Chance constrained programming and robust optimization approaches for uncertain hub location problem in a cooperative competitive environment

Farzin Nourzadeh\textsuperscript{a,1}, Sadoullah Ebrahimnejad\textsuperscript{b,*}, Kaveh Khalili-Damghani\textsuperscript{a,2}, Ashkan Hafezalkotob\textsuperscript{a,3}

\textsuperscript{a} Department of Industrial Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran.
\textsuperscript{b} Department of Industrial Engineering, Karaj Branch, Islamic Azad University, P.O.Box: 31485/313, Karaj, Iran.

\*Corresponding author. Tel.: +98 912 209 9648.

\textsuperscript{1}Tel.: +98 912 6142907.
\textsuperscript{2}Tel.: +98 912 3980373.
\textsuperscript{3}Tel.: +98 912 7785681.

E-mail addresses:
\texttt{st_f_noorzadeh@azad.ac.ir} (F. Nourzadeh), \texttt{ibrahimnejad@kiau.ac.ir} (S. Ebrahimnejad), \texttt{kaveh.khalili@gmail.com} (K. Khalili-Damghani), \texttt{hafezalkotob@iust.ac.ir} (A. Hafezalkotob).
Abstract: In this paper, we propose an integer programming model for Capacitated Multi-Allocation Median Hub Location Problem, which is applied in a both cooperative and competitive environment among airlines. We divide the hubs into six independent categories by comparing the parameters of the ticket price, travel time, and the service quality of hub airports are controlled by follower and leader airlines. In this paper, the degree of importance of time and cost parameters determine by a multivariate Lagrange interpolation method, which can play an important role in allocating travelers to follower airline hubs. Then, based on the seasonal demand of travelers, we consider travel demand as uncertain parameters. To determine the deterministic equivalent forms of this category of hub location models, robust optimization method and chance-constrained programming model are used. Finally, the proposed model test in a case study. Based on the results, a coalition of follower airlines can absorb nearly 2% of travelers of leader airline due to lower travel cost and travel time compared to that of leader airline.

Keywords: Hub Location Problem; Robust Optimization; Chance-Constrained Programming; Cooperative Competitive Environment; Multivariate Lagrange Interpolation Function.

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Highlights

- We determine the best Iranian hubs (airports) to transfer travelers from European countries to East Asian countries based on the least travel time and ticket prices and the most service quality.
- We divide hubs of follower airlines into six independent capture sets by comparing ticket prices, travel time, and the quality of hubs.
- We determine Chance constrained programming and robust optimization approaches for a new type of hub location problem in a cooperative competition environment.
- We determine the degree of importance of travel time and cost by using a multivariate Lagrangian interpolation method.

1. Introduction

Since networks have a hierarchical structure, performing several travels from lower levels in demand spokes (origins) to lower levels in destination spokes is not cost-effective for companies due to low demand level. In these cases, it is better to establish flow from other points called hub [1]. Hub location problem as an indirect transportation problem seeks to designs an efficient network. This applies when there is a need for economies of scale in transportation or communication costs.

Today the hub location problem can be used in transportation problems ([2]), postal service ([3]), order distribution and ([4]), emergency services ([5]), and computer networks ([6]).

In the real world, transportation occurs via hubs in a competitive market, where several service providers compete with each other on gaining more market share. Accordingly, customers can choose one of them based on freight, service delivery time, type of offered services, and so on. Marianov et al. [7] proposed the first hub location problem in a competitive environment. In their model for the hub location of the follower player, the leader player has already selected its hub-and-spoke network. The proposed model uses fewer variables and constraints compared to previous location problems. The flow from origin to destination passes through only one path (including the maximum of two hubs). They showed that using more hubs leads to lower costs. They also presented two models for follower player. In the first model, if the cost of traveler movement by follower player is lower than the leader player, then all of the travelers are attracted to leader player hubs. However, in
the second model, three capture sets are presented based on the compared cost of travels by follower and leader player. The members of those sets will attract 50%, 75%, and 100% of travelers, respectively.

By changing two models of Marianov et al. [7], Wagner [8] reduced the time needed to achieve an optimal solution for problems with more than 50 spokes and 5 hubs. He developed capture sets, and based on the fact that flow from origin to destination passes through one or two paths, he proposed 6 capture sets.

Marianov et al. [7] and Wagner [8] used the values of 0.7, 0.9, and 1.1 as well as 50%, 75%, and 100% in their capture sets. In this paper, a multinomial Lagrange interpolation method use to calculate the exact values determined by them. The members of capture sets of follower airlines determine by comparing parameters of travel cost and travel time between hubs of leader airline and follower airline as well as comparing the service quality of its under-controlled airports. Moreover, the bounds of variables that belong to these sets determine by considering the degree of importance of parameters of travel time and travel cost from the viewpoint of travelers, the quality of airports and airlines, safety, and delays of airlines.

To identify the members of capture sets, Marianov et al. [7] and Wagner [8] just compared the cost of moving traveler by follower and leader player. However, aside from the movement cost of a traveler, there are other parameters such as travel time and service quality of hubs (airports) in the real world, which can be used as the basis for a player and its under-controlled hubs selection. To overcome the aforementioned problem and determine the members of capture sets more accurately, we compare traveler times in addition to the traveler cost of players. Additionally, comparing the service quality of hubs under-controlled of players is the cornerstone of our research.

In real-world applications, ignoring even one small uncertainty in data will result in an optimal solution that can be insignificant and meaningless. Therefore, various approaches were developed in the literature to gain more realistic optimal solutions. Chance-constrained programming and robust optimization are the most important approaches in this area. The approaches presented by Soyster [9], Ben-Tal and Nemirovski [10], and Bertsimas and Sim [11] are the main approaches for robust optimization, which are employed in articles related to hub location under uncertainty. Concerning the possibility of controlling the conservativeness of robust solutions and linearity of the method of Bertsimas and Sim [11], we use this approach and chance-constrained programming to demonstrate the uncertainty in parameters of travel demand to gain more realistic solutions and then compare the results together.

Due to increasing the foreign air travelers to Iran [12], in this paper to organize the travelers and determine the exact location of hubs, we develop the model of Marianov et al. [7] by defining a new type of capture sets under uncertainty for an alliance of airlines with lower market share. In the proposed model, we present an optimal hub-and-spoke network for follower airlines, in which their share of travelers market increase in competition with other airlines through alliance. By comparing parameters of time and cost of travel by follower and leader airline as well as comparing the service quality of its under-controlled airports, we divide hub sets of follower airlines into six categories to generate traveler's decision options and reduce the complexity of modeling. By using a multivariate Lagrange interpolation method, we determine the degree of importance of time and cost of travel from the traveler's perspective. In the proposed model, spokes are demand points that can travel by using hubs of follower airline. Finally, we examine a case study, in which Emirate airline is the leader airline, and follower airline consists of three airlines Mahan Airline, Iran Air, and Aseman Airline. The spokes are selected from a pool of cities, which pass the largest number of travelers from
European countries to East Asian countries by the hubs of Emirate airline. The results show that the follower airline's income has a considerable growth compared to the Marianov et al. [7] model and can absorb nearly 2% of travelers of the leader airline due to lower travel costs and travel time compared to that of leader airline.

In the next section, the literature review is presented. In section 3, a model for the capacitated multiple allocation median hub location in both a cooperative and competitive environment is presented. Then, to determine the degree of importance of travel time and cost, we introduce a multivariate Lagrangian interpolation function. In sections 4 and 5 the model described in the third section will be presented in uncertainty mode by employing robust optimization and chance-constrained programming approaches, separately. In section 6, deterministic equivalent forms that obtain from robust optimization and chance-constrained programming approaches are tested in a case study. Also, we investigate the impact of change in the problem's parameters on the value of the objective function. Concluding remarks are presented in the last section of this paper.

2. Literature Review

The most important research related to the subject of this paper is presented here.

Sasaki and Fukushima [13] proposed a new model called "Stackelberg Hub Location" to explore the competition among a large company and several medium-sized firms, in which space solution was considered as a Continues Space. They suggested that a leader company could lose a huge percentage of travelers regardless of competition strategies. Based on the Stackelberg hub location model ([13]), Sasaki [14] began to design a hub network, in which two companies compete over maximizing their profits and their solution space was like a network. To avoid unprofitable services, he added a flow threshold constraint and considered the model's effect given the abovementioned constraints and players' strategies. The results indicated that the flow threshold constraint was one of the most important factors in hub network design. Sasaki et al. [15] proposed a model for the hub location in a competitive environment. Whereas Sasaki and Fukushima [13] and Sasaki [14] allowed only using one hub in the path of flow passing from origin to destination, they studied more than one hub in their model. Eiselt and Mariano [16] proposed a hub location problem in a competitive environment in which customers based on the parameters of the ticket price and flight time used the gravity-like function to select airlines. In previous models, flow passing from origin to destination should only pass through one path. However, the proposed model allowed flows passing from origin to destination through different paths and hubs. They implemented their model for the follower player as a new entrant player to the Australian Postal network.

Code Sharing Agreement is a commercial agreement in air transportation, in which two or more airlines share a common flight. In this paper, we consider the alliance between Mahan Airline, Iran Air, and Aseman Airline using Code Sharing Agreement. Lin [17] assessed the economic effects of alliance and cooperation between an international airline and a local airline. He suggested that two Stackelberg equilibrium points were achievable when a set of allied airlines was identified as the leader (follower) player. Lin also demonstrated that in addition to an increase in the social welfare of travelers, the alliance might reduce international traveler excess and local direct travelers. Zou and Chen [18] investigated the effects of code sharing and global alliance on the performance of airlines simultaneously. The results from a group of 81 airlines during the 2007–2012 period showed that the profit margin of an airline was positively associated with a few numbers of partners in
code sharing. Yimga [19] investigated the relationship between alliances and on-time performance. He found evidence that code-sharing alliances improve the on-time performance rate and allow for more efficient connections between flights. More efficient connections decrease the total travel time and thereby increase the quality of a flight.

Adibi and Razmi [20] developed 2-stage stochastic programming for formulating stochastic uncapacitated multiple-allocation HLP in Iran. They considered three cases, wherein (1) flow is stochastic, (2) cost is stochastic, and (3) both flow and cost, are stochastic. To evaluate their formulations, they used a case study based on the ten-node network of top cities of the air transportation network in Iran. Boukani et al. [21] developed two mathematical models for the capacitated single and multiple-allocation P-hub median problems. They presented a robust optimization approach to consider uncertainty in hub establishment fixed cost and capacity of each hub. They showed that costs increase when uncertainties are not considered in the model. An integrated hub location and revenue management problem were considered by Tikani et al [22]. They presented a two-stage stochastic programming formulation to maximize the revenue made out of the transportation network and minimize hub installation costs. In the first stage, the hub location, the link between the hub and the non-hub, and the protection level of tickets for different booking classes are determined. The booking limit of tickets can be obtained in the second stage. The demand is captured in a set of discrete scenarios under the average case. Robust optimization is proposed to handle uncertainty in the demands of customer classes. The problem was demonstrated in the airline industry. Their results show the efficiency of their methods on the instances with up to 25 nodes.

Nikoofal and Sadjadi [23] used Bertimas and Sim's approach [11] to propose the robust model of the median hub location problem with uncertain travel costs. They compared the performance of the model with that of the min-max regret approach. Taking advantage of Bertsimas and Sim's approach [11], Ghaffari-Nasab et al. [24] proposed a robust model for the capacitated hub location problem (single allocation-multiple allocation) in which the demand change was uncertain. They used uncertain demand only for capacity constraints and supposed that the objective function of demand is certain. Zetina et al. [25] proposed a robust counterpart for multiple hub location problem in which (like Bertsimas and Sim's approach [11]) the level of conservativeness was controlled by using uncertainty budget. The problem was modeled in three modes: uncertain transportation cost, uncertain demand, and both simultaneously.

Chance constraint programming was first introduced by Charnes and et al. [26]. Gao and Qin [27] developed a chance-constrained programming approach for p-hub center location problems under uncertain setting by characterizing the travel times as uncertain variables. Chance constraint programming is often encountered applications when there is uncertainty in the data and parameters ([28]). It is well known when the random input has a joint normal distribution, and it can be reduced to a convex problem. Thus, it can be solved efficiently via convex programming techniques ([29]). Many problems in various areas, can be formulated as the Chance constraint programing. A series of applications has been reviewed in the literature ([30]).

3. Deterministic model formulation

In this section, we propose a new formulation for the capacitated multi-allocation median hub location problem in both a cooperative and competitive environment. Then, sets, parameters, variables, objective
function, and constraints of the model are presented. The notations used in the proposed deterministic model are presented in Table 1.

{Please insert table 1 about here.}

3.1 Objective function

The first term of the objective function (1) represents the total income from selling tickets to travelers passing from origin \(i\) (per each origin) to destination \(j\) (to all destinations) through hubs \(k\) of follower airline. Accordingly, none of the follower airline's hubs can be the origin or destination of travel. The second term of the objective function (1) represents the total income from selling tickets to travelers who their destinations are one of the hubs of follower airline. The third term of the objective function (1) represents the total income from selling tickets to travelers who their origins are one of the hubs of follower airline. Here is the reason why there is a relationship between ticket prices in the objective function (1) such that \(\beta_1 \leq \gamma_1 \leq \beta_2 \leq \gamma_2\). When travelers use any other origin of follower airline hubs to reach any other destination of follower airline's hubs, their profitability for airlines will be higher, and they deserve more discount. In other words, \(\beta_1 \gamma_1 \leq \gamma_2 \) and \(\beta_1 \gamma_1 \leq \beta_2 \). In this way, because travelers use follower airline's hub in the second part of their travel, airport costs are decreased. Also, because travelers from different origins aim to reach the same destination, taking advantage of a bigger airplane with lower ticket prices seems more logical; therefore \(\beta_1 \leq \gamma_1\).

When the origin or destination is one of follower airline's hubs, the amount of discount is lower due to shorter travel. So, when the origin is follower airline's hub, we have \(\beta_2 \leq \gamma_2\). Because airport costs are lower, travelers from other origins are going to reach the aforementioned destination.

\[
z = \max \sum_{k \in K} \sum_{i \in I} \sum_{j \in J} \sum_{s = 1}^{2} (x_{kj}^i + y_{kj}^i + z_{kj}^i)(\gamma_i \epsilon_{ik} + \beta_i \epsilon_{kj}) + \sum_{i \in I} \sum_{j \in J} \sum_{s = 1}^{2} (x_{kj}^i + y_{kj}^i + z_{kj}^i)(\gamma_j \epsilon_{jk} + \beta_j \epsilon_{kj})
\]

\[
+ \sum_{k \in K} \sum_{i \in I} \sum_{s = 1}^{2} (x_{kj}^i + y_{kj}^i + z_{kj}^i)(\beta_i \epsilon_{kj}) \quad (\beta_1 \leq \gamma_1 \leq \beta_2 \leq \gamma_2)
\]

3.2 Constraints

Constraints (2) to (7) show that the flow is established when a hub exists.

\[
\sum_{k \in N^1} x_{kj}^i \leq W_{ij} \cdot p_1 \quad \forall i \in I, j \in J
\]

\[
\sum_{k \in N^2} x_{kj}^i \leq W_{ij} \cdot p_2 \quad \forall i \in I, j \in J
\]

\[
\sum_{k \in M^1} y_{kj}^i \leq W_{ij} \cdot q_1 \quad \forall i \in I, j \in J
\]

\[
\sum_{k \in M^2} y_{kj}^i \leq W_{ij} \cdot q_2 \quad \forall i \in I, j \in J
\]

\[
\sum_{k \in P^1} z_{kj}^i \leq W_{ij} \cdot r_1 \quad \forall i \in I, j \in J
\]

\[
\sum_{k \in P^2} z_{kj}^i \leq W_{ij} \cdot r_2 \quad \forall i \in I, j \in J
\]

According to Constraint (8), the maximum flow passing from all origins to all destinations through follower airline hubs equals to\([W_{ij} \cdot (U_{i} / U_{f})(S_{i} / S_{f})(T_{i} / T_{f})]\). Also, this constraint shows multiple allocations.
\[
\sum_{s,t}^2 \left( \sum_{k \in K} x_{ij}^s + \sum_{k \in K} y_{ij}^s + \sum_{k \in K} z_{ij}^s \right) \leq W_i \left( \frac{U_j}{U_j} \right) \left( \frac{S_j}{S_j} \right) \left( \frac{I_f}{I_f} \right) \quad \forall i, j
\]  
(8)

Constraint (9) shows that the maximum incoming flow to hub \( K \) equals \( \Gamma_k \).

\[
\sum_{i \in I} \sum_{j \in J} \sum_{s,t}^2 x_{ij}^s + y_{ij}^s + z_{ij}^s \leq \Gamma_k h_k \quad \forall k \in K
\]  
(9)

Constraint (10) shows that the minimum flow passing for using spoke \( k \) as a hub is equal to \( E_k \).

\[
E_k - \sum_{i \in I} \sum_{j \in J} \sum_{s,t}^2 (x_{ij}^s + y_{ij}^s + z_{ij}^s) \leq M (1 - h_k) \quad \forall k \in K \]  
(10)

Constraint (11) guarantees that for each origin \( i \) and destination \( j \), only variables \( x_{ij} \) related to airports \( k \) in set \( K \) can be non-zero, and the remaining variables are equal to zero.

\[
x_{ij}^s = 0 \quad \forall i \in I, j \in J; s = 1,2; k \in K - K
\]  
(11)

Constraint (12) guarantees that for each origin \( i \) and destination \( j \), only variables \( y_{ij} \) related to airports \( k \) in set \( K \) can be non-zero, and the remaining variables are equal to zero.

\[
y_{ij}^s = 0 \quad \forall i \in I, j \in J; s = 1,2; k \in K - K
\]  
(12)

Constraint (13) guarantees that for each origin \( i \) and destination \( j \), only variables \( z_{ij} \) related to airports \( k \) in set \( K \) can be non-zero, and the remaining variables are equal to zero.

\[
z_{ij}^s = 0 \quad \forall i \in I, j \in J; s = 1,2; k \in K - K
\]  
(13)

Constraints (14) and (15) show the domain of variables.

\[
h_k \in \{0,1\} \quad \forall k \in K
\]  
(14)

\[
x_{ij}^1, x_{ij}^2, y_{ij}^1, y_{ij}^2, z_{ij}^1, z_{ij}^2 \in I^+ \cup \{0\} \quad \forall i \in I, j \in J, k \in K
\]  
(15)

3.3 Determining the importance of travel time, travel cost and service quality of hubs

\( f_c, f_t \) and \( f_q \) represent the degree of importance of the parameters of travel time, travel cost, and service quality of hubs for travelers. To determine them, we use data gathered by questionnaires from air travelers with different income levels, ages, and travel purposes. Also, we introduce the interpolation function of importance for time and cost in air travels by using a multivariate Lagrange interpolation method. Lagrange interpolation function with \( m \) variables and degree \( n \) is defined as equation (16) ([31]).

\[
f (X_1, X_2, \ldots, X_m) = \sum_{e=1}^{m} \alpha_e X^{e_i}
\]  
(16)

Where \( e_i = (e_{i1}, \ldots, e_{in}) \) and \( \sum_{i=1}^{m} e_{i} = 1 \).

To uniquely determine \( f \), we require \( \rho = \binom{n+m}{n} \) independent points in equation (17).

\[
(x_{ij}, \ldots, x_{n,i}, f_{j}) \in R^{m+1}, 1 \leq i \leq \rho, f_{j} = f (x_{1,j}, \ldots, x_{n,j})
\]  
(17)
By calculating the degree of importance of travel time, travel cost, and service quality of airport and by taking quality, safety, and travel time delay of follower and leader airlines into account, parameters $p_1, p_2, q_1, q_2$ and $r_1, r_2$ are determined as equations (18).

$$
p_1 = f_c \left( \frac{U_f}{U_i} \right) \left( \frac{S_f}{S_i} \right) \left( \frac{t_f}{t_i} \right)
$$

$$
p_2 = (f_c + f_q) \left( \frac{U_f}{U_i} \right) \left( \frac{S_f}{S_i} \right) \left( \frac{t_f}{t_i} \right)
$$

$$
q_1 = f_i \left( \frac{U_f}{U_i} \right) \left( \frac{S_f}{S_i} \right) \left( \frac{t_f}{t_i} \right)
$$

$$
q_2 = (f_i + f_q) \left( \frac{U_f}{U_i} \right) \left( \frac{S_f}{S_i} \right) \left( \frac{t_f}{t_i} \right)
$$

$$
r_1 = (f_c + f_i) \left( \frac{U_f}{U_i} \right) \left( \frac{S_f}{S_i} \right) \left( \frac{t_f}{t_i} \right)
$$

$$
r_2 = (f_c + f_i + f_q) \left( \frac{U_f}{U_i} \right) \left( \frac{S_f}{S_i} \right) \left( \frac{t_f}{t_i} \right)
$$

4. Robust optimization

In this section, we present the robust counterpart of the model proposed in section 3 by employing Bertsimas and Sim's approach [11]. Given the impact of seasonal changes (such as travel in high season) and weather conditions on the number of travelers, we consider $w_{ij}$ as uncertain. The values of the uncertain parameter $w_{ij}$ select based on a symmetric distribution in the interval $[w_{ij} - w_{ij}, w_{ij} + w_{ij}]$ with an average of $w_{ij}$.

The notations used in the proposed robust model are presented in Table 2.

{Please insert table 2 about here.}

4.1 Robust model formulation

Given the fact that uncertain parameters $w_{ij}$ are included in constraints (2) to (8), we employ Bertsimas and Sim’s approach [11] and by defining variable $H$, we show the abovementioned constraints in the form of constraints (19) to (26):

$$
\sum_{k \in N_{j}^{1}} x^{1}_{kj} - w_{ij} \left( f_c \left( \frac{U_f}{U_i} \right) \left( \frac{S_f}{S_i} \right) \left( \frac{t_f}{t_i} \right) \right) H \leq 0 \quad \forall i \in I, j \in J
$$

$$
\sum_{k \in N_{j}^{2}} x^{2}_{kj} - w_{ij} \left( f_c + f_q \left( \frac{U_f}{U_i} \right) \left( \frac{S_f}{S_i} \right) \left( \frac{t_f}{t_i} \right) \right) H \leq 0 \quad \forall i \in I, j \in J
$$

$$
\sum_{k \in M_{j}^{1}} y^{1}_{kj} - w_{ij} \left( f_i \left( \frac{U_f}{U_i} \right) \left( \frac{S_f}{S_i} \right) \left( \frac{t_f}{t_i} \right) \right) H \leq 0 \quad \forall i \in I, j \in J
$$

$$
\sum_{k \in M_{j}^{2}} y^{2}_{kj} - w_{ij} \left( f_i + f_q \left( \frac{U_f}{U_i} \right) \left( \frac{S_f}{S_i} \right) \left( \frac{t_f}{t_i} \right) \right) H \leq 0 \quad \forall i \in I, j \in J
$$

$$
\sum_{k \in P_{j}^{1}} z^{1}_{kj} - w_{ij} \left( f_c + f_i \left( \frac{U_f}{U_i} \right) \left( \frac{S_f}{S_i} \right) \left( \frac{t_f}{t_i} \right) \right) H \leq 0 \quad \forall i \in I, j \in J
$$

$$
\sum_{k \in P_{j}^{2}} z^{2}_{kj} - w_{ij} \left( f_c + f_i + f_q \left( \frac{U_f}{U_i} \right) \left( \frac{S_f}{S_i} \right) \left( \frac{t_f}{t_i} \right) \right) H \leq 0 \quad \forall i \in I, j \in J
$$
Given the fact that constraints (19) to (26) are defined for each \( i \) and each \( j \), constraints itself subjects to maximum \( i \times j \) constraints. Since each of those constraints have only one \( w_{ij} \), we got \( \Gamma_{ij} \subseteq \{0,1\} \). Based on research by Bertsimas and Sim \([11]\), constraint (19) is turned into constraint (27).

**Theorem 1.** For providing \( H^* \), function (28)

\[
\beta_1(H^*,\Gamma_{ij}) = \max_{\forall (i,j) \in F^1} \left\{ \Gamma_{ij} \left[ w_{ij} \left( f_x \left( U_{ij} \right) \left( S_{ij} \right) \left( t_{ij} \right) \right) H^* + \left( \Gamma_{ij} - \Gamma_{ij} \right) \left[ \Gamma_{ij} \left( f_x \left( U_{ij} \right) \left( S_{ij} \right) \left( t_{ij} \right) \right) H^* \right] \right] \right\}
\]

is equal to the linear programming problem below.

**Proof.** We prove the theorem in two cases:

**Case 1:** \( \Gamma_{ij}^1 \) is an integer.

So \( \Gamma_{ij} \in \{0,1\} \), then \( \Gamma_{ij}^1 = 1 \). In this case, \( \beta_1(H^*,\Gamma_{ij}^1) \) is equal to:

\[
\beta_1(H^*,\Gamma_{ij}^1) = \max_{\forall (i,j) \in F^1} \left\{ \Gamma_{ij} \left[ w_{ij} \left( f_x \left( U_{ij} \right) \left( S_{ij} \right) \left( t_{ij} \right) \right) H^* \right] \right\} = \Gamma_{ij} \left[ w_{ij} \left( f_x \left( U_{ij} \right) \left( S_{ij} \right) \left( t_{ij} \right) \right) H^* \right]
\]

Because the optimization problem is a maximization one, then, we consider \( \Gamma_{ij} = 1 \). Therefore, the problem can become feasible and the optimal value of the objective function is equal to:

\[
w_{ij} \left( f_x \left( U_{ij} \right) \left( S_{ij} \right) \left( t_{ij} \right) \right) H^*
\]

Hence, in this case, equation (29) is correct.

**Case 2:** \( \Gamma_{ij}^1 \) is non-integer.
So $\Gamma_{ij}^1 \in [0,1]$, then $\left| \Gamma_{ij}^1 \right| = 0$. In this case, $\beta_i(H^*, \Gamma_{ij}^1)$ is equal to:

$$\beta_i(H^*, \Gamma_{ij}^1) = \max_{\Gamma_{ij}^1 \in [0,1]} \left\{ (\Gamma_{ij}^1 - 0)\hat{w}_{ij} (f_x, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H^* \right\} = \Gamma_{ij}^1 \hat{w}_{ij} (f_x, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H^*$$

(32)

Because the optimization problem is a maximization one, then we consider $z_{ij} = \Gamma_{ij}^1$. Therefore, the problem can become feasible and the optimal value of objective function is equal to:

$$\Gamma_{ij}^1 \hat{w}_{ij} (f_x, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H^*$$

(33)

Hence, in this case, equation (29) is correct.

By using theorem 1 and taking advantage of Strong duality theorem, it can be shown that the robust counterpart of robust model presented in section 3 is as following:

$$(1, 10-15)$$

$$\sum_{k \in N_{ij}}^0 x_{ij}^1 W_{ij} (f_x, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H + z_{ij}^1 \Gamma_{ij}^1 + \sum_{f \in F^1} P_{ij}^1 \leq 0 \quad \forall i \in I, j \in J$$

$$\sum_{k \in N_{ij}}^0 x_{ij}^2 W_{ij} (f_x, f_q, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H + z_{ij}^2 \Gamma_{ij}^2 + \sum_{f \in F^2} P_{ij}^2 \leq 0 \quad \forall i \in I, j \in J$$

$$\sum_{k \in N_{ij}}^0 y_{ij}^1 W_{ij} (f_x, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H + z_{ij}^3 \Gamma_{ij}^3 + \sum_{f \in F^3} P_{ij}^3 \leq 0 \quad \forall i \in I, j \in J$$

$$\sum_{k \in N_{ij}}^0 y_{ij}^2 W_{ij} (f_x, f_q, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H + z_{ij}^4 \Gamma_{ij}^4 + \sum_{f \in F^4} P_{ij}^4 \leq 0 \quad \forall i \in I, j \in J$$

$$\sum_{k \in N_{ij}}^0 z_{ij}^1 W_{ij} (f_x, f_q, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H + z_{ij}^5 \Gamma_{ij}^5 + \sum_{f \in F^5} P_{ij}^5 \leq 0 \quad \forall i \in I, j \in J$$

$$\sum_{k \in N_{ij}}^0 z_{ij}^2 W_{ij} (f_x, f_q, f_q, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H + z_{ij}^6 \Gamma_{ij}^6 + \sum_{f \in F^6} P_{ij}^6 \leq 0 \quad \forall i \in I, j \in J$$

$$\sum_{s=1}^2 \left( \sum_{k \in N_{ij}}^0 x_{ij}^s + \sum_{k \in N_{ij}}^0 y_{ij}^s + \sum_{k \in N_{ij}}^0 z_{ij}^s \right) - W_{ij} (\frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H + z_{ij}^7 \Gamma_{ij}^7 + \sum_{f \in F^7} P_{ij}^7 \leq 0 \quad \forall i, j$$

$$(34)$$

$$\hat{W}_{ij} (f_x, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H \leq z_{ij}^1 + P_{ij}^1 \quad \forall i \in I, j \in J, f \in F^1$$

$$\hat{W}_{ij} (f_x, f_q, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H \leq z_{ij}^2 + P_{ij}^2 \quad \forall i \in I, j \in J, f \in F^2$$

$$\hat{W}_{ij} (f_x, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H \leq z_{ij}^3 + P_{ij}^3 \quad \forall i \in I, j \in J, f \in F^3$$

$$\hat{W}_{ij} (f_x, f_q, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H \leq z_{ij}^4 + P_{ij}^4 \quad \forall i \in I, j \in J, f \in F^4$$

$$\hat{W}_{ij} (f_x, f_q, f_q, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H \leq z_{ij}^5 + P_{ij}^5 \quad \forall i \in I, j \in J, f \in F^5$$

$$\hat{W}_{ij} (f_x, f_q, f_q, f_q, \frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H \leq z_{ij}^6 + P_{ij}^6 \quad \forall i \in I, j \in J, f \in F^6$$

$$\hat{W}_{ij} (\frac{U_f}{U_l}, \frac{S_f}{S_l}, \frac{t}{t_f})H \leq z_{ij}^7 + P_{ij}^7 \quad \forall i \in I, j \in J, f \in F^7$$

$$1 \leq H \leq 1$$
\[
\begin{align*}
\text{const}_{ij}, P_{ij}^{\text{const}} & \geq 0 \quad \forall i \in I, j \in J, f \in \mathcal{F}^{\text{const}}, \text{const} \in \{1, 2, \ldots, 7\}
\end{align*}
\]

5. Chance-constrained programming

In this section, taking into account the uncertain travel demand, we present the deterministic equivalent of the model presented in section 3 using chance-constraint programming approach. Suppose we have a probable scheduling problem as follows:

\[
\begin{align*}
\text{max} \quad & z = \sum_{i=1}^{n} c_i x_i \\
\text{s.t.} \quad & Ax \leq d \\
& P\left(\sum_{i=1}^{n} a_i x_i \leq \bar{b}\right) \geq 1 - \alpha \\
& x_i \geq 0 \quad i = 1, \ldots, n
\end{align*}
\]

Which \( \bar{b} \) is the independent random variable and has a normal distribution. It can be shown the probable constraint \( P\left(\sum_{i=1}^{n} a_i x_i \leq \bar{b}\right) \geq 1 - \alpha \) is equivalent to the constraint (36) ([32]).

\[
\sum_{i=1}^{n} a_i x_i \leq E(\bar{b}) - Z_{1-\alpha} \sqrt{\text{Var}(\bar{b})} \tag{36}
\]

Equation (36), \( Z_{1-\alpha} \) represents the point of the standard normal distribution, so that: \( P(Z > Z_{1-\alpha}) = \alpha \).

Assuming that the parameter \( W_{ij} \) of the random variable has the normal distribution, it can be deduced from the constraint (36) that the constraints (2) to (8) change in the constraints (37) through (43).

\[
\begin{align*}
\sum_{k \in \mathcal{K}} x_{ij}^{(k)} \leq E(W_{ij}) + p_{ij} - Z_{1-\alpha} p_{ij} \sqrt{\text{Var}(W_{ij})} & \quad \forall i \in I, j \in J \\
\sum_{k \in \mathcal{K}} y_{ij}^{(k)} \leq E(W_{ij}) + p_{ij} - Z_{1-\alpha} p_{ij} \sqrt{\text{Var}(W_{ij})} & \quad \forall i \in I, j \in J \\
\sum_{k \in \mathcal{K}} z_{ij}^{(k)} \leq E(W_{ij}) + q_{ij} - Z_{1-\alpha} q_{ij} \sqrt{\text{Var}(W_{ij})} & \quad \forall i \in I, j \in J \\
\sum_{k \in \mathcal{K}} \hat{z}_{ij}^{(k)} \leq E(W_{ij}) + r_{ij} - Z_{1-\alpha} r_{ij} \sqrt{\text{Var}(W_{ij})} & \quad \forall i \in I, j \in J \\
\sum_{k \in \mathcal{K}} \tilde{z}_{ij}^{(k)} \leq E(W_{ij}) + s_{ij} - Z_{1-\alpha} s_{ij} \sqrt{\text{Var}(W_{ij})} & \quad \forall i \in I, j \in J \\
\sum_{i=1}^{n} \left(\sum_{k \in \mathcal{K}} x_{ij}^{(k)} + y_{ij}^{(k)} + z_{ij}^{(k)} + \hat{z}_{ij}^{(k)} + \tilde{z}_{ij}^{(k)}\right) \leq E(W_{ij}) \frac{U_{ij}}{U_{ij} - S_{ij}} + s_{ij} - Z_{1-\alpha} s_{ij} \sqrt{\text{Var}(W_{ij})} & \quad \forall i, j
\end{align*}
\]

6. Case Study

In this paper, we study a case to examine the proposed model in designing an optimal hub-and-spoke network for a follower airline, in which Emirate Airline is the leader, and an alliance consisted of Mahan Airline, Iran Air, and Aseman Airline can play the role of follower airlines.
6.1 Determining the importance of travel time, travel cost and service quality of hubs

To determine \( f_c \), \( f_t \) and \( f_q \), we distribute 100 questionnaires to collect opinions of Imam Khomeini International Airport's travelers (with different income levels, age, and travel purposes) about the importance of travel cost and travel time.

Age and income levels in the developed questionnaire are indicated in Tables 3.

{Please insert table 3 about here.}

The average of opinions among travelers with the same income, age, and travel purpose about the degree of importance of travel time and cost by types of travel (business, educational, recreational) are summarized in Tables 4, 5, and 6 respectively.

{Please insert tables 4 to 6 about here.}

Using values in Tables 4 to 6 as well as multivariate Lagrange interpolation method, quadratic interpolation functions for travel time and cost in air travels are presented as equations (44) ([31]).

\[
\begin{align*}
\hat{f}_{tw}(x,y) &= -0.016x^2 + 0.0004y^2 + 0.014xy + 1.49x - 0.17y + 20.87 \\
\hat{f}_{cw}(x,y) &= 0.039x^2 + 0.013y^2 - 0.0341xy - 2.57x - 0.051y + 102.34 \\
\hat{f}_{te}(x,y) &= -0.0157x^2 - 0.087y^2 + 0.0042xy + 1.187x + 3.23y - 15.25 \\
\hat{f}_{ce}(x,y) &= -0.0007x^2 - 0.164y^2 - 0.036xy + 0.97x + 2.73y + 53.07 \\
\hat{f}_{th}(x,y) &= -0.18x^2 - 0.148y^2 + 0.456xy + 7.67x - 12.19y + 8.58 \\
\hat{f}_{ch}(x,y) &= 0.11x^2 + 0.064y^2 - 0.327xy - 4.24x + 12.5y + 27.67
\end{align*}
\] (44)

Where \( \hat{f}_{tw} \), \( \hat{f}_{te} \), and \( \hat{f}_{th} \) represent the importance of time in business, educational and recreational travels, respectively and \( \hat{f}_{cw} \), \( \hat{f}_{ce} \), \( \hat{f}_{ch} \) denoting the importance of cost in business, educational and recreational travels respectively.

We use the coefficient of determination \((R^2)\) which is presented as equation (45), not only to demonstrate the accuracy of quadratic Lagrange the interpolation functions in equations (44) but also to show that how much of opinions about the degree of importance of travel time and cost are covered by those functions ([33]).

\[
R^2 = 1 - \frac{\sum_{i=1}^{N}(y_i - \bar{y})^2}{\sum_{i=1}^{N}(y_i - \bar{y})^2}
\] (45)

Where \( N \) is the number of total observations, \( y \) is dependent variable, \( \bar{y} \) is the average values of \( y \), and \( \bar{y} \) is values predicted by interpolation functions. Using equation (45), it can be seen that the more value of \( R^2 \), the higher the accuracy of the interpolation model will be.

Coefficients of determination for functions (44) are derived by substituting values into equation (46). These include:

\[
R^2_{\hat{f}_{tw}} = 0.9994, R^2_{\hat{f}_{te}} = 0.995, R^2_{\hat{f}_{th}} = 0.9996, R^2_{\hat{f}_{cw}} = 0.9999, R^2_{\hat{f}_{ce}} = 0.995, R^2_{\hat{f}_{ch}} = 0.9994.
\] (46)

Given the values for coefficients of determination for functions (44), it can be said that these functions interpolate the points in Tables 4, 5, and 6 very accurately. Additionally, one may consider them as a basis to
calculate the degree of importance of time and cost for travelers with different ages, income levels, and purposes.

Equation (47) calculates the average value of functions (45) in their domain of definition.

\[
\frac{\int_0^{10000} \int_{-13}^{13} f(x, y) \, dx \, dy + \int_{0}^{10000} \int_{-13}^{13} f(x, y) \, dx \, dy}{2}
\]  

(47)

In equation (47), it is assumed that negative values are substituted by zero values. The second term represents this.

Using equation (47), the average values of functions (37) are presented as equations (48) and (49).

\[
\begin{align*}
M_{f_{\alpha}} &= 67.44, M_{f_{\beta}} = 16.01, M_{f_{\gamma}} = 56.91 \\
M_{f_{\alpha}} &= 39.31, M_{f_{\beta}} = 13.36, M_{f_{\gamma}} = 37.89
\end{align*}
\]  

(48)  

(49)

The average values in equations (48) and (49) are equal to 47% and 30%, respectively; therefore, it can be concluded that travel time has a high priority for nearly 47% of travelers. Even so, only 30% of travelers consider travel cost as an important factor. Without loss of generality, one may assume that the remaining 23% is related to factors, including quality of airport, airport services and other factors (\(f_q\) other than time and cost); Therefore, we can conclude that the importance of time (\(f_t\)), cost (\(f_c\)) and service quality (\(f_q\)) for travelers is equal to equation (50).

\[
f_t = 47\% \quad , \quad f_c = 30\% \quad , \quad f_q = 23\%
\]  

(50)

According to equation (51) and assuming that \(\frac{U_L}{U_I} = 0.4\), \(\frac{S_L}{S_I} = 0.2\) and \(\frac{t_L}{t_f} = 0.25\), values of parameters \(p_1, p_2, q_1, q_2, r_1\) and \(r_2\) are presented as equations (43) by substituting the abovementioned values into equations (19):

\[
\begin{align*}
p_1 &= f_c \times \left( \frac{U_L}{U_I} \right) \times \left( \frac{S_L}{S_I} \right) \times \left( \frac{t_L}{t_f} \right) = 0.3 \times (0.4) \times (0.2) \times (0.25) = 0.006 \\
p_2 &= (f_c + f_q) \times \left( \frac{U_L}{U_I} \right) \times \left( \frac{S_L}{S_I} \right) \times \left( \frac{t_L}{t_f} \right) = (0.3 + 0.23) \times (0.4) \times (0.2) \times (0.25) = 0.011 \\
q_1 &= f_t \times \left( \frac{U_L}{U_I} \right) \times \left( \frac{S_L}{S_I} \right) \times \left( \frac{t_L}{t_f} \right) = 0.47 \times (0.4) \times (0.2) \times (0.25) = 0.009 \\
q_2 &= (f_t + f_q) \times \left( \frac{U_L}{U_I} \right) \times \left( \frac{S_L}{S_I} \right) \times \left( \frac{t_L}{t_f} \right) = (0.47 + 0.23) \times (0.4) \times (0.2) \times (0.25) = 0.014 \\
r_1 &= (f_t + f_c) \times \left( \frac{U_L}{U_I} \right) \times \left( \frac{S_L}{S_I} \right) \times \left( \frac{t_L}{t_f} \right) = (0.3 + 0.47) \times (0.4) \times (0.2) \times (0.25) = 0.015 \\
r_2 &= (f_t + f_c + f_q) \times \left( \frac{U_L}{U_I} \right) \times \left( \frac{S_L}{S_I} \right) \times \left( \frac{t_L}{t_f} \right) = (0.3 + 0.47 + 0.23) \times (0.4) \times (0.2) \times (0.25) = 0.02
\end{align*}
\]  

(51)

6.2 Data needed for implementation of the model

Origins, destinations, and hub candidates of follower airlines are presented in Tables 7 and 8.

{Please insert tables 7 and 8 about here.}
The service quality of airports and the capacity of hub candidates as well as the minimum number of passing travelers for being them as hubs are presented in Table 9.

Based on the first row of Table 9, it is obvious that, $V_k < V$ ( $\forall k$ ).

Information summarized in Table 9 is adjusted for a particular period (October 19-30, 2017).

The weekly average number of travelers transferred by Emirate airport in the first six months of the year 2017 is presented in Table 10.

The other necessary input data for the model include:

- The time and cost of passing traveler between origin and destination through Dubai international airport by Emirate Airline.
- The cost of passing one traveler between origin and hub candidate and between hub candidate and destination by follower airline.
- The time of passing traveler between origin and hub candidate and between hub candidate and destination by follower airline.

Also, according to the information provided by Iran Airports & Air Navigation Company, it is assumed that the average travel time delay for transfer traveler in the airport $k$ of follower airline is 60 minutes.

6.3 Implementation of the uncertainty model

Based on the abovementioned information, we implement a robust model and chance-constraint programming as follows.

6.3.1 Implementation of Robust model

We implement a robust model in a situation where all parameters $w_{ij}$ and $\Gamma_k$ are uncertain. By uncertainty of all parameters $w_{ij}$, we mean that:

$$\Gamma_{ij}^{const} = 1, \quad \forall i \in I, j \in J, const \in \{1, 2, ..., 7\}$$ (52)

Also, we suppose that:

$$\hat{w}_{ij} = 0.1 \times w_{ij}, \quad \forall i \in I, j \in J$$ (53)

By using the GAMS software and Baron Solving method, it can be shown that the optimal value of the objective function for $\beta_1 = 0.8$ and $\gamma_1 = 0.9$ is $Z^* = 702,464$$. Whereas the optimal value of Marianov et al. [7] model is $Z^* = 670,263$$. Therefore, it can be concluded that the location of hubs in this paper leads to a significant increase in the incomes of the follower airline.

Table 11 shows the share of each hub of follower airline from leader airline travelers.

According to Table 11, Imam Khomeini and Mashhad airports with 499 and 258 travelers have the largest international traffics among Iranian airports, respectively.
Given the result, it is clear that Iranian airlines are accounted for 2% of travelers of Emirate Airline (i.e. 1510 travelers).

Figures 1(a) and 1(b) show the path of passing traveler through hubs of the follower.

6.3.2 Implementation of chance-constraint programming

First, we show that the random variable parameter $W_{ij}$ has a normal distribution, so that the values of the "P-value" and "AD" have calculated types of distribution functions using the mini-tab software, and we show in Table 12.

Given "AD" and "P-value" values in Table 12, it is concluded that the best distribution for displaying the parameter is the normal distribution. Therefore, taking into account the matrices of $E(W_{ij})$ and $\sqrt{Var(W_{ij})}$ through tables 13 and 14 and placing them in the constraints (38) to (44), the optimal value of the objective function in $Z_{0.95} = 1.645$ is equal to $Z^* = 695,082$.

Therefore, it can be concluded that if random constraints are possible with a probability of 95%, the value of the target function will be $695,082$.

Table 15 shows the share of each hub of follower airline from leader airline travelers.

Since it is not possible to determine with certainty which chance constraint programming and robust optimization methods provide an accurate approximation of the optimal problem [34], therefore, considering that the value of the objective function in the robust optimization method is greater than the random constraint programming, in the following, answer to the robust optimization method is considered the basis of the decision and analysis of the results.

6.3.3 Compare cost and travel time

Based on information about Emirate Airline, it can be said that the cost of passing existing travelers is $1,252,088. Therefore, we conclude that the alliance of Iranian airlines and using their competitive edge not only lead to an increase in their market share by $702,464 but also decrease the ticket costs for travelers by $549,624 dollars. The value of $549,624 is the costs of Emirate Airline ($1,252,088) minus costs of alliance among Iranian airlines ($702,464) for passing 1,510 travelers. If $\beta_1 = 0.8$ and $\gamma_1 = 0.9$, on average, there will be about $364 reduction in ticket price per traveler. Furthermore, it can be said that the time need to move all existing travelers (in Table 11) by Emirate Airline is about 21,000 hours, while this time for the alliance of Iranian airlines is about 19,516 hours. When we use alliance of Iranian airlines and their under-controlled hubs, travel time per traveler is decreased about 59 minutes on average. The value of 59 minutes is the time of Emirate Airline (21,000 h) minus time of alliance among Iranian airlines (19,516 h) for passing 1,510 travelers.
6.3.4 Sensitivity Analysis

To assess the impact of quality and safety of follower airline and the number of their under-controlled hubs, and examine changes in discount factor \((\beta_1, \gamma_1)\) in value of \(z\), We implement model with different values for \(\beta_1, \gamma_1, U_f, U_l, S_f, S_l\) and \(|K|\). The results are presented in Tables 12 to 14.

Table 16 shows the impact of discount factors \(\beta_1, \gamma_1\) of follower airline on the value of the objective function.

Based on the last column of Table 16, it can be said that a 0.1% increase in the discount factor \(\beta_1\), led to an increase in the value of \(z\) by $37,933. However, based on the second column of Table 16, a 0.1% increase in the discount factor \(\gamma_1\), led to an increase in the value of \(z\) by $43,552. Therefore, we conclude that the discount factor \(\gamma_1\) has a higher impact on profitability than the discount factor \(\beta_1\). Also, based on the first column of Table 16 and the cost of passing travelers by Emirate Airline ($1,252,088), it can be concluded that there be a decrease in the ticket price for travelers (429,503 dollars) when follower airlines do not use discount factor in passing travelers.

Table 17 indicates the impact of the number of airports controlled by follower airlines on the value of the objective function.

As shown in Table 17, an increase in the number of hub airports of followers airline leads to an increase in the value of the objective function. By simple calculations, it can be shown that adding one airport to airports controlled by follower airlines lead to a 1% increase in revenue of follower airline.

Table 18 shows the impact of the quality and safety of follower airlines on the value of the objective function.

The first column of Table 18 shows that a 10% increase in the quality of follower airline leads to a 25% increase in the value of the objective function. While a 10% increase in safety of follower airline (second column of Table 18) leads to a 50% increase in revenue of follower airline. Therefore, it can be concluded that the safety of airlines is more effective in traveler attraction and revenue increase. The third column of Table 18 shows that a 10% increase in each of the parameters of quality and safety of follower airline leads a nearly 88% increase in the value of the objective function.

To examine the impact of the number of uncertain parameters, we consider Table 19.

According to Table 19, the more uncertain parameters in the model, the worst value of the objective function. For example, when all 33 parameters \(w_{ij}\) are uncertain, with the following deviation value:

\[
\tilde{w}_{ij} = 0.3 \cdot w_{ij}
\] (54)

Then, the value of the objective function is equal to $485050, which is $217414 lower than the optimal value of a certain model ($702464).
Also, it is clear that as the number of uncertain parameters increase, the value of the objective function decrease accordingly and become more realistic. Table 20 shows the change percentage in value of the objective function of a robust model in comparison with a certain model in various modes (the number of travelers has a 10% deviation from real value).

{Please insert table 20 about here.}

7. Conclusion

In this paper, to organize the travelers and determine the exact location of hubs we develop the model of Marianov et al. [7] by defining a new type of capture sets under uncertainty in which follower airlines attract a percentage of leader airline's travelers. We divide the nominated airports for follower airline's hub into six independent categories by comparing parameters of travel time, travel cost, and service quality of airports of follower airline and leader airline. By employing multivariate Lagrange interpolation function, we determine the degree of importance of time and cost of travel from the traveler's perspective. By using the degree of importance of time and cost of travel, as well as comparing the parameters of quality, safety, and delays between leader airline and follower airline, we allocate travelers to those six-fold sets. Based on the seasonal demand of travelers used by local travelers, we consider the travel demand of hubs as uncertain parameters. To determine the deterministic equivalent forms of this category of hub location models, robust optimization method and chance-constrained programming model are used. The results show that the follower airline's income has a considerable growth compared to the Marianov et al. [7] model and can absorb nearly 2% of travelers of the leader airline due to lower travel costs and travel time compared to that of leader airline. By examining the change in model's parameters, we find that improvement of quality and safety of constituent airlines in follower airlines and an increase in the number of their under-controlled hubs have a positive impact on the revenue of follower airline.

References


Farzin Nourzadeh obtained his B.Sc. degree in Pure Mathematics from Urmia University, Urmia, Iran in 2007 and M.Sc. degree in Applied Mathematics (Operation Research) from Kurdistan University, Sanandaj, Iran in 2009. He is obtained his Ph.D. degree in Industrial Engineering from Islamic Azad University, South - Tehran Branch, Tehran, Iran in 2019. His research interests include Hub location, optimization model, DEA, game theory, mathematical modelling and applied operation research.

Sadoullah Ebrahimnejad is an Associate Professor of Industrial Engineering at Karaj Islamic Azad University (KIAU). He holds a BS in Industrial Engineering from Iran University of Science and Technology, and MS from Amirkabir University of Technology and PhD from Science and Research Branch of the Islamic Azad University (Tehran SRBIAU). His research interests are Fuzzy MADM/MODM, SCM networks, Operation management, Optimization models, Risk management, Multi-criteria network optimization. He published more than 80 papers in reputable academic journals and conferences.

Kaveh Khalili-Damghani is currently an Associate Professor of Industrial Engineering at Islamic Azad University, South - Tehran Branch, Tehran, Iran. He obtained his first Ph.D. degree in Industrial Engineering-Minor in Operation Research from Islamic Azad University, Science and Research Branch, Tehran, Iran in 2008 and second Ph.D. degree in Industrial Management-Minor in Operations and Production Management from Allameh Tabataba'i University, Tehran, Iran in 2012. His M.Sc. in Industrial Engineering –Minor Operations Research from Islamic Azad University, South - Tehran Branch, Tehran, Iran in 2005. His B.Sc. in Industrial Engineering-Minor Industrial Production from Islamic Azad University, South - Tehran Branch, Tehran, Iran in 2003. His current research interests are Mathematical Modelling and Applied Operations Research, Combinatorial Optimization, Soft Computing, Operations and Production Management in Supply Chains, Data Envelopment Analysis for Performance Assessment.

Ashkan Hafezalkotob is currently an Associate Professor of Industrial Engineering at Islamic Azad University, South - Tehran Branch, Tehran, Iran. He obtained his Ph.D. degree in Industrial
Engineering-Social and Economical Systems Engineering from Iran University of Science and Technology, Tehran, Iran in 2012. His M.Sc. in Industrial Engineering-Social and Economical Systems Engineering from Iran University of Science and Technology, Tehran, Iran in 2007. His B.Sc. in Industrial Engineering, Production Engineering from Islamic Azad University, South - Tehran Branch, Tehran, Iran in 2004. His current research interests are Applied Game Theory, Multi Criteria Decision Making, Competition and Cooperation, Logistics, Energy Economics, Operation Research, Meta-Heuristic Methods.

**Figure captions**

Fig 1(a). Hub-and-spoke network of follower airline (uncertainty mode)

Fig 1(b). Hub-and-spoke network of follower airline (uncertainty mode)

**Table captions**

Table 1. The notations used in the proposed deterministic model

Table 2. The notations used in the proposed robust model

Table 3. Age and income levels

Table 4. Degree of the importance of travel time and cost for business travel

Table 5. Degree of the importance of travel time and cost for educational travel

Table 6. Degree of the importance of travel time and cost for recreational travel

Table 7. Origins and destinations

Table 8. Hub candidates of follower airline

Table 9. Service quality, Capacity and Minimum number of passing travelers for being airport as a hub

Table 10. Number of travelers

Table 11. Number of travelers passing from follower's airports

Table 12. Distribution functions

Table 13. Average travel demand

Table 14. Travel standard deviation

Table 15. Number of travelers passing from follower's airports

Table 16. Sensitivity analysis of discount factors

Table 17. Sensitivity analysis of number of airport

Table 18. Sensitivity analysis of quality and safety

Table 19. The impact of uncertain parameters of travel demand

Table 20. The impact of the number of uncertain parameters (travel demand)
Figures

Fig 1. Hub-and-spoke network of follower airline (uncertainty mode)

Tables

Table 1

Sets

\[ N_{ij}^1 = \{ k \in K | c_{ik} + c_{kj} < C_{ij} \cdot t_{ik} + t_k + t_{kj} \geq T_{ij} \cdot V_k < V \} \]
\[ N_{ij}^2 = \{ k \in K | c_{ik} + c_{kj} < C_{ij} \cdot t_{ik} + t_k + t_{kj} \geq T_{ij} \cdot V_k \geq V \} \]
\[ M_{ij}^1 = \{ k \in K | c_{ik} + c_{kj} \geq C_{ij} \cdot t_{ik} + t_k + t_{kj} < T_{ij} \cdot V_k < V \} \]
\[ M_{ij}^2 = \{ k \in K | c_{ik} + c_{kj} \geq C_{ij} \cdot t_{ik} + t_k + t_{kj} < T_{ij} \cdot V_k \geq V \} \]
\[ P_{ij}^1 = \{ k \in K | c_{ik} + c_{kj} < C_{ij} \cdot t_{ik} + t_k + t_{kj} < T_{ij} \cdot V_k < V \} \]
\[ P_{ij}^2 = \{ k \in K | c_{ik} + c_{kj} < C_{ij} \cdot t_{ik} + t_k + t_{kj} < T_{ij} \cdot V_k \geq V \} \]
\[ K^+ = N_{ij}^1 \cup N_{ij}^2 \]
\[ K^- = M_{ij}^1 \cup M_{ij}^2 \]
\[ K^- = P_{ij}^1 \cup P_{ij}^2 \]
\[ K = K^+ \cup K^- \cup K^- \]

Indices

\[ I, i \]
Set and counter of origin points

\[ I, i \]
Set and counter of origin points
\( J, j \) \quad \text{Set and counter of destination points}

\( K, k \) \quad \text{Set and counter of all hubs of follower airline}

**Parameters**

\( c_{ik} \) \quad \text{Ticket price for passing a traveler from origin } i \text{ to hub } k \text{ by follower airline}

\( c_{kj} \) \quad \text{Ticket price for passing a traveler from hub } k \text{ to destination } j \text{ by follower airline}

\( C_{ij} \) \quad \text{Total airfare of passing a traveler from origin } i \text{ to destination } j \text{ through leader hub(s)}

\( t_k \) \quad \text{Average travel time delay for transfer traveler in the airport } k

\( t_{ik} \) \quad \text{Time needed to pass travelers from origin } i \text{ to hub } k \text{ by follower airline}

\( t_{kj} \) \quad \text{Time needed to pass travelers from hub } k \text{ to destination } j \text{ by follower airline}

\( T_{ij} \) \quad \text{Total time needed to pass from origin } i \text{ to destination } j \text{ by leader hub(s)}

\( W_{ij} \) \quad \text{The total flow passing from origin } i \text{ to destination } j \text{, which has already moved by leader}

\( \Gamma_k \) \quad \text{Capacity of hub } k

\( E_k \) \quad \text{Minimum passing flow to select spoke } k \text{ as a hub}

\( V_k \) \quad \text{Service quality of airport } k \text{ controlled by follower airline}

\( V \) \quad \text{Average service quality of airports controlled by leader airline}

\( U_f \) \quad \text{Quality of follower airline}

\( U_l \) \quad \text{Quality of leader airline}

\( S_f \) \quad \text{Safety of follower airline}

\( S_l \) \quad \text{Safety of leader airline}

\( t_f \) \quad \text{The average travel time delay to transfer traveler by follower airline}

\( t_l \) \quad \text{The average travel time delay to transfer traveler by leader airline}

\( f_c \) \quad \text{Importance of cost for traveler}

\( f_t \) \quad \text{Importance of time for traveler}

\( f_q \) \quad \text{Importance of service quality of airport for traveler}

\( M \) \quad \text{Large positive number}

\( 0 \leq p_1 < p_2 \leq 1 \) \quad \text{Reduction factor of } W_{ij} \text{ for sets } N_{ij}^1, N_{ij}^2

\( 0 \leq q_1 < q_2 \leq 1 \) \quad \text{Reduction factor of } W_{ij} \text{ for sets } M_{ij}^1, M_{ij}^2

\( 0 \leq r_1 < r_2 \leq 1 \) \quad \text{Reduction factor of } W_{ij} \text{ for sets } P_{ij}^1, P_{ij}^2

\( 0 < \beta_1, \beta_2, \gamma_1, \gamma_2 \leq 1 \) \quad \text{Discount factor for ticket price}

**Decision variables**

\( h_k \) \quad \text{Equals 1 if spoke } k \text{ is selected as a hub, otherwise, Equals zero}

\( x_{ij}^{1k} \) \quad \text{Amount of passing flow by follower from origin } i \text{ to destination } j \text{ through hub } k \in N_{ij}^1

\( x_{ij}^{2k} \) \quad \text{Amount of passing flow by follower from origin } i \text{ to destination } j \text{ through hub } k \in N_{ij}^2

\( y_{ij}^{1k} \) \quad \text{Amount of passing flow by follower from origin } i \text{ to destination } j \text{ through hub } k \in M_{ij}^1

\( y_{ij}^{2k} \) \quad \text{Amount of passing flow by follower from origin } i \text{ to destination } j \text{ through hub } k \in M_{ij}^2

\( z_{ij}^{1k} \) \quad \text{Amount of passing flow by follower from origin } i \text{ to destination } j \text{ through hub } k \in P_{ij}^1
\[ z_{ij} \] Amount of passing flow by follower from origin \( i \) to destination \( j \) through hub \( k \in P_{ij} \).

### Table 2

**Sets**

\[
F^{\text{const}} = \{ (i, j) | i \in I, j \in J, \mathcal{W}_{ij} \text{ in } \text{const has noise } (\mathcal{W}_{ij} > 0) \}, \text{ const } \in \{1, \ldots, 7\}
\]

\[
\bigcup_{\text{const }=1}^7 |F^{\text{const}}| \leq |I| \times |J|
\]

**Parameters**

\( \mathcal{W}_{ij} \) deviation from \( \mathcal{W}_{ij} \)

\( \Gamma_{ij}^{\text{const}} \) the number of uncertain parameters \( \mathcal{W}_{ij} \) about the constraint of \( \text{const } (\text{const } = 1, \ldots, 7) \)

**Decision variables**

\( P_{ij}^{\text{const}}, z_{ij}^{\text{const}} \) Dual auxiliary variables for constraint \( \text{const } (\text{const } = 1, \ldots, 7) \)

\( H \) the variable defined to include right-hand side vector \( b \) as a column of technical matrix \( A \)

### Table 3

<table>
<thead>
<tr>
<th>Age levels</th>
<th>[13, 20)</th>
<th>[20, 30)</th>
<th>[30, 50)</th>
<th>[50, 75)</th>
<th>More than 75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income levels ($)</td>
<td>Less than 500</td>
<td>[500, 1250]</td>
<td>(1250, 3750]</td>
<td>More than 3750</td>
<td>-</td>
</tr>
</tbody>
</table>

### Table 4

<table>
<thead>
<tr>
<th>Number of travelers</th>
<th>Income levels ($)</th>
<th>Age level</th>
<th>Degree of the importance of travel time (%)</th>
<th>Degree of the importance of travel cost (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>950</td>
<td>25</td>
<td>50</td>
<td>55</td>
</tr>
<tr>
<td>8</td>
<td>6000</td>
<td>40</td>
<td>80</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>1000</td>
<td>43</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>3000</td>
<td>45</td>
<td>70</td>
<td>30</td>
</tr>
<tr>
<td>8</td>
<td>7000</td>
<td>45</td>
<td>90</td>
<td>20</td>
</tr>
<tr>
<td>9</td>
<td>3200</td>
<td>60</td>
<td>75</td>
<td>35</td>
</tr>
</tbody>
</table>

### Table 5

<table>
<thead>
<tr>
<th>Number of travelers</th>
<th>Income levels ($)</th>
<th>Age level</th>
<th>Degree of the importance of travel time (%)</th>
<th>Degree of the importance of travel cost (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>980</td>
<td>18</td>
<td>25</td>
<td>75</td>
</tr>
<tr>
<td>2</td>
<td>480</td>
<td>29</td>
<td>20</td>
<td>85</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>27</td>
<td>30</td>
<td>80</td>
</tr>
<tr>
<td>1</td>
<td>1700</td>
<td>23</td>
<td>35</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>900</td>
<td>33</td>
<td>30</td>
<td>85</td>
</tr>
<tr>
<td>3</td>
<td>2020</td>
<td>40</td>
<td>40</td>
<td>50</td>
</tr>
</tbody>
</table>

### Table 6

<table>
<thead>
<tr>
<th>Number of travelers</th>
<th>Income levels ($)</th>
<th>Age level</th>
<th>Degree of the importance of travel time (%)</th>
<th>Degree of the importance of travel cost (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>900</td>
<td>17</td>
<td>35</td>
<td>55</td>
</tr>
<tr>
<td>9</td>
<td>1000</td>
<td>25</td>
<td>65</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>1100</td>
<td>48</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>450</td>
<td>35</td>
<td>70</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>8000</td>
<td>60</td>
<td>90</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>3000</td>
<td>77</td>
<td>85</td>
<td>30</td>
</tr>
</tbody>
</table>
Currently, Dubai international airport is the only hub of Emirates Airline.

### Table 7

<table>
<thead>
<tr>
<th>Origin Airport</th>
<th>Label</th>
<th>Hamburg</th>
<th>London</th>
<th>Stockholm</th>
<th>Paris</th>
<th>Rome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Destination Airport</td>
<td>Label</td>
<td>Beijing</td>
<td>Bangkok</td>
<td>Kuala Lumpur</td>
<td>Delhi</td>
<td>Karachi</td>
</tr>
</tbody>
</table>

### Table 8

<table>
<thead>
<tr>
<th>Label</th>
<th>Imam Khomeini</th>
<th>Mashhad</th>
<th>Tabriz</th>
<th>Isfahan</th>
<th>Shiraz</th>
<th>Bandar Abbas</th>
<th>Yazd</th>
<th>Zahedan</th>
</tr>
</thead>
</table>

### Table 9

<table>
<thead>
<tr>
<th>Airport</th>
<th>service quality(*)</th>
<th>Capacity</th>
<th>Minimum number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imam Khomeini</td>
<td>3</td>
<td>125,136</td>
<td>2,700</td>
</tr>
<tr>
<td>Mashhad</td>
<td>3</td>
<td>110,469</td>
<td>2,400</td>
</tr>
<tr>
<td>Tabriz</td>
<td>3</td>
<td>33,721</td>
<td>700</td>
</tr>
<tr>
<td>Isfahan</td>
<td>1</td>
<td>57,813</td>
<td>1,300</td>
</tr>
<tr>
<td>Shiraz</td>
<td>1</td>
<td>61,018</td>
<td>1,400</td>
</tr>
<tr>
<td>Bandar Abbas</td>
<td>1</td>
<td>34,009</td>
<td>700</td>
</tr>
<tr>
<td>Yazd</td>
<td>1</td>
<td>21,111</td>
<td>500</td>
</tr>
<tr>
<td>Zahedan</td>
<td>4</td>
<td>18,989</td>
<td>400</td>
</tr>
</tbody>
</table>

* Currently, Dubai international airport is the only hub of Emirates Airline.

### Table 10

<table>
<thead>
<tr>
<th>Number of travelers</th>
<th>$j_1$</th>
<th>$j_2$</th>
<th>$j_3$</th>
<th>$j_4$</th>
<th>$j_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_1$</td>
<td>4020</td>
<td>3350</td>
<td>1340</td>
<td>4020</td>
<td>4020</td>
</tr>
<tr>
<td>$i_2$</td>
<td>4020</td>
<td>2680</td>
<td>1340</td>
<td>4020</td>
<td>3350</td>
</tr>
<tr>
<td>$i_3$</td>
<td>4020</td>
<td>4020</td>
<td>2010</td>
<td>4020</td>
<td>3350</td>
</tr>
<tr>
<td>$i_4$</td>
<td>2010</td>
<td>2010</td>
<td>670</td>
<td>2010</td>
<td>2010</td>
</tr>
<tr>
<td>$i_5$</td>
<td>4020</td>
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<td>1340</td>
<td>4020</td>
<td>4020</td>
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</tbody>
</table>

### Table 11

<table>
<thead>
<tr>
<th>Number of travelers (airport)</th>
<th>$j_1$</th>
<th>$j_2$</th>
<th>$j_3$</th>
<th>$j_4$</th>
<th>$j_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_1$</td>
<td>38($k_2$)</td>
<td>32($k_1$)</td>
<td>34($k_8$)</td>
<td>38($k_1$)</td>
<td>81($k_1$)</td>
</tr>
<tr>
<td></td>
<td>43($k_7$)</td>
<td>36($k_5$)</td>
<td>43($k_7$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i_2$</td>
<td>24($k_2$)</td>
<td>25($k_1$)</td>
<td>13($k_7$)</td>
<td>38($k_4$)</td>
<td>36($k_1$)</td>
</tr>
<tr>
<td></td>
<td>57($k_3$)</td>
<td>29($k_2$)</td>
<td>38($k_4$)</td>
<td>43($k_8$)</td>
<td>32($k_2$)</td>
</tr>
<tr>
<td>$i_3$</td>
<td>43($k_3$)</td>
<td>38($k_2$)</td>
<td>19($k_2$)</td>
<td>38($k_1$)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>38($k_4$)</td>
<td>43($k_6$)</td>
<td>21($k_6$)</td>
<td>43($k_7$)</td>
<td></td>
</tr>
<tr>
<td>$i_4$</td>
<td>28($k_3$)</td>
<td>19($k_1$)</td>
<td>7($k_6$)</td>
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<td></td>
<td>21($k_6$)</td>
<td>21($k_7$)</td>
<td>19($k_2$)</td>
<td></td>
</tr>
<tr>
<td>$i_5$</td>
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<td>43($k_1$)</td>
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<tr>
<td></td>
<td>27($k_3$)</td>
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<td></td>
<td>43($k_1$)</td>
<td></td>
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<td>14($k_6$)</td>
<td>38($k_8$)</td>
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<td></td>
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</table>

### Table 12

<table>
<thead>
<tr>
<th>row</th>
<th>Distribution</th>
<th>AD</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Normal</td>
<td>1.765</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>2</td>
<td>Logistic</td>
<td>1.653</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>3</td>
<td>Box- cox transformation</td>
<td>2.619</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>4</td>
<td>Gamma</td>
<td>4.566</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>5</td>
<td>Logistic</td>
<td>4.69</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>6</td>
<td>Log normal</td>
<td>4.877</td>
<td>&lt; 0.005</td>
</tr>
<tr>
<td>7</td>
<td>Exponential</td>
<td>9.495</td>
<td>&lt; 0.003</td>
</tr>
<tr>
<td>Number of travelers</td>
<td>$j_1$</td>
<td>$j_2$</td>
<td>$j_3$</td>
</tr>
<tr>
<td>---------------------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>$i_1$</td>
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<td>3015</td>
<td>1206</td>
</tr>
<tr>
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<td>3618</td>
<td>2412</td>
<td>1206</td>
</tr>
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<td>3618</td>
<td>3618</td>
<td>1809</td>
</tr>
<tr>
<td>$i_4$</td>
<td>1809</td>
<td>1809</td>
<td>1206</td>
</tr>
<tr>
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<td>3618</td>
<td>1206</td>
</tr>
</tbody>
</table>

<table>
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<tr>
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<th>$j_1$</th>
<th>$j_2$</th>
<th>$j_3$</th>
<th>$j_4$</th>
<th>$j_5$</th>
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</thead>
<tbody>
<tr>
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<td>19</td>
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<td>20</td>
<td>20</td>
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<tr>
<td>$i_2$</td>
<td>20</td>
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<td>14</td>
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<td>13</td>
<td>20</td>
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</table>

<table>
<thead>
<tr>
<th>Number of travelers (airport)</th>
<th>$j_1$</th>
<th>$j_2$</th>
<th>$j_3$</th>
<th>$j_4$</th>
<th>$j_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_1$</td>
<td>38($k_2$)</td>
<td>32($k_1$)</td>
<td>14($k_8$)</td>
<td>38($k_1$)</td>
<td>80($k_1$)</td>
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<tr>
<td>43($k_3$)</td>
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<td>43($k_7$)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$i_2$</td>
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<td>25($k_1$)</td>
<td>12($k_7$)</td>
<td>38($k_4$)</td>
<td>35($k_1$)</td>
</tr>
<tr>
<td>57($k_5$)</td>
<td>29($k_2$)</td>
<td>14($k_4$)</td>
<td>43($k_8$)</td>
<td>31($k_2$)</td>
<td></td>
</tr>
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<td>43($k_7$)</td>
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<td>7($k_6$)</td>
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<tr>
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<td>13($k_7$)</td>
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<td>43($k_1$)</td>
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<tr>
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<td>14($k_6$)</td>
<td>38($k_2$)</td>
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<table>
<thead>
<tr>
<th>Discount factors</th>
<th>$\beta_1 = 1, \gamma_1 = 1$</th>
<th>$\beta_1 = 0, \gamma_1 = 1$</th>
<th>$\beta_1 = 0.9, \gamma_1 = 0.9$</th>
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<tbody>
<tr>
<td>$z$</td>
<td>822585</td>
<td>746016</td>
<td>740397</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$\mathbf{K}$</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tbody>
<tr>
<td>$z$</td>
<td>665843</td>
<td>688690</td>
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<td>702464</td>
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<table>
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<tr>
<th>$U_+, S_+$</th>
<th>$U_-, S_-$</th>
<th>$\mathbf{U}, \mathbf{S}$</th>
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<tbody>
<tr>
<td>0.5, 0.2</td>
<td>0.4, 0.3</td>
<td>0.5, 0.3</td>
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<tr>
<td>$z$</td>
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<td>1053696</td>
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<tr>
<td>$\Gamma_{ij}^{const}$</td>
<td>$w_{ij}$</td>
<td>$z^*$</td>
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<tr>
<td>----------------------</td>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>0.1($w_{ij}$)</td>
<td>658474</td>
<td></td>
</tr>
<tr>
<td>0.2($w_{ij}$)</td>
<td>632802</td>
<td></td>
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<tr>
<td>0.3($w_{ij}$)</td>
<td>602738</td>
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<tr>
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Table 20

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<th>The number of uncertain parameters</th>
<th>Percentage of change in the objective function</th>
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