i$ to unit 3, and $\delta_i x_i$ to unit 4 so that $\sum_{i=1}^n (\alpha_i + \beta_i + \gamma_i + \delta_i) = 1$. In fact, the input entered in unit 1 is equal to the sum of the specific and shared input. That is, the input to unit 1 is equal to $\alpha_i x_i + x^{(1)}$, the input to unit 2 is equal to $\alpha_i x_i + x^{(2)}$, the input to unit 3 is equal to $\alpha_i x_i + x^{(3)}$, and the input to unit 4 is equal to $\alpha_i x_i + x^{(4)}$. Each unit at different levels does not consume all m inputs and does not generate the total s outputs. It distributes its own input from its parent into its subunits in order to

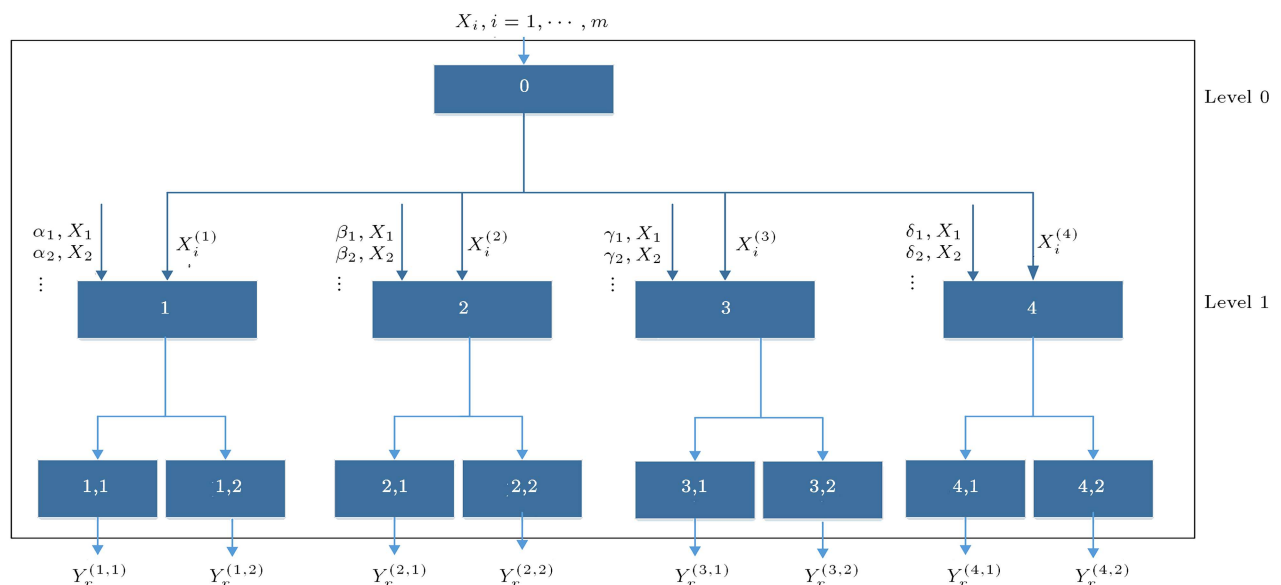


Figure 3. Hierarchical system.

$$\begin{aligned}
 E_K^{BB} &= \max \frac{\sum_{r=1}^s u_r \left(Y_{rk}^{(1,1)} + Y_{rk}^{(1,2)} + Y_{rk}^{(2,1)} + Y_{rk}^{(2,2)} + Y_{rk}^{(3,1)} + Y_{rk}^{(3,2)} + Y_{rk}^{(4,1)} + Y_{rk}^{(4,2)} \right)}{\sum_{i=1}^m v_i X_{ik}}, \\
 \text{s.t.} \\
 \sum_{r=1}^s u_r \left(Y_{rk}^{(1,1)} + Y_{rk}^{(1,2)} + Y_{rk}^{(2,1)} + Y_{rk}^{(2,2)} + Y_{rk}^{(3,1)} + Y_{rk}^{(3,2)} + Y_{rk}^{(4,1)} + Y_{rk}^{(4,2)} \right) - \sum_{i=1}^m v_i X_{ij} &\leq 0, \\
 j = 1, \dots, n; \quad u_r, v_i &\geq \varepsilon; \quad r = 1, \dots, s; \quad i = 1, \dots, m.
 \end{aligned} \tag{30}$$

Box I

generate output. For example, unit (1) receives shared input $\alpha_i x_i$ from the parent unit (0) and has its specific input $x^{(1)}$ as well.

The traditional black box DEA model focuses on the coefficients u_r and v_i to generate the highest ratio of integrated share of outputs generated by the evaluation of the DMU to the total inputs employed given that the ratio for each DMU should be less than or equal to one. In practice, the black box efficiency (E_{BB}) measurement model DMU_K with the assumption of constant returns to scale is expressed by Eq. (30), as shown in Box I. If the internal operations of the hierarchical system units are considered, then network efficiency (E^{NW}) can be measured. Since this system has four subunits at different levels, there will be four constraint sets in the network model. By maximizing the efficiency of the DMU, in unit (0) at level (0), the objective function is as follows:

$$E_k^{NW} = \max \frac{\sum_{r=1}^s u_r Y_{rk}}{\sum_{i=1}^m v_i X_{ik}},$$

s.t.

$$\text{Unit (0): } \sum_{r=1}^s u_r Y_{rj} - \sum_{i=1}^m v_i X_{ij} \leq 0,$$

$$j = 1, \dots, n,$$

$$\text{Unit (1): } \sum_{r=1}^s u_r Y_{rj}^{(1)} - \sum_{i=1}^m v_i (X_{ij}^{(1)} + \alpha_{ij} X_{ij}) \leq 0,$$

$$j = 1, \dots, n,$$

$$\text{Unit (2): } \sum_{r=1}^s u_r Y_{rj}^{(2)} - \sum_{i=1}^m v_i (X_{ij}^{(2)} + \beta_{ij} X_{ij}) \leq 0,$$

$$j = 1, \dots, n,$$

$$\text{Unit (3): } \sum_{r=1}^s u_r Y_{rj}^{(3)} - \sum_{i=1}^m v_i (X_{ij}^{(3)} + \gamma_{ij} X_{ij}) \leq 0,$$

$$j = 1, \dots, n,$$

$$\text{Unit (4): } \sum_{r=1}^s u_r Y_{rj}^{(4)} - \sum_{i=1}^m v_i (X_{ij}^{(4)} + \delta_{ij} X_{ij}) \leq 0,$$

$$j = 1, \dots, n,$$

$$u_r, v_i \geq \varepsilon, \quad r = 1, \dots, s, \quad i = 1, \dots, m. \tag{31}$$

such that:

$$Y_{rj}^{(1)} = Y_{rj}^{(1,1)} + Y_{rj}^{(1,2)},$$

$$Y_{rj}^{(2)} = Y_{rj}^{(2,1)} + Y_{rj}^{(2,2)},$$

$$Y_{rj}^{(3)} = Y_{rj}^{(3,1)} + Y_{rj}^{(3,2)},$$

$$Y_{rj}^{(4)} = Y_{rj}^{(4,1)} + Y_{rj}^{(4,2)}. \tag{32}$$

The objective function is nonlinear, but it can be converted into a linear form by assigning a value of 1 to a denominator and can be eliminated from the numerator of the objective function.

$$E_k^{NW} = \max \sum_{r=1}^s u_r Y_{rk},$$

$$\text{s.t. } \sum_{i=1}^m v_i X_{ik} = 1,$$

$$\text{Unit (0): } \sum_{r=1}^s u_r Y_{rj} - \sum_{i=1}^m v_i X_{ij} \leq 0,$$

$$j = 1, \dots, n,$$

$$\text{Unit (1): } \sum_{r=1}^s u_r Y_{rj}^{(1)} - \sum_{i=1}^m v_i (X_{ij}^{(1)} + \alpha_{ij} X_{ij}) \leq 0,$$

$$\begin{aligned}
 & j = 1, \dots, n, \\
 \text{Unit (2): } & \sum_{r=1}^s u_r Y_{rj}^{(2)} - \sum_{i=1}^m v_i (X_{ij}^{(2)} + \beta_{ij} X_{ij}) \leq 0, \\
 & j = 1, \dots, n, \\
 \text{Unit (3): } & \sum_{r=1}^s u_r Y_{rj}^{(3)} - \sum_{i=1}^m v_i (X_{ij}^{(3)} + \gamma_{ij} X_{ij}) \leq 0, \\
 & j = 1, \dots, n, \\
 \text{Unit (4): } & \sum_{r=1}^s u_r Y_{rj}^{(4)} - \sum_{i=1}^m v_i (X_{ij}^{(4)} + \delta_{ij} X_{ij}) \leq 0, \\
 & j = 1, \dots, n, \\
 & u_r, v_i \geq \varepsilon, \quad r = 1, \dots, s, \quad i = 1, \dots, m. \quad (33)
 \end{aligned}$$

Network model (32) includes additional constraints. The sum of the constraints corresponding to units (1), (2), (3), and (4) is similar to the parent unit (0). Thus, the constraints corresponding to unit (0) are additional and can be eliminated.

$$\begin{aligned}
 E_k^{NW} &= \max \sum_{r=1}^s u_r Y_{rk}, \\
 \text{s. t. } & \sum_{i=1}^m v_i X_{ik} = 1, \\
 \text{Unit (1): } & \sum_{r=1}^s u_r Y_{rj}^{(1)} - \sum_{i=1}^m v_i (X_{ij}^{(1)} + \alpha_{ij} X_{ij}) \leq 0, \\
 & j = 1, \dots, n, \\
 \text{Unit (2): } & \sum_{r=1}^s u_r Y_{rj}^{(2)} - \sum_{i=1}^m v_i (X_{ij}^{(2)} + \beta_{ij} X_{ij}) \leq 0, \\
 & j = 1, \dots, n, \\
 \text{Unit (3): } & \sum_{r=1}^s u_r Y_{rj}^{(3)} - \sum_{i=1}^m v_i (X_{ij}^{(3)} + \gamma_{ij} X_{ij}) \leq 0, \\
 & j = 1, \dots, n, \\
 \text{Unit (4): } & \sum_{r=1}^s u_r Y_{rj}^{(4)} - \sum_{i=1}^m v_i (X_{ij}^{(4)} + \delta_{ij} X_{ij}) \leq 0, \\
 & j = 1, \dots, n, \\
 & u_r, v_i \geq \varepsilon, \quad r = 1, \dots, s, \quad i = 1, \dots, m. \quad (34)
 \end{aligned}$$

3. IFANP-NDEA method

Management in many organizations has a hierarchical

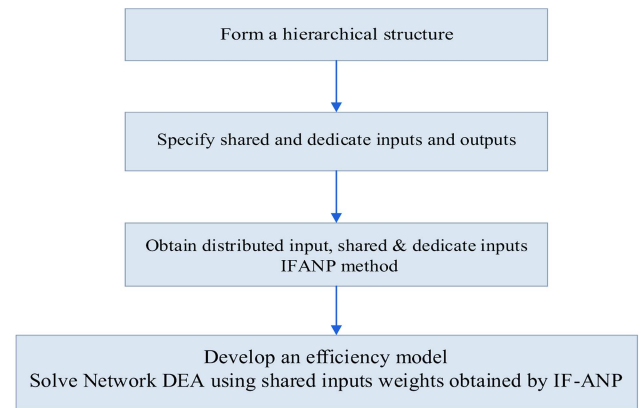


Figure 4. Methodology algorithm.

structure. DEA method has a general disadvantage due to its merely mathematical nature and hence, not incorporating qualitative, inferential, and intuitive indicators. However, this disadvantage can be resolved by some MCDM techniques, including ANP, due to their specific characteristics. In practice, both DEA and ANP methods are used. Both models have their own limitations, but their combination gains benefits of both and tackles their limitations.

The framework of the present study consists of four steps as shown in Figure 4. The Faculty of Basic Sciences of the Islamic Azad University is investigated. Education and research are the two main functions of a university. In many cases, the system consists of several parallel processes; for example, a university includes departments that use multiple inputs to generate multiple outputs. The Charnes, Cooper, and Rhodes (CCR) model can be used to measure the efficiency of a university [11]. If the decision-maker is interested in the performance of a particular department, the model is applicable to the inputs and outputs of the respective department. Theoretically, if all the components of a processes are efficient, the system will be efficient. However, it is possible to design examples that are inconsistent with this assumption in order to evaluate the efficiency of the system and its processes independently. DEA investigates a system as a black box and ignores the internal relations of the processes. The NDEA model is an approach to evaluating the efficiency of systems with several processes and it is able to deal with the processes and the internal relations. The main goal of the NDEA research is to open the black box of a system. It includes the elements of components when computing the efficiency. Figure 5 shows the structure of the Faculty of Basic Sciences with inputs and outputs.

The Faculty of Basic Sciences includes the departments of mathematics, biology, chemistry, and physics. The university can be studied from different aspects of research, educational, administrative, financial, student, and cultural. Each group can be investigated in terms of education and research dimensions.

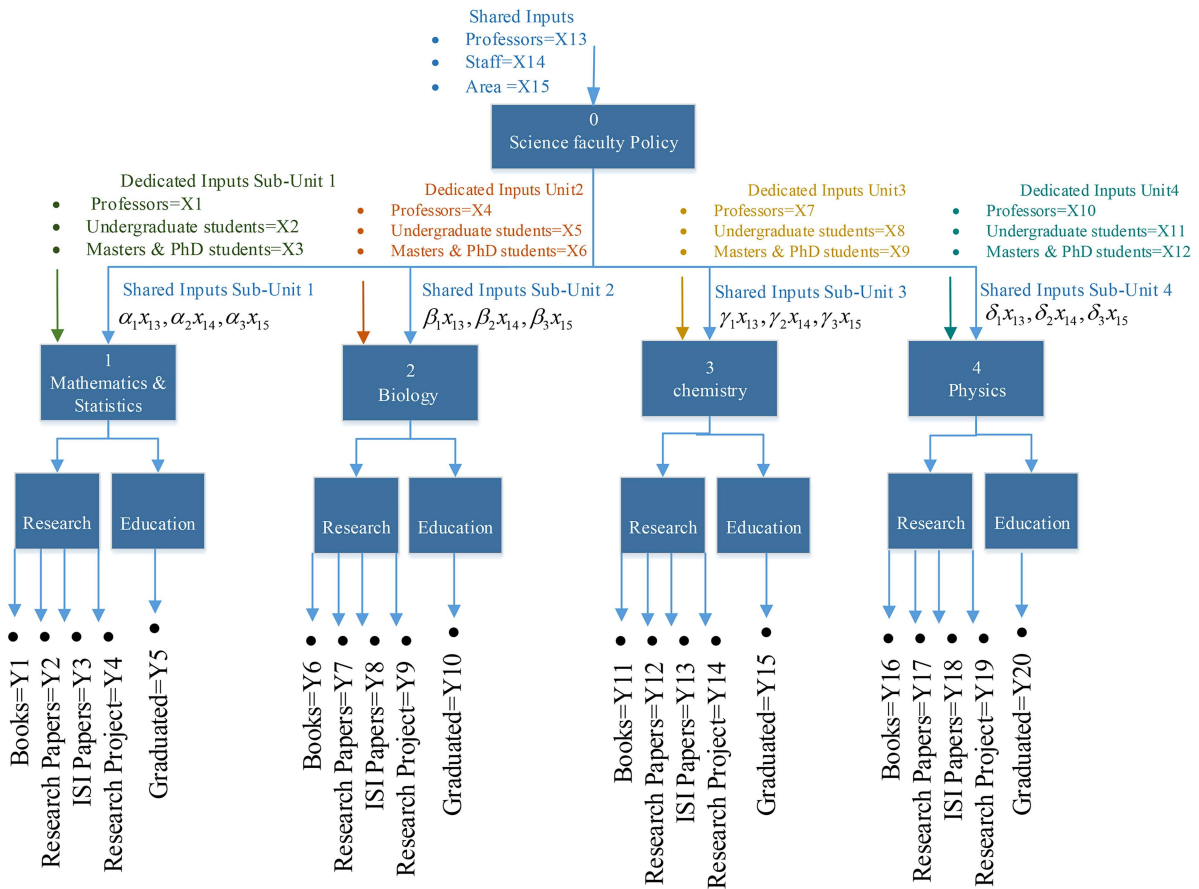


Figure 5. DMU structure with input and output.

Each group has three specific inputs and three shared inputs. Specific inputs include specialized professors as well as bachelor’s, and master’s and PhD students. Shared inputs include general professors, staff, and area. The research outcomes of each group include books, research papers, ISI articles, and research projects and the educational outcome is graduation. In the present study, there are 9 DMUs. Table 1 presents information on the DMUs.

Since the number of DMUs is less than three multiplied by the sum of the inputs and outputs and, in fact, the relation $n \geq 3(m + s)$ is not met, weight limitations should be applied by the experts. The measurement model of network efficiency DMU_k with the assumption of constant returns to scale is as follows:

$$E_k^{NW} = \max \quad u_1 Y_{1k} + u_2 Y_{2k} + u_3 Y_{3k} + u_4 Y_{4k} + u_5 Y_{5k} + u_6 Y_{6k} + u_7 Y_{7k} + u_8 Y_{8k} + u_9 Y_{9k} + u_{10} Y_{10k} + u_{11} Y_{11k} + u_{12} Y_{12k} + u_{13} Y_{13k} + u_{14} Y_{14k} + u_{15} Y_{15k} + u_{16} Y_{16k} + u_{17} Y_{17k} + u_{18} Y_{18k} + u_{19} Y_{19k} + u_{20} Y_{20k};$$

s.t.

$$\begin{aligned} &v_1 X_{1k} + v_2 X_{2k} + v_3 X_{3k} + v_4 X_{4k} + v_5 X_{5k} + v_6 X_{6k} \\ &+ v_7 X_{7k} + v_8 X_{8k} + v_9 X_{9k} + v_{10} X_{10k} \\ &+ v_{11} X_{11k} + v_{12} X_{12k} + v_{13} X_{13k} \\ &+ v_{14} X_{14k} + v_{15} X_{15k} = 1, \\ &(u_1 Y_{1j} + u_2 Y_{2j} + u_3 Y_{3j} + u_4 Y_{4j}) + u_5 Y_{5j} \\ &- (v_1 X_{1j} + v_2 X_{2j} + v_3 X_{3j}) \\ &- (\alpha_1 v_{13} X_{13j} + \alpha_2 v_{14} X_{14j} + \alpha_3 v_{15} X_{15j}) \leq 0, \\ &(u_6 Y_{6j} + u_7 Y_{7j} + u_8 Y_{8j} + u_9 Y_{9j}) + u_{10} Y_{10j} \\ &- (v_4 X_{4j} + v_6 X_{6j} + v_7 X_{7j}) \\ &- (\beta_1 v_{13} X_{13j} + \beta_2 v_{14} X_{14j} + \beta_3 v_{15} X_{15j}) \leq 0, \\ &(u_{11} Y_{11j} + u_{12} Y_{12j} + u_{13} Y_{13j} + u_{14} Y_{14j}) \\ &+ u_{15} Y_{15j} - (v_8 X_{8j} + v_9 X_{9j} + v_{10} X_{10j}) \\ &- (\gamma_1 v_{13} X_{13j} + \gamma_2 v_{14} X_{14j} + \gamma_3 v_{15} X_{15j}) \leq 0, \end{aligned}$$

Table 1. Information for DMUs.

2015-2016	Input and output	Mean	Variance of the entire population (Var.p)	Variance of a sample set of data (Var.s)	Standard deviation of the entire population (Std.p)	Standard deviation of a sample set of data (Std.s)
Mathematics and statistics	X ₁ =Specialized faculty members	22	41	46	6	7
	X ₂ =Undergraduate students	211	24081	27091	155	165
	X ₃ =Master's and PhD students	261	8559	9629	93	98
	Y ₁ =Books	4	1	1	1	1
	Y ₂ =Research papers	26	76	85	9	9
	Y ₃ =International Scientific Indexing (ISI) papers	10	27	30	5	5
	Y ₄ =Research projects	59	668	752	26	27
	Y ₅ =Graduated students	171	2434	2738	49	52
Biology	X ₄ =Specialized faculty members	23	76	86	9	9
	X ₅ =Undergraduate students	524	49498	55685	222	236
	X ₆ =Master's and PhD students	354	9449	10630	97	103
	Y ₆ =Books	3	4	4	2	2
	Y ₇ =Research papers	44	406	457	20	21
	Y ₈ =ISI papers	42	239	269	15	16
	Y ₉ =Research projects	47	488	550	22	23
	Y ₁₀ =Graduated students	210	4508	5071	67	71
Chemistry	X ₇ =Specialized faculty members	28	178	201	13	14
	X ₈ =Undergraduate students	451	41368	46539	203	216
	X ₉ =Master's and PhD students	167	4126	4641	64	68
	Y ₁₁ =Books	3	3	4	2	2
	Y ₁₂ =Research papers	15	22	25	5	5
	Y ₁₃ =ISI papers	39	103	116	10	11
	Y ₁₄ =Research projects	48	722	812	27	28
	Y ₁₅ =Graduated students	55	339	382	18	20
Physics	X ₁₀ =Specialized faculty members	59	224	252	15	16
	X ₁₁ =Undergraduate students	580	22938	25805	151	161
	X ₁₂ =Master's and PhD students	226	6223	7001	79	84
	Y ₁₆ =Books	2	1	2	1	1
	Y ₁₇ =Research papers	5	1	1	1	1
	Y ₁₈ =ISI papers	49	220	248	15	16
	Y ₁₉ =Research projects	323	14200	15975	119	126
	Y ₂₀ =Graduated students	365	11061	12443	105	112
Shared	X ₁₃ =General teachers	28	176	198	13	14
	X ₁₄ =Staff	38	227	255	15	16
	X ₁₅ =Area	538	51407	57833	227	240

$$\begin{aligned}
 &(u_{16}Y_{16j} + u_{17}Y_{17j} + u_{18}Y_{18j} + u_{19}Y_{19j}) \\
 &+ u_{20}Y_{20j} - (v_{10}X_{10j} + v_{11}X_{11j} + v_{12}X_{12j}) \\
 &- (\delta_1v_{13}X_{13j} + \delta_2v_{14}X_{14j} + \delta_3v_{15}X_{15j}) \leq 0, \\
 &u_1, \dots, u_{20} \geq 0; \quad v_1, \dots, v_{15} \geq 0.
 \end{aligned} \tag{35}$$

As shown in the above model, $\alpha_i, \beta_i, \gamma_i, \delta_i, i = 1, 2, 3$ is the distributed input coefficient to each unit. The value of distributed input to each unit is obtained by the IFANP method.

3.1. Calculation of the weights of shared inputs through the IFANP

In this step, the matrix of the paired comparisons should be created. Using Table 2, linguistic variables are converted into triangular fuzzy numbers.

The model has three shared inputs named $x_{13}, x_{14},$ and x_{15} . It is necessary to calculate each coefficient of shared input that is distributed to each unit. The intuitionistic fuzzy paired comparisons are employed to calculate the distribution coefficient. A matrix of paired comparisons was made by a survey of 10 experts. Suppose X is a set of all the experts who participated in the survey.

- $A(x)$ The number of experts who agree;
- $D(x)$ The number of experts who disagree;
- $U(x)$ The number of experts who neither agree nor disagree (no decision).

Suppose that n is the total number of experts who

participated in the survey. We have:

$$\begin{aligned}
 \mu(x) &= \frac{A(x)}{n}; \quad \nu(x) = \frac{D(x)}{n}; \\
 \pi(x) &= 1 - \frac{A(x)}{n} - \frac{D(x)}{n} = \frac{n - A(x) - D(x)}{n}.
 \end{aligned} \tag{36}$$

Table 3 shows the results of the survey of experts for the paired comparisons with the groups for input x_{13} .

As an instance, for input x_{13} , regarding the comparison between the group of biology and the mathematical group, 16 people agreed, shown with the symbol $A(x)$; two experts did not agree, represented by the symbol $D(x)$; and two experts did not participate in voting, indicated by the symbol $U(x)$.

The intuitionistic indicators of certainty, uncertainty, and hesitancy are calculated by the above formula. The results are shown in Table 4. For example, the certainty degree of 0.6 is calculated as follows:

$$\mu(x) = \frac{6}{10} = 0.6. \tag{37}$$

The global weight of each input distributed to each group is given in Table 5.

Given the hierarchical structure, the sum of weights is equal to 1.

$$\begin{aligned}
 \alpha_1 + \alpha_2 + \alpha_3 + \beta_1 + \beta_2 + \beta_3 + \gamma_1 + \gamma_2 + \gamma_3 \\
 + \delta_1 + \delta_2 + \delta_3 = 1.
 \end{aligned} \tag{38}$$

The second stage is about using the weights obtained by the IFANP in the NDEA model. The IFANP-NDEA model is as follows:

Table 2. Triangular fuzzy scale.

Linguistic scale for importance	Triangular fuzzy scale	Triangular fuzzy reciprocal scale
Just equal	(1, 1, 1)	(1, 1, 1)
Equally Important (EI)	(1/2, 1, 3/2)	(2/3, 1, 2)
Weakly More Important (WMI)	(1, 3/2, 2)	(1/2, 2/3, 1)
Strongly More Important (SMI)	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
Very Strongly More Important (VSMI)	(2, 5/2, 3)	(1/3, 2/5, 1/2)
Absolutely More Important (AMI)	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)

Table 3. Paired comparisons of the groups for input x_{13} .

x_{13}	Mathematics			Biology			Chemistry			Physics		
	$A(x)$	$D(x)$	$U(x)$	$A(x)$	$D(x)$	$U(x)$	$A(x)$	$D(x)$	$U(x)$	$A(x)$	$D(x)$	$U(x)$
Mathematics				6	3	1	6	2	2	6	2	2
Biology	6	3	1				6	3	1	6	2	2
Chemistry	6	2	2	6	3	1				6	3	1
Physics	6	2	2	6	2	2	6	3	1			

Table 4. Certainty, uncertainty, and hesitancy degrees.

x_{13}	Mathematics			Biology			Chemistry			Physics		
	$\mu(x)$	$\nu(x)$	$\pi(x)$	$\mu(x)$	$\nu(x)$	$\pi(x)$	$\mu(x)$	$\nu(x)$	$\pi(x)$	$\mu(x)$	$\nu(x)$	$\pi(x)$
Mathematics				0.6	0.3	0.1	0.6	0.2	0.2	0.6	0.2	0.2
Biology	0.6	0.3	0.1				0.6	0.3	0.1	0.6	0.2	0.2
Chemistry	0.6	0.2	0.2	0.6	0.3	0.1				0.6	0.3	0.1
Physics	0.6	0.2	0.2	0.6	0.2	0.2	0.6	0.3	0.1			

Table 5. Global weights.

Inputs	Groups		
	X_{13}	X_{14}	X_{15}
	Mathematics	$\alpha_1 = 0.067$	$\alpha_2 = 0.119$
Biology	$\beta_1 = 0.067$	$\beta_2 = 0.079$	$\beta_3 = 0.080$
Chemistry	$\gamma_1 = 0.084$	$\gamma_2 = 0.079$	$\gamma_3 = 0.081$
Physics	$\delta_1 = 0.066$	$\delta_2 = 0.079$	$\delta_3 = 0.098$

$$E_k^{NW} = \max u_1 Y_{1k} + u_2 Y_{2k} + u_3 Y_{3k} + u_4 Y_{4k} + u_5 Y_{5k} + u_6 Y_{6k} + u_7 Y_{7k} + u_8 Y_{8k} + u_9 Y_{9k} + u_{10} Y_{10k} + u_{11} Y_{11k} + u_{12} Y_{12k} + u_{13} Y_{13k} + u_{14} Y_{14k} + u_{15} Y_{15k} + u_{16} Y_{16k} + u_{17} Y_{17k} + u_{18} Y_{18k} + u_{19} Y_{19k} + u_{20} Y_{20k},$$

s. t.

$$v_1 X_{1k} + v_2 X_{2k} + v_3 X_{3k} + v_4 X_{4k} + v_5 X_{5k} + v_6 X_{6k} + v_7 X_{7k} + v_8 X_{8k} + v_9 X_{9k} + v_{10} X_{10k} + v_{11} X_{11k} + v_{12} X_{12k} + v_{13} X_{13k} + v_{14} X_{14k} + v_{15} X_{15k} = 1,$$

$$(u_1 Y_{1j} + u_2 Y_{2j} + u_3 Y_{3j} + u_4 Y_{4j}) + u_5 Y_{5j} - (v_1 X_{1j} + v_2 X_{2j} + v_3 X_{3j}) - (0.067 * v_{13} X_{13j} + 0.119 * v_{14} X_{14j} + 0.098 * v_{15} X_{15j}) \leq 0,$$

$$(u_6 Y_{6j} + u_7 Y_{7j} + u_8 Y_{8j} + u_9 Y_{9j}) + u_{10} Y_{10j} - (v_4 X_{4j} + v_6 X_{6j} + v_7 X_{7j}) - (0.067 * v_{13} X_{13j} + 0.079 * v_{14} X_{14j} + 0.080 * v_{15} X_{15j}) \leq 0,$$

$$(u_{11} Y_{11j} + u_{12} Y_{12j} + u_{13} Y_{13j} + u_{14} Y_{14j})$$

$$+ u_{15} Y_{15j} - (v_8 X_{8j} + v_9 X_{9j} + v_{10} X_{10j})$$

$$- (0.084 * v_{13} X_{13j} + 0.079 * v_{14} X_{14j}$$

$$+ 0.081 * v_{15} X_{15j}) \leq 0,$$

$$(u_{16} Y_{16j} + u_{17} Y_{17j} + u_{18} Y_{18j} + u_{19} Y_{19j})$$

$$+ u_{20} Y_{20j} - (v_{10} X_{10j} + v_{11} X_{11j} + v_{12} X_{12j})$$

$$- (0.066 * v_{13} X_{13j} + 0.079 * v_{14} X_{14j}$$

$$+ 0.098 * v_{15} X_{15j}) \leq 0,$$

$$5.7 \times v_{13} \leq v_{14}; \quad 3.5 \times v_{15} \leq v_{13};$$

$$5.7 \times v_3 \leq v_1; \quad 2.8 \times v_2 \leq v_3;$$

$$2.9 \times v_6 \leq v_4; \quad 2.5 \times v_5 \leq v_6;$$

$$5.8 \times v_9 \leq v_7; \quad 6.5 \times v_8 \leq v_9;$$

$$6.9 \times v_{12} \leq v_{10}; \quad 4.5 \times v_{11} \leq v_{12};$$

$$5.9 \times u_1 \leq u_3; \quad 5.7 \times u_4 \leq u_1;$$

$$7.5 \times u_2 \leq u_4; \quad 6.9 \times u_6 \leq u_8;$$

$$5.6 \times u_9 \leq u_6; \quad 7.8 \times u_7 \leq u_9;$$

$$4.9 \times u_{11} \leq u_{13}; \quad 3.9 \times u_{14} \leq u_{11};$$

$$5.7 \times u_{12} \leq u_{14}; \quad 4.8 \times u_{16} \leq u_{18};$$

$$6.8 \times u_{19} \leq u_{16}; \quad 5.7 \times u_{17} \leq u_{19};$$

$$\alpha_1, \alpha_2, \alpha_3, \alpha_4 \geq 0,$$

$$u_1, \dots, u_{20} \geq 0.00001; \quad v_1, \dots, v_{15} \geq 0.00001.$$

(39)

In the present study, 9 DMUs were investigated. The results of IFANP-NDEA and classic DEA methods are shown in Table 6.

Table 6. Efficiency results.

DMU	Efficiency of the whole system by black box DEA (E_{BB})	Efficiency of the whole system by IFANP-NDEA (E_{NW})	Efficiency of mathematics by IFANP-NDEA ($E_{NW-Math}$)	Efficiency of biology by IFANP-NDEA ($E_{NW-Biology}$)	Efficiency of chemistry by IFANP-NDEA ($E_{NW-Chemistry}$)	Efficiency of physics by IFANP-NDEA ($E_{NW-Physics}$)
1	1	0.90	0.99	0.83	0.92	0.99
2	1	0.81	0.92	0.68	0.99	0.89
3	1	0.87	0.51	0.99	0.65	0.80
4	1	0.83	0.79	0.99	0.42	0.84
5	1	0.66	0.75	0.67	0.55	0.78
6	1	0.84	0.60	0.91	0.39	0.99
7	1	0.91	0.78	0.93	0.99	0.99
8	1	0.81	0.98	0.62	0.85	0.98
9	1	0.89	0.83	0.98	0.42	0.98

4. Conclusion

Evaluation of a system through the traditional Data Envelopment Analysis (DEA) model is similar to dealing with a black box by only focusing on the evaluation of the input and outputs and ignoring the internal operations and interactions of the processes. Therefore, the traditional DEA may identify a system as an efficient one while all of its process components are not efficient. Of note, there are cases in which all the components of a process are worse than another system while their performance is better. With these said, in order to obtain meaningful efficiency, it is better to consider the internal structure of a system whenever data is available. An approach that considers the component operations in DEA is called Network DEA (NDEA). Systems can have different structures and hence, there are different NDEA models.

The Fuzzy Analytic Network Process (IFANP) method, a combination of Intuitionistic Fuzzy Set (IFS) and Analytic Network Process (ANP) tools, is a powerful technique to obtain the relative importance of the evaluation criteria. The main objective of the present study was to investigate the efficiency of the Faculty of Basic Sciences of Islamic Azad University in two dimensions of education and research using an IFANP-NDEA method. The IFANP technique was employed to obtain the relative weights of the evaluation criteria considered in the process of modeling the problem as well as to rank the faculties based on their corresponding weights in the NDEA model.

The Faculty of Science consists of mathematics, biology, chemistry, and physics departments. Each department was investigated in the educational and research aspects. Each group had three specific inputs and three shared inputs. Specialized inputs included

specialized professors and bachelor's, master's, and PhD students and shared inputs included general professors, staff, and area. The research outputs of each group included books, research papers, ISI articles, and research projects and educational output consisted in graduates.

The efficiency of all DMUs was equal to 1 and the DMUs did not follow any ranking. The DMUs were considered as a black box in calculating the efficiency using the traditional DEA method, that is, the internal structure was ignored. Twelve efficient universities as well as eight inefficient ones were identified.

Furthermore, the efficiency was calculated using the NDEA method by the IFANP-NDEA model (E_{NW}). In this calculation, the weights of shared inputs were obtained using IFANP and then, they were used in the NDEA model. DMU5 with the efficiency value of 0.66 had the lowest efficiency and DMU7 with the efficiency of 0.91 had the highest efficiency. For DMU5, the chemistry department with the efficiency of 0.55 had the lowest efficiency and the physics department with the efficiency of 0.78 had the highest efficiency among the groups. In addition, the internal efficiency of the departments of mathematics ($E_{NW-Math}$), biology ($E_{NW-Biology}$), chemistry ($E_{NW-Chemistry}$), and physics ($E_{NW-Physics}$) was also calculated. The solution to this model was identical in both cases of considering and removing the zero unit.

The presented model allows managers to identify the points of weakness and improve overall efficiency by focusing on deficiencies. According to the results of the numerical example, the proposed model can achieve more significant results than DEA, because it considers the operations of the internal processes and prioritizes the efficient units.

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Appendix

Consider Model (10) again. We use the L_∞ -norm in formula (A.1):

$$\mu_{\tilde{p}}(w) = \min_k \left\{ \begin{array}{ll} \left\| 1 - \frac{R_k w}{d_k} \right\|_\infty; & 0 \leq R_k w \leq d_k, \\ 1; & R_k w \leq 0, \\ 0; & R_k w \geq d_k, \\ |w_1 + \dots + w_n = 1| \end{array} \right\}. \quad (\text{A.1})$$

Then, we will have:

$$\begin{aligned} & \mu_{\tilde{p}}(w_{\max}^*) \\ &= \max_k \min \left\{ \begin{array}{ll} \left\| 1 - \frac{R_k w}{d_k} \right\|_\infty; & 0 \leq R_k w \leq d_k, \\ 1; & R_k w \leq 0, \\ 0; & R_k w \geq d_k, \\ |w_1 + \dots + w_n = 1| \end{array} \right\}. \end{aligned} \quad (\text{A.2})$$

Finally, Model (A.3) is formulated, which is the same as Model (14). Therefore, we convert the multi-objective function into a single-objective one by using the L_∞ -norm.

$$\begin{aligned} & \max \alpha, \\ & \text{s.t. } \alpha \leq 1 - \frac{R_k w}{d_k}, \quad k = 1, \dots, m, \\ & \sum_{i=1}^n w_i = 1, \quad w_i > 0, \quad 0 \leq \alpha \leq 1, \end{aligned} \quad (\text{A.3})$$

$$\alpha = \max_k \min \left\{ \begin{array}{ll} \left\| 1 - \frac{R_k w}{d_k} \right\|_\infty; & 0 \leq R_k w \leq d_k, \\ 1; & R_k w \leq 0, \\ 0; & R_k w \geq d_k, \\ |w_1 + \dots + w_n = 1| \end{array} \right\}. \quad (\text{A.4})$$

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